Distributed Fusion Filtering for Nonlinear Time-Varying Systems Over Amplify-and-Forward Relay Networks: An H_∞ Quantized Framework

Xueyang Meng, Zidong Wang, Fan Wang, and Yun Chen

***Abstract*—This paper is concerned with the distributed fusion filtering problem for a class of nonlinear time-varying systems subject to quantization effects within a finite-horizon H_∞ framework. To improve the communication quality, the amplify-and-forward (AaF) relay mechanism, which accounts for phenomenon of missing measurements, is utilized to schedule the data transmissions from the sensors to the remote filters. The dynamic quantization, as a result of the inherent limit of network bandwidth, is further considered in the communication process from the filters to the fusion center. The main objective of this paper is to propose a distributed fusion scheme that ensures both local and fusion H_∞ performance indices over a finite horizon. A sufficient condition is first established for guaranteeing a prescribed performance constraint on the local filtering error dynamics, and then the corresponding filter gains are calculated by solving a set of recursive matrix inequalities. Subsequently, with the help of the acquired local state estimates, the desired parameters of the fusion filters are designed in terms of the solution to a convex optimization problem. Finally, the effectiveness of the obtained theoretical results is testified by a numerical example.**

***Index Terms*—Distributed fusion filtering, amplify-and-forward relay networks, H_∞ filtering, missing measurements, dynamic quantization, convex optimization.**

Abbreviations and Notations

NTVS	Nonlinear time-varying system
AaF	Amplify-and-forward
DaF	Decode-and-forward
FaF	Filtering-and-forward
AaFRN	Amplify-and-forward relay network
FC	Fusion center
AFES	Augmented filtering error system
STR	Sensor-to-relay
RTF	Relay-to-filter
STF	Sensor-to-filter
R^s	The s-dimensional Euclidean space
$\|y\|$	The Euclidean norm of a vector y
$E\{z\}$	The mathematical expectation of a random variable z
$\mathrm{Prob}\{a\}$	The occurrence probability of the event "a"
A^{-1}	The inverse of A
B^T	The transpose of B
$\mathrm{col}_N\{y_i\}$	The column vector $\begin{bmatrix} y_1^T & y_2^T & \cdots & y_N^T \end{bmatrix}^T$
$\mathrm{diag}\{\cdots\}$	The block-diagonal matrix
$\mathrm{diag}_N\{A_i\}$	The diagonal matrix $\mathrm{diag}\{A_1, A_2, \cdots, A_N\}$ with the components A_i $(i \in \{1, 2, \cdots, N\})$
I	The identity matrix with compatible dimensions
0	The zero matrix with compatible dimensions
$A > 0$	The matrix A is symmetric and positive definite
$*$	The ellipsis for symmetry-induced terms in symmetric block matrices

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I. Introduction

In accordance to the rapidly growing popularity of large-scale sensor networks, the information fusion techniques have been attracting a recurring research interest from both academy and industry with great application potentials in engineering practice such as process control, target tracking, air-traffic control, and environment monitoring [12], [20], [23], [48], [54]. A fundamental issue with information fusion is the multi-sensor fusion filtering that seeks to attain a reliable fusion estimate by integrating the local measurements collected from the sensors or the local state estimates from the individual filters. Till now, the fusion filtering issue has drawn considerable research attention and many efficient fusion schemes have been proposed in the literature, see e.g. [5], [8]–[10], [30], [45], [53].

The fusion filtering strategies can be generally categorized into two types, the centralized and the distributed ones [7], [11], [16], [27], [31], [49], where the former deals with all the sensor measurements directly transmitted to the fusion center (FC), thereby achieving the optimal estimation at the expense of energy consumption. For the latter, the local state estimate of each filter is first calculated and then transmitted to the FC for further processing, and therefore the distributed fusion strategies have distinctive advantages of mitigating the network burden and enhancing the communication reliability

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at the cost of sacrificing certain estimation accuracy. So far, much research effort has been devoted to the investigation on distributed fusion filtering problems especially for time-invariant systems [6], [28], [34], [36]. Note that almost all practical systems are literally time-varying for various reasons (e.g. magnetic field/temperature fluctuation and component aging) and the distributed fusion filtering algorithms are typically implemented on the *finite horizon* in a recursive way [44].

In view of the physical/resource constraints, missing measurements (also called packet losses or dropouts) are often encountered during the signal transmissions which, if not properly handled, would give rise to undesired performance degradation [22], [40]. As such, it is essential to take the influence of missing measurements into account when addressing the filtering/control problems for networked systems [13], [19], [33], [52]. On the other hand, signal quantization is likely to occur owing mainly to the limited network bandwidths, and the quantization-induced errors are known to have adverse impacts on the system performance. In the past few years, signal quantization has received an increasing research interest from both communities of signal processing and control system, and many elegant results have been reported in the literature on networked systems with quantization effects [38], [47].

Pertaining to signal quantization, there have been two well-known quantization mechanisms, namely, the static quantization [14], [43] and the dynamic quantization [4], [21], [25]. Different from its static counterpart with fixed quantization parameters, the dynamic quantization is dependent on adjustable quantizer parameters, and is therefore equipped with more flexibility and less conservatism. To date, a great deal of research attention has been drawn towards the analysis and synthesis problems for networked systems undergoing dynamic quantization, see e.g. [24], [39] and the references therein. For example, the distributed quantized state estimation problem has been studied in [39] for time-varying systems, where a zoom variable is introduced to dynamically adjust the quantization region and the quantization error. Nevertheless, the dynamic-quantization-based distributed fusion filtering problem has not been fully investigated yet due mainly to the mathematical difficulty of incorporating the fusion filtering scheme, and this motivates the current study.

In addition to missing measurements and signal quantization, the network communication quality, which is greatly affected by the transmission distance between the sensors and the receivers, is another vitally important issue for networked systems. Clearly, the transmission distance cannot be infinite because of the limited transmission capability of the sensors. To extend the propagation distance and enhance the communication quality, some efficient relay-based protocols have been put forward to facilitate the signal transmissions over long-distance communication channels, and some widely deployed protocols include the amplify-and-forward (AaF) protocol [17], [29], the decode-and-forward (DaF) protocol [18], [32], and the filtering-and-forward (FaF) protocol [2], [3].

It is worth mentioning that, compared with the DaF and FaF protocols, the AaF mechanism aims to receive and amplify the signal observed by the sensor and then forward the amplified signal to the remote filter, leading to distinctively easy-to-implement feature [1], [41], [50]. The filtering problems over AaF relay networks (AaFRNs) have recently begun to gain particular research attention, see e.g. [26], [35], [37] for some representative results. For instance, a recursive filtering algorithm has been proposed in [35] for a class of discrete systems with stochastic uncertainties over the AaF relay-based protocol. To date, there have been very few results concerning the distributed fusion filtering problems with missing measurements over AaFRNs, not to mention the consideration of dynamic quantization, and this comprises the main motivation of this study.

Concluding the literature review conducted so far, it is theoretically significant and practically important to investigate the distributed fusion filtering problem for nonlinear time-varying systems (NTVSs) subject to dynamic quantizations under the AaF relay-based mechanism catering for missing measurements. In doing so, the following emerging challenges are identified: 1) how to construct a suitable local filter for the underlying NTVS over AaFRNs with multiple missing measurements taken into account? 2) how to analyze the transient behavior of the resultant filtering error dynamics over a finite horizon so as to accommodate the time-varying nature of the overall system? and 3) how to design the local filter gains and further develop a distributed fusion scheme to achieve the expected filtering performances under the dynamic quantization effects?

To overcome challenges highlighted above, the principal contributions of this paper are outlined as follows: 1) the addressed distributed fusion filtering issue is, for the first time, investigated for a general class of NTVSs; 2) several network-traffic-related phenomena, which include the AaF relay communication, the missing measurement and dynamic quantization, are simultaneously considered in the filter design; 3) a sufficient condition is provided to guarantee the prescribed finite-horizon H_∞ performance for the augmented filtering error system (AFES); 4) appropriate gain parameters of the local filters are obtained by solving some recursive linear matrix inequalities; and 5) a novel distributed fusion filtering scheme is established under an H_∞ quantized framework by resorting to the solution to a convex optimization problem.

The rest of this paper is arranged as follows. In Section II, the distributed quantized fusion filtering problem is formulated for an array of NTVSs over AaFRNs. In Section III, suitable local filters are designed to guarantee the prescribed performance constraint of the AFES, and the optimal fusion parameters are calculated by solving a certain convex optimization problem. Section IV provides a numerical example to demonstrate the validity of the developed distributed fusion scheme. Finally, the conclusion is drawn in Section V.

II. Problem Formulation and Preliminaries

A. System model

Consider the following NTVS:

$$\begin{cases} x_{s+1} = f(x_s) + B_s v_s \\ y_{i,s} = h_i(x_s) + D_{i,s} v_s \\ z_s = M_s x_s \end{cases} \tag{1}$$

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where, for $s \in [0, T]$ and $i \in S_1 \triangleq \{1, 2, \cdots, N\}$ with T and N being known positive integers, $x_s \in \mathrm{R}^{n_x}$, $y_{i,s} \in \mathrm{R}^{n_y}$ and $z_s \in \mathrm{R}^{n_z}$ denote, respectively, the system state, the measurement output and the output signal to be estimated; $v_s \in l_2([0, T], \mathrm{R}^{n_v})$ is the noise disturbance; B_s, $D_{i,s}$ and M_s are known time-varying matrices of suitable dimensions.

The nonlinear functions $f(\cdot) : \mathrm{R}^{n_x} \mapsto \mathrm{R}^{n_x}$ and $h_i(\cdot) : \mathrm{R}^{n_x} \mapsto \mathrm{R}^{n_y}$ satisfy $f(0) = 0$, $h_i(0) = 0$ and

$$\begin{aligned} \|f(x_1) - f(x_2)\|^2 &\le a^2 \|x_1 - x_2\|^2 \\ \|h_i(x_1) - h_i(x_2)\|^2 &\le b_i^2 \|x_1 - x_2\|^2 \end{aligned} \tag{2}$$

for any vectors $x_1, x_2 \in \mathrm{R}^{n_x}$, where a and b_i are known positive scalars.

B. Amplify-and-forward relay network

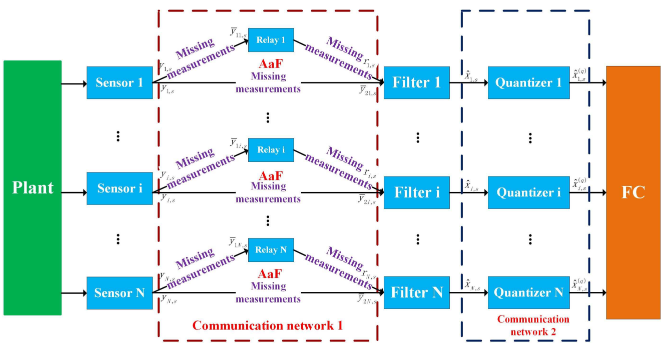

Fig. 1: Distributed quantized fusion filtering over the AaFRN.

In this paper, the AaF relay-based protocol is adopted to facilitate the data transmissions from the sensors to the remote filters. The structure diagram of the underlying system over the AaFRN is presented in Fig. 1. It is observed from Fig. 1 that, based on the deployed AaF relay node, there are three transmission channels between the sensor i and the filter i, namely, the sensor-to-relay (STR) channel, the relay-to-filter (RTF) channel, and the sensor-to-filter (STF) channel. In addition, the phenomenon of missing measurement is taken into account in all the three communication procedures due to probabilistic network congestions.

Let $\bar{y}_{1i,s}$ be the signal received by the relay i via the STR channel, $r_{i,s}$ and $\bar{y}_{2i,s}$ be the signals received by the filter i via the RTF channel and then the STF channel, respectively. The dynamics of $\bar{y}_{1i,s}$, $r_{i,s}$ and $\bar{y}_{2i,s}$ are given by

$$\begin{cases} \bar{y}_{1i,s} = \sqrt{E_{1i}}\beta_{1i,s} y_{i,s} + C_{1i,s}\varrho_{1i,s} \\ r_{i,s} = \sqrt{E_{2i}}\beta_{2i,s}\bar{y}_{1i,s} + C_{2i,s}\varrho_{2i,s} \\ \bar{y}_{2i,s} = \sqrt{E_{3i}}\beta_{3i,s} y_{i,s} + C_{3i,s}\varrho_{3i,s} \end{cases} \tag{3}$$

where E_{li} $(l \in \{1, 2, 3\},\ i \in S_1)$ is the known average signal energy, $\varrho_{li,s} \in l_2([0, T], \mathrm{R})$ is the channel noise, and $C_{li,s}$ is a known matrix with proper dimensions. For $l \in \{1, 2, 3\}$ and $i \in S_1$, the random variables $\beta_{li,s}$ (governing the missing measurement phenomena) are mutually independent with respect to l, i and s, and satisfy the following Bernoulli distributions:

$$\mathrm{Prob}\{\beta_{li,s} = 1\} = \bar{\beta}_{li}, \quad \mathrm{Prob}\{\beta_{li,s} = 0\} = 1 - \bar{\beta}_{li} \tag{4}$$

with $\bar{\beta}_{li}$ being known positive scalars.

Defining $\tilde{y}_{i,s} \triangleq \begin{bmatrix} r_{i,s}^T & \bar{y}_{2i,s}^T \end{bmatrix}^T$, we obtain from (3) that

$$\tilde{y}_{i,s} = \mathbb{E}_{1i}\delta_{1i,s} y_{i,s} + \mathbb{E}_{2i}\delta_{2i,s} C_{i,s}\bar{\varrho}_{i,s} \tag{5}$$

where

$$\mathbb{E}_{1i} \triangleq \mathrm{diag}\{\sqrt{E_{1i}E_{2i}}I, \sqrt{E_{3i}}I\}, \quad \delta_{1i,s} \triangleq [\theta_{i,s} I \quad \beta_{3i,s} I]^T,$$

$$\theta_{i,s} \triangleq \beta_{1i,s}\beta_{2i,s}, \quad \mathbb{E}_{2i} \triangleq \begin{bmatrix} \sqrt{E_{2i}}I & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

$$C_{i,s} \triangleq \mathrm{diag}\{C_{1i,s}, C_{2i,s}, C_{3i,s}\}, \quad \delta_{2i,s} \triangleq \mathrm{diag}\{\beta_{2i,s} I, I, I\},$$

$$\bar{\varrho}_{i,s} \triangleq \begin{bmatrix} \varrho_{1i,s}^T & \varrho_{2i,s}^T & \varrho_{3i,s}^T \end{bmatrix}^T.$$

Remark 1: In this study, both the randomly occurring missing measurements and the channel noises are introduced during the data transmissions over the STR channel, the RTF channel, and the STF channel. Particularly, three sequences of Bernoulli distributed variables with known statistic properties are adopted to characterize the phenomenon of missing measurements. It is obvious to see from (3) that the measurement signal $r_{i,s}$ ($\bar{y}_{2i,s}$, respectively) can be successfully received by the filter i only if $\beta_{1i,s} = \beta_{2i,s} = 1$ ($\beta_{3i,s} = 1$, respectively).

Remark 2: According to the AaF relay-based protocol under consideration, the measurements received at the filter side are collected from both the relay and the sensor through different communication channels, thereby complicating the measurement model. In particular, by augmenting $r_{i,s}$ and $\bar{y}_{2i,s}$, we obtain a new measurement $\tilde{y}_{i,s}$ as presented in (5), which will be used in the local filter i to estimate the system state.

C. Local filter

For notational simplicity, we denote

$$\begin{aligned} &\tilde{f}_{i,s} \triangleq f_s - \hat{f}_{i,s}, && f_s \triangleq f(x_s), \\ &\hat{f}_{i,s} \triangleq f(\hat{x}_{i,s}), && \tilde{h}_{i,s} \triangleq h_{i,s} - \hat{h}_{i,s}, \\ &h_{i,s} \triangleq h_i(x_s), && \hat{h}_{i,s} \triangleq h_i(\hat{x}_{i,s}), \\ &\tilde{\delta}_{1i,s} \triangleq \begin{bmatrix} \tilde{\theta}_{i,s} I & \tilde{\beta}_{3i,s} I \end{bmatrix}^T, && \tilde{\theta}_{i,s} \triangleq \theta_{i,s} - \bar{\theta}_i, \\ &\tilde{\beta}_{3i,s} \triangleq \beta_{3i,s} - \bar{\beta}_{3i}, && \bar{\theta}_i \triangleq \bar{\beta}_{1i}\bar{\beta}_{2i}, \\ &\tilde{\delta}_{2i,s} \triangleq \mathrm{diag}\{\tilde{\beta}_{2i,s} I, 0, 0\}, && \bar{\delta}_{1i} \triangleq \begin{bmatrix} \bar{\theta}_i I & \bar{\beta}_{3i} I \end{bmatrix}^T, \\ &\tilde{\beta}_{2i,s} \triangleq \beta_{2i,s} - \bar{\beta}_{2i}, && \bar{\delta}_{2i} \triangleq \mathrm{diag}\{\bar{\beta}_{2i} I, I, I\}. \end{aligned}$$

Based on the augmented measurement $\tilde{y}_{i,s}$ $(i \in S_1)$, the local filter i is constructed as follows:

$$\begin{cases} \hat{x}_{i,s+1} = \hat{f}_{i,s} + K_{i,s}(\tilde{y}_{i,s} - \mathbb{E}_{1i}\bar{\delta}_{1i}\hat{h}_{i,s}) \\ \hat{z}_{i,s} = M_s \hat{x}_{i,s} \end{cases} \tag{6}$$

where $\hat{x}_{i,s}$ is the estimate of x_s, $\hat{z}_{i,s}$ is the estimate of z_s, and $K_{i,s}$ is the gain matrix to be designed.

Set $e_{i,s} \triangleq x_s - \hat{x}_{i,s}$ and $\tilde{z}_{i,s} \triangleq z_s - \hat{z}_{i,s}$ as the local filtering error and the output signal error, respectively. It is derived from (1) and (6) that

$$\begin{cases} e_{i,s+1} = \tilde{f}_{i,s} + (B_s - K_{i,s}\mathbb{E}_{1i}\delta_{1i,s}D_{i,s})v_s \\ \qquad - K_{i,s}(\mathbb{E}_{1i}\tilde{\delta}_{1i,s}h_{i,s} + \mathbb{E}_{1i}\bar{\delta}_{1i}\tilde{h}_{i,s} \\ \qquad + \mathbb{E}_{2i}\delta_{2i,s}C_{i,s}\bar{\varrho}_{i,s}) \\ \tilde{z}_{i,s} = M_s e_{i,s}. \end{cases} \tag{7}$$

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Furthermore, defining $\xi_{i,s} \triangleq \begin{bmatrix} x_s^T & e_{i,s}^T \end{bmatrix}^T$, one has the following AFES:

$$\begin{cases} \xi_{i,s+1} = \tilde{\mathcal{F}}_{i,s} + \big(R_{1i,s} + R_{2i,s}\tilde{\Delta}_{i,s}\big)\tilde{\mathcal{H}}_{i,s} \\ \qquad\quad + \big(\bar{B}_{i,s} + R_{3i,s}\tilde{\delta}_{1i,s}D_{i,s}\big)v_s \\ \qquad\quad + \big(R_{4i,s} + R_{5i,s}\tilde{\delta}_{2i,s}\big)C_{i,s}\bar{\varrho}_{i,s} \\ \tilde{z}_{i,s} = \mathcal{M}_s\xi_{i,s} \end{cases} \tag{8}$$

where

$$\tilde{\mathcal{F}}_{i,s} \triangleq \begin{bmatrix} f_s^T & \tilde{f}_{i,s}^T \end{bmatrix}^T, \quad \tilde{\mathcal{H}}_{i,s} \triangleq \begin{bmatrix} h_{i,s}^T & \tilde{h}_{i,s}^T \end{bmatrix}^T,$$
$$\tilde{\Delta}_{i,s} \triangleq \text{diag}\{\tilde{\delta}_{1i,s}, 0\}, \quad R_{1i,s} \triangleq \text{diag}\{0, -K_{i,s}\mathbb{E}_{1i}\bar{\delta}_{1i}\},$$
$$R_{2i,s} \triangleq \begin{bmatrix} 0 & 0 \\ -K_{i,s}\mathbb{E}_{1i} & 0 \end{bmatrix}, \quad R_{3i,s} \triangleq \begin{bmatrix} 0 \\ -K_{i,s}\mathbb{E}_{1i} \end{bmatrix},$$
$$\mathcal{M}_s \triangleq [0 \quad M_s], \quad \bar{B}_{i,s} \triangleq \begin{bmatrix} B_s \\ B_s - K_{i,s}\mathbb{E}_{1i}\bar{\delta}_{1i}D_{i,s} \end{bmatrix},$$
$$R_{4i,s} \triangleq \begin{bmatrix} 0 \\ -K_{i,s}\mathbb{E}_{2i}\bar{\delta}_{2i} \end{bmatrix}, \quad R_{5i,s} \triangleq \begin{bmatrix} 0 \\ -K_{i,s}\mathbb{E}_{2i} \end{bmatrix}.$$

We are now in the position to state the first objective of this paper as follows.

O1: For the target plant (1) with missing measurements over AaFRNs, we aim to design a local filter of form (6) such that the following H_∞ performance constraint

$$\mathcal{J}_{1i} = \sum_{s=0}^{T} E\big\{\|\tilde{z}_{i,s}\|^2 - \gamma_i^2\big(\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\big)\big\} - \gamma_i^2\xi_{i,0}^T\mathbf{W}_i\xi_{i,0} < 0 \tag{9}$$

is satisfied for the AFES (8) over the finite horizon $[0, T]$, where $\gamma_i > 0$ is a prescribed disturbance rejection level, $\mathbf{W}_i > 0$ is a known matrix, and $\xi_{i,0}$ $(i \in S_1)$ is a given initial value.

D. Distributed fusion filter

To improve the estimation accuracy of the local state estimate, we intend to adopt a distributed fusion scheme under which the local estimates $\hat{x}_{i,s}$ for all $i \in S_1$ are delivered to the FC in hope of acquiring a fusion estimate with desired estimation accuracy.

Note that, in many practical situations, the communication channels between the local filters and the FC are likely to be bandwidth-constrained. In this paper, the following quantization effect on $\hat{x}_{i,s}$ is considered:

$$\hat{x}_{i,s}^{(q)} \triangleq \text{col}_{n_x}\left\{u_{ij,s}q_{ij}\left(\frac{\hat{x}_{ij,s}}{u_{ij,s}}\right)\right\} \tag{10}$$

where $\hat{x}_{i,s}^{(q)}$ is the quantized signal of $\hat{x}_{i,s}$, $u_{ij,s} > 0$ $(j \in \{1, 2, \cdots, n_x\})$ is an adjustable parameter, $\hat{x}_{ij,s}$ is the j-th component of the state estimate $\hat{x}_{i,s}$, and $q_{ij}(\cdot)$ is a scalar quantization of the following uniform type:

$$q_{ij}(\hat{x}_{ij,s}) = \begin{cases} -d_{ij,s}, & \hat{x}_{ij,s} < -d_{ij,s} \\ d_{ij,s}, & \hat{x}_{ij,s} \geq d_{ij,s} \\ -d_{ij,s} + \dfrac{(2n-1)d_{ij,s}}{p}, & \underline{d}_{ij,s}^{(n)} \leq \hat{x}_{ij,s} < \bar{d}_{ij,s}^{(n)}. \end{cases} \tag{11}$$

Here, for $i \in S_1$ and $n \in \{1, 2, \cdots, p\}$,

$$\underline{d}_{ij,s}^{(n)} \triangleq -d_{ij,s} + \frac{2(n-1)d_{ij,s}}{p}$$

and

$$\bar{d}_{ij,s}^{(n)} \triangleq -d_{ij,s} + \frac{2nd_{ij,s}}{p}$$

where p is a known positive integer implying that the quantization interval $[-d_{ij,s}, d_{ij,s}]$ is uniformly divided into p segments, and $d_{ij,s} > 0$ is the time-varying amplitude of $q_{ij}(\cdot)$ described by

$$d_{ij,s+1} = \varsigma_{ij,s}d_{ij,s} + \hat{x}_{ij,s}^T\mathbf{Q}_{ij,s}\hat{x}_{ij,s} \tag{12}$$

with a known initial value $d_{ij,0}$, a scalar $\varsigma_{ij,s} \in (0, 1)$, and a matrix $\mathbf{Q}_{ij,s} > 0$.

By denoting

$$\mu_{ij,s} \triangleq u_{ij,s}q_{ij}\left(\frac{\hat{x}_{ij,s}}{u_{ij,s}}\right) - \hat{x}_{ij,s}$$

as the component quantization error and $\mu_{i,s} \triangleq \text{col}_{n_x}\{\mu_{ij,s}\}$, one derives that

$$|\mu_{ij,s}| \leq \frac{u_{ij,s}d_{ij,s}}{p} \tag{13}$$
$$\hat{x}_{i,s}^{(q)} = \hat{x}_{i,s} + \mu_{i,s}. \tag{14}$$

Remark 3: On account of the limited bandwidths, the dynamic quantizer (10)-(12) is applied to the communication channels between the individual filters and the FC. Particularly, the local state estimates are subjected to the dynamic quantization effects, where the amplitude $d_{ij,s}$ is iteratively updated by the difference equation (12), and then the quantized estimates are transmitted to the FC. Compared with the extensively utilized static quantization [14], [43], the dynamic quantization under consideration exhibits remarkable flexibility because of the dynamical evolution of $d_{ij,s}$.

Remark 4: It follows from (13) that the component quantization error $\mu_{ij,s}$ is seriously affected by the values of $u_{ij,s}$, $d_{ij,s}$ and p. Notice that the quantization error becomes smaller with a smaller value of $u_{ij,s}$, whereas such a value of $u_{ij,s}$ might result in the quantizer saturation, namely, the signal $\frac{\hat{x}_{ij,s}}{u_{ij,s}}$ to be quantized is out of the interval $[-d_{ij,s}, d_{ij,s}]$. Therefore, it is of importance to select an appropriate $u_{ij,s}$ at each time step with aim to achieve a trade-off between the quantization error and the quantizer saturation.

According to (14), the following distributed fusion filter is constructed:

$$\begin{cases} \bar{x}_s = \sum_{i=1}^{N} c_i\hat{x}_{i,s}^{(q)} \\ \hat{z}_s = M_s\bar{x}_s \end{cases} \tag{15}$$

where $\bar{x}_s$ and $\hat{z}_s$ are, respectively, the fusion estimates of x_s and z_s, and c_i ($\sum_{i=1}^{N} c_i = 1$ and $0 < c_i < 1$) are the fusion parameters to be designed.

Denoting $\bar{e}_s \triangleq x_s - \bar{x}_s$ and $\tilde{z}_s \triangleq z_s - \hat{z}_s$ as the fusion filtering error and the fusion output signal error, respectively,

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one has

$$\begin{cases} \bar{e}_s = \sum_{i=1}^{N} c_i(e_{i,s} - \mu_{i,s}) \\ \tilde{z}_s = \sum_{i=1}^{N} c_i(\tilde{z}_{i,s} - M_s\mu_{i,s}). \end{cases} \tag{16}$$

The second objective of this paper is presented as follows.

O2: By resorting to the designed local filter in **O1**, we shall determine the fusion parameters such that the fusion filtering error system (16) satisfies

$$\begin{aligned} \mathcal{J}_2 = &\sum_{s=0}^{T} E\big\{\|\tilde{z}_s\|^2 - \tilde{\gamma}^2\big(\|v_s\|^2 + \|\bar{\varrho}_s\|^2\big) - 2N\bar{\Lambda}_s\big\} \\ &- \tilde{\gamma}^2\xi_0^T\bar{\mathbf{W}}\xi_0 < 0 \end{aligned} \tag{17}$$

with a positive scalar $\tilde{\gamma} > 0$, where

$$\bar{\varrho}_s \triangleq \text{col}_N\{\bar{\varrho}_{i,s}\}, \quad \xi_0 \triangleq \text{col}_N\{\xi_{i,0}\}, \quad \bar{\mathbf{W}} \triangleq \text{diag}_N\{\mathbf{W}_i\},$$
$$\bar{\Lambda}_s \triangleq \sum_{i=1}^{N} \bar{\Lambda}_{i,s}, \quad \bar{\Lambda}_{i,s} \triangleq \text{tr}\Big\{M_s^T M_s \sum_{j=1}^{n_x} \frac{u_{ij,s}^2 d_{ij,s}^2}{p^2}\Big\}.$$

Remark 5: In this paper, the distributed fusion filtering problem is considered for the NTVS (1), and we are interested in the transient behaviors of the filtering error dynamics over a prescribed time period. To this end, the local and fusion performance indices (9) and (17) are introduced to characterize the transient characteristics of error systems (8) and (16), respectively. Specifically, the index (9) is defined for each filter to assess the transient H_∞ performance of the local filtering error dynamics, and the constraint (17) is put forward to describe the robustness of the distributed fusion filter against the noise disturbances and quantization errors over the finite horizon $[0, T]$.

III. Main results

The distributed quantized H_∞ fusion filtering issue is tackled in this section. To be more specific, a sufficient condition is first established to guarantee that the finite-horizon H_∞ performance (9) is satisfied for the AFES (8), and then the proper local filter parameters are calculated by resorting to the solutions to a set of recursive matrix inequalities. After acquiring the local filters, the distributed fusion filter subject to dynamic quantization effect is designed by solving a certain convex optimization problem.

A. H_∞ performance analysis

The following theorem provides a sufficient condition for (8) to satisfy the H_∞ performance constraint (9).

Theorem 1: Consider the NTVS (1) with the local filter (6). Let the positive scalar γ_i, the positive definite matrix $\mathbf{W}_i$, and the filter gain $K_{i,s}$ $(i \in S_1,\ s \in [0, T])$ be given. The H_∞ performance constraint $\mathcal{J}_{1i} < 0$ is achieved if there exist a positive definite matrix $P_{i,s+1} = \text{diag}\{P_{1i,s+1}, P_{2i,s+1}\}$ and positive scalars $\{\lambda_{1i,s}, \lambda_{2i,s}\}$ satisfying the following inequalities

$$\Phi_{i,s} < 0 \tag{18}$$

$$P_{i,0} < \gamma_i^2\mathbf{W}_i \tag{19}$$

where

$$\Phi_{i,s} \triangleq \begin{bmatrix} \Phi_{i,s}^{(11)} & 0 & 0 & 0 & 0 \\ * & \Phi_{i,s}^{(22)} & \Phi_{i,s}^{(23)} & \Phi_{i,s}^{(24)} & \Phi_{i,s}^{(25)} \\ * & * & \Phi_{i,s}^{(33)} & \Phi_{i,s}^{(34)} & \Phi_{i,s}^{(35)} \\ * & * & * & \Phi_{i,s}^{(44)} & \Phi_{i,s}^{(45)} \\ * & * & * & * & \Phi_{i,s}^{(55)} \end{bmatrix},$$

$$\begin{aligned}
\Phi_{i,s}^{(11)} &\triangleq \lambda_{1i,s}a^2 I + \lambda_{2i,s}b_i^2 I + \mathcal{M}_s^T\mathcal{M}_s - P_{i,s}, \\
\Phi_{i,s}^{(22)} &\triangleq P_{i,s+1} - \lambda_{1i,s}I, \quad \Phi_{i,s}^{(23)} \triangleq P_{i,s+1}R_{1i,s}, \\
\Phi_{i,s}^{(24)} &\triangleq P_{i,s+1}\bar{B}_{i,s}, \quad \Phi_{i,s}^{(25)} \triangleq P_{i,s+1}R_{4i,s}C_{i,s}, \\
\Phi_{i,s}^{(33)} &\triangleq \mathcal{R}_{1i,s} - \lambda_{2i,s}I, \\
\Phi_{i,s}^{(34)} &\triangleq R_{1i,s}^T P_{i,s+1}\bar{B}_{i,s} + \theta_i^*\bar{\Delta}_{1i}^T R_{2i,s}^T P_{i,s+1}R_{3i,s}\mathbf{I}_1 D_{i,s} \\
&\quad + \beta_{3i}^*\bar{\Delta}_{2i}^T R_{2i,s}^T P_{i,s+1}R_{3i,s}\mathbf{I}_2 D_{i,s}, \\
\Phi_{i,s}^{(35)} &\triangleq \big(R_{1i,s}^T P_{i,s+1}R_{4i,s} + \bar{\Theta}_i\bar{\Delta}_{1i}^T R_{2i,s}^T P_{i,s+1}R_{5i,s}\mathbf{I}_3\big)C_{i,s}, \\
\Phi_{i,s}^{(44)} &\triangleq \mathcal{R}_{2i,s} - \gamma_i^2 I, \quad \Phi_{i,s}^{(55)} \triangleq \mathcal{R}_{3i,s} - \gamma_i^2 I, \\
\Phi_{i,s}^{(45)} &\triangleq \bar{B}_{i,s}^T P_{i,s+1}R_{4i,s}C_{i,s} \\
&\quad + \bar{\Theta}_i D_{i,s}^T\mathbf{I}_1^T R_{3i,s}^T P_{i,s+1}R_{5i,s}\mathbf{I}_3 C_{i,s}, \\
\mathcal{R}_{1i,s} &\triangleq R_{1i,s}^T P_{i,s+1}R_{1i,s} + \theta_i^*\bar{\Delta}_{1i}^T R_{2i,s}^T P_{i,s+1}R_{2i,s}\bar{\Delta}_{1i} \\
&\quad + \beta_{3i}^*\bar{\Delta}_{2i}^T R_{2i,s}^T P_{i,s+1}R_{2i,s}\bar{\Delta}_{2i}, \\
\mathcal{R}_{2i,s} &\triangleq \bar{B}_{i,s}^T P_{i,s+1}\bar{B}_{i,s} + D_{i,s}^T\big(\theta_i^*\mathbf{I}_1^T R_{3i,s}^T P_{i,s+1}R_{3i,s}\mathbf{I}_1 \\
&\quad + \beta_{3i}^*\mathbf{I}_2^T R_{3i,s}^T P_{i,s+1}R_{3i,s}\mathbf{I}_2\big)D_{i,s}, \\
\mathcal{R}_{3i,s} &\triangleq C_{i,s}^T R_{4i,s}^T P_{i,s+1}R_{4i,s}C_{i,s} \\
&\quad + \beta_{2i}^* C_{i,s}^T\mathbf{I}_3^T R_{5i,s}^T P_{i,s+1}R_{5i,s}\mathbf{I}_3 C_{i,s},
\end{aligned}$$

$$\bar{\Delta}_{1i} \triangleq \text{diag}\{\mathbf{I}_1, 0\}, \quad \bar{\Delta}_{2i} \triangleq \text{diag}\{\mathbf{I}_2, 0\}, \quad \mathbf{I}_1 \triangleq [I \quad 0]^T,$$
$$\mathbf{I}_2 \triangleq [0 \quad I]^T, \quad \mathbf{I}_3 \triangleq \text{diag}\{I, 0, 0\}, \quad \beta_{2i}^* \triangleq \bar{\beta}_{2i}(1 - \bar{\beta}_{2i}),$$
$$\beta_{3i}^* \triangleq \bar{\beta}_{3i}\big(1 - \bar{\beta}_{3i}\big), \quad \bar{\Theta}_i \triangleq \bar{\theta}_i\big(1 - \bar{\beta}_{2i}\big), \quad \theta_i^* \triangleq \bar{\theta}_i\big(1 - \bar{\theta}_i\big).$$

Proof: Denote $V_s \triangleq \xi_{i,s}^T P_{i,s}\xi_{i,s}$. Then, along the trajectory of (8), we have that

$$\begin{aligned}
&E\{\Delta V_s\} \\
&\triangleq E\{V_{s+1} - V_s\} \\
&= E\big\{\xi_{i,s+1}^T P_{i,s+1}\xi_{i,s+1} - \xi_{i,s}^T P_{i,s}\xi_{i,s}\big\} \\
&= E\big\{\big(\tilde{\mathcal{F}}_{i,s} + R_{1i,s}\tilde{\mathcal{H}}_{i,s} + \bar{B}_{i,s}v_s + R_{4i,s}C_{i,s}\bar{\varrho}_{i,s}\big)^T P_{i,s+1} \\
&\quad \times \big(\tilde{\mathcal{F}}_{i,s} + R_{1i,s}\tilde{\mathcal{H}}_{i,s} + \bar{B}_{i,s}v_s + R_{4i,s}C_{i,s}\bar{\varrho}_{i,s}\big) \\
&\quad + \big(R_{2i,s}\tilde{\Delta}_{i,s}\tilde{\mathcal{H}}_{i,s} + R_{3i,s}\tilde{\delta}_{1i,s}D_{i,s}v_s \\
&\quad + R_{5i,s}\tilde{\delta}_{2i,s}C_{i,s}\bar{\varrho}_{i,s}\big)^T P_{i,s+1}\big(R_{2i,s}\tilde{\Delta}_{i,s}\tilde{\mathcal{H}}_{i,s} \\
&\quad + R_{3i,s}\tilde{\delta}_{1i,s}D_{i,s}v_s + R_{5i,s}\tilde{\delta}_{2i,s}C_{i,s}\bar{\varrho}_{i,s}\big) - \xi_{i,s}^T P_{i,s}\xi_{i,s}\big\}.
\end{aligned} \tag{20}$$

According to the statistic properties of the variables $\beta_{li,s}$, it is easy to verify that

$$\begin{aligned}
&E\big\{\tilde{\mathcal{H}}_{i,s}^T\tilde{\Delta}_{i,s}^T R_{2i,s}^T P_{i,s+1}R_{2i,s}\tilde{\Delta}_{i,s}\tilde{\mathcal{H}}_{i,s}\big\} \\
&= E\big\{h_{i,s}^T\tilde{\delta}_{1i,s}^T\mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1}K_{i,s}\mathbb{E}_{1i}\tilde{\delta}_{1i,s}h_{i,s}\big\} \\
&= E\big\{\text{tr}\big\{\mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1}K_{i,s}\mathbb{E}_{1i}\tilde{\delta}_{1i,s}h_{i,s}h_{i,s}^T\tilde{\delta}_{1i,s}^T\big\}\big\} \\
&= E\big\{\theta_i^* h_{i,s}^T\mathbf{I}_1^T\mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1}K_{i,s}\mathbb{E}_{1i}\mathbf{I}_1 h_{i,s}
\end{aligned}$$

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$$\begin{aligned}&+\beta_{3i}^* h_{i,s}^T \mathbf{I}_2^T \mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1} K_{i,s} \mathbb{E}_{1i} \mathbf{I}_2 h_{i,s}\big\}\\ =& E\big\{\tilde{\mathcal{H}}_{i,s}^T\big(\theta_i^* \bar{\Delta}_{1i}^T R_{2i,s}^T P_{i,s+1} R_{2i,s}\bar{\Delta}_{1i}\\ &+\beta_{3i}^*\bar{\Delta}_{2i}^T R_{2i,s}^T P_{i,s+1} R_{2i,s}\bar{\Delta}_{2i}\big)\tilde{\mathcal{H}}_{i,s}\big\}. \end{aligned} \tag{21}$$

Similar to (21), we have

$$\begin{aligned}&E\big\{v_s^T D_{i,s}^T \tilde{\delta}_{1i,s}^T R_{3i,s}^T P_{i,s+1} R_{3i,s}\tilde{\delta}_{1i,s} D_{i,s} v_s\big\}\\ =& E\big\{v_s^T D_{i,s}^T \tilde{\delta}_{1i,s}^T \mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{1i}\tilde{\delta}_{1i,s} D_{i,s} v_s\big\}\\ =& E\big\{\mathrm{tr}\big\{\mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{1i}\tilde{\delta}_{1i,s} D_{i,s} v_s v_s^T D_{i,s}^T \tilde{\delta}_{1i,s}^T\big\}\big\}\\ =& E\big\{\theta_i^* v_s^T D_{i,s}^T \mathbf{I}_1^T \mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{1i}\mathbf{I}_1 D_{i,s} v_s\\ &+\beta_{3i}^* v_s^T D_{i,s}^T \mathbf{I}_2^T \mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{1i}\mathbf{I}_2 D_{i,s} v_s\big\}\\ =& E\big\{v_s^T D_{i,s}^T\big(\theta_i^*\mathbf{I}_1^T R_{3i,s}^T P_{i,s+1} R_{3i,s}\mathbf{I}_1\\ &+\beta_{3i}^*\mathbf{I}_2^T R_{3i,s}^T P_{i,s+1} R_{3i,s}\mathbf{I}_2\big) D_{i,s} v_s\big\}. \end{aligned} \tag{22}$$

Recalling (4) and the definition of $\tilde{\delta}_{2i,s}$, one has

$$\begin{aligned}&E\big\{\bar{\varrho}_{i,s}^T C_{i,s}^T \tilde{\delta}_{2i,s}^T R_{5i,s}^T P_{i,s+1} R_{5i,s}\tilde{\delta}_{2i,s} C_{i,s}\bar{\varrho}_{i,s}\big\}\\ =& E\big\{\bar{\varrho}_{i,s}^T C_{i,s}^T \tilde{\delta}_{2i,s}^T \mathbb{E}_{2i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{2i}\tilde{\delta}_{2i,s} C_{i,s}\bar{\varrho}_{i,s}\big\}\\ =& E\big\{\mathrm{tr}\big\{\mathbb{E}_{2i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{2i}\tilde{\delta}_{2i,s} C_{i,s}\bar{\varrho}_{i,s}\bar{\varrho}_{i,s}^T C_{i,s}^T\tilde{\delta}_{2i,s}^T\big\}\big\}\\ =& E\big\{\beta_{2i}^*\bar{\varrho}_{i,s}^T C_{i,s}^T \mathbf{I}_3^T \mathbb{E}_{2i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{2i}\mathbf{I}_3 C_{i,s}\bar{\varrho}_{i,s}\big\}\\ =& E\big\{\beta_{2i}^*\bar{\varrho}_{i,s}^T C_{i,s}^T \mathbf{I}_3^T R_{5i,s}^T P_{i,s+1} R_{5i,s}\mathbf{I}_3 C_{i,s}\bar{\varrho}_{i,s}\big\}. \end{aligned} \tag{23}$$

On the basis of the stochastic analysis technique, we obtain

$$\begin{aligned}&E\big\{\tilde{\mathcal{H}}_{i,s}^T\tilde{\Delta}_{i,s}^T R_{2i,s}^T P_{i,s+1} R_{3i,s}\tilde{\delta}_{1i,s} D_{i,s} v_s\big\}\\ =& E\big\{h_{i,s}^T\tilde{\delta}_{1i,s}^T\mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{1i}\tilde{\delta}_{1i,s} D_{i,s} v_s\big\}\\ =& E\big\{\tilde{\mathcal{H}}_{i,s}^T\big(\theta_i^*\bar{\Delta}_{1i}^T R_{2i,s}^T P_{i,s+1} R_{3i,s}\mathbf{I}_1\\ &+\beta_{3i}^*\bar{\Delta}_{2i}^T R_{2i,s}^T P_{i,s+1} R_{3i,s}\mathbf{I}_2\big) D_{i,s} v_s\big\}. \end{aligned} \tag{24}$$

In addition, it is readily derived that

$$\begin{aligned}&E\big\{\tilde{\mathcal{H}}_{i,s}^T\tilde{\Delta}_{i,s}^T R_{2i,s}^T P_{i,s+1} R_{5i,s}\tilde{\delta}_{2i,s} C_{i,s}\bar{\varrho}_{i,s}\big\}\\ =& E\big\{h_{i,s}^T\tilde{\delta}_{1i,s}^T\mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{2i}\tilde{\delta}_{2i,s} C_{i,s}\bar{\varrho}_{i,s}\big\}\\ =& E\big\{\bar{\Theta}_i\tilde{\mathcal{H}}_{i,s}^T\bar{\Delta}_{1i}^T R_{2i,s}^T P_{i,s+1} R_{5i,s}\mathbf{I}_3 C_{i,s}\bar{\varrho}_{i,s}\big\} \end{aligned} \tag{25}$$

and

$$\begin{aligned}&E\big\{v_s^T D_{i,s}^T\tilde{\delta}_{1i,s}^T R_{3i,s}^T P_{i,s+1} R_{5i,s}\tilde{\delta}_{2i,s} C_{i,s}\bar{\varrho}_{i,s}\big\}\\ =& E\big\{v_s^T D_{i,s}^T\tilde{\delta}_{1i,s}^T\mathbb{E}_{1i}^T K_{i,s}^T P_{2i,s+1} K_{i,s}\mathbb{E}_{2i}\tilde{\delta}_{2i,s} C_{i,s}\bar{\varrho}_{i,s}\big\}\\ =& E\big\{\bar{\Theta}_i v_s^T D_{i,s}^T\mathbf{I}_1^T R_{3i,s}^T P_{i,s+1} R_{5i,s}\mathbf{I}_3 C_{i,s}\bar{\varrho}_{i,s}\big\}. \end{aligned} \tag{26}$$

Substituting (21)-(26) into (20) results in

$$E\{\Delta V_s\} = E\big\{\eta_{i,s}^T\bar{\Phi}_{i,s}\eta_{i,s}\big\} \tag{27}$$

where

$$\eta_{i,s} \triangleq \begin{bmatrix}\xi_{i,s}^T & \tilde{\mathcal{F}}_{i,s}^T & \tilde{\mathcal{H}}_{i,s}^T & v_s^T & \bar{\varrho}_{i,s}^T\end{bmatrix}^T,$$

$$\bar{\Phi}_{i,s} \triangleq \begin{bmatrix} -P_{i,s} & 0 & 0 & 0 & 0\\ * & P_{i,s+1} & \Phi_{i,s}^{(23)} & \Phi_{i,s}^{(24)} & \Phi_{i,s}^{(25)}\\ * & * & \mathcal{R}_{1i,s} & \Phi_{i,s}^{(34)} & \Phi_{i,s}^{(35)}\\ * & * & * & \mathcal{R}_{2i,s} & \Phi_{i,s}^{(45)}\\ * & * & * & * & \mathcal{R}_{3i,s}\end{bmatrix}.$$

Based on the properties of nonlinear functions in (2), we arrive at

$$\begin{aligned}\tilde{\mathcal{F}}_{i,s}^T\tilde{\mathcal{F}}_{i,s} &= f_s^T f_s + \tilde{f}_{i,s}^T\tilde{f}_{i,s} \le a^2\xi_{i,s}^T\xi_{i,s},\\ \tilde{\mathcal{H}}_{i,s}^T\tilde{\mathcal{H}}_{i,s} &= h_{i,s}^T h_{i,s} + \tilde{h}_{i,s}^T\tilde{h}_{i,s} \le b_i^2\xi_{i,s}^T\xi_{i,s}.\end{aligned} \tag{28}$$

Combining (18), (27) and (28) yields

$$\begin{aligned}&E\{\Delta V_s\}\\ \le& E\big\{\eta_{i,s}^T\bar{\Phi}_{i,s}\eta_{i,s} - \lambda_{1i,s}\big(\tilde{\mathcal{F}}_{i,s}^T\tilde{\mathcal{F}}_{i,s} - a^2\xi_{i,s}^T\xi_{i,s}\big)\\ &-\lambda_{2i,s}\big(\tilde{\mathcal{H}}_{i,s}^T\tilde{\mathcal{H}}_{i,s} - b_i^2\xi_{i,s}^T\xi_{i,s}\big)\\ &+\big[\|\tilde{z}_{i,s}\|^2 - \gamma_i^2\big(\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\big)\big]\\ &-\big[\|\tilde{z}_{i,s}\|^2 - \gamma_i^2\big(\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\big)\big]\big\}\\ =& E\big\{\eta_{i,s}^T\Phi_{i,s}\eta_{i,s} - \big[\|\tilde{z}_{i,s}\|^2 - \gamma_i^2\big(\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\big)\big]\big\}\\ \le& -E\big\{\big[\|\tilde{z}_{i,s}\|^2 - \gamma_i^2\big(\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\big)\big]\big\}. \end{aligned} \tag{29}$$

Summing up both sides of (29) with respect to s from 0 to T, we have

$$\sum_{s=0}^{T} E\big\{\|\tilde{z}_{i,s}\|^2 - \gamma_i^2\big(\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\big)\big\} \le \xi_{i,0}^T P_{i,0}\xi_{i,0}. \tag{30}$$

Then, it is easily obtained from (19) and (30) that

$$\begin{aligned}&\sum_{s=0}^{T} E\big\{\|\tilde{z}_{i,s}\|^2\big\}\\ \le& \sum_{s=0}^{T}\gamma_i^2 E\big\{\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\big\} + \xi_{i,0}^T P_{i,0}\xi_{i,0}\\ <& \sum_{s=0}^{T}\gamma_i^2 E\big\{\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\big\} + \gamma_i^2\xi_{i,0}^T\mathbf{W}_i\xi_{i,0}. \end{aligned} \tag{31}$$

Hence, the H_∞ performance constraint $\mathcal{J}_{1i} < 0$ is verified, which ends the proof. ■

B. Local H_∞ filter design

In this subsection, we proceed to design the desired local filter for each node $i \in S_1$ to guarantee the H_∞ performance (9).

Theorem 2: Consider the NTVS (1) with the local filter (6). For the given scalar $\gamma_i > 0$ and the weighted matrix $\mathbf{W}_i > 0$ $(i \in S_1)$, the H_∞ performance constraint $\mathcal{J}_{1i} < 0$ is achieved if there exist the positive scalars $\{\lambda_{1i,s}, \lambda_{2i,s}\}$, the positive definite matrix $L_{i,s+1} = \mathrm{diag}\{L_{1i,s+1}, L_{2i,s+1}\}$ and the gain matrix $K_{i,s}$ $(s \in [0,T])$ satisfying (19) and the following inequality

$$\begin{bmatrix} \Phi_{i,s}^{(11)} & 0 & 0 & 0 & 0 & 0\\ * & \Upsilon_{i,s}^{(22)} & 0 & \Upsilon_{i,s}^{(24)} & \Upsilon_{i,s}^{(25)} & 0\\ * & * & \Upsilon_{i,s}^{(33)} & \Upsilon_{i,s}^{(34)} & \Upsilon_{i,s}^{(35)} & \Upsilon_{i,s}^{(36)}\\ * & * & * & \Upsilon_{i,s}^{(44)} & 0 & 0\\ * & * & * & * & \Upsilon_{i,s}^{(55)} & 0\\ * & * & * & * & * & \Upsilon_{i,s}^{(66)}\end{bmatrix} < 0 \tag{32}$$

where the matrices $\{P_{1i,s}, P_{2i,s}\}$ $(s \in [1, T+1])$ involved in $\Phi_{i,s}^{(11)}$ are determined by

$$P_{1i,s} = L_{1i,s}^{-1}, \quad P_{2i,s} = L_{2i,s}^{-1}$$

and

$$\Upsilon_{i,s}^{(22)} \triangleq \mathrm{diag}\{-\lambda_{1i,s}I, -\lambda_{2i,s}I\},$$

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$$\Upsilon_{i,s}^{(24)} \triangleq \begin{bmatrix} I & 0 \\ R_{1i,s}^T & \bar{\Delta}_{1i}^T R_{2i,s}^T \end{bmatrix},$$
$$\Upsilon_{i,s}^{(25)} \triangleq \begin{bmatrix} 0 & 0 \\ \bar{\Delta}_{1i}^T R_{2i,s}^T & \bar{\Delta}_{2i}^T R_{2i,s}^T \end{bmatrix},$$
$$\Upsilon_{i,s}^{(33)} \triangleq \text{diag}\{-\gamma_i^2 I, -\gamma_i^2 I\},$$
$$\Upsilon_{i,s}^{(34)} \triangleq \begin{bmatrix} \bar{B}_{i,s}^T & D_{i,s}^T \mathbf{I}_1^T R_{3i,s}^T \\ C_{i,s}^T R_{4i,s}^T & C_{i,s}^T \mathbf{I}_3^T R_{5i,s}^T \end{bmatrix},$$
$$\Upsilon_{i,s}^{(35)} \triangleq \begin{bmatrix} D_{i,s}^T \mathbf{I}_1^T R_{3i,s}^T & D_{i,s}^T \mathbf{I}_2^T R_{3i,s}^T \\ 0 & 0 \end{bmatrix},$$
$$\Upsilon_{i,s}^{(36)} \triangleq [0 \quad R_{5i,s}\mathbf{I}_3 C_{i,s}]^T,$$
$$\Upsilon_{i,s}^{(44)} \triangleq \text{diag}\{-L_{i,s+1}, -\bar{\Theta}_i^{-1} L_{i,s+1}\},$$
$$\Upsilon_{i,s}^{(55)} \triangleq \text{diag}\{-(\theta_i^* - \bar{\Theta}_i)^{-1} L_{i,s+1}, -(\beta_{3i}^*)^{-1} L_{i,s+1}\},$$
$$\Upsilon_{i,s}^{(66)} \triangleq -(\beta_{2i}^* - \bar{\Theta}_i)^{-1} L_{i,s+1}.$$

Proof: With the aid of $P_{1i,s}^{-1} = L_{1i,s}$, $P_{2i,s}^{-1} = L_{2i,s}$ and the Schur Complement Lemma, the inequality (32) ensures the validity of (18) in Theorem 1. Then, it follows immediately from Theorem 1 that the H_∞ performance index (9) is achieved, and the desired local filter gain $K_{i,s}$ can be calculated by solving the matrix inequality (32). The proof of this theorem is thus complete. ■

C. *Distributed H_∞ fusion filter design*

In this subsection, the distributed H_∞ fusion filter design issue is to be discussed in terms of the solvability of certain matrix inequalities. To begin with, the following theorem presents the determination of the fusion parameters to achieve the fusion performance requirement (17).

Theorem 3: Consider the NTVS (1) with the local filter of form (6). For the given scalar $\gamma_i > 0$ $(i \in S_1)$, the H_∞ performance constraint (17) is achieved for the fusion filtering error system (16) if there exist solutions $\{c_i, \pi\}$ satisfying the following inequality

$$\begin{bmatrix} -\frac{\pi}{2N} & c_1\gamma_1 & c_2\gamma_2 & \cdots & c_N\gamma_N \\ c_1\gamma_1 & -1 & 0 & \cdots & 0 \\ c_2\gamma_2 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_N\gamma_N & 0 & 0 & \cdots & -1 \end{bmatrix} < 0 \quad (33)$$

with $\pi = \tilde{\gamma}^2$ and $\sum_{i=1}^N c_i = 1$.

Proof: We have from (16) that

$$\begin{aligned} \tilde{z}_s^T \tilde{z}_s &= \Big[\sum_{i=1}^N c_i(\tilde{z}_{i,s} - M_s\mu_{i,s})\Big]^T \Big[\sum_{i=1}^N c_i(\tilde{z}_{i,s} - M_s\mu_{i,s})\Big] \\ &\le N\sum_{i=1}^N c_i^2(\tilde{z}_{i,s} - M_s\mu_{i,s})^T(\tilde{z}_{i,s} - M_s\mu_{i,s}) \\ &\le 2N\sum_{i=1}^N c_i^2\big(\tilde{z}_{i,s}^T\tilde{z}_{i,s} + \mu_{i,s}^T M_s^T M_s\mu_{i,s}\big). \end{aligned} \quad (34)$$

Recalling (13), we immediately derive that

$$\begin{aligned} &\mu_{i,s}^T M_s^T M_s \mu_{i,s} \\ &= \text{tr}\{M_s^T M_s \mu_{i,s}\mu_{i,s}^T\} \le \text{tr}\{M_s^T M_s \mu_{i,s}^T \mu_{i,s}\} \le \bar{\Lambda}_{i,s}. \end{aligned} \quad (35)$$

Hence, substituting (35) into (34) yields

$$\tilde{z}_s^T \tilde{z}_s \le 2N\sum_{i=1}^N c_i^2\big(\tilde{z}_{i,s}^T\tilde{z}_{i,s} + \bar{\Lambda}_{i,s}\big). \quad (36)$$

It is easily checked from (33) that $2N\sum_{i=1}^N c_i^2\gamma_i^2 < \tilde{\gamma}^2$. Therefore, based on (9) and (36), we have

$$\begin{aligned} &\sum_{s=0}^T E\{\tilde{z}_s^T\tilde{z}_s\} \\ &\le \sum_{i=1}^N 2Nc_i^2 \sum_{s=0}^T E\{\tilde{z}_{i,s}^T\tilde{z}_{i,s} + \bar{\Lambda}_{i,s}\} \\ &< \sum_{i=1}^N 2Nc_i^2\gamma_i^2\Big(\sum_{s=0}^T E\{\|v_s\|^2 + \|\bar{\varrho}_{i,s}\|^2\} + \xi_{i,0}^T \mathbf{W}_i \xi_{i,0}\Big) \\ &\quad + 2N\sum_{s=0}^T \bar{\Lambda}_s \\ &< \sum_{s=0}^T E\{\tilde{\gamma}^2(\|v_s\|^2 + \|\bar{\varrho}_s\|^2) + 2N\bar{\Lambda}_s\} + \tilde{\gamma}^2\xi_0^T\bar{\mathbf{W}}\xi_0, \end{aligned} \quad (37)$$

which guarantees the H_∞ performance index (17) and thus concludes this proof. ■

Next, by solving a certain convex optimization problem, the fusion parameters are designed in the following theorem to guarantee the H_∞ performance constraint $\mathcal{J}_2 < 0$ with the minimum disturbance attenuation level $\tilde{\gamma}$.

Theorem 4: On the basis of Theorem 3, if the following problem

$$\tilde{\gamma}_{\text{min}} = \min_{c_1, c_2, \cdots, c_N} \tilde{\gamma} \quad (38)$$

subject to (33) is solvable, then there exist optimal fusion parameters with which the fusion filtering error dynamics (16) satisfies the prescribed H_∞ performance with the minimum scalar $\tilde{\gamma}_{\text{min}}$.

Remark 6: Up to now, the distributed quantized H_∞ fusion filtering problem has been thoroughly investigated for NTVSs subject to missing measurements over AaFRNs. Based on the stochastic analysis technique, the completing-the-square technique and the matrix inequality approach, sufficient conditions have been established to pledge the expected local performance index over the finite horizon, and the desired local filters are determined by recursively solving some matrix inequalities. By means of the designed local filters, the distributed fusion filtering with dynamic quantization has been further tackled, and the optimal fusion parameters for guaranteeing the finite-horizon H_∞ performance (17) with the minimum value $\tilde{\gamma}_{\text{min}}$ are calculated in terms of the solution to the optimization problem (38).

Remark 7: Compared with the existing literature on distributed fusion filtering problems, the distinguishing features of this study are exhibited as follows: 1) the addressed distributed fusion filtering problem is new for the considered nonlinear systems undergoing time-varying system parameters, missing measurements, AaF relay protocols, and dynamic quantization effects; 2) multiple relays are deployed in communication networks to facilitate the data transmissions from

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the sensors to the remote filters to improve the communication quality and enhance the signal transmission distance; 3) the local filter parameters are derived in a recursive form by means of the solutions to some linear matrix inequalities; and 4) the optimal fusion parameters are designed by solving a specific convex optimization problem.

IV. A NUMERICAL EXAMPLE

This section provides a numerical example to demonstrate the effectiveness of the proposed distributed fusion filtering method.

Consider the system (1) with the following parameters:

$$\begin{aligned} &D_{1,s}=0.3, \quad D_{2,s}=0.15, \quad D_{3,s}=0.1, \quad D_{4,s}=0.25, \\ &B_s=[-0.15 \quad 0.15]^T, \quad M_s=[0.05 \quad 0.05], \\ &f(x_s)=\left[\begin{array}{c} 0.7x_{1,s}\cos(0.1x_{2,s}) \\ 0.7x_{2,s}\sin(sx_{1,s}) \end{array}\right], \\ &h_1(x_s)=0.15x_{1,s}+0.15x_{2,s}, \quad h_2(x_s)=0.2x_{1,s}+0.2x_{2,s}, \\ &h_3(x_s)=0.25x_{1,s}+0.25x_{2,s}, \quad h_4(x_s)=0.3x_{1,s}+0.3x_{2,s} \end{aligned}$$

where $x_{j,s}$ $(j\in\{1,2\})$ is the j-th element of the system state x_s. It is easy to check that $a=0.7$, $b_1=0.15\sqrt{2}$, $b_2=0.2\sqrt{2}$, $b_3=0.25\sqrt{2}$ and $b_4=0.3\sqrt{2}$.

Set the initial system state x_0 and its estimates as follows:

$$\begin{aligned} &x_0=[2.0 \quad 1.5]^T, \quad \hat{x}_{1,0}=[1.7 \quad 1.6]^T, \\ &\hat{x}_{2,0}=[2.1 \quad 1.4]^T, \quad \hat{x}_{3,0}=[2.2 \quad 1.7]^T, \\ &\hat{x}_{4,0}=[1.6 \quad 1.3]^T. \end{aligned}$$

In this example, the parameters $\beta_{li,s}$ $\big(l=1,2,3\big)$ are selected as $\bar{\beta}_{1i}=0.8$, $\bar{\beta}_{2i}=0.75$ and $\bar{\beta}_{3i}=0.7$, which result in $\beta^*_{1i}=0.16$, $\beta^*_{2i}=0.1875$ and $\beta^*_{3i}=0.21$. The noise disturbances are given by

$$\begin{aligned} &v_s=0.4\cos(0.8s)/\sqrt{s}, \quad \varrho_{1i,s}=0.1/\sqrt{s+1}, \\ &\varrho_{2i,s}=0.15/\sqrt{s+1}, \quad \varrho_{3i,s}=0.2/\sqrt{s+1}. \end{aligned}$$

The other parameters are chosen to be

$$\begin{aligned} &\gamma_1=1.8, \quad \gamma_2=1.7, \quad \gamma_3=1.8, \quad \gamma_4=1.9, \\ &p=40, \quad u_{ij,s}=1, \quad \varsigma_{ij,s}=0.6, \quad d_{ij,0}=3, \\ &\mathbf{Q}_{ij,s}=0.1I, \quad \mathbf{W}_i=15I, \quad E_{1i}=1, \quad E_{2i}=1, \\ &E_{3i}=2, \quad T=15, \quad C_{1i,s}=0.1, \quad C_{2i,s}=0.1, \\ &C_{3i,s}=0.15, \quad P_{i,0}=10I. \end{aligned}$$

Utilizing the Matlab software, the desired local filter parameters $K_{i,s}$ $(i=1,2,3,4)$ are designed at each time instant $s\in[0,15]$, and then the fusion parameters c_i and the minimum value of $\tilde{\gamma}$ are calculated as

$$\begin{aligned} &c_1=0.2635, \quad c_2=0.2634, \quad c_3=0.2493, \\ &c_4=0.2237, \quad \tilde{\gamma}=2.5419. \end{aligned}$$

Let us denote $\bar{\mathbf{E}}_s \triangleq \sqrt{\|\bar{e}_s\|^2}$ as the Euclidean norm of the fusion filtering error $\bar{e}_s$. With the proposed distributed fusion filtering scheme, the simulation results are exhibited in Figs. 2-10. Figs. 2-5 plot the first and second components of the actual system states and their estimates of the local filters. Fig. 6 shows the trajectories of the system state and its fusion estimate. Figs. 7-9 depict the occurrences of missing measurements in different transmission channels. Fig. 10

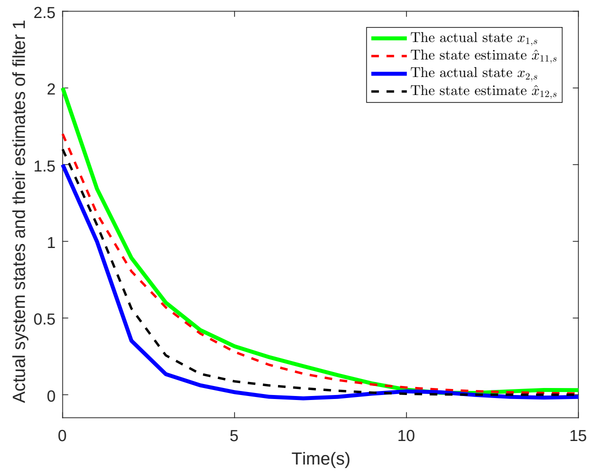

Fig. 2: $x_{j,s}$ and its estimate $\hat{x}_{1j,s}$ $(j=1,2)$.

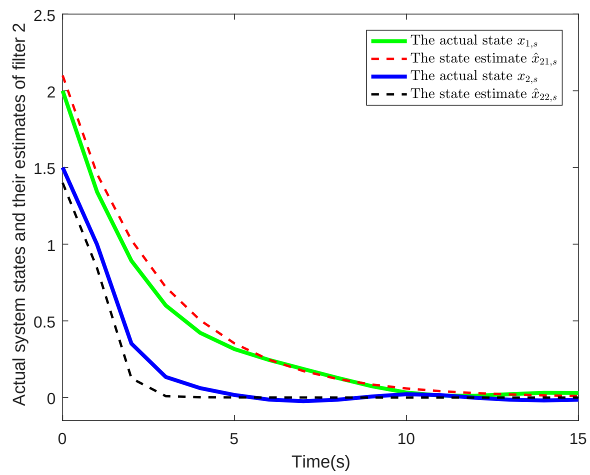

Fig. 3: $x_{j,s}$ and its estimate $\hat{x}_{2j,s}$ $(j=1,2)$.

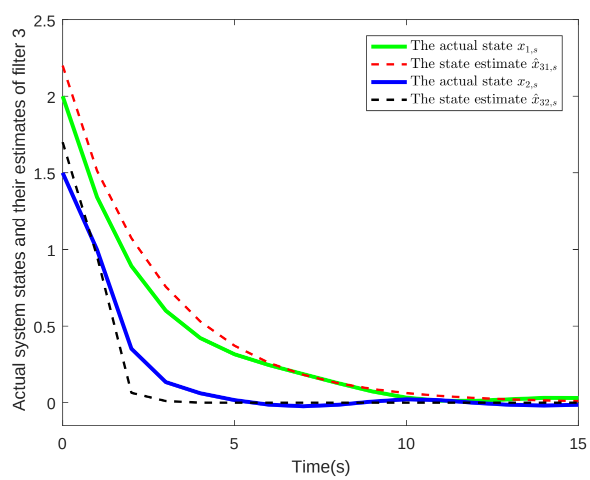

Fig. 4: $x_{j,s}$ and its estimate $\hat{x}_{3j,s}$ $(j=1,2)$.

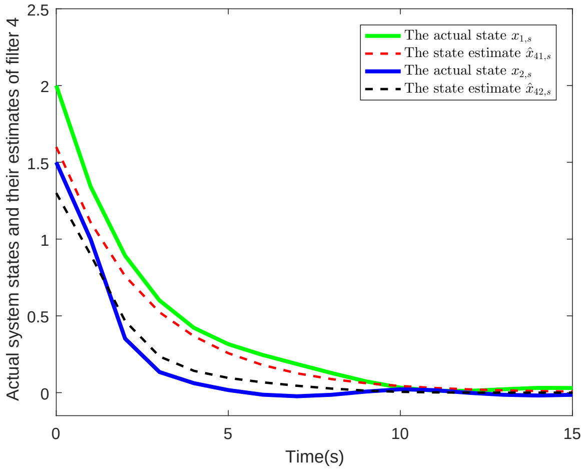

Fig. 5: $x_{j,s}$ and its estimate $\hat{x}_{4j,s}$ $(j = 1, 2)$.

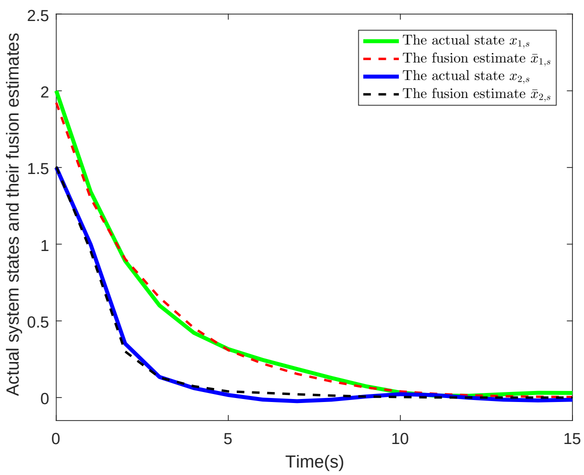

Fig. 6: x_s and its estimate of fusion filter.

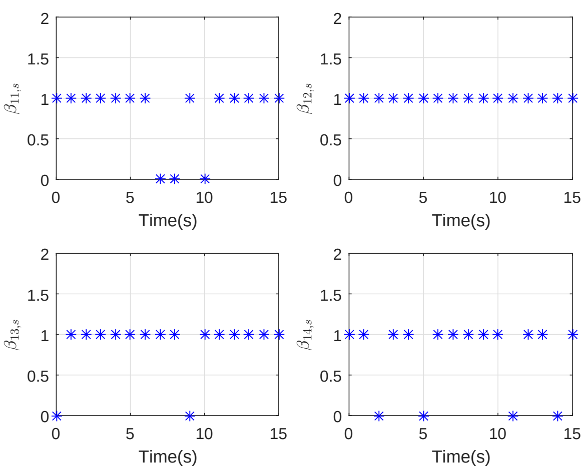

Fig. 7: The values of $\beta_{1i,s}$ $(i = 1, 2, 3, 4)$.

shows the trajectory of $\bar{\mathbf{E}}_s$, which verifies that the developed distributed fusion filter works well for the considered NTVS.

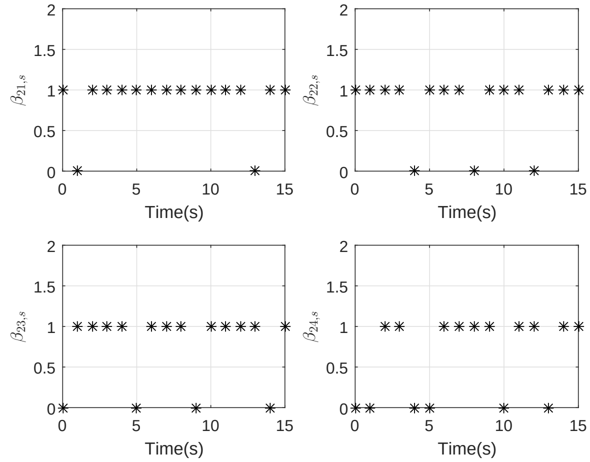

Fig. 8: The values of $\beta_{2i,s}$ $(i = 1, 2, 3, 4)$.

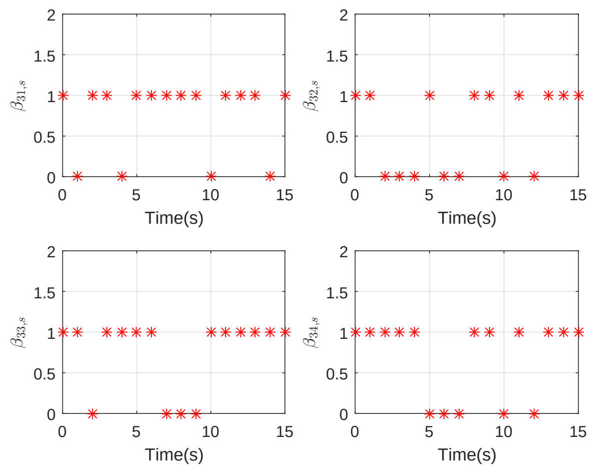

Fig. 9: The values of $\beta_{3i,s}$ $(i = 1, 2, 3, 4)$.

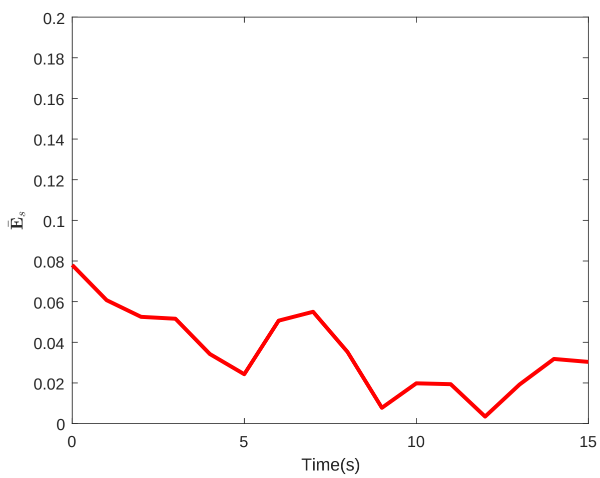

Fig. 10: The trajectory of $\bar{\mathbf{E}}_s$ under dynamic quantization effects.

To further evaluate the influence of quantization mechanism

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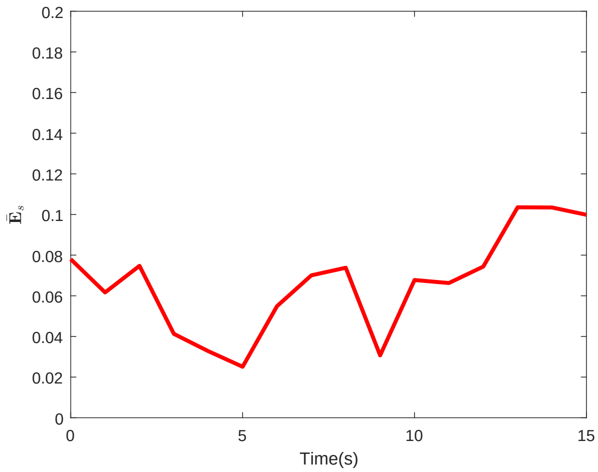

Fig. 11: The trajectory of $\bar{\mathbf{E}}_s$ under statical quantization effects.

on the fusion filtering performance, the static quantization effect is also presented in the simulation. Setting $d_{ij,s} = 3$ without changing the other parameters, the resultant trajectory of $\bar{\mathbf{E}}_s$ is displayed as shown in Fig. 11. It is concluded from Fig. 10 and Fig. 11 that the proposed fusion filtering scheme under the dynamic quantization contributes to a better fusion filtering performance as compared with that under the static quantization.

V. Conclusion

This paper has made one of the first attempts to investigate the distributed H_∞ fusion filtering problem for NTVSs over AaFRNs. The AaF relay communication undergoing randomly occurring missing measurements has been employed to facilitate the data transmissions over the sensors-to-filters channels. The dynamic quantizer has been deployed for each filter-to-FC channel with the limited bandwidth. By using the intensive stochastic analysis and matrix inequality techniques, a sufficient condition has been given to ensure the specific H_∞ performance constraint of the AFES, and the desired local filter gains have been acquired in terms of the feasible solutions to some recursive matrix inequalities. The fusion parameters of the presented distributed fusion scheme have been designed by solving a prescribed optimization problem. Finally, a numerical example has been provided to validate the effectiveness of the proposed fusion filtering strategy. One of the further research topics would be to extend the main results to some more complicated systems with other network-induced phenomena (e.g. the cyber-attacks and the transmission delays [15], [42], [46], [51]). Another future direction is to apply the presented theoretical schemes to more practical scenarios.

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