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Security-Guaranteed Fuzzy Networked State Estimation for 2-D Systems with Multiple Sensor Arrays Subject to Deception Attacks

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***Abstract*—In this paper, the security-guaranteed fuzzy networked state estimation issue is investigated for a class of two-dimensional (2-D) systems with norm-bounded disturbances. Considering the structural specificity of the 2-D systems, the membership function in the Takagi-Sugeno fuzzy model is established to reflect the spatial information. Multiple sensor arrays are utilized to improve the observation diversity and overcome the measurement obstacle induced by geographical restrictions. The network-based deception attacks, occurring in a probabilistic fashion, are characterized by a set of Bernoulli distributed random variables. By resorting to the 2-D fuzzy blending and augmentation operations, the error dynamics of the sth 2-D fuzzy estimator is formulated and, subsequently, the *globally asymptotical stability* of the local error dynamics is studied in virtue of Lyapunov stability theory, fuzzy theory, and stochastic analysis technique. Then, sufficient conditions are derived to ensure the so-called $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$-*security* of the local error dynamics. Furthermore, the estimation fusion problem of the local fuzzy estimators is discussed and the corresponding $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$-*security* is also guaranteed. Finally, an illustrative example is provided to demonstrate the rationality and the effectiveness of the proposed state estimation algorithm.**

***Index Terms*—Two-dimensional systems, Takagi-Sugeno fuzzy model, state estimation, multiple sensor arrays, deception attacks, estimation fusion.**

Notations

$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{R}^{n\times m}$	The set of all $n \times m$ real matrices
$\mathbb{Z}^+$	The set of all nonnegative integers
$\mathbb{Z}^-$	The set of all negative integers
$\\|\cdot\\|$	The Euclidean vector norm in $\mathbb{R}^n$
$\lambda_{\max}(\cdot)$	The maximum eigenvalue
$\lambda_{\min}(\cdot)$	The minimum eigenvalue
I_n	The identity matrix of dimension $n \times n$
M^{-1}	The inverse of M
M^T	The transpose of M
$X \geq Y$	X-Y is positive semi-definite
$X > Y$	X-Y is positive definite

I. Introduction

Ever since the seminal work in [1], the Takagi-Sugeno (T-S) fuzzy model has been attracting a steadily growing interest from the system science and control communities. In particular, benefiting from its distinctive approximation capability, the T-S fuzzy model is well known to be one of the powerful tools to characterize the complicated nonlinear systems. Briefly speaking, by resorting to the fuzzification and defuzzification operations within the T-S framework, a nonlinear system can be approximately described by a set of local linear systems connected via nonlinear membership functions. Such an approach is able to approximate any nonlinear systems with any degree of accuracy in any convex compact region [2], [3]. As a result, the T-S fuzzy model has found successful applications in a variety of realms ranging from control, filtering, parameter estimation, system identification, to model reduction. For example, the H_∞ proportional-integral-derivative-like control problems have been investigated in [4], [8] for T-S fuzzy systems. In [9], an observer-based fuzzy output-feedback controller has been developed for a class of strict-feedback nonlinear systems, where both multiplicative process noises and additive measurement noises are considered.

Over decades, the two-dimensional (2-D) systems have gradually become a research hotpot owing primarily to their extensive application potentials in many industrial fields which include, but are not limited to, multidimensional digital filtering, image processing, and thermal processes [10]–[13]. Up to now, much effort has been devoted to the analysis and synthesis issues for 2-D systems and a rich body of results has been reported in the literature, see [14] and the references therein.

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For example, the robust state estimation problem has been addressed in [15] for a class of 2-D systems within a finite-horizon framework. The sufficient and necessary conditions have been established in [16] to guarantee the stability of 2-D linear systems in continuous, discrete and mixed cases. In [17], the robust H_∞ filtering issue has been investigated for a class of uncertain 2-D discrete systems. In [14], the sliding mode control law has been designed for the 2-D systems under the event-triggered transmission mechanism. Nevertheless, limited work has been done for the 2-D systems due probably to the difficulty in physical modeling and mathematical analysis. As such, it is imperative to build a paradigm for the study of 2-D systems, which constitutes the first motivation of this paper.

Owing to the prominent advantages in light weight, simple installation and easy maintenance, the networked systems (NSs) have received considerable research attention from the engineering and scientific communities [33]–[37]. Accordingly, there has been a great deal of elegant results available in the literature, see e.g. [18]. For a typical NS, the information transmission among system components (e.g. sensors, plants, controllers, and actuators) is usually implemented over a shared communication network [19], [20]. In practical scenarios, it is often the case that the information interactions are prone to the cyber-attacks due to the openness of communication network. The network attacks (including, but are not limited to, data tampering, spoofing, hijacking, and capture-replay) [44]–[46], if not properly handled, would deteriorate the system performance and even destroy the system stability. It is worth pointing out that, compared with other kinds of cyber-attacks, the so-called deception attack caused by tampering/spoofing is more dangerous due to its stealthiness. To this end, the security issue of NSs under deception attacks has begun to attract particular research attention in recent years, see e.g. [47], [48]. Nevertheless, the corresponding security-guaranteed state estimation problem has not been investigated yet for the 2-D fuzzy systems, and this gives rise to another motivation of the current study.

As pointed out in [24], a single sensor array would be sufficient to obtain high-accuracy measurement under ideal conditions. Nevertheless, in real-world applications, the ideal conditions are less likely to be satisfied due to the effects of certain adverse yet ineluctable factors such as non-calibration, offset, and fault [23]. As such, it makes practical sense to consider the case of multiple sensor arrays, where all sensors are geographically distributed in an "array" configuration over different regions of interest [25]. Particularly, the utilization of multiple sensor arrays is capable of improving the observation diversity and overcoming the measurement obstacles incurred by geographical restrictions. Up to now, some elegant research results have been reported on the investigation of multiple sensor arrays. For example, a new technique has been presented in [26] to determine the locations of multiple sensors, where a tradeoff between the information redundancy, the sensor cost, and the process information has been considered.

In the context of state estimation with multiple sensor arrays, the step named estimation fusion has been playing a paramount role in enhancing the authenticity and data availability [27]. The basic idea behind estimation fusion is to combine information from local sources to construct a unified picture and thus achieve better performance than the local setting [28]. In the past several decades, considerable research effort has been devoted to the estimation fusion issues [29]–[31], and some diversified fusion means can be found in [40], [41]. For instance, the multi-sensor fusion problem has been investigated in [32] for a class of clustered sensor networks, where the sequential measurement fusion and estimation fusion have been taken into account. The optimal linear estimation fusion problem has been studied in [38], and three estimation fusion architectures have been discussed. By resorting to the Cholesky factorization and special approximation to the cross-covariance, two computationally effective fusion algorithms have been provided in [39]. However, to the best of our knowledge, the state estimation problem has not been adequately discussed yet for 2-D fuzzy NSs with multiple sensor arrays and deception attacks, not to mention the case where the estimation fusion is also involved.

As is well known, state estimation is a fundamental issue in the areas of systems science and control engineering because, due to physical structure/constraints [5]–[7], some important system states are unobservable from sensor measurements that are likely to be contaminated by noises [21], [22]. So far, a great deal of literature has been available on the observer design for 2-D systems see e.g. [53]. Very recently, the fuzzy-model-based state estimation problem has attracted some initial research attention for nonlinear 2-D systems. For example, with help of T-S fuzzy approximation, the observer design problem has been discussed in [54]–[57] for 2-D systems characterized by the Fornasini-Marchesini model for the purpose of dynamic output-feedback control. Similarly, in [51], [52], the state estimation problem for 2-D Roesser systems has been examined based upon the T-S fuzzy methodology. On the other hand, networked systems have now become increasingly popular and, accordingly, it makes both theoretical and practical sense to look into the fuzzy state estimation problems for 2-D systems, and this motivates our current study.

Summarizing the above discussions, in this paper, we endeavor to deal with the security-guaranteed state estimation problem for a class of 2-D fuzzy NSs with multiple sensor arrays and deception attacks. The main contributions of this paper are highlighted as follows. 1) The security-guaranteed state estimation problem is, for the first time, addressed for the 2-D NSs with multiple sensor arrays and deception attacks. 2) An estimation fusion scheme is developed based on a set of 2-D fuzzy local state estimators. 3) Some sufficient conditions are established to guarantee the *globally asymptotical stability* and the $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$-*security* of the local error dynamics, as well as the $(\varrho_1, \varrho_2, \varrho_3, \rho)$-*security* of the fused counterpart.

The remainder of this paper is organized as follows. Section II formulates the security-guaranteed state estimation problem in the 2-D T-S fuzzy framework. In Section III, the desired state estimator is proposed, and the *globally asymptotical stability* and the prescribed $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$-*security*/$(\varrho_1, \varrho_2, \varrho_3, \rho)$-*security* are discussed. In Section IV, a simulation example is provided to examine the validity of the developed 2-D fuzzy estimation algorithm. Finally, this paper is concluded in Section V.

II. Problem Formulation

Consider a class of T-S fuzzy systems of the following 2-D form:

Plant Rule i:
IF $\theta_1^{(p,q)}$ is $\mathcal{F}_{i1}, \cdots, \theta_j^{(p,q)}$ is $\mathcal{F}_{ij}, \cdots$ and $\theta_p^{(p,q)}$ is $\mathcal{F}_{ip}$,
THEN

$$x(p+1,q+1) = A_{1i}x(p,q+1) + A_{2i}x(p+1,q) + B_{1i}\nu(p,q+1) + B_{2i}\nu(p+1,q), \quad (1)$$

where $x(p,q) \in \mathbb{R}^{n_x} (p,q \in \mathbb{Z}^+)$ denotes the state vector, $\theta_j^{(p,q)} \triangleq [\theta_j(p,q+1),\ \theta_j(p+1,q)]$ $(j = 1,2,\cdots,p)$ is the spatial premise variable at the location (p,q) (which might be state or measurable variable), $\mathcal{F}_{ij}$ is a spatial fuzzy set of rule i corresponding to the spatial input vector $\theta_j^{(p,q)}$, $\mathcal{I}$ is defined as $\mathcal{I} \triangleq \{1,\ 2,\cdots,\ \mathcal{R}\}$ with R being the number of IF-THEN rules, $\nu(p,q) \in \mathbb{R}^{n_\nu}$ is the disturbance input, and A_{1i}, A_{2i}, B_{1i} and B_{2i} are known real constant system matrices with compatible dimensions.

Assumption 1. *The disturbance input $\nu(p,q)$ is bounded by*

$$\|\nu(p,q)\| \le \varrho_1, \quad (2)$$

where ϱ_1 is a given positive scalar.

Considering the structural specificity of the premise variable in 2-D systems, the following spatial membership function is introduced [42]:

$$h_i^{(p,q)} = [h_i(p,q+1),\ h_i(p+1,q)], \quad (3)$$

where

$$h_i(p,q) \triangleq \frac{\Psi_i(p,q)}{\sum_{i=1}^{\mathcal{R}} \Psi_i(p,q)}, \qquad \Psi_i(p,q) \triangleq \prod_{j=1}^{p} \mathfrak{F}_{ij}(\theta_j^{(p,q)}).$$

The component $h_i(p,q)$ of the spatial membership function is actually the normalized membership function. $\mathfrak{F}_{ij}(\theta_j^{(p,q)}) \ge 0$ is the grade of membership of $\theta_j^{(p,q)}$ in $\mathcal{F}_{ij}$. It is not difficult to see that

$$0 \le h_i(p,q) \le 1, \quad \sum_{i=1}^{\mathcal{R}} h_i(p,q) = 1, \quad \forall\ p,q \in \mathbb{Z}^+. \quad (4)$$

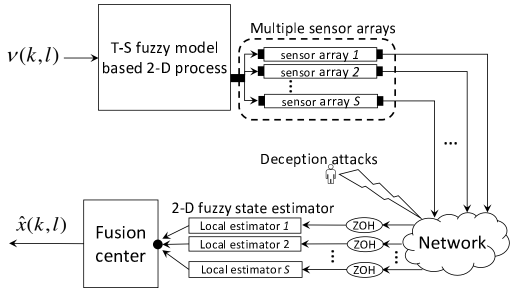

Fig. 1: Block diagram of 2-D fuzzy networked estimation system.

In this paper, as illustrated in Fig. 1, multiple sensor arrays are adopted to improve the observation diversity and overcome the measurement obstacles caused by geographical restrictions. Specifically, the measurement model of the sth sensor array is given by

$$\widetilde{y}^s(p,q) = C_i^s x(p,q) + D_i^s \nu(p,q), \qquad s \in \mathfrak{S} \quad (5)$$

where $\widetilde{y}^s(p,q) \in \mathbb{R}^{n_y}$ denotes the measurement output, C_i^s and D_i^s are given coefficients with appropriate dimensions, and $\mathfrak{S} = \{1,\ 2,\cdots,\ S\}$ with S being the number of sensor arrays.

The measured data in the multiple sensor arrays are all transmitted to the remote endpoint devices (i.e. local state estimator) through a shared communication network (subject to cyber-attacks), thereby achieving the so-called networked deployment concerning the information acquisition process. As such, the underlying system is referred to as the *networked* 2-D fuzzy systems. Note that the measurements transmitted over an open network environment might suffer from the deception attack launched by adversaries. In this case, the actual measurement model can be described by

$$y^s(p,q) = \widetilde{y}^s(p,q) + \beta(p,q)\bar{y}^s(p,q), \quad (6)$$

where $\bar{y}^s(p,q) \triangleq -\widetilde{y}^s(p,q) + \xi(p,q)$, $\xi(p,q) \in \mathbb{R}^{n_y}$ is the deception data injected by the attackers, and $\beta(p,q)$ is a random variable with the following probability distribution:

$$\text{Prob}\{\beta(p,q) = 1\} = \bar{\beta}, \quad \text{Prob}\{\beta(p,q) = 0\} = 1 - \bar{\beta} \quad (7)$$

with $\bar{\beta} \in [0,1)$ being a known scalar. The measurements passing through the network are stored in a set of zero-order-holders (ZOHs), which provide the ready-made data for the subsequent fuzzy state estimation procedure.

Assumption 2. *The deception data $\xi(p,q)$ satisfies*

$$\|\xi(p,q)\| \le \varrho_2, \quad (8)$$

where ϱ_2 is a prescribed positive scalar.

Remark 1. *It is worth mentioning that the combination of multiple sensor arrays and information fusion center would enhance the diversity of available data, thereby improving the reliability and accuracy as confirmed by engineering practice. Compared with the ordinary sensor network, one clearly finds from the construction (5) that characters of the multiple sensor arrays take on large quantity, wide distribution and multifarious collection. It will immediately degrade into the ordinary sensor network once $\mathfrak{S} = 1$. In terms of this view, results obtained in this paper can be applied to ordinary sensor network. On the other hand, the system measurements transmitted via an open communication network are very susceptible to deception attacks. In TCP/IP based network, there are many forms of deception attacks which include, but are not limited to, IP deception, ARP deception, DNS deception, and route source deception. Generally speaking, these various but random attacks can be characterized by the unified model (6) through constructing $\xi(p,q)$ and $\beta(p,q)$. Such a formulation would facilitate the design and analysis of the fuzzy state estimator under deception attacks.*

In this paper, the sth full-order local fuzzy state estimator is constructed of the following form:

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Local State Estimator Rule i:
IF $\theta_1^{(p,q)}$ is $\mathcal{F}_{i1}$, $\cdots$, $\theta_j^{(p,q)}$ is $\mathcal{F}_{ij}$, $\cdots$ and $\theta_p^{(p,q)}$ is $\mathcal{F}_{ip}$, **THEN**

$$\begin{aligned}\hat{x}^s(p+1,q+1) =& A_{1fi}^s\hat{x}^s(p,q+1) + A_{2fi}^s\hat{x}^s(p+1,q)\\ &+ B_{1fi}^s y^s(p,q+1)\\ &+ B_{2fi}^s y^s(p+1,q),\end{aligned} \tag{9}$$

where $\hat{x}^s(p,q)$ is the state vector of the sth state estimator, and A_{1fi}^s, A_{2fi}^s, B_{1fi}^s and B_{2fi}^s are the gain parameters to be determined.

Assumption 3. *The initial boundary conditions of the 2-D fuzzy system (1) and the sth local state estimator (9) are specified by*

$$x(p,q) = \hat{x}^s(p,q) = \begin{cases} \psi^h(p,q); & \text{if } (p,q) \in 0 \times [0,\, b_h] \\ \psi^v(p,q); & \text{if } (p,q) \in [0,\, b_v] \times 0 \\ 0; & \text{if } (p,q) \in 0 \times (b_h,\, \infty) \\ 0; & \text{if } (p,q) \in (b_v,\, \infty) \times 0 \end{cases} \tag{10}$$

with $\psi^h(0,0) = \psi^v(0,0)$, where b_h and b_v are prescribed positive integers. For a known positive scalar ϱ_2, $\psi^h(p,q)$ $\psi^v(p,q)$ are given vectors satisfying

$$\left\| \begin{bmatrix} \psi^h(p,q) \\ \psi^v(p,q) \end{bmatrix} \right\| \leq \varrho_3. \tag{11}$$

Based on the operation of fuzzy blending, the defuzzified output of the 2-D fuzzy system (1) can be represented as

$$\begin{cases} x(p+1,q+1) = \sum_{i=1}^{\mathcal{R}} h_i(p,q+1)\big\{A_{1i}x(p,q+1) \\ \qquad + B_{1i}\nu(p,q+1)\big\} + \sum_{i=1}^{\mathcal{R}} h_i(p+1,q) \\ \qquad \times \big\{A_{2i}x(p+1,q) + B_{2i}\nu(p+1,q)\big\}, \\ \widetilde{y}^s(p,q) = \sum_{i=1}^{\mathcal{R}} h_i(p,q)\big[C_i^s x(p,q) + D_i^s\nu(p,q)\big]. \end{cases} \tag{12}$$

It should be noted that the validity of the 2-D fuzzy model in (1) with (12) were already studied in [59].

Similarly, the defuzzified output of the sth local state estimator is readily obtained as

$$\begin{aligned}\hat{x}^s(p+1,l+1) =& \sum_{i=1}^{\mathcal{R}} h_i(p,q+1)\big\{A_{1fi}^s\hat{x}^s(p,q+1) \\ &+ B_{1fi}^s y^s(p,q+1)\big\} + \sum_{i=1}^{\mathcal{R}} h_i(p+1,q) \\ &\times \big\{A_{2fi}^s\hat{x}^s(p+1,q) + B_{2fi}^s y^s(p+1,q)\big\}.\end{aligned} \tag{13}$$

For brevity, let's choice $h_i \triangleq h_i(p,q)$, $\acute{h}_i \triangleq h_i(p,q+1)$, $\grave{h}_i \triangleq h_i(p+1,q)$ and

$$\sum_{i_1,i_2,\cdots,i_s=1}^{r} h_{i_1}h_{i_2}\cdots h_{i_s} = \sum_{i_1=1}^{r} h_{i_1}\sum_{i_2=1}^{r} h_{i_2}\cdots\sum_{i_s=1}^{r} h_{i_s}$$

for any $s \in \mathbb{Z}^+$. Letting $\tilde{e}^s(p,q) \triangleq x(p,q) - \hat{x}^s(p,q)$, the sth 2-D fuzzy error dynamics can be calculated as follows:

$$\begin{aligned}\tilde{e}^s(p+1,q+1) =& \sum_{i,j,n=1}^{\mathcal{R}} \acute{h}_i\acute{h}_j\acute{h}_n\Big[\big(A_{1i} - A_{1fj}^s\big)x(p,q+1) \\ &+ A_{1fj}^s\tilde{e}^s(p,q+1) + B_{1i}\nu(p,q+1) \\ &- (1-\beta(p,q+1))B_{1fj}^s C_n^s x(p,q+1) \\ &- (1-\beta(p,q+1))B_{1fj}^s D_n^s\nu(p,q+1)\big) \\ &- \beta(p,q+1)B_{1fj}^s\xi(p,q+1)\Big] \\ &+ \sum_{i,j,n=1}^{\mathcal{R}} \grave{h}_i\grave{h}_j\grave{h}_n\Big[\big(A_{2i} - A_{2fj}^s\big)x(p+1,q) \\ &+ A_{2fj}^s\tilde{e}^s(p+1,q) + B_{2i}\nu(p+1,q) \\ &- (1-\beta(p+1,q))B_{2fj}^s C_n^s x(p+1,q) \\ &- (1-\beta(p+1,q))B_{2fj}^s D_n^s\nu(p+1,q)\big) \\ &- \beta(p+1,q)B_{2fj}^s\xi(p+1,q)\Big].\end{aligned} \tag{14}$$

Define $e^s(p,q) \triangleq \Big[x^T(p,q)\ \big(\tilde{e}^s(p,q)\big)^T\Big]^T$ and $\bar{\xi}(p,q) \triangleq \big[\nu^T(p,q)\ \xi^T(p,q)\big]^T$. Then, the augmented 2-D fuzzy error dynamics is obtained as

$$\begin{aligned}e^s(p+1,q+1) =& \sum_{\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{n}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{n}} \\ &\times \Big[\bar{A}_{1\acute{i}\acute{j}}^s e^s(p,q+1) + \bar{B}_{1\acute{i}}\bar{\xi}(p,q+1) \\ &- (1-\beta(p,q+1))\bar{C}_{1\acute{j}\acute{n}}^s e^s(p,q+1) \\ &- (1-\beta(p,q+1))\bar{B}_{1f\acute{j}}^s\bar{\xi}(p,q+1) \\ &- \beta(p,q+1)\widetilde{B}_{1f\acute{j}}^s\bar{\xi}(p,q+1) \\ &+ \bar{A}_{2\grave{i}\grave{j}}^s e^s(p+1,q) + \bar{B}_{2\grave{i}}\bar{\xi}(p+1,q) \\ &- (1-\beta(p+1,q))\bar{C}_{2\grave{j}\grave{n}}^s e^s(p+1,q) \\ &- (1-\beta(p+1,q))\bar{B}_{2f\grave{j}}^s\bar{\xi}(p+1,q) \\ &- \beta(p+1,q)\widetilde{B}_{2f\grave{j}}^s\bar{\xi}(p+1,q)\Big],\end{aligned} \tag{15}$$

where

$$\bar{A}_{1\acute{i}\acute{j}}^s \triangleq \begin{bmatrix} A_{1\acute{i}} & 0 \\ A_{1\acute{i}} - A_{1f\acute{j}}^s & A_{1f\acute{j}}^s \end{bmatrix},\ \bar{B}_{1\acute{i}} \triangleq \begin{bmatrix} B_{1\acute{i}} & 0 \\ B_{1\acute{i}} & 0 \end{bmatrix},$$

$$\bar{A}_{2\grave{i}\grave{j}}^s \triangleq \begin{bmatrix} A_{2\grave{i}} & 0 \\ A_{2\grave{i}} - A_{2f\grave{j}}^s & A_{2f\grave{j}}^s \end{bmatrix},\quad \bar{B}_{2\grave{i}} \triangleq \begin{bmatrix} B_{2\grave{i}} & 0 \\ B_{2\grave{i}} & 0 \end{bmatrix},$$

$$\bar{B}_{1f\acute{j}}^s \triangleq \begin{bmatrix} 0 & 0 \\ B_{1f\acute{j}}^s D_{\acute{n}}^s & 0 \end{bmatrix},\quad \bar{B}_{2f\grave{j}}^s \triangleq \begin{bmatrix} 0 & 0 \\ B_{2f\grave{j}}^s D_{\grave{n}}^s & 0 \end{bmatrix},$$

$$\widetilde{B}_{1f\acute{j}}^s \triangleq \begin{bmatrix} 0 & 0 \\ 0 & B_{1f\acute{j}}^s D_{\acute{n}}^s \end{bmatrix},\quad \widetilde{B}_{2f\grave{j}}^s \triangleq \begin{bmatrix} 0 & 0 \\ 0 & B_{2f\grave{j}}^s D_{\grave{n}}^s \end{bmatrix},$$

$$\bar{C}_{1\acute{j}\acute{n}}^s \triangleq \begin{bmatrix} 0 & 0 \\ B_{1f\acute{j}}^s C_{\acute{n}}^s & 0 \end{bmatrix},\quad \bar{C}_{2\grave{j}\grave{n}}^s \triangleq \begin{bmatrix} 0 & 0 \\ B_{2f\grave{j}}^s C_{\grave{n}}^s & 0 \end{bmatrix}.$$

Assumption 4. *The initial boundary condition (10) as well as the stochastic variables $\nu(p,q)$ and $\beta(p,q)$ $(p,q \in \mathbb{Z}^+)$ are mutually independent.*

Now, let us present two relevant definitions.

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Definition 1. *(Globally asymptotical stability) The sth 2-D fuzzy error dynamics (15) with $\bar{\xi}(p,q) \equiv 0$ is said to be globally asymptotically stable in the mean-square sense if*

$$\lim_{p+q\to\infty} \mathbb{E}\left\{\left\|e^s(p,q)\right\|^2\right\} = 0 \tag{16}$$

holds for the initial condition (10). In this case, the 2-D state estimator (9) is said to be a globally asymptotically stable fuzzy state estimator for the target system (1).

Definition 2. *($(\varrho_1, \varrho_2, \varrho_3, \rho_s)$-security) [43], [49] Given the positive constant scalars ϱ_1, ϱ_2, ϱ_3 and ρ_s. The sth error dynamics (15) with $\bar{\xi}(p,q) \neq 0$ is said to be $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$-secure in the mean-square sense if*

$$\mathbb{E}\left\{\left\| \begin{array}{c} e^s(\mathcal{T}, q+1) \\ e^s(p+1, \mathcal{T}) \end{array} \right\|^2\right\} < \mathcal{T}{\rho_s}^2,\ p,q \in \mathbb{Z}^+,\ s \in \mathfrak{S} \tag{17}$$

holds for any given integer $\mathcal{T} \in \mathbb{Z}^+$ under the conditions (2), (8) and (11).

The input-to-state stability, which was introduced in [58], has been widely used in analyzing nonlinear systems with exogenous inputs. Such a concept bridges the gap between input-output and state-space approaches, and has therefore gained a growing popularity in recent years. In this paper, Definition 2 can be considered to be a security-adapted version of the traditional notion of input-to-state stability. Similar to input-to-state stability (with bounded input and bounded state), in Definition 2, the trajectories of the error dynamics (15) are bounded in terms of size of the input (i.e. ϱ_1, ϱ_2 and ϱ_3) for sufficiently large times. Actually, the bounded state can also be regarded as a sort of stability criterion (i.e. the so-called security), which is a very important performance index for cyber-physical systems against malicious attacks. In this sense, the security notion (reflected in Definition 2) is of clear engineering insight.

The main purpose of this paper is to design a set of full-order local fuzzy state estimators of the form (9) for the 2-D T-S model (1). More specifically, we are looking for a set of estimator gain parameters to guarantee the *globally asymptotical stability* and the $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$*-security* of the 2-D fuzzy error dynamics (15).

Before ending this section, the following lemma is introduced, which will be used in the subsequent analysis.

Lemma 1. *Let $R \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix. For any real vectors $X_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}} \in \mathbb{R}^n$ and $X_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}} \in \mathbb{R}^n$ with $\acute{i}, \acute{j}, \acute{m}, \grave{i}, \grave{j}, \grave{m}, \acute{a}, \acute{b}, \acute{c}, \grave{a}, \grave{b}, \grave{c} \in \mathbb{S}$, we have*

$$\begin{aligned} &\sum_{\acute{i},\acute{j},\acute{m},\acute{a},\acute{b},\acute{c},\grave{i},\grave{j},\grave{m},\grave{a},\grave{b},\grave{c}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{m}}\acute{h}_{\acute{a}}\acute{h}_{\acute{b}}\acute{h}_{\acute{c}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}\grave{h}_{\grave{a}}\grave{h}_{\grave{b}}\grave{h}_{\grave{c}} \\ &\qquad \times X^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}} R X_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}} \\ &\leq \sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}} X^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}} R X_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}, \end{aligned}$$

where $\acute{h}_t \geq 0$, $\grave{h}_t \geq 0$, and $\sum_{t=1}^{r} \acute{h}_t = \sum_{t=1}^{r} \grave{h}_t = 1$ with $t \in \mathbb{S}$.

III. Main Results

In this section, we first derive some sufficient conditions to ensure the *globally asymptotical stability* and the $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$*-security* of the 2-D fuzzy error dynamics (15). Then, we discuss the design of the desired local state estimators as well as the corresponding estimation fusion problem.

A. Stability and $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$-security

The following theorem presents a sufficient condition under which the closed-loop 2-D fuzzy system (9) is *globally asymptotically stable in the mean-square sense*.

Theorem 1. *Let the sate estimator gains A^s_{1fi}, A^s_{2fi}, B^s_{1fi} and B^s_{2fi} be given. The sth 2-D fuzzy error dynamics (15) with $\bar{\xi}(p,q) \equiv 0$ is globally asymptotically stable in the mean-square sense if there exist matrices $Q^h > 0$ and $Q^v > 0$ such that the following matrix inequalities hold:*

$$\Lambda^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}} < 0, \quad \acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n} \in \mathcal{I},\ s \in \mathfrak{S} \tag{18}$$

where

$$\begin{aligned} \Lambda^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}} &\triangleq \begin{bmatrix} \Lambda_{11} & * \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix}, \\ \Lambda_{11} &\triangleq \Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big)^T \\ &\quad \times \Big(Q^h + Q^v\Big)\Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big) \\ &\quad + \bar{\beta}(1-\bar{\beta})\Big(\bar{C}^s_{1\acute{j}\acute{n}}\Big)^T\Big(Q^h + Q^v\Big)\bar{C}^s_{1\acute{j}\acute{n}} - Q^h, \\ \Lambda_{21} &\triangleq \Big(\bar{A}^s_{2\grave{i}\grave{j}} + (\bar{\beta}-1)\bar{C}^s_{2\grave{j}\grave{n}}\Big)^T\Big(Q^h + Q^v\Big) \\ &\quad \times \Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big), \\ \Lambda_{22} &\triangleq \Big(\bar{A}^s_{2\grave{i}\grave{j}} + (\bar{\beta}-1)\bar{C}^s_{2\grave{j}\grave{n}}\Big)^T\Big(Q^h + Q^v\Big(\bar{A}^s_{2\grave{i}\grave{j}} \\ &\quad + (\bar{\beta}-1)\bar{C}^s_{2\grave{j}\grave{n}}\Big) + \bar{\beta}(1-\bar{\beta})\Big(\bar{C}^s_{2\grave{j}\grave{n}}\Big)^T \\ &\quad \times \Big(Q^h + Q^v\Big)\bar{C}^s_{2\grave{j}\grave{n}} - Q^v. \end{aligned}$$

Proof: Choose a Lyapunov-like functional of the following form

$$V(p,q) = V^h(p,q) + V^v(p,q), \tag{19}$$

where

$$\begin{aligned} V^h(p,q) &\triangleq (e^s(p,q))^T Q^h e^s(p,q), \\ V^v(p,q) &\triangleq (e^s(p,q))^T Q^v e^s(p,q). \end{aligned}$$

Letting $\bar{\xi}(p,q) \equiv 0$, along the trajectory of error dynamics (9), the difference of $V(p,q)$ can be obtained by

$$\mathcal{I}(p,q) = \Delta V^h(p,q) + \Delta V^v(p,q), \tag{20}$$

where

$$\begin{aligned} \Delta V^h(p,q) &\triangleq \mathbb{E}\big\{V^h(p+1,q+1) - V^h(p,q+1)\big|\hbar(p,q)\big\}, \\ \Delta V^v(p,q) &\triangleq \mathbb{E}\big\{V^v(p+1,q+1) - V^v(p+1,q)\big|\hbar(p,q)\big\}, \\ \hbar(p,q) &\triangleq \big\{e^s(p,q+1), e^s(p+1,q)\big\}. \end{aligned}$$

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Considering Lemma 1, it follows from (15) and (20) that

$$\begin{aligned}\Delta V^h(p,q) =&(e^s(p+1,q+1))^T Q^h e^s(p+1,q+1)\\ &-(e^s(p,q+1))^T Q^h e^s(p,q+1)\\ =&\sum_{\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{\imath}}\acute{h}_{\acute{\jmath}}\acute{h}_{\acute{n}}\grave{h}_{\grave{\imath}}\grave{h}_{\grave{\jmath}}\grave{h}_{\grave{n}}\\ &\times\mathbb{E}\Big\{\Big[\bar{A}^s_{1\acute{\imath}\acute{\jmath}}e^s(p,q+1)\\ &-(1-\beta(p,q+1))\bar{C}^s_{1\acute{\jmath}\acute{n}}e^s(p,q+1)\\ &+\bar{A}^s_{2\grave{\imath}\grave{\jmath}}e^s(p+1,q)\\ &-(1-\beta(p+1,q))\bar{C}^s_{2\grave{\jmath}\grave{n}}e^s(p+1,q)\Big]^T Q^h\\ &\times\Big[\bar{A}^s_{1\acute{\imath}\acute{\jmath}}e^s(p,q+1)\\ &-(1-\beta(p,q+1))\bar{C}^s_{1\acute{\jmath}\acute{n}}e^s(p,q+1)\\ &+\bar{A}^s_{2\grave{\imath}\grave{\jmath}}e^s(p+1,q)-(1-\beta(p+1,q))\\ &\times\bar{C}^s_{2\grave{\jmath}\grave{n}}e^s(p+1,q)\Big]-(e^s(p,q+1))^T Q^h\\ &\times e^s(p,q+1)\Big|\hbar(p,q)\Big\},\end{aligned}\tag{21}$$

$$\begin{aligned}\Delta V^v(p,q) =&(e^s(p+1,q+1))^T Q^v e^s(p+1,q+1)\\ &-(e^s(p+1,q))^T Q^v e^s(p+1,q)\\ =&\sum_{\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{\imath}}\acute{h}_{\acute{\jmath}}\acute{h}_{\acute{n}}\grave{h}_{\grave{\imath}}\grave{h}_{\grave{\jmath}}\grave{h}_{\grave{n}}\\ &\times\mathbb{E}\Big\{\Big[\bar{A}^s_{1\acute{\imath}\acute{\jmath}}e^s(p,q+1)\\ &-(1-\beta(p,q+1))\bar{C}^s_{1\acute{\jmath}\acute{n}}e^s(p,q+1)\\ &+\bar{A}^s_{2\grave{\imath}\grave{\jmath}}e^s(p+1,q)\\ &-(1-\beta(p+1,q))\bar{C}^s_{2\grave{\jmath}\grave{n}}e^s(p+1,q)\Big]^T Q^v\\ &\times\Big[\bar{A}^s_{1\acute{\imath}\acute{\jmath}}e^s(p,q+1)\\ &-(1-\beta(p,q+1))\bar{C}^s_{1\acute{\jmath}\acute{n}}e^s(p,q+1)\\ &+\bar{A}^s_{2\grave{\imath}\grave{\jmath}}e^s(p+1,q)-(1-\beta(p+1,q))\bar{C}^s_{2\grave{\jmath}\grave{n}}\\ &\times e^s(p+1,q)\Big]-(e^s(p+1,q))^T Q^v\\ &\times e^s(p+1,q)\Big|\hbar(p,q)\Big\}.\end{aligned}\tag{22}$$

Then, we arrive at

$$\begin{aligned}&\mathbb{E}\{\mathcal{I}(p,q)\}\\ \le&\sum_{\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{\imath}}\acute{h}_{\acute{\jmath}}\acute{h}_{\acute{n}}\grave{h}_{\grave{\imath}}\grave{h}_{\grave{\jmath}}\grave{h}_{\grave{n}}c\mathbb{E}\Big\{\Big[\bar{A}^s_{1\acute{\imath}\acute{\jmath}}e^s(p,q+1)\\ &-(1-\beta(p,q+1))\bar{C}^s_{1\acute{\jmath}\acute{n}}e^s(p,q+1)+\bar{A}^s_{2\grave{\imath}\grave{\jmath}}e^s(p+1,q)\\ &-(1-\beta(p+1,q))\bar{C}^s_{2\grave{\jmath}\grave{n}}e^s(p+1,q)\Big]^T Q^h\Big[\bar{A}^s_{1\acute{\imath}\acute{\jmath}}e^s(p,q+1)\\ &-(1-\beta(p,q+1))\bar{C}^s_{1\acute{\jmath}\acute{n}}e^s(p,q+1)+\bar{A}^s_{2\grave{\imath}\grave{\jmath}}e^s(p+1,q)\\ &-(1-\beta(p+1,q))\bar{C}^s_{2\grave{\jmath}\grave{n}}e^s(p+1,q)\Big]+\Big[\bar{A}^s_{1\acute{\imath}\acute{\jmath}}e^s(p,q+1)\\ &-(1-\beta(p,q+1))\bar{C}^s_{1\acute{\jmath}\acute{n}}e^s(p,q+1)+\bar{A}^s_{2\grave{\imath}\grave{\jmath}}e^s(p+1,q)\\ &-(1-\beta(p+1,q))\bar{C}^s_{2\grave{\jmath}\grave{n}}e^s(p+1,q)\Big]^T Q^v\\ &\times\Big[\bar{A}^s_{1\acute{\imath}\acute{\jmath}}e^s(p,q+1)-(1-\beta(p,q+1))\bar{C}^s_{1\acute{\jmath}\acute{n}}e^s(p,q+1)\\ &+\bar{A}^s_{2\grave{\imath}\grave{\jmath}}e^s(p+1,q)-(1-\beta(p+1,q))\bar{C}^s_{2\grave{\jmath}\grave{n}}e^s(p+1,q)\Big]\\ &-(e^s(p,q+1))^T Q^h e^s(p,q+1)\\ &-(e^s(p+1,q))^T Q^v e^s(p+1,q)\Big|\hbar(p,q)\Big\}\\ =&\sum_{\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{\imath}}\acute{h}_{\acute{\jmath}}\acute{h}_{\acute{n}}\grave{h}_{\grave{\imath}}\grave{h}_{\grave{\jmath}}\grave{h}_{\grave{n}}\\ &\times\mathbb{E}\Big\{\eta^T(p,q)\Lambda^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\eta(p,q)\Big\},\end{aligned}\tag{23}$$

where $\eta(p,q)\triangleq\big[(e^s(p,q+1))^T \quad (e^s(p+1,q))^T\big]^T$.

For any positive integers $\mathcal{T}_h$ and $\mathcal{T}_v$, summing up both sides of (23) for p and q varying from 0 to, respectively, $\mathcal{T}_h$ and $\mathcal{T}_h$, one has

$$\begin{aligned}&\sum_{q=0}^{\mathcal{T}_v}\sum_{p=0}^{\mathcal{T}_h}\mathbb{E}\{\mathcal{I}(p,q)\}\le\lambda_{\max}\left(\Lambda^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\right)\sum_{q=0}^{\mathcal{T}_v}\sum_{p=0}^{\mathcal{T}_h}\\ &\times\sum_{\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{\imath}}\acute{h}_{\acute{\jmath}}\acute{h}_{\acute{n}}\grave{h}_{\grave{\imath}}\grave{h}_{\grave{\jmath}}\grave{h}_{\grave{n}}\mathbb{E}\left\{\big\|\eta(p,q)\big\|^2\right\}.\end{aligned}\tag{24}$$

Then, it follows from the nonnegativeness of $V(p,q)$ that

$$\begin{aligned}&\sum_{q=0}^{\mathcal{T}_v}\sum_{p=0}^{\mathcal{T}_h}\sum_{\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}=1}^{r} \acute{h}_{\acute{\imath}}\acute{h}_{\acute{\jmath}}\acute{h}_{\acute{n}}\grave{h}_{\grave{\imath}}\grave{h}_{\grave{\jmath}}\grave{h}_{\grave{n}}\mathbb{E}\left\{\big\|\eta(p,q)\big\|^2\right\}\\ \le&\frac{1}{\lambda_{\max}\left(\Lambda^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\right)}\mathbb{E}\left\{\sum_{q=0}^{N}V_1^h(0,q+1)\right.\\ &\left.+\sum_{p=0}^{M}V_1^v(p+1,0)\right\}.\end{aligned}$$

Furthermore, based on the finite initial boundary condition (10), we conclude that

$$\begin{aligned}\lim_{\mathcal{T}_v,\mathcal{T}_v\to+\infty}&\sum_{q=0}^{\mathcal{T}_v}\sum_{p=0}^{\mathcal{T}_h}\sum_{\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}=1}^{r} \acute{h}_{\acute{\imath}}\acute{h}_{\acute{\jmath}}\acute{h}_{\acute{n}}\grave{h}_{\grave{\imath}}\grave{h}_{\grave{\jmath}}\grave{h}_{\grave{n}}\\ &\times\mathbb{E}\left\{\big\|\eta(p,q)\big\|^2\right\}<\infty,\end{aligned}\tag{25}$$

which infers $\lim_{p+l\to\infty}\mathbb{E}\left\{\big\|e^s(p,q)\big\|^2\right\}=0$. Therefore, the sth 2-D fuzzy error dynamics (15) with $\bar{\xi}(p,q)\equiv0$ is globally asymptotically stable in the mean-square sense, which completes the proof. ■

Theorem 1 provides a sufficient condition to guarantee the *globally asymptotical stability* of the sth error dynamics. In what follows, we are going to discuss the $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-*security* with the help of stochastic analysis techniques and matrix theory.

Theorem 2. *Let the sate estimator gains A^s_{1fi}, A^s_{2fi}, B^s_{1fi} and B^s_{2fi} be given. The sth 2-D fuzzy error dynamics (15) with $\nu(p,q)\neq0$ is $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-secure in the mean-square sense if there exist matrices $Q^h>0$ and $Q^v>0$ such that the following matrix inequalities hold for any $\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}\in\mathcal{I}$*

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and $s \in \mathfrak{S}$*:*

$$\begin{cases} \overline{\Lambda}^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}} < 0, & \text{(26a)} \\ \dfrac{\bar{\varrho}_{1,2}^2 + \max\Big\{\lambda_{\max}(Q^h), \lambda_{\max}(Q^v)\Big\}\varrho_3^2}{\min\Big\{\lambda_{\min}(Q^h), \lambda_{\min}(Q^v)\Big\}} < \rho_s^2, & \text{(26b)} \end{cases}$$

where $\bar{\varrho}_{1,2} \triangleq \sqrt{2\varrho_1^2 + 2\varrho_2^2}$*,*

$$\overline{\Lambda}^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}} \triangleq \begin{bmatrix} \Lambda_{11} & * & * & \\ \Lambda_{21} & \Lambda_{22} & * & \\ \overline{\Lambda}_{31} & \overline{\Lambda}_{32} & \overline{\Lambda}_{33} & * \\ \overline{\Lambda}_{41} & \overline{\Lambda}_{42} & \overline{\Lambda}_{43} & \overline{\Lambda}_{44} \end{bmatrix} < 0,$$

$$\begin{aligned}
\overline{\Lambda}_{31} \triangleq & \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big)^T \Big(Q^h + Q^v\Big) \\
& \times \Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big), \\
\overline{\Lambda}_{32} \triangleq & \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big)^T \Big(Q^h + Q^v\Big) \\
& \times \Big(\bar{A}^s_{2\grave{i}\grave{j}} + (\bar{\beta}-1))\bar{C}^s_{2\grave{j}\grave{n}}\Big), \\
\overline{\Lambda}_{33} \triangleq & \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big)^T \Big(Q^h + Q^v\Big) \\
& \times \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big) \\
& + (\bar{\beta}-\bar{\beta}^2)\Big(\bar{B}^s_{1f\acute{j}}\Big)^T \Big(Q^h + Q^v\Big)\bar{B}^s_{1f\acute{j}} \\
& + (\bar{\beta}-\bar{\beta}^2)\Big(\widetilde{B}^s_{1f\acute{j}}\Big)^T \Big(Q^h + Q^v\Big)\widetilde{B}^s_{1f\acute{j}} - I, \\
\overline{\Lambda}_{41} \triangleq & \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^s_{2f\grave{j}} - \bar{\beta}\widetilde{B}^s_{2f\grave{j}}\Big)^T \Big(Q^h + Q^v\Big) \\
& \times \Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big), \\
\overline{\Lambda}_{42} \triangleq & \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^s_{2f\grave{j}} - \bar{\beta}\widetilde{B}^s_{2f\grave{j}}\Big)^T \Big(Q^h + Q^v\Big) \\
& \times \Big(\bar{A}^s_{2\grave{i}\grave{j}} + (\bar{\beta}-1))\bar{C}^s_{2\grave{j}\grave{n}}\Big), \\
\overline{\Lambda}_{43} \triangleq & \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^s_{2f\grave{j}} - \bar{\beta}\widetilde{B}^s_{2f\grave{j}}\Big)^T \Big(Q^h + Q^v\Big) \\
& \times \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big), \\
\overline{\Lambda}_{44} \triangleq & \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^s_{2f\grave{j}} - \bar{\beta}\widetilde{B}^s_{2f\grave{j}}\Big)^T \Big(Q^h + Q^v\Big) \\
& \times \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^s_{2f\grave{j}} - \bar{\beta}\widetilde{B}^s_{2f\grave{j}}\Big) \\
& + (\bar{\beta}-\bar{\beta}^2)\Big(\bar{B}^s_{2f\grave{j}}\Big)^T \Big(Q^h + Q^v\Big)\bar{B}^s_{2f\grave{j}} \\
& + (\bar{\beta}-\bar{\beta}^2)\Big(\widetilde{B}^s_{2f\grave{j}}\Big)^T \Big(Q^h + Q^v\Big)\widetilde{B}^s_{2f\grave{j}} - I.
\end{aligned}$$

Proof: Recalling Assumptions 1-2, it is not difficult to see that

$$\begin{aligned} & \begin{cases} -\nu^T(p,q)\nu(p,q) + \varrho_1^2 > 0, \\ -\xi^T(p,q)\xi(p,q) + \varrho_2^2 > 0, \end{cases} \\ \Longleftrightarrow \quad & -\bar{\xi}^T(p,q)\bar{\xi}(p,q) + \varrho_1 + \varrho_2 > 0. \end{aligned} \tag{27}$$

Similar to the proof of Theorem 1, the difference of $V(p,q)$ along the trajectories of error dynamics (9) with $\bar{\xi}(p,q) \neq 0$ can be calculated as

$$\mathcal{I}(p,q) = \Delta V^h(p,q) + \Delta V^v(p,q), \tag{28}$$

where

$$\begin{aligned}
& \Delta V^h(p,q) \\
\triangleq & \mathbb{E}\big\{(e^s(p+1,q+1))^T Q^h e^s(p+1,q+1) \\
& - (e^s(p,q+1))^T Q^h e^s(p,q+1) \big| \hbar(p,q)\big\} \\
= & \sum_{\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{n}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{n}} \\
& \times \mathbb{E}\Big\{\Big[\Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big)e^s(p,q+1) \\
& + \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big)\bar{\xi}(p,q+1) \\
& + \Big(\bar{A}^s_{2\grave{i}\grave{j}} + (\bar{\beta}-1))\bar{C}^s_{2\grave{j}\grave{n}}\Big)e^s(p+1,q) \\
& + \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^s_{2f\grave{j}} - \bar{\beta}\widetilde{B}^s_{2f\grave{j}}\Big)\bar{\xi}(p+1,q)\Big]^T Q^h \\
& \times \Big[\Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big)e^s(p,q+1) \\
& + \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big)\bar{\xi}(p,q+1) \\
& + \Big(\bar{A}^s_{2\grave{i}\grave{j}} + (\bar{\beta}-1))\bar{C}^s_{2\grave{j}\grave{n}}\Big)e^s(p+1,q) \\
& + \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^s_{2f\grave{j}} - \bar{\beta}\widetilde{B}^s_{2f\grave{j}}\Big)\bar{\xi}(p+1,q)\Big] \\
& + (\bar{\beta}-\bar{\beta}^2)(e^s(p,q+1))^T\Big(\bar{C}^s_{1\acute{j}\acute{n}}\Big)^T Q^h \bar{C}^s_{1\acute{j}\acute{n}} e^s(p,q+1) \\
& + (\bar{\beta}-\bar{\beta}^2)\bar{\xi}^T(p,q+1)\Big(\bar{B}^s_{1f\acute{j}}\Big)^T Q^h \bar{B}^s_{1f\acute{j}}\bar{\xi}(p,q+1) \\
& + (\bar{\beta}-\bar{\beta}^2)\bar{\xi}^T(p,q+1)\Big(\widetilde{B}^s_{1f\acute{j}}\Big)^T Q^h \widetilde{B}^s_{1f\acute{j}}\bar{\xi}(p,q+1) \\
& + (\bar{\beta}-\bar{\beta}^2)(e^s(p+1,q))^T\Big(\bar{C}^s_{2\grave{j}\grave{n}}\Big)^T Q^h \bar{C}^s_{2\grave{j}\grave{n}} e^s(p+1,q) \\
& + (\bar{\beta}-\bar{\beta}^2)\bar{\xi}^T(p+1,q)\Big(\bar{B}^s_{2f\grave{j}}\Big)^T Q^h \bar{B}^s_{2f\grave{j}}\bar{\xi}(p+1,q) \\
& + (\bar{\beta}-\bar{\beta}^2)\bar{\xi}^T(p+1,q)\Big(\widetilde{B}^s_{2f\grave{j}}\Big)^T Q^h \widetilde{B}^s_{2f\grave{j}}\bar{\xi}(p+1,q) \\
& - (e^s(p,q+1))^T Q^h e^s(p,q+1)\Big|\hbar(p,q)\Big\}
\end{aligned}$$

and

$$\begin{aligned}
& \Delta V^v(p,q) \\
\triangleq & \mathbb{E}\big\{(e^s(p+1,q+1))^T Q^v e^s(p+1,q+1) \\
& - (e^s(p+1,q))^T Q^v e^s(p+1,q) \big| \hbar(p,q)\big\} \\
= & \sum_{\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{n}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{n}} \\
& \times \mathbb{E}\Big\{\Big[\Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big)e^s(p,q+1) \\
& + \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big)\bar{\xi}(p,q+1) \\
& + \Big(\bar{A}^s_{2\grave{i}\grave{j}} + (\bar{\beta}-1))\bar{C}^s_{2\grave{j}\grave{n}}\Big)e^s(p+1,q) \\
& + \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^s_{2f\grave{j}} - \bar{\beta}\widetilde{B}^s_{2f\grave{j}}\Big)\bar{\xi}(p+1,q)\Big]^T Q^v \\
& \times \Big[\Big(\bar{A}^s_{1\acute{i}\acute{j}} + (\bar{\beta}-1)\bar{C}^s_{1\acute{j}\acute{n}}\Big)e^s(p,q+1) \\
& + \Big(\bar{B}_{1\acute{i}} + (\bar{\beta}-1)\bar{B}^s_{1f\acute{j}} - \bar{\beta}\widetilde{B}^s_{1f\acute{j}}\Big)\bar{\xi}(p,q+1) \\
& + \Big(\bar{A}^s_{2\grave{i}\grave{j}} + (\bar{\beta}-1))\bar{C}^s_{2\grave{j}\grave{n}}\Big)e^s(p+1,q)
\end{aligned}$$

$$
\begin{aligned}
&+ \Big(\bar{B}_{2\grave{i}} + (\bar{\beta}-1)\bar{B}^{s}_{2f\grave{j}} - \bar{\beta}\widetilde{B}^{s}_{2f\grave{j}}\Big)\bar{\xi}(p+1,q)\Big] \\
&+ (\bar{\beta}-\bar{\beta}^2)(e^s(p,q+1))^T\Big(\bar{C}^{s}_{1\acute{j}\acute{n}}\Big)^T Q^v \bar{C}^{s}_{1\acute{j}\acute{n}} e^s(p,q+1) \\
&+ (\bar{\beta}-\bar{\beta}^2)\bar{\xi}^T(p,q+1)\Big(\bar{B}^{s}_{1f\acute{j}}\Big)^T Q^v \bar{B}^{s}_{1f\acute{j}}\bar{\xi}(p,q+1) \\
&+ (\bar{\beta}-\bar{\beta}^2)\bar{\xi}^T(p,q+1)\Big(\widetilde{B}^{s}_{1f\acute{j}}\Big)^T Q^v \widetilde{B}^{s}_{1f\acute{j}}\bar{\xi}(p,q+1) \\
&+ (\bar{\beta}-\bar{\beta}^2)(e^s(p+1,q))^T\Big(\bar{C}^{s}_{2\grave{j}\grave{n}}\Big)^T Q^v \bar{C}^{s}_{2\grave{j}\grave{n}} e^s(p+1,q) \\
&+ (\bar{\beta}-\bar{\beta}^2)\bar{\xi}^T(p+1,q)\Big(\bar{B}^{s}_{2f\grave{j}}\Big)^T Q^v \bar{B}^{s}_{2f\grave{j}}\bar{\xi}(p+1,q) \\
&+ (\bar{\beta}-\bar{\beta}^2)\bar{\xi}^T(p+1,q)\Big(\widetilde{B}^{s}_{2f\grave{j}}\Big)^T Q^v \widetilde{B}^{s}_{2f\grave{j}}\bar{\xi}(p+1,q) \\
&- (e^s(p,q+1))^T Q^v e^s(p,q+1)\Big|\hbar(p,q)\Big\}.
\end{aligned}
$$

Then, it follows from (27) that

$$
\begin{aligned}
\mathcal{I}(p,q) \leq& \Delta V^h(p,q) + \Delta V^v(p,q) - \bar{\xi}^T(p+1,q)\bar{\xi}(p,q+1) \\
&- \bar{\xi}^T(p+1,q)\bar{\xi}(p+1,q) + \bar{\varrho}^2_{1,2} \\
=& \mathbb{E}\sum_{\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{n}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{n}}\Big(\bar{\eta}(p,q)^T(p,q) \\
&\times \overline{\Lambda}^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}}\bar{\eta}(p,q)(p,q) + \bar{\varrho}^2_{1,2}\Big|\hbar(p,q)\Big),
\end{aligned} \tag{29}
$$

where $\bar{\eta}(p,q) \triangleq \begin{bmatrix}\eta^T(p,q) & \bar{\xi}^T(p,q+1) & \bar{\xi}^T(p+1,q)\end{bmatrix}^T$.

Based on the fact that $\overline{\Lambda}^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}} < 0$, one has

$$
\begin{aligned}
\mathcal{I}(p,q) \leq& \sum_{\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{n}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{n}}\mathbb{E}\Big\{ \\
&- \lambda_{\min}\Big(-\overline{\Lambda}^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}}\Big)\|\eta(p,q)\|^2 + \bar{\varrho}^2_{1,2}\Big\}.
\end{aligned} \tag{30}
$$

Recalling the definition of the Lyapunov-like functional in (19), it is not difficult to verify that

$$
\begin{aligned}
\mathbb{E}\{V(p,q)\} \leq& \mathbb{E}\Big\{\lambda_{\max}\big(Q^h\big)\|e^s(p,q+1)\|^2 \\
&+ \lambda_{\max}\big(Q^v\big)\|e^s(p+1,q)\|^2\Big\} \\
\leq& \max\Big\{\lambda_{\max}\big(Q^h\big), \lambda_{\max}\big(Q^v\big)\Big\}\|\eta(p,q)\|^2
\end{aligned} \tag{31}
$$

and

$$
\begin{aligned}
\mathbb{E}\{V(p,q)\} \geq& \mathbb{E}\Big\{\lambda_{\min}\big(Q^h\big)\|e^s(p,q+1)\|^2 \\
&+ \lambda_{\min}\big(Q^v\big)\|e^s(p+1,q)\|^2\Big\} \\
\geq& \min\Big\{\lambda_{\min}\big(Q^h\big), \lambda_{\min}\big(Q^v\big)\Big\}\|\eta(p,q)\|^2.
\end{aligned} \tag{32}
$$

For any integer $\mathcal{T} \in \mathbb{Z}^+$, summing up both sides of the inequality (30) for p and q varying from 0 to $\mathcal{T}-1$ yields

$$
\begin{aligned}
&\sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\mathbb{E}\{\mathcal{I}(p,q)\} \\
=& \mathbb{E}\Big\{\sum_{q=0}^{\mathcal{T}-1}\Big(V^h(\mathcal{T},q+1) - V^h(0,q+1)\Big) \\
&+ \sum_{p=0}^{\mathcal{T}-1}\Big(V^v(p+1,\mathcal{T}) - V^v(p+1,0)\Big)\Big\} \\
\leq& \mathbb{E}\Big\{\sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\sum_{\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{n}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{n}} \\
&\times\Big[-\lambda_{\min}\Big(-\overline{\Lambda}^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}}\Big)\|\eta(p,q)\|^2 + \bar{\varrho}^2_{1,2}\Big]\Big\},
\end{aligned} \tag{33}
$$

which can be rewritten as

$$
\begin{aligned}
&\mathbb{E}\Big\{\sum_{\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{n}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{n}}\mathbb{E}\Big\{\sum_{q=0}^{\mathcal{T}-1}\Big(V^h(\mathcal{T},q+1) \\
&- V^h(0,q+1)\Big)\Big\} \\
&+ \sum_{p=0}^{\mathcal{T}-1}\Big(V^v(p+1,\mathcal{T}) - V^v(p+1,0)\Big)\Big\} \\
\leq& \mathbb{E}\Big\{\sum_{\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}=1}^{\mathcal{R}} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{n}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{n}}\sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\Big[\\
&- \lambda_{\min}\Big(-\overline{\Lambda}^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}}\Big)\|\eta(p,q)\|^2 + \bar{\varrho}^2_{1,2}\Big]\Big\}
\end{aligned}
$$

or, equivalently,

$$
\begin{aligned}
&\mathbb{E}\Big\{\sum_{q=0}^{\mathcal{T}-1}\Big(V^h(\mathcal{T},q+1) - V^h(0,q+1)\Big) \\
&+ \sum_{p=0}^{\mathcal{T}-1}\Big(V^v(p+1,\mathcal{T}) - V^v(p+1,0)\Big)\Big\} \\
=& \mathbb{E}\Big\{\sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\frac{1}{\mathcal{T}}\Big(e^s(\mathcal{T},q+1)Q^h e^s(\mathcal{T},q+1) \\
&+ e^s(p+1,\mathcal{T})Q^v e^s(p+1,\mathcal{T}) \\
&- e^s(\mathcal{T},0)Q^h e^s(\mathcal{T},0) - e^s(0,\mathcal{T})Q^v e^s(0,\mathcal{T})\Big)\Big\} \\
\leq& \mathbb{E}\Big\{\sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\Big[-\lambda_{\min}\Big(-\overline{\Lambda}^{s,\acute{i},\acute{j},\acute{n},\grave{i},\grave{j},\grave{n}}\Big)\|\eta(p,q)\|^2 \\
&+ \bar{\varrho}^2_{1,2}\Big]\Big\} \\
\leq& \sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\bar{\varrho}^2_{1,2}.
\end{aligned} \tag{34}
$$

From (11), (26b), (31), (32) and (34), we obtain

$$
\begin{aligned}
&\mathbb{E}\Big\{\sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\Big(\|e^s(\mathcal{T},q+1)\|^2 + \|e^s(p+1,\mathcal{T})\|^2\Big)\Big\} \\
\leq& \frac{1}{\min\Big\{\lambda_{\min}\big(Q^h\big), \lambda_{\min}\big(Q^v\big)\Big\}}\mathbb{E}\Big\{\sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\Big(\mathcal{T}\bar{\varrho}^2_{1,2} \\
&+ \max\Big\{\lambda_{\max}\big(Q^h\big), \lambda_{\max}\big(Q^v\big)\Big\} \\
&\times\Big(\|e^s(0,q+1)\|^2 + \|e^s(p+1,0)\|^2\Big)\Big\}
\end{aligned}
$$

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$$\leq \mathbb{E}\Bigg\{ \sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\Bigg(\mathcal{T}\Big(\bar{\varrho}_{1,2}^2+\max\Big\{\lambda_{\max}\big(Q^h\big),\lambda_{\max}\big(Q^v\big)\Big\} \times \varrho_3^2\Big)\Bigg)\Big/\min\Big\{\lambda_{\min}\big(Q^h\big),\lambda_{\min}\big(Q^v\big)\Big\}\Bigg\}$$
$$<\mathbb{E}\Bigg\{\sum_{q=0}^{\mathcal{T}-1}\sum_{p=0}^{\mathcal{T}-1}\mathcal{T}\rho_s^2\Bigg\}, \tag{35}$$

which implies

$$\mathbb{E}\left\{\left\|\begin{matrix} e^s(\mathcal{T},q+1)\\ e^s(p+1,\mathcal{T})\end{matrix}\right\|^2\right\}<\mathcal{T}\rho_s^2. \tag{36}$$

Therefore, according to Definition 2, we can conclude that the error dynamics (15) with $\bar{\xi}(p,q)\neq 0$ is $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-*secure* in the mean-square sense. The proof is now complete. ∎

Up to now, some sufficient conditions have been derived in Theorems 1-2 to guarantee the *globally asymptotic stability* and the $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-*security* of the local error dynamics. Next, we shall deal with the estimation fusion issues of these local state estimators. In this regard, an estimation fusion scheme will be provided in the next corollary. Meanwhile, it should be pointed out that the matrix inequalities (26a) are actually unsolvable due to the product terms of matrix variables. To address such a problem and design the gain parameters, a new theorem (Theorem 3) will be offered in the following subsection.

B. *Fusion of local state estimators and parameter design*

In this paper, the fused state estimate at fusion center is expressed by

$$\hat{x}(p,q)=\sum_{s=1}^{S}\alpha_s\hat{x}^s, \tag{37}$$

where α_s $(0\leq\alpha_s\leq 1)$ are the fusion coefficients satisfying $\sum_{s=1}^{S}\alpha_s=1$. Similar to the definitions of $\tilde{e}^s(p,q)$ and $e^s(p,q)$, we let $\tilde{x}(p,q)\triangleq x(p,q)-\hat{x}(p,q)$ and $e(p,q)\triangleq\Big[x^T(p,q)\ \big(\tilde{x}(p,q)\big)^T\Big]^T$.

Corollary 1. *Let the positive scalars ϱ_1 in (2), ϱ_2 in (8), ϱ_3 in (11), ρ_s in (17) and ρ be given. Based on the sth local 2-D fuzzy state estimator (9) with the globally asymptotical stability and $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-security, the fused state estimation error $e(p,q)$ obtained from the fusion mechanism (37) is $(\varrho_1,\varrho_2,\varrho_3,\rho)$-secure if there exist parameters α_s $(0\leq\alpha_s\leq 1$ and $s\in\mathfrak{S})$ such that the following constraint holds*

$$\sum_{s=1}^{S}\alpha_s{\rho_s}^2<\rho^2. \tag{38}$$

Proof: In light of (37), the augmented error vector $e(p,q)$ can be rewritten as

$$\begin{aligned} e(p,q)&=\begin{bmatrix} x(p,q)\\ x(p,q)-\hat{x}(p,q)\end{bmatrix}\\ &=\begin{bmatrix} x(p,q)\\ x(p,q)-\sum_{s=1}^{S}\alpha_s\hat{x}^s\end{bmatrix}\\ &=\begin{bmatrix} \sum_{s=1}^{S}\alpha_s x(p,q)\\ \sum_{s=1}^{S}\alpha_s x(p,q)-\sum_{s=1}^{S}\alpha_s\hat{x}^s\end{bmatrix}\\ &=\sum_{s=1}^{S}\alpha_s\begin{bmatrix} x(p,q)\\ x(p,q)-\hat{x}^s(p,q)\end{bmatrix}\\ &=\sum_{s=1}^{S}\alpha_s e^s(p,q). \end{aligned} \tag{39}$$

Then, it is easy to obtain that

$$\begin{aligned} &\left\|\begin{matrix} e(\mathcal{T},q+1)\\ e(p+1,\mathcal{T})\end{matrix}\right\|^2\\ =&\left\|\begin{matrix} \sum_{s=1}^{S}\alpha_s e^s(\mathcal{T},q+1)\\ \sum_{s=1}^{S}\alpha_s e^s(p+1,\mathcal{T})\end{matrix}\right\|^2\\ =&\Bigg[\ \Big(\sum_{s=1}^{S}\alpha_s e^s(\mathcal{T},q+1)\Big)^T\quad \Big(\sum_{s=1}^{S}\alpha_s e^s(p+1,\mathcal{T})\Big)^T\ \Bigg]^T\\ &\times\begin{bmatrix} \sum_{s=1}^{S}\alpha_s e^s(\mathcal{T},q+1)\\ \sum_{s=1}^{S}\alpha_s e^s(p+1,\mathcal{T})\end{bmatrix}\\ =&\Big(\sum_{s=1}^{S}\alpha_s e^s(\mathcal{T},q+1)\Big)^T\sum_{s=1}^{S}\alpha_s e^s(\mathcal{T},q+1)\\ &+\Big(\sum_{s=1}^{S}\alpha_s e^s(p+1,\mathcal{T})\Big)^T\sum_{s=1}^{S}\alpha_s e^s(p+1,\mathcal{T}). \end{aligned} \tag{40}$$

By using Lemma 1 again, we have

$$\begin{aligned} \left\|\begin{matrix} e(\mathcal{T},q+1)\\ e(p+1,\mathcal{T})\end{matrix}\right\|^2 \leq&\sum_{s=1}^{S}\alpha_s\Big[\big(e^s(\mathcal{T},q+1)\big)^T e^s(\mathcal{T},q+1)\Big]\\ &+\sum_{s=1}^{S}\alpha_s\Big[\big(e^s(p+1,\mathcal{T})\big)^T e^s(p+1,\mathcal{T})\Big]\\ =&\sum_{s=1}^{S}\alpha_s\Big[\big(e^s(\mathcal{T},q+1)\big)^T e^s(\mathcal{T},q+1)\\ &+\big(e^s(p+1,\mathcal{T})\big)^T e^s(p+1,\mathcal{T})\Big]\\ =&\sum_{s=1}^{S}\alpha_s\left\|\begin{matrix} e^s(\mathcal{T},q+1)\\ e^s(p+1,\mathcal{T})\end{matrix}\right\|^2. \end{aligned} \tag{41}$$

Note that the $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-*security* of the sth local state estimator is guaranteed under the conditions (26a)-(26b) in Theorem 2. Then, it follows from (17) that

$$\left\|\begin{matrix} e(\mathcal{T},q+1)\\ e(p+1,\mathcal{T})\end{matrix}\right\|^2\leq\sum_{s=1}^{S}\alpha_s\mathcal{T}{\rho_s}^2=\mathcal{T}\sum_{s=1}^{S}\alpha_s{\rho_s}^2. \tag{42}$$

Subsequently, it is obtained from condition (38) in Corollary 1 and (42) that

$$\left\|\begin{matrix} e(\mathcal{T},q+1)\\ e(p+1,\mathcal{T})\end{matrix}\right\|^2\leq\mathcal{T}\rho^2, \tag{43}$$

which implies that the fused state estimation error $e(p,q)$ is $(\varrho_1,\varrho_2,\varrho_3,\rho)$-*secure*. The proof is complete. ■

Now, we are in a position to design the gain parameters and the fusion coefficients.

Theorem 3. *Let the positive scalars ϱ_1 in (2), ϱ_2 in (8), ϱ_3 in (11), ρ_s in (17) and ρ, as well as the nonzero slack matrix $\mathcal{S}$ be given. The gain matrices of the local 2-D fuzzy state estimator (9) satisfying globally asymptotical stability and $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-security in the mean-square sense can be readily obtained if there exist matrices Q^h and Q^v, and parameters A^s_{1fi}, A^s_{2fi}, B^s_{1fi}, B^s_{2fi}, and $\tilde{\alpha}_s$ ($0\le\tilde{\alpha}_s\le 1$ and $s\in\mathfrak{S}$) such that the following matrix inequalities hold:*

$$\begin{cases} \Theta^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}<0, & \text{(44a)}\\ \bar{\varrho}^2_{1,2}+\max\Big\{\lambda_{\max}\big(Q^h\big),\lambda_{\max}\big(Q^v\big)\Big\}\varrho_3^2 \\ \qquad -\min\Big\{\lambda_{\min}\big(Q^h\big),\lambda_{\min}\big(Q^v\big)\Big\}\rho_s^2<0, & \text{(44b)}\\ -Q^h<0, & \text{(44c)}\\ -Q^v<0, & \text{(44d)}\\ \begin{bmatrix} -\rho^2 & * & * & * & * \\ \tilde{\alpha}_1\rho_1 & -1 & * & * & * \\ \tilde{\alpha}_2\rho_2 & 0 & -1 & * & * \\ & \cdots & & \ddots & * \\ \tilde{\alpha}_S\rho_S & 0 & 0 & 0 & -1 \end{bmatrix}<0, & \text{(44e)} \end{cases}$$

where $\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}\in\mathcal{I}$, $s\in\mathfrak{S}$ and

$$\Theta^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\triangleq\begin{bmatrix}\Theta_{11}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}} & * \\ \Theta_{21}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}} & \Theta_{22}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\end{bmatrix},$$

$$\Theta_{11}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\triangleq\begin{bmatrix} -Q^h & * & * & * & * & * \\ 0 & -Q^v & * & * & * & * \\ 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & -I & * & * \\ \Theta_{5,1} & \Theta_{5,2} & \Theta_{5,3} & \Theta_{5,4} & \Theta_5 & * \\ \Theta_{6,1} & 0 & 0 & 0 & 0 & \Theta_6 \end{bmatrix},$$

$$\Theta_{21}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\triangleq\begin{bmatrix} 0 & 0 & \Theta_{7,3} & 0 & 0 & 0 \\ 0 & 0 & \Theta_{8,3} & 0 & 0 & 0 \\ 0 & \Theta_{9,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Theta_{10,4} & 0 & 0 \\ 0 & 0 & 0 & \Theta_{11,4} & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
\Theta_{22}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}} &\triangleq\mathrm{diag}\{\Theta_6,\Theta_6,\Theta_6,\Theta_6,\Theta_6\},\ \Theta_{8,3}\triangleq\widetilde{B}^s_{1f\acute{\jmath}},\\
\Theta_{5,1} &\triangleq\bar{A}^s_{1\acute{\imath}\acute{\jmath}}+(\bar{\beta}-1)\bar{C}^s_{1\acute{\jmath}\acute{n}},\ \Theta_{6,1}\triangleq\bar{C}^s_{1\acute{\jmath}\acute{n}},\\
\Theta_{5,2} &\triangleq\bar{A}^s_{2\grave{\imath}\grave{\jmath}}+(\bar{\beta}-1))\bar{C}^s_{2\grave{\jmath}\grave{n}},\ \Theta_{7,3}\triangleq\bar{B}^s_{1f\acute{\jmath}},\\
\Theta_{5,3} &\triangleq\bar{B}_{1\acute{\imath}}+(\bar{\beta}-1)\bar{B}^s_{1f\acute{\jmath}}-\bar{\beta}\widetilde{B}^s_{1f\acute{\jmath}},\\
\Theta_{5,4} &\triangleq\bar{B}_{2\grave{\imath}}+(\bar{\beta}-1)\bar{B}^s_{2f\grave{\jmath}}-\bar{\beta}\widetilde{B}^s_{2f\grave{\jmath}},\\
\Theta_5 &\triangleq\mathcal{S}\big(Q^h+Q^v\big)\mathcal{S}^T-\mathcal{S}^T-\mathcal{S},\\
\Theta_6 &\triangleq\frac{\mathcal{S}\big(Q^h+Q^v\big)\mathcal{S}^T-\mathcal{S}^T-\mathcal{S}}{\bar{\beta}-\bar{\beta}^2},\\
\Theta_{9,2} &\triangleq\bar{C}^s_{2\grave{\jmath}\grave{n}},\ \Theta_{10,4}\triangleq\bar{B}^s_{2f\grave{\jmath}},\ \Theta_{11,4}\triangleq\widetilde{B}^s_{2f\grave{\jmath}}.
\end{aligned}$$

In this case, the $(\varrho_1,\varrho_2,\varrho_3,\rho)$-security of the fused estimation is ensured, and the fusion coefficients α_s in (37) are derived by letting $\alpha_s=\tilde{\alpha}_s^2$.

Proof: With the help of Schur Complement [50], the matrix inequalities $\overline{\Lambda}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}<0$ in Theorem 2 can be rewritten as

$$\overline{\Theta}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}<0, \tag{45}$$

where

$$\overline{\Theta}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\triangleq\begin{bmatrix}\overline{\Theta}_{11}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}} & * \\ \overline{\Theta}_{21}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}} & \overline{\Theta}_{22}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\end{bmatrix},$$

$$\overline{\Theta}_{11}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\triangleq\begin{bmatrix} -Q^h & * & * & * & * & * \\ 0 & -Q^v & * & * & * & * \\ 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & -I & * & * \\ \Theta_{5,1} & \Theta_{5,2} & \Theta_{5,3} & \Theta_{5,4} & \overline{\Theta}_5 & * \\ \Theta_{6,1} & 0 & 0 & 0 & 0 & \overline{\Theta}_6 \end{bmatrix},$$

$$\overline{\Theta}_{21}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}\triangleq\begin{bmatrix} 0 & 0 & \Theta_{7,3} & 0 & 0 & 0 \\ 0 & 0 & \Theta_{8,3} & 0 & 0 & 0 \\ 0 & \Theta_{9,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Theta_{10,4} & 0 & 0 \\ 0 & 0 & 0 & \Theta_{11,4} & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
\overline{\Theta}_{22}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}} &\triangleq\mathrm{diag}\{\overline{\Theta}_6,\overline{\Theta}_6,\overline{\Theta}_6,\overline{\Theta}_6,\overline{\Theta}_6\},\\
\overline{\Theta}_5 &\triangleq-\big(Q^h+Q^v\big)^{-1},\\
\overline{\Theta}_6 &\triangleq-\frac{1}{\bar{\beta}-\bar{\beta}^2}\big(Q^h+Q^v\big)^{-1}.
\end{aligned}$$

Noting that $Q^h>0$ and $Q^v>0$, one has

$$\begin{aligned}
&\big(Q^h+Q^v\big)^{-1}+\mathcal{S}\big(Q^h+Q^v\big)\mathcal{S}^T-\mathcal{S}^T-\mathcal{S}\\
=&\Big[\mathcal{S}-\big(Q^h+Q^v\big)^{-1}\Big]\big(Q^h+Q^v\big)\mathcal{S}^T\\
&-\Big[\mathcal{S}-\big(Q^h+Q^v\big)^{-1}\Big]\\
=&\Big[\mathcal{S}^T-\big(Q^h+Q^v\big)^{-1}\Big]^T\big(Q^h+Q^v\big)\\
&\times\Big[\mathcal{S}^T-\big(Q^h+Q^v\big)^{-1}\Big]\\
>&0\\
\Longleftrightarrow&-\big(Q^h+Q^v\big)^{-1}<\mathcal{S}\big(Q^h+Q^v\big)\mathcal{S}^T-\mathcal{S}^T-\mathcal{S},
\end{aligned} \tag{46}$$

which implies $\overline{\Theta}^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}<\Theta^{s,\acute{\imath},\acute{\jmath},\acute{n},\grave{\imath},\grave{\jmath},\grave{n}}$. Then, the condition (26a) in Theorem 2 is ensured by (44a). Meanwhile, it is easy to see that the scalar inequality (26b) in Theorem 2 is equivalent to (44b). The positivity of matrices Q^h and Q^v in Theorem 2 is satisfied according to the constraints (44c) and (44d). In other words, the conditions in Theorem 2 are all guaranteed by (44a)-(44d). As such, the globally asymptotical stability and the $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-*security* of the sth local 2-D fuzzy error dynamics (15) are simultaneously ensured.

On the other hand, by noting that $\alpha_s=\tilde{\alpha}_s^2$, it can be easily checked from (44e) and Schur Complement [50] that

$$\sum_{s=1}^{S}\tilde{\alpha}_s^2{\rho_s}^2=\sum_{s=1}^{S}\alpha_s{\rho_s}^2<\rho^2, \tag{47}$$

which implies that the $(\varrho_1,\varrho_2,\varrho_3,\rho)$-*security* of the fused estimation is guaranteed. The proof is now complete. ■

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Remark 2. *It is worth mentioning that the product terms of matrix variables have been avoided in Theorem 3 by utilizing the slack matrix technique. In this paper, the slack matrix is selected prior to solving the linear matrix inequalities (44a)-(44d). The main advantages of such a manipulation are twofold. Firstly, it is not necessary to choose a special structure for the slack matrix $\mathcal{S}$ since there is no product term composed of $\mathcal{S}$ and the gain matrices (e.g. $S\bar{A}^s_{1\acute{i}\acute{j}}$, $S\bar{B}^s_{1f\acute{j}}$, and $Sz\bar{C}^s_{1\acute{j}\acute{n}}$). Secondly, the gain matrices of the sth local fuzzy estimator can be directly derived without any extra manipulations such as the contragradient transformation and the reversible transformation. In this sense, the known slack matrix can facilitate the gain design of the desired 2-D fuzzy estimator. More specifically, the $\mathcal{S}$ is required to be nonzero in Theorem 3, and its simple structure (e.g. diagonal form) could further reduce the computational burden. In other words, the matrix $\mathcal{S}$ should be chosen to be nonzero, structurally simple and practically meaningful. It is seen from Theorem 2 that the security of the sth 2-D fuzzy error dynamics (15) is heavily dependent on the amplitudes of both $\nu(p,q)$ and $\xi(p,q)$, which means that an excessive large intensity of the disturbance/deception signals (e.g. $\|\nu(p,q)\| > \varrho_1$ or $\|\xi(p,q)\| > \varrho_2$) might result in the undesired insecurity for the estimation error system.*

Remark 3. *In this paper, we have dealt with the security-guaranteed state estimation problem for a class of 2-D fuzzy NSs with multiple sensor arrays and deception attacks. Compared to the existing results, our main results exhibit the following distinct characteristics: 1) a novel security-guaranteed state estimation problem is investigated where an estimation fusion scheme is developed based on a set of 2-D fuzzy local state estimators; and 2) the information about the globally asymptotic stability, the $(\varrho_1,\varrho_2,\varrho_3,\rho_s)$-security of the local error dynamics, and $(\varrho_1,\varrho_2,\varrho_3,\rho)$-security of the fused counterpart are all reflected in Theorems 1-3.*

IV. Illustrative Example

Consider a 2-D T-S fuzzy system in the form of (1) with the number of **IF-THEN** rules being $\mathcal{R} = 2$. The detailed parameters are given as follows:

$$A_{11} = \begin{bmatrix} -0.52 & 0.02 \\ 0.02 & -0.51 \end{bmatrix},\ A_{12} = \begin{bmatrix} -0.43 & 0.11 \\ 0.08 & -0.54 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -0.33 & 0.05 \\ 0.03 & -0.32 \end{bmatrix},\ A_{22} = \begin{bmatrix} -0.30 & 0.04 \\ 0.02 & -0.41 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} -0.25 & 0.05 \\ 0.07 & -0.53 \end{bmatrix},\ B_{12} = \begin{bmatrix} -0.23 & 0.12 \\ 0.01 & -0.62 \end{bmatrix},$$
$$B_{21} = \begin{bmatrix} -0.34 & 0.14 \\ 0.02 & -0.32 \end{bmatrix},\ B_{22} = \begin{bmatrix} -0.45 & 0.01 \\ 0.01 & -0.52 \end{bmatrix}.$$

The number of multiple sensor arrays is taken as $S = 3$, and the coefficient matrices in the measurement model (5) are given by

$$C_{11} = \begin{bmatrix} -0.26 & 0.06 \\ 0.02 & -0.41 \end{bmatrix},\ C_{12} = \begin{bmatrix} -0.18 & 0.01 \\ -0.03 & -0.26 \end{bmatrix},$$
$$C_{13} = \begin{bmatrix} -0.39 & 0.02 \\ 0.01 & -0.33 \end{bmatrix},\ C_{21} = \begin{bmatrix} -0.30 & 0.02 \\ 0.03 & -0.62 \end{bmatrix},$$
$$C_{22} = \begin{bmatrix} -0.15 & 0.01 \\ 0.02 & -0.41 \end{bmatrix},\ C_{23} = \begin{bmatrix} -0.27 & 0.03 \\ 0.01 & -0.30 \end{bmatrix},$$
$$D_{11} = \begin{bmatrix} -0.42 & 0.02 \\ -0.04 & -0.56 \end{bmatrix},\ D_{12} = \begin{bmatrix} -0.13 & 0.01 \\ -0.02 & -0.24 \end{bmatrix},$$
$$D_{13} = \begin{bmatrix} -0.21 & 0.02 \\ 0.01 & -0.12 \end{bmatrix},\ D_{21} = \begin{bmatrix} -0.32 & 0.03 \\ -0.12 & -0.21 \end{bmatrix},$$
$$D_{22} = \begin{bmatrix} -0.21 & 0 \\ 0.01 & -0.32 \end{bmatrix},\ D_{23} = \begin{bmatrix} -0.43 & 0.01 \\ 0 & -0.21 \end{bmatrix}.$$

The success probability of the deception attacks is assumed to be $\bar{\beta} = 0.32$. The constants in conditions (2), (8), (11) and (38) are, respectively, set as $\rho_1 = 1.2$, $\rho_2 = 0.7$, $\rho_3 = 1.1$ and $\rho = 2$.

Let us define $\mathcal{S} = \text{diag}\{\mathcal{S}_{11},\ \mathcal{S}_{22}\}$, where

$$\mathcal{S}_{11} = \begin{bmatrix} -0.0008 & 0 \\ 0 & -0.0009 \end{bmatrix},\ \mathcal{S}_{22} = \begin{bmatrix} -0.0006 & 0 \\ 0 & -0.0007 \end{bmatrix}.$$

By solving the matrix constraints (44a)-(44e) in Theorem 3 with the help of control toolbox of MATLAB software, we can obtain

$$Q_{h11} = \begin{bmatrix} 36.19 & -8.95 \\ -8.95 & 59.95 \end{bmatrix},\ Q_{h21} = \begin{bmatrix} 6.56 & -7.96 \\ -7.96 & 27.90 \end{bmatrix},$$
$$Q_{h22} = \begin{bmatrix} 35.72 & -8.79 \\ -8.79 & 59.20 \end{bmatrix},\ Q_{v11} = \begin{bmatrix} 33.14 & -1.81 \\ -1.81 & 48.57 \end{bmatrix},$$
$$Q_{v21} = \begin{bmatrix} 4.05 & -1.46 \\ -1.46 & 16.95 \end{bmatrix},\ Q_{v22} = \begin{bmatrix} 32.93 & -1.72 \\ -1.72 & 48.18 \end{bmatrix}.$$

Meanwhile, the gain matrices of three local state estimators are readily obtained as follows:

$$A^1_{1f1} = \begin{bmatrix} -0.27 & 0.04 \\ 0.04 & -0.35 \end{bmatrix},\ A^2_{1f1} = \begin{bmatrix} -0.28 & 0.04 \\ 0.04 & -0.35 \end{bmatrix},$$
$$A^3_{1f1} = \begin{bmatrix} -0.29 & 0.05 \\ 0.02 & -0.41 \end{bmatrix},\ A^1_{1f2} = \begin{bmatrix} -0.25 & 0.07 \\ 0.06 & -0.35 \end{bmatrix},$$
$$A^2_{1f2} = \begin{bmatrix} -0.26 & 0.10 \\ 0.14 & -0.56 \end{bmatrix},\ A^3_{1f2} = \begin{bmatrix} -0.26 & 0.08 \\ 0.06 & -0.40 \end{bmatrix},$$
$$A^1_{2f1} = \begin{bmatrix} -0.14 & 0.04 \\ 0.02 & -0.24 \end{bmatrix},\ A^2_{2f1} = \begin{bmatrix} -0.15 & 0.03 \\ 0.03 & -0.24 \end{bmatrix},$$
$$A^3_{2f1} = \begin{bmatrix} -0.15 & 0.03 \\ 0.02 & -0.28 \end{bmatrix},\ A^1_{2f2} = \begin{bmatrix} -0.15 & 0.04 \\ 0.02 & -0.19 \end{bmatrix},$$
$$A^2_{2f2} = \begin{bmatrix} -0.17 & 0.06 \\ 0.01 & -0.37 \end{bmatrix},\ A^3_{2f2} = \begin{bmatrix} -0.16 & 0.04 \\ 0.03 & -0.20 \end{bmatrix},$$
$$B^1_{1f1} = \begin{bmatrix} -0.38 & 0.03 \\ 0.15 & -0.47 \end{bmatrix},\ B^2_{1f1} = \begin{bmatrix} -0.46 & 0.13 \\ 0.26 & -0.77 \end{bmatrix},$$
$$B^3_{1f1} = \begin{bmatrix} -0.20 & 0.06 \\ 0.20 & -0.51 \end{bmatrix},\ B^1_{1f2} = \begin{bmatrix} -0.32 & 0.07 \\ 0.12 & -0.41 \end{bmatrix},$$
$$B^2_{1f2} = \begin{bmatrix} -0.54 & 0.13 \\ -0.01 & -0.85 \end{bmatrix},\ B^3_{1f2} = \begin{bmatrix} -0.27 & 0.09 \\ 0.17 & -0.45 \end{bmatrix},$$
$$B^1_{2f1} = \begin{bmatrix} -0.50 & 0.01 \\ 0.17 & -0.34 \end{bmatrix},\ B^2_{2f1} = \begin{bmatrix} -0.77 & 0.10 \\ 0.21 & -0.62 \end{bmatrix},$$
$$B^3_{2f1} = \begin{bmatrix} -0.38 & 0.06 \\ 0.18 & -0.66 \end{bmatrix},\ B^1_{2f2} = \begin{bmatrix} -0.43 & 0.01 \\ 0.11 & -0.19 \end{bmatrix},$$
$$B^2_{2f2} = \begin{bmatrix} -0.58 & 0.15 \\ 0.81 & -0.97 \end{bmatrix},\ B^3_{2f2} = \begin{bmatrix} -0.32 & 0.09 \\ 0.10 & -0.37 \end{bmatrix}.$$

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The fusion coefficients are acquired as $\alpha_1 = 0.23$, $\alpha_2 = 0.44$ and $\alpha_3 = 0.33$, and the condition (38) is thus satisfied with $\sum_{s=1}^{S} \alpha_s {\rho_s}^2 = 0.9461 < \rho^2 = 4$.

To simulate the dynamic process of the fuzzy state estimators, the normalized membership functions are chosen as

$$h_1(p,q) = \frac{\sin(x_1(p,q)) + pq}{2 + \cos(x_1(p,q)) + pq}, \quad h_2(p,q) = 1 - h_1(p,q).$$

The disturbance input and deception signal are, respectively, selected as $\nu(p,q) = [\text{arccot}(\frac{1}{pq}) \quad \text{arccot}(\frac{1}{p+q})]^T$ and $\xi(p,q) = \begin{bmatrix} 0.23 & 0.56 \end{bmatrix}^T$. In addition, we have $\psi^h(p,q) = 30n(p,q) \begin{bmatrix} \text{arccot}(pq) & \cos(pq) \end{bmatrix}^T$ and $\psi^v(p,q) = 30n(p,q) \begin{bmatrix} \text{arccot}(pq) & \cos(p+q) \end{bmatrix}^T$, where $n(p,q)$ is a normally distributed stochastic variable with mean zero and variance $\sigma^2 = 1$. $\psi^h(0,0) = \psi^v(0,0) = 0$. Moreover, we set $b_h = 45$ and $b_v = 50$. It can be verified that the conditions (2), (8) and (11) are met under the aforementioned disturbance input, deception attack and boundary conditions.

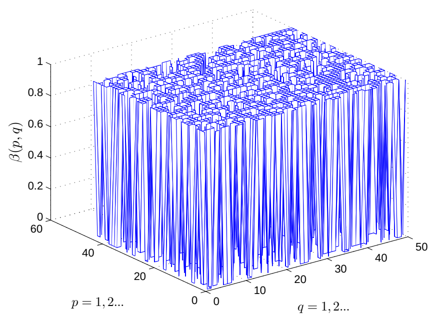

Fig. 2: The binary white sequence $\beta(p,q)$.

The value of the random binary white sequence $\beta(p,q)$ is shown in Fig. 2, where $\beta(p,q) = 1$ means that the deception attack is successful. The error trajectory of the fused estimation is depicted in Fig. 3, which also implies the $(\varrho_1, \varrho_2, \varrho_3, \rho)$-*security* of the 2-D fuzzy error dynamics (15). To demonstrate the advantages of the proposed estimation fusion scheme, the fused estimation error in case of nonconvergent error of the 3rd local estimator (Fig. 4) is shown in Fig. 5, which clearly illustrates that the fusion approach can still function even though one of the local estimators lost efficacy. In order to analyze the impact of the intensity of the network attack on the system performance, the fused estimation results in different probabilities of occurrence of $\beta(p,q)$ (i.e. $\bar{\beta} = 0.12$ and $\bar{\beta} = 0.92$) are revealed in Fig. 6, where the index p is fixed to 40 for sake of examination convenience. Fig. 7 shows the actual occurrence of $\beta(p,q)$ when $p = 40$. Figs. 6-7 visibly indicate that the fused estimation error is inferior to the case of higher $\bar{\beta}$. To implement a comparison with the case of no multiple sensor arrays, the matrices C_1^1, C_1^3, C_2^1, C_2^3, D_1^1, D_1^3, D_2^1 and D_2^3 are all set to be zero, which means only a single sensor array is used during the measurement procedure. Fig. 8 illustrates the state trajectories of the fuzzy estimation error under such case, from which we see a single sensor measurement may induce a nonconvergent estimation

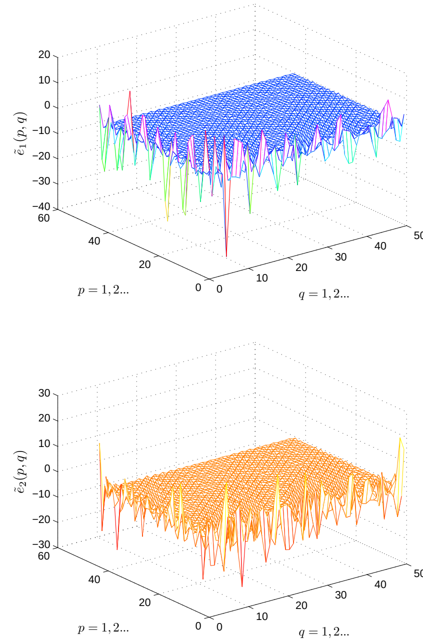

Fig. 3: Estimation error under the proposed fusion scheme.

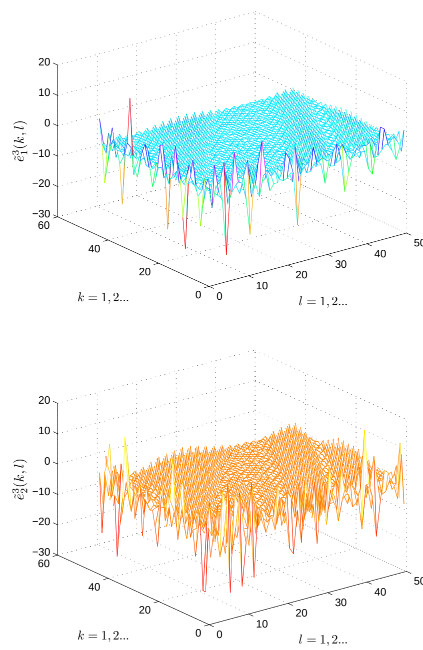

Fig. 4: Nonconvergent estimation error of local fuzzy estimator (s=3).

error. Performance improvement offered by the technique of multiple sensor arrays is confirmed by the result comparison on Fig. 3 and Fig. 8. Moreover, the emanative state plotted in Fig. 9 is the estimation error by overlooking deception attacks and fuzzy rules, and the comparison effects substantiate the studied fuzzy estimation solution.

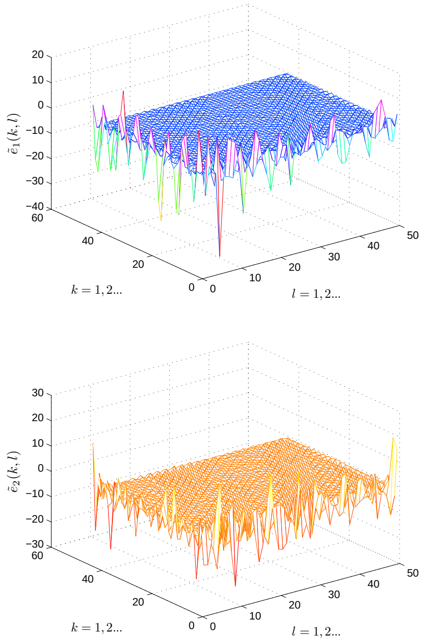

Fig. 5: Estimation error under the proposed fusion scheme in case of nonconvergent error of the 3rd local estimator.

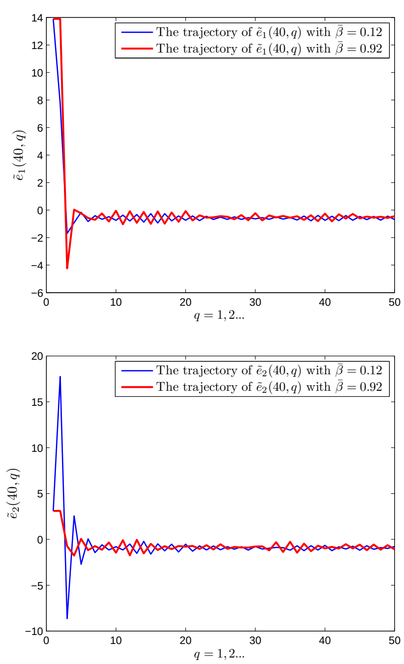

Fig. 6: Fused Estimation error with different $\bar{\beta}$.

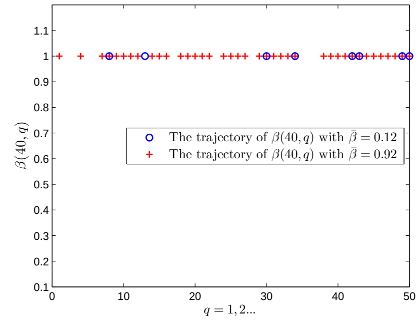

Fig. 7: The binary white sequence $\beta(p,q)$ with different $\bar{\beta}$.

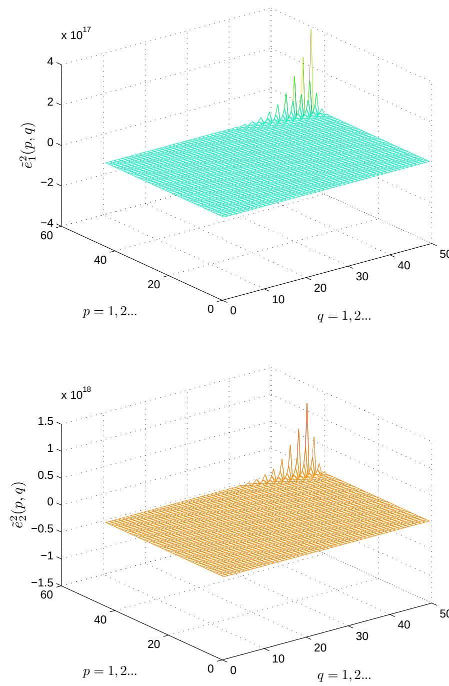

Fig. 8: Estimation error under a single sensor.

V. Conclusions

In this paper, the security-guaranteed fuzzy state estimation problem has been tackled for a class of 2-D NSs with multiple sensor arrays and deception attacks. In order to enhance the observation diversity and overcome the measurement obstacles, multiple sensor arrays have been employed in this paper. A Bernoulli distributed white sequence with known statistical property has been introduced to characterize the deception attacks launched by malicious attackers. Accordingly, a set of local 2-D fuzzy state estimators has been constructed, and some sufficient conditions have been derived to guarantee the *globally asymptotical stability* and $(\varrho_1, \varrho_2, \varrho_3, \rho_s)$-*security* of the local error dynamics in the mean-square sense. Moreover, the estimation fusion problem has also been considered for the developed local fuzzy estimators, and a sufficient condition has been established to ensure the $(\varrho_1, \varrho_2, \varrho_3, \rho)$-*security* of the fused estimation system. Finally, the proposed 2-D fuzzy state estimation scheme has been validated via a numerical example.

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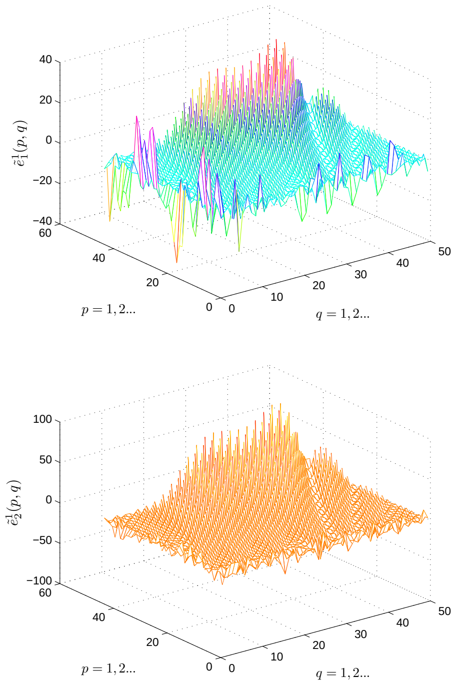

Fig. 9: Estimation error in case of ignoring network attacks and fuzzy rules.

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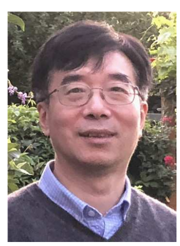

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