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A Novel Dynamic Multiobjective Optimization Algorithm with Hierarchical Response System

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***Abstract*—In this paper, a novel dynamic multi-objective optimization algorithm (DMOA) is proposed based on a designed hierarchical response system (HRS). Named as HRS-DMOA, the proposed algorithm mainly aims at integrating merits from the mainstream ideas of dynamic behavior handling (i.e., the diversity-, memory-, and prediction-based methods) so as to make flexible responses to environmental changes. In particular, by two pre-defined thresholds, the environmental changes are quantified as three levels. In case of a slight environmental change, the previous Pareto set-based refinement strategy is recommended, while the diversity-based re-initialization method is applied in case of a dramatic environmental change. For changes occurring in a medium level, the transfer-learning-based response is adopted to make full use of the historical searching experiences. The proposed HRS-DMOA is comprehensively evaluated on a series of benchmark functions, and the results show an improved comprehensive performance as compared with four popular baseline DMOAs in terms of both convergence and diversity, which also outperforms other two state-of-the-art DMOAs in 10 out of 14 testing cases, exhibiting the competitiveness and superiority of the algorithm. Finally, extensive ablation studies are carried out, and from the results, it is found that as compared with randomly selecting the response methods, the proposed HRS enables more reasonable and efficient responses in most cases. In addition, the generalization ability of the proposed HRS as a flexible plug-and-play module to handle dynamic behaviors is proven as well.**

***Index terms*—** Dynamic multiobjective optimization algorithm (DMOA); transfer learning (TL); hierarchical response system; evolutionary algorithm

I. Introduction

Dynamic multiobjective optimization problems (DMOPs) composed of conflicting objective functions are inevitably encountered in many real-world scenes [9], [14], [62], [66], [67], where both the objectives and constraints may change with time [5], [12], and this has attracted wide research attention to the design of effective dynamic multiobjective optimization algorithms (DMOAs) [6], [15], [20], [32], [34], [44]. The population-based evolutionary algorithms have been proven to be effective under various optimization scenarios in searching for the optimal solutions [2], [4], [7], [38], [40], [54], [56]. Particularly, owing to the wide existence of dynamic behaviors, the DMOAs are required to timely update the obtained Pareto solutions to ensure the convergence in each environment. In order to track the time-varying Pareto front, it is of vital significance for the DMOAs to make effective responses to the environmental changes, which is quite a challenging issue.

To address above basic and important issue in handling DMOPs, the existing DMOAs can be generally divided into the memory- [45], [59], prediction- [42], [48]–[50], [57], [70], and diversity-based methods [12], [35], [36]. Additionally, some hybrid algorithms have also been proposed [8], [21], [31], [53], [69], and in particular, a novel trend of developing DMOAs has emerged recently, which combines the memory mechanism and the prediction method, where the transfer learning (TL) technique has been adopted to make full use of history knowledge to accelerate convergence in a new environment, see [25], [26], [33], [39], [55], [60] for some successful applications. For a clear inspection of above mainstream algorithms, their main ideas are summarized in Table I (see Section II-B for more discussions of their advantages and disadvantages).

TABLE I: Overview of the mainstream DMOAs

Mainstream algorithms	Main idea
Diversity-based	Enrich or maintain diversity of the population for sufficient search
Memory-based	Recall the useful historical Pareto solutions in the new environment
Prediction-based	Predict the varying Pareto set in advance to accelerate convergence
Hybrid methods	Apply several different strategies to collaboratively handle changes

It should be highlighted that all of the existing mainstream DMOAs have already been proven effective in many cases, which are popular and reliable in coping with the environmental changes. In [35], a novel coevolutionary multi-swarm particle swarm optimizer has been proposed to solve DMOPs, where once a change is detected, 20% of the swarm is generated randomly to enhance the population diversity, and the experimental results have shown that the proposed algorithm performs well in the rapidly changing environment. In [45], an explicit memory has been adopted to store non-dominated solutions, where a novel minimum distance search-based updating technique is used. When there is an environmental change, the stored solutions are reused in later stages for population initialization, and from the results, the proposed algorithm has shown its competitiveness in tracking the true Pareto front. An individual-based self-learning prediction method has been

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proposed in [42], whose major innovation is the employment of adjustable reference points, which can effectively alleviate the situation of inaccurate prediction caused by the non-uniform Pareto front. From the experiments, it is found that the proposed algorithm can well balance the convergence and diversity. Similarly, an adaptive reference vector-based adjustment strategy has been introduced in [70] along with a linear prediction strategy, whose effectiveness is demonstrated on twelve functions with diverse dynamic characteristics.

Aiming at recovering diversity in a short time, the authors in [21] have devised a subspace-based diversity maintenance strategy, which can identify gaps between population distribution to maintain the diversity regardless of the environmental changes. Moreover, another layered prediction strategy has been proposed in [21], which benefits making prompt responses and improving the accuracy of predicted evolutionary direction. Similarly, a hybrid DMOA of prediction- and diversity-based strategy has been developed in [69], where it is regarded that too much reliance on the prediction method may reduce the diversity. Therefore, the authors have employed several mechanisms to generate different seed sets, which obtains satisfactory results on series of benchmark evaluations, and the proposed algorithm is also promising to handle the unpredictable DMOPs. Additionally, the memory- and prediction-based strategy have been combined in [31], where the similarity among the environmental changes is compared. If the current detected change is dissimilar to any historical records, then individuals in the new environment are predicted based on the differential population center in the previous two environments. Otherwise, memory mechanism is adopted to deal with the similar changes, and the experimental results have shown the robustness of the proposed hybrid strategy.

Recently, more and more advanced DMOAs have been proposed. To cope with the irregular patterns in stochastic changes, a Mahalanobis distance-based approach has been developed in [22] to estimate the correlation between the current environment and the previous ones, which performs significantly better than some latest algorithms when handling the stochastic DMOPs. In [61], a knowledge-guided Bayesian (KGB) classification method has been proposed to make robust prediction, which follows the same idea of TL-based DOMAs to sufficiently exploit the history information, and the effectiveness of the proposed KGB is demonstrated on different test suites.

In [60], the authors have developed a clustering difference-based TL method to solve DMOPs, where the k-means algorithm is applied to divide the population into five clusters, whose centroid is then adopted to construct the target domain by the first-order difference. By increasing the similarity between source and target domains, the phenomenon of negative transfer is expected to be alleviated. For the same purpose, a knowledge reconstruction method has been proposed in [17], where the fuzzy neural network is applied to extract domain knowledge from two successive Pareto sets, which is evaluated and screened via a pre-designed mechanism. Consequently, the suitable knowledge can be selected to guide the evolution, and the proposed algorithm has presented both better convergence and diversity performance in comparison to some other algorithms.

A noticeable issue is that the most existing methods have paid a great many of attention to designing effective response strategies, while the in-depth analysis is relatively limited on the dynamic behaviors (e.g., the changing severity), whose precise characterization is conducive to tracking the varying Pareto front [18], [46]. Even in those emerging hybrid algorithms that have applied different novel response methods, few attention has been paid to the quantification of the environmental changes, which motivates us to cover this research gap. Additionally, in case of dynamic behaviors, it would be tough to figure out the useful historical experiences in the absence of further analysis about response-making; in cases of some slight changes, introducing the new individuals seems unnecessary and moreover, the prediction-based responses might lead the evolution to an inappropriate direction when dramatic changes exist in the environment. In this regard, the DMOAs should be sensitive to the environmental changes to enable both timely and appropriate responses. Accounting for the respective merits of the previously mentioned methods, a promising way to endow the DMOAs with comprehensive performances is to make efficient and rational integration of diverse response mechanisms.

Motivated by above discussions, in this study, we aim to design a novel DMOA capable of quantitatively measuring the environmental changes so as to take appropriate response strategies accordingly. To be specific, in case of a slight environmental change, the previous Pareto set-based refinement strategy is recommended to save unnecessary computations and track the almost unchanged Pareto front. On the contrary, in case of a dramatic change, an intuitive idea (of introducing individuals in the new environment) is adopted due to the fact that searching for Pareto solutions in a totally different environment can be directly regarded as solving a new problem. In addition to above two extreme cases, the TL-based response strategy is applied to accelerate the convergence, whose main idea is to transfer useful history knowledge to provide a high-quality initial population in the new environment. It is noticeable that different from those hybrid methods where several types of advanced strategies have been designed, the main innovation of this study is to rationally adopt appropriate responses based on the quantification of the changes, which is supposed to encourage and promote more in-depth investigations of the dynamic behaviors in DMOPs.

In this paper, a novel dynamic multi-objective optimization algorithm is proposed based on a hierarchical response system (HRS), where the environmental changes are quantified and divided into three levels. The proposed algorithm (named as the HRS-DMOA) is, in essence, a two-stage algorithm where 1) half of the non-dominated solutions in the previous environment are firstly selected as the sensors to quantify the dynamic behaviors; and accordingly, based on two predefined thresholds, 2) three response strategies can be later adopted to generate the initial population in the new environment to accelerate the convergence. In particular, the diversity-based strategy is adopted in case of severe changes, and when the changes are in a medium level, the novel frontier of TL-based response method is considered, which combines the popular ideas of memory and prediction in solving DMOPs. It should be pointed out that the flexible response strategies can make the proposed HRS-DMOA adaptive to different situations, which enables the responses to have little blindness so as to efficiently handle the dynamic behaviors.

The major contributions of this paper are listed as follows.

1. The dynamic behaviors in the DMOPs are characterized both qualitatively and quantitatively in the proposed HRS-DMOA.

2. Advantages complementation of the mainstream response methods in existing DMOAs is realized in the developed hierarchical response system.

3. The established flexible response modes can endow the proposed algorithm with adaptivity (to diverse situations) and strong generalization ability.

4. The proposed HRS is proven a reliable and effective plug-and-play module to solve DMOPs, which can be integrated with different static optimizer.

The remainder of this paper is organized as follows. Some preliminaries of this work are provided in Section II. The proposed HRS-DMOA is elaborated in Section III, experimental results and discussions are presented in Section IV. Finally, conclusions are drawn in Section V.

II. Preliminaries

In this section, some preliminaries related to this study are provided. To make this paper easy to follow, annotations of the frequently used symbols are presented in Table II.

TABLE II: Major symbols and corresponding annotations

Symbols	Annotations
$\boldsymbol{x}, \boldsymbol{z}$	Decision variable
$\boldsymbol{F}$	Objective function set
f_i	The i-th objective
$g(\cdot)$	Inequality constraints
$h(\cdot)$	Equality constraints
m	The number of objective functions
n	The dimension of decision space
t	Time variable
n_t	Change severity
τ_t	Change frequency
τ	The maximum generation
PS_t	Pareto set at time t
PF_t	Pareto front at time t
CD	Change degree
LT	Lower-threshold of CD
HT	Higher-threshold of CD
D_s	Source domain
D_t	Target domain
$Q(\cdot)$	Quality factor
P_{ini}	Initialized population
N	Population size

A. Formulation of DMOP

Without loss of generality, a minimized DMOP is defined as:

$$\begin{cases} \min\ \boldsymbol{F}(\boldsymbol{x},t)=\{f_1(\boldsymbol{x},t),\ f_2(\boldsymbol{x},t),\ \ldots,\ f_m(\boldsymbol{x},t)\} \\ \qquad s.t.\ g(\boldsymbol{x},t)\le 0,\ h(\boldsymbol{x},t)=0 \end{cases} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the decision vector, $\boldsymbol{F}(\cdot)$ is a set of m objective functions, and $g(\cdot)$ and $h(\cdot)$ are inequality and equality constraints, respectively. t is the time variable that can occur in both objectives and constraints. Based on the above formulation, some definitions are provided as follows.

Definition 1: Dynamic Pareto domination.

At time t, the decision vector $\boldsymbol{x}_1$ dominates $\boldsymbol{x}_2$ (denoted as $\boldsymbol{x}_1 \prec_t \boldsymbol{x}_2$) only in conditions of:

$$\begin{cases} \forall\ i \in \{1,\ 2,\ \ldots,\ m\},\ f_i(\boldsymbol{x}_1,t) \le f_i(\boldsymbol{x}_2,t) \\ \exists\ j \in \{1,\ 2,\ \ldots,\ m\},\ f_j(\boldsymbol{x}_1,t) < f_j(\boldsymbol{x}_2,t) \end{cases} \tag{2}$$

Definition 2: Dynamic Pareto set (PS).

Pareto set at time t (denoted as PS_t) is composed of current Pareto solutions $\boldsymbol{x}$ that cannot be dominated by any other decision vector $\boldsymbol{x}' \in \mathbb{R}^n$, which is defined as

$$PS_t = \{\boldsymbol{x}^* | \neg\exists\ \boldsymbol{x} \in \mathbb{R}^n,\ \boldsymbol{x} \prec_t \boldsymbol{x}^*\} \tag{3}$$

Definition 3: Dynamic Pareto front (PF).

Pareto front at time t (denoted as PF_t) can be obtained via mapping PS_t to the objective space:

$$PF_t = \{\boldsymbol{F}(\boldsymbol{x},t) | \boldsymbol{x} \in PS_t\} \tag{4}$$

B. Responses to Dynamic Behaviors

The existence of the dynamic behaviors makes it difficult to handle DMOPs, and thus has aroused great research interests on how to make effective responses to the environmental changes. When the population enters a new environment, diversity enhancement is a natural response manner. In [12], population initialization in the new environment has been realized by random generation and Gaussian mutation. As a result, partial previous individuals in the population are replaced by the newly generated ones so that the diversity is increased. Another intuitive response strategy is to directly adopt the obtained Pareto solutions in the previous environment (as the initial population) to search for the updated Pareto solution in the new environment such that population state is maintained and no extra computations are required for subsequent evolvement. It is worth mentioning that the above methods are somehow blind due to their common procedure of simply re-starting the search without analysis on the environmental changes, which may lead to poor convergence. Though, in some situations with dramatic environmental changes, the large diversity in the population is able to facilitate a thorough search in the new environment.

The memory-based methods aim to store and recall the historical searching experiences to form the initial population in the new environment [59], and thus are feasible to solve those periodically changing DMOPs. In dynamic situations without the periodic changes, it is hard to realize rapid but efficient

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responses with the stored solutions in the memory, which motivates the development of the prediction-based methods due to correlations between consecutive environments may benefit further exploration on the stored solutions. For example, in [72] and [68], the knee- and center-points of the population have been employed to predict the trace of PS, respectively. In the predictive strategy proposed in [28], different types of special points have been sufficiently utilized with three mechanisms. An ensemble learning-based prediction strategy has been proposed in [52], which aims at overcoming the shortcomings of inaccurate and unstable prediction. Similarly, several sub-prediction models have been integrated in [16], and the proposed ensemble method is applied to estimate the fitness of individuals in the new environment, which has presented better robustness than the single prediction model. Above methods have concentrated on developing various novel prediction strategies, which can accelerate the convergence in the new environment and accordingly save some searching efforts, while as pointed out in some work, the prediction model may also suffer from the inaccurate tracking of PS and the extra computational costs [16], [52], especially when the change is either too severe or slight.

Considering that those historical experiences may still have some reference values to search in the new environment, the TL technique has paved a feasible path in solving the DMOPs by combining the memory- and prediction-based strategy, whose idea is to transfer the useful knowledge to assist learning in similar tasks. Under the assumption that there may be inherent associations among individuals in successive environments, developing TL-based DMOAs is an emerging and promising direction [51], where an important concern is to alleviate the negative transfer phenomenon, which may lead the evolution towards wrong directions.

Based on above discussions, it is clear that each kind of the mainstream strategy in solving DMOPs has the own advantages and disadvantages without a certain one always being the best choice. More importantly, it seems feasible and promising to realize the complementary performance by quantifying the environmental changes. As previously mentioned, if the environment is dramatically changed, it might be better to re-initialize the population to restart searching than to figure out the useful history solutions to accomplish the initialization. On the contrary, in case of a very slight change, it might be unnecessary to apply any advanced prediction models. Consequently, in this paper, we aim to realize an efficient and rational integration of different response strategies based on the quantification of environmental changes, which provides a novel idea to develop a competitive DMOA with comprehensive performance.

III. Methodology

In this section, the proposed HRS-DMOA is elaborated, whose main idea is to grade the environmental changes via some quantification procedure so that hierarchical responses can be adopted accordingly.

A. Environmental Change Quantification

As previously mentioned, one of the most important issues in handling DMOPs is to make effective responses to the dynamic behaviors therein, of which the premise is to detect the environmental changes. The popular change detection manner contains the population- and sensor-based methods [1], [44], and in this study, we hold the belief that it is important to not only detect but also quantify the environmental changes, which is conducive to adopting appropriate response strategies. In this regard, half of the non-dominated solutions in the previous environment are selected to form the sensor set S, which is used to estimate the change degree (denoted by CD) of objective function as [46]:

$$CD_i = \sum_{j \in S} \frac{f_{i,j}(t) - f_{i,j}(t-1)}{f_{i,j}(t-1) + \mu},\ i = 1, 2, ..., m \tag{5}$$

where m is the number of objective functions, $f_{i,j}(t)$ denotes the i-th fitness value of sensor j in environment t, and $\mu = 0.001$ is a smoothing value that avoids the denominator equaling to zero. Then, the overall change degree of the environment is defined as:

$$CD = \lambda \max_{1 \leq k \leq m} \{CD_k\} \tag{6}$$

where λ is an amplification factor, which is set to $m - 1$ as the same in [46].

B. Hierarchical Response System

In the proposed HRS-DMOA, the essence of response is to re-initialize the population to start searching in a new environment, where the major concern is how to improve reliability of the initialized population. Hence, a hierarchical response mode is taken to realize tailored and appropriate reactions to environmental changes in different extents. To be specific, based on the change degree CD of environment in Eq. (6), two predefined thresholds LT and HT are used to divide CD into three levels, and accordingly, the response modes of refinement (if $CD < LT$), TL (if $LT \leq CD \leq HT$), and re-initialization (if $CD > HT$) are adopted.

When CD is fewer than the lower-threshold LT, it is deemed that the change is negligible, that is, the new environment (denoted as t) is similar to the previous one (denoted as $t-1$). Therefore, PS_{t-1} is encouraged to keep improving the convergence and searching for the Pareto solutions, where mutation operators are adopted to further supplement the diversity. If CD is larger than the higher-threshold HT, then the change is regarded so dramatic that most of the previous searching experience is no longer useful. As a result, such situation is treated as a new optimization problem and the evolution is re-started, where only a few solutions in PS_{t-1} are reserved in the re-initialized population. In particular, if CD falls between LT and HT, then the TL-based response is applied, where knowledge acquired in previous environments is used to train a prediction model, and more details are presented in the next subsection.

It is noticeable that, in essence, LT and HT determine the probability of selecting above three response strategies, and

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in this paper, (LT, HT) is set to $(0.03, 0.78)$. In addition, the framework of the proposed hierarchical response system is summarized in Algorithm. 1.

Algorithm 1 Hierarchical response system

Require:
Predefined lower-threshold LT and higher-threshold HT
Ensure:
Response strategy
Calculate change degree (CD) according to Eqs. (5) and (6)
If $CD < LT$
  Refinement
else if $CD > HT$
  Re-initialization
else
  TL
EndIf

C. TL-based Population Initialization

The essence of transfer learning is to apply the learned knowledge from the source domain D_s to assist solving related but not the same tasks in the target domain D_t. In the proposed HRS-DMOA, the main idea of the TL-based response is to make full use of the historical searching experiences to initialize population in the new environment, which is required to have high quality to accelerate the convergence [26]. Consequently, PS in the previous environment with some mutations is selected as the source domain D_s. In addition, to guarantee knowledge is transferred towards a correct direction, another group of transfer reference points (TRPs) is screened in the new environment to form the target domain D_t. To be specific, a local search strategy is applied to assign each individual with a quality factor $Q(\cdot)$ as [3]:

$$Q(\boldsymbol{x}) = \min_{\boldsymbol{z}\in P\setminus\{\boldsymbol{x}\}} \max \Big(f_j(\boldsymbol{z},t) - f_j(\boldsymbol{x},t)\Big),\ j \in \{1, 2, \cdots, m\} \tag{7}$$

where P stands for the population, and the individuals with larger $Q(\cdot)$ value are deemed to have better quality. Then, based on Eq. (7), tournament is performed between two populations in the new environment to obtain a set of TRPs to form the D_t. The details are displayed in Algorithm. 2, where simulated binary crossover and polynomial mutation operators are applied to further enhance the diversity.

Let $T = \{X, Y\}$ denote the training set, where $X = D_s \cup D_t$ and $Y = \{0,\ 1\}$ is the ground-truth label determined by the domination relationship at the new time t ($Y = 1$ for non-dominated individuals). In the proposed HRS-DMOA, the transfer learning process is realized via the sample-based TrAdaboost technique [11], where several base learners are trained by approximating the mapping from X to Y, and the weight of training samples is updated according to the weighted errors on D_t of the trained classifier. Then, the ensemble learning method is adopted to form a strong classifier $H(\cdot)$ which is later used to generate an initial population with high quality in the new environment. In Algorithm. 3, the implementation details of above TL-based initialization strategy are presented.

Algorithm 2 Construction of the target domain

Require:
Two populations $P_1 = \{\boldsymbol{x}_i^1\}_{i=1}^N$ and $P_2 = \{\boldsymbol{x}_i^2\}_{i=1}^N$ in the new environment, mutation probability p_m
Ensure:
Target domain D_t
Initialize $D_t = \emptyset$
For n from 1 to N
  Crossover $\boldsymbol{x}_n^1$ and $\boldsymbol{x}_n^2$ to generate $\tilde{\boldsymbol{x}}_n^1$ and $\tilde{\boldsymbol{x}}_n^2$
  Define competition group $G_c = \{\boldsymbol{x}_n^1, \boldsymbol{x}_n^2, \tilde{\boldsymbol{x}}_n^1, \tilde{\boldsymbol{x}}_n^2\}$
  Calculate $Q(\boldsymbol{x}),\ \boldsymbol{x} \in G_c$ according to Eq. (7)
  $D_t \leftarrow \arg\max_{\boldsymbol{x}\in G_c} Q(\boldsymbol{x})$
EndFor
For each individual $\tilde{\boldsymbol{x}} \in D_t$
  Generate a random number $rand$
  If $rand < p_m$
    $\tilde{\boldsymbol{x}}'$ = mutation($\tilde{\boldsymbol{x}}$)
    If $Q(\tilde{\boldsymbol{x}}') \geq Q(\tilde{\boldsymbol{x}})$
      Substitute $\tilde{\boldsymbol{x}}$ with $\tilde{\boldsymbol{x}}'$
    EndIf
  EndIf
EndFor
Return D_t

D. Overall Framework of HRS-DMOA

The overall flowchart of the proposed HRS-DMOA is shown in Fig. 1, where the main contributions (i.e., the proposed HRS) are illustrated in the red dashed box. In addition, the support vector machine (SVM) [10] is adopted as the base learner in the TL-based response strategy, and the MOEA/D [64] is employed as the static optimizer, which is a representative multi-objective problem solver that decomposes the problem into several scalar sub-problems. Since that the MOEA/D has been successfully applied in various situations, it is deemed that MOEA/D is competent in searching for the Pareto solutions in each individual environment when solving a DMOP.

Explanations of the proposed hierarchical response modes are further summarized as follows.

(1) *Refinement*. At time t, polynomial mutation is performed on PS_{t-1} to enhance the diversity, and P_{ini} is obtained by selection from the augmented PS_{t-1}.
(2) *TL*. At time t, the TL-based initialization is applied to generate P_{ini}, where the augmented PS_{t-1} is adopted as the source domain, and a group of TRPs is screed to form the target domain (see Section III-C for details).
(3) *Re-initialization*. At time t, only a few of the individuals in PS_{t-1} are reserved in P_{ini}, whereas the rest of P_{ini} is directly generated by random initialization.

It is noticeable that the TL technique is only adopted when the environmental changes are regarded in a medium level. The merits of this setting contain mainly two aspects. On one hand, if the environment slightly changes, it seems

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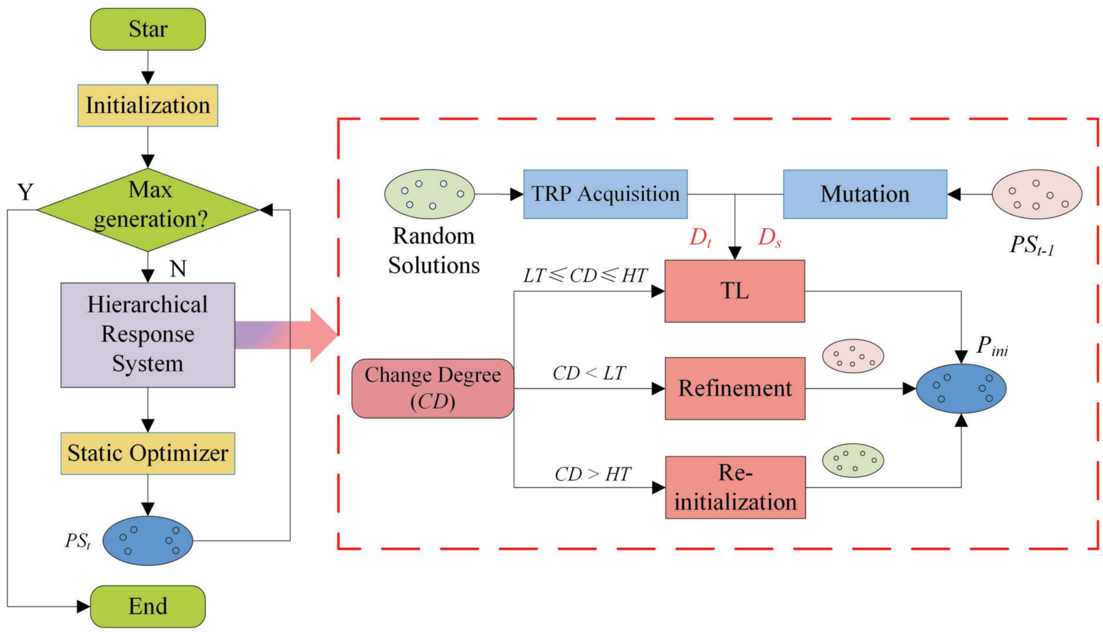

Fig. 1: Flowchart of HRS-DMOA, where the proposed HRS (in the red dashed box) is responsible for generating the initial population (P_{ini}) to the static optimizer, which accomplishes the searching in each environment and can be any multi-objective problem solver (the MOEA/D is employed in this study). By comparing the calculated change degree (CD) with the two thresholds, the responses based on TL, refinement, and re-initialization are adopted when $LT \leq CD \leq HT$, $CD < LT$, and $CD > HT$, respectively. Additionally, the "TRP Acquisition" refers to the construction of target domain D_t.

unnecessary to introduce the prediction model, which will no doubts burden the computation. On the other hand, dramatic environmental changes may lead to few correlations between the successive environments, and in this case, the negative transfer phenomenon easily occurs. As a result, the advantages of different strategies are effectively integrated in the proposed hierarchical response system, which is, in fact, a rational combination of them. In Algorithm. 4, a brief pseudocode of the proposed HRS-DMOA is presented for a clear view.

Remark 1: In the first environment $t = 1$, the initial population P_{ini} is directly generated by the static optimizer $Opt(\cdot)$. In other cases, P_{ini} is provided to $Opt(\cdot)$ by the adopted response strategy.

E. Complexity Analysis

Given that N is the population size, m is the number of objectives and n is the dimension of decision vector. The following cases are mainly considered to estimate the computational complexity of the proposed HRS-DMOA.

1) MOEA/D is used to search for Pareto solutions in each environment, whose computational complexity is $O(NmT)$ [64], where T is the size of applied neighborhood.

2) The computational complexity is $O(n)$ for both polynomial mutation and simulated binary crossover.

3) The computational complexity is $O(N)$ when calculating the fitness value of the sensors in a new environment.

4) The computational complexity is $O(Nm)$ when calculating the quality factor $Q(\cdot)$ to select transfer reference points to form D_t.

5) In the TL-based initialization, SVM is employed as the base learner, whose computational complexity is $O(N^2mn)$ [10].

6) In the re-initialization response mode, the computational complexity is $O(Nmn)$.

7) In the refinement response mode, only polynomial mutation operation and a selection process are involved. Hence, the computational complexity is $O(Nn) + O(N)$.

In general, N is far larger than m and n. Consequently, the computational complexity of the proposed HRS-DMOA is $O(N^2mn)$.

IV. Experimental Results and Discussions

In this section, benchmark evaluations are performed to validate the effectiveness of the proposed HRS-DMOA in solving DMOPs, and sufficient ablation studies are carried out to verify the superiority of the developed HRS.

A. Experimental Environment

The proposed HRS-DMOA is comprehensively evaluated on 14 DMOPs in the CEC2018 test suites [27], where the provided benchmark DMOPs can well characterize the properties of dynamic problems in various real-world optimization scenarios, such as time-varying PS and disconnected PF, etc. The 14 benchmark functions are named as DF1 to DF14, and one can find more information of the applied test suites in [27]. For the evaluation metrics, the inverted generational distance (IGD) and the hypervolume (HV) are adopted. To be specific, IGD reflects the convergence of algorithms by measuring distance between the obtained PF and the real one.

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Algorithm 3 Transfer learning-based initialization

Require:
The number of base learner N_b, source domain D_s, target domain D_t, and population size N

Ensure:
Initialized population P_{ini} in the new environment

Calculate $n_s = |D_s|$ and $n_t = |D_t|$
Obtain training set $T = \{X, Y\}$ where $X = D_s \cup D_t$ and corresponding ground-truth label $Y = \{y_i\}_{i=1}^{n_s+n_t}$
Initialize weight for each training sample by

$$\omega(\boldsymbol{x}) = \begin{cases} 1/n_s,\ \boldsymbol{x} \in D_s \\ 1/n_t,\ \boldsymbol{x} \in D_t \end{cases}$$

For j from 1 to N_b
  Weight normalization by $\omega(\boldsymbol{x}) = \omega(\boldsymbol{x}) / \sum_{\boldsymbol{z}} \omega(\boldsymbol{z})$
  Train a base learner L_j with $(T; \omega)$
  Calculate error ε_j of trained L_j on target domain D_t according to $\varepsilon_j = \sum\limits_{(\boldsymbol{x},y)\in D_t} \omega(\boldsymbol{x})|L_j(\boldsymbol{x}) - y|$
  Define $\beta_j = \frac{\varepsilon_j}{1-\varepsilon_j}$ and $\beta = \frac{1}{1+\sqrt{ln(n_s^2)/N_b}}$
  Update weight for each training sample based on

$$\omega(\boldsymbol{x}) = \begin{cases} \omega(\boldsymbol{x}) \cdot \beta^{\ |L_j(\boldsymbol{x})-y|},\ (\boldsymbol{x}, y) \in D_s \\ \omega(\boldsymbol{x})\ / \beta_j^{\ |L_j(\boldsymbol{x})-y|},\ (\boldsymbol{x}, y) \in D_t \end{cases}$$

EndFor
Integrate base learners to form a strong classifier as $H(\boldsymbol{x}) = \text{sgn}[\sum_{j=1}^{N_b} -ln(\beta_j)L_j(\boldsymbol{x})]$
Generate a selection pool P_s in the new environment ($|P_s| >> N$)
Screen high-quality solutions from P_s based on $\tilde{P}_s = \{\boldsymbol{z}|\ H(\boldsymbol{z}) = 1,\ \boldsymbol{z} \in P_s\}$ until $|\tilde{P}_s| = N$
Return $P_{ini} = \tilde{P}_s$

Algorithm 4 Framework of HRS-DMOA

Require:
Static optimizer $Opt(\cdot)$, thresholds LT and HT, optimization problem $\boldsymbol{F}$, and the time variables $t = \{1, 2, ..., \lfloor \frac{\tau}{\tau_t} \rfloor\}$

Ensure:
Pareto sets $\{PS_t\}$ of all environments

Initialization procedure
While $t \leq \frac{\tau}{\tau_t}$
  If $t = 1$
    $PS_t \leftarrow Opt(\boldsymbol{F}, t)$
  else
    Determine the response mode via Algorithm. 1
    Generate the P_{ini} by the adopted strategy
    $PS_t \leftarrow Opt(\boldsymbol{F}, t, P_{ini})$
  EndIf
  $t \leftarrow t + 1$
EndWhile
Return $\{PS_t\}_{t=1}^{\lfloor \frac{\tau}{\tau_t} \rfloor}$

Considering the time-varying property of DMOPs, IGD at time t is calculated as:

$$IGD_t = \frac{\sum_{\boldsymbol{x}\in \tilde{PF_t}} dist(\boldsymbol{x}, PF_t)}{|\tilde{PF_t}|} \tag{8}$$

where $dist(\cdot)$ is the Euclidean distance and $\tilde{PF_t}$ stands for the true PF at time t. HV refers to the volume of the hypercube surrounded by individuals in the obtained PF and corresponding reference point in the objective space. Let $\nu(\cdot)$ denote the volume of the mentioned hypercube, then HV at time t is given as:

$$HV_t = \bigcup_{\boldsymbol{x}\in PF_t} \nu(\boldsymbol{x}) \tag{9}$$

In addition, another commonly used performance indicator MS (short for the maximum spread) is adopted to comprehensively evaluate the diversity of the five DMOAs, which measures the scope that the true PF is covered by the obtained one. The larger MS, the better diversity, whose definition is given as:

$$MS_t = \sqrt{\frac{1}{m}\sum_{k=1}^{m}\left[\frac{\min\{F_k^{max}, f_k^{max}\} - \max\{F_k^{min}, f_k^{min}\}}{F_k^{max} - F_k^{min}}\right]^2} \tag{10}$$

where F_k^{max} and F_k^{min}, f_k^{max} and f_k^{min} represent the maximum and minimum values of the k-th objective in the true and obtained PF at time t, respectively.

In the average level, the above three indicators are reported and calculated as:

$$\begin{aligned} MIGD &= \frac{1}{\tau}\sum_{t=1}^{\tau} IGD_t \\ MHV &= \frac{1}{\tau}\sum_{t=1}^{\tau} HV_t \\ MMS &= \frac{1}{\tau}\sum_{t=1}^{\tau} MS_t \end{aligned} \tag{11}$$

where τ is the maximum generation.

Moreover, to further verify the competitiveness of the proposed HRS-DMOA, other four popular DMOAs are employed as baseline models for comparison, including the second version of the dynamic non-dominated sorting genetic algorithm (NSGA)-II [12], the change-responsive NSGA-II [46], the TL-based DMOA in [25], and the knee-point-based imbalanced TL-DMOA in [26]. For convenience, above four algorithms are denoted as DNSGA-II-B, CR-DMOEA, Tr-DMOEA, and KT-DMOEA, respectively. Notice that the response strategy based on diversity is applied in the former two methods, and the TL-based response strategy (which can be regarded as the combination of the memory- and prediction-based ones) is adopted with different implementation manners in the latter two algorithms. Hence, above four models are employed as baselines for comparison to validate whether the proposed HRS-DMOA can realize better comprehensive performance via the rational integration of different mainstream strategies.

In addition, the dynamic behavior of DMOPs is depicted as $t = \frac{1}{n_t} \cdot \lfloor \frac{\tau}{\tau_t} \rfloor$, where the number of the environments is set to 50 ($\tau = 50 \times \tau_t$), τ_t and n_t stand for the frequency and severity of change, respectively. On each benchmark, three groups of evaluation are performed based on different dynamic parameters, including $(n_t = 5, \tau_t = 10)$, $(n_t = 10, \tau_t = 5)$ and $(n_t = 10, \tau_t = 10)$. For a fair comparison, all algorithms are evaluated with the same parametric conditions on each benchmark, where the dimension of test problems is set to 10, the population size for bi- and tri-objective optimization problem is set to 100 and 150, respectively. 10 epochs of search in each individual environment is fixed for all DMOAs, that is, the searching times for Pareto solutions by the applied static optimizer in all algorithms are the same and, by doing so, the effectiveness of the responses to the environmental changes can be reflected to some extents. Moreover, considering that while applying the TL technique to make responses, it is inevitable to consume some evaluations in advance at the new environment to establish the target domain, for those algorithms that do not involve extra evaluations, six more epochs of search for the static optimizer are added to promote the fairness of comparison.

All algorithms have run 20 times individually on each benchmark to alleviate the influence of randomness, and results in average level are reported along with the Wilcoxon rank sum test at the significance level of 0.05 [13]. Comparison results in terms of MIGD, MHV, and MMS are presented in Table III, Table IV, and Table V, respectively, where "+/-" indicates that the proposed HRS-DMOA is significantly better/worse than the corresponding algorithm, and "=" denotes there is no significant difference between the two algorithms in a statistical sense.

B. Benchmark Evaluations Results

1) Convergence: As reported in Table III, HRS-DMOA achieves 26 out of 42 best results in terms of MIGD, which performs significantly better than KT-DMOEA, Tr-DMOEA, CR-DMOEA and DNSGA-II-B in 32, 35, 29 and 39 cases, respectively, reflecting the reliability of the proposed method in adapting to the varying environments in DMOPs. According to the Wilcoxon rank sum test, there are 135 cases where HRS-DMOA performs significantly better than other algorithms, which shows that the proposed method is a competitive DMOA with outstanding convergence. Notice that on the problems DF7, the DNSGA-II-B performs slightly better than the proposed HRS-DMOA in two situations, which may mainly due to the time-varying PF ranges of DF7 are in dissimilar scales, and simultaneously the corresponding PS center is fixed, thereby making it extremely hard to realize efficient knowledge transfer towards a proper direction.

In addition, for the tri-objective problem DF11, not only the centroid of its PF oscillates by expanding and shrinking with multiple scales, but also the density of solutions changes with time, which brings great challenges to the algorithms. On one hand, the population is required to rapidly converge with a considerable diversity so that the obtained PF can well cover the true one. On the other hand, the population should also realize timely escape from the local optimum. Such a high requirement on maintaining diversity in the objective-space makes DF11 quite hard to handle, whereas the proposed HRS-DMOA still ranks second on DF11 and performs only slightly worse than the KT-DMOEA, which shows the competitiveness of our method.

2) Diversity: According to Table IV, HRS-DMOA achieves half of the best results in terms of MHV, and the sub-optimal algorithm Tr-DMOEA wins in 14 cases, which implies the advantage of the proposed algorithm regarding to the comprehensive performance. Furthermore, as displayed in Table V, it is found that the proposed HRS-DMOA obtains 22 best MMS out of the 42 results, and in totally 106 cases, our method performs significantly better than other comparison models. Combining the results reported in both Table IV and Table V, it can be concluded that the proposed algorithm yields considerable diversity as well. While on the problem DF6, the MHV of our method is not so satisfactory as compared with other algorithms, which may due to that the time-varying PF of DF6 has the geometric property of long-tails and knee-regions. By observing the response manner of our method when solving DF6, it is found that in almost 65.3% of the total 50 environment, the initial population is randomly generated within the re-initialization mode. Simultaneously, it can be inferred that the negative transfer phenomenon may occur in the left 34.7% cases where the TL-based response is adopted, as in the unstable environment it is hard to accurately identify the useful knowledge. Consequently, the random initialization is performed in most cases, and recalling the historical experiences makes few contributions, which collaboratively lead to a poor performance.

It is also noticeable that the PF of problem DF2 keeps unchanged in a period of time (while the PS changes), which requires the algorithms to not only monitor environmental changes, but also maintain the population diversity to obtain considerable result. From the reported results, with different dynamic parameter settings, the proposed HRS-DMOA obtains almost all the best MHV and MMS on DF2, exhibiting the superiority of HRS in handling dynamic behaviors with multiple concerns.

To sum up, the proposed HRS-DMOA ranks first in most testing cases, which is competent in handling complex dynamic behaviors in DMOPs with satisfactory comprehensive performance. Moreover, according to the above results, it is found that the DNSGA-II-B has achieved few best results in terms of MIGD although a competitive diversity is presented, which indicates that the diversity-based response strategy does have some limitations in practice. Meanwhile, in various dynamic situations, it is also hard to guarantee the performances of DMOAs with only TL-based strategy. Hence, it can be concluded that the designed HRS is effective and practical, which successfully integrates the merits of different response strategies.

C. Comparisons with Other Hybrid DMOA

In this subsection, two additional hybrid methods SVM-DMOA [24] and PPS-RM-MEDA [71] are employed for comparisons. In Table VI, the results on DF series testing problems are presented, where the number in parenthesis denotes the rank (from 1st to 3rd) of corresponding algorithm, and the data of above two comparison models are cited from [37] and [58], respectively.

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TABLE III: Performance of five DMOAs on MIGD

Problems	$n_t,\ \tau_t$	Algorithms				
		DNSGA-II-B [12]	Tr-DMOEA [25]	CR-DMOEA [46]	KT-DMOEA [26]	HRS-DMOA
DF1	5, 10	0.4305±1.18e-01(+)	0.2919±1.25e-01(+)	0.0849±1.22e-02(+)	0.1427±2.17e-02(+)	**0.0740±1.62e-02**
	10, 5	2.3232±7.40e-01(+)	0.2727±6.34e-02(+)	0.1102±5.41e-02(=)	0.1326±2.27e-02(+)	**0.0859±1.94e-02**
	10, 10	2.0923±6.29e-01(+)	0.3594±6.94e-02(+)	0.1038±2.54e-02(=)	0.1261±1.94e-02(+)	**0.0884±1.80e-02**
DF2	5, 10	0.2667±7.12e-02(+)	0.5891±1.00e-01(+)	**0.0242±4.11e-03(-)**	0.1093±1.29e-02(+)	0.0505±1.68e-02
	10, 5	1.2338±5.04e-01(+)	0.5183±7.52e-02(+)	**0.0347±1.87e-02(=)**	0.1196±1.04e-02(+)	0.0419±1.31e-02
	10, 10	1.3458±5.50e-01(+)	0.5227±7.06e-02(+)	**0.0402±1.66e-02(=)**	0.1095±1.09e-02(+)	0.0482±1.37e-02
DF3	5, 10	0.8344±1.86e-01(+)	0.7486±3.67e-01(=)	0.6393±3.50e-01(=)	0.8366±8.29e-02(+)	**0.5182±1.33e-01**
	10, 5	2.4443±8.07e-01(+)	0.6780±1.75e-01(=)	**0.3807±7.25e-02(-)**	0.7350±1.24e-01(+)	0.5680±1.44e-01
	10, 10	2.7178±7.92e-01(+)	0.8917±4.27e-01(=)	**0.3943±1.18e-01(-)**	0.7128±9.16e-02(=)	0.5916±1.57e-01
DF4	5, 10	1.6360±2.44e-01(+)	3.7776±4.93e-01(+)	1.2742±1.41e-01(+)	1.6231±1.43e-01(+)	**0.3620±7.49e-02**
	10, 5	1.8601±3.52e-01(+)	5.1859±2.01e-01(+)	1.3296±1.47e-01(+)	1.7109±9.97e-02(+)	**0.4296±6.97e-02**
	10, 10	1.7133±3.27e-01(+)	5.4790±2.25e-01(+)	1.3684±1.81e-01(+)	1.7089±1.14e-01(+)	**0.4310±7.83e-02**
DF5	5, 10	0.3309±5.93e-02(+)	0.1653±2.79e-02(+)	0.0698±5.30e-02(+)	0.4060±4.98e-02(+)	**0.0361±7.22e-03**
	10, 5	1.5103±5.20e-01(+)	0.1605±3.31e-02(+)	0.0594±2.09e-02(+)	0.3623±6.79e-02(+)	**0.0367±9.04e-03**
	10, 10	1.4960±4.17e-01(+)	0.1916±3.56e-02(+)	0.0655±4.32e-02(+)	0.3501±6.63e-02(+)	**0.0366±5.65e-03**
DF6	5, 10	6.2760±1.45e+00(+)	2.2846±6.85e-01(+)	2.7808±2.50e+00(=)	3.2694±3.06e-01(+)	**1.5302±6.95e-01**
	10, 5	1.8258±6.45e-01(=)	3.9235±1.05e+00(+)	5.9046±4.39e+00(+)	3.7916±4.24e-01(+)	**1.7784±6.44e-01**
	10, 10	1.8768±1.04e+00(+)	1.5666±4.28e-01(+)	6.8773±3.12e+00(+)	3.9172±3.90e-01(+)	**1.2066±6.06e-01**
DF7	5, 10	2.8578±5.82e-01(+)	3.5497±9.14e-01(+)	2.3553±7.25e-01(+)	2.9380±7.64e-01(+)	**0.9094±2.12e-01**
	10, 5	**1.1209±1.35e-01(-)**	3.6926±9.29e-01(+)	9.1002±7.34e+00(+)	4.4453±7.07e-01(+)	1.2286±1.74e-01
	10, 10	**1.1154±1.53e-01(=)**	2.1510±7.00e-01(+)	5.7568±2.25e+00(+)	2.9671±4.29e-01(+)	1.1298±1.53e-01
DF8	5, 10	0.3023±5.64e-02(+)	0.6467±1.64e-02(+)	0.9582±1.32e-01(+)	1.0937±1.53e-02(+)	**0.1373±6.89e-02**
	10, 5	0.2783±5.55e-02(+)	0.5996±2.09e-02(+)	1.0822±1.38e-01(+)	1.0736±2.32e-02(+)	**0.1232±3.16e-02**
	10, 10	0.2749±5.93e-02(+)	0.6068±2.63e-02(+)	1.0353±1.20e-01(+)	1.0904±1.78e-02(+)	**0.1108±3.15e-02**
DF9	5, 10	1.1643±2.73e-01(+)	2.1294±2.93e-01(+)	0.2699±1.11e-01(=)	0.6778±7.75e-02(+)	**0.2189±3.09e-02**
	10, 5	1.1467±2.80e-01(+)	3.1613±5.44e-01(+)	0.2797±1.12e-01(+)	0.6427±1.09e-01(+)	**0.1993±3.56e-02**
	10, 10	1.0891±2.99e-01(+)	3.1626±4.40e-01(+)	0.2953±1.40e-01(+)	0.6567±8.32e-02(+)	**0.1965±2.64e-02**
DF10	5, 10	0.8708±1.70e-01(+)	**0.1923±3.90e-02(-)**	0.4345±8.85e-02(+)	0.3086±1.91e-02(+)	0.2661±7.70e-02
	10, 5	1.2816±3.47e-01(+)	**0.1280±1.72e-02(-)**	0.3933±7.34e-02(+)	0.2836±2.15e-02(-)	0.3283±2.28e-02
	10, 10	1.2348±3.28e-01(+)	**0.1158±1.45e-02(-)**	0.3504±6.57e-02(=)	0.2946±1.33e-02(-)	0.3230±2.76e-02
DF11	5, 10	0.7717±1.56e-01(+)	0.3433±3.16e-02(+)	0.3868±5.11e-03(+)	0.1636±5.54e-03(+)	**0.1492±3.68e-03**
	10, 5	0.8730±1.55e-01(+)	0.4038±4.48e-02(+)	0.4847±8.07e-03(+)	**0.1634±8.18e-03(-)**	0.3677±2.78e-03
	10, 10	0.9039±9.84e-02(+)	0.3728±4.60e-02(=)	0.4776±1.52e-02(+)	**0.1643±8.74e-03(-)**	0.3666±2.20e-03
DF12	5, 10	0.8208±6.17e-02(+)	2.5764±1.73e-01(+)	**0.3076±1.14e-02(-)**	0.6093±5.28e-02(=)	0.5108±1.78e-01
	10, 5	0.8656±7.23e-02(+)	1.4244±9.61e-02(+)	**0.3051±3.42e-03(=)**	0.6321±5.39e-02(=)	0.4643±1.81e-01
	10, 10	0.9036±7.61e-02(+)	1.4266±1.16e-01(+)	**0.3074±3.26e-03(-)**	0.6358±7.12e-02(=)	0.6236±1.69e-01
DF13	5, 10	0.5057±1.04e-01(+)	0.3911±3.42e-02(+)	0.2554±1.82e-02(+)	0.4067±4.16e-02(+)	**0.2409±7.86e-03**
	10, 5	1.6747±4.90e-01(+)	0.3841±2.29e-02(+)	0.3052±1.93e-02(+)	0.3789±3.80e-02(+)	**0.2556±1.20e-02**
	10, 10	1.6450±6.22e-01(+)	0.4262±4.73e-02(+)	0.3031±9.74e-03(+)	0.3741±3.30e-02(+)	**0.2527±1.26e-02**
DF14	5, 10	0.4126±1.07e-01(+)	0.2410±5.64e-02(+)	0.1246±2.11e-02(+)	0.1310±1.44e-02(+)	**0.0972±4.35e-03**
	10, 5	3.0028±8.52e-01(+)	0.2174±5.35e-02(+)	0.1570±2.06e-02(+)	0.1253±1.16e-02(=)	**0.1216±3.71e-03**
	10, 10	3.0825±1.14e+00(+)	0.1938±3.51e-02(+)	0.1614±2.42e-02(+)	**0.1210±1.32e-02(=)**	0.1231±3.97e-03
+ / - / =	\	39 / 1 / 2	35 / 3 / 4	29 / 4 / 9	32 / 4 / 6	\

As can be seen from Table VI, the proposed HRS-DMOA ranks first and second in 13 and 14 cases, respectively. In particular, on the complex tri-objective benchmark problems DF13 and DF14, our method yields considerable convergence performance, which owes to that the changing severity is taken into account when making response to the environmental changes, and it enables the generated initial population well adapt to the new environment. It should be pointed out that the static optimizer can also make great contribution to the results, and according to our algorithm configuration, it is difficult for the static optimizer to always obtain sufficient high-quality Pareto solutions within only 10 epochs of search in each environment, which accounts for that our algorithm may present performance declination as compared with the optimal one in some situations.

In addition, the proposed HRS-DMOA is further compared with the MoE [43], which is a state-of-the-art DMOA that has employed multiple prediction mechanism. To make a fair comparison, the basic experimental settings are made the same

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TABLE IV: Performance of five DMOAs on MHV

Problems	$n_t,\ \tau_t$	Algorithms				
		DNSGA-II-B [12]	Tr-DMOEA [25]	CR-DMOEA [46]	KT-DMOEA [26]	HRS-DMOA
DF1	5, 10	0.1838±5.31e-02(+)	0.2526±7.37e-02(+)	0.4500±1.54e-02(=)	0.3811±1.63e-02(+)	**0.4587±3.15e-02**
	10, 5	0.0089±3.60e-02(+)	0.2493±3.75e-02(+)	0.4241±6.92e-02(=)	0.3909±1.69e-02(+)	**0.4633±3.03e-02**
	10, 10	0.0108±3.39e-02(+)	0.2100±4.08e-02(+)	0.4236±3.82e-02(+)	0.3940±1.57e-02(+)	**0.4697±4.19e-02**
DF2	5, 10	0.2317±7.41e-02(+)	0.5531±1.11e-01(+)	0.6872±8.42e-03(=)	0.5862±1.04e-02(+)	**0.6990±2.31e-02**
	10, 5	0.0516±1.14e-01(+)	**0.6777±3.12e-02(-)**	0.6743±2.50e-02(-)	0.5895±9.76e-03(+)	0.6595±2.02e-02
	10, 10	0.0465±8.94e-02(+)	0.6439±8.38e-02(+)	0.6661±1.66e-02(+)	0.5976±1.28e-02(+)	**0.6818±1.76e-02**
DF3	5, 10	0.0305±4.37e-02(+)	**0.1724±2.57e-02(=)**	0.0878±7.50e-02(=)	0.1495±1.06e-02(-)	0.1196±8.54e-02
	10, 5	0.0087±3.91e-02(+)	**0.2009±1.54e-02(=)**	0.1628±5.45e-02(-)	0.1655±1.40e-02(-)	0.1069±6.22e-02
	10, 10	0.0050±2.33e-03(+)	**0.2013±1.74e-02(=)**	0.1609±7.74e-02(=)	0.1634±7.47e-03(=)	0.1304±5.94e-02
DF4	5, 10	0.1625±3.79e-02(+)	0.0209±2.64e-02(+)	**0.6013±5.63e-02(-)**	0.4960±3.08e-02(=)	0.5218±3.35e-02
	10, 5	0.1489±6.04e-02(+)	0.0188±1.61e-02(+)	0.5185±5.31e-02(=)	0.4840±3.13e-02(+)	**0.5193±2.72e-02**
	10, 10	0.1813±5.76e-02(+)	0.0027±2.89e-03(+)	**0.5709±5.39e-02(-)**	0.4705±3.54e-02(+)	0.5216±2.87e-02
DF5	5, 10	0.2538±4.22e-02(+)	0.4330±3.43e-02(+)	0.4971±5.73e-02(=)	0.2335±2.46e-02(+)	**0.5293±1.22e-02**
	10, 5	0.0282±6.65e-02(+)	0.4303±3.15e-02(+)	0.4999±3.00e-02(+)	0.2719±2.37e-02(+)	**0.5306±1.69e-02**
	10, 10	0.0166±4.68e-02(+)	0.4055±2.88e-02(+)	0.4932±5.37e-02(+)	0.2777±2.35e-02(+)	**0.5284±1.08e-02**
DF6	5, 10	0.0012±3.19e-03(+)	**0.9480±7.03e-02(-)**	0.2471±4.22e-02(-)	0.0271±1.19e-02(=)	0.0268±1.13e-02
	10, 5	0.0635±9.28e-02(=)	**0.8041±1.08e-01(-)**	0.1745±4.44e-02(-)	0.0299±1.34e-02(=)	0.0260±9.25e-03
	10, 10	0.0785±6.97e-02(=)	**0.8315±1.03e-01(-)**	0.2412±2.41e-02(-)	0.0289±1.22e-02(=)	0.0275±1.22e-02
DF7	5, 10	0.1285±3.17e-02(=)	**0.9479±9.08e-02(-)**	0.0124±1.25e-02(+)	0.2334±2.03e-02(-)	0.1269±1.62e-02
	10, 5	0.1400±3.17e-02(=)	**0.8986±7.02e-02(-)**	0.0315±2.15e-02(+)	0.2692±3.50e-02(-)	0.1347±3.30e-02
	10, 10	0.1398±2.99e-02(=)	**0.8392±7.24e-02(-)**	0.0379±1.78e-02(+)	0.2471±2.12e-02(-)	0.1382±2.96e-02
DF8	5, 10	0.6909±3.88e-02(+)	0.9042±1.10e-02(-)	0.9340±1.51e-02(+)	0.9115±6.78e-03(-)	**0.9532±1.28e-02**
	10, 5	0.6484±5.18e-02(+)	0.8954±1.22e-02(-)	0.9439±1.84e-02(=)	0.9133±5.97e-03(-)	**0.9442±1.69e-02**
	10, 10	0.6445±3.04e-02(+)	0.8988±7.67e-03(-)	**0.9402±1.69e-02(-)**	0.9123±7.45e-03(-)	0.9255±1.82e-02
DF9	5, 10	0.0555±3.79e-02(+)	**0.5656±2.55e-02(-)**	0.3232±9.77e-02(=)	0.1698±1.88e-02(+)	0.3161±3.47e-02
	10, 5	0.0495±3.63e-02(+)	0.1104±3.65e-02(+)	0.3020±9.94e-02(=)	0.1958±2.34e-02(+)	**0.3391±4.16e-02**
	10, 10	0.0634±4.50e-02(+)	0.1294±1.90e-02(+)	0.2753±1.20e-01(=)	0.1842±1.53e-02(+)	**0.3443±3.07e-02**
DF10	5, 10	0.0371±1.37e-01(+)	0.6399±2.59e-02(+)	**0.9115±3.26e-02(=)**	0.6142±2.17e-02(+)	0.8791±1.36e-01
	10, 5	0.0530±1.75e-01(+)	0.8712±1.91e-02(+)	0.9066±1.73e-02(=)	0.6605±1.57e-02(+)	**0.9150±1.83e-02**
	10, 10	0.0407±1.40e-01(+)	0.8841±1.14e-02(+)	0.9161±1.81e-02(+)	0.6581±1.26e-02(+)	**0.9247±1.76e-02**
DF11	5, 10	0.1093±2.03e-01(+)	0.1336±1.54e-02(+)	0.4875±8.77e-03(+)	0.2193±2.94e-03(+)	**0.7670±1.62e-02**
	10, 5	0.0561±1.91e-01(+)	0.1402±2.17e-02(+)	0.6350±1.03e-02(+)	0.2224±3.19e-03(+)	**0.7775±9.94e-03**
	10, 10	0.0548±1.90e-01(+)	0.1414±1.33e-02(+)	0.6307±2.01e-02(+)	0.2231±2.20e-03(+)	**0.7728±1.65e-02**
DF12	5, 10	**0.9800±1.55e-02(-)**	0.7480±4.12e-02(+)	0.8960±8.09e-03(-)	0.7784±1.38e-02(+)	0.8375±3.47e-02
	10, 5	**0.9486±3.99e-02(-)**	0.6676±4.37e-02(+)	0.9089±3.54e-03(-)	0.7988±6.90e-03(=)	0.8371±5.48e-02
	10, 10	**0.9649±2.56e-02(-)**	0.6630±6.25e-02(+)	0.9075±6.50e-03(-)	0.7957±8.83e-03(+)	0.8141±6.23e-02
DF13	5, 10	0.4643±1.16e-01(=)	**0.5326±2.97e-02(-)**	0.5135±2.02e-02(-)	0.4044±2.37e-02(+)	0.4549±1.63e-02
	10, 5	0.0933±1.08e-01(+)	**0.5011±1.81e-02(-)**	0.3020±2.54e-02(+)	0.4223±2.17e-02(+)	0.4506±1.17e-02
	10, 10	0.1045±1.08e-01(+)	**0.4889±1.12e-02(-)**	0.2953±1.99e-02(+)	0.4232±2.11e-02(+)	0.4544±6.67e-03
DF14	5, 10	0.0281±1.39e-02(+)	0.3516±6.45e-02(+)	0.4226±3.28e-02(+)	0.4063±1.91e-02(+)	**0.4884±1.03e-02**
	10, 5	0.0027±1.22e-02(+)	0.3733±5.11e-02(+)	0.4116±1.59e-02(+)	0.4276±1.65e-02(+)	**0.4801±9.70e-03**
	10, 10	0.0018±8.25e-03(+)	0.3960±3.58e-02(+)	0.4096±1.58e-02(+)	0.4326±1.38e-02(+)	**0.4791±1.01e-02**
+ / - / =	\	33 / 3 / 6	25 / 14 / 3	17 / 12 / 13	28 / 8 / 6	\

as mentioned in [21], and the comparison results on JY series problems [23] are displayed in Table VII, where data of the algorithm MoE are cited from [21].

According to Table VII, the proposed HRS-DMOA wins the state-of-the-art MoE in 15 cases, and on problems of JY1 and JY5, the MoE yields better performance than our method. Particularly, on these two problems, the results obtained by the proposed HRS-DMOA merely reach the level of 10^{-1}, whose major reason is that the main focus of this study is to realize rational and efficient integration of the mainstream strategies, thus only several basic response methods are applied in the proposed HRS-DMOA, which may limit the algorithm performance as compared with the meticulously designed advanced strategies in MoE.

It is also noticeable that on the rest five JY problems, the proposed HRS-DMOA obtains the best results under all dynamic parameter settings, which is a promising and inspiring result as it implies that even the basic methods could realize

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TABLE V: Performance of five DMOAs on MMS

Problems	$n_t,\ \tau_t$	Algorithms				
		DNSGA-II-B [12]	Tr-DMOEA [25]	CR-DMOEA [46]	KT-DMOEA [26]	HRS-DMOA
DF1	5, 10	0.7769±5.89e-02(+)	0.7443±1.39e-01(+)	0.8912±8.68e-02(=)	0.7694±1.82e-02(+)	**0.9084±4.65e-02**
	10, 5	0.9113±6.31e-02(=)	0.9064±2.50e-02(=)	0.8752±1.00e-01(+)	0.7654±2.86e-02(+)	**0.9137±3.56e-02**
	10, 10	0.8646±1.08e-01(+)	0.8606±4.52e-02(+)	0.8781±8.70e-02(+)	0.7701±4.12e-02(+)	**0.9040±2.95e-02**
DF2	5, 10	0.8513±4.88e-02(+)	0.5661±1.10e-01(+)	0.9174±4.79e-02(=)	0.7866±2.54e-02(+)	**0.9216±3.17e-02**
	10, 5	0.8047±5.25e-02(+)	0.5548±8.51e-02(+)	0.9271±6.30e-02(=)	0.7382±2.55e-02(+)	**0.9373±2.77e-02**
	10, 10	0.8112±7.64e-02(+)	0.5510±1.25e-01(+)	0.8940±7.61e-02(=)	0.7531±2.40e-02(+)	**0.9297±4.15e-02**
DF3	5, 10	0.3554±9.02e-02(+)	0.5532±3.83e-02(-)	0.3715±1.51e-01(+)	**0.5541±3.14e-02(-)**	0.4140±1.09e-01
	10, 5	**0.8493±1.52e-01(-)**	0.6196±7.01e-02(-)	0.4165±1.06e-01(=)	0.5297±7.05e-02(-)	0.4244±1.04e-01
	10, 10	**0.8780±1.76e-01(-)**	0.6380±4.57e-02(-)	0.4259±1.28e-01(=)	0.4953±4.04e-02(=)	0.4879±8.93e-02
DF4	5, 10	0.4351±5.95e-02(+)	0.5384±1.22e-01(+)	0.7662±8.33e-02(+)	0.4607±2.44e-02(+)	**0.9429±3.37e-02**
	10, 5	0.3068±8.30e-02(+)	0.5556±9.31e-02(+)	0.7601±1.01e-01(+)	0.4760±3.10e-02(+)	**0.9235±5.10e-02**
	10, 10	0.2836±8.99e-02(+)	0.3909±6.13e-02(+)	0.7400±1.00e-01(+)	0.4613±2.41e-02(+)	**0.9060±4.01e-02**
DF5	5, 10	0.9698±4.51e-04(=)	0.9637±3.60e-04(+)	0.9292±1.31e-01(=)	0.8683±1.95e-02(+)	**0.9808±1.22e-02**
	10, 5	0.9686±2.53e-03(+)	0.9719±1.62e-04(=)	0.9551±6.32e-02(=)	0.8760±1.80e-02(+)	**0.9754±1.23e-02**
	10, 10	0.9491±1.70e-03(+)	0.9779±3.22e-04(=)	0.9661±4.31e-02(=)	0.8626±2.51e-02(+)	**0.9788±1.26e-02**
DF6	5, 10	**0.9988±2.65e-03(-)**	0.9940±4.71e-03(=)	0.8729±1.88e-01(+)	0.7458±3.74e-02(+)	0.9552±6.03e-02
	10, 5	**0.9997±1.34e-03(=)**	0.9948±3.50e-03(-)	0.9153±2.39e-01(+)	0.7610±4.17e-02(+)	0.9913±2.73e-02
	10, 10	**0.9999±5.24e-04(-)**	0.9927±3.42e-03(=)	0.9989±2.31e-03(-)	0.7573±3.31e-02(+)	0.9576±6.68e-02
DF7	5, 10	0.8251±3.56e-02(+)	0.8178±1.27e-01(=)	**0.9127±5.43e-02(=)**	0.7114±4.08e-02(+)	0.8878±3.69e-02
	10, 5	0.7918±2.53e-02(+)	0.6464±1.32e-01(+)	0.8258±6.04e-02(+)	0.6057±4.41e-02(+)	**0.8778±3.10e-02**
	10, 10	0.7999±3.07e-02(+)	0.6598±8.29e-02(+)	0.8061±5.45e-02(+)	0.6271±5.98e-02(+)	**0.8727±2.46e-02**
DF8	5, 10	**0.9321±2.87e-02(-)**	0.6392±2.43e-02(+)	0.3383±1.04e-01(+)	0.2264±1.37e-02(+)	0.8508±3.56e-02
	10, 5	**0.9674±2.15e-02(-)**	0.6378±1.65e-02(+)	0.3006±1.46e-01(+)	0.2200±1.05e-02(+)	0.8671±1.43e-02
	10, 10	**0.9683±2.06e-02(-)**	0.6421±2.04e-02(+)	0.3689±1.49e-01(+)	0.2238±1.22e-02(+)	0.8739±2.29e-02
DF9	5, 10	**0.9946±1.28e-02(-)**	0.9530±4.65e-02(=)	0.7751±1.87e-01(+)	0.8053±3.60e-02(+)	0.9261±3.81e-02
	10, 5	**0.9983±2.14e-03(-)**	0.9759±8.82e-03(=)	0.8130±1.73e-01(+)	0.7497±3.13e-02(+)	0.9414±3.94e-02
	10, 10	**0.9992±1.60e-03(-)**	0.9707±1.53e-02(-)	0.9023±1.29e-01(=)	0.7809±5.64e-02(+)	0.9170±4.44e-02
DF10	5, 10	0.8952±1.15e-02(+)	0.9087±3.85e-03(-)	0.6635±1.44e-01(+)	0.8159±3.05e-02(+)	**0.9102±2.08e-02**
	10, 5	0.8826±3.44e-02(+)	0.8935±5.35e-05(+)	0.7003±1.98e-01(+)	0.8094±2.06e-02(+)	**0.9191±2.88e-02**
	10, 10	0.8850±3.65e-02(+)	0.8772±8.68e-09(+)	0.8149±1.20e-01(+)	0.8027±2.25e-02(+)	**0.9153±2.44e-02**
DF11	5, 10	0.9725±2.86e-02(+)	0.9882±1.28e-02(=)	0.8530±1.44e-02(+)	0.9606±5.77e-03(+)	**0.9986±1.39e-03**
	10, 5	0.7343±1.99e-02(+)	**0.9883±5.09e-03(-)**	0.7523±6.39e-03(+)	0.9592±6.01e-03(-)	0.9523±4.30e-03
	10, 10	0.7247±4.19e-02(+)	**0.9920±4.66e-03(-)**	0.7639±2.22e-02(+)	0.9612±9.65e-03(-)	0.9513±2.03e-03
DF12	5, 10	0.5946±1.16e-01(+)	0.7350±8.84e-02(+)	0.5985±5.25e-02(+)	0.6382±3.05e-02(+)	**0.7943±6.41e-02**
	10, 5	0.7137±9.75e-02(+)	0.6971±4.75e-02(+)	0.5514±7.51e-03(+)	0.6026±3.71e-02(+)	**0.7986±7.50e-02**
	10, 10	0.7094±5.77e-02(=)	0.6731±4.06e-02(+)	0.5543±7.65e-03(+)	0.6036±4.31e-02(+)	**0.7499±9.66e-02**
DF13	5, 10	**0.9852±2.21e-02(-)**	0.9962±1.61e-03(-)	0.9265±5.56e-02(+)	0.8499±2.20e-02(+)	0.9526±2.01e-02
	10, 5	**0.9982±5.58e-03(-)**	0.9968±1.07e-03(-)	0.9470±4.43e-02(=)	0.8431±1.68e-02(+)	0.9504±2.83e-02
	10, 10	**0.9997±1.14e-03(-)**	0.9966±4.10e-03(-)	0.9184±5.90e-02(+)	0.8378±1.61e-02(+)	0.9563±1.89e-02
DF14	5, 10	0.8935±1.65e-02(+)	0.9245±1.06e-02(+)	0.7087±8.65e-02(+)	0.9215±1.40e-02(=)	**0.9326±1.13e-02**
	10, 5	**0.9930±3.10e-02(-)**	0.9415±1.63e-02(-)	0.6398±7.91e-02(+)	0.9139±1.12e-02(-)	0.8729±1.12e-02
	10, 10	**0.9926±3.28e-02(-)**	0.9582±4.06e-02(-)	0.6268±5.33e-02(+)	0.9126±1.44e-02(-)	0.8723±1.23e-02
+ / - / =	\	23 / 15 / 4	20 / 13 / 9	29 / 1 / 12	34 / 6 / 2	\

considerable overall performance via rational integration. In addition, it can be inferred that to sufficiently exploit the merits of those meticulously designed advanced methods, an in-depth analysis on the specific situation is necessary and helpful, which indicates the reliability and superiority of the core idea in the proposed quantification-based hierarchical scheme. Simultaneously, above inference motivates us to provide more thorough insights on the dynamic behaviors, and to seek potential integration of some state-of-the-art response strategies.

D. Generalization Ability of HRS

As a comprehensive approach to deal with the dynamic behaviors in DMOPs, the proposed HRS is proven competitive in above presented experimental results. To further validate the effectiveness and generalization ability of HRS, in this subsection, another well-known algorithm NSGA-II is

TABLE VI: Comparisons with other hybrid algorithms on DF series problems in terms of MIGD

Problems	n_t, τ_t	Algorithms		
		SVM-DMOA [24]	PPS-RM-MEDA [71]	HRS-DMOA
DF1	10, 10	0.4310±4.85e-02(3)	**0.0365±7.34e-03**(1)	0.0884±1.80e-02(2)
	10, 5	1.3278±8.55e-02(3)	0.0949±2.11e-02(2)	**0.0859±1.94e-02**(1)
DF2	10, 10	0.2962±2.26e-02(3)	**0.0408±8.58e-03**(1)	0.0482±1.37e-02(2)
	10, 5	0.8960±6.07e-02(3)	0.0917±7.86e-03(2)	**0.0419±1.31e-02**(1)
DF3	10, 10	0.5159±5.47e-02(2)	**0.2012±6.02e-03**(1)	0.5916±1.57e-01(3)
	10, 5	1.2482±2.07e-01(3)	**0.2809±3.57e-02**(1)	0.5680±1.44e-01(2)
DF4	10, 10	**0.1219±3.83e-03**(1)	1.0118±1.78e-02(3)	0.4310±7.83e-02(2)
	10, 5	**0.1990±4.03e-02**(1)	1.0861±1.48e-02(3)	0.4296±6.97e-02(2)
DF5	10, 10	0.1482±4.32e-02(2)	1.1942±1.30e-02(3)	**0.0366±5.65e-03**(1)
	10, 5	1.2815±2.70e-01(2)	1.3787±6.25e-02(3)	**0.0367±9.04e-03**(1)
DF6	10, 10	3.9768±5.77e-01(3)	3.0892±3.07e-01(2)	**1.2066±6.06e-01**(1)
	10, 5	9.5840±1.09e+00(3)	6.5176±5.38e-01(2)	**1.7784±6.44e-01**(1)
DF7	10, 10	**0.5220±5.70e-02**(1)	2.9774±2.83e-01(3)	1.1298±1.53e-01(2)
	10, 5	**0.6959±1.22e-01**(1)	5.7018±5.14e-01(3)	1.2286±1.74e-01(2)
DF8	10, 10	**0.0752±1.12e-02**(1)	0.8673±7.93e-03(3)	0.1108±3.15e-02(2)
	10, 5	0.1940±1.42e-02(2)	0.7860±2.10e-02(3)	**0.1232±3.16e-02**(1)
DF9	10, 10	0.4151±6.63e-02(2)	1.5801±1.13e-02(3)	**0.1965±2.64e-02**(1)
	10, 5	1.0983±1.63e-01(2)	1.6691±3.88e-02(3)	**0.1993±3.56e-02**(1)
DF10	10, 10	0.6037±3.84e-02(3)	**0.1389±1.84e-03**(1)	0.3230±2.76e-02(2)
	10, 5	0.6188±3.34e-02(3)	**0.1874±2.68e-03**(1)	0.3283±2.28e-02(2)
DF11	10, 10	0.6717±2.15e-03(3)	**0.1565±1.09e-02**(1)	0.3666±2.20e-03(2)
	10, 5	0.6933±3.06e-03(3)	**0.1783±7.75e-03**(1)	0.3677±2.78e-03(2)
DF12	10, 10	**0.3829±3.86e-03**(1)	1.1756±5.61e-03(3)	0.6236±1.69e-01(2)
	10, 5	**0.4262±6.10e-03**(1)	1.1760±5.61e-03(3)	0.4643±1.81e-01(2)
DF13	10, 10	0.5733±5.07e-02(2)	1.3815±2.58e-02(3)	**0.2527±1.26e-02**(1)
	10, 5	1.4883±2.57e-01(2)	1.6325±3.88e-02(3)	**0.2556±1.20e-02**(1)
DF14	10, 10	0.2556±5.73e-02(2)	0.8579±7.83e-03(3)	**0.1231±3.97e-03**(1)
	10, 5	0.8887±2.44e-01(2)	1.0334±4.22e-02(3)	**0.1216±3.71e-03**(1)

TABLE VII: Comparisons with the state-of-the-art MoE algorithm on JY series problems in terms of MIGD

Problems	n_t, τ_t	Algorithms	
		MoE [43]	HRS-DMOA
JY1	10, 10	**2.43e-2±4.24e-3**	1.16e-1±2.05e-2
	10, 20	**9.36e-3±6.17e-4**	1.10e-1±1.99e-2
	10, 30	**7.01e-3±2.43e-4**	1.01e-1±1.80e-2
JY2	10, 10	1.68e-1±1.63e-3	**3.11e-2±1.39e-3**
	10, 20	1.64e-1±1.46e-4	**3.28e-2±1.51e-3**
	10, 30	1.63e-1±7.22e-5	**3.20e-2±1.87e-3**
JY3	10, 10	3.16e-1±6.93e-3	**1.51e-2±2.91e-2**
	10, 20	3.12e-1±1.88e-3	**1.56e-2±2.97e-2**
	10, 30	3.13e-1±2.77e-3	**1.44e-2±2.46e-2**
JY4	10, 10	1.51e-1±1.78e-3	**9.09e-2±2.60e-2**
	10, 20	1.36e-1±2.86e-4	**7.43e-2±1.76e-2**
	10, 30	1.35e-1±8.84e-5	**8.47e-2±2.01e-2**
JY5	10, 10	**9.89e-3±4.54e-4**	1.51e-1±4.95e-2
	10, 20	**7.59e-3±1.99e-4**	1.73e-1±6.03e-2
	10, 30	**7.12e-3±1.70e-4**	1.65e-1±3.97e-2
JY6	10, 10	1.64e+0±1.35e-1	**5.34e-1±6.53e-1**
	10, 20	9.45e-1±1.01e-1	**3.96e-1±6.13e-1**
	10, 30	5.86e-1±4.90e-2	**2.90e-1±1.40e-1**
JY7	10, 10	2.69e+0±5.09e-1	**1.20e+0±1.88e-1**
	10, 20	2.27e+0±8.53e-1	**1.31e+0±2.65e-1**
	10, 30	2.20e+0±6.58e-1	**1.30e+0±1.96e-1**

employed as the static optimizer, and the integrated HRS-NSGA-II is evaluated on the 14 benchmark problems with $(n_t, \tau_t, \tau) = (10, 10, 200)$. Moreover, the evaluation results are compared with the baseline algorithm D-NSGA-II and other two state-of-the-art methods, namely KGB [61] and TCD [60], respectively, which are integrated to the NSGA-II algorithm as well to form the DMOAs.

According to the results reported in Table VIII, it is found that our HRS-NSGA-II yields 10 best MIGD out of the 14 benchmarks, which is also an inspiring result. On one hand, it exhibits the superiority of the proposed HRS in comparison to other advanced response approaches to the environmental changes; on the other hand, the proposed HRS is demonstrated a flexible plug-and-play module with considerable generalization ability, which can be integrated with any other static optimizer to effectively solve the DMOPs.

E. Sensitivity Analysis of Thresholds LT and HT

In the proposed HRS-DMOA, the thresholds LT and HT play important roles in grading the change degree to adopt different response strategies. Hence, in this subsection, influences of the two thresholds are investigated.

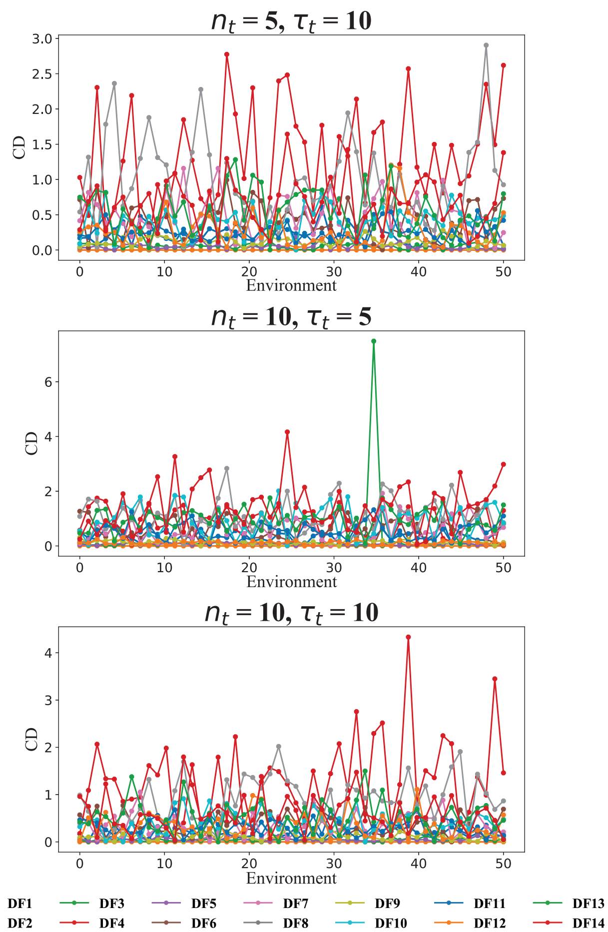

Fig. 2: Change degree of 14 DF problems.

As previously mentioned that, in essence, LT and HT determine the proportion of selecting three response strategies, which also depends on the change degree CD of the environment. In Fig. 2, the changing curves of CD are illustrated in the three given dynamic conditions of all test problems. Based on the CD values shown in Fig. 2, three proportions of adopting refinement, TL and re-initialization response modes are

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TABLE VIII: Comparisons against other DMOAs with NSGA-II serving as the static optimizer in terms of MIGD

Problems	Algorithms			
	KGB-NSGA-II	TCD-NSGA-II	DNSGA-II-B	HRS-NSGA-II
DF1	0.3113±1.39e-03	**0.1887±4.93e-02**	0.7182±1.54e-01	0.2257±4.71e-02
DF2	0.2198±1.64e-03	**0.1146±1.74e-02**	0.4638±1.12e-01	0.1380±1.91e-02
DF3	0.9593±1.19e-02	0.5371±6.67e-02	1.0556±2.24e-01	**0.4983±2.02e-01**
DF4	3.1503±2.84e-01	**0.1487±4.20e-02**	2.0526±2.28e-01	0.1812±1.76e-02
DF5	0.4004±3.05e-03	0.3427±1.46e-01	0.6813±6.67e-02	**0.0232±4.40e-03**
DF6	19.8500±4.97e+00	6.6381±6.96e-01	6.0010±1.07e+00	**3.3446±9.41e-01**
DF7	**0.4575±5.85e-03**	8.0032±3.77e+00	1.1040±1.23e-01	0.5021±5.62e-02
DF8	0.2130±2.51e-04	0.3057±2.54e-02	0.4431±1.08e-01	**0.0441±8.75e-03**
DF9	0.5932±6.34e-03	0.5954±1.42e-01	2.2331±5.79e-01	**0.2155±3.39e-02**
DF10	0.1892±4.71e-04	0.1814±1.70e-02	1.3771±3.14e-01	**0.1317±8.44e-03**
DF11	10.3910±3.06e-02	0.2670±4.09e-02	1.0163±2.24e-01	**0.2108±1.73e-02**
DF12	0.5650±3.03e-03	4.4655±5.09e+00	0.7770±6.64e-02	**0.2290±2.53e-02**
DF13	0.5854±2.49e-03	0.7456±2.24e-01	0.6401±1.10e-01	**0.2603±1.30e-02**
DF14	0.2267±5.72e-04	0.8972±5.40e-01	0.5049±1.23e-01	**0.1907±3.05e-03**

investigated, including $10\% : 80\% : 10\%,\ 15\% : 70\% : 15\%$ and $20\% : 60\% : 20\%$, where the corresponding (LT, HT) takes value from $\{(0.01, 0.97),\ (0.03, 0.78),\ (0.05, 0.68)\}$, respectively. Using IGD as the evaluation metric, algorithm performances with different settings of (LT, HT) are shown in Fig. 3, where the dynamic parameters of all 14 benchmark functions are fixed at $\tau_t = 10,\ n_t = 10$.

As is shown, when LT and HT are set to 0.03 and 0.78, the algorithm presents a more stable performance. To be specific, poor convergence is found on problems DF3 and DF6 when $LT = 0.01,\ HT = 0.97$. It may because that the two thresholds are too marginalized to enable the hierarchical response system timely adapt to the varying environments such as the concave/convex change on a certain objective function. On the contrary, if LT and HT are set to 0.05 and 0.68, the designed response system becomes highly sensitive to the environmental changes, which can be reflected from the sharp variation tendency of the IGD value on problems DF12 and DF13. Under this circumstance, once a disconnected time-varying PF occurs, the algorithm is likely to treat it as a dramatic change and accordingly take the re-initialization strategy, which may cause adverse effects on the convergence. Therefore, it is also inappropriate to set too centralized thresholds, which cannot take full advantages of the hierarchical response system. As a result, $LT = 0.03$ and $HT = 0.78$ are recommended and they are adopted in other reported experiments in this study.

In addition, we also make another attempt to adaptively select the response modes (rather than based on the fixed thresholds), which benefits further strengthening the connections between the response manner and the severity of environmental changes. To be specific, by taking the average value of CD (in Eq. (5)) regarding to the cardinality of sensor set S, an environmental changing degree is obtained as $E_{cd} = \frac{1}{|S|}\max\{CD_i\}\ (i = 1, 2, ..., m)$, which is deemed as the objective-wise influences of the current change to all sensors in average level. Meanwhile, one can also obtain the maximum changing degree for each sensor (S_{cd}) in objective-wise by:

$$S_{cd} = \max\ \frac{f_{i,j}(t) - f_{i,j}(t-1)}{f_{i,j}(t-1) + \mu},\ i = 1, 2, ..., m \tag{12}$$

where j stands for the sensor individual.

Accordingly, if it satisfies that $S_{cd} \geq E_{cd}$, it can be regarded that the corresponding individual is greatly influenced by the fluctuation in environment, which is not quite a reliable solution in the new environment. By calculating the proportion of those unreliable sensors to the whole sensor set, severity of the changes can be reflected. Then, considering the three optional response modes, the interval $(0,\ 1)$ is uniformly divided into three sub-intervals, thereby realizing the adaptive selection of different response modes. (1) If more than two thirds of sensors are unreliable, then the change is deemed severe and the re-initialization strategy is applied; (2) in case of fewer than one third of the sensors are unreliable, the refinement mode is adopted; (3) otherwise, the TL-based response is taken.

Benchmark valuation results obtained by the HRS with fixed thresholds $(LT, HT) = (0.03, 0.78)$ and the HRS with adaptive thresholds (denoted as HRS*) are displayed in Table IX, where it is found that the adaptive thresholds also enable the proposed HRS to effectively handle the dynamic behaviors with satisfactory convergence. Moreover, according to the rank sum test results, HRS* presents equivalent performance as compared to HRS in six cases, and the former even outperforms the latter on DF3, which is a promising finding that motivates us to explore other adaptive threshold setting manners and, by doing so, the subjectivity in determining response modes can be effectively eliminated so as to establish a highly generalized model that can adapt to diverse changing situations by connecting the change severity with the responses.

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Fig. 3: Influences of different LT and HT on IGD values ($\tau_t = 10,\ n_t = 10$).

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TABLE IX: Fixed thresholds vs. adaptive thresholds in terms of MIGD where $(n_t, \tau_t, \tau) = (10, 10, 200)$

Problems	HRS(fixed thresholds)	HRS*(adaptive thresholds)
DF1	**0.0778±4.95e-02**	0.0889±4.85e-02(=)
DF2	**0.0562±1.85e-02**	0.0586±2.80e-02(=)
DF3	0.5146±1.88e-01	**0.2561±7.48e-02(-)**
DF4	**0.3584±8.47e-02**	0.6212±2.35e-01(+)
DF5	**0.0417±8.90e-03**	0.0438±2.09e-02(=)
DF6	**1.3116±7.17e-01**	2.2021±1.63e+00(=)
DF7	**0.4297±5.99e-02**	0.5761±2.01e-01(+)
DF8	**0.1268±6.17e-02**	0.1423±1.14e-01(=)
DF9	**0.2216±4.22e-02**	0.2687±5.79e-02(+)
DF10	**0.2659±5.77e-02**	0.3966±8.04e-02(+)
DF11	**0.1586±1.44e-02**	0.5913±4.04e-02(+)
DF12	0.3370±1.22e-01	**0.2974±4.21e-03(=)**
DF13	**0.2506±2.24e-02**	0.2905±1.55e-02(+)
DF14	**0.1043±9.09e-03**	0.2141±4.88e-03(+)
+ / - / =	\	7 / 1 / 6

F. Investigation on Single TL-based Response Mode

Recently, developing the TL-based DMOAs has become the novel frontier of solving DMOPs [51], which can realize the seamless combination of the memory- and prediction-based response strategy. In this regard, the proposed HRS further integrates the merits of diversity enhancement when making responses. To validate whether such designing is useful, an algorithm variant with single response namely SR-DMOA is investigated, where only the TL-based response is adopted regardless of the change degree of environment. The comparison results in terms of both the MIGD indicator and the average runtime are reported in Table X, where dynamic parameters are set to $\tau_t = 10,\ n_t = 10$.

TABLE X: Performance comparison against single TL-based response in terms of MIGD and average runtime $(\tau_t = 10,\ n_t = 10)$

Problems	MIGD		Average runtime (seconds)	
	HRS-DMOA	SR-DMOA	HRS-DMOA	SR-DMOA
DF1	**0.1336**	0.1653	**2.4201**	4.2164
DF2	**0.0597**	0.1148	**1.2067**	3.025
DF3	**0.1871**	0.1888	**1.5432**	2.4524
DF4	**0.6758**	0.9439	2.192	**1.9164**
DF5	0.3324	**0.2464**	**2.4921**	3.3526
DF6	1.6638	**0.9891**	**3.352**	3.482
DF7	31.4148	**28.246**	**2.4963**	2.728
DF8	**0.8603**	1.3746	3.9365	**3.6527**
DF9	1.521	**1.0455**	**2.708**	3.064
DF10	**0.2062**	0.2168	6.2245	**5.4015**
DF11	**0.1722**	0.2034	6.0723	**5.6432**
DF12	**0.8925**	0.9125	**2.4647**	2.7542
DF13	**0.2813**	0.2832	7.5568	**5.7128**
DF14	**0.1367**	0.1613	7.2126	**5.7605**

According to Table X, SR-DMOA only achieves 4 best MIGD results out of 14 problems, and in none of the cases that SR-DMOA can outperform the original HRS-DMOA in terms of both MIGD and runtime. It is noticeable that on problems DF1, DF2, DF3 and DF12, the proposed HRS-DMOA spends less time and achieves better performance than the variant SR-DMOA, whose reason lies in that if the environment only has slight changes, taking TL-based response will increase the computational burden with little performance enhancement. On the contrary, if the TL method is used in case of dramatic changes, the negative transfer phenomenon may occur that can lead to poor convergence. Consequently, it can be concluded that the designed HRS can enable flexible responses to the varying environments, which effectively combines the merits of different mainstream strategies to pursue comprehensive performance improvement.

G. Ablation Study on HRS

In this subsection, extensive ablation studies are carried out to validate the effectiveness of the proposed HRS. In particular, five bi- (DF1-DF5) and tri-objective (DF10-DF14) problems are selected for performance evaluation in this group of experiments, where the dynamic parameters are set as $(n_t, \tau_t) = (5, 10)$, and each algorithm is run 20 times to report the MIGD results in average level.

1) Influences of the Quantification-based Hierarchy: Firstly, in the proposed HRS, selection criterion of the different response modes is based on the quantification of environmental changes. To validate whether the quantification-based response-making is effective, another variant of the proposed algorithm is designed, which is named RRS-DMOA that takes the same three response modes with random probability.

TABLE XI: Effectiveness of the quantification-based hierarchical response modes in terms of MIGD

Problems	Algorithms	
	HRS-DMOA	RRS-DMOA
DF1	**0.0657±8.28e-03**	0.8467±4.68e-01
DF2	**0.0474±1.55e-02**	0.5827±1.72e-01
DF3	**0.2795±1.32e-01**	0.6866±2.29e-01
DF4	**0.6621±3.52e-02**	1.2031±1.48e-01
DF5	**0.0639±1.78e-02**	1.8872±4.68e-01
DF10	**0.1820±9.42e-03**	0.2170±2.25e-02
DF11	**0.1486±2.84e-03**	0.1899±1.10e-02
DF12	**0.4715±1.15e-01**	0.5050±3.26e-02
DF13	**0.2710±1.16e-02**	2.0003±3.20e-01
DF14	**0.1342±3.63e-03**	1.2288±2.74e-01

According to the results displayed in Table XI, it is found that the proposed HRS-DMOA outperforms the variant RRS-DMOA on all of the adopted test problems with overwhelming advantages. In particular, only on the four problems of DF3 and DF10-DF12 can random response modes obtain similar results as the hierarchical ones. In other cases, the proposed HRS presents significant convergence improvement, which may due to that although the RRS can also exploit merits of different response strategies, the randomness-based response-making lacks analysis on the specific environment. Hence, it can lead to the inappropriate responses, like to transfer previous knowledge in the extremely fluctuating situations. Particularly, on problem DF5, an improvement on MIGD by

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TABLE XII: Ablation study on different response modes in terms of MIGD

		HRS	HRS_v1	HRS_v2	HRS_v3
Response modes	Refinement	✓	✗	✓	✓
	TL	✓	✓	✗	✓
	Re-initialization	✓	✓	✓	✗
Problems	DF1	**0.0657±8.28e-03**	0.0844±2.34e-02	0.2940±6.52e-02	0.0872±1.78e-02
	DF2	**0.0474±1.55e-02**	0.0577±1.57e-02	0.2801±1.29e-01	0.0553±1.83e-02
	DF3	0.2795±1.32e-01	**0.1732±1.78e-02**	0.8096±1.45e-01	0.2659±1.27e-01
	DF4	0.6621±3.52e-02	0.7027±4.44e-02	**0.6396±1.91e-01**	0.7253±9.31e-02
	DF5	**0.0639±1.78e-02**	0.0897±3.92e-02	2.1753±8.03e-02	0.0732±2.88e-02
	DF10	**0.1820±9.42e-03**	0.1939±8.98e-03	0.2442±2.83e-02	0.2355±1.32e-01
	DF11	**0.1486±2.84e-03**	0.1508±2.34e-03	0.1817±9.70e-03	0.1507±3.41e-03
	DF12	0.4715±1.15e-01	**0.4171±4.10e-03**	0.5271±2.42e-02	0.5421±1.35e-01
	DF13	**0.2710±1.16e-02**	0.2876±1.64e-02	4.1604±1.79e-01	0.2806±9.69e-03
	DF14	**0.1342±3.63e-03**	0.1372±4.26e-03	1.6557±4.68e-02	0.1360±4.02e-03

28.53% is achieved by the quantification-based hierarchical responses, which indicates that it is necessary to analyze the environment so as to make flexible and efficient responses. In addition, from the scatter plot illustrated in Fig. 4, it can be observed that the proposed HRS-DMOA can track the time-varying PF better than RRS-DMOA, which demonstrates the superiority of the quantification-based response-making manner.

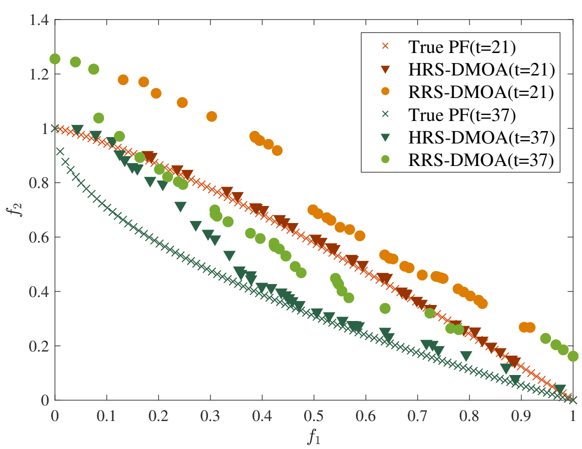
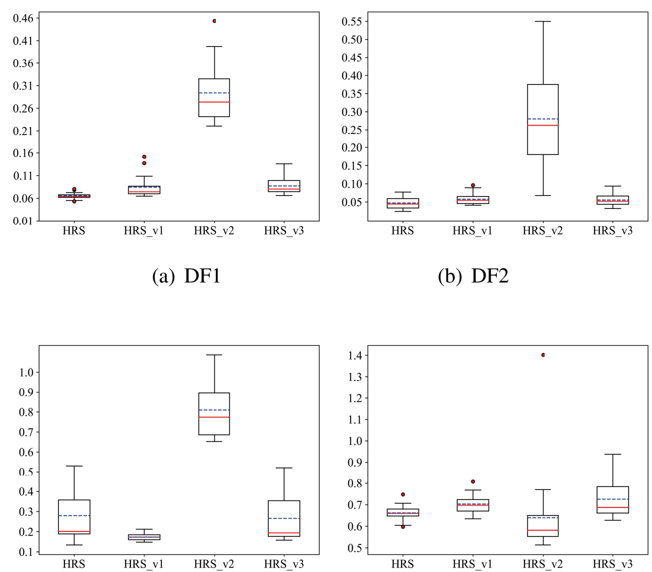


Fig. 4: Scatter plots on DF1 of the two algorithms (HRS-DMOA vs. RRS-DMOA) at the 21- and 37-th environments.

2) Effectiveness of the Response Modes: Secondly, in the proposed HRS, three response modes are adopted, and to verify the effectiveness of them, three variants of the proposed algorithm are designed by removing each mode from the HRS respectively, which are termed as HRS_v1 (without refinement), HRS_v2 (without TL), and HRS_v3 (without re-initialization). The comparison results are reported in Table XII, where "✓" and "✗" denote whether a certain response mode is contained in corresponding algorithm or not. Moreover, the box plots regarding to MIGD of problems DF1-DF4 are illustrated in Fig. 5, and the PF obtained by the four algorithms is visualized in Fig. 6.



(a) DF1 (b) DF2

(c) DF3 (d) DF4

Fig. 5: Box plots on four benchmark problems of the four algorithms, where the red circles refer to some outliers, the red solid and blue dashed lines represent the median and average value, respectively.

According to the above presented results, we have following findings:

1) On most benchmark problems, the original HRS yields the best convergence performance, which validates that the proposed HRS can flexibly handle different dynamic behaviors, and some variances in 10^{-3} level also present a stable performance.

2) When the TL-based response is removed, the HRS_v2 presents the most severe performance declination in many

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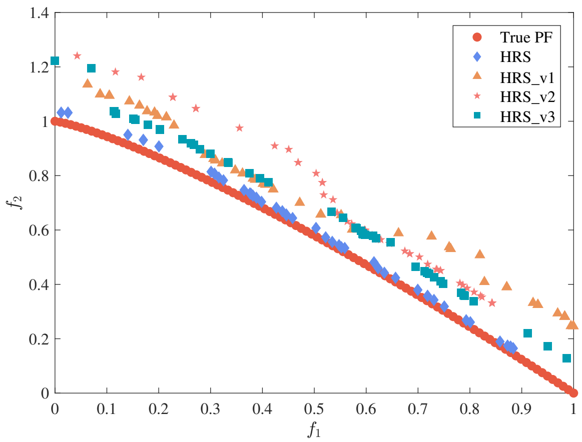

Fig. 6: Scatter plots on DF1 of the four hierarchical response systems at the 21-th environment.

cases such as on problem DF1, where the MIGD is as nearly five times as that obtained by HRS, which indicates the importance of the TL-based response mode.

3) On tri-objective problems DF10 and DF11, the four algorithms obtain similar results, while slight convergence improvement can be achieved by the original HRS, which implies that the complementary performance is realized in the proposed algorithm.

4) The response made by the proposed HRS may not always be the best choice, as can be seen that on DF3 and DF4, the variants HRS_v1 and HRS_v2 obtain the best result, respectively. The reason may be that the thresholds of HRS are manually set.

5) Without the re-initialization mode, none of the best MIGD is obtained by the HRS_v3, while in some cases like DF2 and DF14, the HRS_v3 can slightly outperform the HRS_v1 and HRS_v2. Hence, it can be concluded that the diversity-based response can still be helpful.

6) From the results on DF4, it can be inferred that the TL-based response may even cause adverse effects due to the negative transfer phenomenon, which simultaneously implies the superiority of the rational integration of diverse strategies as none of the single response can always be the best.

Based on above discussions, it can be concluded that the proposed HRS does realize the systematical integration of different mainstream response strategies, which is effective in handling various dynamic behaviors in DMOPs. Moreover, the quantification of environmental changes is also proven necessary to some extents, which motivates us to develop other novel schemes to comprehensively quantify the dynamic behaviors in our future work.

H. Outlook for Future Work

In addition to above satisfactory results, it is worth pointing out that there are still spaces for improvements on the proposed HRS-DMOA, where the most important issue is that some state-of-the-art novel strategies are not well considered in the proposed HRS.

In future, we aim to 1) propose other advanced response strategies to the environmental changes; 2) seek potential integration of some state-of-the-art DMOAs for further performance enhancement; 3) develop a learnable DMOP optimizer that can realize adaptive parameter configuration; 4) employ other advanced population-based heuristic algorithms as the static optimizer [29], [47], [63], which can improve the efficiency of obtaining the Pareto solutions; 5) investigate more comprehensive quantification manners of environmental changes so as to provide in-depth and thorough insights on dynamic behaviors, which is also beneficial to the research on other dynamic systems [19], [65]; 6) improve and apply our algorithm to some real-world optimization problems such as the complex system modeling [30] and the influence maximization of complex networks [41] so as to validate and enhance the engineering practicality of the proposed HRS-DMOA.

V. Conclusions

In this paper, a novel DMOA has been proposed based on a hierarchical response system, whose main idea is to realize rational and efficient integration of the mainstream methods so as to enhance comprehensive performance. The environmental changes have been quantified as three levels and, by doing so, different response modes can be adopted accordingly to flexibly handle various dynamic behaviors in DMOPs.

Benchmark evaluations have been carried out on 14 DMOPs, and the results show that the proposed HRS-DMOA outperforms other four popular baseline DMOAs in terms of both convergence and diversity. In addition, extensive ablation studies have been carried out to validate the superiority of the proposed HRS. On one hand, as compared with randomly taking different responses, the quantification-based hierarchical response-making has shown overwhelming advantages, which proves the necessity of in-depth analysis on the dynamic behaviors in DMOPs. On the other hand, performance declination to different extents has been observed when some certain response mode is removed from the HRS, which demonstrates the effectiveness of integrating different mainstream methods.

Although some satisfactory results have been obtained, several important issues of the proposed HRS-DMOA deserve further attention, which mainly include (1) insufficient concerns on integrating novel state-of-the-art response strategies; (2) manual intervention on the threshold parameters of HRS.

In future, we tend to propose some novel response methods with other advanced techniques, and to develop a learnable optimizer that can avoid the manual parameterization is also a feasible and innovative direction. Then, how to comprehensively quantify the dynamic behaviors from the aspects of both changing severity and frequency can provide us with a more thorough understanding on the essence of the DMOPs. In addition, we are prone to integrate the proposed HRS to other swarm-intelligence-based static optimizer, and it is also promising to apply the HRS-DMOA to more real-world optimization scenes to validate its practicality.

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