This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


# Maximum Correntropy Filtering for Complex Networks With Uncertain Dynamical Bias: Enabling Component-Wise Event-Triggered Transmission

Weihao Song, Zidong Wang, Zhongkui Li, Qing-Long Han and Dong Yue

***Abstract*—This paper is concerned with the maximum correntropy filtering problem for a class of nonlinear complex networks subject to non-Gaussian noises and uncertain dynamical bias. With aim to utilize the constrained network bandwidth and energy resources in an efficient way, a component-wise dynamic event-triggered transmission (DETT) protocol is adopted to ensure that each sensor component independently determines the time instant for transmitting data according to the individual triggering condition. The principal purpose of the addressed problem is to put forward a dynamic event-triggered recursive filtering scheme under the maximum correntropy criterion such that the effects of the non-Gaussian noises can be attenuated. In doing so, a novel correntropy-based performance index (CBPI) is first proposed to reflect the impacts from the component-wise DETT mechanism, the system nonlinearity and the uncertain dynamical bias. The CBPI is parameterized by deriving upper bounds on the one-step prediction error covariance and the equivalent noise covariance. Subsequently, the filter gain matrix is designed by means of maximizing the proposed CBPI. Finally, an illustrative example is provided to substantiate the feasibility and effectiveness of the developed maximum correntropy filtering scheme.**

***Index Terms*—Complex networks, maximum correntropy filtering, non-Gaussian noises, dynamical bias, dynamic event-triggered protocol.**

## I. Introduction

Along with the burgeoning information technologies with applications in network sciences, the past several decades have witnessed a steadily growing research interest in complex networks that are capable of modeling a wide variety of real-world systems. Some representative complex networks include, but are not limited to, social networks, airport networks, World Wide Web, electric power grids, metabolic networks, neural networks, and genetic regulatory networks [6], [33], [36], [48]. Roughly speaking, a typical complex network is constituted by a group of interplayed dynamical nodes where each individual node shares its local information with its neighbors with aim to accomplish certain missions in a collaborative manner.

Inspired by their theoretical significance and extensive applicability, the dynamical analysis issues for complex networks have drawn a large amount of research attention leading to many excellent results reported in the literature, see e.g. [35], [42]. For example, the output synchronization issue has been considered in [41] for uncertain general complex networks based on the neural sliding-mode pinning control strategy. In [29], the finite/fixed-time synchronization issue has been tackled for complex networks subject to stochastic disturbances by means of a unified pinning controller with regulatable power parameters.

There has been an ever-increasing demand for acquiring the states of underlying system plants in general practice such as monitoring and control of process engineering. Correspondingly, the state estimation (or filtering) issues have become a research focus for several decades and a great number of effective estimation/filtering schemes have been put forward for various systems with different performance requirements, see e.g. [3], [4], [23], [49] and the references therein. In particular, in the context of complex networks, the relevant results have been obtained based primarily on the Kalman-like (or extended-Kalman-like) filtering approach [11], [25], the $H_\infty$ filtering approach [43], [50], and the set-membership filtering approach [27], [39]. It should be noted that most existing results are predominantly applicable to Gaussian noises as well as energy- or norm-bounded noises.

In engineering practice, owing for example to the signal reflection and impulsive electromagnetic interference, non-Gaussian noises are frequently encountered in many practical scenarios such as underwater acoustic localization and ballistic target tracking [18], [21]. As such, much effort has been devoted to the treatment of non-Gaussian noises and several effective strategies have been developed, for instance, the celebrated particle filter [2], the Gaussian sum filter [1] and the variational Bayesian filter [57], to name just a few. Among others, the maximum correntropy filter [5], [16] has stood out as an easy-to-implement yet efficacious approach to handle

This work was supported in part by the National Natural Science Foundation of China under Grants 62203016, 61973006, T2121002 and 61933007, the Beijing Natural Science Foundation of China under Grant JQ20025, the China Postdoctoral Science Foundation under Grant 2021TQ0009, the Royal Society of the UK, and the Alexander von Humboldt Foundation of Germany.
*(Corresponding author: Zhongkui Li.)*

W. Song and Z. Li are with the State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Engineering Science, College of Engineering, Peking University, Beijing 100871, China. (Emails: weihao.song@pku.edu.cn, zhongkli@pku.edu.cn)

Z. Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk)

Q.-L. Han is with the School of Science, Computing and Engineering Technologies, Swinburne University of Technology, Melbourne, VIC 3122, Australia. (Email: qhan@swin.edu.au).

D. Yue is with the College of Automation and College of Artificial Intelligence, the Jiangsu Engineering Laboratory of Big Data Analysis and Control for Active Distribution Network, and the Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing 210023, China. (Email: medongy@vip.163.com).

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


the impulsive non-Gaussian noises and measurement outliers, where the core idea lies in the fact that the correntropy, as an information theoretic criterion, is able to reflect the higher-order statistics of the probability density function [28].

Due to its distinguishing merits of flexibility (in filter design), robustness (against outliers) and computational cost, the maximum correntropy filtering (MCF) scheme has begun to stir some initial attention and several excellent results have been reported in recent years [38], [40], [45], [46]. For example, a maximum correntropy Kalman filter, combined with the fixed-point method, has been developed in [5] for linear systems. In [22], the Chandrasekhar-type recursion has been derived for the maximum correntropy Kalman filtering scheme proposed in [16]. It is worth mentioning that limited work has been done towards the complex networks so far. One of the few available results is the maximum correntropy filter developed for *linear* complex networks undergoing random occurring cyber-attacks [37]. When it comes to *nonlinear* complex networks with engineering-oriented complexities, the corresponding MCF issue has not been thoroughly studied yet.

It has now been widely recognized that, in most practical applications, the system dynamics and/or sensor measurements might be affected by not only the modeled noises but also the unknown inputs (e.g. unknown disturbances, unmodeled dynamics, component faults and imperfect calibrations) [17], [19], [34]. Such unknown inputs, if not delicately tackled, are likely to incur severe performance deterioration and even failure of certain tasks (e.g. target tracking). In this regard, noticeable attention has been paid to the investigation on joint state and input estimation issues, see [9], [52], [53] and the references therein. It should be noted that, a kind of unknown inputs, namely, stochastic bias, has attracted a particular research interest in the literature [10], [15], [19], where a prerequisite for most existing results is that the bias dynamics can be precisely determined, which might not be the case in practice for many reasons such as environment-induced parameter perturbations. Taking the typical target tracking problem as an example, the actuators (e.g. rudders) of the targets of interest might undergo the uncertain dynamical bias in a rather complicated environment as a result of abrupt external disturbances, random operation errors and electromagnetic interferences [44]. Consequently, it makes practical sense to investigate the filtering problem for complex networks subject to *uncertain* dynamical bias.

With the nowadays popularity of networked systems, the network-induced imperfections (e.g. network traffic congestion) has grabbed considerable research attention from both academia and industry, see e.g. [14], [27], [55], and [54] for a recent survey. In particular, the so-called event-triggered transmission protocol has proven to be an effective approach to mitigating unnecessary network traffic, thereby extenuating the occurrence of network-induced imperfections [8], [20], [26], [30], [47], [56]. In comparison with its static counterpart, the dynamic event-triggered transmission (DETT) protocol has better potentials in economizing the limited network resources since its threshold can be dynamically adjusted, see e.g. [7], [12], [13], [24], [32] for some DETT-based results. It should be stressed that in most existing results, the measured outputs have been assumed to share a common triggering condition, which is sometimes unreasonable since the output magnitude and the required update frequency of each individual component might be different. Towards this direction, a component-wise DETT protocol has been proposed in investigating the state estimation problem for linear systems with non-uniform sampling, which allows for different triggering conditions for different components [58]. Nevertheless, the corresponding results for nonlinear complex networks have been very few (if not none), let alone the consideration of uncertain dynamical bias and maximum correntropy criterion.

Motivated by the aforementioned discussions, in this paper, we endeavor to handle the MCF problem for a class of nonlinear complex networks in the presence of non-Gaussian noises and uncertain dynamical bias under component-wise DETT protocol. In doing so, we foresee the following three essential challenges: 1) how to establish an appropriate model for complex networks with uncertain dynamical bias and non-Gaussian noises, where the measurement transmissions are scheduled by the component-wise DETT protocol? 2) how to design a correntropy-based performance index (CBPI) that quantifies the joint influence from the considered system complexities and transmission scheduling mechanism? and 3) how to deal with the analytical complexity induced by the component-wise DETT protocol? As such, the primary objective of this current investigation is to overcome the challenges listed above.

The major contributions of this paper are highlighted from the following three aspects: 1) the MCF problem is, for the first time, investigated for nonlinear and non-Gaussian complex networks subject to uncertain dynamical bias under the component-wise DETT protocol; 2) a novel CBPI is established, by resorting to the upper bounds on the prediction error covariance and the equivalent noise covariance, to take into account the impacts from uncertain dynamical bias and component-wise DETT protocol; and 3) an easy-to-implement algorithm is developed to realize the resource-saving filter, which is suitable for online computations with desired robustness against non-Gaussian noises.

The rest of this paper is outlined as follows. Section II specifies the problem under investigation, and presents the component-wise DETT mechanism as well as the MCF scheme. In Section III, the proposed CBPI is explicitly expressed by determining two weighted matrices, based on which the filter gain matrix is designed. In Section IV, simulation results are exhibited to verify the effectiveness of the proposed filtering algorithm. Finally, some concluding remarks are summarized in Section V.

**Notation**. The notations adopted throughout this paper are fairly standard. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean vector space. The superscripts $T$ and $-1$ stands for, respectively, the operation of transpose and inverse. $\|x\|$ and $\|x\|_A \triangleq (x^TAx)^{1/2}$ represent, respectively, the Euclidean norm and the weighted norm of a vector $x \in \mathbb{R}^n$, where $A$ is a positive definite matrix. $\mathbb{E}\{\cdot\}$ refers to the mathematical expectation operator. $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix. $0$ and $I$ denote, respectively, the zero matrix and the identity matrix of compatible dimensions. Other notations will be interpreted as

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


the need arises.

## II. Problem Formulation

Consider the following class of complex networks composed of $N$ coupled nodes with dynamical biases:

$$\bar{x}_{i,s+1} = \bar{A}_{i,s}\bar{x}_{i,s}+\bar{f}_s(\bar{x}_{i,s})+\sum_{j=1}^{N} d_{ij}\bar{\Gamma}\bar{x}_{j,s}+B_{i,s}z_{i,s}+\zeta_{i,s} \quad (1)$$

where $\bar{x}_{i,s} \in \mathbb{R}^{\bar{n}}$ denotes the state vector of the $i$-th node at time instant $s$, $D = [d_{ij}]_{N\times N}$ represents the coupled configuration matrix and $\bar{\Gamma} = \text{diag}\{\gamma_1,\gamma_2,\cdots,\gamma_{\bar{n}}\}$ stands for the inner coupling matrix. $\bar{A}_{i,s}$ and $B_{i,s}$ are the known matrices with appropriate dimensions. $\zeta_{i,s}$ refers to the process noise with zero mean and covariance $\bar{Q}_{i,s} > 0$. $z_{i,s} \in \mathbb{R}^k$ is the random bias with the following dynamics

$$z_{i,s+1} = (G_{i,s} + \Delta G_{i,s})z_{i,s} + \eta_{i,s} \quad (2)$$

where $G_{i,s}$ represents the known bias transition matrix and $\Delta G_{i,s}$ implies the perturbation term satisfying

$$\mathbb{E}\{\Delta G_{i,s+1}\} = 0,$$
$$\mathbb{E}\{(\Delta G_{i,s+1})(\Delta G^T_{i,s+1})\} \le \tau_i I$$

where $\tau_i$ is a given positive scalar, $\eta_{i,s}$ denotes the noise with zero mean and covariance $S_{i,s} > 0$. The known nonlinear function $\bar{f}_s(\cdot)$ satisfies $\bar{f}_s(0) = 0$ and

$$\left\|\bar{f}_s(u) - \bar{f}_s(v) - \bar{F}_s(u-v)\right\| \le \kappa_s\|u-v\| \quad (3)$$

where $u, v \in \mathbb{R}^{\bar{n}}$, $\bar{F}_s$ is a known matrix, and $\kappa_s \ge 0$ is a known scalar.

The measurement model of the $i$-th node is characterized by

$$y_{i,s} = \bar{C}_{i,s}\bar{x}_{i,s} + \nu_{i,s} \quad (4)$$

where $y_{i,s} \in \mathbb{R}^m$ denotes the measurement output and $\nu_{i,s}$ stands for the measurement noise with zero mean and covariance $R_{i,s} > 0$. Throughout this paper, we assume that $\zeta_{i,s}$, $\eta_{i,s}$, $\nu_{i,s}$, $\bar{x}_{i,0}$, $z_{i,0}$ and the elements of $\Delta G_{i,s}$ are mutually independent in $i$ and $s$.

Denote

$$x_{i,s} = \begin{bmatrix}\bar{x}^T_{i,s} & z^T_{i,s}\end{bmatrix}^T, A_{i,s} = \begin{bmatrix}\bar{A}_{i,s} & B_{i,s}\\ 0 & G_{i,s}\end{bmatrix},$$
$$\Delta A_{i,s} = \begin{bmatrix}0 & 0\\ 0 & \Delta G_{i,s}\end{bmatrix}, \Gamma = \begin{bmatrix}\bar{\Gamma} & 0\\ 0 & 0\end{bmatrix},$$
$$f_s(x_{i,s}) = \begin{bmatrix}\bar{f}_s(\bar{x}_{i,s})^T & 0\end{bmatrix}^T, \omega_{i,s} = \begin{bmatrix}\zeta^T_{i,s} & \eta^T_{i,s}\end{bmatrix}^T,$$
$$C_{i,s} = \begin{bmatrix}\bar{C}_{i,s} & 0\end{bmatrix}.$$

Then, the augmented dynamics can be obtained as follows:

$$\begin{cases} x_{i,s+1} = (A_{i,s} + \Delta A_{i,s})x_{i,s} + f_s(x_{i,s}) \\ \qquad\qquad + \sum_{j=1}^{N} d_{ij}\Gamma x_{j,s} + \omega_{i,s} \\ y_{i,s} = C_{i,s}x_{i,s} + \nu_{i,s}. \end{cases} \quad (5)$$

### A. Component-wise dynamic event-triggered mechanism

In this paper, to ameliorate the utilization efficiency of the limited network bandwidth and energy resources, the component-wise DETT mechanism is employed to schedule the process of data transmission. To this end, the measurements of the $i$-th node are rewritten by

$$y_{i,s} = \begin{bmatrix}y_{i,1,s} & y_{i,2,s} & \cdots & y_{i,m,s}\end{bmatrix}^T$$

where $y_{i,l,s}$ $(l = 1, 2, \cdots, m)$ denotes the measurement output of the $l$-th sensor component for the $i$-th node.

Define the event generator function $g(r_{i,l,s}, \pi_{i,l}, \rho_{i,l}, \xi_{i,l,s})$ as follows [7], [32]:

$$g(r_{i,l,s}, \pi_{i,l}, \rho_{i,l}, \xi_{i,l,s}) = r^2_{i,l,s} - \pi_{i,l} - \frac{1}{\rho_{i,l}}\xi_{i,l,s} \quad (6)$$

where $r_{i,l,s} = y_{i,l,s} - \check{y}_{i,l,s}$, and $\check{y}_{i,l,s}$ refers to the transmitted measurement at latest triggering time instant. $\pi_{i,l}$ and $\rho_{i,l}$ are, respectively, the prescribed triggering threshold and adjustable parameter. $\xi_{i,l,s}$ represents an auxiliary variable governed by the following dynamics

$$\xi_{i,l,s} = \delta_{i,l}\xi_{i,l,s-1} - r^2_{i,l,s-1} + \pi_{i,l} \quad (7)$$

where $\delta_{i,l}$ denotes a predefined parameter and the initial auxiliary variable satisfies $\xi_{i,l,0} \ge 0$.

For the convenience of subsequent analysis, define an indicator variable $\lambda_{i,l,s}$ as follows:

$$\lambda_{i,l,s} = \begin{cases} 1, & g(r_{i,l,s}, \pi_{i,l}, \rho_{i,l}, \xi_{i,l,s}) > 0; \\ 0, & g(r_{i,l,s}, \pi_{i,l}, \rho_{i,l}, \xi_{i,l,s}) \le 0. \end{cases} \quad (8)$$

Then, the available measurement at the filter end, denoted by $\bar{y}_{i,l,s}$, can be written as

$$\bar{y}_{i,l,s} = y_{i,l,s} - (1 - \lambda_{i,l,s})r_{i,l,s}. \quad (9)$$

In what follows, let us denote

$$\bar{y}_{i,s} = \begin{bmatrix}\bar{y}_{i,1,s} & \bar{y}_{i,2,s} & \cdots & \bar{y}_{i,m,s}\end{bmatrix}^T,$$
$$r_{i,s} = \begin{bmatrix}r_{i,1,s} & r_{i,2,s} & \cdots & r_{i,m,s}\end{bmatrix}^T,$$
$$\Lambda_{i,s} = \text{diag}\left\{\lambda_{i,1,s}, \lambda_{i,2,s}, \cdots, \lambda_{i,m,s}\right\}.$$

Accordingly, the compact form of the available measurements can be reformulated by

$$\bar{y}_{i,s} = y_{i,s} - (I - \Lambda_{i,s})r_{i,s}. \quad (10)$$

### B. Maximum-correntropy-based filtering scheme

In this paper, a similarity measure named correntropy is considered to enhance the robustness against the non-Gaussian noises. Compared with the widely used minimum mean square error criterion that only utilizes the second-order statistics, the utilization of correntropy can capture not only the second-order statistics but also the higher-order ones, thereby exhibiting great potentials in improving the filtering performance. More specifically, given any two scalar random variables $X$ and $Y$, the correntropy $V(X,Y)$ is defined as follows [5], [28]:

$$V(X,Y) = \mathbb{E}\{\epsilon(X,Y)\} = \iint_{x,y} \epsilon(x,y)f_{X,Y}(x,y)dxdy \quad (11)$$

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


where $\epsilon(X,Y)$ denotes the kernel function, and $f_{X,Y}(x,y)$ is the joint probability density function of $X$ and $Y$. In this paper, the kernel function is chosen to be the most popular Gaussian type, i.e.,

$$\epsilon(x,y) = G_{\chi}(e) = \exp(-\frac{e^2}{2\chi^2}) \tag{12}$$

where $e = x - y$ and $\chi > 0$ refers to the bandwidth of kernel size.

Considering the fact that it is usually difficult (if not impossible) to obtain the analytical expression of the joint density $f_{X,Y}(x,y)$ and only a few number of samples $\{x_i, y_i\}_{i=1}^{M}$ are accessible, the correntropy is approximately evaluated as follows:

$$V(X,Y) \approx \frac{1}{M}\sum_{i=1}^{M}\exp(-\frac{(x_i - y_i)^2}{2\chi^2}). \tag{13}$$

It is clear that in the case of Gaussian kernel, the correntropy reaches its maximum value only when $X = Y$.

In this paper, the filter for the $i$-th node is constructed with the following two-stage recursive form:

$$\hat{x}_{i,s|s-1} = A_{i,s-1}\hat{x}_{i,s-1} + f_{s-1}(\hat{x}_{i,s-1}) + \sum_{j=1}^{N} d_{ij}\Gamma\hat{x}_{j,s-1} \tag{14}$$

$$\hat{x}_{i,s} = \hat{x}_{i,s|s-1} + K_{i,s}\left(\bar{y}_{i,s} - C_{i,s}\hat{x}_{i,s|s-1}\right) \tag{15}$$

where $\hat{x}_{i,s|s-1}$ and $\hat{x}_{i,s}$ stand for, respectively, the one-step prediction and state estimate of the $i$-th node at time instant $s$, and $K_{i,s}$ denotes the filter gain matrix to be designed. Note that if the available measurements $\bar{y}_{i,s}$ are replaced by $y_{i,s}$ herein, the resultant filter is customized for the case without component-wise DETT mechanism.

In what follows, the CBPI is constructed for the purpose of designing the filter gain, i.e.,

$$\begin{aligned} J(x_{i,s}) = & G_{\chi}\left(\|x_{i,s} - \hat{x}_{i,s|s-1}\|_{\mathcal{P}^{-1}_{i,s|s-1}}\right) \\ & + G_{\chi}\left(\|\bar{y}_{i,s} - C_{i,s}x_{i,s}\|_{\mathcal{R}^{-1}_{i,s}}\right) \end{aligned} \tag{16}$$

where $\mathcal{P}_{i,s|s-1}$ and $\mathcal{R}_{i,s}$ denote the weighted matrices, which will be determined in the subsequent section.

On the basis of the maximum correntropy criterion, the desired filter gain $K_{i,s}$ can be obtained by solving the following optimization problem:

$$\hat{x}_{i,s} = \arg\max_{x_{i,s}} J(x_{i,s}). \tag{17}$$

*Remark 1:* It should be pointed out that, in most existing literature (e.g. [16], [46]), the one-step prediction error covariance and the measurement noise covariance are introduced as the weighted matrices in the CBPI with aim to realize the minimum-variance estimation. Nevertheless, due mainly to the existence of the component-wise DETT mechanism as well as the uncertain dynamical bias, it would be a rather challenging task to parameterize the accurate correntropy dynamics in a similar way. To this end, a novel performance index of the form (16) is proposed in this paper, where the latest transmitted measurements $\bar{y}_{i,s}$ (instead of the newly measured outputs $y_{i,s}$) are utilized. In comparison with the standard CBPI, our proposed one with weighted matrices $\mathcal{P}_{i,s|s-1}$ and $\mathcal{R}_{i,s}$ (which are actually the corresponding upper bounds of the one-step prediction covariance and equivalent noise covariance) is able to attenuate the joint effects of the component-wise DETT mechanism and uncertain dynamical bias.

*Remark 2:* It is worthwhile to mention that in this paper, our attention is mainly focused on the *remote* state estimation problem, where the sensors and the remote filters are not deployed in the same positions. Consider a scenario where the small unmanned aerial vehicles are utilized as sensors to detect or track the targets of interest, and the corresponding measurement data is transmitted to the ground control station (including the filters) for further processing. To prolong the lifespan of sensors, the component-wise DETT mechanism can be exploited to select the necessary data transmissions. Meanwhile, the real-time data transmissions among remote filters can be retained to guarantee the reliable state estimation. When the energy supply of filters becomes a major concern, it is of practical significance to ameliorate the utilization efficiency of limited resources by averting frequent data transmissions among filters, which deserves further investigations.

The primary purpose of this paper is to develop a recursive filter of the structure (14)-(15) for the uncertain non-Gaussian complex networks described by (1)-(4) under the component-wise DETT mechanism (6)-(7) as well as the maximum correntropy criterion (16)-(17).

## III. Maximum-Correntropy-Based Filter Design and Discussion

In this section, a recursive filtering scheme is developed based on the maximum correntropy criterion. To be more specific, we first establish an explicit expression for the proposed performance index by determining the weighted matrices $\mathcal{P}_{i,s|s-1}$ and $\mathcal{R}_{i,s}$, and then design the filter gain matrix by maximizing the proposed performance index.

To begin with, let us denote

$$\begin{aligned} &\tilde{x}_{i,s|s-1} = x_{i,s} - \hat{x}_{i,s|s-1}, \quad \tilde{x}_{i,s} = x_{i,s} - \hat{x}_{i,s} \\ &P_{i,s|s-1} = \mathbb{E}\{\tilde{x}_{i,s|s-1}\tilde{x}^T_{i,s|s-1}\}, \quad P_{i,s} = \mathbb{E}\{\tilde{x}_{i,s}\tilde{x}^T_{i,s}\} \end{aligned} \tag{18}$$

where $\tilde{x}_{i,s|s-1}$ and $\tilde{x}_{i,s}$ represent, respectively, the one-step prediction error and the filtering error for the $i$-th node at time instant $s$. $P_{i,s|s-1}$ and $P_{i,s}$ denote, respectively, the corresponding one-step prediction error covariance and filtering error covariance.

According to (5), (14) and (18), the dynamics of the one-step prediction error can be given by

$$\begin{aligned} \tilde{x}_{i,s|s-1} = & A_{i,s-1}\tilde{x}_{i,s-1} + \Delta A_{i,s-1}x_{i,s-1} + \tilde{f}_{s-1}(x_{i,s-1}) \\ & + \sum_{j=1}^{N} d_{ij}\Gamma\tilde{x}_{j,s-1} + \omega_{i,s-1} \end{aligned} \tag{19}$$

where $\tilde{f}_{s-1}(x_{i,s-1}) = f_{s-1}(x_{i,s-1}) - f_{s-1}(\hat{x}_{i,s-1})$.

Recalling the definition of prediction error covariance in (18) and the fact that $\omega_{i,s-1}$ is independent of $\tilde{x}_{i,s-1}$, $\tilde{x}_{j,s-1}$

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


and $x_{i,s-1}$, it is not difficult to obtain that

$$\begin{aligned}
&P_{i,s|s-1}\\
=&\mathbb{E}\{\tilde{x}_{i,s|s-1}\tilde{x}_{i,s|s-1}^T\}\\
=&A_{i,s-1}P_{i,s-1}A_{i,s-1}^T+\mathbb{E}\{\Delta A_{i,s-1}x_{i,s-1}x_{i,s-1}^T\Delta A_{i,s-1}^T\}\\
&+\mathbb{E}\{\tilde{f}_{s-1}(x_{i,s-1})\tilde{f}_{s-1}(x_{i,s-1})^T\}+Q_{i,s-1}\\
&+\mathbb{E}\{(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})^T\}\\
&+\mathbb{E}\{A_{i,s-1}\tilde{x}_{i,s-1}\tilde{f}_{s-1}(x_{i,s-1})^T\}+\mathbb{E}\{\tilde{f}_{s-1}(x_{i,s-1})\\
&\times\tilde{x}_{i,s-1}^TA_{i,s-1}^T\}+\mathbb{E}\{A_{i,s-1}\tilde{x}_{i,s-1}(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})^T\}\\
&+\mathbb{E}\{(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})\tilde{x}_{i,s-1}^TA_{i,s-1}^T\}+\mathbb{E}\{\tilde{f}_{s-1}(x_{i,s-1})\\
&\times(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})^T\}+\mathbb{E}\{(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})\tilde{f}_{s-1}(x_{i,s-1})^T\}
\end{aligned} \tag{20}$$

where $Q_{i,s-1}=\mathrm{diag}\{\bar{Q}_{i,s-1},S_{i,s-1}\}$.

On the other hand, subtracting (15) from $x_{i,s}$ and using (10) lead to the following dynamics of filtering error:

$$\begin{aligned}
&\tilde{x}_{i,s}\\
=&\tilde{x}_{i,s|s-1}-K_{i,s}\left(y_{i,s}-(I-\Lambda_{i,s})r_{i,s}-C_{i,s}\hat{x}_{i,s|s-1}\right)\\
=&(I-K_{i,s}C_{i,s})\tilde{x}_{i,s|s-1}-K_{i,s}\nu_{i,s}+K_{i,s}(I-\Lambda_{i,s})r_{i,s}.
\end{aligned} \tag{21}$$

In view of (18) and (21), the filtering error covariance can be derived as follows:

$$\begin{aligned}
&P_{i,s}\\
=&\mathbb{E}\{\tilde{x}_{i,s}\tilde{x}_{i,s}^T\}\\
=&(I-K_{i,s}C_{i,s})P_{i,s|s-1}(I-K_{i,s}C_{i,s})^T+K_{i,s}R_{i,s}K_{i,s}^T\\
&+\mathbb{E}\{K_{i,s}(I-\Lambda_{i,s})r_{i,s}r_{i,s}^T(I-\Lambda_{i,s})K_{i,s}^T\}\\
&+\mathbb{E}\{(I-K_{i,s}C_{i,s})\tilde{x}_{i,s|s-1}r_{i,s}^T(I-\Lambda_{i,s})K_{i,s}^T\}\\
&+\mathbb{E}\{K_{i,s}(I-\Lambda_{i,s})r_{i,s}\tilde{x}_{i,s|s-1}^T(I-K_{i,s}C_{i,s})^T\}\\
&-\mathbb{E}\{K_{i,s}\nu_{i,s}r_{i,s}^T(I-\Lambda_{i,s})K_{i,s}^T\}\\
&-\mathbb{E}\{K_{i,s}(I-\Lambda_{i,s})r_{i,s}\nu_{i,s}^TK_{i,s}^T\}.
\end{aligned} \tag{22}$$

*Remark 3:* The accurate expressions of the one-step prediction error covariance and the filtering error covariance have been parameterized in (20) and (22). Nevertheless, the simultaneous presence of the component-wise DETT mechanism, uncertain dynamical bias and nonlinearities renders it really troublesome to calculate the accurate covariances. In this regard, an alternative yet effective approach is to seek upper bounds on the error covariance matrices as the performance criteria. It is worth mentioning that, compared with the existing literature with traditional static event-triggered mechanism, the introduction of the auxiliary variable $\xi_{i,l,s}$ would contribute to additional complexity in handling the term $\mathbb{E}\{r_{i,s}r_{i,s}^T\}$ and the cross terms related to $r_{i,s}$ since the auxiliary variable is not accessible from the filter end. On the other hand, by introducing a diagonal indicator matrix $\Lambda_{i,s}$ (with diagonal elements being 1 or 0), the effects of the component-wise event-triggered transmissions have been clearly reflected in the dynamics of filtering error covariance.

To deal with the complexity induced by the component-wise DETT mechanism, the following proposition provides a feasible approach to establishing an upper bound for the term $\mathbb{E}\{r_{i,s}r_{i,s}^T\}$.

*Proposition 1:* For the $i$-th node, let $0<\delta_{i,l}<1$ and $\rho_{i,l}>\frac{1}{\delta_{i,l}}$ $(l=1,2,\cdots,m)$. The solution to the following recursion

$$\Xi_{i,l,s}=\delta_{i,l}\Xi_{i,l,s-1}+\pi_{i,l} \tag{23}$$

with initial condition $\Xi_{i,l,0}=\xi_{i,l,0}\geq 0$ is always an upper bound on the auxiliary variable $\xi_{i,l,s}$, i.e., $0\leq\xi_{i,l,s}\leq\Xi_{i,l,s}$. In addition, the following inequality

$$r_{i,s}r_{i,s}^T\leq\upsilon_{i,s}I \tag{24}$$

holds, where

$$\upsilon_{i,s}\triangleq\sum_{l=1}^m\left(\pi_{i,l}+\frac{1}{\rho_{i,l}}\Xi_{i,l,s}\right).$$

*Proof:* Following the similar line of [7], it is clear that, if $0<\delta_{i,l}<1$ and $\rho_{i,l}>\frac{1}{\delta_{i,l}}$ $(l=1,2,\cdots,m)$ hold, one has $\xi_{i,l,s}\geq 0$.

Next, we will prove $\xi_{i,l,s}\leq\Xi_{i,l,s}$ by using the mathematical induction. For $s=0$, the result is obviously true. Assume that $\xi_{i,l,s-1}\leq\Xi_{i,l,s-1}$ holds. Then, at time instant $s$, noting that $r_{i,l,s-1}^2\geq 0$, it follows from (7) that

$$\begin{aligned}
\xi_{i,l,s}&=\delta_{i,l}\xi_{i,l,s-1}-r_{i,l,s-1}^2+\pi_{i,l}\\
&\leq\delta_{i,l}\Xi_{i,l,s-1}+\pi_{i,l}=\Xi_{i,l,s}
\end{aligned} \tag{25}$$

which ends the proof of the first part.

In what follows, we move on to the proof of the second part. Recalling the property of the DETT mechanism (6)-(8) as well as the inequality (25), it is not difficult to see that

$$r_{i,l,s}^2\leq\pi_{i,l}+\frac{1}{\rho_{i,l}}\xi_{i,l,s}\leq\pi_{i,l}+\frac{1}{\rho_{i,l}}\Xi_{i,l,s}. \tag{26}$$

Then, we can obtain that

$$r_{i,s}r_{i,s}^T\leq\|r_{i,s}\|^2I=\sum_{l=1}^m r_{i,l,s}^2I\leq\sum_{l=1}^m\left(\pi_{i,l}+\frac{1}{\rho_{i,l}}\Xi_{i,l,s}\right)I. \tag{27}$$

The proof is now complete. ■

For convenience of presentation, define

$$F_s=\begin{bmatrix}\bar{F}_s&0\\0&0\end{bmatrix},\quad\bar{I}=\begin{bmatrix}I&0\\0&0\end{bmatrix}.$$

Based on the results in Proposition 1 as well as the parameterized one-step prediction error covariance and filtering error covariance in (20) and (22), we are going to establish their corresponding upper bounds in the following proposition.

*Proposition 2:* For the $i$-th node, given positive scalars $\alpha_{i,j}$ $(j=1,2,\cdots,5)$ and $\beta_{i,j}$ $(j=1,2)$, the solutions to

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


the following two difference equations

$$\begin{aligned}&\mathcal{P}_{i,s|s-1}\\&=(1+\alpha_{i,3}+\alpha_{i,4})A_{i,s-1}\mathcal{P}_{i,s-1}A_{i,s-1}^T\\&\quad+\tau_i\mathrm{tr}\{(1+\alpha_{i,1})\mathcal{P}_{i,s-1}+(1+\alpha_{i,1}^{-1})\hat{x}_{i,s-1}\hat{x}_{i,s-1}^T\}I\\&\quad+(1+\alpha_{i,3}^{-1}+\alpha_{i,5})\Big\{(1+\alpha_{i,2})\kappa_s^2\mathrm{tr}\{\bar{I}\mathcal{P}_{i,s-1}\bar{I}\}I\\&\quad+(1+\alpha_{i,2}^{-1})F_{s-1}\mathcal{P}_{i,s-1}F_{s-1}^T\Big\}+Q_{i,s-1}\\&\quad+(1+\alpha_{i,4}^{-1}+\alpha_{i,5}^{-1})N\sum_{j=1}^N d_{ij}^2\Gamma\mathcal{P}_{j,s-1}\Gamma\end{aligned}\tag{28}$$

and

$$\begin{aligned}&\mathcal{P}_{i,s}\\&=(1+\beta_{i,1})(I-K_{i,s}C_{i,s})\mathcal{P}_{i,s|s-1}(I-K_{i,s}C_{i,s})^T\\&\quad+(1+\beta_{i,2})K_{i,s}R_{i,s}K_{i,s}^T\\&\quad+(1+\beta_{i,1}^{-1}+\beta_{i,2}^{-1})\upsilon_{i,s}K_{i,s}(I-\Lambda_{i,s})K_{i,s}^T\end{aligned}\tag{29}$$

with initial conditions $\mathcal{P}_{i,0}=P_{i,0}>0$ represent, respectively, the upper bounds on the one-step prediction error covariance and filtering error covariance, i.e.,

$$P_{i,s|s-1}\leq\mathcal{P}_{i,s|s-1},\ \ P_{i,s}\leq\mathcal{P}_{i,s}.$$

*Proof:* To begin with, let us analyze the unknown terms on the right-hand side of (20) one by one. Concentrating on the second term on the right-hand side of (20) and recalling the elementary inequality $xy^T+yx^T\leq\alpha_{i,1}xx^T+\alpha_{i,1}^{-1}yy^T$ for $x,y\in\mathbb{R}^n$ and any positive scalar $\alpha_{i,1}$, it is not difficult to obtain that

$$\begin{aligned}&\mathbb{E}\{\Delta A_{i,s-1}x_{i,s-1}x_{i,s-1}^T\Delta A_{i,s-1}^T\}\\&\leq\tau_i\mathrm{tr}\{(1+\alpha_{i,1})P_{i,s-1}+(1+\alpha_{i,1}^{-1})\hat{x}_{i,s-1}\hat{x}_{i,s-1}^T\}I.\end{aligned}\tag{30}$$

In light of (3), the third term on the right-hand side of (20) can be rearranged as follows:

$$\begin{aligned}&\mathbb{E}\{\tilde{f}_{s-1}(x_{i,s-1})\tilde{f}_{s-1}(x_{i,s-1})^T\}\\&\leq\mathbb{E}\{(1+\alpha_{i,2})\|f_{s-1}(x_{i,s-1})\\&\quad-f_{s-1}(\hat{x}_{i,s-1})-F_{s-1}\tilde{x}_{i,s-1}\|^2I\\&\quad+(1+\alpha_{i,2}^{-1})F_{s-1}\tilde{x}_{i,s-1}\tilde{x}_{i,s-1}^TF_{s-1}^T\}\\&\leq\mathbb{E}\{(1+\alpha_{i,2})\kappa_s^2\|\bar{I}\tilde{x}_{i,s-1}\|^2I\\&\quad+(1+\alpha_{i,2}^{-1})F_{s-1}\tilde{x}_{i,s-1}\tilde{x}_{i,s-1}^TF_{s-1}^T\}\\&=(1+\alpha_{i,2})\kappa_s^2\mathrm{tr}\{\bar{I}P_{i,s-1}\bar{I}\}I\\&\quad+(1+\alpha_{i,2}^{-1})F_{s-1}P_{i,s-1}F_{s-1}^T\end{aligned}\tag{31}$$

where $\alpha_{i,2}$ is a positive scalar.

Now, we focus on the fifth term on the right-hand side of (20). Based on the aforementioned elementary inequality, it is straightforward to see that

$$\begin{aligned}&\mathbb{E}\left\{(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})^T\right\}\\&\leq N\sum_{j=1}^N d_{ij}^2\Gamma P_{j,s-1}\Gamma.\end{aligned}\tag{32}$$

Subsequently, let us move on to deal with the cross terms in (20). Selecting the positive scalars $\alpha_{i,3}$, $\alpha_{i,4}$ and $\alpha_{i,5}$, one has

$$\begin{aligned}&\mathbb{E}\{A_{i,s-1}\tilde{x}_{i,s-1}\tilde{f}_{s-1}(x_{i,s-1})^T\}\\&\quad+\mathbb{E}\{\tilde{f}_{s-1}(x_{i,s-1})\tilde{x}_{i,s-1}^TA_{i,s-1}^T\}\\&\leq\alpha_{i,3}A_{i,s-1}P_{i,s-1}A_{i,s-1}^T\\&\quad+\alpha_{i,3}^{-1}\mathbb{E}\{\tilde{f}_{s-1}(x_{i,s-1})\tilde{f}_{s-1}(x_{i,s-1})^T\},\end{aligned}\tag{33}$$

$$\begin{aligned}&\mathbb{E}\{A_{i,s-1}\tilde{x}_{i,s-1}(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})^T\}\\&\quad+\mathbb{E}\{(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})\tilde{x}_{i,s-1}^TA_{i,s-1}^T\}\\&\leq\alpha_{i,4}A_{i,s-1}P_{i,s-1}A_{i,s-1}^T\\&\quad+\alpha_{i,4}^{-1}\mathbb{E}\{(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})^T\},\end{aligned}\tag{34}$$

and

$$\begin{aligned}&\mathbb{E}\{\tilde{f}_{s-1}(x_{i,s-1})(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})^T\}\\&\quad+\mathbb{E}\{(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})\tilde{f}_{s-1}(\tilde{x}_{i,s-1})^T\}\\&\leq\alpha_{i,5}\mathbb{E}\{\tilde{f}_{s-1}(x_{i,s-1})\tilde{f}_{s-1}(\tilde{x}_{i,s-1})^T\}\\&\quad+\alpha_{i,5}^{-1}\mathbb{E}\{(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})(\sum_{j=1}^N d_{ij}\Gamma\tilde{x}_{j,s-1})^T\}.\end{aligned}\tag{35}$$

Substituting (30)-(35) into (20) results in

$$\begin{aligned}&P_{i,s|s-1}\\&\leq(1+\alpha_{i,3}+\alpha_{i,4})A_{i,s-1}P_{i,s-1}A_{i,s-1}^T\\&\quad+\tau_i\mathrm{tr}\{(1+\alpha_{i,1})P_{i,s-1}+(1+\alpha_{i,1}^{-1})\hat{x}_{i,s-1}\hat{x}_{i,s-1}^T\}I\\&\quad+(1+\alpha_{i,3}^{-1}+\alpha_{i,5})\Big\{(1+\alpha_{i,2})\kappa_s^2\mathrm{tr}\{\bar{I}P_{i,s-1}\bar{I}\}I\\&\quad+(1+\alpha_{i,2}^{-1})F_{s-1}P_{i,s-1}F_{s-1}^T\Big\}+Q_{i,s-1}\\&\quad+(1+\alpha_{i,4}^{-1}+\alpha_{i,5}^{-1})N\sum_{j=1}^N d_{ij}^2\Gamma P_{j,s-1}\Gamma.\end{aligned}\tag{36}$$

Next, let us consider the cross terms on the right-hand side of (22). Similar to the above derivations, the cross terms satisfy the following relationships:

$$\begin{aligned}&\mathbb{E}\{(I-K_{i,s}C_{i,s})\tilde{x}_{i,s|s-1}r_{i,s}^T(I-\Lambda_{i,s})K_{i,s}^T\}\\&\quad+\mathbb{E}\{K_{i,s}(I-\Lambda_{i,s})r_{i,s}\tilde{x}_{i,s|s-1}^T(I-K_{i,s}C_{i,s})^T\}\\&\leq\beta_{i,1}(I-K_{i,s}C_{i,s})P_{i,s|s-1}(I-K_{i,s}C_{i,s})^T\\&\quad+\beta_{i,1}^{-1}\mathbb{E}\{K_{i,s}(I-\Lambda_{i,s})r_{i,s}r_{i,s}^T(I-\Lambda_{i,s})K_{i,s}^T\}\end{aligned}\tag{37}$$

and

$$\begin{aligned}&-\mathbb{E}\{K_{i,s}\nu_{i,s}r_{i,s}^T(I-\Lambda_{i,s})K_{i,s}^T\}\\&\quad-\mathbb{E}\{K_{i,s}(I-\Lambda_{i,s})r_{i,s}\nu_{i,s}^TK_{i,s}^T\}\\&\leq\beta_{i,2}K_{i,s}R_{i,s}K_{i,s}^T\\&\quad+\beta_{i,2}^{-1}\mathbb{E}\{K_{i,s}(I-\Lambda_{i,s})r_{i,s}r_{i,s}^T(I-\Lambda_{i,s})K_{i,s}^T\}.\end{aligned}\tag{38}$$

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


Recalling (24) and the fact that $(I-\Lambda_{i,s})(I-\Lambda_{i,s}) = (I-\Lambda_{i,s})$, one has

$$\begin{aligned}&\mathbb{E}\{K_{i,s}(I-\Lambda_{i,s})r_{i,s}r_{i,s}^T(I-\Lambda_{i,s})K_{i,s}^T\}\\ &\le \upsilon_{i,s}K_{i,s}(I-\Lambda_{i,s})K_{i,s}^T.\end{aligned} \tag{39}$$

Then, substituting (37)-(39) into (22) leads to

$$\begin{aligned}&P_{i,s}\\ \le\ &(1+\beta_{i,1})(I-K_{i,s}C_{i,s})P_{i,s|s-1}(I-K_{i,s}C_{i,s})^T\\ &+(1+\beta_{i,2})K_{i,s}R_{i,s}K_{i,s}^T\\ &+(1+\beta_{i,1}^{-1}+\beta_{i,2}^{-1})\upsilon_{i,s}K_{i,s}(I-\Lambda_{i,s})K_{i,s}^T.\end{aligned} \tag{40}$$

Based on the relationships (36) and (40) as well as the mathematical induction method, it is not difficult to verify that $\mathcal{P}_{i,s|s-1}$ and $\mathcal{P}_{i,s}$ are, respectively, the upper bounds on the one-step prediction error covariance and the filtering error covariance. The proof is now complete. ∎

In view of Proposition 2, the weighted matrix $\mathcal{P}_{i,s|s-1}$ is parameterized by the one-step prediction error covariance. Next, we further determine the form of another weighted matrix $\mathcal{R}_{i,s}$ by considering the effect of the component-wise DETT mechanism. From (10), it is clear that

$$\begin{aligned}&\bar{y}_{i,s}-C_{i,s}x_{i,s}\\ &=y_{i,s}-(I-\Lambda_{i,s})r_{i,s}-C_{i,s}x_{i,s}\\ &=C_{i,s}x_{i,s}+\nu_{i,s}-(I-\Lambda_{i,s})r_{i,s}-C_{i,s}x_{i,s}\\ &=\nu_{i,s}-(I-\Lambda_{i,s})r_{i,s}.\end{aligned} \tag{41}$$

Then, one has

$$\begin{aligned}&\mathbb{E}\{(\bar{y}_{i,s}-C_{i,s}x_{i,s})(\bar{y}_{i,s}-C_{i,s}x_{i,s})^T\}\\ =\ &\mathbb{E}\{(\nu_{i,s}-(I-\Lambda_{i,s})r_{i,s})(\nu_{i,s}-(I-\Lambda_{i,s})r_{i,s})^T\}\\ \le\ &(1+\beta_{i,2})R_{i,s}+(1+\beta_{i,2}^{-1})\upsilon_{i,s}(I-\Lambda_{i,s})\\ \triangleq\ &\mathcal{R}_{i,s},\end{aligned} \tag{42}$$

which can be regarded as an upper bound on the equivalent measurement noise covariance.

By resorting to the weighted matrices $\mathcal{P}_{i,s|s-1}$ and $\mathcal{R}_{i,s}$ determined in (28) and (42), we are able to parameterize the explicit expression of the CBPI (16). Accordingly, motivated by [16], the MCF scheme is designed as follows.

Bearing in mind the fact that the design purpose of filter (i.e., gain $K_{i,s}$) is to maximize the CBPI (16), we take partial derivative with respect to $x_{i,s}$ and let

$$\frac{\partial J(x_{i,s})}{\partial x_{i,s}}=0.$$

Then, it is obtained that

$$\begin{aligned}&-\frac{1}{\chi^2}G_\chi\left(\|x_{i,s}-\hat{x}_{i,s|s-1}\|_{\mathcal{P}_{i,s|s-1}^{-1}}\right)\\ &\qquad\times\mathcal{P}_{i,s|s-1}^{-1}(x_{i,s}-\hat{x}_{i,s|s-1})\\ &+\frac{1}{\chi^2}G_\chi\left(\|\bar{y}_{i,s}-C_{i,s}x_{i,s}\|_{\mathcal{R}_{i,s}^{-1}}\right)\\ &\qquad\times C_{i,s}^T\mathcal{R}_{i,s}^{-1}(\bar{y}_{i,s}-C_{i,s}x_{i,s})=0.\end{aligned} \tag{43}$$

For ease of analysis, let us denote

$$U_{i,s}=\frac{G_\chi\left(\|\bar{y}_{i,s}-C_{i,s}x_{i,s}\|_{\mathcal{R}_{i,s}^{-1}}\right)}{G_\chi\left(\|x_{i,s}-\hat{x}_{i,s|s-1}\|_{\mathcal{P}_{i,s|s-1}^{-1}}\right)}. \tag{44}$$

Then, (43) can be rewritten by

$$\mathcal{P}_{i,s|s-1}^{-1}(x_{i,s}-\hat{x}_{i,s|s-1})=U_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}(\bar{y}_{i,s}-C_{i,s}x_{i,s}), \tag{45}$$

which, by adding the zero term

$$U_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}C_{i,s}\hat{x}_{i,s|s-1}-U_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}C_{i,s}\hat{x}_{i,s|s-1},$$

can be further reformulated as

$$\begin{aligned}&(\mathcal{P}_{i,s|s-1}^{-1}+U_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}C_{i,s})x_{i,s}\\ &=(\mathcal{P}_{i,s|s-1}^{-1}+U_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}C_{i,s})\hat{x}_{i,s|s-1}\\ &\quad+U_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}(\bar{y}_{i,s}-C_{i,s}\hat{x}_{i,s|s-1}).\end{aligned} \tag{46}$$

Clearly, we can obtain that

$$\hat{x}_{i,s}=\hat{x}_{i,s|s-1}+K_{i,s}(\bar{y}_{i,s}-C_{i,s}\hat{x}_{i,s|s-1}) \tag{47}$$

where

$$K_{i,s}=(\mathcal{P}_{i,s|s-1}^{-1}+U_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}C_{i,s})^{-1}U_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}. \tag{48}$$

It should be pointed out that $x_{i,s}$ is also involved in the expression of $U_{i,s}$ (and hence in $K_{i,s}$), which means that it would be really tricky to derive an analytic form of the state estimate $\hat{x}_{i,s}$ via (47). To this end, the fixed-point iterative algorithm is introduced in [5] with the iterative initial state being the one-step prediction. In this paper, following the similar line of [22], [45], the fixed-point algorithm with only one iteration is employed to guarantee the calculation efficiency. Specifically, the one-step prediction $\hat{x}_{i,s|s-1}$ is utilized to approximate $x_{i,s}$ in both the numerator and denominator on the right-hand side of (44), which implies that

$$\hat{U}_{i,s}=G_\chi\left(\|\bar{y}_{i,s}-C_{i,s}\hat{x}_{i,s|s-1}\|_{\mathcal{R}_{i,s}^{-1}}\right). \tag{49}$$

Then, the filter gain matrix can be calculated by

$$K_{i,s}=(\mathcal{P}_{i,s|s-1}^{-1}+\hat{U}_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}C_{i,s})^{-1}\hat{U}_{i,s}C_{i,s}^T\mathcal{R}_{i,s}^{-1}. \tag{50}$$

To facilitate the future implementation, the developed MCF algorithm under the component-wise DETT mechanism is summarized in Algorithm 1.

*Remark 4:* It should be emphasized that, for the traditional Kalman-like filtering algorithms based on the variance-constrained approach, the filter gain matrix is designed to minimize the trace of the filtering error covariance or its upper bound by letting

$$\frac{\partial \mathrm{tr}(\mathcal{P}_{i,s})}{\partial K_{i,s}}=0.$$

In this case, recalling the expression of $\mathcal{P}_{i,s}$ in (29), one has

$$\begin{aligned}&(1+\beta_{i,1})(-2\mathcal{P}_{i,s|s-1}C_{i,s}^T+2K_{i,s}C_{i,s}\mathcal{P}_{i,s|s-1}C_{i,s}^T)\\ &+2(1+\beta_{i,2})K_{i,s}R_{i,s}\\ &+2(1+\beta_{i,1}^{-1}+\beta_{i,2}^{-1})\upsilon_{i,s}K_{i,s}(I-\Lambda_{i,s})=0.\end{aligned}$$

Then, the filter gain matrix for the $i$-th node can be obtained as follows:

$$\begin{aligned}K_{i,s}=&(1+\beta_{i,1})\mathcal{P}_{i,s|s-1}C_{i,s}^T\Big[(1+\beta_{i,1})C_{i,s}\mathcal{P}_{i,s|s-1}C_{i,s}^T\\ &+(1+\beta_{i,2})R_{i,s}\\ &+(1+\beta_{i,1}^{-1}+\beta_{i,2}^{-1})\upsilon_{i,s}(I-\Lambda_{i,s})\Big]^{-1}.\end{aligned} \tag{51}$$

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


**Algorithm 1** MCF algorithm under the component-wise DETT mechanism.

***Step* 1. Algorithm initialization**
For each node $i$ $(i = 1, 2, \cdots, N)$, set the initial estimates $\hat{x}_{i,0}$ and initial covariance matrices $\mathcal{P}_{i,0}$. Choose the positive scalars $\alpha_{i,j}$ $(j = 1, 2, \cdots, 5)$ and $\beta_{i,j}$ $(j = 1, 2)$, and select the proper kernel bandwidth $\chi$. The maximum time instant is set to be $K$.

***Step* 2. One-step prediction**
Calculate the one-step prediction $\hat{x}_{i,s|s-1}$ and the upper bound $\mathcal{P}_{i,s|s-1}$ on the prediction error covariance based on (14) and (28), respectively.

***Step* 3. State estimate update**
- Collect the available measurements $\bar{y}_{i,s}$ under scheduling of the component-wise DETT scheme (6) and (7).
- Calculate the filter gain matrix $K_{i,s}$ based on (42), (49) and (50).
- Update the state estimate $\hat{x}_{i,s}$ and the upper bound $\mathcal{P}_{i,s}$ on the filtering error covariance based on (15) and (29), respectively.

***Step* 4.** If $s < K$, then set $s = s + 1$ and go to ***Step* 2**; otherwise stop the recursion.

It is clear that the above derivations only make use of the second-order moment information of the filtering error, and hence the resulted filter would be inapplicable to the scenarios with non-Gaussian noises. Frankly speaking, such a problem greatly motivates this current investigation.

*Remark 5:* It should also be mentioned that the number of required floating point operations has been widely utilized to approximately evaluate the computational complexity of a given algorithm. Following the similar line of [5], we can conclude that the computational complexity of the proposed MCF algorithm (composed of (14), (15), (28), (29), (42), (49) and (50)) is $C_1 = 12n^3 + (14+4N)n^2 + 12mn^2 + 4m^2 + 6nm^2 + (3N+4)n + 6mn + 9m + \Delta_f + N + 22 + O(m^3) + 2O(n^3)$ (where $n = \bar{n} + k$ and $\Delta_f$ denote, respectively, the state dimension of the augmented system (5) and the computational complexity determined by the specific form of nonlinear function $f_s(\cdot)$), and that of the variance-constrained filtering algorithm (comprised by (14), (15), (28), (29) and (51)) is $C_2 = 12n^3 + (13+4N)n^2 + 12mn^2 + 3m^2 + 6nm^2 + (3N+4)n + 2mn + 8m + \Delta_f + N + 23 + O(m^3)$. In summary, the proposed algorithm has moderate computational complexity in comparison with the traditional variance-constrained one, while guaranteeing a substantial performance improvement under the non-Gaussian environment.

*Remark 6:* So far, considerable research enthusiasms have been devoted to the state estimation or filtering problem for complex networks with various network-induced phenomena, and a rich body of elegant results has been reported in recent years. Compared with the existing literature, this paper may shed some new insights from the following three aspects: 1) the problem under investigation is new in the sense that it takes into simultaneous account the non-Gaussian noises, the system nonlinearity, the uncertain dynamical bias and the component-wise DETT protocol; 2) the maximum correntropy criterion is adopted with a novel performance index which embraces the effects of uncertain dynamical bias and component-wise DETT protocol; and 3) the proposed filtering algorithm inherits the recursive form of the Kalman-like filters and exhibits appealing robustness against the non-Gaussian noises under the component-wise DETT mechanism, which is therefore suitable for the online implementations in practice.

## IV. Simulation Validations

In this section, an illustrative example is presented to demonstrate the effectiveness of the developed filtering methodology. Consider the complex network (1)-(4) with six nodes as well as the following parameters:

$$\bar{A}_{1,s} = \begin{bmatrix} 0.85 + 0.1\sin(s) & 0.6 \\ -0.2 & 0.85 \end{bmatrix}, \ B_{1,s} = \begin{bmatrix} 0.4 & 0.5 \\ 0.4 & -0.25 \end{bmatrix}$$
$$\bar{A}_{2,s} = \begin{bmatrix} 0.7 & -0.05 + 0.1\sin(s) \\ -0.3 & 0.7 \end{bmatrix}, \ B_{2,s} = \begin{bmatrix} 0.8 & 0.4 \\ 0.3 & -0.2 \end{bmatrix}$$
$$\bar{A}_{3,s} = \begin{bmatrix} 0.8 & 0.4 \\ -0.4 & 0.65 + 0.2\cos(s) \end{bmatrix}, \ B_{3,s} = \begin{bmatrix} 0.7 & -0.1 \\ 0.2 & 0.3 \end{bmatrix},$$
$$\bar{A}_{4,s} = \begin{bmatrix} 0.8 + 0.1\cos(s) & 0.4 \\ -0.2 & 0.75 \end{bmatrix}, \ B_{4,s} = \begin{bmatrix} 0.6 & -0.1 \\ 0.2 & 0.2 \end{bmatrix},$$
$$\bar{A}_{5,s} = \begin{bmatrix} 0.9 & 0.1 \\ -0.1 + 0.1\sin(s) & 0.7 \end{bmatrix}, \ B_{5,s} = \begin{bmatrix} 0.5 & -0.1 \\ 0.1 & 0.3 \end{bmatrix},$$
$$\bar{A}_{6,s} = \begin{bmatrix} 0.85 & -0.05 + 0.1\cos(s) \\ -0.1 & 0.75 \end{bmatrix}, \ B_{6,s} = \begin{bmatrix} 0.6 & -0.1 \\ 0.1 & 0.2 \end{bmatrix},$$
$$G_{1,s} = \begin{bmatrix} 0.5 & 0.4 \\ -0.3 & 0.6 \end{bmatrix}, \ G_{2,s} = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.3 \end{bmatrix},$$
$$G_{3,s} = \begin{bmatrix} 0.6 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}, \ G_{4,s} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}, \ G_{5,s} = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.2 \end{bmatrix},$$
$$G_{6,s} = \begin{bmatrix} 0.4 & -0.1 \\ 0.1 & 0.2 \end{bmatrix}, \ \bar{C}_{1,s} = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.3 \end{bmatrix},$$
$$\bar{C}_{2,s} = \begin{bmatrix} 1.8 & 0 \\ 0 & 1.2 \end{bmatrix}, \ \bar{C}_{3,s} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.4 \end{bmatrix}, \ \bar{C}_{4,s} = \begin{bmatrix} 1.6 & 0 \\ 0 & 1.4 \end{bmatrix},$$
$$\bar{C}_{5,s} = \begin{bmatrix} 1.9 & 0 \\ 0 & 1.5 \end{bmatrix}, \ \bar{C}_{6,s} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.2 \end{bmatrix}, \ \bar{\Gamma} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

The nonlinear function is described by

$$\bar{f}_s(\bar{x}_{i,s}) = \begin{bmatrix} 0.1 & -0.01 + 0.05\cos(s) \\ 0.01 & 0.15 \end{bmatrix} \bar{x}_{i,s} + \begin{bmatrix} 0.08\sin(\bar{x}^1_{i,s}) \\ 0.1\sin(\bar{x}^2_{i,s}) \end{bmatrix}$$

where $\bar{x}^l_{i,s}$ $(l = 1, 2)$ denotes the $l$th component of the state $\bar{x}_{i,s}$. Other modeling parameters are selected as $\tau_i = 0.01$, $d_{ij} = 0.2$ $(i \neq j)$ and $d_{ii} = -1$ for $i = 1, 2, \ldots, 6$.

In the simulation, the initial conditions are chosen as $\bar{x}_{1,0} = [0 \ \ 0]^T$, $\bar{x}_{2,0} = [-1 \ \ 1]^T$, $\bar{x}_{3,0} = [2 \ \ -1]^T$, $\bar{x}_{4,0} = [-1 \ \ 2]^T$, $\bar{x}_{5,0} = [-2 \ \ -1]^T$, $\bar{x}_{6,0} = [0 \ \ -1]^T$, $z_{1,0} = [-1 \ \ -2]^T$, $z_{2,0} = [2 \ \ 3]^T$, $z_{3,0} = [2 \ \ -4]^T$, $z_{4,0} = [2 \ \ -3]^T$, $z_{5,0} = [-1 \ \ 2]^T$, $z_{6,0} = [1 \ \ 2]^T$ and $P_{i,0} = 2I$ $(i = 1, 2, \ldots, 6)$. For the component-wise DETT protocol, we let $\rho_{i,l} = 4$, $\delta_{i,l} = 0.9$, $\xi_{i,l,0} = 1$ for $i = 1, 2, \ldots, 6$ and $l = 1, 2$. Moreover, $\pi_{1,1} = 1$, $\pi_{1,2} = 2$, $\pi_{2,1} = 2$, $\pi_{2,2} = 1$, $\pi_{3,1} = 1$, $\pi_{3,2} = 1$, $\pi_{4,1} = 1.5$, $\pi_{4,2} = 2$, $\pi_{5,1} = 1$, $\pi_{5,2} = 1.2$, $\pi_{6,1} = 1.2$

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


and $\pi_{6,2} = 1$. The root mean square error (RMSE) on state estimate is calculated over 500 Monte Carlo runs to facilitate the evaluation/comparison of filtering performance.

In what follows, two scenarios with different noise conditions are taken into account to compare the tracking performance of the proposed MCF algorithm (abbreviated as PMCF) and the variance-constrained filtering algorithm (denoted as VCF) in Remark 4.

***Case 1:*** In this case, both the process noise and measurement noise are Gaussian mixture noises, i.e.,

$$\zeta_{i,s} \sim 0.9\mathcal{N}(0, \text{diag}\{0.01, 0.01\}) + 0.1\mathcal{N}(0, \text{diag}\{1, 1\}),$$
$$\eta_{i,s} \sim 0.9\mathcal{N}(0, \text{diag}\{0.001, 0.001\}) + 0.1\mathcal{N}(0, \text{diag}\{0.1, 0.1\}),$$
$$\nu_{i,s} \sim 0.8\mathcal{N}(0, \text{diag}\{0.5, 0.5\}) + 0.2\mathcal{N}(0, \text{diag}\{500, 500\}).$$

The simulation results are shown in Figs. 1-4. It can be observed from Figs. 1-3 that compared with the VCF method, our proposed algorithm can well track the true state trajectories of three representative nodes, i.e. nodes 1, 3 and 5. In Fig. 4, the triggering instants for the first node are displayed, which verifies that the sensor components have different triggering rates under the component-wise DETT protocol.

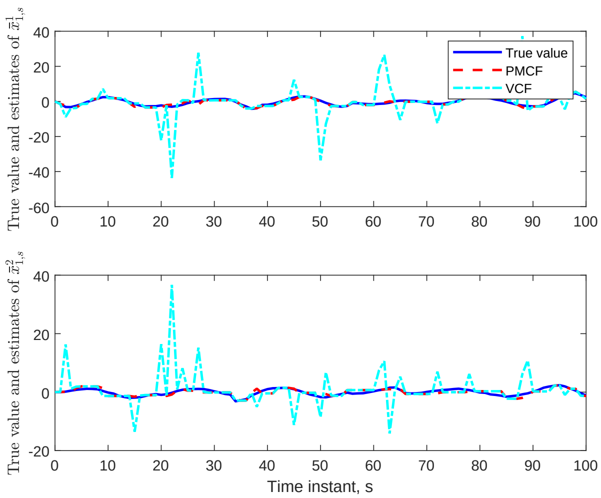



Fig. 1: True value and state estimates for node 1 in ***Case 1***.

The average RMSEs on state estimate, calculated over the total simulation times, are summarized in Table I, where only the results for nodes 1, 3 and 5 are displayed for brevity. Obviously, in this case, the kernel bandwidth can be neither too large nor too small. When the kernel bandwidth becomes very larger, the performance of the proposed algorithm is similar to that of the VCF method. It should be pointed out that when the kernel bandwidth is set as $\chi = 0.08$ in this case, the proposed algorithm has a noticeable performance improvement over the traditional method based on variance-constrained strategy.

To examine the effect of the triggering parameters onto the filtering performance and resource consumption, the average transmission rate is defined as the mean of the transmission rate (the ratio of the triggering times to total simulation times) over 500 Monte Carlo runs. The simulation results in terms of average RMSEs and transmission rates are provided in Table II, from which we are able to conclude that a proper

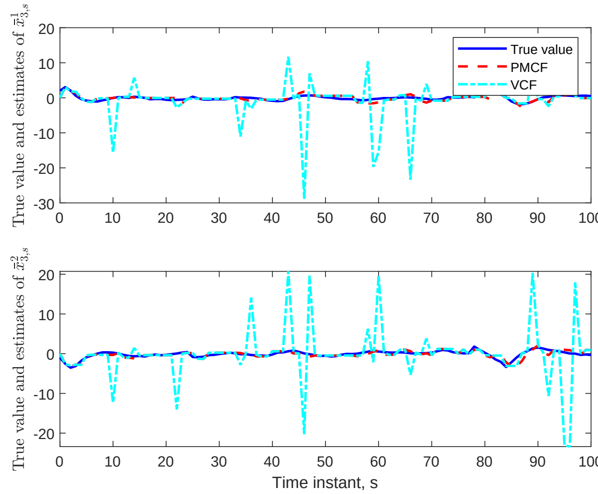



Fig. 2: True value and state estimates for node 3 in ***Case 1***.

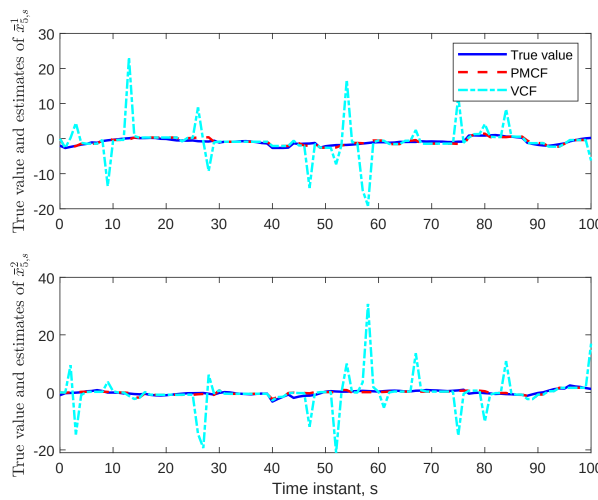



Fig. 3: True value and state estimates for node 5 in ***Case 1***.

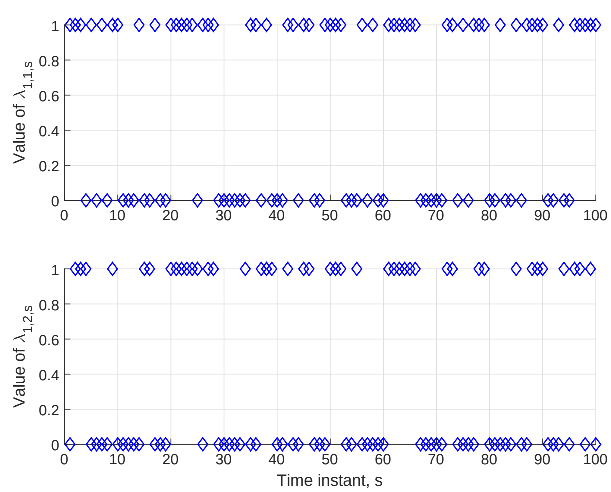



Fig. 4: Triggering instants for node 1 in ***Case 1***.

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


TABLE I: Performance comparisons for different algorithms and parameters under Gaussian mixture noises.

| | Node 1 | | Node 3 | | Node 5 | |
|---|---|---|---|---|---|---|
| | $\mathrm{RMSE}_1^1$ | $\mathrm{RMSE}_1^2$ | $\mathrm{RMSE}_3^1$ | $\mathrm{RMSE}_3^2$ | $\mathrm{RMSE}_5^1$ | $\mathrm{RMSE}_5^2$ |
| VCF | 8.2005 | 7.4317 | 6.4892 | 6.9062 | 5.0592 | 6.1273 |
| PMCF ($\chi = 0.01$) | 1.6610 | 1.0168 | 0.8645 | 0.7956 | 0.8902 | 0.6973 |
| PMCF ($\chi = 0.08$) | 0.9584 | 0.8249 | 0.8456 | 0.7044 | 0.5898 | 0.5891 |
| PMCF ($\chi = 0.5$) | 4.2965 | 3.0973 | 4.3674 | 2.7436 | 2.0805 | 1.7407 |
| PMCF ($\chi = 5$) | 8.1172 | 7.3320 | 6.3757 | 6.7044 | 4.9704 | 5.8088 |
| PMCF ($\chi = 10$) | 8.1419 | 7.3618 | 6.4129 | 6.7666 | 4.9972 | 5.9080 |

TABLE II: Average RMSEs and transmission rates for different triggering parameters under Gaussian mixture noises.

| | | $\rho_{i,l} = 1.2$ | $\rho_{i,l} = 4$ | $\rho_{i,l} = \infty$ |
|---|---|---|---|---|
| Node 1 | $\mathrm{RMSE}_1^1$ | 1.0465 | 0.9584 | 0.8358 |
| | $\mathrm{RMSE}_1^2$ | 0.9180 | 0.8249 | 0.7127 |
| | $\mathrm{TR}_1^1$ | 49.65% | 54.62% | 66.37% |
| | $\mathrm{TR}_1^2$ | 38.44% | 41.60% | 51.31% |
| Node 2 | $\mathrm{RMSE}_2^1$ | 0.9543 | 0.8686 | 0.7283 |
| | $\mathrm{RMSE}_2^2$ | 0.8779 | 0.8010 | 0.6956 |
| | $\mathrm{TR}_2^1$ | 38.59% | 41.74% | 52.57% |
| | $\mathrm{TR}_2^2$ | 41.22% | 45.87% | 59.14% |
| Node 3 | $\mathrm{RMSE}_3^1$ | 0.9020 | 0.8456 | 0.7735 |
| | $\mathrm{RMSE}_3^2$ | 0.7565 | 0.7044 | 0.6381 |
| | $\mathrm{TR}_3^1$ | 45.58% | 51.02% | 63.31% |
| | $\mathrm{TR}_3^2$ | 44.81% | 50.21% | 62.61% |
| Node 4 | $\mathrm{RMSE}_4^1$ | 0.7556 | 0.6980 | 0.6368 |
| | $\mathrm{RMSE}_4^2$ | 0.7736 | 0.7137 | 0.6319 |
| | $\mathrm{TR}_4^1$ | 41.58% | 45.66% | 56.33% |
| | $\mathrm{TR}_4^2$ | 36.81% | 39.88% | 49.53% |
| Node 5 | $\mathrm{RMSE}_5^1$ | 0.5928 | 0.5898 | 0.5611 |
| | $\mathrm{RMSE}_5^2$ | 0.6190 | 0.5891 | 0.5515 |
| | $\mathrm{TR}_5^1$ | 42.33% | 47.15% | 59.74% |
| | $\mathrm{TR}_5^2$ | 38.14% | 42.67% | 55.08% |
| Node 6 | $\mathrm{RMSE}_6^1$ | 0.6299 | 0.5976 | 0.5639 |
| | $\mathrm{RMSE}_6^2$ | 0.6897 | 0.6666 | 0.6321 |
| | $\mathrm{TR}_6^1$ | 38.31% | 42.83% | 55.62% |
| | $\mathrm{TR}_6^2$ | 38.66% | 43.79% | 56.52% |

triggering parameter can be chosen to cater for the practical engineering requirements. On the other hand, it is worth mentioning that when $\rho_{i,l} = \infty$, the component-wise DETT protocol reduces to its component-wise static counterpart. Clearly, the dynamic version has great potentials in relaxing the network communication burden.

***Case 2:*** In this case, the process noise is of the Gaussian form and the measurement noise is shot noise, i.e.,

$$\zeta_{i,s} \sim \mathcal{N}(0, \mathrm{diag}\{0.01, 0.01\}),$$
$$\eta_{i,s} \sim \mathcal{N}(0, \mathrm{diag}\{0.001, 0.001\}),$$
$$\nu_{i,s} \sim \mathcal{N}(0, \mathrm{diag}\{0.5, 0.5\}) + \text{Shot noise}.$$

For the first sensor component of node 1, one realization of the measurement noise at the time instants when shot noise occurs is depicted in Fig. 5. The simulation results are summarized in Figs. 6-12, which again verify the effectiveness of the proposed filtering scheme in terms of dealing with the typical shot non-Gaussian noises and saving the limited network resources.

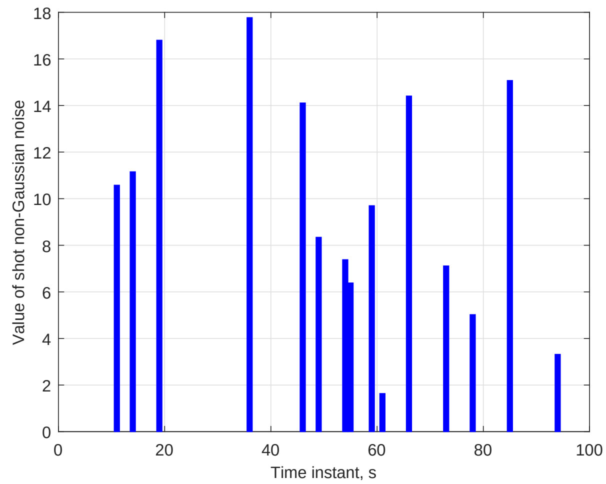



Fig. 5: One realization of the measurement noise at the time instants when shot noise occurs.

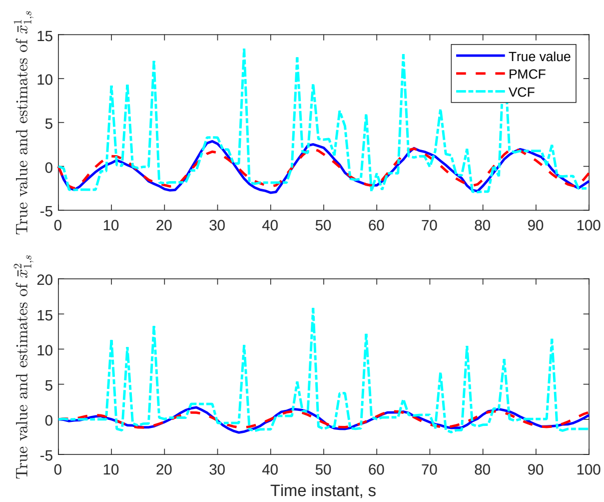



Fig. 6: True value and state estimates for node 1 in ***Case 2***.

## V. Conclusions

In this paper, the MCF problem has been dealt with for a class of nonlinear and non-Gaussian complex networks subject to uncertain dynamical bias under the DETT mechanism. To guarantee that each sensor component can determine its own triggering instant in an independent manner, the component-wise DETT protocol has been exploited to govern the process of data transmission. By resorting to the established upper bounds on the one-step prediction error covariance and the equivalent noise covariance, a novel CBPI has been parameterized to extenuate the influence of the non-Gaussian noises and the component-wise DETT protocol. Accordingly, a recursive filter has been designed based on the maximum

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems

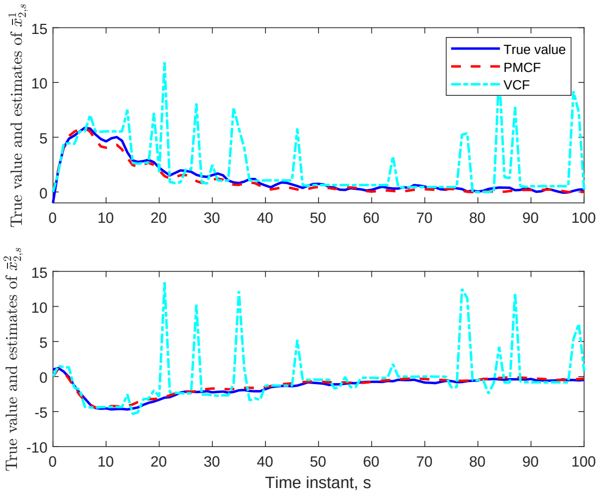




Fig. 7: True value and state estimates for node 2 in ***Case 2***.

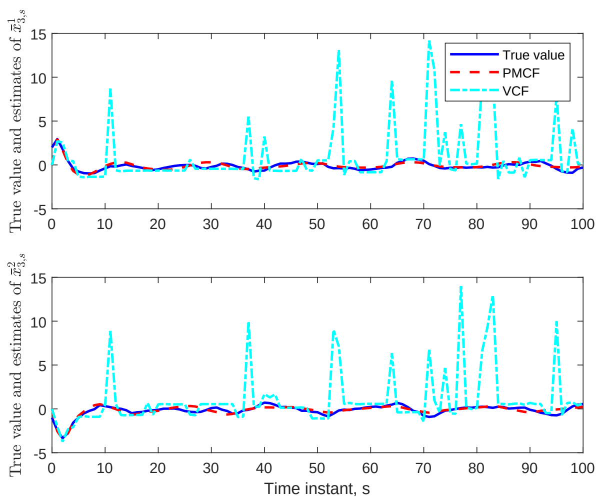



Fig. 8: True value and state estimates for node 3 in ***Case 2***.

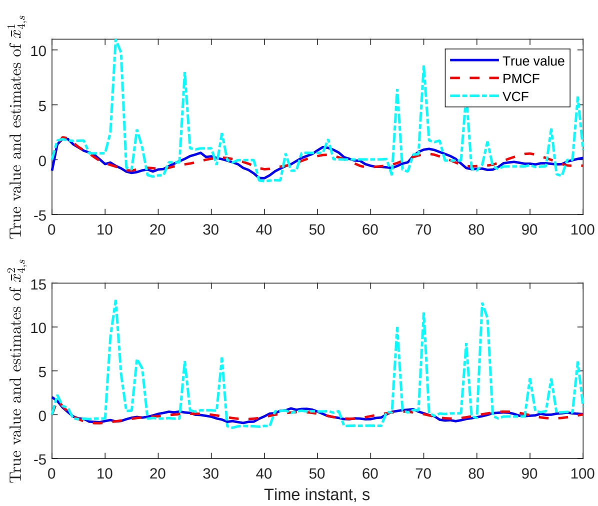



Fig. 9: True value and state estimates for node 4 in ***Case 2***.

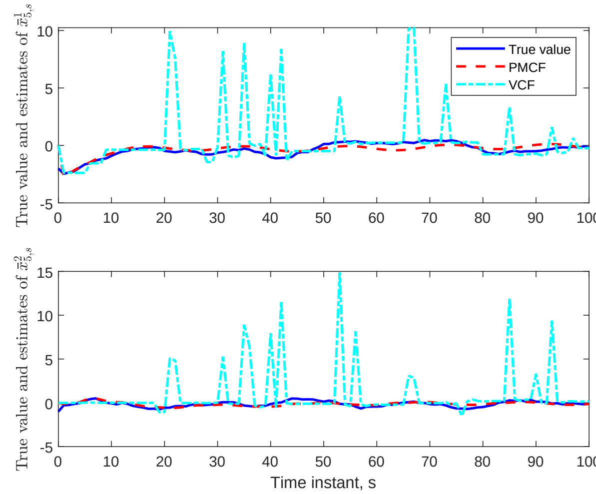



Fig. 10: True value and state estimates for node 5 in ***Case 2***.

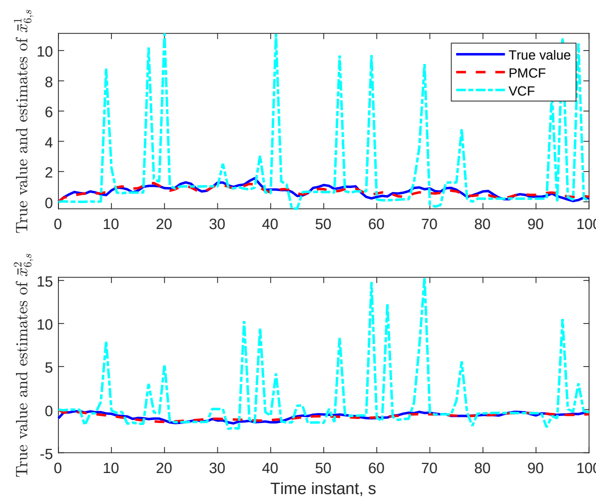



Fig. 11: True value and state estimates for node 6 in ***Case 2***.

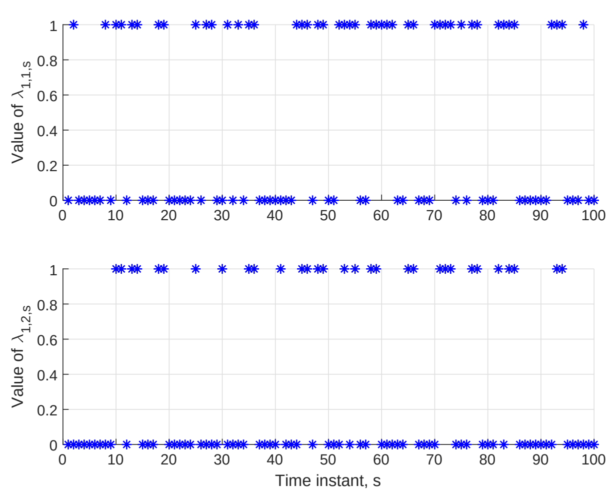



Fig. 12: Triggering instants for node 1 in ***Case 2***.

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


correntropy criterion, which exhibits distinguishing advantages in terms of not only the robustness against non-Gaussian noises, but the resource saving as well. Finally, simulation results have been given to demonstrate the effectiveness of the proposed MCF algorithm. In the future, two possible research directions would be 1) investigating the influences of the node couplings onto the filtering performance; and 2) exploring the extensions of the developed results to other systems, such as the recurrent time-varying neural networks [31], the wireless sensor networks [53], and the multi-agent systems [51].

## References


[1] D. Alspach and H. Sorenson, Nonlinear Bayesian estimation using Gaussian sum approximations, *IEEE Transactions on Automatic Control*, vol. 17, no. 4, pp. 439–448, 1972.

[2] M. S. Arulampalam, S. Maskell, N. Gordon and T. Clapp, A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking, *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174–188, 2002.

[3] M. Basin and P. Rodriguez-Ramirez, Sliding mode filter design for nonlinear polynomial systems with unmeasured states, *Information Sciences*, vol. 204, pp. 82–91, 2012.

[4] R. Caballero-Águila, A. Hermoso-Carazo and J. Linares-Pérez, Networked fusion estimation with multiple uncertainties and time-correlated channel noise, *Information Fusion*, vol. 54, pp. 161–171, 2020.

[5] B. Chen, X. Liu, H. Zhao and J. C. Principe, Maximum correntropy Kalman filter, *Automatica*, vol. 76, pp. 70–77, 2017.

[6] E. R. Pinto, E. G. Nepomuceno and A. S. L. O. Campanharo, Individual-based modelling of animal brucellosis spread with the use of complex networks, *International Journal of Network Dynamics and Intelligence*, vol. 1, no. 1, pp. 120–129, 2022.

[7] X. Ge, Q.-L. Han and Z. Wang, A dynamic event-triggered transmission scheme for distributed set-membership estimation over wireless sensor networks, *IEEE Transactions on Cybernetics*, vol. 49, no. 1, pp. 171–183, 2019.

[8] Y. Han, W. Lu and T. Chen, Event-triggered scheduling for pinning networks of coupled dynamical systems under stochastically fast switching, *IET Control Theory & Applications*, vol. 15, no. 12, pp. 1673–1685, 2021.

[9] C.-S. Hsieh, Unbiased minimum-variance input and state estimation for systems with unknown inputs: A system reformation approach, *Automatica*, vol. 84, pp. 236–240, 2017.

[10] J. Hu, Z. Wang and G.-P. Liu, Delay compensation-based state estimation for time-varying complex networks with incomplete observations and dynamical bias, *IEEE Transactions on Cybernetics*, vol. 52, no. 11, pp. 12071–12083, 2022.

[11] J. Hu, Z. Wang, G.-P. Liu and H. Zhang, Variance-constrained recursive state estimation for time-varying complex networks with quantized measurements and uncertain inner coupling, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 6, pp. 1955–1967, 2020.

[12] S. Hu, X. Chen, J. Qiu, F. Zhao, X. Jiang and Y. Du, Dynamic event-triggered bipartite consensus of multiagent systems with estimator and cooperative-competitive interactions, *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 7, pp. 3309–3313, 2022.

[13] S. Hu, J. Qiu, X. Chen, F. Zhao and X. Jiang, Dynamic event-triggered control for leader-following consensus of multiagent systems with the estimator, *IET Control Theory & Applications*, vol. 16, no. 4, pp. 475–484, 2022.

[14] Z. Hu, J. Hu, H. Tan, J. Huang and Z. Cao, Distributed resilient fusion filtering for nonlinear systems with random sensor delay under Round-Robin protocol, *International Journal of Systems Science*, vol. 53, no. 13, pp. 2786–2799, 2022.

[15] M. Ignagni, Optimal and suboptimal separate-bias Kalman estimators for a stochastic bias, *IEEE Transactions on Automatic Control*, vol. 45, no. 3, pp. 547–551, 2000.

[16] R. Izanloo, S. A. Fakoorian, H. S. Yazdi and D. Simon, Kalman filtering based on the maximum correntropy criterion in the presence of non-Gaussian noise, in *Proceedings of 2016 Annual Conference on Information Science and Systems*, Princeton, NJ, USA, 2016, pp. 500–505.

[17] J.-Y. Keller and D. D. J. Sauter, Kalman filter for discrete-time stochastic linear systems subject to intermittent unknown inputs, *IEEE Transactions on Automatic Control*, vol. 58, no. 7, pp. 1882–1887, 2013.

[18] J. Kim, M. Tandale, P. K. Menon and E. Ohlmeyer, Particle filter for ballistic target tracking with glint noise, *Journal of Guidance, Control, and Dynamics*, vol. 33, no. 6, pp. 1918–1921, 2010.

[19] P. S. Kim, Separate-bias estimation scheme with diversely behaved biases, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 38, no. 1, pp. 333–339, 2002.

[20] M. Kooshkbaghi and H. J. Marquez, Event-triggered discrete-time cubature Kalman filter for nonlinear dynamical systems with packet dropout, *IEEE Transactions on Automatic Control*, vol. 65, no. 5, pp. 2278–2285, 2020.

[21] R. J. Kozick and B. M. Sadler, Maximum-likelihood array processing in non-Gaussian noise with Gaussian mixtures, *IEEE Transactions on Signal Processing*, vol. 48, no. 12, pp. 3520–3535, 2000.

[22] M. V. Kulikova, Chandrasekhar-based maximum correntropy Kalman filtering with the adaptive kernel size selection, *IEEE Transactions on Automatic Control*, vol. 65, no. 2, pp. 741–748, 2020.

[23] M. Li, J. Liang and F. Wang, Robust set-membership filtering for two-dimensional systems with sensor saturation under the Round-Robin protocol, *International Journal of Systems Science*, vol. 53, no. 13, pp. 2773-2785, 2022.

[24] Q. Li, B. Shen, Z. Wang and W. Sheng, Recursive distributed filtering over sensor networks on Gilbert-Elliott channels: A dynamic event-triggered approach, *Automatica*, vol. 113, art. no. 108681, 2020.

[25] W. Li, Y. Jia and J. Du, Resilient filtering for nonlinear complex networks with multiplicative noise, *IEEE Transactions on Automatic Control*, vol. 64, no. 6, pp. 2522–2528, 2019.

[26] W. Li, Y. Niu and Z. Cao, Event-triggered sliding mode control for multi-agent systems subject to channel fading, *International Journal of Systems Science*, vol. 53, no. 6, pp. 1233-1244, 2022.

[27] C. Liu, L. Yang, J. Tao, Y. Xu and T. Huang, Set-membership filtering for complex networks with constraint communication channels, *Neural Networks*, vol. 152, pp. 479–486, 2022.

[28] W. Liu, P. P. Pokharel and J. C. Principe, Correntropy: Properties and applications in non-Gaussian signal processing, *IEEE Transactions on Signal Processing*, vol. 55, no. 11, pp. 5286–5298, 2007.

[29] X. Liu, D. W. C. Ho, Q. Song and W. Xu, Finite/fixed-time pinning synchronization of complex networks with stochastic disturbances, *IEEE Transactions on Cybernetics*, vol. 49, no. 6, pp. 2398–2403, 2019.

[30] W. Lu, Y. Han and T. Chen, Synchronization in networks of linearly coupled dynamical systems via event-triggered diffusions, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 26, no. 12, pp. 3060–3069, 2015.

[31] W. Lu, R. Zheng and T. Chen, Centralized and decentralized global outer-synchronization of asymmetric recurrent time-varying neural network by data-sampling, *Neural Networks*, vol. 75, pp. 22–31, 2016.

[32] L. Ma, Z. Wang, C. Cai, and F. E. Alsaadi, Dynamic event-triggered state estimation for discrete-time singularly perturbed systems with distributed time-delays, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 9, pp. 3258–3268, 2020.

[33] M. E. J. Newman, The structure and function of complex networks, *SIAM Review*, vol. 45, no. 2, pp. 167–256, 2003.

[34] L. D. L. Perera, W. S. Wijesoma and M. D. Adams, SLAM with joint sensor bias estimation: Closed form solutions on observability, error bounds and convergence rates, *IEEE Transactions on Control Systems Technology*, vol. 18, no. 3, pp. 732–740, 2010.

[35] M. S. Radenković and M. Krstić, Distributed adaptive consensus and synchronization in complex networks of dynamical systems, *Automatica*, vol. 91, pp. 233–243, 2018.

[36] H. Shen, Y. Men, J. Cao and J. H. Park, $H_\infty$ filtering for fuzzy jumping genetic regulatory networks with Round-Robin protocol: A hidden-Markov-model-based approach, *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 1, pp. 112–121, 2020.

[37] H. Song, D. Ding, H. Dong and X. Yi, Distributed filtering based on Cauchy-kernel-based maximum correntropy subject to randomly occurring cyber-attacks, *Automatica*, vol. 135, art. no. 110004, 2022.

[38] H. Song, D. Ding, H. Dong and Q.-L. Han, Distributed maximum correntropy filtering for stochastic nonlinear systems under deception attacks, *IEEE Transactions on Cybernetics*, vol. 52, no. 5, pp. 3733–3744, 2022.

[39] H. Song, P. Shi, C.-C. Lim, W.-A. Zhang and L. Yu, Set-membership estimation for complex networks subject to linear and nonlinear bounded attacks, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 1, pp. 163–173, 2020.

[40] T. Su, Z. Wang, L. Zou and F. E. Alsaadi, Maximum-correntropy-based Kalman filtering for time-varying systems with randomly occurring uncertainties: An event-triggered approach, *International Journal of Robust and Nonlinear Control*, vol. 31, no. 5, pp. 1582–1599, 2021.

[41] O. J. Suarez, C. J. Vega, E. N. Sanchez, G. Chen, J. S. Elvira-Ceja and D. I. Rodriguez, Neural sliding-mode pinning control for output synchronization for uncertain general complex networks, *Automatica*, vol. 112, art. no. 108694, 2020.

[42] J. Suo and N. Li, Observer-based synchronisation control for discrete-time delayed switched complex networks with coding-decoding approach, *International Journal of Systems Science*, vol. 53, no. 13, pp. 2711–2728, 2022.

[43] H. Tao, H. Tan, Q. Chen, H. Liu and J. Hu, $H_\infty$ state estimation for memristive neural networks with randomly occurring DoS attacks, *Systems Science & Control Engineering*, vol. 10, no. 1, pp. 154–165, 2022.

[44] F. Wang, Z. Wang, J. Liang and X. Liu, Recursive state estimation for two-dimensional shift-varying systems with random parameter perturbation and dynamical bias, *Automatica*, vol. 112, art. no. 108658, 2020.

[45] G. Wang, N. Li and Y. Zhang, Maximum correntropy unscented Kalman and information filters for non-Gaussian measurement noise, *Journal of the Franklin Institute*, vol. 354, no. 18, pp. 8659–8677, 2017.

[46] G. Wang, Y. Zhang and X. Wang, Maximum correntropy Rauch-Tung-Striebel smoother for nonlinear and non-Gaussian systems, *IEEE Transactions on Automatic Control*, vol. 66, no. 3, pp. 1270–1277, 2021.

[47] X. Wang, Y. Sun and D. Ding, Adaptive dynamic programming for networked control systems under communication constraints: A survey of trends and techniques, *International Journal of Network Dynamics and Intelligence*, vol. 1, no. 1, pp. 85–98, 2022.

[48] D. J. Watts and S. H. Strogatz, Collective dynamics of ‘small-world’ networks, *Nature*, vol. 393, pp. 440–442, 1998.

[49] P. Wen, X. Li, N. Hou and S. Mu, Distributed recursive fault estimation with binary encoding schemes over sensor networks, *Systems Science & Control Engineering*, vol. 10, no. 1, pp. 417–427, 2022.

[50] Y. Xu, C. Liu, J.-Y. Li, C.-Y. Su and T. Huang, Finite-horizon $H_\infty$ state estimation for time-varying neural networks with periodic inner coupling and measurements scheduling, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 1, pp. 211–219, 2020.

[51] X. Yi, W. Lu and T. Chen, Pull-based distributed event-triggered consensus for multiagent systems with directed topologies, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 1, pp. 71–79, 2017.

[52] S. Z. Yong, M. Zhu and E. Frazzoli, A unified filter for simultaneous input and state estimation of linear discrete-time stochastic systems, *Automatica*, vol. 63, pp. 321–329, 2016.

[53] D. Yu, Y. Xia, L. Li and C. Zhu, Distributed consensus-based estimation with unknown inputs and random link failures, *Automatica*, vol. 122, art. no. 109259, 2020.

[54] X.-M. Zhang, Q.-L. Han, X. Ge, D. Ding, L. Ding, D. Yue and C. Peng, Networked control systems: A survey of trends and techniques, *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 1, pp. 1–17, 2020.

[55] Y. Zhao, X. He, L. Ma and H. Liu, Unbiasedness-constrained least squares state estimation for time-varying systems with missing measurements under Round-Robin protocol, *International Journal of Systems Science*, vol. 53, no. 9, pp. 1925–1941, 2022.

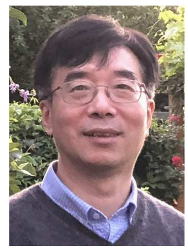
[56] R. Zheng, X. Yi, W. Lu and T. Chen, Stability of analytic neural networks with event-triggered synaptic feedbacks, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 27, no. 2, pp. 483–494, 2016.

[57] H. Zhu, G. Zhang, Y. Li and H. Leung, An adaptive Kalman filter with inaccurate noise covariances in the presence of outliers, *IEEE Transactions on Automatic Control*, vol. 67, no. 1, pp. 374–381, 2022.

[58] L. Zou, Z. Wang and D. Zhou, Moving horizon estimation with non-uniform sampling under component-based dynamic event-triggered transmission, *Automatica*, vol. 120, art. no. 109154, 2020.


**Weihao Song** received the B.S. degree in flight vehicle design and engineering in 2016 and the Ph.D. degree in aeronautical and astronautical science and technology in 2021, both from Beijing Institute of Technology, Beijing, China.

From May 2019 to May 2020, he was a Visiting Scholar with the Department of Computer Science, Brunel University London, London, U.K. He is currently a Postdoctoral Researcher with the College of Engineering, Peking University. His research interests include Bayesian state estimation, distributed state estimation, nonlinear filtering, and networked control systems.



**Zidong Wang** (Fellow, IEEE) received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently a Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the UK. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published more than 700 papers in international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, the Editor-in-Chief for *Systems Science & Control Engineering*, and an Associate Editor for 12 international journals including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a Member of the Academia Europaea, a Member of the European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a Fellow of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

**Zhongkui Li** (Senior Member, IEEE) received the B.S. degree in space engineering from the National University of Defense Technology, China, in 2005, and his Ph.D. degree in dynamics and control from Peking University, China, in 2010.

Since 2013, Dr. Li has been with the Department of Mechanics and Engineering Science, College of Engineering, Peking University, China, where he is currently an Associate Professor with tenure. He has authored the book entitled *Cooperative Control of Multi-Agent Systems: A Consensus Region Approach* (CRC press, 2014) and has published a number of journal papers. His current research interests include cooperative control of multi-agent systems, networked control systems, control of autonomous unmanned systems.

Dr. Li was the recipient of the State Natural Science Award of China (Second Prize) in 2015, the Yang Jiachi Science and Technology Award in 2015, and the National Excellent Doctoral Thesis Award of China in 2012. His coauthored papers received the IET Control Theory & Applications Premium Award in 2013 and the Best Paper Award of *Journal of Systems Science & Complexity* in 2012. He was selected into the Changjiang Scholars Program (Young Scholar), Ministry of Education of China, in 2017. He serves as an Associate Editor of *IEEE Transactions on Automatic Control*, *Nonlinear Analysis: Hybrid Systems*, and the Conference Editorial Board of IEEE Control Systems Society.

This article has been accepted for publication in a future issue of this conference proceedings, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/tnnls.2023.3302190, IEEE Transactions on Neural Networks and Learning Systems


**Qing-Long Han** (Fellow, IEEE) received the B.Sc. degree in Mathematics from Shandong Normal University, Jinan, China, in 1983, and the M.Sc. and Ph.D. degrees in Control Engineering from East China University of Science and Technology, Shanghai, China, in 1992 and 1997, respectively.

Professor Han is Pro Vice-Chancellor (Research Quality) and a Distinguished Professor at Swinburne University of Technology, Melbourne, Australia. He held various academic and management positions at Griffith University and Central Queensland University, Australia. His research interests include networked control systems, multi-agent systems, time-delay systems, smart grids, unmanned surface vehicles, and neural networks.

Professor Han was awarded The 2021 Norbert Wiener Award (the Highest Award in systems science and engineering, and cybernetics) and The 2021 M. A. Sargent Medal (the Highest Award of the Electrical College Board of Engineers Australia). He was the recipient of The 2021 IEEE/CAA Journal of Automatica Sinica Norbert Wiener Review Award, The 2020 IEEE Systems, Man, and Cybernetics (SMC) Society Andrew P. Sage Best Transactions Paper Award, The 2020 IEEE Transactions on Industrial Informatics Outstanding Paper Award, and The 2019 IEEE SMC Society Andrew P. Sage Best Transactions Paper Award.

Professor Han is a Member of the Academia Europaea (The Academy of Europe). He is a Fellow of The International Federation of Automatic Control and a Fellow of The Institution of Engineers Australia. He is a Highly Cited Researcher in both Engineering and Computer Science (Clarivate Analytics). He has served as an AdCom Member of IEEE Industrial Electronics Society (IES), a Member of IEEE IES Fellows Committee, and Chair of IEEE IES Technical Committee on Networked Control Systems. He is Co-Editor-in-Chief of IEEE Transactions on Industrial Informatics, Deputy Editor-in-Chief of IEEE/CAA JOURNAL OF AUTOMATICA SINICA, Co-Editor of Australian Journal of Electrical and Electronic Engineering, an Associate Editor for 12 international journals, including the IEEE TRANSACTIONS ON CYBERNETICS, IEEE INDUSTRIAL ELECTRONICS MAGAZINE, Control Engineering Practice, and Information Sciences, and a Guest Editor for 14 Special Issues.

**Dong Yue** (Fellow, IEEE) is currently a Professor and the Dean of the Institute of Advanced Technology and College of Automation & AI, Nanjing University of Posts and Telecommunications. He is the Chair of IEEE IES Technical Committee on NCS and Applications and Chair of IEEE PES Smart Grid & Emerging Technologies Satellite Committee-China. He has served as the Associate Editor of IEEE Industrial Electronics Magazine, IEEE Transactions on Industrial Informatics, IEEE Transactions on Systems, Man and Cybernetics: Systems, IEEE Transactions on Neural Networks and Learning Systems, Journal of the Franklin Institute, and International Journal of Systems Sciences, and the Guest Editor of Special Issue on New Trends in Energy Internet: Artificial Intelligence-Based Control, Network Security and Management, and IEEE Transactions on Systems, Man, and Cybernetics: Systems. He is a Fellow of the IEEE for his contribution to network-based control and its applications to power systems. Up to now, he has published more than 250 papers in international journals and 4 books in Springer. He holds more than 90 patents. His research interests include analysis and synthesis of networked control systems, multi-agent systems, optimal control of power systems, and Internet of Things.