



PERSISTENCE AND SEASONALITY IN THE US INDUSTRIAL PRODUCTION INDEX

**Guglielmo Maria Caporale¹, Luis A. Gil-Alana^{2,3,*},
Carlos Poza⁴ and Alvaro Baños Izquierdo⁵**

¹Brunel University London
UK

²Faculty of Economics
University of Navarra
NCID, DATAI, Pamplona
Spain

³Department of Economics
Universidad Francisco de Vitoria
Madrid, Spain
e-mail: alana@unav.es

⁴Universidad Francisco de Vitoria
Madrid, Spain

Received: September 20, 2023; Accepted: November 24, 2023

2020 Mathematics Subject Classification: C22, E23, E32.

Keywords and phrases: industrial production index, seasonality, persistence, fractional integration, time series.

*Corresponding author

Communicated by K. K. Azad

How to cite this article: Guglielmo Maria Caporale, Luis A. Gil-Alana, Carlos Poza and Alvaro Baños Izquierdo, Persistence and seasonality in the US industrial production index, *Advances and Applications in Statistics* 92(7) (2025), 963-972.

<https://doi.org/10.17654/0972361725041>

This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

Published Online: May 13, 2025

⁵University of Navarra
DATAI, Pamplona, Spain

Abstract

This paper uses a seasonal long-memory model to capture the behaviour of the US industrial production index (IPI) over the period 1919Q1-2022Q4. This series is found to display a large value of the periodogram at the zero, long-run frequency, and to exhibit an order of integration around 1. When first differences (of either the original data or their logged values) are taken, evidence of seasonality is obtained; more specifically, deterministic seasonality is rejected in favour of a seasonal fractional integration model with an order of integration equal to 0.14 for the original data and 0.29 for their logged values, which implies the presence of a seasonal long-memory mean reverting pattern.

1. Introduction

Understanding economic fluctuations is crucial for the design of effective macroeconomic policies. Policy makers use a variety of demand and supply indicators to monitor economic activity and to identify trends and seasonal patterns (see [1]). On the demand side, these include private consumption, retail sales, car registrations, electricity consumption, etc.; on the supply side, the most informative series are gross capital formation, which is available at a quarterly frequency, as well as industrial production, electricity production, and capacity utilisation in the industrial sector, which are released at a monthly frequency (see [2]).

The present study focuses on the industrial production index (IPI), which is normally thought to be a good proxy for aggregate production and also to be informative about seasonality in the economy. According to [3]: “*The index of industrial production (IPI) is probably the most important and widely analyzed high-frequency indicator, given the relevance of manufacturing activity as a driver of the whole business cycle*”. In this paper a long-memory seasonal model is estimated to capture the behaviour of the

IPI and to obtain evidence on both its degree of persistence and seasonal patterns.

The rest of the paper is organised as follows: Section 2 provides a brief review of the relevant literature; Section 3 presents the empirical analysis; Section 4 offers some concluding comments.

2. Literature Review

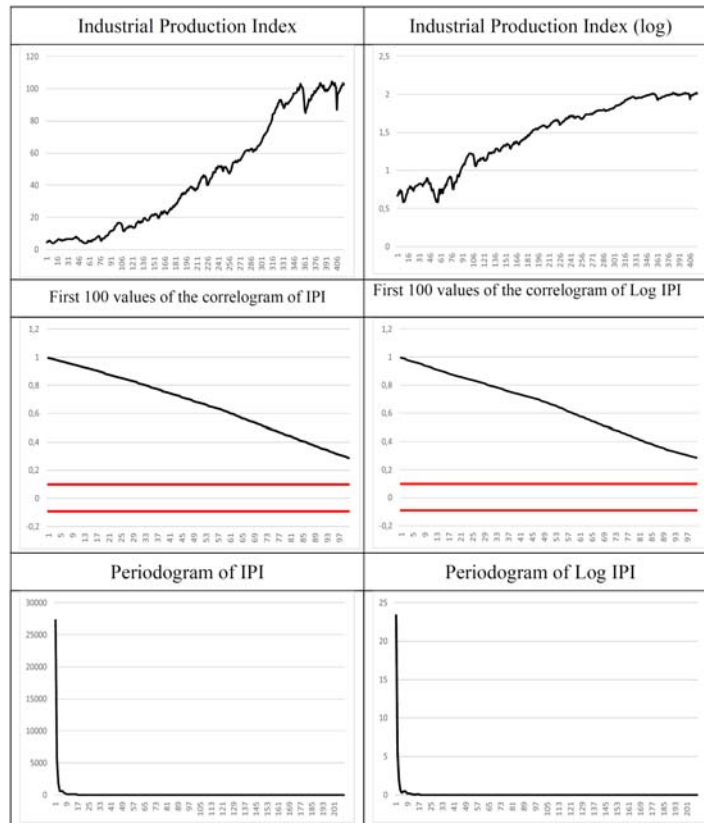
Numerous studies have analysed the behaviour of the IPI because of its usefulness as an indicator of economic activity. For instance, [3] assessed the forecasting performance of various models for the Italian IPI including a univariate ARIMA model, a dynamic single-equation model with a few indicators, a dynamic multiple-equation model disaggregated by sector, an average of bivariate autoregressive distributed lag model forecasts, h-step forecasts and sequential one-step forecasts of a static factor model, and generalized dynamic factor models with fixed rules and optimal criteria respectively to determine the number of factors. [4] instead analysed business survey data for France, Germany, and Italy and estimated dynamic factor and unobserved components models. [5] modeled monthly IPI in Turkey, Brazil, and the G7 economies over the period from 1990 to 2017 using linear, quadratic, cubic, and hyperbolic specifications as well as non-linear ones (Weibull, Negative Exponential, Brody, Gompertz, Logistic, Von Bertalanffy, Richards), while [6] examined persistence in the Indian IPI by carrying out augmented Dickey-Fuller (ADF), Phillips Perron (PP), and KPSS tests.

An important feature of the IPI often overlooked in the existing literature is its seasonality. [7] showed that cross-sectional aggregation or structural changes can result in fractional orders of integration at the seasonal frequencies. Therefore [8] proposed a framework allowing for unit and fractional roots at both the seasonal and long-run frequencies. In particular, they analysed the behaviour of the IPI in four Latin American countries, namely Brazil, Argentina, Colombia, and Mexico, and found evidence of long-memory behaviour in the seasonal component in the two former

economies. The present study also uses a framework allowing for long memory in the seasonal component, as explained below.¹

3. Empirical Analysis

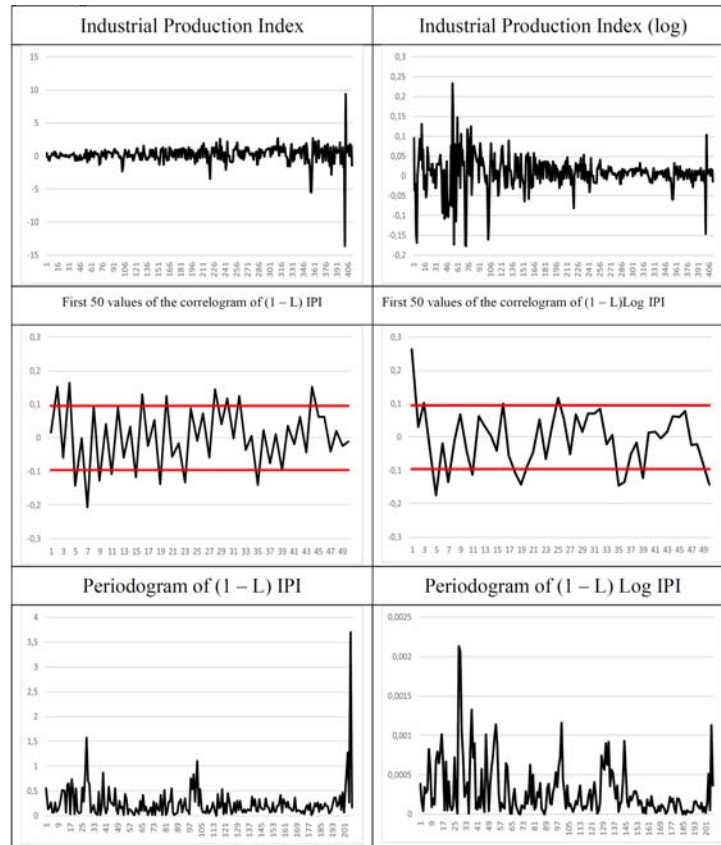
We use quarterly, seasonally unadjusted data on the US Industrial Production Index, for the sample period from 1919Q1 to 2022Q4, which have been obtained from the St. Louis Federal Reserve Bank database.



The red lines in the correlograms refer to the 95% confidence bands for no autocorrelation.

Figure 1. Plots of IPI and log IPI with their correlograms and periodograms.

¹Note that seasonal long-memory models have also been used for GDP ([9], M1 [10]), tourism series [11], inflation [12, 13], climatological series [14], and energy consumption [15].



The red lines in the correlograms refer to the 95% confidence bands for no autocorrelation.

Figure 2. First differences of IPI and log IPI with their correlograms and periodograms.

Figure 1 displays both the original data and their logged values together with their respective correlograms and periodograms, the latter exhibiting a large value at the zero, long-run frequency. Figure 2 shows instead the first differenced series, a seasonal pattern being clearly visible.

Given the large value of the periodogram at the long-run, zero frequency we focus first on the degree of integration of the series at this frequency. Standard unit root tests [16-18] provide evidence of unit roots in all cases (these results are not reported). However, it is well known that such tests have very low power if the true data generating process (DGP) is in fact

fractionally integrated (see, e.g., [19-21]); therefore we allow for the possibility of fractional degrees of integration by estimating a model of the following form:

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1-L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

where y_t stands for the observed time series, β_0 and β_1 are the intercept and the coefficient on a linear time trend respectively, and x_t is assumed to be $I(d)$, where d is another parameter to be estimated from the data. As for the error term u_t , this is assumed to be in turn a white noise and a (weakly) autocorrelated process, where the non-parametric approach of Bloomfield [22] (which is an approximation to AR structures based on the spectral density function) is used first, and then, given the quarterly frequency of the data, a seasonal AR(1) process is also considered of the following form:

$$u_t = \rho u_{t-12} + \varepsilon_t, \quad t = 1, 2, \dots, \quad (2)$$

where ε_t is a white noise process. The estimated values of d together with their 95% confidence bands are reported in Table 1 for three different specifications, namely: (i) without deterministic terms, (ii) with a constant, and (iii) with both a constant and a linear time trend.

Table 1. Estimates at the long-run or zero frequency

(i) Original data					
Type of errors	No deterministic terms		An intercept		An intercept and a time trend
White noise	1.03	(0.96, 1.10)	1.03	(0.97, 1.11)	1.03 (0.97, 1.11)
Bloomfield	1.07	(0.97, 1.25)	1.07	(0.96, 1.24)	1.08 (0.97, 1.25)
Seasonal AR1	1.03	(0.97, 1.10)	1.03	(0.96, 1.11)	1.04 (0.96, 1.12)
(ii) Logged values					
Type of errors	No deterministic terms		An intercept		An intercept and a time trend
White noise	1.04	(0.96, 1.13)	1.20	(1.10, 1.31)	1.19 (1.10, 1.30)
Bloomfield	0.98	(0.84, 1.14)	0.90	(0.80, 1.08)	0.90 (0.80, 1.08)
Seasonal AR1	1.02	(0.96, 1.13)	1.20	(1.11, 1.30)	1.19 (1.10, 1.30)

In bold, evidence of unit roots at the 95% level.

In the majority of cases the unit root null hypothesis cannot be rejected. The only exception is the logged series with white noise and seasonal AR disturbances when deterministic terms are included in the model. Given the overwhelming evidence in favour of the presence of unit roots, first differences are then taken of both the raw data and their logged values, the latter being a measure of the growth rate.

After removing the long-run frequency, seasonality is still present in the data as shown by the correlograms and periodograms of the first differenced series displayed in Figure 2. To capture it, we adopt the following specification:

$$y_t = \beta + \sum_{s=1}^4 \gamma_s D_{st} + x_t, \quad (1 - L^4)^d x_t = u_t, \quad (3)$$

where u_t is again a seasonal AR(1) process.

Table 2. Estimated coefficients for a seasonally fractionally integrated process

Series	d	β	γ_1	γ_2	γ_3	γ_4
Original	0.14 (0.04, 0.32)	-0.05768 (-0.30)	-0.05701 (-0.30)	0.17705 (0.93)	-0.09637 (-0.50)	0.00005 (0.16)
Logged values	0.29 (0.02, 0.65)	-0.00719 (-0.67)	0.00314 (0.32)	-0.00107 (-0.10)	0.00632 (0.64)	-0.00001 (-0.44)

The values in parenthesis are the t -values of the estimated coefficients.

Table 2 reports the estimated coefficients. It can be seen that, for both the original and logged values, the deterministic terms are statistically insignificant in all cases, which represents evidence against deterministic seasonality. The seasonal AR coefficient, is insignificant for the original data (0.0006) while significant for the logged ones (-0.2456); also, the seasonal fractional parameter d is positive and below 0.5 in both cases (0.14 for the original series and 0.29 for the logged one), which implies the presence of stationary seasonal long-memory in both series, the effects of shocks being mean reverting with a hyperbolic rate of decay to zero.

4. Conclusions

This paper uses a seasonal long-memory model to capture the behaviour of the US Industrial Production Index (IPI) over the period 1919Q1-2022Q4. This series is found to display a large value of the periodogram at the zero, long-run frequency, and to exhibit an order of integration around 1. When first differences (of either the original data or their logged values) are taken, evidence of seasonality is obtained; more specifically, deterministic seasonality is rejected in favour of a seasonal fractional integration model with an order of integration equal to 0.14 for the original data and 0.29 for their logged values, which implies the presence of a seasonal long-memory mean reverting pattern. These findings confirm the importance of allowing for (stochastic) seasonality when modelling IPI, which is a very useful proxy for aggregate economic activity often used by policy makers and agents to monitor developments in the economy.

References

- [1] The Economist, Guide to Economic Indicators, Making Sense of Economics, 6th ed., Bloomberg Press, 2007.
- [2] C. Poza, *Nálisis macroeconómico de países*, Thomson Reuters Aranzadi, 2020.
- [3] G. Bulligan, R. Golinelli and G. Parigi, Forecasting industrial production: the role of information and methods, IFC Bulletin 33 (2010), 227-235.
- [4] G. Bruno and C. I. Lupi, Forecasting Euro-Area Industrial Production using Business Surveys Data, ISAE Istituto di Studi e Analisi Economica, 2003.
- [5] N. Öksüz Nariñç, Modeling and model comparison for industrial production index of Turkey, Brazil and G7 countries, *Int. J. Sci. Res. and Man.* 6(4) (2018), 1-11.
- [6] P. Dua and T. Mishra, Presence of persistence in industrial production: the case of India, *Indian Economic Review* 34 (1999), 23-38.
- [7] P. M. Lildholdt, Sources of Seasonal Fractional Integration in Macroeconomic Time Series, Centre for Analytical Finance, University of Aarhus, Working Paper, No. 125, 2002.

- [8] B. Candelon and L. A. Gil-Alana, Seasonal and long-run fractional integration in the Industrial Production Indexes of some Latin American countries, *J. Pol. Mod.* 26 (2004), 301-313.
- [9] L. A. Gil-Alana, Seasonal long memory in the aggregate output, *Economics Letters* 74(3) (2002), 333-337. [https://doi.org/10.1016/S0165-1765\(01\)00556-0](https://doi.org/10.1016/S0165-1765(01)00556-0).
- [10] L. A. Gil-Alana, Seasonal long memory in the US monthly monetary aggregate, *Appl. Econ. Lett.* 8(9) (2001), 573-575. DOI: 10.1080/13504850010026078.
- [11] L. A. Gil-Alana, F. Perez De Gracia and J. Cuñado, Seasonal fractional integration in the Spanish tourism quarterly time series, *J. Trav. Res.* 42(4) (2004), 408-414. <https://doi.org/10.1177/0047287503258843>.
- [12] J. Arteche, The analysis of seasonal long memory, the case of Spanish inflation, *Oxf. Bull. Econ. Stat.* 69(6) (2007), 749-772.
- [13] J. Arteche, Standard and seasonal long memory in volatility: An application to Spanish inflation, *Emp. Econ.* 42 (2012), 693-712. <https://doi.org/10.1007/s00181-010-0446-8>.
- [14] O. S. Yaya, L. A. Gil-Alana and A. A. Akomolafe, Long memory, seasonality and time trends in the average monthly rainfall in major cities of Nigeria, *CBN J. Appl. Stat.* 6(2) (2015), 39-58.
- [15] O. B. Adekoya, Long memory in the energy consumption by source of the United States: Fractional integration, seasonality effect and structural breaks, *Estudios Econ.* 47(1) (2020), 31-48.
- [16] D. A. Dickey and W. A. Fuller, Distribution of the estimators for autoregressive time series with a unit root, *J. Amer. Stat. Assoc.* 74(366) (1979), 427-431. Doi:10.1080/01621459.1979.10482531.
- [17] P. C. B. Phillips and P. Perron, Testing for a unit root in time series regression, *Biometrika* 75(2) (1988), 335-346.
- [18] G. Elliot, T. J. Rothenberg and J. H. Stock, Efficient tests for an autoregressive unit root, *Econometrica* 64 (1996), 813-836.
- [19] F. X. Diebold and G. D. Rudebusch, On the power of Dickey-fuller tests against fractional alternatives, *Econ. Lett.* 35 (1991), 155-160. [https://doi.org/10.1016/01651765\(91\)90163-F](https://doi.org/10.1016/01651765(91)90163-F).
- [20] U. Hassler and J. Wolters, On the power of unit root tests against fractional alternatives, *Econ. Lett.* 45 (1994), 1-5.

- [21] D. Lee and P. Schmidt, On the power of the KPSS test of stationary against fractionally integrated alternatives, *J. Econometrics* 73 (1996), 285-302.
- [22] P. Bloomfield, An exponential model in the spectrum of a scalar time series, *Biometrika* 60 (1973), 217-226.