# Supplementary material

# A dynamic flotation model for predictive control incorporating froth physics. Part II: Model calibration and validation

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## Tables



**Note:** In both tables, $i$ refers to the number of mineralogical classes, while $K$ refers to the number of bubble size classes in the pulp phase. A detailed model development is presented in Part I of this paper.

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Table 1: Variables of the model, nomenclature, units, and total number of variables of the dynamic model.

| No. | Variable | Symbol | Units | Number of variables |
|---|---|---|---|---|
| 1 | Air recovery | $\alpha$ | $-$ | 1 |
| 2 | Axial dispersion | $D_{axial}$ | $-$ | 1 |
| 3 | Bubble size in the froth (overflowing) | $d_{b,froth}$ | $m$ | 1 |
| 4 | Bubble surface area flux | $S_b$ | $1/s$ | 1 |
| 5 | Bursting rate | $v_b$ | $m/s$ | 1 |
| 6 | Concentrate flowrate | $Q_{conc}$ | $m^3/s$ | 1 |
| 7 | Control valve output for the tailings flowrate | $u_v$ | $-$ | 1 |
| 8 | Entrainment factor | $ENT_i$ | $-$ | $i$ |
| 9 | Feed flowrate | $Q_{feed}$ | $m^3/s$ | 1 |
| 10 | Flotation rate constant | $k_i$ | $1/s$ | $i$ |
| 11 | Froth depth | $h_f$ | $m$ | 1 |
| 12 | Froth recovery | $R_{f,i}$ | $-$ | $i$ |
| 13 | Froth velocity over the lip | $v_f$ | $m/s$ | 1 |
| 14 | Froth-phase air recovery | $\alpha^*$ | $-$ | 1 |
| 15 | Gas flowrate into the cell (total) | $Q_{air,in}$ | $m^3/s$ | 1 |
| 16 | Gas flowrate for each bubble size class | $Q^k_{air,in}$ | $m^3/s$ | $K$ |
| 17 | Gas free pulp height | $h_0$ | $m$ | 1 |
| 18 | Gas hold-up for each bubble size class | $\varepsilon^k_0$ | $-$ | $K$ |
| 19 | Gas velocity out of the pulp for each bubble size class | $v^k_{g,\text{out pulp}}$ | $m/s$ | $K$ |
| 20 | Gas volume of each bubble size class in the pulp phase | $V^k_{gas}$ | $m^3$ | $K$ |
| 21 | Head grade (mineral concentration) | $C_{i,f}$ | $kg/m^3$ | $i$ |
| 22 | Interfacial gas rate | $v^*_g$ | $m/s$ | 1 |
| 23 | Length of Plateau borders per volume of foam | $\lambda_{out}$ | $1/m^2$ | 1 |
| 24 | Mean bubble size in the interface | $d_{b,int}$ | $m$ | 1 |
| 25 | Particle size per mineralogical class | $d_{p,i}$ | $m$ | $i$ |
| 26 | Physical parameter | $k_1$ | $m/s$ | 1 |
| 27 | Proportion of bubble size classes in the pulp | $\Psi^k_{d_{b,pulp}}$ | $-$ | $K$ |
| 28 | Pulp flowrate out of the cell | $Q_{pulp,out}$ | $m^3/s$ | 1 |
| 29 | Pulp height | $h_p$ | $m$ | 1 |
| 30 | Pulp volume in the cell | $V_{pulp}$ | $m^3$ | 1 |
| 31 | Residence time in the froth | $\tau_f$ | $s$ | 1 |
| 32 | Settling velocity | $v_{set}$ | $m/s$ | $i$ |
| 33 | Slurry content | $\epsilon$ | $-$ | 1 |
| 34 | Slurry density | $\rho_{pulp}$ | $kg/m^3$ | 1 |
| 35 | Slurry viscosity | $\mu_{pulp}$ | $kg/ms$ | 1 |
| 36 | Solid concentration in the tailings | $C_{i,tailings}$ | $kg/s$ | $i$ |
| 37 | Solid flowrate in the concentrate due to entrainment | $m_{i,ENT}$ | $kg/s$ | $i$ |
| 38 | Solid flowrate in the concentrate due to true flotation | $m_{i,TF}$ | $kg/s$ | $i$ |

**Table 1 continued from previous page**

| | | | | |
|---|---|---|---|---|
| 39 | Solid mass flowrate in the feed | $m_{i,feed}$ | $kg/s$ | $i$ |
| 40 | Solid mass flowrate in the tailings | $m_{i,tailings}$ | $kg/s$ | $i$ |
| 41 | Solid mass in the pulp phase | $M_i$ | $kg$ | $i$ |
| 42 | Superficial gas velocity | $j_g$ | $m/s$ | 1 |
| 43 | Tailings flowrate | $Q_{tailings}$ | $m^3/s$ | 1 |
| 44 | Total gas hold-up | $\varepsilon_{0,total}$ | $-$ | 1 |
| 45 | Total gas velocity out of the pulp | $v_{g,\text{out pulp}}^{total}$ | $m/s$ | 1 |
| 46 | Total gas volume in the pulp phase | $V_{gas}$ | $m^3$ | 1 |
| | | | **Total number of variables** | $29 + 12\ i + 5\ K$ |

Table 2: Model equations for each variable, including their classification. The last column refers to the total number of equations for each variable.

| Variable | Equation | Type of equation | Number of equation(s) |
|---|---|---|---|
| Conservation of mass | $\frac{dM_i}{dt} = m_{i,feed} - m_{i,tailings} - m_{i,TF} - m_{i,ENT}$ | Phenomenological | $i$ |
| | *Pulp phase* | | |
| Air flowrate of each bubble size class | $Q^k_{air,in} = Q_{air,in}\Psi^k_{d_{b,pulp}}$ | Semi-empirical | $K$ |
| Flowrate out of the flotation tank | $Q_{\text{pulp,out}} = Q_{\text{tailings}} + Q_{\text{conc}}$ | Phenomenological | 1 |
| Gas holdup | $\frac{d\left(\frac{\varepsilon_0^k}{1-\varepsilon_{0,\text{total}}}\right)}{dt} = \frac{1}{h_0}\left(\frac{Q^k_{\text{air},in}}{A} - v^k_{\text{g,out pulp}}\varepsilon_0^k\right) - \frac{1}{h_0}\left(\frac{Q_{\text{feed}}}{A_{\text{cell}}} - \frac{Q_{\text{pulp,out}}}{A_{\text{cell}}}\right)\left(\frac{\varepsilon_0^k}{1-\varepsilon_{0,\text{total}}}\right)$ | Phenomenological | $K$ |
| Gas volume of each bubble size class | $V^k_{gas} = \frac{\varepsilon_0^k}{1-\varepsilon_0^k}h_0A_{cell}$ | Phenomenological | $K$ |
| Gas-free pulp height | $\frac{dh_0}{dt} = \frac{Q_{feed}}{A_{cell}} - \frac{Q_{pulp,out}}{A_{cell}}$ | Phenomenological | 1 |
| Proportion of each bubble size class | $\Psi^k_{d_{b,pulp}} = \frac{(\text{frequency bubble size class})_k}{\sum_{k=1}^K (\text{frequency bubble size class})_k}$ | Empirical | $K$ |
| Pulp density | $\rho_{\text{pulp}} = \phi\rho_{\text{sol}} + (1-\phi)\rho_{\text{water}}$ | Phenomenological | 1 |
| Pulp height | $h_p = \frac{h_0}{1-\varepsilon_{0,\text{ total}}}$<br>ODE form (combining Eqs. for $h_0$ and gas holdup):<br>$\frac{dh_p}{dt} = \frac{1}{A_{cell}}\left(Q_{feed} - Q_{pulp,out}\right)\left(\frac{1}{1-\sum_{k=1}^K \varepsilon_0^k}\right) + \frac{h_0}{\left(1-\sum_{k=1}^K \varepsilon_0^k\right)}\sum_{k=1}^K \frac{d\varepsilon_0^k}{dt}$ | Phenomenological | 1 |
| Pulp viscosity | $\mu_{\text{pulp}} = \mu_{\text{water}}\exp\left(\frac{2.5\phi}{1-0.609\phi}\right)$ | Empirical | 1 |
| Pulp volume (liquid + gas) | $V_{pulp} = h_0A_{cell} + V_{gas}$ | Phenomenological | 1 |
| Solid mass flowrate in the feed | $m_{i,feed} = C_{i,f}Q_{feed}$ | Phenomenological | $i$ |
| Solid mass flowrate in the tailings | $m_{i,tailings} = C_{i,tailings}Q_{tailings}$ | Phenomenological | $i$ |
| Superficial gas velocity | $j_g = Q_{air}A_{cell}$ | Phenomenological | 1 |
| Tailings particle concentration | $C_{i,tailings} = \frac{M_i}{V_{pulp}}$ | Phenomenological | $i$ |
| Total gas holdup | $\varepsilon_0^{total} = \sum_{k=1}^K$ | Phenomenological | 1 |
| Total gas volume in the pulp phase | $V_{gas} = \sum_{k=1}^K V^k_{gas}$ | Phenomenological | 1 |
| Total upward gas velocity | $v^{\text{total}}_{g,\text{out pulp}} = \sum_{k=1}^K v^k_{g,\text{out pulp}}\varepsilon_0^k$ | Semi-empirical | 1 |
| Upward gas velocity | $v^k_{g,\text{out pulp}} = \frac{g\rho_{\text{pulp}}(d^k_{b,pulp})^2}{18\mu_{\text{pulp}}\left(1-\varepsilon_0^k\right)^{1.39}}$ | Semi-empirical | $K$ |
| Volumetric fraction of solids | $\phi = \frac{\rho_{\text{water}}}{\rho_{\text{water}}-\rho_{\text{solids}}+\frac{\rho_{\text{solids}}}{\Phi_{solids}}}$ | Phenomenological | 1 |

**Table 2 continued from previous page**

| Variables | Equations | Classification | Number of equation(s) |
|---|---|---|---|
| | *Pulp-froth interface* | | |
| Interfacial bubble size | $d_{b,int} = \frac{\sum_{k=1}^{K} v_{gas,\text{out pulp}}^{k} \varepsilon_0^k}{\sum_{k=1}^{K} \frac{v_{gas,\text{out pulp}}^{k} \varepsilon_0^k}{d_{b,pulp}^k}}$ | Phenomenological | 1 |
| Interfacial gas velocity | $v_g^* = \frac{dh_p}{dt} + v_{g,\text{ out pulp}}^{\text{total}}$ | Phenomenological | 1 |
| | *Froth phase* | | |
| Air recovery | $\alpha = \frac{v_f l_{lip} h_{over}}{Q_{air,in}}$ | Phenomenological | 1 |
| Air recovery as a function of busting rate | $\alpha^* = \frac{v_g^* - v_b}{v_g^*}$ | Phenomenological | 1 |
| Axial dispersion | $D_{\text{axial}} = \frac{j_g^{1.5}}{\sqrt{k_1\left(\sqrt{3}-\frac{\pi}{2}\right)Pe}}$ | Phenomenological | 1 |
| Bubble surface area flux | $S_b = \frac{6v_g^*}{d_{b,int}}$ | Phenomenological | 1 |
| Bursting rate | $v_b = a + bj_g + cj_g^2$ | Semi-empirical | 1 |
| Concentrate flowrate | $Q_{conc} = \{ \begin{array}{ll} \frac{A_{cell} v_g^{*2} \lambda_{out}(1-\alpha^*)\alpha^*}{k_1} & \text{if } \alpha < 0.5 \\ \frac{A_{cell} v_g^{*2} \lambda_{out}}{4k_1} & \text{if } \alpha \geq 0.5 \end{array}$ | Phenomenological | 1 |
| Entrainment factor | $ENT_i \approx \{ \begin{array}{ll} \exp\left(\frac{-v_{set,i}^{1.5} h_f}{D_{axial}\sqrt{v_g^*(1-\alpha^*)}}\right) & \text{if } \alpha < 0.5 \\ \exp\left(\frac{-2v_{\text{set},i}^{1.5} h_f}{D_{\text{axial}}\sqrt{v_g^*}}\right) & \text{if } \alpha \geq 0.5 \end{array}$ | Phenomenological | $i$ |
| Froth height | $h_f = h_T - h_p$ | Phenomenological | 1 |
| Froth recovery | $R_{F,i} = \{ \begin{array}{ll} \left(\frac{\alpha^*(1-\alpha^*)v_g^*}{v_{\text{set },i}}\right)^{\frac{f}{2}} \left(\frac{d_{b,\text{ int}}}{d_{\text{bfroth}}}\right)^f & \text{if } \alpha < 0.5 \\ \left(\frac{v_g^*}{v_{\text{set},i}}\right)^{\frac{f}{2}} \left(\frac{d_{b,\text{ int}}}{d_{\text{bfroth}}}\right)^f & \text{if } \alpha \geq 0.5 \end{array}$ | Phenomenological | $i$ |
| Froth residence time | $\tau_f = \frac{h_f}{v_g^*}$ | Phenomenological | 1 |
| Length Plateau border out of the froth per volume | $\lambda_{out} = \frac{k_\lambda}{d_{\text{bfroth out}}^2}$ | Phenomenological | 1 |
| Overflowing froth bubble size | $ODE : \frac{d}{dt} d_{b,froth} = C d_{b,froth}^{1-n}$<br>$Analytical solution : d_{\text{bfroth out}} = \left(nC\tau_f + d_{b,int}^n\right)^{1/n}$ | Phenomenological | 1 |
| Overflowing froth velocity | $v_f = \frac{Q_{\text{concentrate}}}{\epsilon l_{\text{lip}}\ h_{\text{over}}}$ | Phenomenological | 1 |
| Physical constant | $k_1 = \frac{\rho_{\text{pulp}} g}{3C_{PB}\mu_{\text{pulp}}}$ | Phenomenological | 1 |
| Settling velocity | $v_{set,i} = \frac{g(\rho_{\text{solid},i} - \rho_{\text{water}})d_{p,i}^2}{18\mu_{\text{pulp}}} \frac{(1-\phi)^{4.65}}{3}$ | Phenomenological | $i$ |

**Table 2 continued from previous page**

| Variables | Equations | Classification | Number of equation(s) |
|---|---|---|---|
| Slurry content | $\epsilon \approx \begin{cases} \frac{v_g^*}{k_1}\left(1-\alpha^*\right)\lambda_{out} \text{ if } \alpha < 0.5 \\ \frac{v_g^*}{2k_1}\lambda_{\text{out}} \text{ if } \alpha \geq 0.5 \end{cases}$ | Phenomenological | 1 |
| Solid mass flowrate in the concentrate due to entrainment | $m_{i,ENT} = Q_{\text{conc}} ENT_i C_{\text{tailings},i}$ | Phenomenological | $i$ |
| Solid mass flowrate in the concentrate due to true flotation | $m_{i,\text{true flotation}} = V_{\text{cell}} k_i R_{f,i} C_{\text{tailings},i}$ | Phenomenological | $i$ |
| Specific rate constant | $k_i = P_i S_b$ | Phenomenological | $i$ |
| | | **Total number of equations:** | 26+10$i$+5$K$ |