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Joint Radar-Communication Beamforming Considering Both Transceiver Hardware Impairments and Imperfect CSI

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***Abstract*—This paper studies multi-user multi-input multi-output (MU-MIMO) beamforming designs under different channel uncertainties for integrated sensing and communication (ISAC) system with hardware impairments. The Cramér-Rao Bound (CRB) of radar sensing is minimized by considering the energy and the signal-to-noise-plus-interference ratio (SINR) requirements of Internet of Things Devices (IoTDs). Specifically, we first reformulate the CRB by using the Schur complement theorem. To handle the non-convex constraints, we employ the S-procedure and the Bernstein-type inequality for approximation. Subsequently, we adopt the Gaussian randomization algorithm to seek the rank-one optimums. Numerical results indicate that the proposed transmission designs are more robust in the presence of both hardware impairments and imperfect CSI.**

***Index Terms*—Integrated sensing and communication, Cramér-Rao Bound, hardware impairments, imperfect channel state information (CSI).**

I. Introduction

INTEGRATED sensing and communication (ISAC) has been widely recognized as a crucial technique for the next generation of mobile communication networks. ISAC techniques are expected to support many emerging applications, e.g., unmanned aerial vehicles (UAVs) [1].

In order to support simultaneous target sensing and information transmission, various designs have been discussed. The authors in [2] investigated the reuse of transmitted waveform for both target sensing and multi-user communication in the ISAC system. An optimization is proposed to design a desired radar beampattern, while considering the constraints of signal-to-noise-plus-interference ratio (SINR) at users and power budget. In [3], the authors introduced a framework that places particular emphasis on interference alignment in scenarios involving multiple communication users and multiple radar users. On the other hand, some works focused on the estimation accuracy of the target's parameter. The researchers in [4] studied the estimation of the azimuth angle of a target. The primary focus is on minimizing the Cramér-Rao bound (CRB), which serves as a lower limit on the variance for all unbiased estimators. This minimization is carried out while ensuring the fulfilment of communication requirements in both point and extended target scenarios. The goal is to enhance the precision of radar target measurements by minimizing the CRB. However, the majority of existing research assumed perfect transceiver hardware and accurate channel state information (CSI), which proves overly idealistic in practical scenarios. It is noted that hardware ageing, digital-to-analog converters (DACs), and analog-to-digital converters (ADCs) imperfections unavoidably result in hardware impairments, which can be modelled as additive Gaussian distribution, whose variance is proportional to the signal power [5] [6]. Besides, obtaining perfect CSI information in real-world scenarios is also highly challenging.

Against the above background, we design a framework for multi-user multi-input multi-output (MU-MIMO) ISAC beamforming in the presence of transceiver hardware impairments and imperfect CSI. This framework is commonly found in UAV communication and sensing, where UAVs are regarded as aerial base stations (BSs) catering to various data-intensive scenarios, including concerts, football games, disasters, and emergency situations [1].

II. System Model

A. System Model

We consider a MIMO ISAC system equipped with N_t transmit antennas and N_r receive antennas, where the BS communicates with K downlink single-antenna Internet of things devices (IoTDs) while detecting a UAV which has been seen as a point target as depicted in Fig. 1. Notice that the set of IoTDs is defined as $\mathcal{K} = \{1, 2, \dots, K\}$. Besides we set $N_t \leq N_r$ to enhance the aperture and angle resolution of the radar antenna array. Let $\mathbf{x}(t) \in \mathbb{C}^{N_t \times 1}$ be the transmit signal from the BS at time slot t. $T > N_t$ is the number of time-domain snapshots. Then, the intended signal transmitted from the BS can be modelled as $\mathbf{x}(t) = \mathbf{W}\mathbf{s}(t) + \mathbf{z}_b(t)$, where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_k] \in \mathbb{C}^{N_t \times K}$ is the beamforming matrix to be designed. $\mathbf{s}(t) \in \mathbb{C}^{K \times 1}$ denotes the corresponding transmission data vector at time slot t. We further assume

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that the data flows are independent of each other, which is $\frac{1}{T}\sum_{t=1}^{T}\mathbf{s}(t)\mathbf{s}(t)^H = \mathbf{I}_K$. $\mathbf{z}_b(t) \in \mathbb{C}^{N_t \times 1}$ represents the additional noise resulting from hardware impairments on the BS side. Specifically, each element of $\mathbf{z}_b(t)$ is an independent

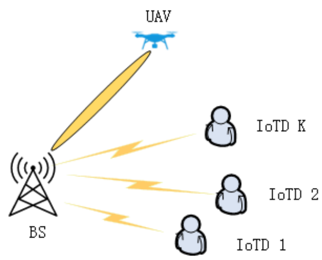

Fig. 1: Integrated sensing and communication system

zero-mean Gaussian random variable, with the variance of the kth entry being proportional to the transmit power of the kth antenna of the BS. The generated noise can be modeled as $\mathbf{z}_b(t) \sim \mathcal{CN}(0, k_b diag\{\sum_{i=1}^{K}\mathbf{w}_i\mathbf{w}_i^H\})$, where $k_b \in [0,1]$ indicates the hardware impairment coefficient of the BS. Then the covariance matrix of the downlink ISAC signal can be written as $\mathbf{R}_X = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}(t)\mathbf{x}^H(t) = \sum_{i=1}^{K}\mathbf{W}_i + k_b diag\left\{\sum_{i=1}^{K}\mathbf{W}_i\right\}$, where $\mathbf{W}_i = \mathbf{w}_i\mathbf{w}_i^H$.

B. Communication Signal Model

The received signal matrix for the kth IoTD at time $t \in \{1,...,T\}$ is formulated as

$$y_k(t) = \mathbf{h}_k^H\mathbf{x}(t) + \eta_k(t) + z_k(t) = \tilde{y}_k(t) + z_k(t), \quad (1)$$

where $\tilde{y}_k(t) = \mathbf{h}_k^H\mathbf{x}(t) + \eta_k(t)$, $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$ is the channel between the BS and the kth IoTD. $\eta_k(t) \sim \mathcal{CN}(0, \sigma_{C,k}^2)$ represents the additive white Gaussian noise (AWGN) at IoTDs. $z_k(t)$ represents the receiving distortion noise at the kth IoTD, following a zero-mean Gaussian distribution with variance proportional to the power of the received signal, i.e., $z_k(t) \sim \mathcal{CN}(0, k_k\mathbb{E}|\tilde{y}_k(t)|^2)$, where $k_k \in [0,1]$ is the hardware impairment coefficient of the kth IoTD. Then, we have $\mathbb{E}|\tilde{y}_k(t)|^2 = \sum_{i=1}^{K}\mathbf{h}_k^H\mathbf{W}_i\mathbf{h}_k + k_b\mathbf{h}_k^H diag\{\sum_{i=1}^{K}\mathbf{W}_i\}\mathbf{h}_k + \sigma_{C,k}^2$. Thus, the SINR per frame for the kth IoTD is given by (2) at the top of the next page.

C. Radar Signal Model

By transmitting signal $\mathbf{x}(t)$ for sensing, the reflected echo signal matrix at the receiving array of the BS at time $t \in \{1,...,T\}$ is given as

$$\mathbf{y}_R(t) = \mathbf{G}\mathbf{x}(t) + \mathbf{z}_R(t), \quad (3)$$

where $\mathbf{x}(t) \in \mathbb{C}^{N_t \times 1}$ is a known and determined transmission waveform. Due to the high probability of line of sight (LoS) between the BS and UAV, the response matrix can be expressed as $\mathbf{G} = \alpha\mathbf{b}(\theta)\mathbf{a}(\theta)^H = \alpha\mathbf{A}(\theta)$, where $\mathbf{A}(\theta) = \mathbf{b}(\theta)\mathbf{a}(\theta)^H$. $\alpha \in \mathbb{C}$ denotes the reflection coefficient dependent on the return path loss of the target and the radar scattering cross-sectional area (RCS). θ is the azimuth angle of the UAV relative to the BS. $\mathbf{z}_R$ is the interference and noise term, where the variance of each element is σ_R^2. The vectors $\mathbf{b}(\theta) \in \mathbb{C}^{N_r \times 1}$ and $\mathbf{a}(\theta) \in \mathbb{C}^{N_t \times 1}$ serve as the directional vectors for the receiving array and the transmitting array, respectively. We assume that the number of antennas is even and the center of the uniform linear array (ULA) antennas is the reference point. Then the transmitting guide vector can be written as $\mathbf{a}(\theta) = \left[e^{-j\frac{N_t-1}{2}\pi\sin\theta}, e^{-j\frac{N_t-3}{2}\pi\sin\theta}, \ldots, e^{j\frac{N_t-1}{2}\pi\sin\theta}\right]^T$, and its derivative can be expressed as $\dot{\mathbf{a}}(\theta) = \frac{\partial\mathbf{a}(\theta)}{\partial\theta} = \left[-ja_1\frac{N_t-1}{2}\pi\cos\theta, \ldots, ja_{N_t}\frac{N_t-1}{2}\pi\cos\theta\right]^T$, where a_i is the ith element of $\mathbf{a}(\theta)$. Similar to the transmitting array, $\dot{\mathbf{b}}(\theta)$ denotes the derivative of the receiving guide vector. It is easy to confirm that $\mathbf{a}^H(\theta)\dot{\mathbf{a}}(\theta) = 0, \mathbf{b}^H(\theta)\dot{\mathbf{b}}(\theta) = 0, \forall\theta$. For the point target, the CRB of estimating the angle θ was derived in [7] in detail, which is provided by (4) at the top of the next page, where $\dot{\mathbf{A}}(\theta) = \frac{\partial\mathbf{A}(\theta)}{\partial\theta} = \dot{\mathbf{b}}(\theta)\mathbf{a}(\theta)^H + \mathbf{b}(\theta)\dot{\mathbf{a}}(\theta)^H$.

D. Channel Uncertainty and CSI Error Models

In practical scenarios, there are dense obstructions between the BS and IoTDs, which poses a challenge in obtaining accurate direct CSI through channel estimation techniques. Consequently, we assume that the direct channel is imperfect. It is then modeled as $\mathbf{h}_k = \hat{\mathbf{h}}_k + \Delta\mathbf{h}_k$, where $\Delta\mathbf{h}_k$ is the corresponding channel estimation error. According to [8], we consider two distinct channel uncertainty scenarios.

1) Bounded CSI Error Model: In this model, the CSI error can be written as $||\Delta\mathbf{h}_k|| \le \epsilon_k, \forall k \in \mathcal{K}$, where ϵ_k is the radii of the unknown region of CSI error already known for the BS.

2) Statistical CSI Error Model: The outage probability-constrained transmission design takes into account the following statistical CSI error model. In this model, $\Delta\mathbf{h}_k$ follows the CSCG distribution [9], i.e., $\Delta\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_k), \forall k \in \mathcal{K}$, where $\mathbf{\Sigma}_k \in \mathbb{C}^{N_t \times N_t}$ is the positive semi-definite matrix.

III. Joint Beamforming Design For Point Target

In this section, we investigate the beamforming where imperfect CSI and transceiver hardware impairments are present.

A. Joint Beamforming Design With Bounded CSI Error

Define $||\Delta\mathbf{h}_k||_2 \le \epsilon_k$, the problem is formulated as

$$\text{(P1):} \min_{\{\mathbf{w}_i\}_{i=1}^K} CRB(\theta) \quad (5)$$
$$\text{s.t.} \quad SINR_k \ge \Gamma_k, \forall k \in \mathcal{K}, \quad (6)$$
$$\sum_{i=1}^{K}||\mathbf{w}_i||^2 \le P_T, \quad (7)$$

where Γ_k denotes the required SINR level for the kth user. P_T is the transmit power budget. By using $\mathbf{h}_k = \hat{\mathbf{h}}_k + \Delta\mathbf{h}_k, \forall k \in \mathcal{K}$, (6) can be rewritten as

$$\hat{\mathbf{h}}_k^H\mathbf{T}_k\hat{\mathbf{h}}_k + 2Re\left\{\hat{\mathbf{h}}_k^H\mathbf{T}_k\Delta\mathbf{h}_k\right\} + \Delta\mathbf{h}_k^H\mathbf{T}_k\Delta\mathbf{h}_k - (1+k_k)\sigma_C^2 \ge 0, \forall k \in \mathcal{K}, \quad (8)$$

where $\mathbf{T}_k = \frac{1}{\Gamma_k}\mathbf{W}_k - \sum_{i=1,i\ne k}^{K}\mathbf{W}_i - k_k\sum_{i=1}^{K}\mathbf{W}_i - (1+k_k)k_b diag\{\sum_{i=1}^{K}\mathbf{W}_i\}$. Problem (P1) is hard to deal with due

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$$SINR_k = \frac{\mathbf{h}_k^H \mathbf{W}_k \mathbf{h}_k}{\sum_{i=1,i\neq k}^K \mathbf{h}_k^H \mathbf{W}_i \mathbf{h}_k + k_k \sum_{i=1}^K \mathbf{h}_k^H \mathbf{W}_i \mathbf{h}_k + (1+k_k)k_b \mathbf{h}_k^H diag\{\sum_{i=1}^K \mathbf{W}_i\}\mathbf{h}_k + (1+k_k)\sigma_{C,k}^2} \quad (2)$$

$$CRB(\theta) = \frac{\sigma_R^2 \operatorname{Tr}\left(\mathbf{A}(\theta)\mathbf{R}_X \mathbf{A}^H(\theta)\right)}{2|\alpha|^2 T \left(\operatorname{Tr}\left(\dot{\mathbf{A}}(\theta)\mathbf{R}_X \dot{\mathbf{A}}^H(\theta)\right) \operatorname{Tr}\left(\mathbf{A}(\theta)\mathbf{R}_X \mathbf{A}^H(\theta)\right) - \left|\operatorname{Tr}\left(\mathbf{A}(\theta)\mathbf{R}_X \dot{\mathbf{A}}^H(\theta)\right)\right|^2 \right)} \quad (4)$$

to the non-convex objective function and constraints. By introducing the auxiliary variable t and using Schur complement [10], the objective function can be equivalently represented as

$$\min_{\{\mathbf{w}_i\}_{i=1}^K, t} \quad -t \quad (9)$$

$$\text{s.t.} \quad \begin{bmatrix} \operatorname{Tr}(\dot{\mathbf{A}}^H \dot{\mathbf{A}} \mathbf{R}_x) - t & \operatorname{Tr}(\dot{\mathbf{A}}^H \mathbf{A} \mathbf{R}_x) \\ \operatorname{Tr}(\mathbf{A}^H \dot{\mathbf{A}} \mathbf{R}_x) & \operatorname{Tr}(\mathbf{A}^H \mathbf{A} \mathbf{R}_x) \end{bmatrix} \succeq \mathbf{0}, \quad (10)$$

where $\mathbf{R}_x$ is the covariance matrix of the downlink signal, which has been defined above. $\mathbf{A}$ and $\dot{\mathbf{A}}$ denote $\mathbf{A}(\theta)$ and $\dot{\mathbf{A}}(\theta)$ respectively. Note that (8) is still non-convex due to the uncertainty of CSI. Fortunately, the S-procedure offers a means to address this issue [11]. Define the quadratic function of variable $\mathbf{u} \in \mathbb{C}^{N\times 1}$ as

$$f_i(\mathbf{u}) = \mathbf{u}^H \mathbf{V}_i \mathbf{u} + 2Re\{\mathbf{s}_i^H \mathbf{u}\} + v_i, i = 0, \ldots, M, \quad (11)$$

where $\mathbf{V}_i = \mathbf{V}_i{}^H$, and the condition $f_i(\mathbf{u}) \geq 0, i = 1, ..., M \Rightarrow f_0(\mathbf{u}) \geq 0$ holds if and only if $\forall i, \varpi_i \geq 0$ exists such that

$$\begin{bmatrix} \mathbf{V}_0 & \mathbf{v}_0 \\ \mathbf{v}_0^H & v_0 \end{bmatrix} - \sum_{i=1}^M \varpi_i \begin{bmatrix} \mathbf{V}_i & \mathbf{s}_i \\ \mathbf{s}_i^H & v_i \end{bmatrix} \succeq 0. \quad (12)$$

By using the S-procedure, we can recast the parameters as $M = 1, \mathbf{V}_0 = \mathbf{T}_k, \mathbf{v}_0 = \mathbf{T}_k^H \hat{\mathbf{h}}, v_0 = \hat{\mathbf{h}}_k^H \mathbf{T}_k \hat{\mathbf{h}}_k - (1+k_k)\sigma_{C,k}^2, \mathbf{V}_1 = -\mathbf{I}_{N_t}, v_1 = \epsilon_k^2, \mathbf{s}_i = [0, ..., 0]^T \in \mathbb{R}^{N_t \times 1}, \mathbf{u} = \Delta \mathbf{h}_k$. Then, (8) can be rewritten as

$$\begin{bmatrix} \mathbf{T}_k + \omega_k \mathbf{I}_{N_t} & \mathbf{a}_k \\ \mathbf{a}_k^H & c_k - \epsilon_k^2 \end{bmatrix} \succeq 0, \forall k \in \mathcal{K}, \quad (13)$$

where $\mathbf{a}_k = \mathbf{T}_k^H \hat{\mathbf{h}}$, $c_k = \hat{\mathbf{h}}_k^H \mathbf{T}_k \hat{\mathbf{h}}_k - (1+k_k)\sigma_{C,k}^2$. $\boldsymbol{\omega} = [\omega_1, ..., \omega_K] \geq 0$ represents the slack variable.

Hence problem (P1) can be formulated as

$$\text{(P2):} \quad \min_{\{\mathbf{W}_i\}_{i=1}^K, t, \boldsymbol{\omega}} \quad -t \quad (14)$$

$$\text{s.t.} \quad \begin{bmatrix} \operatorname{Tr}(\dot{\mathbf{A}}^H \dot{\mathbf{A}} \mathbf{R}_x) - t & \operatorname{Tr}(\dot{\mathbf{A}}^H \mathbf{A} \mathbf{R}_x) \\ \operatorname{Tr}(\mathbf{A}^H \dot{\mathbf{A}} \mathbf{R}_x) & \operatorname{Tr}(\mathbf{A}^H \mathbf{A} \mathbf{R}_x) \end{bmatrix} \succeq \mathbf{0}, \quad (15)$$

$$\begin{bmatrix} \mathbf{T}_k + \omega_k \mathbf{I}_{N_t} & \mathbf{a}_k \\ \mathbf{a}_k^H & c_k - \epsilon_k^2 \end{bmatrix} \succeq 0, \forall k \in \mathcal{K}, \quad (16)$$

$$\boldsymbol{\omega} \geq 0, \quad (17)$$

$$\operatorname{Tr}\left\{\sum_{i=1}^K \mathbf{W}_i\right\} \leq P_T, \quad (18)$$

$$\mathbf{W}_k \succeq \mathbf{0}, \forall k \in \mathcal{K}, \quad (19)$$

$$\operatorname{rank}(\mathbf{W}_k) = 1, \forall k \in \mathcal{K}. \quad (20)$$

To ensure the recovery of the optimal beamforming vectors $\mathbf{w}_k^{opt}$ through the eigenvalue decomposition of the corresponding optimal rank-one matrices $\mathbf{W}_k^{opt}$, (19) and (20) are imposed. Note that problem (P2) is a standard semidefinite programming (SDP) by dropping the constraint (20). Hence (P2) can be solved by the CVX tool. However, it should be noted that the optimal solution obtained by the CVX does not necessarily satisfy the rank-one constraint. It is necessary to combine the Gaussian randomization technique to restore the solution satisfying the rank-one constraint which is summarized in Algorithm 1. Specifically, the algorithm independently generates a set of feasible solutions that satisfy the constraints and subsequently selects the one with the optimal performance as the solution.

Algorithm 1 Gaussian Randomization Algorithm.

Initialize: $n = 1$, the number of feasible solutions N_{MAX}, $CRB = 1000$.
repeat
  repeat
    Create $\mathbf{w}_k, \forall k \in \mathcal{K}$ with variance of 1.
    Get $\mathbf{V}_k, \forall k \in \mathcal{K}$ as the Cholesky decomposition of $\mathbf{W}_k, \forall k \in \mathcal{K}$.
    Set $\mathbf{w}_k = \mathbf{V}_k^H \mathbf{w}_k, \forall k \in \mathcal{K}$.
  until $\mathbf{w}_k, \forall k \in \mathcal{K}$ satisfies (6) and (7).
  Calculate $CRB^{[n]}$.
  if $CRB^{[n]} < CRB$.
    Update $CRB = CRB^{[n]}$.
    Set $\mathbf{w}_k^{opt} = \mathbf{w}_k, \forall k \in \mathcal{K}$.
  end if
  n=n+1.
until $n > N_{MAX}$.
output $\mathbf{w}_k^{opt}, \forall k \in \mathcal{K}$.

B. Joint Beamforming Design With Statistical CSI Error

The main difference between this and the previous section is the CSI error model. Compared with the previous optimization problem, the initial optimization problem under the statistical error model can be expressed as follows

$$\text{(P3):} \quad \min_{\{\mathbf{w}_i\}_{i=1}^K} \quad CRB(\theta) \quad (21)$$

$$\text{s.t.} \quad \Pr\{SINR_k \geq \Gamma_k\} \geq 1 - \tau_k, \forall k \in \mathcal{K}, \quad (22)$$

$$\sum_{i=1}^K ||\mathbf{w}_i||^2 \leq P_T, \quad (23)$$

where $\boldsymbol{\tau} = [\tau_1, ..., \tau_K] \geq 0$ indicates the SINR interrupt probability. The utilization of the outage constraint (22) is aimed at guaranteeing the outage probability for each user. This ensures that the kth IoTD can achieve successful data reception with a sufficiently high SINR, exceeding $1 - \tau_k$. Obviously, there is no simple closed-form expression for the above

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constraint (22). In order to solve this problem, we introduce the Bernstein-Type Inequality [12] which will be used in the later derivations. Assume $f(\mathbf{u}) = \mathbf{u}^H\mathbf{V}\mathbf{u} + 2Re\left\{\mathbf{v}^H\mathbf{u}\right\} + \nu$, where $\mathbf{V} \in \mathbb{C}^{N\times N}$, $\mathbf{v} \in \mathbb{C}^{N\times 1}$, $\nu \in \mathbb{R}$ and $\mathbf{u} \in \mathbb{C}^{N\times 1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$. Then for any $\tau \in [0, 1]$, we have the following relationship

$$\begin{aligned}&\Pr\{f(\mathbf{u}) \geq 0\} \geq 1 - \tau \\ &\Rightarrow \mathrm{Tr}\{\mathbf{V}\} - \sqrt{2\ln(1/\tau)}x + \ln(\tau)\lambda_{\max}^{+}\{-\mathbf{V}\} + v \geq 0,\end{aligned} \tag{24}$$

where x is slack variable, and $\lambda_{\max}^{+}\{-\mathbf{V}\} = \max\{\lambda_{\max}\{-\mathbf{V}\}, 0\}$. Then the above inequality (24) can be approximately decomposed into three inequalities

$$\begin{cases} \mathrm{Tr}\{\mathbf{V}\} - \sqrt{2\ln(1/\tau)}x + \ln(\tau)y + v \geq 0, \\ \sqrt{\|\mathbf{V}\|_F^2 + 2\|\mathbf{v}\|_2^2} \leq x, \\ y\mathbf{I}_N + \mathbf{V} \succeq 0, y \geq 0, \end{cases} \tag{25}$$

where y is the slack variable introduced. Then, we study the beamforming that takes into account both hardware impairments and statistical CSI error. Then constraint (22) can be transformed into $\Pr\left\{\mathbf{h}_k^H\mathbf{T}_k\mathbf{h}_k - \left(1 + k_k\sigma_{C,k}^2\right) \geq 0\right\}$, where $\mathbf{T}_k = \frac{1}{\Gamma_k}\mathbf{W}_k - \sum_{i=1,i\neq k}^{K}\mathbf{W}_i - k_k\sum_{i=1}^{K}\mathbf{W}_i - (1 + k_k)k_b diag\{\sum_{i=1}^{K}\mathbf{W}_i\}$. For convenience, assuming $\mathbf{\Sigma}_k = \xi_k^2\mathbf{I}_{N_t}$, then the statistical CSI error is transformed to $\Delta\mathbf{h} = \xi_k\mathbf{p}_k$, $\mathbf{p}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_t})$. Then we have

$$\begin{aligned}&\Pr\left\{\mathbf{h}_k^H\mathbf{T}_k\mathbf{h}_k - \left(1 + k_k\sigma_{C,k}^2\right) \geq 0\right\} \\ &= \Pr\left\{\left(\hat{\mathbf{h}}_k^H + \Delta\mathbf{h}_k^H\right)\mathbf{T}_k\left(\hat{\mathbf{h}}_k + \Delta\mathbf{h}_k\right) - \left(1 + k_k\sigma_{C,k}^2\right) \geq 0\right\} \\ &= \Pr\left\{\mathbf{p}_k^H\mathbf{U}_k\mathbf{p}_k + 2\,\mathrm{Re}\left\{\mathbf{u}_k^H\mathbf{p}_k\right\} + c_k \geq 0\right\},\end{aligned} \tag{26}$$

where $\mathbf{U}_k = \xi_k^2\mathbf{T}_k$, $\mathbf{u}_k = \xi_k\mathbf{T}_k^H\hat{\mathbf{h}}_k$, $c_k = \hat{\mathbf{h}}_k^H\mathbf{T}_k\hat{\mathbf{h}}_k - \left(1 + k_k\sigma_{C,k}^2\right)$.

Then, utilizing the Bernstein-Type Inequality, we can handle (26) and transform it into

$$\begin{cases} \xi_k^2\,\mathrm{Tr}\left(\mathbf{T}_k\right) - \sqrt{2\ln\left(1/\tau_k\right)}x_k + \ln\left(\tau_k\right)y_k + c_k \geq 0, \\ \left\| \begin{matrix} \xi_k^2\,\mathrm{vec}\left(\mathbf{T}_k\right) \\ \xi_k\sqrt{2}\mathbf{T}_k^H\hat{\mathbf{h}}_\mathbf{k} \end{matrix} \right\| \leq x_k, \\ y_k\mathbf{I}_{N_t} + \xi_k^2\mathbf{T}_k \succeq 0, y_k \geq 0, \end{cases} \tag{27}$$

where $\mathbf{x} = [x_1, \ldots, x_K]^{\mathrm{T}}$ and $\mathbf{y} = [y_1, \ldots, y_K]^{\mathrm{T}}$ are auxiliary variables.

Then, problem (P3) can be translated as

$$\text{(P4)}: \min_{\{\mathbf{W}_i\}_{i=1}^{K}, t, \mathbf{x}, \mathbf{y}} \quad -t \tag{28}$$

$$\text{s.t.} \quad \begin{bmatrix} \mathrm{Tr}(\dot{\mathbf{A}}^H\mathbf{A}\mathbf{R}_X) - t & \mathrm{Tr}(\dot{\mathbf{A}}^H\mathbf{A}\mathbf{R}_X) \\ \mathrm{Tr}(\mathbf{A}^H\mathbf{A}\mathbf{R}_X) & \mathrm{Tr}(\mathbf{A}^H\mathbf{A}\mathbf{R}_X) \end{bmatrix} \succeq \mathbf{0}, \tag{29}$$

$$\xi_k^2\,\mathrm{Tr}\left(\mathbf{T}_k\right) - \sqrt{2\ln\left(1/\tau_k\right)}x_k + \ln\left(\tau_k\right)y_k + c_k \geq 0, \forall k \in \mathcal{K}, \tag{30}$$

$$\left\| \begin{matrix} \xi_k^2\,\mathrm{vec}\left(\mathbf{T}_k\right) \\ \xi_k\sqrt{2}\mathbf{T}_k^H\hat{\mathbf{h}}_k \end{matrix} \right\| \leq x_k, \forall k \in \mathcal{K}, \tag{31}$$

$$y_k\mathbf{I}_{N_t} + \xi_k^2\mathbf{T}_k \succeq 0, y_k \geq 0, \forall k \in \mathcal{K}, \tag{32}$$

$$\mathrm{Tr}\left\{\sum_{i=1}^{K}\mathbf{W}_i\right\} \leq P_T, \tag{33}$$

$$\mathbf{W}_k \succeq 0, \forall k \in \mathcal{K}, \tag{34}$$

$$\mathrm{rank}\left(\mathbf{W}_k\right) = 1, \forall k \in \mathcal{K}. \tag{35}$$

This problem is still hard to solve due to the non-convex constraints (35). Similar to the previous section, it can be solved by the CVX tool by dropping the constraint (35). After that, the Algorithm 1 can be used to restore the best solution.

IV. Simulation Results

In this section, we validate our analytical results through simulations. Without loss of generality, we set $N_t = 8$, $N_r = 24$ and consider there are $K = 10$ IoTDs. The power budget is $P = 30$ dBm, the noise power is set to $\sigma_{C,k}^2 = \sigma_R^2 = 0$ dBm and the ISAC frame length is set as $T = 30$. The communication channel is assumed as Rayleigh fading following the standard assumption, i.e., each entry of the channel matrix $\{\mathbf{h}_k\}_{k=1}^{K}$ follows i.i.d. complex Gaussian distribution, with zero mean and unit variance. We assume the target angle is $\theta = 0°$ and the reflection coefficient $\alpha = 1$. In the statistic CSI error model, the variance of $\Delta\mathbf{h}_k$ is defined as $\xi_k^2 = \delta^2||vec(\hat{\mathbf{h}}_k)||^2$, where $\delta \in [0, 1)$ is used to measure the channel uncertainty level. For the bounded CSI error model, the radii of the uncertainty regions are set as $\varepsilon_k = \sqrt{\frac{\xi_k^2}{2}F_{2N_t}^{-1}(1-\tau)}$, where $F_{2N_t}^{-1}(\cdot)$ represents the inverse cumulative distribution function (CDF) of the Chi-square distribution with degrees of freedom equal to $2N_t$. According to [12], this bounded CSI error model provides a fair comparison between the performance of the worst-case design and the outage-constrained design. The minimum SINR of all IoTDs is set as the same values. Besides, we compare the performance of our proposed design with the following benchmark schemes.

1) SNR maximization: The received SNR of the sensing signal is maximized while considering the communications requirement and energy budget in the presence of hardware impairments and imperfect CSI. It is noted that the SNR is a widely used metric to evaluate the detection and estimation performance. Specifically, the average SNR of the echo signals is $SNR = \frac{1}{T}\sum_{t=1}^{T}\frac{\|\mathbf{G}\mathbf{x}(t)\|^2}{\sigma_\mathrm{R}^2} = \frac{tr(\mathbf{G}\mathbf{R}_x\mathbf{G}^H)}{\sigma_\mathrm{R}^2}$.

2) Equal power transmission (EPT): In this scheme, the transmit power of each antenna is identical. Specifically, the power budget constraint in (P1)-(P4) is set as $diag\left\{\sum_{i=1}^{K}\mathbf{W}_i\right\} = \frac{P_T}{N_t}\mathbf{1}$, which has been widely used in sensing and communication services [13].

Fig. 2 (a) and Fig. 2 (b) present the root-CRB of the target angle versus the minimum SINR under different hardware impairment coefficients in the bounded CSI model and statistical CSI model, respectively. The channel uncertainty level is $\delta = 0.01$. It is observed that the root-CRB increases when the minimum SINR becomes large under different hardware impairment coefficients and CSI error models, which implies the tradeoff between radar and communication performance. Besides, increasing the hardware impairment coefficients will increase the root-CRB under different CSI error models, which means that the hardware impairments will increase the error of unbiased radar estimation of the target. On the other hand, when the SINR threshold is low, the performance gap between scenarios with no hardware impairment and those with existing hardware impairment is small in the above three designs. The reason is that in this scenario, meeting the SINR requirement

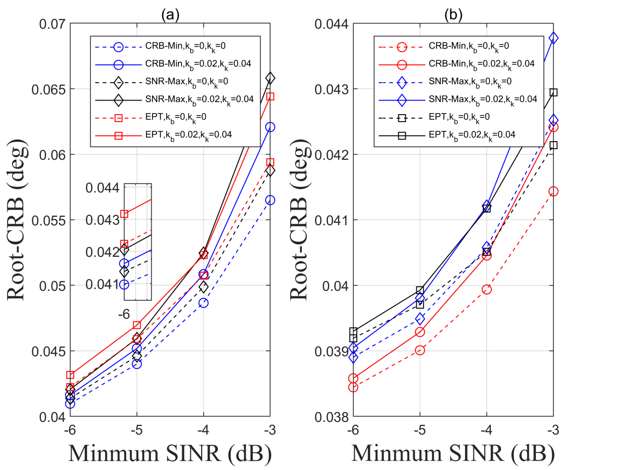

Fig. 2: The tradeoff between radar and communication performance under different hardware impairment coefficients in (a) bounded CSI model and (b) statistical CSI model.

is relatively straightforward and the effect of the hardware impairments is small. However, when the SINR threshold becomes large, the gap is significant. Moreover, Fig. 2 shows that the proposed CRB minimization designs achieve a lower CRB compared with the SNR maximization design and EPT design, which means that our proposed CRB minimization designs are more robust.

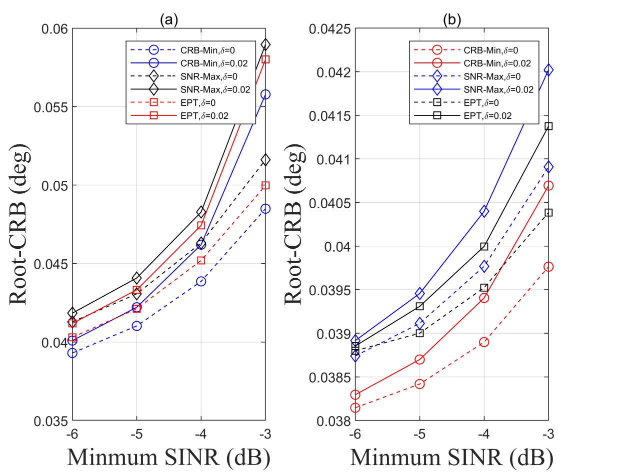

Fig. 3: The tradeoff between radar and communication performance under different channel uncertainty level in (a) bounded CSI model and (b) statistical CSI model.

Fig. 3 (a) and Fig. 3 (b) show the root-CRB of the target angle versus the minimum SINR under different levels of channel uncertainty in the bounded CSI model and statistical CSI model, respectively. The hardware impairment coefficients are set as $k_b = 0.01$ and $k_k = 0.02$, respectively. In Fig. 3 we can observe that increasing the channel uncertainty level in both the bounded CSI model and the statistical CSI model can raise the root-CRB, and the performance gap becomes more significant as the minimum SINR increases. This is due to the negative impact of channel uncertainty on communication services, making it challenging to meet the SINR requirement. Furthermore, it is also observed that the proposed CRB minimization design achieves the lowest CRB compared with the SNR maximization design and EPT design.

V. Conclusion

This paper studied the beamforming under different channel uncertainties for the ISAC system with hardware impairments. Our goal is to minimize the CRB while meeting the SINR requirements of the IoTDs and adhering to the power budget of the BS in both the bounded and statistical CSI error models. Specifically for the bounded CSI error model, the S-Procedure was adopted to solve the infinite inequality constraints with unknown CSI error. For the statistic CSI error model, the Berstein-type inequality was adopted to solve the minimum SINR outage probability constraints. Moreover Shur complement theorem was adopted to deal with non-convex objective function. Finally we changed these non-convex optimization problems into convex problems that are easy to be solved by CVX. The numerical simulation results showed that the hardware impairments and channel uncertainty have a negative effect on the ISAC system and the proposed beamforming designs are more robust compared to other designs.

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