



Modelling Loans to Non-Financial Corporations in the Eurozone: A Long-Memory Approach

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Abstract This paper uses fractional integration and cointegration methods to analyze the long-run relationship between loans to non-financial corporations, real gross domestic product, real gross fixed capital formation, the cost of borrowing differential between long- and short-term rates, and a proxy for the cost of debt, securities, and equity issuance. The analysis includes four Eurozone countries, namely Germany, France, Italy, and Spain, and spans the most recent decades. More precisely, fractional integration and cointegration models are estimated to investigate the persistence of the series as well as their long-run relationships and short-run dynamics using both unrestricted and restricted specifications. The univariate results are heterogeneous, the highest degrees of integration being found in the case of loans to non-financial corporations, whilst the multivariate ones provide evidence of a single fractional cointegration vector as well as of a lower adjustment speed to the long-run equilibrium compared to previous studies in all four countries. Moreover, both the short- and long-run response of loans to exogenous shocks to real gross domestic product and the cost of borrowing differentials differs across countries because of country-specific factors.

Keywords Non-financial corporations · NFCs · Loans · Eurozone · Long-memory · Fractional integration and cointegration

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Introduction

Credit plays an important role in the economy. In particular, the amount of loans provided to non-financial corporations (NFCs) is an indicator of the investment and spending decisions of the banking sector and thus also provides useful information to policy makers. Within the Eurozone in particular, bank lending is one of the major sources of financing to NFCs, with European firms heavily relying on bank lending to finance investment, especially in the case of small and medium-sized enterprises that have few alternatives to address their external financing needs (Revoltella et al., 2014). Credit is normally found to be highly correlated with asset prices and hence can help understand financial cycles. It also has an important role in the transmission of monetary policy to the real side of the economy. Loans are a key component on the asset side of the balance sheet of Eurozone banks, and thus a significant counterpart to monetary aggregates. Consequently, corporate lending and financing to NFCs are important measures to consider for assessing the monetary policy stance.

Therefore, detailed knowledge of the factors determining corporate loan decisions is crucial for understanding monetary developments and the setting of monetary policy in the Eurozone. Credit growth for NFCs in this area has trended downwards, especially because of the accession of new member states that were more severely affected by the global financial crisis (GFC) of 2007–2008. Stagnation of bank lending can be a severe constraint on economic growth in Europe where it plays a much more important role in financing the corporate sector than, for instance, in the United States (U.S.). In the wake of the 2007–2008 crisis, the capacity of many banks to lend to relatively high-risk sectors and to young, innovative firms was seriously hindered by capital constraints and a strong deterioration in the quality of the assets on their balance sheets. However, prior to the 2019 coronavirus disease (COVID-19) outbreak, lending to NFCs showed clear signs of recovery in the four largest Eurozone countries because of the decreasing influence of various demand-side and supply-side factors related to the global financial crisis of 2007–2008 which depressed lending levels. Given their importance in the European context, this paper analyses the determinants of the amount of loans provided to NFCs in four countries belonging to the Eurozone, namely Germany, France, Italy, and Spain. The selection of countries was made on the basis of the availability of relatively long time series, which are required to identify long-run relationships, and also to make the results comparable to those of other studies that provided evidence on the same set of countries (e.g., Focarelli & Rossi, 1998; Levieuge 2017; Dajcman, 2023).

The empirical framework is based on fractional integration and cointegration methods since most macroeconomic series appear to exhibit long-memory or long-range dependence, namely, the autocorrelations do not decay exponentially but rather according to a hyperbolic shape. This makes $I(d)$ processes the most appropriate to model them since it allows shocks to have long-lasting effects. After testing

for the degree of fractional integration of loans to NFCs and their main determinants, the long-run equilibrium relationships are examined using both unrestricted and restricted fractionally cointegrated vector autoregressive (FCVAR) models (e.g., Johansen & Nielsen, 2010, 2012), since such models are shown to outperform standard cointegrated vector autoregressive (CVAR) models.

Literature Review

Since the early 1990s a vast literature has developed on modelling credit to the private sector, especially within the central banking community because of its policy relevance. A common feature of these studies is the econometric framework used. Owing to the typically non-stationary nature of loans and their determinants, a vector error correction model (VECM) is normally estimated.

Sørensen et al. (2010) was the first to use Johansen's (1992) methodology to explain the long-term behavior of loans to NFCs in the Eurozone and identified three cointegrating relationships. Previous studies generally modelled credit to the private sector. For instance, Hofmann (2001) estimated a four-variable VECM for eight Eurozone countries from 1980 to 1998 and was unable to detect any cointegration relationships. Hülsewig (2003) analyzed German data using a five-variable VECM. He found two cointegrating relationships which he interpreted as the credit demand and the credit supply equilibria, with credit demand reverting rather slowly to its long-run equilibrium and supply effects through their impact on lending rates being insignificant. Calza et al. (2006) estimated a four-variable VECM for the Eurozone and detected one cointegration relationship interpreted as the credit demand equilibrium. Gambacorta and Rossi (2010) investigated possible non-linearities in the response of bank lending to monetary policy shocks in the Eurozone over the period 1985–2005 by means of an asymmetric vector error correction model (AVECM) involving four endogenous variables. They found that the effect on credit, gross domestic product (GDP) and the price of monetary policy tightening was larger than that of monetary policy easing. This result supported the existence of an asymmetric credit channel in the Eurozone.

Other studies focused on business lending in individual Eurozone countries. Focarelli and Rossi (1998) specified a five-variable VECM model and found three cointegrating relationships, namely loan demand, a relationship between investment and borrowing requirements and the lending rate equaling risk-free government bond yields. Bridgen and Mizen (1999) investigated interactions between investment, money holding and bank borrowing by private NFCs and identified long-run relationships for investment, money and borrowing, with the dynamics indicating the existence of feedback from money and credit disequilibria on investment. Bridgen and Mizen (2004) found equilibrium relationships for investment, lending and money with causal linkages running from money and lending to investment and from money to lending in a dynamic model. Kakes (2000) analysed the role of bank lending in the monetary transmission mechanism in Germany following a sectoral approach and distinguishing between corporate lending and household lending. Kakes (2000) reported that banks respond to a monetary contraction by adjusting their security holdings rather than by

reducing their loan portfolio. Finally, Plašil et al. (2012) showed that Czech banks had to restrict credit significantly when the financial crisis hit.

Other studies (e.g., Busch et al., 2010; Tamasi & Vilagi, 2011) estimated vector autoregressive (VAR) models with theory-based restrictions imposed on the impulse response functions to identify different types of shocks. Ferrari et al. (2013) presented evidence suggesting survey indicators of credit conditions can be useful for macroprudential purposes. De Bondt et al. (2010) examined the information content of the Eurozone Bank Lending Survey for aggregate credit and output growth, which suggests that both price and non-price conditions and terms of credit matter for credit and business cycles.

More recently, Levieuge (2017) estimated both VAR and VECM specifications for bank loans to NFCs in France and reported that the former outperforms the latter and that the growth rate of equity prices is the best predictor of such loans. In another recent paper, Pitoňáková (2018) investigated the factors influencing the demand for private sector loans in the euro area, and concluded that loans are negatively related to the producer price index and real interest rates, but are positively related to the industrial production index. Finally, Dajcman (2023) examined the impact of uncertainty shocks on the demand for business loans in individual euro area countries by estimating impulse response functions in the context of a Bayesian VAR model.

Methodology

The analysis involved two steps. First, the stochastic properties of loans to NFCs and their determinants were examined by means of both standard unit root tests and fractional integration methods (specifically, the exact maximum likelihood (EML) estimator of Sowell (1992) and the tests of Robinson (1994) based on the Lagrange multiplier (LM) principle). Second, the economic relationships linking them were investigated in the context of both standard and fractional cointegration multivariate models (in the latter case, the recently introduced FCVAR approach of Johansen and Nielsen (2012) was implemented). These methods are outlined next.

Long Memory Processes

An important characteristic of many economic and financial time series is their non-stationary nature, which can be described by a variety of models. Until the 1980s, the standard approach was to use deterministic (linear or quadratic) functions of time, thus assuming that the residuals from the regression model were $I(0)$ stationary. Later on, especially after the seminal work of Nelson and Plosser (1982), a general consensus was reached that the non-stationary component of most series was stochastic, and unit roots (or first differences, $I(1)$) were most appropriate for them. However, the $I(1)$ case is merely one particular model that can describe such behavior. In fact, the number of differences required to achieve $I(0)$ stationarity is not necessarily an integer but could be any point on the real line, including fractional values. In the latter case, the process is said to be fractionally integrated or $I(d)$. Long

memory is a feature of observations that are far apart in time, but highly correlated. This can be captured by fractionally integrated or I(d) models similar to the one in Eq. (1), where d can be any real value, L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is I(0), defined here as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency:

$$(1 - L)^d x_t = u_t, t = 0, \pm 1, \dots \quad (1)$$

Although fractional integration can also occur at other frequencies away from zero, as in the case of seasonal and cyclical fractional models, the series used for this analysis does not have these features. Hence, standard I(d) models were estimated as shown in Eq. (1). The idea of fractional integration was introduced by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981), although Adenstedt (1974) had already shown awareness of its representation. The polynomial $(1 - L)^d$ in Eq. (1) can be expressed in terms of its binomial expansion, such that, for all real d , x_t depends not only on a finite number of past observations, but on the whole of its past history. In this context, d plays a crucial role since it indicates the degree of dependence of the series. The higher the value of d is, the higher the level of association between the observations will be.

More precisely, one can distinguish between several cases depending on the value of d . Specifically, if $d = 0$, $x_t = u_t$, then x_t is said to be "short memory" or I(0), and if the observations are (weakly) autocorrelated (e.g. AR), then the values in the autocorrelations decay exponentially fast. If $d > 0$, x_t is said to exhibit long memory, so called because of the strong association between observations far apart in time. In this case, if d belongs to the interval (0, 0.5), x_t is still covariance stationary, while $d > 0.5$ implies non-stationarity. Finally, if $d < 1$, the series is mean-reverting. Therefore, the effects of shocks disappear in the long run. If $d \geq 1$, they persist forever. Hence the value of this parameter provides very useful information to policymakers.

There exist several methods to estimate and test the fractional differencing parameter d . Some of them are parametric while others are semi-parametric and can be specified in the time or in the frequency domain. Sowell (1992) analyzed the exact maximum likelihood (EML) estimator of the parameters of the autoregressive fractionally integrated moving average (ARFIMA) model in the time domain using a recursive procedure that permits a quick evaluation of the likelihood function. Doornik and Ooms (2003) refined this likelihood-based procedure. Then, Doornik and Ooms (2004) applied this method to model inflation data in the United Kingdom (UK) and the U.S.

Other parametric methods to estimate d in the frequency domain were proposed by, among others, Fox and Taqqu (1986) and Dahlhaus (1989). The small sample properties of these and other estimators were examined by Hauser (1999). A semi-parametric frequency domain estimator is the log-periodogram estimator proposed by Geweke and Porter-Hudak (1983). Other semi-parametric methods have been put forward by Velasco (1999a, 1999b) and Phillips and Shimotsu (2004, 2005) among others. Another approach widely employed in the empirical literature and in the present study is the parametric testing procedure of Robinson (1994), which is a LM test based on the

Whittle function in the frequency domain. Robinson (1994) showed that, under certain very general regularity conditions, its LM-based statistic converges asymptotically to a standard $N(0, 1)$ distribution, and this limit behaviour holds independently of the use of exogenous regressors (or deterministic terms) and the specific modelling assumptions about the $I(0)$ disturbances. The tests of Robinson (1994) were applied to an extended version of the Nelson and Plosser (1982) dataset in Gil-Alana and Robinson (1997) to test for unit roots and other long-memory processes when the singularity at the spectrum occurs at the zero frequency, as is the case in the series analyzed herein. Such tests have not been previously used to analyze the provision of loans to NFCs as in the present study. The results of the fractional integration analysis are reported in Table 1.

Fractional Cointegration

The FCVAR model was introduced by Johansen (2008) and further developed by Johansen and Nielsen (2010, 2012). It is a generalization of the Johansen (1995) CVAR model which allows for fractional processes of order d that cointegrate with order $d-b$. To introduce the FCVAR model, one can start with the well-known, non-fractional, CVAR model. Let $Y_t, t = 1, \dots, T$ be a p -dimensional $I(1)$ time series. The CVAR model can then be expressed as in Eq. (2):

$$\Delta Y_t = \alpha \beta' Y_{t-1} + \sum_{i=1}^k \Gamma_i \Delta Y_{t-i} + \varepsilon_t = \alpha \beta' L Y_t + \sum_{i=1}^k \Gamma_i \Delta L^i Y_t + E_t. \quad (2)$$

The simplest way to derive the FCVAR model is to replace the difference and lag operators Δ and L in Eq. (2) with their fractional counterparts, Δ^b and $L_b = 1 - \Delta^b$ respectively, to obtain the fractionally differentiated model in Eq. (3). By considering the cointegration order as in $Y_t = \Delta^{d-b} X_t$, one obtains a cointegrated model where ε_t is a p -dimensional independent and identically distributed vector with mean zero and covariance matrix Ω (Eq. (4)):

$$\Delta^b Y_t = \alpha \beta' L_b Y_t + \sum_{i=1}^k \Gamma_i \Delta^b L_b^i Y_t + \varepsilon_t \quad (3)$$

$$\Delta^d X_t = \alpha \beta' L_b \Delta^{d-b} X_t + \sum_{i=1}^k \Gamma_i \Delta^b L_b^i X_t + \varepsilon_t. \quad (4)$$

The parameters have the same interpretation as in the CVAR model. In particular, α and β are $p \times r$ matrices, where $0 \leq r \leq p$. The columns of β are the cointegrating relationships in the system corresponding to the long-run equilibria. The parameters Γ_i govern the short-run behavior of the variables and the coefficients α represent the speed of adjustment towards equilibrium for each of the variables. Thus, the FCVAR model permits simultaneous modelling of the long-run equilibria, the adjustment responses to deviations from those and the short-run dynamics of the system. As an intermediate step towards the final model, a version of the

Table 1 Descriptive statistics of the variables used in the integration and cointegration analysis

FRANCE	Mean	Median	Min	Max	St.dev	Skewness	Kurtosis
LOANS	3886.590	3699.860	2625.900	5402.100	863.996	0.287	1.898
RGFCF	116,321.200	114,094.100	98,549.580	143,627.700	10,514.050	0.618	2.765
RGDP	515,267.200	513,036.000	448,128.600	581,943.000	32,302.950	0.078	2.339
DIFF	0.596	0.585	-0.380	1.930	0.674	0.189	1.720
DEBT	2.272	2.389	-0.305	4.690	1.605	-0.145	1.487
GERMANY	Mean	Median	Min	Max	St.dev	Skewness	Kurtosis
LOANS	3714.445	3613.115	3440.240	4264.580	238.765	1.068	2.832
RGFCF	135,932.900	137,145.500	97,610.150	164,697.100	16,375.620	-0.258	2.399
RGDP	675,497.900	673,721.300	576,658.900	757,540.100	53,016.200	-0.049	1.806
DIFF	0.137	0.145	-0.650	1.180	0.425	0.359	2.626
DEBT	1.900	1.690	-0.607	4.505	1.707	0.056	1.468
ITALY	Mean	Median	Min	Max	St.dev	Skewness	Kurtosis
LOANS	1885.446	1958.985	0.000	2164.050	349.938	-3.916	21.268
RGFCF	78,893.290	77,708.200	59,923.640	96,631.590	9627.918	0.073	1.885
RGDP	400,654.900	401,645.300	336,892.800	431,420.300	15,555.780	-0.765	5.229
DIFF	0.209	0.170	-1.020	1.380	0.513	0.072	2.790
DEBT	3.342	3.833	0.616	6.537	1.472	-0.225	2.019
SPAIN	Mean	Median	Min	Max	St.dev	Skewness	Kurtosis
LOANS	2019.355	1917.875	1324.090	2612.160	3430.205	0.245	2.116
RGFCF	5.61E + 10	5.51E + 10	4.39E + 10	7.24E + 10	7.91E + 09	0.509	2.430
RGDP	269,158.500	268,978.500	230,586.300	304,440.000	17,008.606	-0.018	2.572
DIFF	0.153	0.070	-0.820	1.380	0.377	0.943	4.855
DEBT	3.038	3.681	0.078	6.795	1.711	-0.159	1.874

The table shows the descriptive statistics for all the covariates employed in the analysis (Loans to Non-Financial Corporations (LOANS), real gross capital fixed formation (RGFCF), real GDP (RGDP), the differential between the cost of borrowing for new long- and short-term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT)). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data ((Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

model in Eq. (4) with $d = b$ and a constant mean term inside the cointegrating vector (Eq. (5)) is considered:

$$\Delta^d X_t = \alpha(\beta' L_d X_t + \rho') + \sum_{i=1}^k \Gamma_i \Delta^d L_d^i X_t + \varepsilon_t. \quad (5)$$

Johansen and Nielsen (2012) and Nielsen and Morin (2014) discussed estimation and inference of this model, the latter providing computational codes for the calculation of estimators and test statistics. It is noteworthy that fractional differencing is defined in terms of an infinite series, but any actual sample will include only a finite number of observations. To calculate the fractional differences, one should assume that X_t was zero before the start of the sample. The bias introduced by this assumption was analyzed by Johansen and Nielsen (2016) using higher-order expansions. They showed that it can be completely avoided by including a level parameter μ

that shifts each of the series by a constant. The final estimated model, which will be tested against the standard CVAR one (Eq. (2)) by means of a log-likelihood test is given by Eq. (6):

$$\Delta^d(X_t - \mu) = L_d \alpha \beta' (X_t - \mu) + \sum_{i=1}^k \Gamma_i \Delta^d L_d^i (X_t - \mu) + \varepsilon_t. \quad (6)$$

The asymptotic analysis in Johansen and Nielsen (2016) shows that the maximum likelihood estimators of $(d, \alpha, \Gamma, \dots, \Gamma_2)$ are asymptotically normal, while the maximum likelihood estimator of (β, ρ) is asymptotically mixed normal when $d_0 < 1/2$ and asymptotically normal when $d_0 > 1/2$. FCVAR models were recently estimated for forecasting commodity returns by Dolatabadi et al. (2018), for forecasting political opinion polls by Nielsen and Shibaev (2016) and Jones et al. (2014), and for commodity futures markets by Dolatabadi et al. (2014).

Data

The analysis focuses on four Eurozone economies, namely Germany, France, Italy and Spain. The following quarterly series are examined: real loans to non-financial corporations (NFC), real gross capital fixed formation (RGFCF), real GDP (RGDP), the differential between the cost of borrowing for new long- and short-term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample starts in 2003Q1 and ends in 2022Q4. The series were obtained from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), with the exception of the cost of borrowing for new long- and short-term loans used to calculate DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024).

Online Supplemental Appendix (OSA) Figs. 1 through 20 display the time series plots of the series for each of the four countries examined. Real NFC loans generally trend upwards throughout the sample period, except in Spain, where they have decreased since 2010. Real gross fixed capital formation has been increasing steadily in France and Germany but has declined in Italy and Spain since the middle of the first decade of this century, and fluctuated the most in Germany and Italy. The behavior of real GDP shows the effects of the 2007–2008 GFC. After rising steadily in all four countries, it fell substantially during the crisis before gradually recovering. Finally, long and short borrowing rates and private sector debt have steadily declined during the past two decades, though since 2021 long-term rates have spiked. Table 1 reports descriptive statistics for the variables used in the integration and cointegration analyses. It can be seen that RGFCF is a very sizeable component of RGDP. Also, as a percentage of RGDP, loans are at their highest in France and their lowest in Spain. The cost of borrowing and the cost of debt are highest in France and Italy, respectively, and lowest in Germany in both cases. Further, RGFCF and RGDP are the most volatile series, and there is evidence of either negative skewness, positive skewness or leptokurtosis in all cases.

The aim of the analysis is to test for the existence of a long-run relationship linking these variables, namely:

$$LOANS_t = f(RGDP, RGFCF, DIFF, DEBT) \quad (7)$$

where RGDP and RGFCF capture the overall state of the economy and correspond to structural and cyclical components, respectively, DIFF measures the cost of short-vis-à-vis long-term lending, and DEBT captures the cost of alternative sources of financing including equity issuance. (Eq. 7). The priors of the analysis are the following. RGDP and RGFCF should have a positive effect on loans, the latter being a component of the former and acting as a scale variable (e.g., Focarelli & Rossi, 1998). DIFF, which measures the cost of borrowing (including bank lending), is expected to have a negative impact (Calza et al., 2003). DEBT should have a positive effect, since an increase in the cost of alternative sources of financing provides an incentive to resort to bank loans instead.

Empirical Results

Order of integration

As a first step, standard unit root tests were conducted, specifically the ADF test developed by Dickey and Fuller (1979) and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests to determine if a time series is stationary. The results (available upon request) support the hypothesis of unit roots or non-stationarity in virtually all cases. However, it is well known that unit root tests have very low power against specific alternatives such as structural breaks (Campbell & Perron, 1991); trend-stationary models (DeJong et al., 1992), regime-switching (Nelson et al., 2001), or fractional integration (Diebold & Rudebusch, 1991; Hassler & Wolters, 1994; Lee & Schmidt, 1996). Therefore, in the present study a more general fractional integration framework is considered, which includes the classic unit root models as a particular case of interest.

The fractional integration parameter d is obtained by estimating Eq. (8), where y_t is the observed time series; β_0 and β_1 are the coefficients on the intercept and the linear time trend, respectively, and the disturbance term u_t is $I(0)$ and assumed to be white noise¹:

$$y_t = \beta_0 + \beta_1 t + x_t, (1 - L)^d x_t = u_t, t = 1, 2, \dots \quad (8)$$

Table 2 displays the Whittle estimates of d along with the 95% confidence intervals of the non-rejection values of d using Robinson's (1994) parametric approach.

¹ Note that the $I(0)$ u_t term also allows for (weakly) ARMA-types of autocorrelations, with very similar results in this case.

Table 2 Long memory-fractional integration analysis

	No regressors	With Intercept	With Intercept and Trend
RGDP			
Germany	0.96(0.84, 1.10)	0.65(0.59, 0.69)	0.36* (0.31, 0.46)
France	0.83(0.77, 0.94)	0.62(0.60, 0.74)	0.41* (0.38, 0.49)
Italy	0.96(0.83, 1.11)	0.44(0.34, 0.55)	0.44* (0.37, 0.58)
Spain	0.84(0.76, 0.97)	0.57(0.53, 0.67)	0.56* (0.47, 0.72)
RGFCF			
Germany	0.77(0.64, 0.92)	0.44(0.38, 0.53)	0.22* (0.17, 0.33)
France	0.80(0.72, 0.88)	0.66(0.55, 0.68)	0.46* (0.40, 0.55)
Italy	0.91(0.80, 0.99)	0.57* (0.49, 0.67)	0.59(0.49, 0.69)
Spain	1.07(0.94, 1.21)	1.13 (1.04, 1.22)	1.16(1.04, 1.23)
LOANS			
Germany	1.34(1.28, 1.37)	1.34(1.28, 1.44)	1.41 (1.28, 1.48)
France	1.33(1.26, 1.37)	1.39(1.27, 1.47)	1.36 (1.29, 1.45)
Italy	1.39(1.31, 1.39)	1.36(1.33, 1.41)	1.39 (1.30, 1.38)
Spain	1.60* (1.55, 1.67)	1.60(1.52, 1.64)	1.62(1.54, 1.67)
DIFF			
Germany	1.01(0.84, 1.22)	1.00 (0.82, 1.23)	(0.83, 1.23)
France	1.01(0.84, 1.25)	1.11 (0.91, 1.39)	1.11(0.92, 1.39)
Italy	0.65(0.50, 0.87)	0.64* (0.49, 0.85)	0.67(0.55, 0.85)
Spain	0.30(0.19, 0.45)	0.30* (0.20, 0.45)	0.31(0.20, 0.46)
DEBT			
Germany	1.20(1.04, 1.40)	1.17 (0.98, 1.45)	1.17(0.97, 1.45)
France	1.20(1.04, 1.41)	1.19 (0.99, 1.45)	1.19(0.99, 1.44)
Italy	1.23(1.07, 1.42)	1.25 (1.05, 1.50)	1.25(1.05, 1.50)
Spain	1.18(0.98, 1.36)	1.14 (0.99, 1.33)	1.14(0.99, 1.34)

This table reports the estimated values of d with their corresponding confidence intervals in brackets for each of the specifications considered (Loans to Non-Financial Corporations (LOANS), real gross capital fixed formation (RGFCF), real GDP (RGDP), the differential between the cost of borrowing for new long- and short-term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT)). The sample ranges from 2003Q1 to 2022Q4. The values in bold are those for the models selected on the basis of the statistical significance of the regressors. The asterisks indicate evidence of mean reversion at the 95% level. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

The estimates of d are reported for the three standard cases of no regressors in the undifferenced regression (i.e., $\beta_0 = \beta_1 = 0$ in Eq. (8)), an intercept (β_0 unknown and $\beta_1 = 0$), and an intercept with a linear time trend (both β_0 and β_1 unknown). The coefficients in bold are those for the selected model according to the statistical significance of the regressors. Note that the approach of Robinson (1994) is based on the null differenced model, which is $I(0)$ by construction, and thus the t-values are still valid in the differenced regression model. In other words, under the null

hypothesis in Eq. (9), Eq. (8) becomes $\tilde{y}_t = \beta_0 \tilde{1}_t + \beta_1 \tilde{t}_t + u_t$, where $\tilde{t}_t = (1 - L)^{d_0} 1_t$, and $\tilde{t}_t = (1 - L)^{d_0} t_t$, and, given that u_t is $I(0)$ by construction, standard t-tests apply.

$$H_0 : d = d_o. \quad (9)$$

In the case of LOANS, the estimates of d are all significantly above 1 in the four countries examined, which suggests the presence of a unit root or a higher integrated process.

Concerning RGDP, the time trend coefficient is significant in all four countries and the estimates of d are all in the $(0, 1)$ interval, ranging from 0.36 for Germany to 0.56 for Spain. Concerning Germany, France and Italy, the results support the stationary assumption since d is found to be significantly below 0.5. Finally, for Spain the confidence intervals include both stationary and nonstationary values.

Regarding RGFCF, the time trend coefficient is statistically significant in the case of Germany and France but insignificant for the other two countries (Italy and Spain). The estimated value of the fractional coefficient d is heterogeneous across countries: 0.22 for Germany and 0.46 for France, in both cases supporting the hypothesis of stationarity with long memory; 0.57 for Italy, which implies non-stationarity, and finally 1.13 for Spain, where the unit root null hypothesis could not be rejected.

For DIFF, evidence of unit roots is found in the case of Germany and France, while mean reversion (i.e., $d < 1$) occurs in Italy ($d = 0.64$) and Spain ($d = 0.30$). Finally, in the case of DEBT, the time trend is insignificant, and the unit root null cannot be rejected for Germany, France and Spain, while Italy is the only country with evidence of $d > 1$.

Fractional Cointegration

Next, the estimates for the FCVAR models are examined. As mentioned previously, to account for the initial bias value of zero, the approach of Johansen and Nielsen (2016) was followed by incorporating a level parameter (μ in Eq. (6)), which shifts each series by a non-zero constant to be estimated. Furthermore, following Johansen and Nielsen (2012) and Nielsen and Morin (2014), and given the heterogeneous results of the fractional tests, the fractional parameter d and the fractional exponent b are restricted to be equal, so that the cointegrating order of the fractional processes of the series analyzed is given by $CI(d - b = 0)$, as shown in Eq. (5).

First, the number of lags was chosen using the Akaike information criterion (AIC) and Bayesian information criterion (BIC), both of which suggest a parsimonious model with no more than two lags in each of the four estimations. Serial correlation was tested using a white noise test. Next, the number of rank relations between the variables was determined using a likelihood ratio (LR) test for the cointegrating rank, as outlined in Johansen (1995) for the CVAR case and Johansen (2008) for the FCVAR case. Then, LR tests were performed of the adequacy of the CVAR model (where the null hypothesis is $d = b = 1$, i.e., the order of cointegration is $CI(0)$ and fractional differentiation is not necessary) against the FCVAR one (the alternative

hypothesis being $d = b \neq 1$, i.e., fractional differentiation is required to find a stationary long-run equilibrium of order $CI(1)$.

Finally, LR tests of a set of overidentifying restrictions were carried out for both the FCVAR and CVAR models in order to choose the best specification. In particular, the null that α and β are equal to 0 was tested. Failure to reject this hypothesis in the case of α implies that the corresponding variable adjusts over time towards the long-run equilibrium, while in the case of β the implication is that the variable enters the long-run equilibrium relationship.²

France

The results for the LR fractional cointegration test for France, as well as the estimated fractional parameter and the analysis of the residuals for the unrestricted model, are shown in Table 3. Table 4, column 1, reports the LR test statistic for comparing a standard CVAR against the FCVAR alternative in the case of France. This implies that the null, no statistically significant difference between the likelihood of the two competing models, should be rejected, and that the FCVAR should be preferred. The unrestricted FCVAR results (the estimated α s and β s) are reported in Eq. (10).³ The LR test for the cointegrating rank in Table 4 implies the existence of at most one cointegrating relationship. Equation (10) shows that loans are the only variable converging towards the long-run equilibrium after an exogenous shock has occurred, though at a very slow rate ($\alpha=0.021$), while the others diverge as indicated by their positive α coefficients. By normalizing with respect to loans, the long-run equilibrium vector is obtained as given by Eq. (11), with positive coefficients on RGDP and RGFCF as expected, and a positive one on DIFF:

$$\Delta^{\hat{d}} \begin{bmatrix} \text{LOANS}_t - \mu_1 \\ \text{RGDP}_t - \mu_2 \\ \text{RGFCF}_t - \mu_3 \\ \text{DIFF}_t - \mu_4 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.021 \\ 0.092 \\ 0.225 \\ 0.356 \end{bmatrix} \begin{bmatrix} 1.000 \\ -4.256 \\ -0.264 \\ -0.190 \end{bmatrix} (X_t - \hat{\mu}) + \sum_{i=1}^2 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (10)$$

$$\text{LOANS}_t = \mu u + 4.256 \text{RGDP}_t + 0.264 \text{RGFCF}_t + 0.190 \text{DIFF}_t + v_t. \quad (11)$$

Restricted model for France

This section reports a set of LR tests for restrictions concerning the long-run significance of the determinants of loans being considered (setting the β coefficient on each of the covariates equal to 0) and the weak exogeneity of each series (setting

² Figures reporting a graphical rendition of the characteristic polynomials for both the CVAR and the FCVAR models in each country analysed are reported in the OSA. OSA Figs. 1 to 5 report loans to NFC, GDP, GFCF, differentials in the cost of borrowing from long-term and short-term loans and the cost of debt in France. OSA Figs. 6–10 refer to Germany; OSA Figs. 11–15 refer to Italy and OSA Figs. 16–20 refer to Spain.

³ p-values are not reported for the unrestricted models. They are only reported for the selected final specification.

Table 3 Fractional cointegration analysis for France

Likelihood Ratio Tests				
Rank	d	b	LR statistic	P-value
0	0.766	0.766	53.378	0.005
1	0.780	0.780	20.036	0.358
2	1.149	1.149	8.372	0.882
3	1.272	1.272	1.756	0.880
4	1.193	1.193
White Noise Tests				
Variable	Q	P-value	LM	P-value
All variables	22.290	0.899	-	-
LOANS	3.121	0.210	4.260	0.119
RGDP	0.047	0.977	0.025	0.987
RGFCF	0.784	0.676	0.877	0.645
DIFF	0.241	0.887	1.684	0.431
Value for $d = b$				
	estimate	s.e		
	0.736	0.082		

The upper part of the table shows the results for the Trace LR statistic estimated for $d = b$, together with the relevant probability values. The middle part shows the white noise test results on the residuals of the model, with a null of absence of residual autocorrelation. The lower part of the table shows the final estimate for the fractional parameters and their statistical precision. The variables employed are Loans to Non-Financial Corporations (LOANS), real GDP (RGDP), the differential between the cost of borrowing for new long- and short -term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

each α coefficient equal to 0). The final specification is given by Eq. (12), which shows the estimates of the α and β parameters as well as the corresponding p-values (in parentheses) of the LR tests of the null of a zero coefficient. The normalized long-run equilibrium is given by Eq. (13). The significant coefficients are also shown in Table 4, Columns 2 and 3, alongside the CVAR-FCVAR test.

Table 4 Hypothesis testing for France

	H_1^{FCvsC}	$H_2^{\beta^{DIFF}}$	$H_3^{\alpha^{GDP}}$
df	1	1	1
LR	5.504	2.855	0.686
P-value	0.019	0.091	0.408

The table shows the results for the Likelihood Ratio tests. The first column compares the likelihood of an FCVAR against a canonical CVAR. The subsequent columns show the LR tests for the α and β parameters where the 0 null could not be rejected. The variables employed are Loans to Non-Financial Corporations (LOANS), real GDP (RGDP), the differential between the cost of borrowing for new long- and short -term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

The long-run relationship now only includes RGDP and DIFF as determinants of loans, the latter now having a negative coefficient as expected, though of a small magnitude. The adjustment coefficient on LOANS is almost double in size, having increased from -0.021 to -0.049 (Eq. 12). However, it is still much lower than the corresponding estimate of -0.286 reported by Levieuge (2017) on the basis of standard CVAR analysis. This is in fact the only negative adjustment coefficient, which implies that loans are the only endogenous variable in the model:

$$\Delta^{\hat{d}} \begin{bmatrix} \text{LOANS}_t - \mu_1 \\ \text{RGDP}_t - \mu_2 \\ \text{RGFCF}_t - \mu_3 \\ \text{DIFF}_t - \mu_4 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.049 \\ (0.001) \\ 0.000 \\ (0.408) \\ 0.094 \\ (0.064) \\ 0.459 \\ (0.397) \end{bmatrix}, \begin{bmatrix} 1.000 \\ (0.000) \\ -4.215 \\ (0.048) \\ 0.000 \\ (0.091) \\ 0.088 \\ (0.008) \end{bmatrix} (X_t - \hat{\mu}) + \sum_{i=1}^2 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (12)$$

$$\text{LOANS}_t = \mu u + 4.215 \text{RGDP}_t - 0.088 \text{DIFF}_t + v_t. \quad (13)$$

Germany

The results for the LR test for model selection for Germany, as well as the estimated fractional parameter and the analysis of any correlation left in the residuals of the unrestricted model estimates, are reported in Table 5. Only 1 lag is required in the German case, there is no evidence of serial correlation, and the LR test supports an FCVAR specification rather than a CVAR one, as can be seen in Table 6, Column 1. The rank test implies that there are up to three cointegrating vectors, but only the one corresponding to a demand-driven equilibrium relationship is selected. The estimated value for both d and b is 0.517, which implies slow convergence towards the long-run equilibrium and the existence of an $I(0)$ cointegrating relationship.

Equation (14) shows that all the α coefficients are positive, but none of them are statistically significant. However, the normalized cointegrating vector indicates that there exists a long-run equilibrium relationship with a positive coefficient on RGDP and DEBT and a negative one on DIFF, consistently with the priors (Eq. 15):

$$\Delta^{\hat{d}} \begin{bmatrix} \text{LOANS}_t - \mu_1 \\ \text{RGDP}_t - \mu_2 \\ \text{DIFF}_t - \mu_3 \\ \text{DEBT}_t - \mu_4 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} 0.018 \\ 0.011 \\ 0.187 \\ 1.404 \end{bmatrix}, \begin{bmatrix} 1.000 \\ 2.127 \\ 0.315 \\ -0.330 \end{bmatrix} (X_t - \hat{\mu}) + \sum_{i=1}^2 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (14)$$

$$\text{LOANS}_t = \mu u + 2.127 \text{RGDP}_t - 0.315 \text{DIFF}_t + 0.330 \text{DEBT}_t + v_t. \quad (15)$$

Table 5 Fractional cointegration analysis for Germany

Likelihood Ratio Tests				
Rank	d	b	LR statistic	P-value
0	0.964	0.964	87.712	0.000
1	0.518	0.518	45.218	0.000
2	0.387	0.387	27.440	0.000
3	0.151	0.151	1.427	0.232
4	0.183	0.183	-	-
White Noise Tests				
Variable	Q	P-value	LM	P-value
All variables	11.826	0.756	-	-
LOANS	0.297	0.597	0.194	0.660
RGDP	0.000	0.984	0.000	0.986
DIFF	0.677	0.411	0.331	0.565
DEBT	1.197	0.274	1.670	0.196
Values for $d = b$				
	estimate	s.e		
	0.517	0.053		

The upper part of the table shows the results for the Trace LR statistic estimated for $d = b$, together with the relevant probability values. The middle part shows the white noise test results on the residuals of the model, with a null of absence of residual autocorrelation. The lower part of the table shows the final estimate for the fractional parameters and their statistical precision. The variables employed are Loans to Non-Financial Corporations (LOANS), real GDP (RGDP), the differential between the cost of borrowing for new long- and short -term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

Restricted model for Germany

Again, the starting point is to test for the significance of the long-run β coefficients as well of the dynamic adjustment α coefficients to obtain a restricted model which only includes the relevant determinants of loans and considers the weak exogeneity

Table 6 Hypothesis testing for Germany

	H_1^{FCvsC}	$H_2^{\beta^{DIFF}}$	$H_3^{\alpha^{DIFF}}$
df	1	1	1
LR	24.111	0.179	0.050
P-value	0.000	0.091	0.823

The table shows the results for the Likelihood Ratio tests. The first column compares the likelihood of an FCVAR against a canonical CVAR. The subsequent columns show all the LR tests for the α and β parameters where the 0 null could not be rejected. The variables employed are Loans to Non-Financial Corporations (LOANS), real GDP (RGDP), the differential between the cost of borrowing for new long- and short -term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

properties of the variables of interest. Table 6 reports, in Columns 2 and 3, the LR tests for the α and β parameters where the 0 null could not be rejected, while Eq. (16) shows the final restricted model together with the p-values from the LR tests in parentheses, and Eq. (17) displays the final equilibrium relationship normalized on LOANS.

Evidence is found of short-run exogeneity for DIFF (since the null that the corresponding α coefficient is equal to 0 cannot be rejected), while RGDP and DEBT exhibit high speeds of convergence (-0.297 and -0.272 respectively). The corresponding estimate for loans, equal to -0.079 , is much closer to the values reports by previous studies on Europe (e.g. Calza et al. (2006) with a -0.075) (Eq. 16).

Concerning the long-run equilibrium determinants (Eq. 17), the estimated coefficient of 0.726 for RGDP is only marginally significant at the 10% level with a p-value equal to 0.051, while the coefficient on DEBT is equal to 0.108, and is highly significant as indicated by its p-value. However, both these coefficients are inconsistent with the priors. DEBT does not adjust towards the long-run equilibrium and thus can be considered weakly exogenous:

$$\Delta^{\hat{d}} \begin{bmatrix} \text{LOANS}_t - \mu_1 \\ \text{RGDP}_t - \mu_2 \\ \text{DIFF}_t - \mu_3 \\ \text{DEBT}_t - \mu_4 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.079 \\ (0.001) \\ -0.297 \\ (0.000) \\ 0.000 \\ (0.823) \\ -0.272 \\ (0.828) \end{bmatrix}, \begin{bmatrix} 1.000 \\ (0.010) \\ 0.726 \\ (0.051) \\ 0.000 \\ (0.673) \\ 0.108 \\ (0.000) \end{bmatrix} (X_t - \hat{\mu}) + \sum_{i=1}^2 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (16)$$

$$\text{LOANS}_t = \mu u - 0.726 \text{RGDP}_t - 0.108 \text{DEBT}_t + v_t. \quad (17)$$

Italy

In Table 7, Column 1, the LR test implies again that an FCVAR specification is preferable to a CVAR one, and again there is no evidence of serial correlation. Furthermore, the rank test suggests that there exists a single cointegrating vector. Table 7, as before, also reports the residual analysis of the unrestricted model for Italy. As in the case of Germany, there is evidence of $I(1)$ behaviour for the individual series and $I(0)$ for the long-run relationship, the value of d and b being 0.655.

In the unrestricted model given by Eq. (18), loans adjust to the long-run equilibrium rather slowly, the corresponding α coefficient being equal to -0.010 . Note that this result implies a much slower convergence rate compared to the adjustment coefficient of -0.072 estimated by Calza et al. (2003) for Italy, and of -0.060 reported by Casolaro et al. (2006) for the Euro Area. DIFF is the only variable with a positive α coefficient (Eq. 19):

Table 7 Fractional cointegration analysis for Italy

Likelihood Ratio Tests				
Rank	d	b	LR statistic	P-value
0	0.803	0.803	114.333	0.000
1	0.655	0.655	67.491	0.000
2	0.304	0.304	20.127	0.017
3	0.010	0.010	4.447	0.349
4	0.295	0.295	0.081	0.776
5	0.309	0.309	-	-
White Noise Tests				
Variable	Q	P-value	LM	P-value
All variables	24.695	0.480	-	-
LOANS	0.311	0.577	0.726	0.394
RGDP	2.055	0.152	0.486	0.485
RGFCF	1.224	0.269	0.624	0.430
DIFF	0.000	0.996	0.009	0.925
DEBT	0.165	0.685	0.113	0.737
Values for $d = b$				
	estimate		s.e	
	0.655		0.005	

The upper part of the table shows the results for the Trace LR statistic estimated for $d = b$, together with the relevant probability values. The middle part shows the white noise test results on the residuals of the model, with a null of absence of residual autocorrelation. The lower part of the table shows the final estimate for the fractional parameters and their statistical precision. The variables employed are Loans to Non-Financial Corporations (LOANS), real gross capital fixed formation (RGFCF), real GDP (RGDP), the differential between the cost of borrowing for new long- and short -term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

$$\Delta^{\hat{d}} \begin{bmatrix} \text{LOANS}_t - \mu_1 \\ \text{RGDP}_t - \mu_2 \\ \text{RGFCF}_t - \mu_3 \\ \text{DIFF}_t - \mu_4 \\ \text{DEBT}_t - \mu_5 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.010 \\ -0.116 \\ -0.300 \\ 0.026 \\ -0.845 \end{bmatrix} \begin{bmatrix} 1.000 \\ 1.687 \\ -1.662 \\ -0.276 \\ 0.148 \end{bmatrix} (X_t - \hat{\mu}) + \sum_{i=1}^2 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (18)$$

$$\text{LOANS}_t = \mu u - 1.687 \text{RGDP}_t + 1.662 \text{RGFCF}_t + 0.276 \text{DIFF}_t - 0.148 \text{DEBT}_t + v_t. \quad (19)$$

Restricted model for Italy

In Table 8, the final specification is discussed based on tests for the significance of the long-run parameters and of the adjustment ones, the latter being weak

Table 8 Hypothesis testing for Italy

	H_1^{FCysC}	$H_2^{\beta^{GDP}}$	$H_3^{\alpha^{DIFF}}$
df	1	1	1
LR	19.373	1.067	0.291
P-value	0.000	0.302	0.590

The table shows the results for the Likelihood Ratio tests. The first column compares the likelihood of an FCVAR against a canonical CVAR. The subsequent columns show all the LR tests for the α and β parameters where the 0 null could not be rejected. The variables employed are Loans to Non-Financial Corporations (LOANS), real gross capital fixed formation (RGFCF), real GDP (RGDP), the differential between the cost of borrowing for new long- and short-term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

exogeneity tests. The estimates for the restricted model together with the p-values of the LR tests are shown in Eq. (20), whilst the normalized long-run equilibrium is given by Eq. (21). In this case RGDP does not enter the cointegrating relationship, whilst its component RGFCF does, and DIFF and DEBT do as well, with a positive and negative coefficient, respectively. The estimated α on loans is equal to -0.013 , which implies slightly faster convergence compared to the unrestricted model, while the adjustment coefficient on DIFF is positive and only slightly bigger in absolute terms compared to the unrestricted model, and the biggest negative coefficient (-0.940) is the one on DEBT, with DIFF being the only exogenous variable. Note that the adjustment coefficient on LOANS is very low and insignificant (the p-value for the LR test is 0.317):

$$\Delta^{\hat{d}} \begin{bmatrix} \text{LOANS}_t - \mu_1 \\ \text{RGDP}_t - \mu_2 \\ \text{RGFCF}_t - \mu_3 \\ \text{DIFF}_t - \mu_4 \\ \text{DEBT}_t - \mu_5 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.013 \\ (0.317) \\ -0.111 \\ (0.000) \\ -0.305 \\ (0.000) \\ 0.000 \\ (0.590) \\ -0.940 \\ (0.042) \end{bmatrix}, \begin{bmatrix} 1.000 \\ (0.069) \\ 0.000 \\ (0.302) \\ -1.289 \\ (0.007) \\ -0.248 \\ (0.024) \\ 0.140 \\ (0.001) \end{bmatrix} (X_t - \hat{\mu}) + \sum_{i=1}^2 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (20)$$

$$\text{LOANS}_t = \mu u + 1.289 \text{RGFCF}_t + 0.248 \text{DIFF}_t - 0.140 \text{DEBT}_t + \nu_t. \quad (21)$$

Spain

The results for the LR tests for Spain, as well as the estimated fractional parameter of the unrestricted model and the serial correlation tests are reported in Table 9. Again, the LR test supports the FCVAR model (Table 10, column 1) and the rank

test suggests a single cointegrating vector. Finally, the order of integration (d and b) is equal to 0.633.

In the unrestricted model given by Eq. (22), the adjustment coefficients are negative in the case of LOANS and RGFCF, the former estimate being close to previous Euro Area estimates and those in the literature based on CVAR specifications (e.g., Calza et al. (2006), who model the stock of private sector bank loans for the Euro Area as a function of an inflation index and the cost of borrowing, estimating an error correction coefficient of around -0.075). In the normalized cointegrating vector given by Eq. (23), RGDP and DIFF have a positive and sizeable impact, while the coefficient on DEBT (also positive) is rather small, and puzzlingly RGFCF appears to have a negative effect:

$$\Delta^{\hat{d}} \begin{bmatrix} \text{LOANS}_t - \mu_1 \\ \text{RGDP}_t - \mu_2 \\ \text{RGFCF}_t - \mu_3 \\ \text{DIFF}_t - \mu_4 \\ \text{DEBT}_t - \mu_5 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.086 \\ 0.198 \\ -0.142 \\ 0.799 \\ 0.258 \end{bmatrix} / \begin{bmatrix} 1.000 \\ -1.666 \\ 0.465 \\ -0.214 \\ -0.006 \end{bmatrix} (X_t - \hat{\mu}) + \sum_{i=1}^2 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (22)$$

$$\text{LOANS}_t = \mu u + 1.666\text{RGDP}_t - 0.465\text{RGFCF}_t + 0.214\text{DIFF}_t + 0.006\text{DEBT}_t + \nu_t. \quad (23)$$

Restricted model for Spain

Once again, the starting point is to test for the significance of both the long-run β coefficients and the adjustment α ones to obtain a restricted specification with a better fit. Both are found to be insignificant in the case of DEBT, while the estimated adjustment coefficient is now slightly higher (-0.082) in the case of loans and insignificant in the case of RGFCF (Eq. 24).

The long-run relationship includes RGDP, DIFF and RGFCF, the latter again with a negative coefficient (Eq. 25). Note that the former two have sizeable and positive adjustment coefficients (0.189 and 0.789, respectively), which implies divergence from the long-run equilibrium:

$$\Delta^{\hat{d}} \begin{bmatrix} \text{LOANS}_t - \mu_1 \\ \text{RGDP}_t - \mu_2 \\ \text{RGFCF}_t - \mu_3 \\ \text{DIFF}_t - \mu_4 \\ \text{DEBT}_t - \mu_5 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.082 \\ (0.002) \\ 0.189 \\ (0.005) \\ -0.136 \\ (0.013) \\ 0.789 \\ (0.007) \\ 0.000 \\ (0.690) \end{bmatrix} / \begin{bmatrix} 1.000 \\ (0.000) \\ -1.529 \\ (0.029) \\ 0.419 \\ (0.243) \\ -0.228 \\ (0.011) \\ 0.000 \\ (0.724) \end{bmatrix} (X_t - \hat{\mu}) + \sum_{i=1}^2 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (24)$$

$$\text{LOANS}_t = \mu u + 1.529\text{RGDP}_t - 0.419\text{RGFCF}_t + 0.228\text{DIFF}_t + \nu_t. \quad (25)$$

Table 9 Fractional cointegration analysis for Spain

Likelihood Ratio Tests				
Rank	d	b	LR statistic	P-value
0	0.804	0.804	105.667	0.000
1	0.632	0.632	55.122	0.000
2	0.541	0.541	37.969	0.000
3	0.392	0.392	23.278	0.000
4	0.010	0.010	0.055	0.814
5	0.010	0.010	-	-
White Noise Tests				
Variable	Q	P-value	LM	P-value
All variables	14.621	0.958	-	-
LOANS	0.890	0.345	0.721	0.396
RGDP	0.048	0.826	0.011	0.916
RGFCF	0.691	0.406	0.097	0.755
DIFF	0.014	0.907	0.010	0.921
DEBT	0.123	0.726	0.107	0.744
Values for $d = b$				
	estimate		s.e	
	0.633		0.048	

The upper part of the table shows the results for the Trace LR statistic estimated for $d = b$, together with the relevant probability values. The middle part shows the white noise test results on the residuals of the model, with a null of absence of residual autocorrelation. The lower part of the table shows the final estimate for the fractional parameters and their statistical precision. The variables employed are Loans to Non-Financial Corporations (LOANS), real gross capital fixed formation (RGFCF), real GDP (RGDP), the differential between the cost of borrowing for new long- and short -term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

Conclusions

This paper investigates the determinants of loans to NFCs in four countries belonging to the Euro Zone (Germany, France, Italy and Spain). The findings are of interest not only to academics, but also to practitioners and European policy makers since this type of financing is much more important for the corporate sector in Europe than elsewhere. The modelling approach is based on fractional integration and cointegration methods, which have the advantage of considering the possible long-memory properties of the series of interest. Specifically, univariate models are estimated for the individual series and both unrestricted and restricted FCVAR models to examine linkages between them.

Table 10 Hypothesis testing for Spain

	H_1^{FCvsC}	$H_2^{\beta^{DEBT}}$	$H_3^{\alpha^{DEBT}}$
df	1	1	1
LR	18.571	0.125	0.159
P-value	0.000	0.724	0.690

The table shows the results for the Likelihood Ratio tests. The first column compares the likelihood of an FCVAR against a canonical CVAR. The subsequent columns show all the LR tests for the α and β parameters where the 0 null could not be rejected. The variables employed are Loans to Non-Financial Corporations (LOANS), real gross capital fixed formation (RGFCF), real GDP (RGDP), the differential between the cost of borrowing for new long- and short-term loans in the Euro Area (DIFF), and the cost of debt, security and equity issuance (DEBT). The sample ranges from 2003Q1 to 2022Q4. Data sources: The series were from the Federal Reserve Economic Data (Federal Reserve Bank of St. Louis, 2024), except for DIFF, which was taken from the European Central Bank Data Portal (European Central Bank, 2024)

The univariate results are very heterogeneous across the variables and the countries, with the highest degrees of integration being found in the case of loans to NFCs. As for the multivariate results, evidence of fractional cointegration was found in the case of Germany, France, Spain, and Italy. The estimated speed of adjustment of loans in the restricted models (based on appropriate overidentifying restrictions) is either very close to (Germany and Spain) or slightly slower (France and Italy) than the estimates reported in previous studies based on standard CVAR models at both the national and European level (e.g., Calza et al., 2003, 2006; Levieuge, 2017). Regarding the factors driving loans to NFCs in the long run, the evidence varies across countries, with DEBT being the only determinant in the case of Germany, RGDP playing the main role in the case of France, and a range of factors being relevant in the case of both Italy and Spain. In particular, the cost of bank lending relative to other sources of financing clearly matters, and thus policy measures affecting either can have an impact on loans.

Future research should estimate systems with a wider set of variables capturing both the demand and supply side as well as the impact of policy measures (Gambacorta & Rossi, 2010). Moreover, structural breaks should be investigated using a variety of methods allowing for a gradual evolution or sudden shifts of the parameters. Nonlinearities reflecting the existence of thresholds should also be examined.

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Data Availability Data are available from the authors upon request.

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