ECCM18, Athens, Greece
24-28 June 2018

Instabilities due to Strain-Softening Solved using the SPH Method

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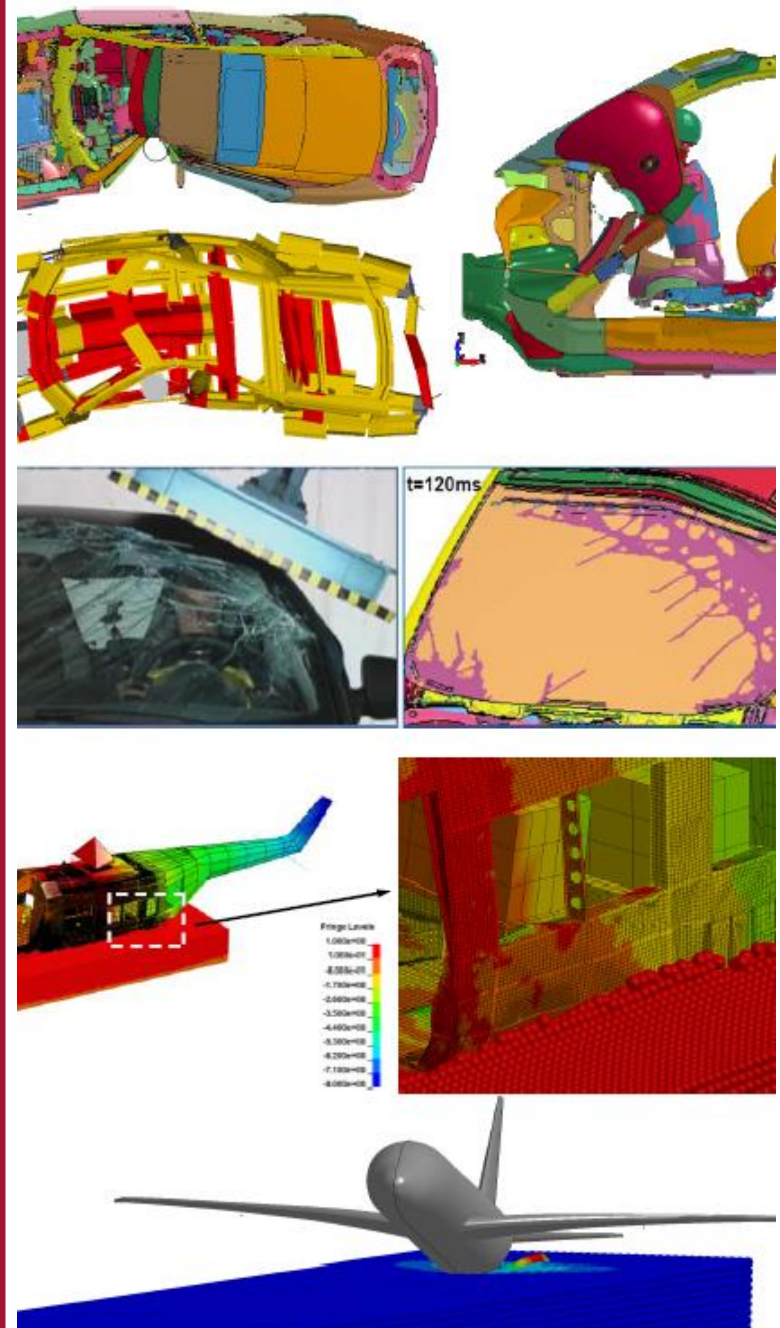

Outline

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- Computational Models for Strain Softening Problem
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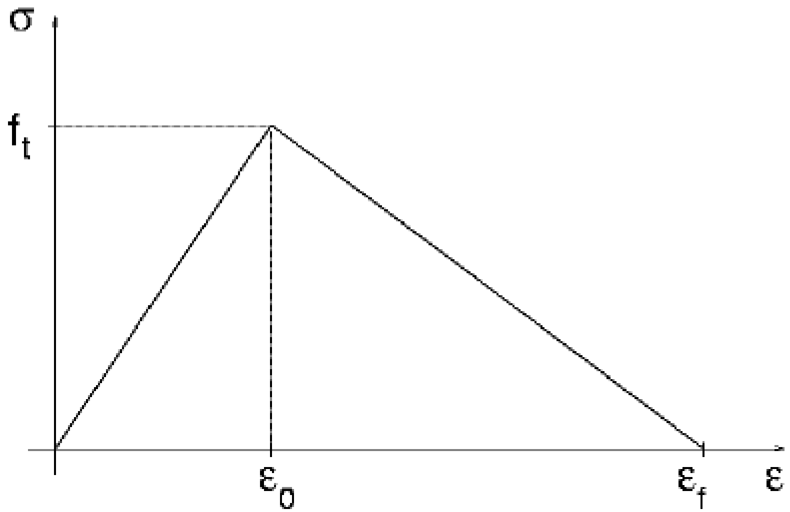
  - SPH Results for wave propagation problem
  - SPH Results for cube impact on a flat plate
- Conclusions and Future Work

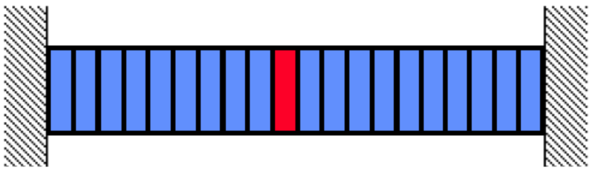
Acknowledgement: The project leading to this presentation has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 636549.

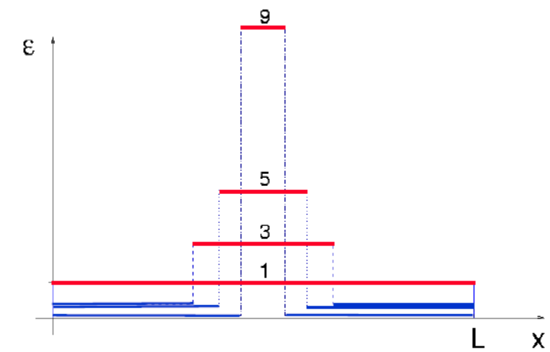
Introduction

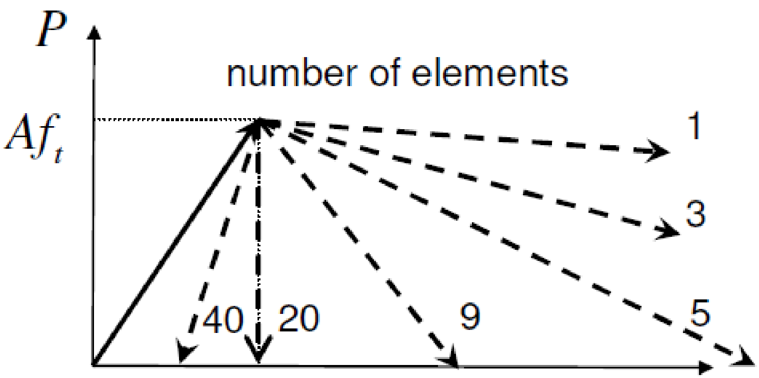
- EXTREME project aims at development novel material characterisation and in-situ measurement methods, **material models** and **improvement of computational methods** for the design and manufacture aerospace composite structures under EXTREME dynamic loadings leading to a significant reduction of weight, design and certification cost.
- This work is aimed at development of technique to address the **localisation** and **material softening** problem, including a treatment of material softening due to damage, in order to reduce and ultimately remove mesh dependency;
- Strain softening effects have been analysed in FEM and SPH method using CDM approach;

Strain Softening Problem







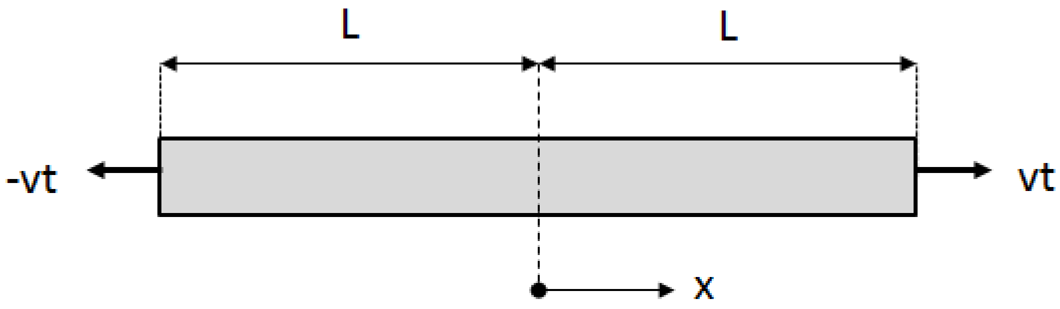


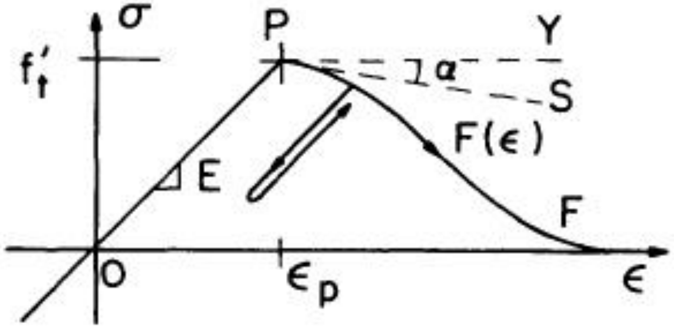
Strain Softening Problem

- Damage in the local continuum damage mechanics (CDM) is modelled as a degeneration of material properties induced by mechanical loading, which can result in strain softening behaviour;
- This can lead to an ill-posed boundary value problem, where the local governing hyperbolic differential equations at a point become elliptic, which induces a numerical instability;
- This instability is mesh-sensitive and manifests itself as non-physical deformation of the softening continuum;
- This work primarily considered the strain-softening effects in the SPH spatial discretisation, and compares those with the classical CDM-FEM solutions;

Strain Softening Problem

- One-dimensional dynamic strain softening problem, for which an exact analytical solution for wave propagation was known (Bažant and Belytschko, 1985)





$$\dot{\varepsilon}_{ij}\dot{\sigma}_{ij} > 0$$ Bifurcation criterion

$$\dot{\sigma}_{ij} = \mathbb{C}_{ijkl}\dot{\varepsilon}_{kl}$$ Rate form of constitutive equation

Material stability criterion

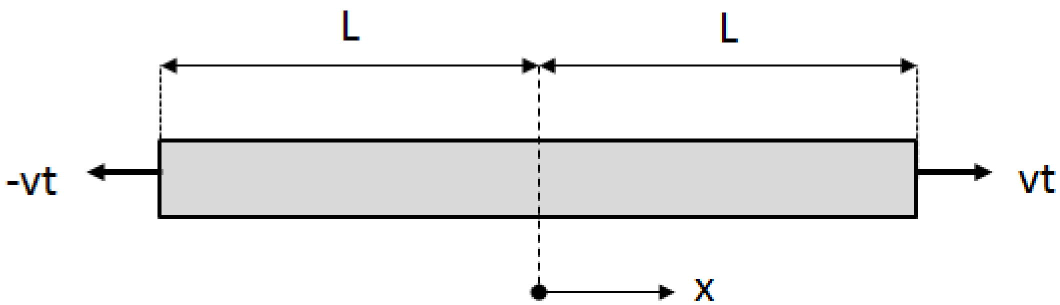
$$\boxed{\dot{\varepsilon}_{ij}\mathbb{C}_{ijkl}\dot{\varepsilon}_{kl} = 0}$$

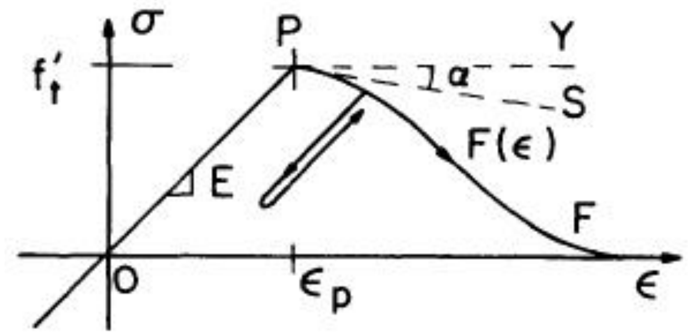
$$c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$ Elastic response

Softening response

$$c^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0 \qquad c^2 = \frac{F'(\varepsilon)}{\rho}$$

Strain Softening Problem





$$u(x,t) = -v\left\langle t - \frac{x+L}{c}\right\rangle + v\left\langle t + \frac{x-L}{c}\right\rangle$$

Analytical solution for displacement field

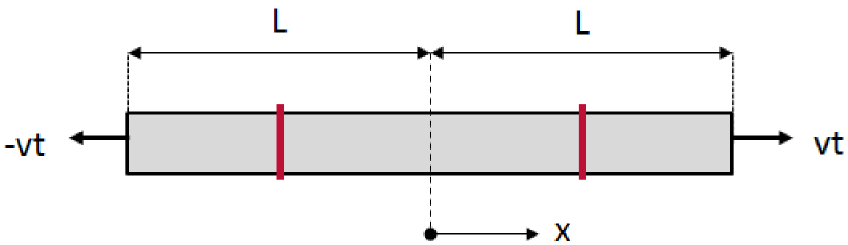
$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{v}{c}\left[H\left(t - \frac{x+L}{c}\right) + H\left(t + \frac{x-L}{c}\right)\right]$$

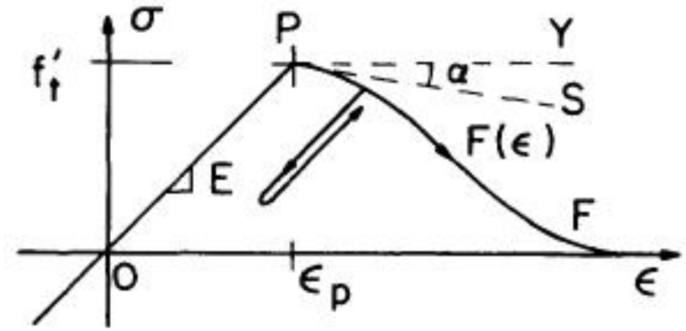
Strain field before the wave superposition in the midsection of the bar

$$\varepsilon_x = \frac{v}{c}\left[H\left(t - \frac{x+L}{c}\right) - H\left(t + \frac{x-L}{c}\right) + 4v\langle t - L/c\rangle \delta(x)\right]$$

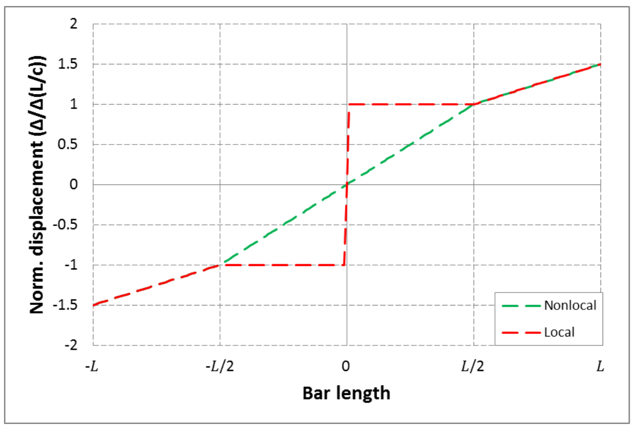
Strain field outside of the softening zone

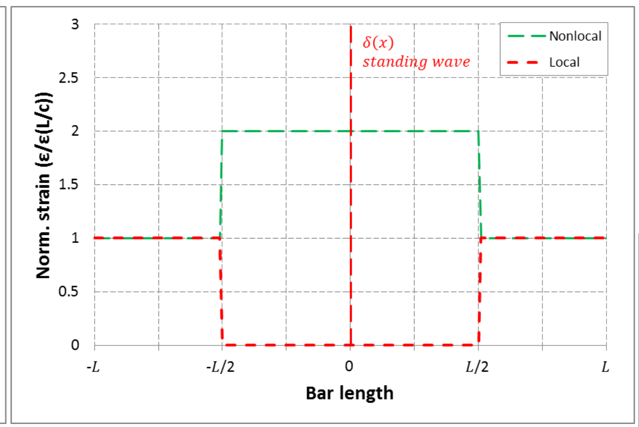
Strain Softening Problem





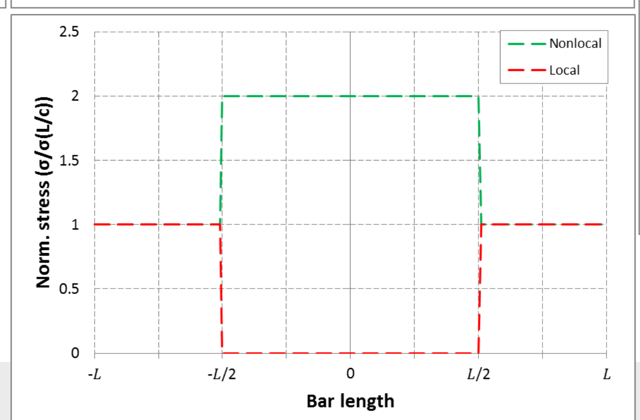
Geometry and loading of softening bar Bažant and Belytschko (1985)

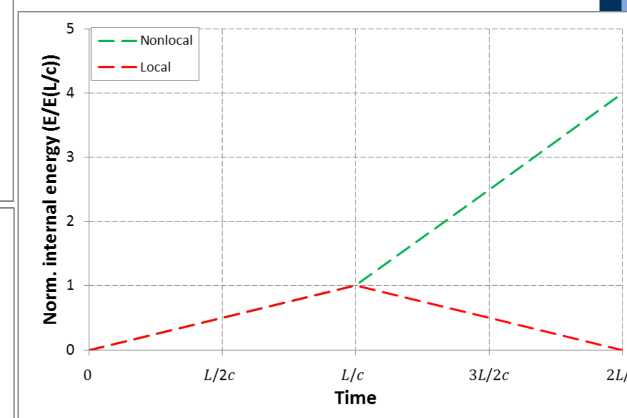




Local and nonlocal solutions at the time instance

$$t = \frac{3}{2}\frac{L}{c}$$



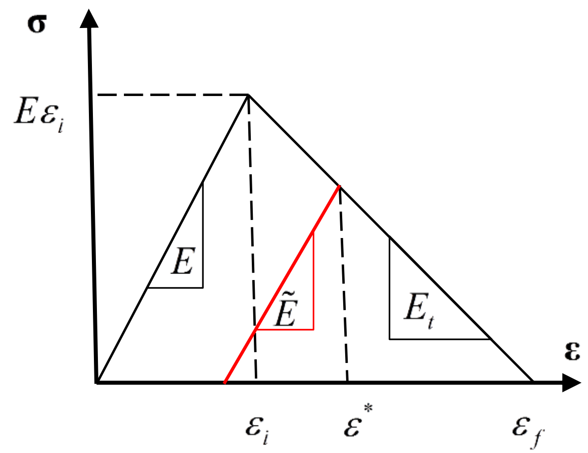


Constitutive Model with Isotropic Damage

- Bilinear constitutive law based on local continuum damage mechanics approach;

$$\bar{\sigma} = \frac{\sigma}{1-\omega} \qquad \omega = 1 - \frac{\varepsilon_i \left(\varepsilon_f - \varepsilon^*\right)}{\varepsilon^* \left(\varepsilon_f - \varepsilon_i\right)}$$

$$E_T = -\frac{E\varepsilon_i}{\varepsilon_f - \varepsilon_i}$$

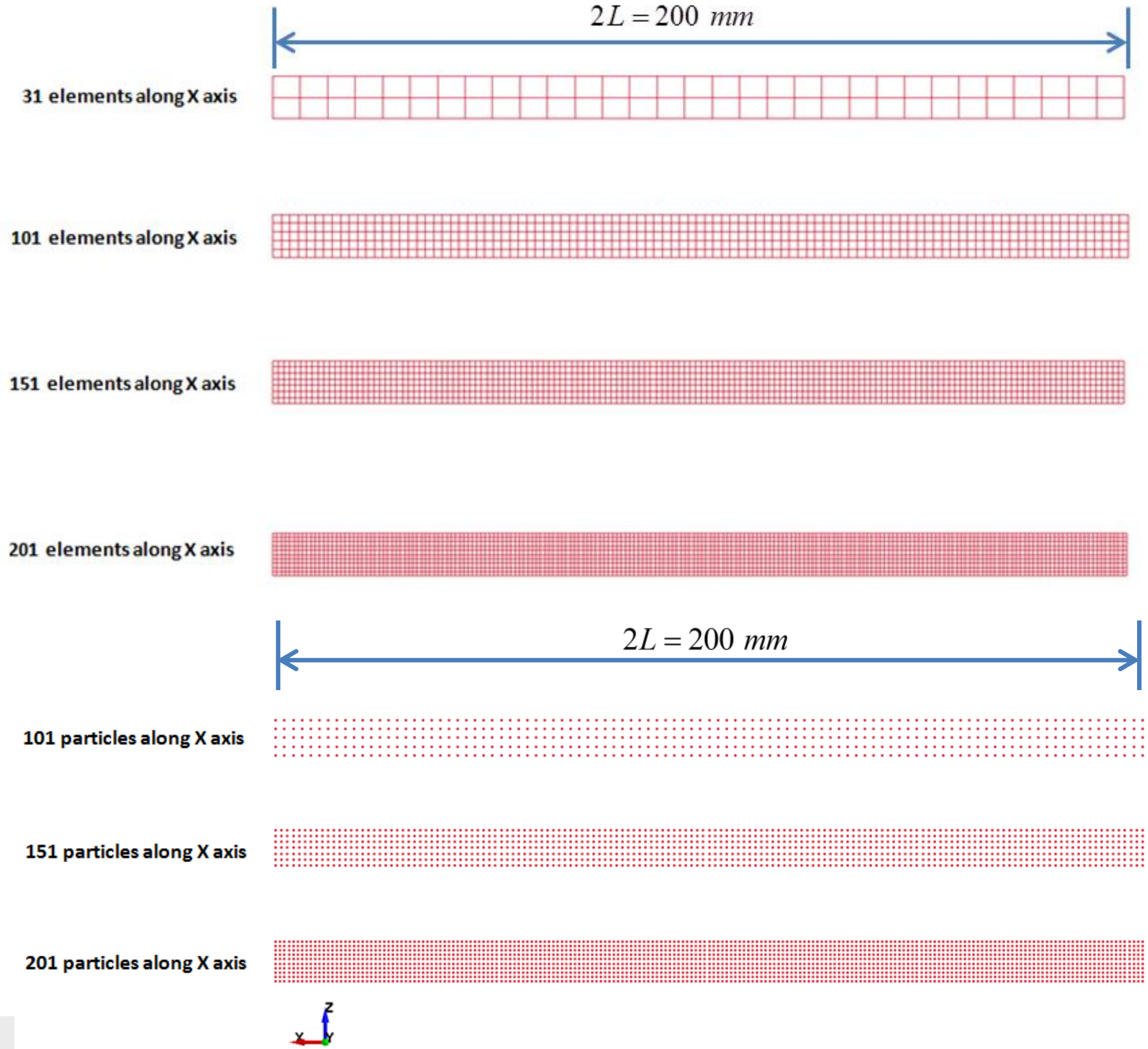


- Two discretisation methods:
  - Finite Elements Method (FEM);
  - Smooth Particle Hydrodynamics (SPH);

Numerical Experiments
FEM and SPH Models

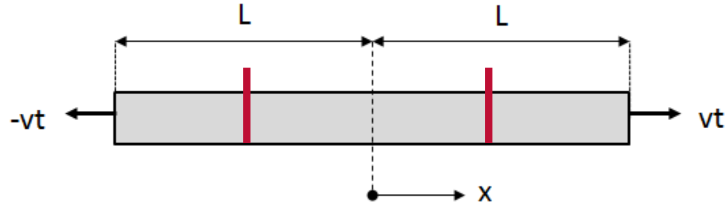
Spatial discretisation used in the FEM (DYNA3D) simulation of the strain-softening bar

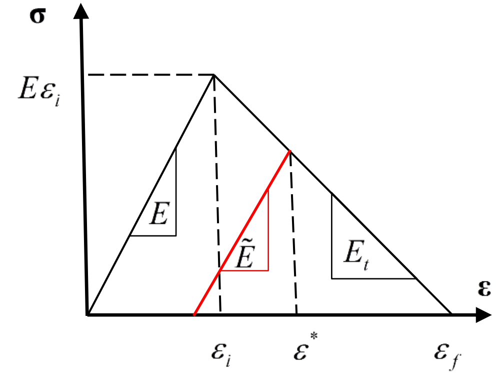
Particle discretisation in SPH (MCM) of strain-softening bar

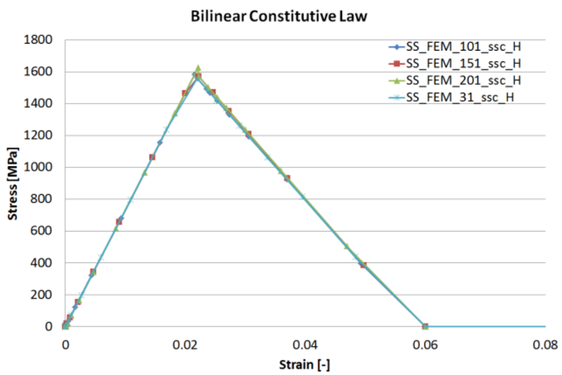


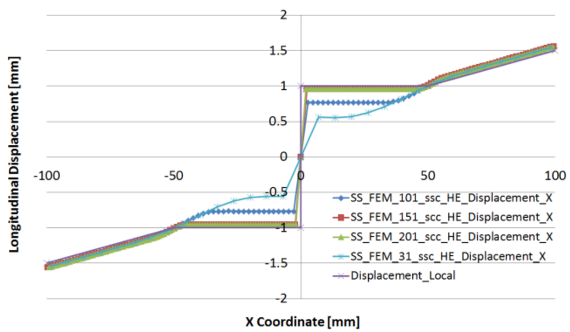
Numerical Experiments
FEM Results

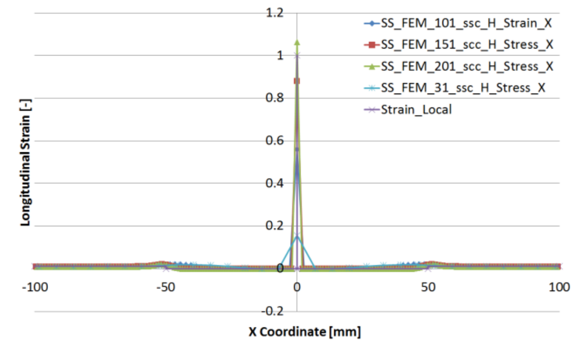
Bilinear law implemented in FEM using a damage parameter and classic CDM approach

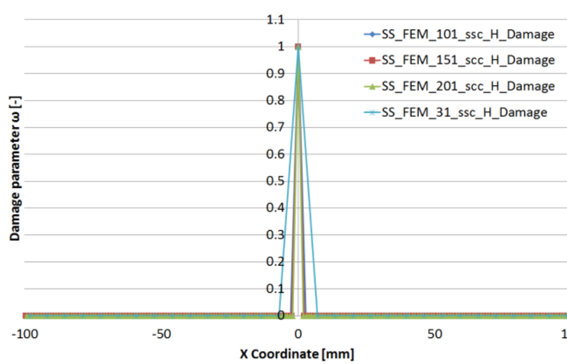


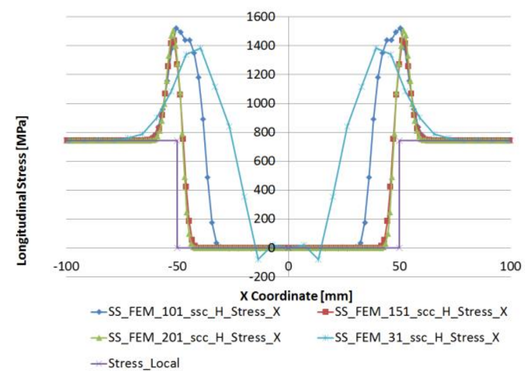






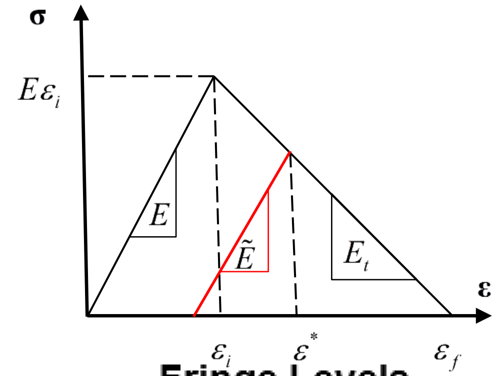


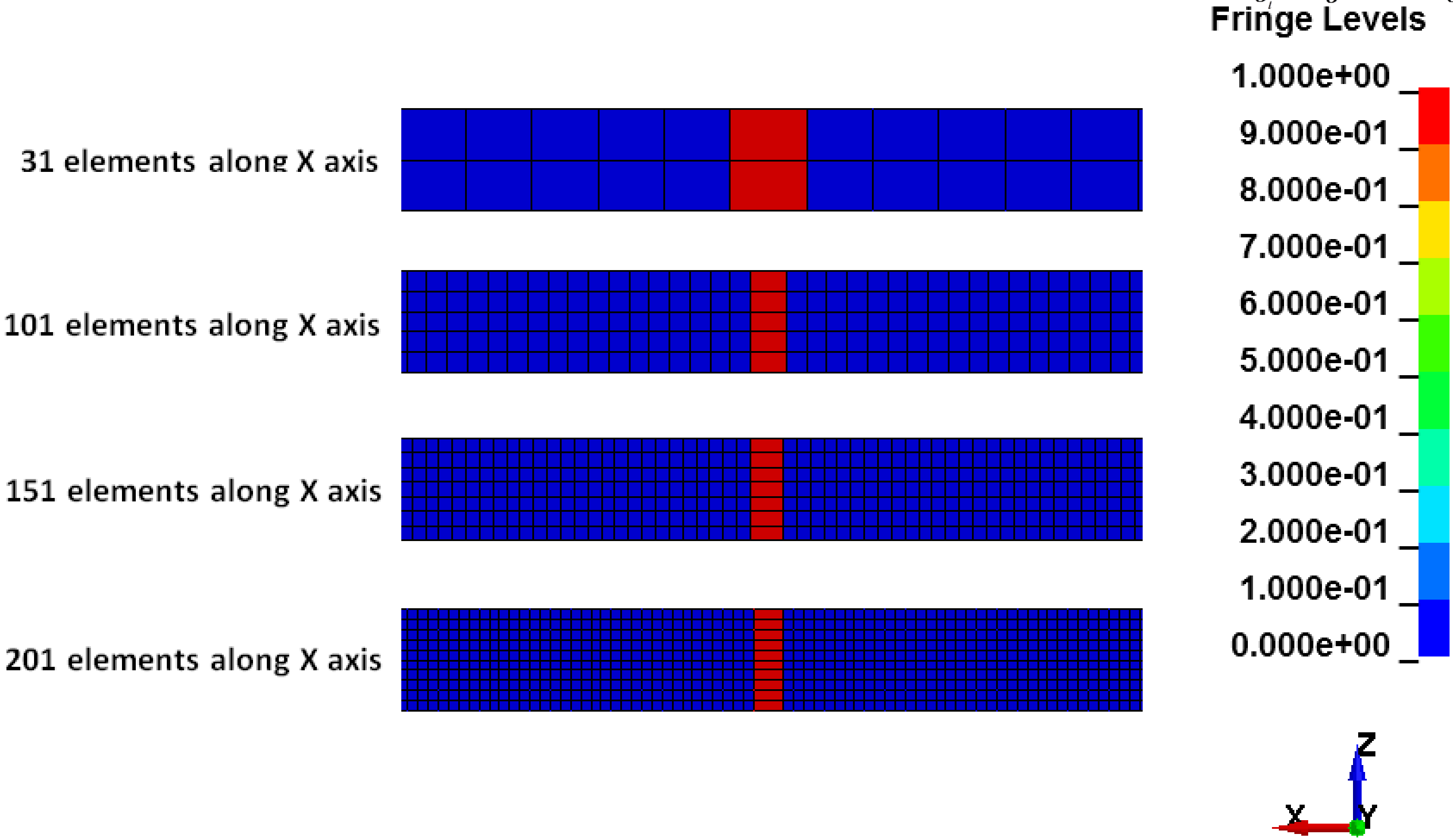




Damage localised in a single element

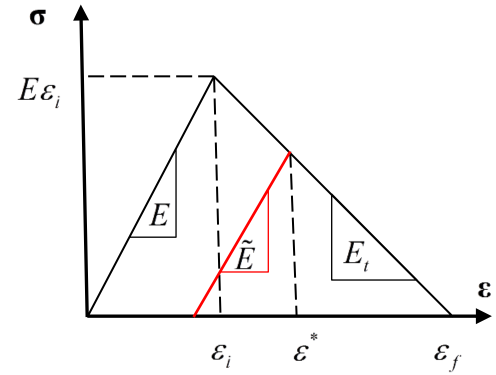
Numerical Experiments
FEM Results





Damage localised in a single element

Numerical Experiments
Smooth Particle Hydrodynamics



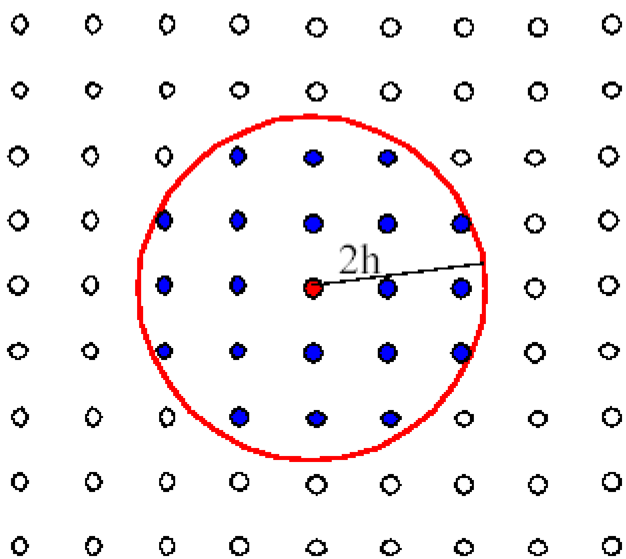
- SPH is a meshless particle method, where the motion of a continuum is described by the movement of a finite number of discrete particles, which are used in the spatial discretisation of the state variables;

$$\langle f(\mathbf{x})\rangle = \int_\Omega f(\mathbf{x}')W(|\mathbf{x}-\mathbf{x}'|,h)\,d\mathbf{x}'$$

$$\langle f(\mathbf{x}_I)\rangle \approx \sum_J^N \frac{m_J}{\rho_J} f(\mathbf{x}_J)\left[W(|\mathbf{x}_I-\mathbf{x}_J|,h)\right]$$

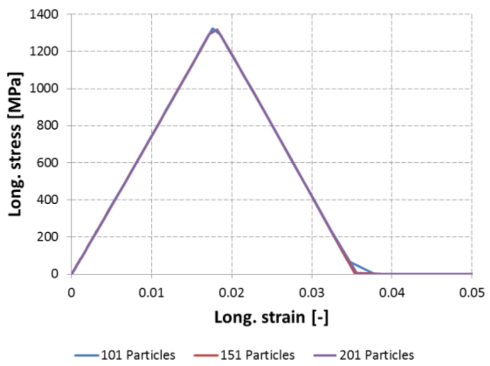
$$\nabla f(\mathbf{x}_I) = \sum_J \frac{m_J}{\rho_J} f(\mathbf{x}_J)\left[\nabla W(|\mathbf{x}_I-\mathbf{x}_J|,h)\right]$$

$$\langle \mathbf{F}_i\rangle = -\sum_{j=1}^{np} \frac{m_j}{\rho_j^0}(\mathbf{v}_i-\mathbf{v}_j)\otimes\nabla_{x_i^0} W(|\mathbf{x}_i^0-\mathbf{x}_j^0|,h^0)$$



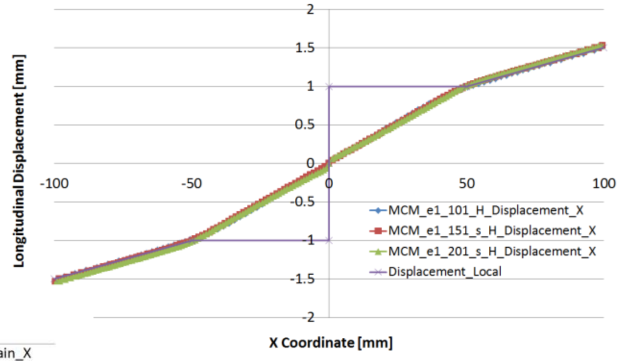
$$h = \lambda\Delta p$$

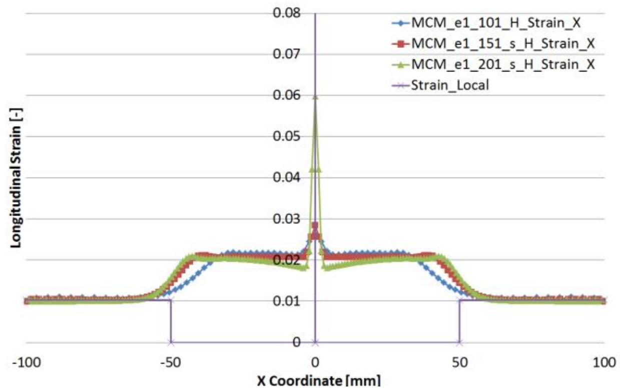
Numerical Experiments
SPH Results of Experiment 1 – inter-particle distance



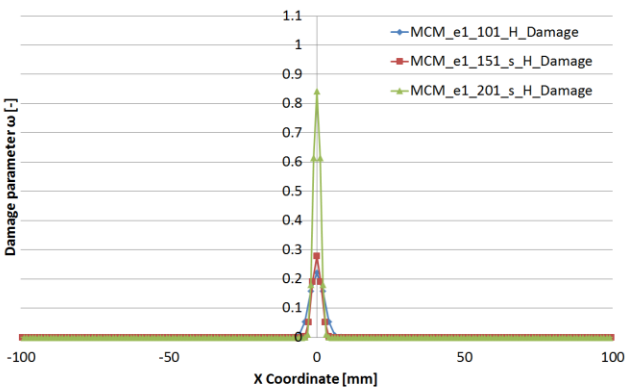
Longitudinal stress vs. longitudinal strain curves for the central particle for different values of Δp, SPH

Analytical solution and the SPH-experiment 1 numerical results for longitudinal displacement at $t = 3/2 \cdot L/c_e$

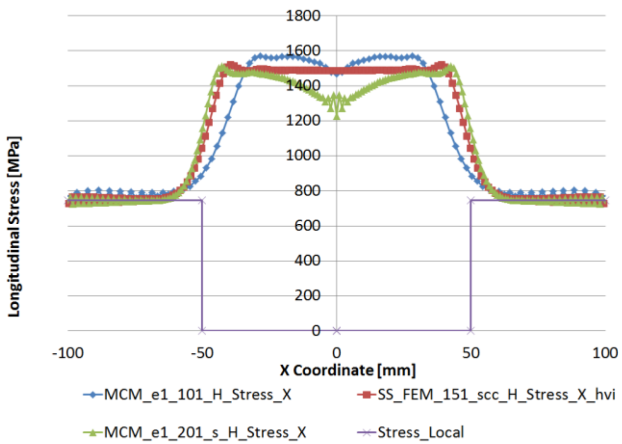




Analytical solution and the SPH-experiment 1 numerical results for longitudinal strain at $t = 3/2 \cdot L/c_e$

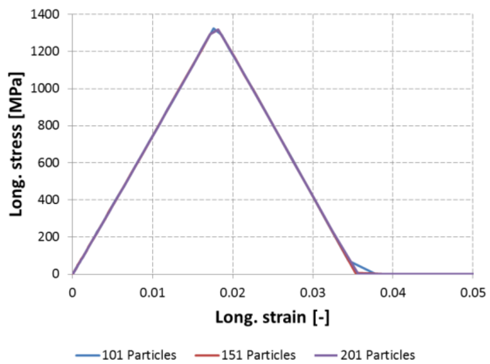


Damage distribution for different particle densities, SPH-experiment 1, response time $t = 3/2 \cdot L/c_e$



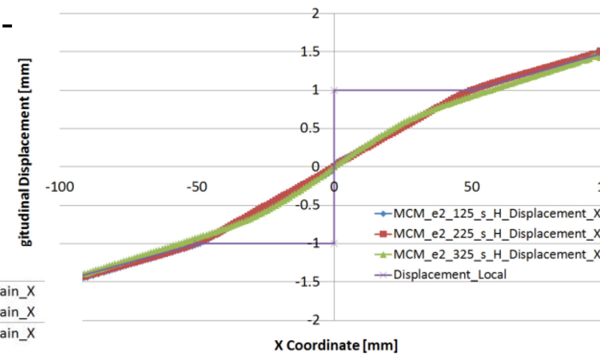
Analytical solution and the SPH-experiment 1 numerical results for longitudinal stress at $t = 3/2 \cdot L/c_e$

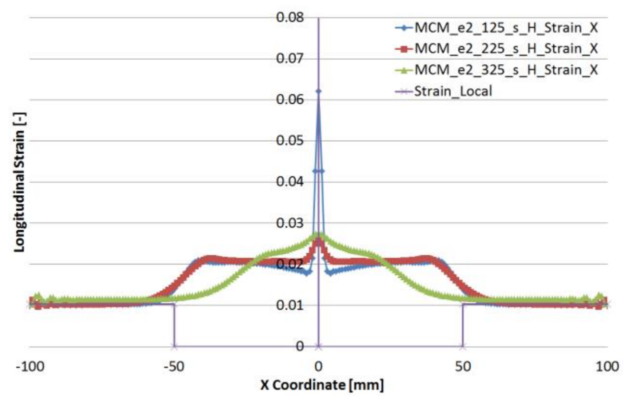
Numerical Experiments
SPH Results of Experiment 2 – averaging



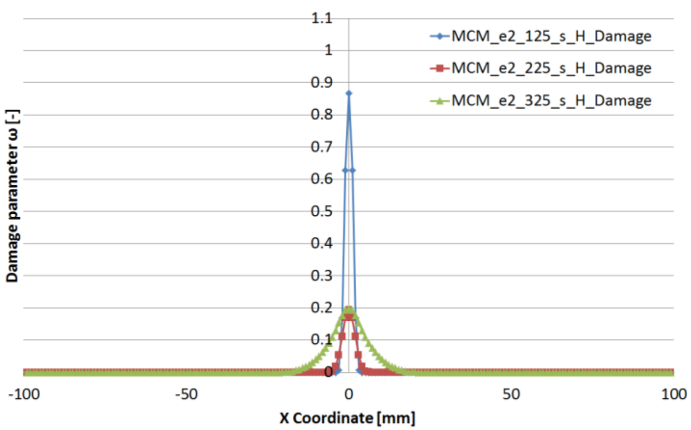
Longitudinal stress vs. longitudinal strain curves for the central particle for different values of λ, SPH

Analytical solution and the SPH-experiment 2 numerical results for longitudinal displacement at $t = 3/2 \cdot L/c_e$

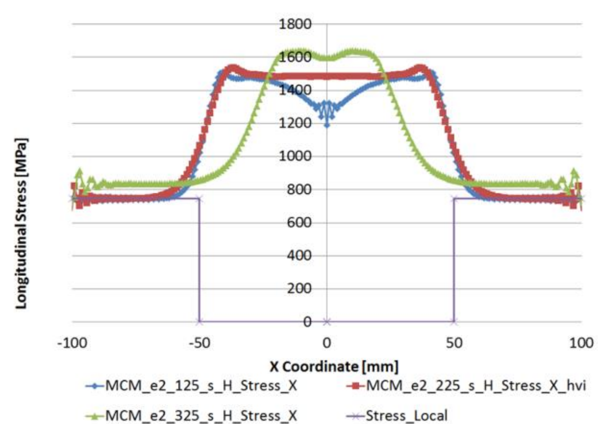




Analytical solution and the SPH-experiment 2 numerical results for longitudinal strain at $t = 3/2 \cdot L/c_e$

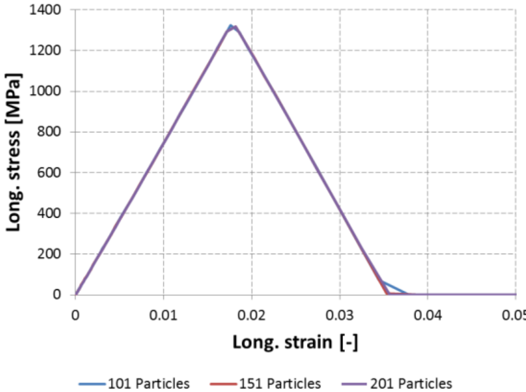


Damage distribution for different particle densities, SPH-experiment 2, response time $t = 3/2 \cdot L/c_e$



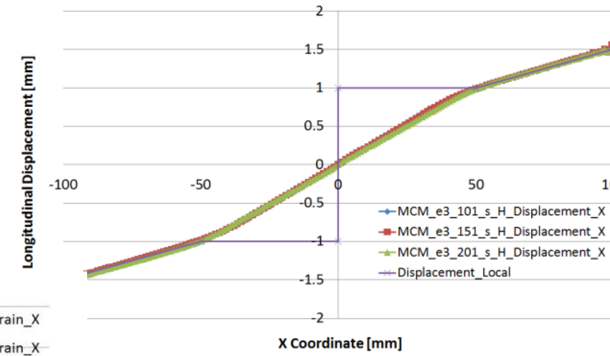
Analytical solution and the SPH-experiment 2 numerical results for longitudinal stress at $t = 3/2 \cdot L/c_e$

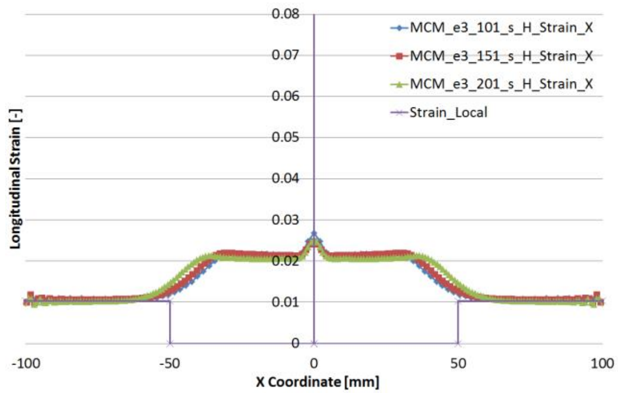
Numerical Experiments
SPH Results of Experiment 3 – constant smooth- length



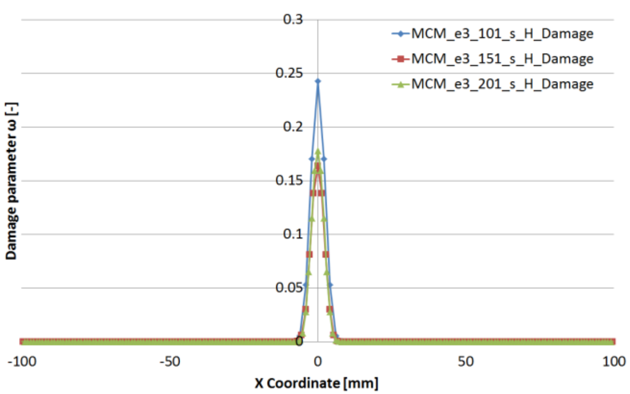
Longitudinal stress vs. longitudinal strain curves for the central particle for different values of Δp, SPH

Analytical solution and the SPH-experiment 3 numerical results for longitudinal displacement at $t = 3/2 \cdot L/c_e$

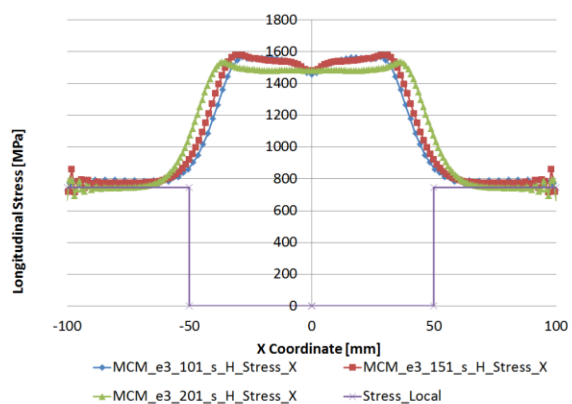




Analytical solution and the SPH-experiment 3 numerical results for longitudinal strain at $t = 3/2 \cdot L/c_e$



Damage distribution for different particle densities, SPH-experiment 3, response time $t = 3/2 \cdot L/c_e$



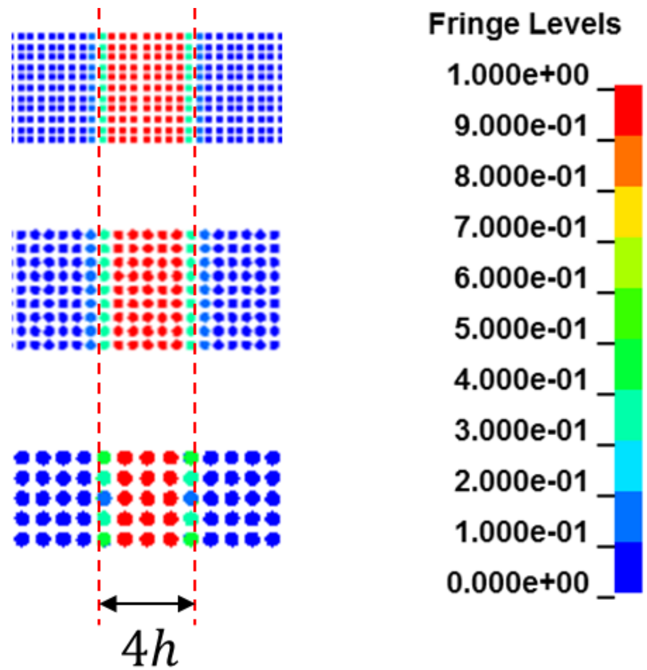
Analytical solution and the SPH-experiment 3 numerical results for longitudinal stress at $t = 3/2 \cdot L/c_e$

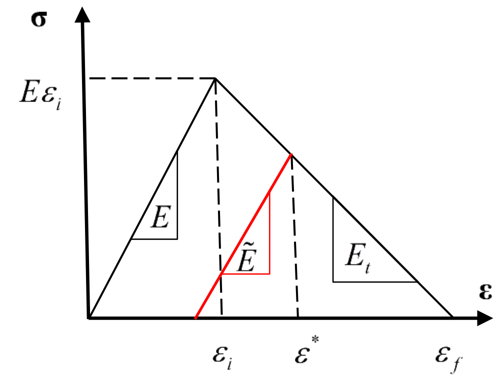
Numerical Experiments
SPH Results of Experiment 3 – constant smooth- length

$$\Delta p = \frac{200mm}{201}$$

$$\Delta p = \frac{200mm}{151}$$

$$\Delta p = \frac{200mm}{101}$$

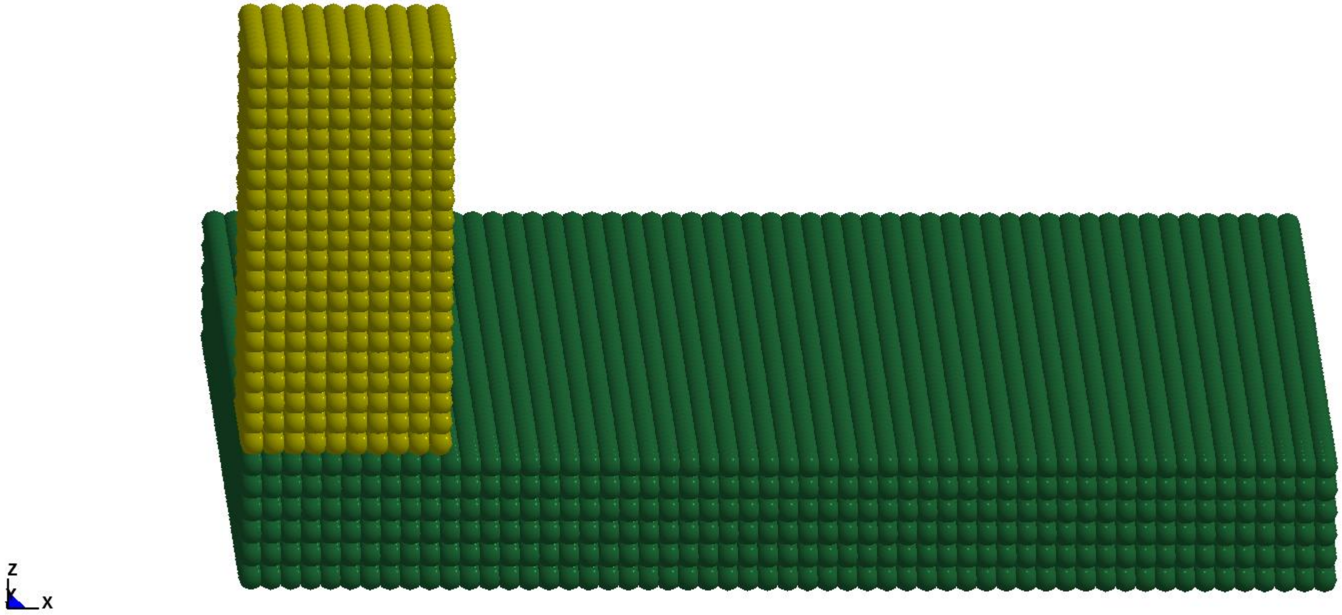




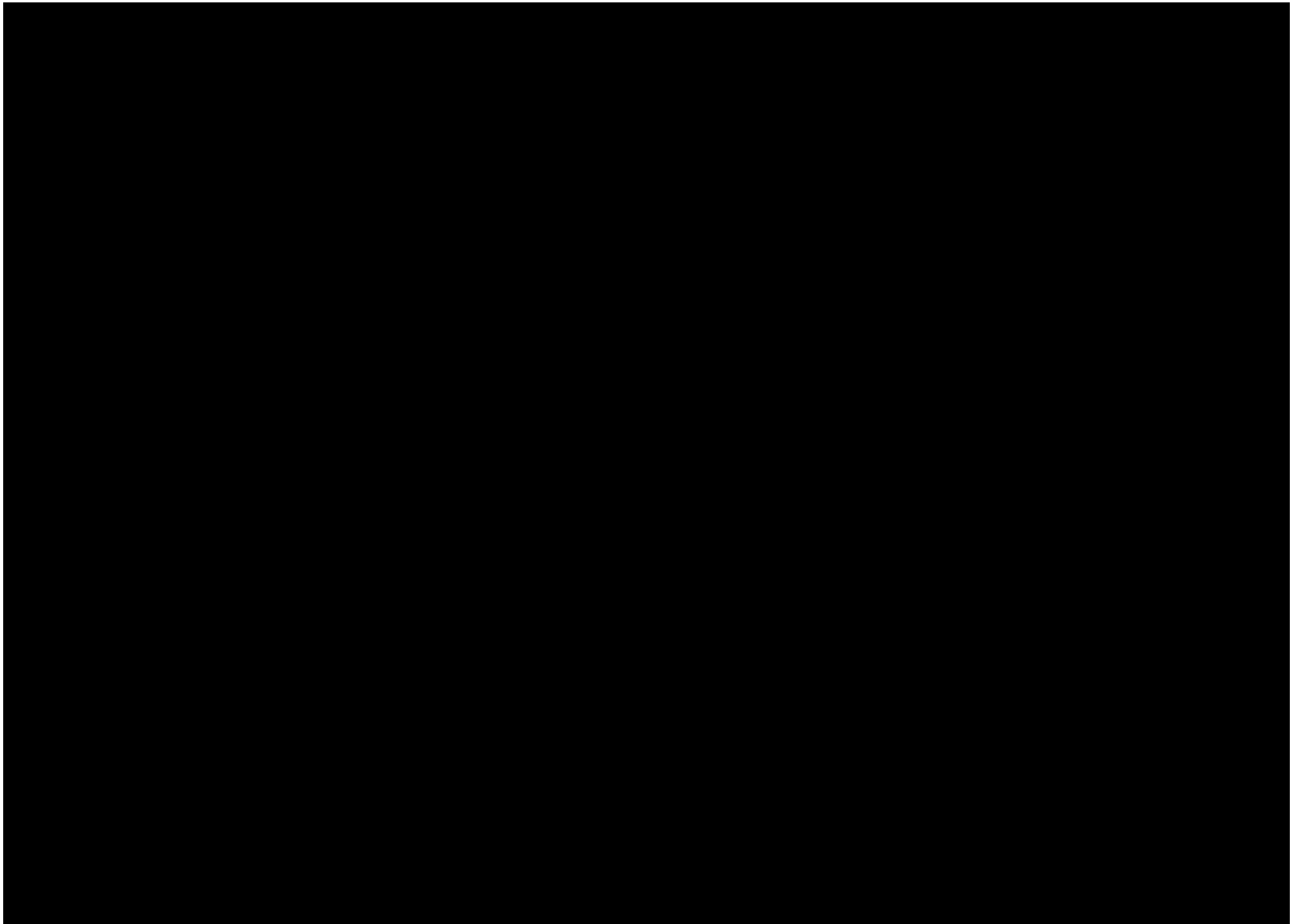
Damage distributed within a limited zone with 4h size around the bar symmetry plane at response time $t = \frac{3}{2}\frac{L}{c}$

Modelling of cube impact

- SPH code tested in a flat faced projectile impact on a 3.175mm target plate;
- Impact velocity 240m/s – 10% above the ballistic limit;



Modelling of cube impact



Conclusions and Future Work

- The strain softening process in the classic CDM combined with FEM is localised in a single element and cannot propagate;
- Localisation effects related to material strain-softening are not present with the SPH method – the method inherently nonlocal;
- Size of the softening zone was defined by the smoothing length and for a fixed smoothing length h, the stain softening was independent of the particle density;
- The future work is aimed at the orthotropic material formulation and application to composites;

Thank you for your attention

Instabilities due to Strain-Softening Solved using the SPH Method

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