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Distributed Kalman Filtering Under Two-Bitrate Periodic Coding Strategies

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***Abstract*—This paper is concerned with the problem of distributed Kalman filtering over sensor networks under two-bitrate periodic coding strategies. Initially, the optimal estimates for sensor individuals are acquired using the conventional Kalman filter. Subsequently, the information pair, consisting of the local estimate and the corresponding covariance, is exchanged among their immediate neighbors to achieve cooperative estimation. Due to the constrained network bandwidth, a vector/matrix quantization approach is formulated to quantize the information pair. The output of this quantization establishes a conservative bound for the actual covariance. A two-bitrate periodic coding strategy is proposed, where the encoded bits of the quantizer outputs are divided into two separate parts, namely the most significant and least significant bits, following a periodic transmission principle. It is demonstrated that the estimation preserves a consistency property over the sensor networks as the reported error covariance always serves as an upper bound for the actual error covariance. It is shown that the mean-square estimation errors are bounded when certain conditions regarding collective observability and network connectivity are satisfied. Finally, the effectiveness of the proposed algorithm is verified through a numerical example.**

***Index Terms*—Kalman filter, distributed filter, sensor network, signal quantization, periodic coding strategies, performance analysis.**

I. Introduction

In recent years, considerable attention has been given to wireless sensor networks (WSNs) due to their diverse applications including multiple autonomous robots, industrial monitoring, battlefield surveillance, intelligent transportation, islanded microgrids, and so on [1], [23], [37], [47]. WSNs consist of numerous small-sized sensing devices that are spatially distributed and equipped with wireless radio transceivers, enabling them to collect various environmental phenomena and share information across networks. However, these individual sensors are relatively inexpensive and possess limited capabilities to handle complex sensing tasks in harsh environments on their own. As a result, the problem of distributed estimation has emerged as a crucial challenge in both industry and academia with aim to establish a collaborative information processing mechanism.

The field of signal processing and control engineering has a long history of centralized multi-sensor fusion problems [4], [7], [8], [16], [27], [39]. In contrast to the centralized approach, where a fusion center is responsible for processing measurements from all sources, the decentralized framework distributes the computational burdens among all the sensors in the network [9], [11], [12], [26]. In this framework, each sensor independently calculates its estimates using only locally available information, such as local measurements and transmitted signals from neighboring sensors. The distributed Kalman filter has gained significant research interest due to its robustness, scalability, and energy efficiency. Numerous results have been reported in this area over the past decades [19], [20], [28], [31]–[33], [41], [42]. The distributed Kalman filter typically consists of two stages: a local update stage using the Kalman filter and an information consensus stage that fuses the neighboring messages.

In general, research on the distributed Kalman filter can be categorized into three main areas: consensus on estimates [22], [35], [36], consensus on measurements [13], [25], [43], and consensus on information [5], [29], [44]. These categories are based on the specific types of information shared over networks using consensus techniques. Consensus on estimates involves sharing local state estimates among sensors, allowing for information fusion to improve the performance of the local Kalman filter [22], [35], [36]. Consensus on measurements, on the other hand, focuses on sharing measurements or innovations among sensors to achieve consensus [13], [25], [43]. Consensus on information involves sharing information vectors and matrices, which includes the correlation information between estimates, to enhance the overall estimation performance [5], [29], [44].

It is important to note that consensus on measurements and consensus on information have complementary strengths and weaknesses. The stability of consensus on measurements depends on the number of consensus steps, while the other category does not have this requirement. However, consensus on information tends to have lower performance compared to consensus on measurements because it discards the correlation information between estimates. To address these trade-offs, a hybrid consensus strategy that combines consensus on infor-

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mation and consensus on measurements has been proposed. This hybrid approach aims to leverage the benefits of both strategies and mitigate their drawbacks [6], [21].

As mentioned earlier, the interaction of information among sensor nodes is crucial for implementing the distributed Kalman filter. The energy/bandwidth constraints of communication links in sensor networks require a reduction of the amount of data transmission, which can be achieved through quantization methods. Some preliminary results have appeared on the quantized Kalman filters, see e.g. [30], [34], [38]. For instance, in [38], the innovations from distributed sensors have been quantized into a single bit, and a recursive distributed filter has then been established to minimize the mean-square error. To date, most existing results have primarily focused on vector quantization, such as quantizing estimates or innovations using a scalar quantizer with one quantizer per component. These methods assume that accurate covariance matrices are always available at the receiver. However, in order to implement information fusion in the distributed Kalman filter, both the estimate vector and the covariance matrix need to be shared over networks. Therefore, an appropriate quantization method should be adopted for both the vector and the matrix to ensure consistent estimation.

In digital communication, the outputs of the quantizer need to be encoded into a finite binary string before transmission, where the bitrate refers to the number of bits that can be reliably transmitted per unit of time via digital networks. Note that communication processes are inherently constrained by bitrate due to limited network resources, and such a constraint could significantly degrade system performance. So far, extensive research efforts have been devoted to addressing bitrate constraints for various control and estimation objectives in both stochastic and deterministic settings, see e.g. [3], [14], [17], [48]. The critical bitrate for networked stabilization problems has been investigated in [17], with subsequent work focusing on robustness considerations in [14]. In [48], a bitrate allocation mechanism under a total constraint has been developed for multi-sensor systems to achieve desired performance. More recently, the period-two coding scheme, which originates from data compression in the signal processing domain, has garnered research attention [46]. Unlike previous frameworks where quantized bits are fully transmitted at each iteration, period-two coding schemes divide the bit string into two parts that are transmitted separately, thereby enabling periodic bitrate assignment and helping manage quantization errors [2].

Based on the preceding discussions, our objective is to investigate a distributed Kalman filter that consists of a Kalman filtering stage and an information fusion stage. This investigation builds upon the findings of the Kalman consensus filter [6], [29]. It should be noted that in the information fusion stage, the information pair from sensor nodes, comprising an estimate vector and covariance matrix, needs to be shared over the network. To address the resource constraints of wireless communication, we propose a two-bitrate periodic coding strategy for efficient transmission. First, we introduce a vector/matrix quantization approach. For the vector case, the dither quantizer is employed individually for each component and, for the matrix case, a diagonal dominant method is exploited to design the quantization scheme for each entry. Subsequently, the quantized outputs are encoded into a binary string, which is further divided into the most significant and least significant bits for transmission in a period-two manner.

The main contributions of our work can be summarized as follows:

1) We propose a two-bitrate periodic coding strategy for the distributed Kalman filter. This strategy involves dividing the messages into two parts with different bitrates and transmitting them periodically based on their significance. By doing so, the amount of communication data exchanged among sensor nodes is significantly reduced.
2) The proposed filter ensures an unbiased and consistent estimate. Specifically, the reported covariance computed by each sensor based on local information always serves as an upper bound for the actual error covariance. This allows for real-time evaluation of the estimation accuracy.
3) We establish the boundedness of mean-square estimation errors, which explicitly depends on network connectivity and collective observability. This analysis provides insights into the factors influencing the estimation performance.

These contributions collectively enhance the efficiency and accuracy of distributed Kalman filtering over sensor networks, addressing the challenges posed by limited communication resources and ensuring reliable estimation results.

The paper is outlined as follows. In Section II, the mathematical description of sensor networks and the conventional distributed Kalman filter is introduced. Subsequently, in Section III, a quantization mechanism for the information pair is presented, and a novel distributed Kalman filter with two-bitrate periodic coding strategies is developed. Section IV undertakes further performance analysis, revealing the unbiasedness, consistency, and boundedness of the proposed distributed Kalman filter. In Section V, the simulation experiments are presented to evaluate the performance of the proposed filter. Finally, Section VI concludes the paper with remarks.

Notations. Throughout the paper, the notation used is fairly standard. The matrix inequality $A \geq B$ ($A > B$) represents that $A - B$ is positive semi-definite (definite). $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ matrices, respectively. I represents an identity matrix of appropriate dimensions and $\mathbf{1} \in \mathbb{R}^N$ is a column vector with all entries equal to one. $\mathbb{Z}$ represents the set of positive integer numbers, $\mathbb{Z}_+$ is the set of positive odd numbers, and $\mathbb{S}^n_+$ represents the set of all real $n \times n$ positive definite matrices.

II. Problem Formulation

A. System models

In this section, we formulate the distributed filtering problem over the sensor networks. Firstly, we consider a dynamical system described by an n-dimensional state-space model at time k as follows:

$$x_{k+1} = Ax_k + w_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector of the system, $w_k \in \mathbb{R}^n$ is the random process noise, and $A \in \mathbb{R}^{m\times n}$ is the state-transition matrix.

The dynamical system is monitored by N intelligent sensors with the local observation model described by

$$y_{k,i} = C_i x_k + v_{k,i} \tag{2}$$

for $i = 1, 2, \cdots, N$, where $y_{k,i} \in \mathbb{R}^m$ is the i-th sensor's observation at instant k, $v_{k,i} \in \mathbb{R}^m$ is the random observation noise, and $C_i \in \mathbb{R}^{m\times n}$ is the local observation matrix of sensor i.

Assumption 1: The process and observation noises $\{w_k\}_{k=1}^{\infty}$ and $\{v_{k,i}\}_{k=1}^{\infty}$ are mutually uncorrelated white Gaussian random variables with zero mean values and bounded covariances $Q > 0$ and $R_i > 0$.

Assumption 2: The initial state x_1 obeys Gaussian distribution with expectation $\bar{x}_1$ and covariance $\Sigma_1 > 0$, and is uncorrelated with the process and observation noises $\{w_k\}_{k=1}^{\infty}$ and $\{v_{k,i}\}_{k=1}^{\infty}$.

The communication structure plays a crucial role in sensor networks as it defines the interaction mechanism among spatially dispersed sensors. In our scenario, the sensor network topology is described by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{S})$ with a vertex set $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$, an edge set $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$, and a weighted matrix $\mathcal{S} = [\pi_{ij}]_{N\times N}$. In this context, an edge $(v_i, v_j) \in \mathcal{E}$ signifies that the i-th node can receive messages from the j-th node, and vice versa. The weighted matrix $\mathcal{S}$ is symmetric and doubly-stochastic, meaning that each row and column sum up to 1, and its nonnegative elements π_{ij} satisfy the property $\pi_{ij} > 0 \iff (v_i, v_j) \in \mathcal{E}$. To denote the set of neighbors of vertex v_i, we use $N_i \triangleq j : (v_i, v_j) \in \mathcal{E}$.

B. Distributed Kalman filtering

It should be noted that individuals in sensor networks face inherent hardware constraints, such as limited sensing, processing, and storage capabilities. As a result, they cannot accurately reconstruct the system state using only local observations. Given the communication topology described in the previous subsection, each node has the ability to combine its local estimates with information received from neighboring nodes. This cooperative approach allows sensors to enhance their local performance and successfully accomplish complex sensing tasks. Consequently, the two-stage distributed Kalman filter [6], [29] is formulated which is detailed below.

In the first stage, a conventional Kalman filter is employed as follows by each sensor to generate a posterior estimation based on its local observations:

Time-Update:

$$\begin{aligned} \hat{x}_{k|k-1,i} &= A\hat{x}_{k-1,i}, \\ P_{k|k-1,i} &= AP_{k-1,i}A' + Q \end{aligned} \tag{3}$$

Measurement-Update:

$$\begin{aligned} \hat{x}_{k,i}^{KF} &= \hat{x}_{k|k-1,i} + K_{k,i}(y_{k,i} - C_i\hat{x}_{k|k-1,i}), \\ P_{k,i}^{KF} &= P_{k|k-1} - \ K_{k,i}C_iP_{k|k-1,i}, \\ K_{k,i} &= P_{k|k-1,i}C_i'(C_iP_{k|k-1,i}C_i' + R_i)^{-1} \end{aligned} \tag{4}$$

where $K_{k,i} \in \mathbb{R}^{n\times m}$ is the Kalman filtering gain, and $\hat{x}_{k|k-1,i}$ and $\hat{x}_{k,i}^{KF}$ are the one-step prediction and the posteriori estimate of the state vector x_k at i-th sensor with the respective corresponding error covariances $P_{k|k-1,i}$ and $P_{k,i}^{KF}$. $\hat{x}_{k-1,i}$ and $P_{k-1,i}$ are the estimate of x_{k-1} and the corresponding error covariance. Moreover, We define a shorthand notation

$$\mathrm{KF}(\star,\star) : (\mathbb{R}^n, \mathbb{S}_+^n) \to (\mathbb{R}^n, \mathbb{S}_+^n)$$

to represent state/covariance update via Kalman filter (3)-(4), i.e.,

$$(\hat{x}_{k,i}^{KF}, P_{k,i}^{KF}) = \mathrm{KF}(\hat{x}_{k-1,i}, P_{k-1,i})$$

where measurement $y_{k,i}$ is omitted in the above notation just for presentation brevity.

In the second stage, the information fusion strategy based on covariance intersection is employed, where each sensor node interacts with its neighboring nodes, exchanging their local estimates $\hat{x}_{k,i}^{KF}$ and the corresponding error covariances $P_{k,i}^{KF}$. Such pair of information, $(\hat{x}_{k,i}^{KF}, P_{k,i}^{KF})$, is referred to as the *information pair*. Upon receiving the information pairs from all neighboring sensors, the i-th sensor utilizes the covariance intersection methods to generate the fused estimate $\hat{x}_{k,i}$ as follows:

$$\begin{aligned} P_{k,i} &= \Big(\sum_{j\in N_i} \pi_{ij}(P_{k,j}^{KF})^{-1}\Big)^{-1}, \\ \hat{x}_{k,i} &= P_{k,i}\Big(\sum_{j\in N_i} \pi_{ij}(P_{k,j}^{KF})^{-1}\hat{x}_{k,j}^{KF}\Big). \end{aligned} \tag{5}$$

Similarly, we also define the following shorthand notation

$$\mathrm{CI}_i(\star,\star) : \big(\{\mathbb{R}^n, ..., \mathbb{R}^n\}, \{\mathbb{S}_+^n, ...\mathbb{S}_+^n\}\big) \to (\mathbb{R}^n, \mathbb{S}_+^n)$$

to represent information fusion via covariance intersection (5) as follows:.

$$(\hat{x}_{k,i}, P_{k,i}) = \mathrm{CI}_i\left(\big\{\hat{x}_{k,j}^{KF}\big\}_{j\in N_i}, \big\{P_{k,j}^{KF}\big\}_{j\in N_i}\right).$$

The aforementioned distributed Kalman filter is summarized in Algorithm I, which consists of a Kalman iteration and information exchange steps. The Kalman filter utilizes the locally sensed measurement information to compute the locally optimal estimate. In order to minimize communication and computation overhead among nodes, the cross-correlations between neighboring sensors are disregarded. Hence, the fusion rule employed in this paper is the covariance intersection method, which is known for its robustness in handling unknown correlations among different sources of information.

In this distributed implementation, the sensor nodes do not possess knowledge about the estimates and covariance matrices of their neighbors. Therefore, to enable cooperation among adjacent nodes across the network, all nodes must broadcast their local information pairs $(\hat{x}_{k,i}^{KF}, P_{k,i}^{KF})$ obtained from the local Kalman iteration, which allows for the exchange of information and facilitates the collaborative estimation process.

Due to the power and bandwidth constraints inherent in sensor networks, it is necessary to quantize the local information of each sensor before exchanging the data. It is

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Algorithm I: *Distributed Kalman Filtering*

Consider the state-space model (1)-(2). Initialize $\hat{x}_{1,i} = \bar{x}_1$ and $P_{1,i} = \Sigma_1$ for all nodes $i \in \mathcal{V}$. At each time $k = 2, 3 \cdots$, repeat

Step 1. With the newly collected measurement $y_{k,i}$, intelligent sensors utilize the standard Kalman filter to update the local estimates as follows:

$$(\hat{x}_{k,i}^{KF}, P_{k,i}^{KF}) = \mathrm{KF}(\hat{x}_{k-1,i}, P_{k-1,i}).$$

Step 2. Intelligent sensors fuse the local and neighboring information by utilizing the covariance intersection method as follow

$$(\hat{x}_{k,i}, P_{k,i}) = \mathrm{CI}_i\left(\{\hat{x}_{k,j}^{KF}\}_{j\in N_i}, \{P_{k,j}^{KF}\}_{j\in N_i}\right).$$

important to note that existing literature on quantized filtering (e.g., [24], [38]) primarily focuses on quantizing the information vector, such as measurements, innovations, or estimates. Consequently, these approaches are not applicable when the covariance matrices of the neighboring sensors are also available. Therefore, it becomes crucial to develop an appropriate quantization mechanism specifically tailored for the information pair. Based on this motivation, the objective of this paper is to develop a modified distributed Kalman filtering approach incorporating coding strategies to effectively reduce resource consumption. Subsequently, a comprehensive analysis of its performance is conducted, providing insights into its efficacy and capabilities.

III. Distributed Kalman filtering under coding strategies

In this section, we will introduce the quantization approach for the information pair and propose a distributed filtering with two-bitrate periodic coding strategies.

A. Quantization for information pairs

To begin with, a subtractive b-bit dithered quantizer, denoted as $\mathcal{Q}_b(\cdot)$, is introduced which takes a scalar signal $x \in \mathbb{R}$ and a dither signal $d \in \mathbb{R}$ as input to generate the output as follows:

$$\mathcal{Q}_b(x) = Q_b(x+d) - d,$$

where $Q_b(\cdot)$ is a standard uniform quantizer. More specifically, for any scalar input $x \in \mathbb{R}$ in the interval $[-\zeta, \zeta]$, we have the quantizing function:

$$Q_b(x) = k\Delta_b, \ \left(k - \frac{1}{2}\right)\Delta_b \le x < \left(k + \frac{1}{2}\right)\Delta_b. \quad (6)$$

It is easy to see that the quantization level k can be expressed by a b-bit string and the quantization step Δ_b is given by

$$\Delta_b = \frac{\zeta}{2^{b-1}}.$$

Moreover, we can rewrite the quantized output as follows:

$$Q_b(x) = x + q(x),$$

where $q(x) \in \mathbb{R}$ stands for the quantization error. Based on the input, we know that $q(x)$ is a deterministic variable satisfying the following relationship

$$-\frac{\Delta_b}{2} \le q(x) < \frac{\Delta_b}{2}.$$

For the aforementioned subtractive b-bit dithered quantizer $\mathcal{Q}_b(\cdot)$, the following property is true.

Lemma 1 ([45]): Assume that the dither signal d is a white random process independent from x with the characteristic function, denoted as $\Phi(\cdot)$, satisfying

$$\Phi\left(\frac{\pi 2^b}{\zeta} s\right) = 0, \quad \text{for } s = \pm 1, \pm 2, \ldots$$

Then, the quantization error

$$\varepsilon = \mathcal{Q}_b(x) - x$$

is uniformly distributed on $[-\frac{\Delta_b}{2}, \frac{\Delta_b}{2})$ and independent of the input signal x.

Vector Quantization: In the previous discussion, we have presented the subtractive b-bit dithered quantizer for scalar input, and this approach can be readily extended to the vector input case by considering one quantizer per component. Note that $\hat{x}_{k,i}^{KF}$ shall be transmitted via the communication channel, and thus the quantized signals would be $\mathcal{Q}_b(\hat{x}_{k,i}^{KF})$. Moreover, in light of the results in [2], [45], we conclude that the quantization error of the vector quantizer $\varepsilon_{k,i} = \mathcal{Q}_b(\hat{x}_{k,i}^{KF}) - \hat{x}_{k,i}^{KF}$ satisfies

$$\mathbb{E}\{\varepsilon_{k,i}\} = 0, \ \mathrm{Cov}(\varepsilon_{k,i}) = \frac{\zeta^2}{3 \times 2^{2b}} I \triangleq S_b.$$

Matrix Quantization: As the local sensor lacks prior knowledge of the error covariance matrices $P_{k,i}^{KF}$ associated with the neighboring sensors' $\hat{x}_{k,i}^{KF}$, it becomes necessary to transmit these error covariance matrices in practical applications. One seemingly viable method for matrix quantization is to quantize each component of the matrix individually. However, this component-wise quantization of the matrix fails to accurately evaluate the uncertainty of the estimate and often leads to a loss of semi-positive definiteness. To address these limitations, we propose a matrix quantization approach, referred to as

$$\mathbf{Q}_b(\cdot): \quad \mathbb{S}_+^n \to \mathbb{S}_+^n,$$

for quantizing the positive semi-definite matrices $X \in \mathbb{S}_+^n$. In this case, the off-diagonal coefficients of X are quantized based on the following rules:

$$\left[\mathbf{Q}_b(X)\right]_{ij} = \begin{cases} Q_b(X_{ij}), & i < j \\ \left[\mathbf{Q}_b(X)\right]_{ji}, & i > j \end{cases} \quad (7)$$

with the diagonal ones being

$$\left[\mathbf{Q}_b(X)\right]_{ii} = \left\lceil X_{ii} + \sum_{j=1, j\neq i}^{n} \left|Q_b(X_{ij}) - X_{ij}\right| \right\rceil, \quad (8)$$

where $\left[\mathbf{Q}_b(X)\right]_{ij}$ and X_{ij} represent the (i,j)-th component of matrices $\mathbf{Q}_b(X)$ and X, respectively. $Q_b(\cdot)$ is a standard uniform quantizer defined in (6), and $\lceil \cdot \rceil$ rounds up to nearest quantization level.

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Based on the findings in [15], it has been established that the rounding method applied to diagonal coefficients in (8) ensures a diagonally dominant quantization error matrix. This condition is sufficient to guarantee the semi-positive definiteness of the quantization error matrix. Consequently, it can be asserted that the quantized covariance matrix $\mathbf{Q}_b(X)$ obtained through the matrix quantization process described in (7)-(8) conservatively bounds the error covariance matrix X, which highlights the effectiveness of the proposed matrix quantization method in providing a reliable estimation of the error covariance matrix.

B. Two-bitrate periodic coding

We now propose a two-bitrate periodic coding strategy within the framework depicted in Fig. 1. Building upon the vector and matrix quantization approach discussed earlier, we quantize the information pairs of the sensor nodes using a component-wise b-bit quantizer. Inspired by data compression techniques in signal processing [46], we adopt a two-bitrate periodic transmission mechanism under which the b-bit quantized scalar signal x is divided into two parts: the most significant bits (MSB) and the least significant bits (LSB). These two parts are transmitted separately at odd and even instants, respectively. The MSB represents the bits in a multiple-bit binary string with the largest value, while the LSB represents the bits with the smallest value. For example, in the binary number 01100110, the MSB is 011 (the first three bits), and the LSB is 110 (the last three bits). To refer to the most significant and least significant r bits of an input signal x, we introduce the notation $\mathrm{MSB}_r(x)$ and $\mathrm{LSB}_r(x)$, respectively, where $r \in \mathbb{Z}$.

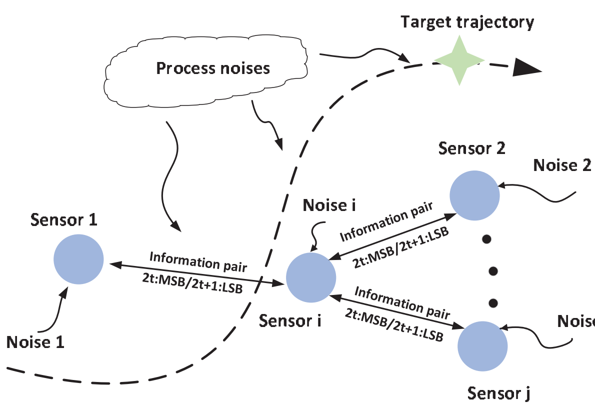

Fig. 1. Distributed Kalman filtering under two-bitrate periodic coding strategies. The most significant bits of the local information pair is transmitted at even instants while the least significant bits of the local information pair is transmitted at odd instants.

For the sake of brevity, we assume that b is an even number. In the quantized vector and matrix, each component's most significant $(b/2+r)$ bits ($r \in \mathbb{Z}$ and $0 \le r \le b/2$) are transmitted at even instants. Specifically, when $k = 2t$, given the prior information $(\hat{x}_{2t-1,i}, P_{2t-1,i})$ and the real-time measurement $y_{2t,i}$, the intelligent sensor updates the local estimates according to the Kalman filter:

$$(\hat{x}_{2t,i}^{KF}, P_{2t,i}^{KF}) = KF(\hat{x}_{2t-1,i}, P_{2t-1,i}). \tag{9}$$

We utilize $p_{2t,i}$ and $\mathscr{P}_{2t,i}$ to represent the $(b/2+r)$-bit transmitted messages as follows:

$$\begin{aligned} p_{2t,i} &= \mathrm{MSB}_{b/2+r}\left\{Q_b\big(\hat{x}_{2t,i}^{KF} + d_{2t,i}\big)\right\}, \\ \mathscr{P}_{2t,i} &= \mathrm{MSB}_{b/2+r}\left\{\mathbf{Q}_b\big(P_{2t,i}^{KF}\big)\right\}, \end{aligned}$$

where the dither $d_{2t,i}$ is a white random process independent from $\hat{x}_{2t,i}^{KF}$ with a probability density possessing characteristic function $\Phi(\cdot)$ satisfying

$$\Phi\Big(\frac{\pi 2^b}{\zeta}s\Big) = 0, \quad \text{for } s = \pm 1, \pm 2, \ldots$$

For the i-th node, we denote

$$m_{2t,j} = \mathscr{D}(p_{2t,j}), \quad M_{2t,j} = \mathscr{D}(\mathscr{P}_{2t,j}),$$

for $j \in N_i$ and $j \neq i$, where $\mathscr{D}(\cdot)$ transfers the binary strings to decimal. $m_{2t,i}$ and $M_{2t,i}$ represent the reconstructed channel output. As for $j = i$, we have that $m_{2t,i} = \hat{x}_{2t,i}^{KF}$ and $M_{2t,i} = P_{2t,i}^{KF}$ because each node could get the accurate local information pair. After that, the covariance intersection method is applied with respect to the reconstructed information pair $(m_{2t,j}, M_{2t,j})$ as follows:

$$(\hat{x}_{2t,i}^{*}, P_{2t,i}^{*}) = \mathrm{CI}_i\Big(\{m_{2t,j}\}_{j\in N_i}, \{M_{2t,j} + S_{b,j}\}_{j\in N_i}\Big)$$

where $S_{b,j}$ is equal to S_b for $j \neq i$, and 0 otherwise.

Next, consider the scenario that the least significant $(b/2-r)$ bits ($r \in \mathbb{Z}$ and $0 \le r \le b/2$) are transmitted at odd instants. That is, when $k = 2t+1$, we have

$$\begin{aligned} p_{2t+1,i} &= \mathrm{LSB}_{b/2-r}\left\{Q_b\big(\hat{x}_{2t,i}^{KF} + d_{2t,i}\big)\right\}, \\ \mathscr{P}_{2t+1,i} &= \mathrm{LSB}_{b/2-r}\left\{\mathbf{Q}_b\big(P_{2t,i}^{KF}\big)\right\}. \end{aligned}$$

With both the most significant and least significant bits, the received information from the neighboring nodes, i.e., $j \in N_i$ and $j \neq i$, can be reconstructed as follows:

$$\begin{aligned} m_{2t+1,j} &= \mathscr{D}\left(p_{2t,i} + 2^{-(b/2-r)} p_{2t+1,j}\right) - d_{2t,j}, \\ M_{2t+1,j} &= \mathscr{D}\left(\mathscr{P}_{2t,j} + 2^{-(b/2-r)} \mathscr{P}_{2t+1,j}\right), \end{aligned} \tag{10}$$

where $s_1 + 2^{-(b/2-r)} s_2$ concatenates two binary numbers with the lower $(b/2-r)$ bit as s_2. Moreover, we have $m_{2t+1,i} = \hat{x}_{2t+1,i}^{KF}$ and $M_{2t+1,i} = P_{2t+1,i}^{KF}$. Subsequently, the covariance intersection method is applied to fuse the updated neighboring information as follows:

$$\begin{aligned} &(\hat{x}_{2t+1,i}^{*}, P_{2t+1,i}^{*}) \\ &= \mathrm{CI}_i\Big(\{m_{2t+1,j}\}_{j\in N_i}, \{M_{2t+1,j} + S_{b,j}\}_{j\in N_i}\Big), \end{aligned} \tag{11}$$

Finally, each node utilizes the newly collected measurement $y_{2t+1,i}$ to further update the local estimate

$$(\hat{x}_{2t+1,i}, P_{2t+1,i}) = KF(\hat{x}_{2t+1,i}^{*}, P_{2t+1,i}^{*}).$$

The distributed Kalman filtering under two-bitrate periodic coding strategies can be summarized in Algorithm II. The fundamental concept behind these strategies is to effectively manage the quantization error by assigning a specific number of transmission bits in a periodic manner. The most significant bits, which carry crucial information, are transmitted first to provide an initial rough state estimate. Subsequently, the reception of the least significant bits allows for accurate reconstruction of the quantized signals, leading to improved estimation quality.

Algorithm II: *Distributed Kalman Filtering Under Two-Bitrate Periodic Coding Strategies*

Consider the state-space model (1)-(2). Initialize $\hat{x}_{1,i} = \bar{x}_1$ and $P_{1,i} = \Sigma_1$ for all nodes $i \in \mathcal{V}$. At each time $k = 2, 3 \cdots$, repeat:

For the even instants $k = 2t$:
Step 1. The standard Kalman filter is utilized to update the local estimates as follows::

$$(\hat{x}^{KF}_{2t,i}, P^{KF}_{2t,i}) = KF(\hat{x}_{2t-1,i}, P_{2t-1,i}) \tag{12}$$

Step 2. The covariance intersection method is adopted as follow

$$(\hat{x}_{2t,i}, P_{2t,i}) = \mathrm{CI}_i\left(\{m_{2t,j}\}_{j\in N_i}, \{M_{2t,j} + S_{b,j}\}_{j\in N_i}\right)$$

For the odd instants $k = 2t + 1$
Step 1. The covariance intersection method is adopted as follow

$$(\hat{x}^*_{2t+1,i}, P^*_{2t+1,i}) = \mathrm{CI}_i\left(\{m_{2t+1,j}\}_{j\in N_i}, \{M_{2t+1,j} + S_{b,j}\}_{j\in N_i}\right) \tag{13}$$

Step 2. The standard Kalman filter is utilized to update the local estimates as follows:

$$(\hat{x}_{2t+1,i}, P_{2t+1,i}) = KF(\hat{x}^*_{2t+1,i}, P^*_{2t+1,i})$$

IV. Performance analysis

In this section, we will demonstrate some fundamental properties of the proposed distributed Kalman filtering under two-bitrate periodic coding strategies. Specifically, we will establish that the proposed filter yields an unbiased and consistent estimate. Additionally, we will show that the dynamics of the estimation error are bounded, ensuring the stability of the estimation process.

A. Unbiasedness

To begin with, we will demonstrate that the proposed filter, with the vector/matrix quantization approach and two-bitrate periodic coding strategies, is unbiased for all sensor nodes.

Theorem 1: Given the dynamical system (1)-(2) and the proposed distributed filter in Algorithm II, the estimates $\hat{x}_{k,i}$ are unbiased for each sensor node at instants $k \in \mathbb{Z}_+$, i.e.,

$$\mathbb{E}\{\hat{x}_{k,i} - x_k\} = 0, \quad i \in \mathcal{V} \tag{14}$$

Proof: We will proceed by applying the induction method on k. For $k = 1$, the conclusion (14) follows immediately from the initial condition $\hat{x}_{1,i} = \bar{x}_1$, for $i \in \mathcal{V}$. Letting the estimates of all the sensors be unbiased at instant $k = 2t - 1$, i.e., $\mathbb{E}\{\hat{x}_{2t-1,i} - x_{2t-1}\} = 0$, it remains to show that the unbiasedness also holds for $k = 2t + 1$.

According to (5) and (11), it follows that

$$P^*_{2t+1,i}\left(\sum_{j\in N_i} \pi_{ij}\left(M_{2t+1,j} + S_{b,j}\right)^{-1}\right) = 1.$$

Therefore, one has

$$\begin{aligned}&\hat{x}^*_{2t+1,i} - x_{2t} \\ &= P^*_{2t+1,i}\left(\sum_{j\in N_i} \pi_{ij}\left(M_{2t+1,j} + S_{b,j}\right)^{-1}(m_{2t+1,j} - x_{2t})\right).\end{aligned} \tag{15}$$

Moreover, it has been shown in (10) that, at instant $2t+1$, the most significant and least significant bits of quantized signal $\hat{x}^{KF}_{2t,j} + d_{2t,j}$ have been received and the dither subtraction has also been done. From the properties of the dither quantizer, we can rewrite $m_{2t+1,j}$ for $j \in N_i$ and $j \neq i$ as follows:

$$m_{2t+1,j} = \hat{x}^{KF}_{2t,j} + \varepsilon_{2t,j} \tag{16}$$

where $\varepsilon_{2t,j} \in \mathbb{R}^n$ is the quantization error with every component i.i.d. uniformly distributed on $[-\frac{\Delta_b}{2}, \frac{\Delta_b}{2})$ and independent of the input sequence $\hat{x}^{KF}_{2t,j}$ in light of Lemma 1.

According to (9), the individual estimate $\hat{x}^{KF}_{2t,i}$ can be rearranged as follows:

$$\begin{aligned}\hat{x}^{KF}_{2t,i} &= A\hat{x}_{2t-1,i} - K_{2t,i}(y_{2t,i} - C_iA\hat{x}_{2t-1,i}) \\ &= A\hat{x}_{2t-1,i} - K_{2t,i}C_iA(x_{2t-1,i} - \hat{x}_{2t-1,i}) \\ &\quad - K_{2t,i}(C_iw_{2t-1} + v_{2t,i}),\end{aligned}$$

where the second equality follows directly from substituting (1)-(2). Noting the facts that 1) $x_{2t-1,i}$ is an unbiased estimate of x_{2t-1} and 2) the process/measurement noises w_{2t-1} and $v_{2t,i}$ are of zero mean, we have

$$\mathbb{E}\left\{\hat{x}^{KF}_{2t,i} - A\hat{x}_{2t-1,i}\right\} = 0. \tag{17}$$

From (16)-(17), it is straightforward to see that

$$\begin{aligned}\mathbb{E}\{m_{2t+1,j} - x_{2t}\} &= \mathbb{E}\{\hat{x}^{KF}_{2t,j} + \varepsilon_{2t,j} - x_{2t}\} \\ &= \mathbb{E}\{A\hat{x}_{2t-1,i} - Ax_{2t-1}\} = 0.\end{aligned}$$

Substituting the above equality into (15) yields $\mathbb{E}\{\hat{x}^*_{2t+1,i} - x_{2t}\} = 0$. Furthermore, the relationship $\mathbb{E}\{\hat{x}_{2t+1,i} - A\hat{x}^*_{2t+1,i}\} = 0$ can be proven by noting that

$$\begin{aligned}\hat{x}_{2t+1,i} &= A\hat{x}^*_{2t+1,i} - K_{2t+1,i}(y_{2t+1,i} - C_iA\hat{x}^*_{2t+1,i}) \\ &= A\hat{x}^*_{2t+1,i} - K_{2t+1,i}C_iA(x_{2t,i} - \hat{x}^*_{2t+1,i}) \\ &\quad - K_{2t+1,i}(C_iw_{2t} + v_{2t+1,i})\end{aligned}$$

As a consequence, we have

$$\mathbb{E}\{\hat{x}_{2t+1,i} - x_{2t+1}\} = \mathbb{E}\{A\hat{x}^*_{2t+1,i} - Ax_{2t}\} = 0,$$

which ends the proof. ■

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B. Consistency

In this subsection, we will investigate the consistency of the proposed distributed filter. To begin, let us introduce the definition of consistency.

Definition 1: Let $\hat{x}$ be an unbiased estimate of the random vector x and P be an estimate of the corresponding error covariance. The information pair $(\hat{x}, P)$ is said to be consistent if the following relationship holds: $\mathbb{E}\{(x-\hat{x})(x-\hat{x})'\} \leq P$.

As per the definition stated above, consistency implies that the estimate is unbiased and the reported error covariance serves as an upper bound for the actual error covariance. It is important to note that the distributed networks under consideration provide suboptimal estimates, as each node does not rely on cross-covariance matrices, and the received information pairs inevitably suffer from inaccuracies due to quantization errors. Therefore, maintaining consistency of the estimates is crucial for evaluating the system's performance, as inconsistency can potentially lead to issues such as divergence.

Before proceeding further, we introduce a useful lemma that confirms the conservative bounding property of the quantized covariance matrix with respect to the input covariance matrix.

Lemma 2: ([15]) For any $X \in \mathbb{S}_+^n$, the matrix quantizer $\mathbf{Q}_b(X)$ defined in (7)-(8) has a positive semidefinite quantization error matrix $\Lambda_b(X) \triangleq \mathbf{Q}_b(X) - X \geq 0$ and therefore provides a conservative upper bound for the input matrix, i.e., $\mathbf{Q}_b(X) \geq X$ holds for all $X \in \mathbb{S}_+^n$.

To facilitate the subsequent analysis, we need to introduce two functions $h(\cdot)$ and $g(\cdot)$: $\mathbb{S}_+^n \to \mathbb{S}_+^n$ as follows:

$$
\begin{aligned}
h(X) &\triangleq AXA' + Q \\
g(X) &\triangleq X - XC'(CXC'+R)^{-1}CX
\end{aligned}
$$

The actual estimate/prediction error covariances of the estimate $\hat{x}_{k,i}$ are denoted by

$$
\begin{aligned}
P_{k,i}^{act} &\triangleq \mathbb{E}\{(x_k - \hat{x}_{k,i})(x_k - \hat{x}_{k,i})'\} \\
P_{k|k-1,i}^{act} &\triangleq \mathbb{E}\{(x_k - \hat{x}_{k|k-1,i})(x_k - \hat{x}_{k|k-1,i})'\}
\end{aligned}
$$

We are now ready to demonstrate that, at each iteration, if the prior estimate is consistent, then the updated estimate obtained using the Kalman filter with the latest collected measurement will also be consistent.

Lemma 3: For any consistent estimate $(\hat{x}_{k,i}, P_{k,i})$ of the state vector x_k, the iterative update via Kalman filter (3)-(4), i.e., $(\hat{x}_{k+1,i}, P_{k+1,i}) = KF(\hat{x}_{k,i}, P_{k,i})$, also provides a consistent estimate $(\hat{x}_{k+1,i}, P_{k+1,i})$ of the state vector x_{k+1}.

Proof: The i-th sensor updates the local estimate based on the measurements $y_{k+1,i}$ according to the Kalman filter

$$(\hat{x}_{k+1,i}, P_{k+1,i}) = KF(\hat{x}_{k,i}, P_{k,i}).$$

Obviously, the dynamics of the actual estimate error can be calculated as

$$
\begin{aligned}
&\hat{x}_{k+1,i} - x_{k+1} \\
&= \hat{x}_{k+1|k,i} - x_{k+1} + K_{k+1,i}(y_{k+1,i} - C_i\hat{x}_{k+1|k,i}) \\
&= (I - K_{k+1,i}C_i)(\hat{x}_{k+1|k,i} - x_{k+1}) + K_{k+1,i}v_{k+1,i},
\end{aligned}
$$

with the corresponding actual estimation error covariance given by

$$
\begin{aligned}
&P_{k+1,i}^{act} \\
&= (I - K_{k+1,i}C_i)\mathbb{E}\{(\hat{x}_{k+1|k,i} - x_{k+1})(\hat{x}_{k+1|k,i} - x_{k+1})'\} \\
&\quad \times (I - K_{k+1,i}C_i)' + K_{k+1,i}R_iK_{k+1,i}' \\
&= (I - K_{k+1,i}C_i)P_{k+1|k,i}^{act}(I - K_{k+1,i}C_i)' \\
&\quad + K_{k+1,i}R_iK_{k+1,i}'.
\end{aligned} \tag{18}
$$

Since $(\hat{x}_{k,i}, P_{k,i})$ is a consistent estimate of the state vector x_k, one has $P_{k,i}^{act} \leq P_{k,i}$. Noting that $h(\cdot)$ is monotonically increasing function, we have

$$P_{k+1|k,i}^{act} = h(P_{k,i}^{act}) \leq h(P_{k,i}) = P_{k+1|k,i}.$$

Substituting the above into (18), it is apparent that

$$
\begin{aligned}
P_{k+1,i}^{act} \leq &(I - K_{k+1,i}C_i)P_{k+1|k,i}(I - K_{k+1,i}C_i)'. \\
&+ K_{k+1,i}R_iK_{k+1,i}'
\end{aligned}
$$

Recalling the definition of $K_{k+1,i}$ in (4), i.e.,

$$K_{k+1,i} = P_{k+1|k,i}C_i'(C_iP_{k+1|k,i}C_i' + R_i^{-1})^{-1},$$

we have

$$P_{k+1,i}^{act} \leq g(P_{k+1|k,i}) = g \circ h(P_{k,i}) = P_{k+1,i}^{KF}$$

Furthermore, by noting that Kalman filter is an unbiased estimator, it is easy to see that $\hat{x}_{k+1,i}$ is an unbiased estimate of x_{k+1}. As a consequence, we can draw the conclusion that $(\hat{x}_{k+1,i}, P_{k+1,i})$ is a consistent estimate of x_{k+1}, which completes the proof. ■

The following theorem demonstrates that, regardless of the cross-covariance between neighboring sensors, the proposed distributed Kalman filtering under two-bitrate periodic coding strategies ensures the consistency of the fused estimate.

Theorem 2: Given the dynamical system (1)-(2) and the proposed distributed filter in Algorithm II, the information pair $(\hat{x}_{k,i}, P_{k,i})$ is a consistent estimate of the state vector x_k (for $k \in \mathbb{Z}_+$) in that

$$\mathbb{E}\{(\hat{x}_{k,i} - x_k)(\hat{x}_{k,i} - x_k)'\} \leq P_{k,i}, \quad \forall i \in \mathcal{V},\ k \in \mathbb{Z}_+. \tag{19}$$

Proof: The consistency will be proven via the induction method. The initial conditions $\hat{x}_{1,i} = \bar{x}_1$ and $P_{1,i} = \Sigma_1$ (for $i \in \mathcal{V}$) guarantee that $\mathbb{E}\{(\hat{x}_{1,i} - x_1)(\hat{x}_{1,i} - x_1)'\} = P_{1,i}$. Suppose that, at instant $k = 2t-1$, the information pair $(\hat{x}_{2t-1,i}, P_{2t-1,i})$ is a consistent estimate of the state vector x_{2t-1}, i.e.,

$$P_{2t-1,i}^{act} = \mathbb{E}\{(\hat{x}_{2t-1,i} - x_{2t-1})(\hat{x}_{2t-1,i} - x_{2t-1})'\} \leq P_{2t-1,i} \tag{20}$$

for $i \in \mathcal{V}$. Now, we would like to show that $(\hat{x}_{2t+1,i}, P_{2t+1,i})$ is a consistent estimate of the state vector x_{2t+1}.

At time instant $k = 2t$, the sensors first update their information based on their local measurements $y_{2t,i}$ via the Kalman filter as follows:

$$(\hat{x}_{2t,i}^{KF}, P_{2t,i}^{KF}) = KF(\hat{x}_{2t-1,i}, P_{2t-1,i}).$$

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Applying the result of Lemma 3, we can easily see that $(\hat{x}^{KF}_{2t,i}, P^{KF}_{2t,i})$ is a consistent estimate of the state vector x_{2t}, i.e.,

$$P^{act}_{2t,i} \leq P^{KF}_{2t,i}.$$

In the subsequent analysis, we will concentrate on the information interaction and processing at the i-th sensor. By examining the definition of (10), it is evident that the i-th sensor has received all the transmitted bits of $Q_{2b}(\hat{x}^{KF}_{2t,j}+d_{2t,j})$ and subsequently subtracted the dither signals $d_{2t,j}$ at time instant $k = 2t + 1$. As a result, $m_{2t+1,j}$ can be expressed as follows:

$$m_{2t+1,j} = \hat{x}^{KF}_{2t,j} + q_{2t,j},$$

where $q_{2t,j}$ is the quantization error. It is easy to see that $q_{2t,i} = 0$ as $m_{2t+1,i} = \hat{x}^{KF}_{2t,i}$. As for $j \in N_i/\{i\}$, based on Lemma 1, we conclude that each component of $q_{2t,j}$ is i.i.d. uniformly distributed and independent of the input sequence $\hat{x}^{KF}_{2t,j}$. Therefore, one has

$$\mathrm{Cov}(m_{2t+1,j}) = \mathrm{Cov}(\hat{x}^{KF}_{2t,j}) + S_{b,j} \leq P^{KF}_{2t,j} + S_{b,j}. \quad (21)$$

Moreover, it can be derived from (10) that

$$M_{2t+1,j} = \mathscr{D}\left(\mathscr{P}_{2t,j} + 2^{-(b/2-r)}\mathscr{P}_{2t+1,j}\right) = \mathbf{Q}_b\left(P^{KF}_{2t,j}\right).$$

According to Lemma 2, it is ensured that the quantized covariance matrix $\mathbf{Q}_{2b}(X)$ conservatively bounds the error covariance matrix X, which implies that

$$P^{KF}_{2t,j} \leq \mathbf{Q}_{2b}(P^{KF}_{2t,j}) = M_{2t+1,j}.$$

Therefore, one has

$$\mathrm{Cov}(m_{2t+1,j}) \leq M_{2t+1,j} + S_{2b,j}, \quad j \in N_i/\{i\},$$

which further indicates that the received information pair from j-th sensor $(m_{2t+1,j}, M_{2t+1,j} + S_{b,j})$ is indeed a consistent estimate of x_{2t}.

Noticing that

$$m_{2t+1,i} = \hat{x}^{KF}_{2t,i},\ M_{2t+1,i} = P^{KF}_{2t,i},\ S_{b,i} = 0$$

holds for $j = i$, the pair $(m_{2t+1,i}, M_{2t+1,i} + S_{2b,i})$ is also a consistent estimate of x_{2t} as $P^{act}_{2t,j} \leq P^{KF}_{2t,j}$.

According to the covariance intersection method (13), we can see

$$P^*_{2t+1,i} = \left(\sum_{j\in N_i} \pi_{ij}(M_{2t+1,j} + S_{b,j})^{-1}\right)^{-1},$$

$$\hat{x}^*_{2t+1,i} = P^*_{2t+1,i}\left(\sum_{j\in N_i} \pi_{ij}(M_{2t+1,j} + S_{b,j})^{-1} m_{2t+1,j}\right).$$

Furthermore, it has been proved in [10] that, if each information pair $(m_{2t+1,j}, M_{2t+1,j}+S_{2b,j})$ of the state vector x_{2t} for $j = N_i$ is consistent, then the covariance intersection fusion method preserves the consistency property, and therefore we have

$$\mathbb{E}\{(\hat{x}^*_{2t+1,i} - x_{2t})(\hat{x}^*_{2t+1,i} - x_{2t})'\} \leq P^*_{2t+1,i},$$

which means that $(\hat{x}^*_{2t+1,i}, P^*_{2t+1,i})$ is a consistent estimate of x_{2t}. To this end, the pair $(\hat{x}_{2t+1,i}, P_{2t+1,i})$ can be calculated via the Kalman filter as follows:

$$(\hat{x}_{2t+1,i}, P_{2t+1,i}) = KF(\hat{x}^*_{2t+1,i}, P^*_{2t+1,i}).$$

Using Lemma 3 once again, we can see that $(\hat{x}_{2t+1,i}, P_{2t+1,i})$ is a consistent estimate of x_{2t+1}, which finally concludes the proof. ■

C. Boundedness

In this subsection, we aim to establish sufficient conditions for the boundedness of the proposed distributed filter. To accomplish this aim, we introduce some preliminary assumptions.

Assumption 3: The system matrix A is invertible.

Assumption 4: The undirected graph $\mathcal{G}$ is connected, i.e. for any pair of vertices $v_i, v_j \in \mathcal{V}$, there exists at least a path from v_i to v_j and vice versa.

Assumption 5: The sensor network is collectively observable, i.e., (A, C) is observable, where $C = [C'_1, C'_2, \cdots, C'_N]'$.

The above assumptions, which are also assumed in [5], [6], are considered to be quite mild for distributed algorithms. The invertibility of the system matrix A is typically guaranteed by discretizing the continuous-time system matrix, making this assumption hold trivially. Assumption 5 imposes a necessary requirement on the connectivity of the sensor networks, as the successful implementation of distributed algorithms relies on the premise that information from every sensor can be disseminated across the network. Regarding Assumption 5, collective observability serves as a fundamental condition that emphasizes the observability of the entire network, regardless of whether individual sub-systems with local measurements are observable or not. By satisfying these assumptions, the distributed filter can operate effectively and achieve reliable estimation results in the sensor network.

Some useful lemmas are presented as follow.

Lemma 4 (Gershgorin's Theorem [18]): All the eigenvalues of the matrix $X \in \mathbb{R}^{n\times n}$ are located in the union of n discs as follows:

$$\bigcup_{i=1}^{n}\left\{z \in \mathbb{C} : |z - X_{ii}| \leq \sum_{j\neq i} |X_{ij}|\right\},$$

where X_{ij} is the (i, j)-th entry of X and $\mathbb{C}$ represents the set of the complex numbers.

Lemma 5: Given the matrix quantization function $\mathbf{Q}_b(\cdot)$, all the eigenvalues of the quantization error matrix, defined as $\Lambda_b(X) \triangleq \mathbf{Q}_b(X) - X$, satisfy the following condition

$$\lambda_k(\Lambda_b(X)) \leq n\Delta_b,$$

for $X \in \mathbb{S}^n_+$ and $k \in \{1, 2, ..., n\}$, where $\lambda_k(X)$ represents the k-th eigenvalue of the matrix X.

Proof: According to the matrix quantization approach (7)-(8), the quantization error matrix $\Lambda_b(X)$ is of the non-diagonal elements

$$[\Lambda_b(X)]_{ij} = Q_b(X_{ij}) - X_{ij},$$

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and the diagonal elements

$$[\Lambda_b(X)]_{ii} = \left\lceil X_{ii} + \sum_{j\neq i} \left| Q_b(X_{ij}) - X_{ij} \right| \right\rceil - X_{ii}$$
$$= \left\lceil X_{ii} + \sum_{j\neq i} \left| [\Lambda_b(X)]_{ij} \right| \right\rceil - X_{ii},$$

In light of Gershgorin's Theorem in Lemma 4, it is clear that

$$|\lambda_k(\Lambda_b(X)) - [\Lambda_b(X)]_{ii}| \leq \sum_{j\neq i} \left| [\Lambda_b(X)]_{ij} \right|$$

holds for $i, k \in \{1, 2, ..., n\}$. Therefore, one has

$$\lambda_k(\Lambda_b(X)) \geq [\Lambda_b(X)]_{ii} - \sum_{j\neq i} \left| [\Lambda_b(X)]_{ij} \right| \geq 0$$

and

$$\lambda_k(\Lambda_b(X)) \leq \sum_{j\neq i} \left| [\Lambda_b(X)]_{ij} \right| + [\Lambda_b(X)]_{ii}$$
$$\leq (n-1)\frac{\Delta_b}{2} + (n-1)\frac{\Delta_b}{2} + \Delta_b = n\Delta_b$$

where the second inequality follows directly from quantization rules, i.e.,

$$|Q_b(X_{ij}) - X_{ij}| \leq \frac{\Delta_b}{2}.$$

The proof is now complete. ■

The boundedness of the distributed filter is presented in the following theorem.

Theorem 3: Given the dynamical system (1)-(2) and the proposed distributed filter in Algorithm II, under the Assumptions 1-5, there exist bounded positive definite matrices $\mathscr{P}_i$ such that

$$P_{k,i} < \mathscr{P}_i, \quad \forall i \in \mathcal{V}, \ k \in \mathbb{Z}_+$$

Consequently, the mean-square estimation error of the sensor network is always bounded, i.e.,

$$\sup_{k\in\mathbb{Z}} \mathbb{E}\Big\{(\hat{x}_{k,i} - x_k)'(\hat{x}_{k,i} - x_k)\Big\} < \infty. \tag{22}$$

Proof: To establish the boundedness of the estimation covariance, we will employ the inductive method. First, let us examine the initial condition, which ensures that $P_{1,i} = \Sigma_1 < \infty$ holds for $i \in \mathcal{V}$. Suppose that the reported estimation covariance is bounded at instant $k = 2t-1$, i.e., $P_{2t-1,i} < \infty$. In light of the time-update stage of the Kalman filter (3), one has

$$P^{-1}_{2t|2t-1,i}$$
$$= (AP_{2t-1,i}A' + Q)^{-1}$$
$$= (A^{-1})'(P_{2t-1,i} + A^{-1}Q(A^{-1})')^{-1}A^{-1}.$$

Since $P_{2t-1,i} < \infty$, there always exists a positive scalar $\beta_1 > 0$ such that

$$A^{-1}Q(A^{-1})' \leq \beta_1 P_{2t-1,i}.$$

By denoting $\gamma_1 = (1+\beta_1)^{-1}$, it is straightforward to derive that

$$P^{-1}_{2t|2t-1,i} \geq \gamma_1(A^{-1})'P^{-1}_{2t-1,i}A^{-1}.$$

According to the measurement-update of the Kalman filter (4), we have

$$\begin{aligned}\left(P^{KF}_{2t,i}\right)^{-1} &= P^{-1}_{2t-1|t,i} + C_iR_i^{-1}C_i' \\ &\geq \gamma_1(A^{-1})'P^{-1}_{2t-1,i}A^{-1} + C_iR_i^{-1}C_i'.\end{aligned} \tag{23}$$

At instant $k = 2t+1$, we have from (13) that

$$P^*_{2t+1,i} = \Big(\sum_{j\in N_i} \pi_{ij}(M_{2t+1,j} + S_{b,j})^{-1}\Big)^{-1}, \tag{24}$$

where the received matrix $M_{2t+1,j}$ can be rewritten into the following form

$$M_{2t+1,j} = \mathbf{Q}_b\left(P^{KF}_{2t,j}\right) = P^{KF}_{2t,j} + \Lambda_b\left(P^{KF}_{2t,j}\right).$$

Based on Lemma 5, it is clear that the quantization error matrix $\Lambda_b\left(P^{KF}_{2t,j}\right)$ is bounded, and therefore there always exists a positive scalar $\beta_2 > 0$ such that

$$\Lambda_b\left(P^{KF}_{2t,j}\right) + S_{b,j} \leq \beta_2 P^{KF}_{2t,j},$$

holds, and thus we have

$$P^{KF}_{2t,j} + \Lambda_b\left(P^{KF}_{2t,j}\right) + S_{b,j} \leq (1+\beta_2)P^{KF}_{2t,j}.$$

Denoting $\gamma_2 = (1+\beta_2)^{-1}$, we can rearrange (24) as follows:

$$\begin{aligned}&\left(P^*_{2t+1,i}\right)^{-1} \\ &= \sum_{j\in N_i, j\neq i} \pi_{ij}\left(P^{KF}_{2t,j} + \Lambda_b\left(P^{KF}_{2t,j}\right) + S_{b,j}\right)^{-1} + \pi_{ii}(P^{KF}_{2t,i})^{-1} \\ &\geq \gamma_2 \sum_{j\in N_i} \pi_{ij}(P^{KF}_{2t,j})^{-1}.\end{aligned}$$

Substituting (23) into the above equation yields

$$\begin{aligned}\left(P^*_{2t+1,i}\right)^{-1} \geq &\gamma_1\gamma_2 \sum_{j\in N_i} \pi_{ij}(A^{-1})'P^{-1}_{2t-1,j}A^{-1} \\ &+ \gamma_2 \sum_{j\in N_i} \pi_{ij}C_jR_j^{-1}C_j'.\end{aligned} \tag{25}$$

Furthermore, the following inequality is satisfied

$$\begin{aligned}&(P_{2t+1,i})^{-1} \\ &\geq \gamma_3(A^{-1})'\left(P^*_{2t+1,i}\right)^{-1}(A^{-1}) + C_i(R_i^{-1})^{-1}C_i',\end{aligned} \tag{26}$$

where $\gamma_3 > 0$ is a positive scalar. By denoting $\gamma = \min(\gamma_1\gamma_2\gamma_3, \gamma_2\gamma_3)$ and substituting (25) , one can rewrite (26) as follows:

$$\begin{aligned}&(P_{2t+1,i})^{-1} \\ &\geq \gamma \sum_{j\in N_i} \pi_{ij}(A^{-2})'P^{-1}_{2(t-1)+1,j}A^{-2} \\ &\quad + \gamma \sum_{j\in N_i} \pi_{ij}(A^{-1})'C_jR_j^{-1}C_j'(A^{-1}) + C_iR_i^{-1}C_i' \\ &\geq \gamma^2 \sum_{j=1}^{N}\sum_{s=1}^{N} \pi_{ij}\pi_{js}(A^{-4})'P^{-1}_{2(t-2)+1,s}A^{-4}\end{aligned}$$

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$$+\gamma^2\sum_{j=1}^{N}\sum_{s=1}^{N}\pi_{ij}\pi_{js}(A^{-3})'C_sR_s^{-1}C_s'A^{-3}$$
$$+\gamma\sum_{j\in N_i}\pi_{ij}(A^{-2})'C_jR_j^{-1}C_j'A^{-2}$$
$$+\gamma\sum_{j\in N_i}\pi_{ij}(A^{-1})'C_jR_j^{-1}C_j'(A^{-1})+C_iR_i^{-1}C_i'.$$

For representation convenience, we denote $\pi_{ij}^{(\kappa)}$ as the (i,j)-th entry of $\mathcal{S}^\kappa$, which is the successive multiplication of the weighted matrix $\mathcal{S}$ for κ times. Subsequently, via a series of mathematical manipulations, one has

$$\sum_{j=1}^{N}\sum_{s=1}^{N}\pi_{ij}\pi_{js}(A^{-4})'P_{2(t-2)+1,s}^{-1}A^{-4}$$
$$=\sum_{j=1}^{N}\pi_{ij}^{(2)}(A^{-4})'P_{2(t-2)+1,j}^{-1}A^{-4}$$

and therefore

$$P_{2t+1,i}^{-1}$$
$$\geq\gamma^2\sum_{j=1}^{N}\pi_{ij}^{(2)}(A^{-4})'P_{2(t-2)+1,j}^{-1}A^{-4}$$
$$+\gamma^2\sum_{j=1}^{N}\pi_{ij}^{(2)}(A^{-3})'C_jR_j^{-1}C_j'A^{-3}$$
$$+\gamma\sum_{j=1}^{N}\pi_{ij}(A^{-2})'C_jR_j^{-1}C_j'A^{-2}$$
$$+\gamma\sum_{j=1}^{N}\pi_{ij}(A^{-1})'C_jR_j^{-1}C_j'(A^{-1})+C_iR_i^{-1}C_i'$$

By recursively applying the above inequality κ times, we arrive at

$$(P_{2t+1,i})^{-1}$$
$$\geq\gamma^\kappa\sum_{j=1}^{N}\pi_{ij}^{(\kappa)}(A^{-2\kappa})'P_{2(t-\kappa)+1,j}^{-1}A^{-2\kappa}$$
$$+\sum_{l=1}^{2\kappa}\sum_{j=1}^{N}\gamma^{\kappa-\lfloor\frac{l}{2}\rfloor}\pi_{ij}^{(\kappa-\lfloor\frac{l}{2}\rfloor)}(A^{-2\kappa+l})'C_jR_j^{-1}C_j'A^{-2\kappa+l}.$$

In fact, when the communication topology is connected, we have that the weighted matrix $\mathcal{S}$ is primitive, and all the entries $\pi_{ij}^{(s)}$ are positive for s greater than a certain $\epsilon\in\mathbb{Z}$ [29]. Therefore, by choosing $\kappa>\epsilon+n$, it can be verified that

$$P_{2t+1,i}^{-1}$$
$$\geq\sum_{l=1}^{2(\kappa-\epsilon-n)}\sum_{j=1}^{N}\gamma^{\kappa-\lfloor\frac{l}{2}\rfloor}\pi_{ij}^{(\kappa-\lfloor\frac{l}{2}\rfloor)}(A^{-2\kappa+l})'C_jR_j^{-1}C_j'A^{-2\kappa+l}.$$

Since $\gamma^{\kappa-\lfloor\frac{l}{2}\rfloor}>0$ and $\pi_{ij}^{(\kappa-\lfloor\frac{l}{2}\rfloor)}>0$, for $l\in\{1,2,\cdots,2(\kappa-\epsilon-n)\}$, we can see that, with the collective observability, the following condition holds

$$P_{2t+1,i}^{-1}\geq\mathscr{P}_i^{-1}>0,\quad\text{for } i\in\mathcal{V}.$$

where

$$\mathscr{P}_i^{-1}=\sum_{l=1}^{2(\kappa-\epsilon-n)}\sum_{j=1}^{N}\gamma^{\kappa-\lfloor\frac{l}{2}\rfloor}\pi_{ij}^{(\kappa-\lfloor\frac{l}{2}\rfloor)}$$
$$\times(A^{-2\kappa+l})'C_jR_j^{-1}C_j'A^{-2\kappa+l}.$$

As a consequence, according to the consistency property presented in Theorem 2, we have that

$$\mathbb{E}\{(\hat{x}_{2t+1,i}-x_{2t+1})(\hat{x}_{2t+1,i}-x_{2t+1})'\}\leq P_{2t+1,i}\leq\mathscr{P}_i<\infty,$$

which also indicates that the mean-square estimation error of the distributed filter (22) is bounded at even times. Moreover, it can be trivially verified that the estimate at odd time is also bounded as the error dynamics would not tend to infinity in a finite step. The proof is now complete. ■

Remark 1:

It is worth pointing out that β_2 is associated with the quantization error. A larger quantization error result in an increase in β_2, causing a decrease in γ, ultimately leading to an increase in the upper bound $\mathscr{P}_i$.

Remark 2: In this paper, we have dealt with the problem of distributed Kalman filtering over sensor networks under two-bitrate periodic coding strategies. Compared to the rich literature on communication-protocol-based filter design, our main results stand out by exhibiting the following distinguishing feature: 1) a novel approach is developed for distributed Kalman filtering in sensor networks by utilizing two-bitrate periodic coding strategies which effectively reduce data transmission and resource requirements while preserving estimation accuracy; 2) our proposed filter consistently provides unbiased estimates, ensuring that the reported error covariance always bounds the actual error covariance, which allows for real-time assessment of estimation accuracy; and 3) conditions are established for the boundedness of mean-square estimation errors by taking into account network connectivity and collective observability, thereby providing insights into the stability and reliability of the distributed estimation process. In summary, our work offers a novel approach to distributed Kalman filtering through addressing the challenges of resource-constrained sensor networks. By leveraging two-bitrate periodic coding strategies, we achieve accurate and efficient estimation while considering the network's connectivity and observability.

V. Simulation Example

In this section, we present a numerical example to evaluate the performance of the proposed information-weighted distributed filtering approach with low-bitrate coding strategies.

We consider a scenario where a group of heterogeneous intelligent sensors collaboratively tracks the trajectory of a moving target plant. The dynamics of the plant are modeled using a discrete-time constant acceleration model as follows:

$$x_{k+1}=\begin{bmatrix}1 & dt & 0 & 0\\ 0 & 1.01 & 0 & 0\\ 0 & 0 & 1 & dt\\ 0 & 0 & 0 & 1.01\end{bmatrix}x_k+w_k$$

where the state $x_k\in\mathbb{R}^4$ consists of position and velocity along the coordinate axes, $dt=0.01$ is the discretization sampling interval and w_k is a Gaussian disturbance with the covariance $Q = dtI$. Assume that the initial state of the moving target x_0 obeys the Gaussian distribution with mean $[0\ 2\ 0\ 2]'$ and covariance $\mathrm{diag}[1\ 0.13\ 1\ 0.13]$.

In this sensor network, there are a total of 70 nodes. Among these nodes, 10 nodes are capable of measuring the position on the x-axis, 10 nodes can measure the position on the y-axis, 10 nodes can measure the velocity on the x-axis, and 10 nodes can measure the velocity on the y-axis. The remaining 30 nodes have the capability to process and transmit local information but do not have direct measurement capabilities. The observation model for the i-th node can be represented as follows:

$$y_{k,i} = C_i x_k + v_{k,i}$$

where the measurement noise v_k obeys the Gaussian disturbance with zero mean and the covariance $R_i = 1$. The measurement matrices are chosen to be

$$C_i = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}, \text{ for } i = 1, \cdots, 10, \text{ Sensor A}$$
$$C_i = \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}, \text{ for } i = 11, \cdots, 20, \text{ Sensor B}$$
$$C_i = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}, \text{ for } i = 21, \cdots, 30, \text{ Sensor C}$$
$$C_i = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}, \text{ for } i = 31, \cdots, 40, \text{ Sensor D}$$
$$C_i = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}, \text{ for } i = 41, \cdots, 70, \text{ Non-Sensor Node}$$

The sensor nodes in the network are randomly distributed within the square region $[0, 6] \times [0, 6]$. Each node can communicate with its neighboring nodes within a radius of 2 units. The network topology is depicted in Fig. 2, illustrating the connectivity and collective observability of the sensor network. The depicted network topology clearly shows that the sensor nodes are connected, meaning that there is a communication path between any two nodes in the network. Additionally, the network is collectively observable, which implies that the combined information from all nodes can provide sufficient observability of the underlying system. Fig. 2 visually confirms that the network topology satisfies the necessary requirements for effective information exchange and collaborative estimation in the sensor network.

The weighted matrix $\mathcal{S} = [\pi_{ij}]_{N\times N}$ of the network are selected as follows

$$\pi_{ij} = \begin{cases} 1/\mathrm{deg}_i, & (i,j) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases}$$

where deg_i is the degree of the node i. Throughout the simulation, we choose the quantization interval $\zeta = 10$, the quantization bit $2b = 12$ and thus the quantization step $\Delta_{2b} = \frac{\zeta}{2^{2b-1}} = 0.0049$. According to [2], the dither signal $d_{k,i}$ is chosen as a white noise process uniformly distributed on the interval $[-\frac{\Delta_{2b}}{2}, \frac{\Delta_{2b}}{2}]$, which turn out to satisfy characteristic function condition in Lemma 1. Furthermore, the bitrate assignments are carried out with $r = 2$. Therefore, for each quantized entries, at even time the $b + r = 8$ most significant bits are transmitted, while at odd time the $b - r = 4$ least significant bits are transmitted.

To verify that the quantized covariance matrix conservatively bounds the original matrix, we define the minimum eigenvalue of the quantization error matrix as

$$\mathbf{Eig}(i) = \min_{k\in\mathbb{Z}/\mathbb{Z}_+} \min \mathrm{eig}\left(\mathbf{Q}_b(P_{k,i}^{KF}) - P_{k,i}^{KF}\right),$$

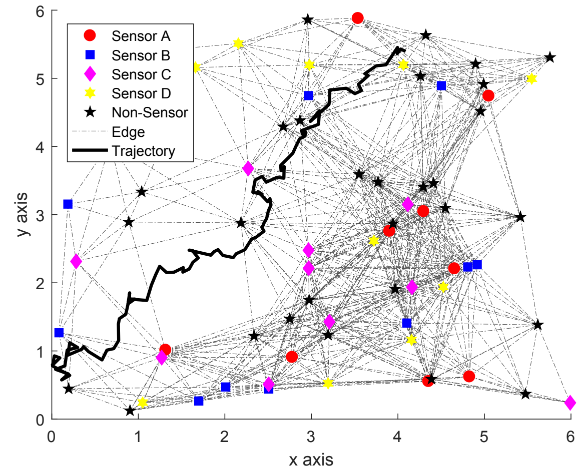

Fig. 2. Sensor networks. Sensors A and Sensor B measure the position of the moving target on the x-axis and y-axis, respectively. Sensors C and Sensor D measure the velocity of the moving target on the x-axis and y-axis, respectively. Non-Sensor Node can only process and transmit the local information.

Now, the value of $\mathbf{Eig}(i)$ for each node at odd instants is presented in TABLE V, from which it is clear to see that all the eigenvalues are larger than zero and therefore $\mathbf{Q}_b(P_{k,i}^{KF}) \geq P_{k,i}^{KF}$, for $k \in \mathbb{Z}/\mathbb{Z}_+$.

TABLE I
THE MINIMUM EIGENVALUE OF THE QUANTIZATION ERROR MATRIX

i-th sensor	1	2	3	4	$\cdots$	70
$\mathbf{Eig}(i)(10^{-3})$	0.0766	0.0947	0.1115	0.0598	$\cdots$	0.0735

Furthermore, the empirical mean-square error (EMSE), averaged over all the nodes, of the proposed distributed filter at per iteration is defined as follow:

$$\mathbf{EMSE}(k) = \frac{1}{NT_{MC}} \sum_{t=1}^{T_{MC}} \sum_{i=1}^{N} \|x_k^{(t)} - \hat{x}_{k,i}^{(t)}\|^2$$

where the superscript "(t)" represents that the value is obtained in the t-th trail experiment. Moreover, the analytical mean-square error (AMSE) of the proposed distributed filter at per iteration is defined as follow:

$$\mathbf{AMSE}(k) = \frac{1}{N} \sum_{i=1}^{N} \mathrm{trace}(P_{k,i})$$

We conduct 1000 independent simulations of the proposed filter using Algorithm 2 and present the simulation results in Fig. 3, which illustrates the behaviors of the EMSE and AMSE at odd instants. From Fig. 3, it can be observed that, despite the lack of local observability in the sensor network, the proposed distributed filter consistently provides accurate estimates with bounded mean-square estimation errors. Moreover, we can see that the AMSE, which represents the analytical error covariance, is always an upper bound of the EMSE, indicating that the reported error covariance is a reliable estimate of the true estimation accuracy.

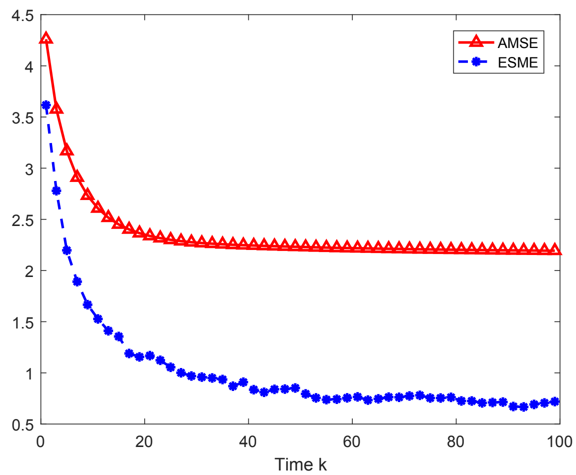

Fig. 3. The behaviors of the $\mathbf{EMSE}(k)$ and $\mathbf{AMSE}(k)$ at odd instants.

VI. Conclusion

In this paper, a novel distributed Kalman filter has been presented for estimating linear discrete-time systems in sensor networks. The proposed filter has combined local Kalman filter updates with information fusion using the covariance intersection method. To reduce bandwidth requirements, quantization methods have been employed for estimates and covariance matrices prior to transmission. A two-bitrate periodic coding strategy has been utilized, transmitting the most significant and least significant bits separately based on a two-periodic principle. The unbiasedness and consistency of the proposed filter have been demonstrated, enabling real-time estimation accuracy evaluation. The boundedness of estimation error dynamics has been established based on collective observability and network topology connectivity. Numerical examples have confirmed the effectiveness of the proposed distributed Kalman filter. A future research topic would be the selection of quantization step to manage the quantization errors for each sensor based on the different estimation accuracy in order to balance the global performance and the constrained network resources.

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