Factor Based Pension Portfolio Strategies for Sustainable Withdrawals

By

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Abstract

This thesis studies the use of factor-based investment strategy to create pension fund portfolios for sustainable withdrawals. The first empirical chapter focuses on identifying factor-based anomalies in the UK stock market. This chapter identifies size, book to market ratio and profitability as factors that explain portfolios/stock excess returns other than the market factor, the sole predictor as stipulated by the capital asset pricing model (CAPM). In addition to these factors, this chapter explores the volatility factor which is currently gaining attention within the academic and professional sectors. Using the FTSE350 universe, the chapter then shows with the CAPM model the presence of size effect (small stocks outperforming large stocks in risk adjusted terms with smaller CAPM beta), profitability effect (high profitable stocks outperforming low profitable stocks in risk adjusted terms with smaller CAPM beta) and the volatility effect (low volatility stocks outperforming high volatility stocks in risk adjusted terms with smaller CAPM beta). The value effect (value stocks outperforming growth stocks in risk adjusted terms) was not identified. This chapter also finds evidence that the size, book to market and volatility factors could produce the observed excess return in an investment portfolio over the market portfolio in the UK stock market, but the profitability factor could not. However, in risk adjusted terms, only low volatility portfolios produced significant returns in excess of the market. Finally, this first empirical chapter documents that the returns of all portfolios constructed from the mentioned factors had a tendency of mean reversion. The chapter also examines the potential impact of transaction cost by using a methodology and statistic that give a reasonable indication of the scale of the likely costs of replicating an index which can then be used in observing if the potential cost of transaction will make the replication worthwhile; the chapter finds that transaction cost would not have changed the order of outcome observed.

The second empirical chapter further investigates the effect of combining the factors that were studied in the previous chapter. I constructed 64 portfolios (and considered 53 of them) based on different combinations of these factors and observed that 23 of these factors-based portfolios outperformed the market portfolio. However, only 7 portfolios produced significant risk adjusted returns in excess of the market, 5 of which also produced observed excess return. Interestingly, all the 7 outperforming

portfolios were constructed with the low volatility factor. This chapter also proposes a new ad hoc measure to capture the relative stability of an investment portfolio. The objective of this measure is to create a theoretical framework to assess the stability of a portfolio. This measure considers both the speed of mean reversion of the portfolio return and the 'distance' of deviation of the return from its mean. The chapter finds that 10 factor-constructed portfolios had better return stability than the market portfolio. Interestingly, all of these portfolios had a common characteristic—they were all constructed with the low volatility factor in the factor combinations.

The third empirical chapter applies the results from the second study to pension fund investment. Using Monte-Carlo simulation, I explore how well the factor-based portfolios can provide sustainable pension-income withdrawals in addition to other performance indicators during the drawdown phase of an individual's retirement. I identify four portfolios that could sustain a withdrawal rate of up to 10% with strong levels of success. Again, very interestingly, all of these portfolios were constructed with the low volatility factor; this seems to suggest that the low volatility factor is indeed a driver of return sustainability. In addition, I revisit the proposed relative stability measure introduced earlier and find that it has varying consistencies with the performance indicators of investment portfolios during the withdrawal phase in a pension fund. The chapter also finds that all else being equal, when the returns of a portfolio are highly persistent (slower reversion speed), a better success rate will likely be achieved; this is also the case when the deviation of shock is smaller, all else being equal.

Overall, the findings of this thesis contribute to the literature in identifying factors that are particularly relevant to the withdrawal sustainability in pension fund investment. It provides evidence that constructing pension fund portfolios using low volatility and value factors tilts can generally provide a more stable and secure withdrawal experience to pension fund clients.

Declaration of Originality

"This thesis and work to which it refers are the results of my own efforts. Any ideas, data, image or text resulting from the work of others (whether published or unpublished) are fully identified as such within the work and attributed to their originator in the text, bibliography or in footnotes. This thesis has not been submitted in whole or in part for any other academic degree or professional qualification. I agree that the University has the right to submit my work to a plagiarism detection service for originality checks. Whether or not drafts have been assessed, the University reserves the right to require an electronic version of the final document (as submitted) for assessment as above."

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Any shortcomings of this work are my sole responsibility.

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1 Introduction

Retirement savings are part of a complex financial planning behaviour and require a multidimensional model that involves individuals to take certain decisions (Husin and Rahman, 2013). In fact, they require individuals to act and make decisions on something that they find hard to visualise, since savings for retirement require prompt action for something that will be used in the future. Global demographic changes and retirement savings adequacy have shown that voluntary pension is a new paradigm for the retirement system. According to Blake (2003), the greatest impediment to having a decent pension in retirement is inadequate pension savings made during the working lifetime. There is a strong case for arguing that only with sufficient mandatory minimum contributions into a funded pension scheme (with credits given to those on very low earnings) can a decent pension be achieved. The topics of retirement, saving and pensions continue to be the focus of interest among both policymakers and the media.

There are broadly 2 pension vehicles: the defined contribution plan (DC) and the defined benefit plan (DB). A defined benefit (DB) plan offers an assured income at retirement, and this is related to individuals' pensionable salary and length of service whilst a defined contribution (DC) plan (also known as money purchase plan) is effectively a savings holding which is invested for capital growth and/or income with the objective to fund the retirement phase of financial planning. This pension scheme provides a pension income to an individual after retirement from a fund built-up from a series of regular contributions during the period of employment. The financial risk is taken by the member of the scheme and there is no guarantee of a fixed benefit level at retirement (like the DB plan). While the DB is usually an occupational plan, the DC can be a personal or occupational plan.

In the UK, prior to 2015, a pensioner had to purchase an annuity (guaranteed income from an insurance company) with whatever value of his pension fund at age 75. This is to ensure that after this age, there will be a steady flow of guaranteed income for the pensioner. This restriction was relaxed after the introduction of the pension's freedom legislation in April 2015, which significantly changed the pension investment landscape. The abolition of this restriction created wider flexibilities to pension fund investors, and a consequence of this is that pension advisers now need to offer portfolio solutions to their clients for the provision of a sustainable and sufficient income through the entire retirement period with a variety of available DC plans.

According to the ONS (office of national statistics), total private pension wealth in Great Britain was £6.1 trillion in April 2016 to March 2018 (accounting for 42% of all wealth in Great Britain) up from £3.6 in July 2006 to June 2008, after adjusting for inflation, and over half of this wealth is held in pension in payment (i.e., pension plans that are already been used to generate income)[1]. Membership of defined contribution (DC) occupational pension schemes was 22.4 million at the end of 2019, compared with 18.3 million for funded defined benefit and hybrid (DBH) pension schemes. Data also shows that an increased proportion of individuals with DC pensions was the main driver of the overall increase in active private pension membership rates[2]. According to the FCA[3], total number of pension plans accessed for the first time in 2019/20 increased by 3% to 674,000 compared to 2018/19 (652,000) as shown in the number of pensions accessed table below and out of this, there were 194,000 pension plans entering into an ongoing drawdown in 2018/19 and 197,000 in 2019/20. Annuity purchases (DC plans used to purchase a guaranteed income) have had a steady decline – down 6% to 69,500 in 2019/20. Plans fully withdrawn at first time of access in 2019/20 increased by 5% to 375,500 while the total number of plans fully withdrawn in 2019/20 remained steady at around 440,000 for the year with a value withdrawn of just under £5.7 billion. The number of plans accessed for the first time using the uncrystallised fund pension lump sum (UFPLS) method (but not fully withdrawn) increased to about 32,000 in 2019/20 from about 27,000 in 2018/19. As shown in the number of pensions accessed by fund size figure below, in 2019/20, about 23% of this drawdown plans were from pension fund of between £50,000 and £100,000, about 18% and 19% were from fund of £30,000 to £50,000 and £10,000 to £30,000. About 14% of this drawdown plans were from fund worth over £250,000. In the same year span (2019/20), 9 out of 10 plans that were fully withdrawn were from plans worth under

[1] Office of National Statistics:
https://www.ons.gov.uk/peoplepopulationandcommunity/personalandhouseholdfinances/incomeandwealth/bulletins/pensionwealthingreatbritain/april2016tomarch2018#toc

[2] *Office of National Statistics:*
https://www.ons.gov.uk/economy/investmentspensionsandtrusts/articles/ukpensionsurveys/redevelopmentand2019results#:~:text=The%20FSPS%20estimates%20that%20total,private%20sectors%20(18.3%20million).

[3] *Financial Conduct Authority:* https://www.fca.org.uk/data/retirement-income-market-data-2019-20

£30,000. About 83% of the plans used to purchase an annuity were from funds worth less than £100,000. Over 61% of plans entering partial UFPLS were from funds worth less than £50,000. The regular withdrawal rates table below shows that in 2019/2020 42% of regular withdrawals were withdrawn at an annual rate of 8% or more of the pot value (40% in 2018/19). All this information points to the fact that DC plans are becoming more prominent, and as a result, the solution to withdrawal sustainability is a crucial topic in pension fund management which requires more research to investigate.

Number of pension plans accessed for the first time.

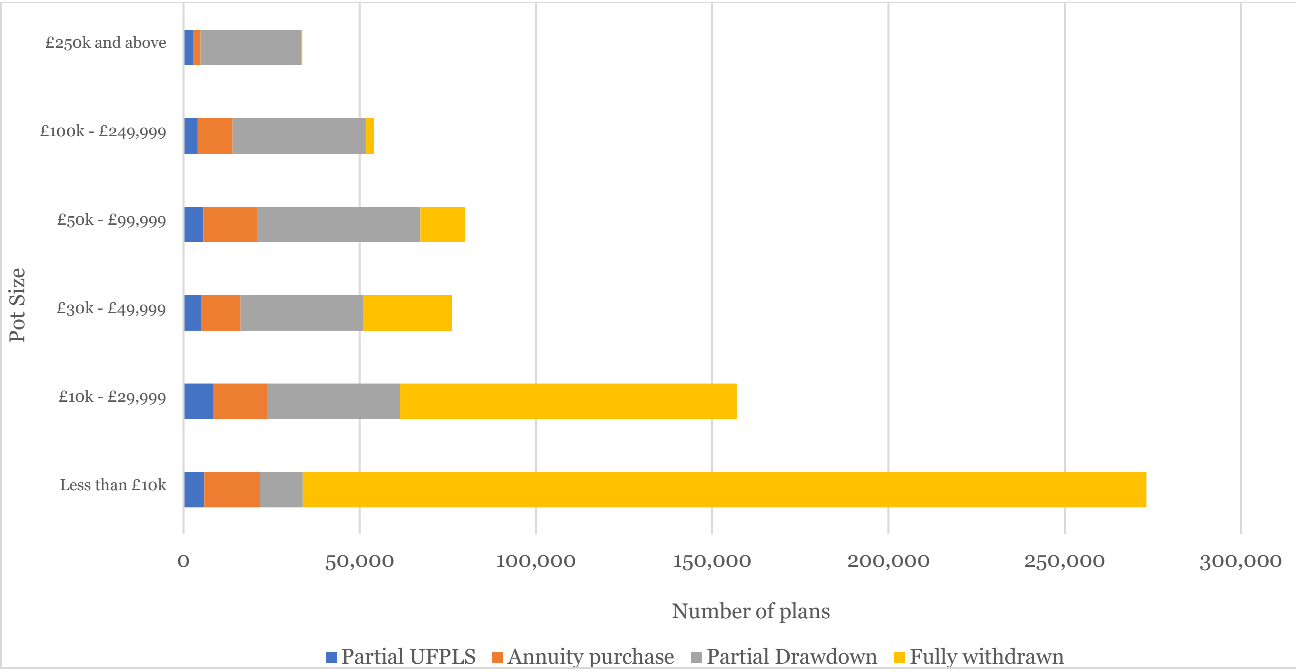
Method of access	Oct 2019 - Mar 2020	Apr - Sept 2019	Oct 2018 - Mar 2019	Apr - Sept 2018
Plans used to buy an annuity	31,138	38,381	35,987	37,990
Plans entering income drawdown and not fully withdrawn	95,493	101,625	983,28	95,830
Plans with first uncrystallised fund pension lump sum (UFPLS) payment and not fully withdrawn*	16,167	15,497	13,366	13,371
Plans fully withdrawn**	174,415	201,115	169,303	187,819
Total plans accessed for the first time	317,213	356,618	316,984	335,010

Source: FCA Retirement Income Data

**Both small pot and UFPLS are other types of flexible withdrawals allowed under pension regulations and aimed at creating a means to cashing in small, valued DC plans or to partially strip part or all of a larger DC plan.*

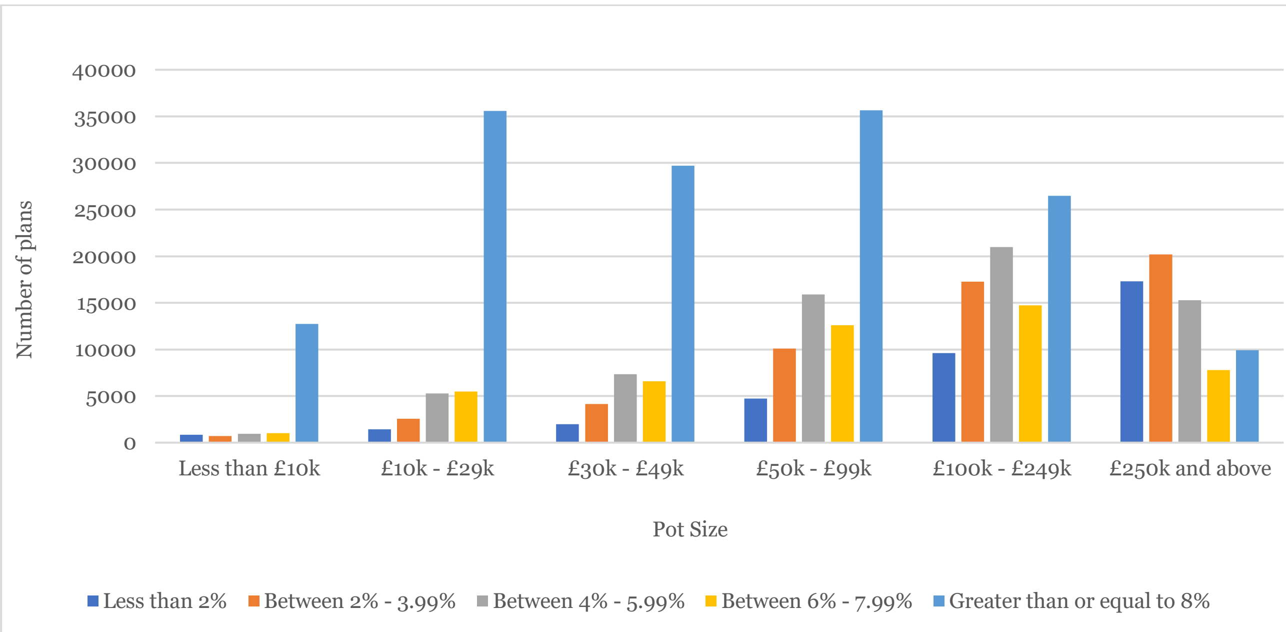
***By plan holders accessing their plans for the first time via small pot lump sum, drawdown or UFPLS.*

Number of pension plans accessed in 2019/20 by pot size and method of access.



Source: FCA Retirement Income Data

Regular withdrawal rates by pot size in 2019/20



Source: FCA Retirement Income Data. Firms reporting 750 plans or more with regular withdrawals in 2019/20

There are two phases in a pension fund investment: During the accumulation (or pre-retirement) phase, the scheme member and/or their employer contribute to the pension fund, which is invested in a portfolio of assets (in line with a particular investment strategy) with a particular risk profile. In the distribution (or post-retirement) phase the pensioner receives periodic income from the fund to provide support in old age.

Broadly speaking, the investment strategies generally employed are either passive or active. Passive investment strategy is where the portfolio is designed (in terms of assets and weightings) in line with a reference index. Passive investing believes it is impossible to outperform markets consistently; therefore, they embrace mirroring a market or index in constructing an asset portfolio. The passive approach has gained traction in recent years because it is clear to many investors that high fees for active management does not always translate into better net performance. Active investment strategy on the other hand involves making educated bets that some shares will do better than others; however, long term data reveals that the majority of active investment managers do not outperform an underlying index for enough time to compensate for the higher fees charged according to Johnson and Bryan (2018)

The debate on whether to use passive or active investment strategies has a middle ground, often called the smart beta strategies or factor-based strategies. A factor-based strategy constructs a portfolio passively using a specific set of transparent rules (e.g., assets which meet certain criteria), but at the same time actively adjusting or rebalancing the portfolio on a regular basis by adding new assets that meet the criteria and removing existing ones that no longer do, following the changes in market conditions. These middle-ground strategies provide more flexibilities to enable investors with particular objectives and/or risk profile to meet their targets while maintaining the fees to a relatively low level and is particularly suitable to DC plans for offering flexibilities to their clients.

There are two main considerations for a pension fund investment:

a) How much should be saved or accumulated to create a sufficient pot for decumulation?

(Decumulation is used interchangeably as pension withdrawal henceforth). This encompasses the two fundamental objectives of traditional portfolio investment, namely, capital growth and income generation.

b) What withdrawal strategy or withdrawal rate should be adopted during retirement to maintain the sustainability of withdrawals for a long-term period? It is this consideration that makes a third investment objective apparent: the return stability/sustainability of a portfolio.

Yet, as Merton (2014) states, there is a clear gap in literature in the construction of investment portfolios for both the pre-retirement and retirement phases in the study of retirement planning, even more obvious is the lack of insight into the construction of retirement income strategies to provide appropriate solutions in line with the changing (more flexible) retirement savings environment. Hence, the motivation for this research is to explore how an investment strategy (factor investing) may provide a better withdrawal experience. This study also proposes and examines the use of a relative stability measure (which is based on the presence of mean reversion of the portfolio returns) as an indicator of withdrawal stability/sustainability of a portfolio. The absence of this feature (mean reversion) will imply that portfolio returns follow a random walk which is unfavourable to the sustainability in pension fund withdrawal.

This research constructs portfolios from FTSE 350, using factor investment strategy, and then assesses the ability of these portfolios to sustain withdrawals at various rates. Although this research perspective is relatively new, existing literature has already showed that size, book to market ratio and profitability are effective factors for factor-based portfolio strategies. Therefore, to provide further contribution, an additional factor, volatility, is also examined in this study. The key aim of this thesis is to fill the gap in understanding the decumulation phase of pension fund investment to enhance the withdrawal experience for the investors.

In chapter 3, I revisit the topic of explaining excess return in a portfolio, by considering three well established and researched factors in portfolio investment- size, book to market ratio, and profitability, and a less explored one-stock volatility, using data from the UK market. The result from this chapter shows that the returns of portfolios formed from the exposure to these factors have mean reverting qualities (using the Dickey-Fuller model). The chapter then shows that generally and in absolute terms, portfolios of small stocks (lower MEs) and high book to market ratio portfolios (value stock) have historically performed better than their respective counterparts (large sized and low BE/ME stocks); in addition, whilst some studies (such as Van Dijk (2011) have identified a reversal of this trend in recent times, the returns of small and value stocks have sustained this outperformance even in recent times. Also, similar to existing literature, the results of the profitability factor show that high profitable stocks have better average returns than low profitable stocks. However, in risk adjusted terms, the size and profitability effects were observed while the value effect was not. The findings for the volatility factor confirm that low volatile stocks do produce higher average risk adjusted returns than high volatile stocks. To summarise, the size, book to market ratio, and volatility factors all contributes to absolute returns in excess of the market, but the profitability factor does not. In addition, this chapter assesses the potential impact of transaction cost and finds that this cost will not likely change the order of performance (outperformance).

Although it has been established that the excess return may be harvested efficiently with a passive strategy, there is also strong evidence supporting the existence of factor premiums, and previous researchers have shown that one factor is unlikely to capture all the available excess return. Chapter 4 of this study combines these factors to form two and three sort portfolios. The chapter finds that portfolios constructed from a combination of factors produced even higher absolute returns in excess of the market. I identify 23 combined factor portfolios which generated this excess return, out of which 15 portfolios were constructed with the low volatility as one of the factors. Seven of these portfolios also produced significant risk adjusted returns in excess of the market.

Using the Dickey-Fuller model, chapter 4 again tests the portfolios for the presence of mean reversion; whilst all the mentioned portfolios are mean reverting, some are reverting faster than others. The proportionality factor of reversion (β – speed of reversion), effectively indicates how quickly a series (portfolio return) will revert to its historical mean return when it wanders away from it. However, using this measure alone is incomplete as it ignores the 'distance' of deviation from the mean. Hence, not just how fast but how far the series can revert. Therefore, the chapter proposes the use of an ad hoc measure that considers the average deviation from the mean; $\frac{|\beta|}{\hat{\sigma}}$, where $\hat{\sigma}$ denotes one unit deviation of shock.

By implication, the higher this measure, the more stable the portfolio returns is expected to be. The results show that 9 combined factor portfolios have a stability measure larger than that of the market portfolio, and 2 of these also produce returns in excess of the market. The results also show that these 9 portfolios are all constructed with the low volatility factor; the 2 portfolios which also produce risk adjusted returns in excess of the market are constructed together with the value factor. From the 8 single factor portfolios created, only the low volatility portfolio has a stability measure better than that of the market and it also outperformed the market in risk adjusted terms.

Following the theoretical study on assessing stability, chapter 5 of this thesis empirically tests the performance of the portfolios during the decumulation phase of a pension fund using Monte Carlo simulation, and then further compares the results with the predictions from the relative stability measure suggested in the previous chapter.

Chapter 5 finds that diversified portfolios constructed based on one individual factor offer the potential to provide better sustainability of up to 6% withdrawal rate (with at least 95% success) compared to the FTSE350 market. But the portfolios constructed based on a combination of factors perform even better, sustaining a withdrawal rate at 10% successfully in most of the simulated paths. Four particular portfolios are identified to be the most successful ones in term of withdrawal sustainability; *H,Lv* (value and low volatility stocks), *S,H,Lv* (small, value and low volatility stocks), *S,Lp,Lv* (small, low profitability and low volatility stocks) and *H,Lp,Lv* (value, low profitability and low volatility stocks). The result of this chapter indicates that the low volatility factor *Lv* as well as the value factor *H* appears to be the key drivers of sustainable withdrawals. Furthermore, conditional on a withdrawal-failure simulated path, the four most successful portfolios sustain withdrawals for a relatively long period of time before failing. And these four portfolios also produce good bequest funds (residual portfolio fund usually left as death benefit for beneficiaries), up to 57% of the portfolio with the largest bequest value. In the assessment of the proposed relative stability measure, the finding shows that the measure has a positive correlation with both the success rate and failure time, but the correlation of the latter is considerably stronger than the former. In addition, the chapter finds that portfolios with high persistence of returns (slower speed of mean reversion) are likely to be more successful during withdrawals, all else being equal. This is also observed to be the case with portfolios with low deviation of shock, all else being equal. Furthermore, conditional on failure and all else being equal, low persistent (faster speed of mean reversion) and smaller deviation of shock portfolios will sustain withdrawals for longer period of time.

This thesis is organised as follows: Chapter 2 reviews the relevant literature on the key areas of this study, including the types of UK pensions, portfolio strategies and models, factors of excess return, withdrawal strategies and sustainability. Chapter 3 to Chapter 5 present the three main research topics of this thesis, as stated above. Finally, chapter 6 provides conclusions and the final remarks.

2 Literature Review

This chapter reviews key studies relating to four main areas of this research. It highlights key findings relating to factors of excess return; a major departure from the widely known capital asset pricing model (CAPM), strategies resulting from exposure to these factors, pension withdrawal strategies and investigating sustainable withdrawal rates. Studies researching the use of factor investment for the purpose of withdrawals was however a gap identified in available literature.

This literature is presented as follows: Section 2.1 provides an introductory presentation of the key types of pension schemes in the UK, Section 2.2 gives an overview of the various factors of excess return, Section 2.3 presents literature on factor investing, Section 2.4 identifies literature on portfolio and withdrawal strategies, and Section 2.5 reviews literature on investigating sustainable withdrawal rates.

2.1 UK Retirement Saving Schemes

According to Blake (2003), the UK was one of the first countries in the world to develop formal private pension arrangements (beginning in the 18th century) and was also one of the first to begin the process of reducing systematically unfunded state provision in favour of funded private provision (beginning in 1980). In the UK, there are two main tiers of pension provision; primarily, there is the fully indexed (increasing) basic state pension and the supplementary pensions which are provided by the state parastatals, employers and private sector. According to the department of works and pensions (DWP 2013), eligibility for the state pension is based on the years of national insurance contributions made by employees and for individuals who are not working but receiving state benefits, they will accrue qualifying credits years. With the Pensions and Lifetime Savings Association (2021) estimating that it will cost an individual and extra £1,560 a year over the state pension to maintain the minimum standard of living, although the state pension is a guaranteed indexed income, it is generally insufficient to cover (on average) the cost of living in retirement hence the need for and importance of having the supplementary source of pension income. Global demographic changes and retirement savings adequacy have emphasised this voluntary (supplementary) pension saving as a new paradigm for the retirement system.

In the UK, there are broadly 2 supplementary pension vehicles: the defined contribution plan (DC) and the defined benefit plan (DB). While the DB is usually an occupational plan (offered by the state establishments and the private sector), the DC can be a personal or occupational plan. The DB plan has certain peculiarities similar to the guaranteed nature of benefits received with the state pension but the defined contribution plan (often known as money purchase plans) are effectively a savings holding which is invested for capital growth and/or income with the objective to fund the retirement phase of financial planning. The Pensions Regulator (2021) report shows that there has been an increasing decline of the DB types of pensions being offered in the private sector predominantly because of the increasing cost of running these types of pensions. Even within state establishments (and some private employers) who have continued to offer this type of pension, there has been several reforms to address this rising cost; one particular one is the use of an employee's average pensionable salary through the period of service rather than the final salary (salary at the time employment stopped) in the estimation of pension benefits. In a bid to make more people save more for their retirement, the government introduced the auto enrolment policy in 2012 where employers and employees now have to contribute a minimum amount into a pension plan offered by the employer. This has further driven up the subscription to DC plans.

The pension's freedom legislation in April 2015 significantly changed the pension investment landscape. Prior to this time, a DC holder must purchase an annuity (a guaranteed income) with the DC pot at age 75; this need was relaxed with the introduction of this legislation. One of the consequences of this relaxation was that it effectively created an alternative for pension investors who are prepared to take investment risks via DC plans. While the legislation provides a number of flexibilities, it did not take away the fundamental fact that an investors pension plan is meant to provide an income in retirement, and the state pension alone will generally not be sufficient. This DC flexibilities have also made the transfer of DB plans into DC arrangements more attractive to individuals willing to accept the risks involved. Therefore, as a result of this change, advisers now need to offer investment solutions for DC plans providing sustainable income to last through retirement.

2.1.1 Defined Benefit Plan.

Defined benefit pension schemes (often referred to as final salary pension schemes) is a type of occupational scheme (which means it is usually provided by an employer) and is one that promises to pay an income based on your final salary when a member leaves or retires from that company. Unlike other schemes, the amount the member receives in retirement is guaranteed by the scheme, paid directly based on the normal retirement date of the scheme and, as a result, the member does not have to decide how to access the pension pot at any time. Both the employer and member make contributions into the plan and this money is invested into various investment vehicles over time. However, unlike other type of pension schemes, the amount the member pays in is irrelevant when calculating the retirement income. This is because the amount of income received in retirement is guaranteed when the member initially agrees on membership.

One broad categorisation of defined benefit pension schemes are the ones which are based on the members final salary when they retire, while the other is based on the average salary the member receives throughout their career at the sponsors employment. The latter has recently become the preferred choice for employers as the structure makes them comparatively cheaper to run. The value of the pension received will also rise in value, known as index-linking, which means that the income the member receives will rise each year, often in line with inflation, although the actual increase will often be capped.

When a member leaves employment of the scheme sponsor and has not decided to take benefits yet whether or not the minimum pension age (55yrs) has been attained, the status of the plan is referred to as "deferred" and there are statutory guaranteed increases applicable until when benefits are taken at which point the index linking increases (described above) kicks in. The current pension regulation allows for deferred members to transfer the value of their pension plan into flexible arrangements within a defined contribution scheme. This will require the actuary's valuation of the plan based on a number of factors and this is offered to the member as a transfer value. Consequently, a member who takes up this offer loses the guarantees from the DB plan and assumes several risks amongst which includes "investment risk" within the DC plan.

2.1.2 Defined Contribution Plan.

Defined contribution (DC) schemes have contributions invested and the resulting fund is used to buy a guaranteed pension (annuity) and/or generate a flexible income at retirement. The value of the DC fund (and ultimately the benefits payable from the DC plan) depends on the amount of contributions paid, the investment return achieved less any fees and charges, and the cost of buying the benefits. It is easy to recognise that the investment product, assets and investment strategy will all have a direct impact in determining the investment return achieved. The common products purchased with the DC fund will usually include annuities and mutual funds or a combination of both.

The investment strategies employed by advisers are broadly the same as those used within other investment vehicles and they include:

Life Styling strategies where the allocation to risky assets is tapered down and substituted with less risky assets as the client approaches retirement. Pablo, Stéphanie and Juan (2010) assessed various lifestyle strategies in the presence of other dynamic strategies and concluded that there is no "one size fits all". According to their findings, lifestyle-strategies have a better outcome when the ultimate fund is used to purchase an annuity compared to using the fund for programmed withdrawals. Dynamic strategies seem to work better for generating income withdrawals.

Passive investment strategy is where the portfolio is designed both in assets (stocks) and weightings in line with a reference index. Passive Investing believe it is impossible to outperform markets consistently therefore, they embrace mirroring a market or index. The "passive" approach has gained traction in recent years because evidence appear to show that high fees for active management don't always translate into better net performance. Long term data reveal that majority of active investment managers do not outperform an underlying index for enough time to compensate for the higher fees charged (Morningstar Active/Passive Barometer (2018)).

Active investment strategy usually involves making educated bets that some shares will do better than others. However, despite the best research in the world, stocks can sometimes fall short of expectation and data suggest this is a hit or miss approach.

It was the pioneering asset pricing work some 60 years earlier of Harry Markowitz who introduced the world of investment management to the phrase 'Modern Portfolio Theory' in the 1950's (Markowitz (1959)) and the work of Eugene Fama which popularised the notion of the Efficient Market Hypothesis (EMH), (Fama 1970) that essentially formed the intellectual basis for a style of investing that has become known invariably as 'passive investing' or 'index tracking'. However, subsequent works suggest that there is more to investing on an indexed basis than simply allocating investor wealth according to market capitalization weights. The debate on whether to use passive or active investment strategies now includes a middle ground, smart beta strategies, which generally are transparent and rules-based like passive strategies while focused on achieving exposures like active strategies. One of such types of strategy is known as *factor investing*.

2.2 Factors of Excess Return

The capital asset-pricing model of Sharpe (1964), Lintner (1965), and Black (1972) has long shaped the way academics and practitioners think about average returns and risk. The central prediction of the model is that the market portfolio of invested wealth is mean-variance efficient in the sense of Markowitz (1959). The efficiency of the market portfolio implies that (a) expected returns on securities are a positive linear function of their market Betas (the slope in the regression of a security's return on the market's return), and (b) market Betas suffice to describe the cross-section of expected returns.

Following the development of this model as the investment paradigm, in their efforts to test its predictions, academics began exploring a range of rules-based investment strategy. Soon after the CAPM had become a benchmark model (for the academic community at least) evidence began to emerge that questioned its key predictions. Researchers started investigating the nature of the risk-return relationship and at the same time began experimenting with certain rules-based investment strategies that seemed to produce returns over and above what could be expected as a result of exposure to 'market beta risk'. These experiments show the existence of other betas, that is, other sources of systematic risk to which investors could get exposure to earn returns.

Over time there has been several empirical observations and enhancements of the Sharpe-Lintner-Black model, and the smart beta strategy (factor-based strategies) is effectively a modern manifestation of this. The first empirical tests of the CAPM by Black, Jensen and Scholes (1972), Miller and Scholes (1972), and Fama and MacBeth (1973) found that although higher risk is rewarded with higher return, it is not rewarded enough. The first main challenge of the CAPM was the finding of Banz (1981) which established that market equity, ME (a stock's price times shares outstanding), adds to the explanation of the cross-section of average returns provided by market betas, the studies show that small stocks had higher returns than large stocks, even after correcting for the fact that the average small stock is riskier than the average large stock; this is the size effect.

Subsequently, the positive relation between leverage and average return documented by Bhandari (1988) is another example of this. Bhandari finds that leverage helps explain the cross-section of average stock returns in tests that include size (ME) as well as market beta. Barr et al (1985) find that average returns on U.S. stocks are positively related to the ratio of a firm's book value of common equity, BE, to its market value, ME (the value effect). Chan, Hamao, and Lakonishok (1991) find that book-to-market equity, BE/ME, also has a strong role in explaining the cross-section of average returns on Japanese stocks. Basu (1983) also showed that earnings-price ratios (E/P) help explain the cross-section of average returns on U.S. stocks in tests that also include the presence of size and market Beta.

It was in the nineties that the true failure of the CAPM became clearly visible. Fama and French (1992) find that market beta is entirely unpriced in the cross-section of stock returns when size and market beta are properly disentangled from each other. Quoting from the abstract of their paper: “when the tests allow for variation in beta that is unrelated to size, the relation between market beta and average return is flat, even when beta is the only explanatory variable”. Anomalies such as size and value imply that the CAPM may need to be augmented with some additional factors. They evaluated the joint roles of market Beta, size, E/P, leverage, and book-to-market equity in the cross-section of average returns on NYSE, AMEX, and NASDAQ stocks. Their work marked the beginning of joint examination of various suspecting factors considered to explain excess return. In summary, their results are: (a) market beta does not seem to help explain the cross-section of average stock returns, and (b) the combination of size and book-to-market equity seems to absorb the roles of leverage and E/P in average stock returns. This became known as the 3-factor model.

Fama French (2014) used the dividend discount model to show evidence that a measure of profitability and investment are worthy additions to the 3-factor model. In their first draft in 2015 (and later published in 2017), Fama French examined this 5-factor model on international markets and found that in some regions including Europe, the role of the investment factor of the five-factor model may largely be to absorb the low average returns of high investment small stocks. During their test of individual factors, they established that the investment factor premium, is redundant for Europe and Japan, and its role is marginal for Asia Pacific. Thus, for three of four regions, dropping the investment premium from the five-factor model has little effect on the description of average returns, at least for 1990-2015. Their factor spanning tests also suggest that the profitability factor premium is important for describing North America, European, and Asia Pacific average returns. Blitz, Baltussen and Van Vliet (2019) also show a flat, or even slightly negative relation between risk and return, which implies that, over a more up to date sample period, the conclusion of Fama and French (1992) that beta is not a priced factor still holds.

There has been several extensions, modification, and additions to these findings. For example, Blitz (2012) argues that investors should allocate strategically not only to the risk premiums offered by traditional asset classes, but also to factor premiums. More specifically, the study identifies value, momentum, and low volatility as the three key factor premiums in the equity market in addition to that provided by traditional asset classes and argues for a sizable and well-diversified allocation to these three factors, based on data spanning the 1963-2009 period.

2.2.1. The size effect.

From about the late 1970s, some academics saw the opportunity to test the predictions of the CAPM (Capital Asset Pricing Model) paradigm. It became well known that US small cap stocks had outperformed their large cap equivalents substantially over the preceding decades. Fama French (2006) show that the size premium in average returns in the sub period of 1926-1963 is 0.20% per month and 0.24% for 1963-2004. The CAPM explanation for this outperformance was relatively straightforward: if small cap stocks produced a higher return than large cap stocks, it was because small cap stocks were riskier and had higher CAPM betas than large cap stocks. This explanation of the outperformance then would have been entirely consistent with the EMH (Efficient Market Hypothesis)/CAPM paradigm. Banz (1981) published a paper that tested this hypothesis. Unfortunately for the paradigm, Banz finds the complete opposite. Not only did Banz find that small cap US stocks outperformed large cap US stocks he found that they did so even though on average they had lower betas than the large cap stocks. Banz reports that stocks in the quintile portfolio with the smallest market capitalization earn a risk-adjusted return that is 0.40% per month higher than the remaining firms. Fama–MacBeth (1973) regressions show a negative and significant relation between returns and market value. The size effect is not linear and is most pronounced for the smallest firms in their sample. Banz conjectures that many investors do not want to hold small stocks because of insufficient information, leading to higher returns on these stocks. This argument is reminiscent of the investor recognition hypothesis developed by Merton (1987).

Reinganum (1981) analysed the size effect in a sample of 566 NYSE and Amex firms over the period 1963–1977. He finds that the smallest size decile outperforms the largest by 1.77% per month. Brown et al. (1983) re-examines the size effect using the Reinganum data and find that there is an approximately linear relation between the average daily return on 10 size-based portfolios and the logarithm of the average market capitalization. Keim (1983) reports a size premium of no less than 2.5% per month in a broader sample of NYSE and Amex firms over the period 1963–1979. Keim shows that small firms have higher betas than large firms, but the difference cannot fully explain the return differential. Based on 20 size-sorted portfolios using a very large sample of firms, Lamoureux and Sanger (1989) find a size premium of 2.0% per month for Nasdaq stocks and of 1.7% for NYSE/Amex stocks over the period 1973–1985. They document that small firms have a lower beta than large firms on Nasdaq.

Van Dijk (2011) survey suggests that the international evidence on the size premium is remarkably consistent. Small firms outperform large firms in 18 of the 19 countries investigated, and also, in a sample of emerging markets and in Europe. The monthly size premium in these countries ranges from 0.13% for the Netherlands to 5.06% for Australia. In 14 out of 19 countries, the size premium lies between 0.4% and 1.2% per month. He also highlighted in this survey that risk, liquidity and investor behaviour are some of the explanation's literature has provided for this anomaly.

This evidence appeared to be a direct challenge to the testable conclusions of the CAPM, and, at the same time seemed to identify another risk factor: size. However, in recent years a consensus seems to have developed that the size effect has disappeared. Several studies report that small firms have not outperformed big firms after the early 1980s. Dichev (1998) surprisingly, finds that firms with high bankruptcy risk earn substantially lower than average returns since 1980. Chan, Karceski, and Lakonishok (2000) show that for the 15-year period from 1984 through 1998, the annual return on the Russell 1000 Index of large cap stocks was 17.71 percent, compared with 11.22 percent for the Russell 2000 Index of small-cap stocks. In only four years out of 15 did the Russell 2000 beat the Russell 1000. The outperformance of large-cap stocks is even more striking if only the most recent period is considered.

Chan, Karceski, and Lakonishok (2000) identify three possibilities to account for the recent relative price performance of the different equity classes—the "rational-asset pricing" explanation, the "new-paradigm" explanation, and the "behavioural" or "institutional" explanation:

Rational-asset-pricing models look to shifts in expected cash flows or discount rates as reasons for changes in equity valuations. The underlying assumptions are that investors make rational, informed decisions and markets are informationally efficient. In this view, large-cap growth stocks had a sequence of unanticipated positive shocks, possibly the results of technological innovations, changes in corporate control mechanisms, or other revisions in investors' expectations of future profits. As a result, these stocks have performed unexpectedly well in recent years. Along the same lines, this implies that small-cap stocks have done poorly because of prolonged negative surprises to current or expected future profitability. If a string of unexpected temporary shocks is the correct explanation for the relatively poor price performance of small cap, then the future should more closely resemble the long-term past (since the shocks are temporary). Therefore, unless a shift has occurred in relative riskiness, small-cap stocks will outperform large-cap stocks in the future. Investors holding small cap will be rewarded for bearing the higher risk of such stocks.

The new paradigm stipulates that recent large-scale and widespread technological advance have put to rest the conventional approach to valuation in selected industries. Companies that are in the forefront of innovation and that have exhibited dazzling growth rates in the past will continue to soar, in defiance of the low average returns they have historically earned. The implication of this is that the technology sector represents an attractive investment and investors should not be deterred by valuations that are high by historical standards. Additionally, this view maintains that investing in large companies provides benefits because of their economies of scale. Unlike the rational-asset-pricing view, the new-paradigm view suggests that the superior returns of large-cap stocks will persist for some time in the future. One connotation of the new-paradigm argument is that market prices have not fully incorporated all the future benefits from technological innovation. Given the market's slow response to information, money is still on the table. Hence, investors should continue to chase cutting-edge companies in the computer, Internet, and networking sectors, despite their high current valuations.

The behavioural explanation accepts that a remarkable spate of technological innovations has marked recent years and these advances have helped fuel the dazzling rise in stock prices in some sectors. It explains that the market's response, once under way, fed on itself similar as discussed in further details by Shleiffer (1999). It is perhaps natural for investors to get excited about successful companies and companies in innovative fields, such as electronic commerce. In this view, as certain equity classes took off and others fell out of favour in the 1990s, investors overreacted, thereby pushing returns away from their long-term patterns.

2.2.2. Value Effect.

In the wake of the Fama French (1992) study, academics shifted their attention to the ratio of book value to market value of equity and company size as the leading explanatory variables for the cross-section of average stock returns. This work built on earlier studies of stock market "anomalies." Basu (1977), for example, show that stocks with low P/Es (price earnings ratio) subsequently tend to have higher average returns than stocks with high P/Es.

The main benchmarks used in academic studies is the book value to market value of equity (BV/MV)—and this ratio has become an important indicator of a portfolio's orientation toward either growth or value. Fama French (1992) used this ratio in their work and the highest ranked portfolio was dubbed the "value" portfolio and the lowest ranked was dubbed the "glamour" (growth) portfolio. Their results show that value portfolio generated an average monthly return of 1.83 percent compared with the average monthly return on the companion growth portfolio of 0.30 percent. Lakonishok, Shleifer, and Vishny (1994) provide similar findings based on NYSE and Amex stocks. The superior returns of the value sorts in their work persisted even after the authors controlled for differences in size.

Although BV/MV has garnered the lion's share of attention as an indicator of value-growth orientation, it is by no means without shortcomings. To take an example from market conditions as of mid-2002 according to Chan and Lakonishok (2004), a stock such as AOL-Time Warner would generally be classified as a "cheap" stock in terms of the book-to-market ratio. By many other yardsticks, such as earnings or dividends relative to price, however, the stock would look less attractive from the value standpoint. This disparity suggests that other measures might also serve as the bases for the investment strategy.

Basu (1977) demonstrates the 'PE (Price Earning) effect', showing that investing in low PE stocks could generate higher returns relative to that could have been earned by investing in high PE stocks, and with less systematic risk. But he also found that this PE effect was closely related to the size effect documented by Banz (1981). The review by Chan and Lakonishok (2004) show that the E/P ratio produced smaller spreads between extreme portfolios, and they stated that the narrower spreads associated with the earnings yield, E/P, may be a result of the noisy nature of earnings. Another valuation indicator that has generally received less attention in academic research is the ratio of cash flow to price (CF/P). In its simplest form, cash flow is measured as earnings plus depreciation. Chan and Lakonishok (2004) show that portfolios formed on the basis of this investment strategy generate relatively larger return spreads than portfolios based on BV/MV.

Although the evidence on returns is relatively uncontroversial, the situation is far less settled when it comes to providing an explanation for the differences between the performance of value and growth portfolios. While Fama French (1996) argue that stocks with high BV/MVs are more prone to financial distress and are hence riskier than growth stocks, Lakonishok, Shleifer, and Vishny (1994) argue against the "metaphysical" approach to risk in which higher average returns on an investment strategy must necessarily reflect some source of risk. Following a conventional approach, they argued that risk does not explain the differences in returns and suggested that cognitive biases underlying investor behaviour and the agency costs of professional investment management were at the root of the rewards to value investing. Yet another explanation for the returns to value investing rested on methodological issues of data-selection bias (see Kothari, Shanken, and Sloan 1995).

Chan and Lakonishok, (2004) also show that value investing appears to be alive and well in U.S, and non-U.S. markets. They conclude that the bulk of the empirical research documenting the superiority of value investing stops short of the late 1990s, which were not kind to value stocks. Growth stocks rocketed in value in those years, but careful examination suggests that the differences in performance between value and growth in the late 1990s were not grounded in fundamental patterns of profitability growth. The most plausible interpretation of the events of the late 1990s is that investor sentiment reached exaggerated levels of optimism about the prospects for technology, media, and telecommunications stocks. The resulting valuations are hard to reconcile with economic logic.

Chan, Karceski, and Lakonishok (2000) put forward that the returns to value stocks have also faltered recently in comparison with those to growth stocks. For example (using US data), large-cap value stocks from 1990 through 1998 earned a mean return of 17.2 percent, falling short of large-cap growth stocks by 2.9 bps a year on average. According to their results overall, value stocks were outpaced by growth stocks (in each case combining large-cap, mid-cap, and small-cap stocks in proportion to their market values) by 1.1 bps a year on average over 1990– 1998. The average underperformance was 1.6 bps for 1994–1998 and 3.3 bps for 1996–1998. Performance in 1998 was notably disastrous for both small-cap and value stocks. In that year, large-cap growth stocks experienced their highest return (41 percent) of the 29 years they covered. The possible explanation they gave for the size effect change (the “rational-asset pricing” explanation, the “new-paradigm” explanation, and the “behavioural” or “institutional” explanation) also applied to this recent change in the value effect.

2.2.3. Profitability.

Fama French (2006) used the dividend discount valuation model to show the validity of the profitability factor (broadly measured as the operating profit of a firm) as an explanatory variable of expected returns however, this was not a virgin territory. Haugen and Baker (1996) and Cohen, Gompers, and Vuolteenaho (2002) find that controlling for book-to-market equity, average returns are positively related to profitability.

Accruals are the non-cash component of earnings. They represent adjustments made to cash flows to generate a profit measure that is largely unaffected by the timing of receipts and payments of cash. Prior research uncovers two anomalies: expected returns increase in profitability and decrease in accruals. Sloan (1996) documents that accruals are negatively related to future profitability, and higher accruals predict lower stock returns. Xie (2001) shows that the market not only prices, but also overprices abnormal accruals. He further suggested that the overpricing of total accruals that Sloan (1996) documents is due largely to abnormal accruals. Fairfield, Whisenant, and Yohn (2003) also made findings that are largely in line with this.

There have been different approaches used to define the measure of profitability. Ball et al (2015) show that cash-based operating profitability (a measure that excludes accruals) outperforms measures of profitability that include accruals. A common measure of profitability used in the asset pricing literature is earnings before extraordinary items but after interest, depreciation, taxes, and preferred dividends plus depreciation (e.g., Fama French, 1996). This measure includes "working capital" accruals such as changes in accounts payable, accounts receivable, and inventory.

Novy-Marx (2013) shows that gross profit scaled by book value of total assets predicts the cross section of average returns. He concludes that it outperforms other measures of profitability such as bottom-line net income, cash flows, and dividends. One potential explanation for the measure's predictive ability is that its numerator (gross profit) is a cleaner measure of economic profitability. According to Ball et al (2015), an alternative explanation lies in the measure's deflator. They find that in terms of predictive power, net income equals gross profit when they have consistent deflators. Deflating profit by the book value of total assets results in a variable that is the product of profitability and the ratio of the market value of equity to the book value of total assets, which is priced.

2.2.4. Volatility.

One of the main tenets of modern portfolio theory is that, as long as an investor holds a well-diversified portfolio of risky securities then over time the higher the inherent expected risk in that portfolio the higher should be the expected return. If high risk should lead over time to higher return, then one could expect that stocks that produce returns with low volatility should generate lower returns over time than stocks that generate a higher volatility.

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972) implies that there exists a positive linear relation between expected returns on securities and their market betas, and variables other than market beta should not capture the cross-sectional variation in expected returns. Therefore, their model, which is effectively an expression of risk adjusted return implies that stocks with higher risk should produce higher return (risk adjusted return). There has been a lively debate on the existence and direction of a trade-off between idiosyncratic risk and the cross section of expected stock returns. Although some studies find a positive relation between idiosyncratic volatility and expected returns at the firm or portfolio level, often the cross-sectional relation has been found insignificant, and sometimes even negative. The measure of volatility employed have often been sighted as a reason for this difference in result. Broadly, Idiosyncratic Volatility = Total Variance – Market Variance. Early empirical tests of the low volatility anomaly focused on total volatility.

Haugen and Heins (1975) first showed that firms with a low standard deviation of past returns outperformed those with a high standard deviation. Baker and Haugen (2012) and Blitz and Vliet (2007) demonstrate that this effect is present in equity markets throughout the world. However, most of the recent literature focuses on idiosyncratic volatility. Tinic and West (1986) and Malkiel and Xu (1997) provide empirical evidence that portfolios with higher idiosyncratic volatility have higher average returns. However, they do not present any significance levels for their idiosyncratic volatility premiums. Lehmann (1990) also finds a statistically significant, positive coefficient on idiosyncratic volatility over his full sample period, but he shows that the coefficient on idiosyncratic risk changes sign in different econometric specifications.

More recently, Malkiel and Xu (2002) find a significantly positive relation between idiosyncratic risk and the cross section of expected returns at the firm level. However, their main findings are not based on a measure of an individual stock's idiosyncratic volatility. Andrew, Hodrick, Xing, and Zhang (2006) (AHXZ hereafter) measure idiosyncratic volatility of individual stocks based on the three-factor Fama-French (1993) model and then generate portfolios of stocks sorted by the individual stocks' idiosyncratic volatility. In contrast to Malkiel and Xu (2002), AHXZ (2006) show that stocks with low idiosyncratic risk earn high average returns, and the average return differential between quintile portfolios of the lowest and highest idiosyncratic risk is about 1.06% per month. Contrary to the existing literature, AHXZ (2006) indicate a strong negative relation between idiosyncratic volatility and expected stock returns. Fu (2009) shows that idiosyncratic risk varies substantially over time and suggests that the existing literature cannot identify a positive relation because the conditional idiosyncratic volatility in earlier studies does not capture the time varying property.

Timotheos and Nikolaos (2008), using data from the UK market examined the predictive ability of various measures of idiosyncratic risk and provide evidence which suggests that: (a) it is the idiosyncratic volatility of small capitalization stocks that matters for asset pricing and (b) that small stocks idiosyncratic volatility predicts the small capitalization premium component of market returns and is unrelated to either "pure" market risk or the value premium.

While the anomaly consensus has grown (and is still growing), a clear explanation remains elusive. The paper by Bali and Cakici (2008) sheds light on the methodological differences in previous studies that mainly led the existing literature to present conflicting evidence. They find strong evidence that 4 reasons play critical roles in determining the presence and significance of a cross-sectional relation between idiosyncratic risk and expected returns; (a) the data frequency used to calculate idiosyncratic risk, (b) the weighting scheme adopted for generating average portfolio returns, (c) the breakpoints utilized to sort stocks into quintile (or decile) portfolios, (d) using a screen for size, price, and liquidity.

One potential explanation for the anomaly is investor preference for lottery-type payoffs. Shefrin and Statman (2000) create a model where investors mentally divide their portfolio into two groups, bonds and lottery tickets. They will overprice and accept low expected returns from highly idiosyncratic assets if the possibility of very large payoffs exists. Barberis and Huang (2008) claim that this investor desire for asymmetric payoffs causes return skewness to be priced by the market. Boyer, Mitton, and Vornick (2010) confirm that returns are negatively associated with a stock's expected idiosyncratic skewness. Bali, Cakici, and Whitelaw (2011), Han and Kumar (2012) also show similar results.

Fu (2009) and Huang, Liu, Rhee, and Zhang (2010) both claim that the negative relationship between idiosyncratic volatility and return is related to short-term return reversals. They control for a stock's return during the past month in their models and find idiosyncratic volatility then has little effect. In addition, these reversals are concentrated in smaller stocks, calling into question the economic relevance of the anomaly. On the other hand, Chen, Jiang, Xu, and Yao (2012) find return reversals do not explain the negative relationship between idiosyncratic volatility and return.

Other explanations for the volatility anomaly include Wong (2011), who finds that accounting for a combination of earnings shocks and earnings momentum eliminates the idiosyncratic volatility anomaly; Hsu, Kudoh, and Yamada (2012), who show that inflated earnings forecasts from sell-side analysts cause high volatility stocks to become overpriced and have low returns; Johnson (2004), who claims that unpriced information risk (proxied by analyst forecast dispersion) causes a negative relationship between return and idiosyncratic risk even without market frictions or irrational agents; Hou and Loh (2012) simultaneously test many of the prior explanations and find that lottery preferences, short-term return reversals, and earnings shocks together explain 60 to 80% of the negative relationship between returns and idiosyncratic volatility. Lottery preference alone can explain 48 to 67% of the relationship.

According to Blitz, Van Vliet and Baltussen (2019), the most popular explanations for the low-risk effect can be grouped into five main categories: (i) constraints, (ii) relative performance objectives, (iii) agency issues, (iv) skewness preference, and (v) behavioural biases:

Constraints. The intuition behind this explanation is that in the world of the CAPM there is only one efficient portfolio, and investors simply lever or de-lever this portfolio based on their degree of risk aversion. In the presence of leverage constraints, however, investors looking to increase their return are forced to tilt their portfolios towards high-beta securities. In support of this explanation, Frazzini and Pedersen (2014) find that when leverage constraints are tighter, the low-risk anomaly tends to be stronger.

Relative performance objectives. A second explanation for the low-risk effect is the focus on performance relative to others instead of absolute performance. The CAPM assumes that investors only care about absolute returns, but in reality, many investors focus on beating the market average. Blitz and Van Vliet (2007), Falkenstein (2009), and Baker, Bradley, and Wurgler (2011) argue that if the CAPM would hold, then low-risk stocks are unattractive for benchmark-relative investors, because they involve high tracking error and lower expected return.

Agency issues. The low-risk effect has also been attributed to agency issues. Karceski (2002) argues that profit-maximizing asset managers have a strong incentive to create high-beta products due to the highly asymmetric nature of the flow-performance relationship documented by Sirri and Tufano (1998). Baker and Haugen (2012) generalize this agency problem and argue that all portfolio managers and their analysts implicitly or explicitly have option-like reward structures, which incentivizes them to focus on high-risk assets.

Skewness preference. Another explanation for the low-risk effect is a preference for lottery-like payoffs or positive skewness as explained in Blitz and van Vliet (2007), Baker, Bradley and Wurgler (2011), Ilmanen (2012), and Hsu and Chen (2017).

Behavioural biases. A fifth explanation for the low-risk effect is behavioural biases, such as attention-grabbing bias, representativeness bias, and overconfidence, which cause investors to irrationally 'prefer' higher risk stocks over lower risk stocks. Blitz and van Vliet (2007), and Baker, Bradley and Wurgler (2011) explain this in their studies.

There has been a notion that the low-risk effect was simply a manifestation of the well-known value effect however, Blitz and Van Vliet (2007), Frazzini and Pederson (2014), and Walkshäusl (2014) all find that the alpha of low-risk stocks is not explained when controlling for value (and other factors), for the universes and sample periods in their studies. Arnott, Beck, Kalesnik, and West (2016), argue that low-risk (and other smart beta) indices have been designed based on relatively short-back test periods, and obtain a large part of their return from multiple expansion of their holdings that is not sustainable in the long run. Their work suggested that investors should be aware that low-risk stocks can go through long cycles of being either value- or growth-tilted.

A challenge was made by Novy-Marx (2014) suggesting that the low-risk anomaly is explained by profitability factors. Blitz and Vidojevic (2017) acknowledge that the low-risk effect appears to be subsumed by the profitability factor in time-series regressions. They go on to argue, however, that if it were true that the CAPM relation holds when accounting for interactions with the profitability factor, then it should be possible to construct portfolios which exhibit a clear positive relation between market beta and return, provided one controls for the profitability characteristics of these portfolios. Applying the Fama and MacBeth (1973) cross-sectional regressions approach they find that all factors in the five-factor model are priced, except market beta. In other words, it is not possible to construct high-beta portfolios with a high return and low-beta portfolios with a low return, whether one controls for profitability or not. Based on this finding they conclude that it is premature to assume that the low-risk effect is explained by profitability or other factors. Additional evidence was provided by Blitz, Van Vliet and Baltussen (2019).

Compared to the Fama-French factors, Blitz, Van Vliet and Baltussen (2019) find that the volatility premium is not only stable through time, but also large in magnitude. Whilst many anomalies are known to be concentrated in small-cap stocks and therefore difficult to exploit in reality, Auer and Schuhmacher (2015) show that the low-risk anomaly is strongly present among the largest, most liquid U.S. stocks. Asness, Frazzini, and Pedersen (2014) and Baker, Bradley and Taliaferro (2014) show the existence of this low-risk effect within industries and countries.

There has also been evidence of the low-risk effect on other asset classes. Carvalho, Dugnolle, Lu, and Moulin (2014) and Israel, Palhares, and Richardson (2015) document a low-risk effect within the investment grade corporate bond market, and Houweling and Van Zundert (2017) show that the effect is not only present among investment grade corporate bonds but also among high yield corporate bonds. Generally, there appears to be a low-risk effect within every asset class, however the relation between risk and return only seems to be positive when entire asset classes are considered relative to each other, since stocks have higher returns than bonds, and corporate bond returns are higher than government bond returns, in the long run.

Whilst it can be seen thus far that there are other factors other than the CAPM beta that could generate excess returns, adopting these factors into a portfolio strategy means that the returns achieved must be able to demonstrate its viability as a trading strategy.

2.2.5 Smart Beta Strategy

Often referred to as alternative betas, enhanced indexes, quantamental indexes, strategic beta etc., Malkiel (2014) states that there is no universally accepted definition of smart beta strategies. What most people who use the term have in mind is that it may be possible to achieve greater-than market returns using a variety of relatively passive investment strategies that involve no more risk than would be assumed by investing in a low-cost total stock market index fund, which, by definition, has a beta of one. This view extends that one doesn't have to be a stock picker, as most active managers are, to be able to beat the market rather, one can manage a relatively passive (low turnover) portfolio to accomplish good results more dependably without assuming any extra risk and at a fee well below that charged by active managers. The approach is to tilt the portfolio in some direction (for example, using alternative weighting schemes relative to an index or simply creating an index to reflect tilts towards certain factor(s)) such as value versus growth, smaller versus larger companies, relatively strong stocks versus weak, and low-volatility stocks versus high volatility ones. Different smart beta strategies such as equally weighted, global minimum variance, equal risk contribution and maximum diversified ratio have often been proposed as alternatives to the cap weighted index.

Smart beta strategies are related to multifactor models of asset pricing. If one assumes that the beta of the capital asset pricing model (CAPM) is an incomplete measure of risk, the tilts listed above can be considered as additional risk factors. By tilting the portfolio toward smaller companies, for example, the investor is making a bet that the risk premium that is available from smaller companies can enhance returns. Effectively, smart beta strategies rely on a type of active management. They do not try to select individual stocks but rather tilt the portfolio toward various characteristics that have historically appeared to generate larger than market returns. Malkiel (2014) further concludes that in general, the records of smart beta funds and ETFs have been spotty. Many smart beta ETFs have failed to produce reliable excess returns, although a few have beaten the market over the lifetime of the funds. To the extent that some smart beta strategies have generated greater than market returns, those excess returns should be interpreted as a reward for assuming extra risk. In departing from the market portfolio investors are taking on a different set of risks.

Dat Le (2023) finds that a proportion of the fast-paced growth of smart beta funds can be attributed to the investor migration from closet factor active mutual funds (funds that closely mirror the holdings of their benchmarks while still charging active-management fees) to smart beta ETFs. Using a sample of US domestic equity active mutual funds and smart beta ETFs from 2000 to 2019, Dat Le (2023) find that smart beta ETFs offer factor exposures at lower fees and therefore higher risk-adjusted returns than closet factor funds. Denys (2016) find that roughly 60% of smart beta ETF categories assessed outperformed their declared benchmarks on a total return basis, with only value category offering benchmark-adjusted returns meaningfully different from zero. Two groups, volatility and value significantly outperformed their risk-adjusted benchmarks, the rest have not yielded risk-adjusted returns meaningfully different from zero. Mateus, Mateus and Soggiu (2020) in their sample of 152 US smart beta ETFs over 2000 to 2017 find that as per the risk-adjusted performance about 40% of Smart Beta ETFs outperformed their related traditional ETFs after expenses. Their analysis of performance persistence conducted based on the relative performance of smart beta ETFs show that the performance of winners and losers does persists in the year ahead (the persistence in performance was documented in 7 out of 9 peer categories). The result of Maguire, Moffett and Maguire (2018) reinforce the effectiveness of smart beta strategies and demonstrate that combining multiple strategies simultaneously can yield better performance than that achieved by any single component in isolation.

However, Rompotis (2019) concludes that smart beta ETFs cannot outperform the market. The study finds that, the influence of market factors on ETF performance is modest whereas performance is significantly related to contemporaneous and lagged premiums and intraday volatility. Jordan and Ausloos (2021) assessed a sample of EU-domicile smart beta ETFs and find that they do not seem to enhance the capabilities of traditional cap-weighted investment products by tilting the index towards particular factors in attempt to harvest risk premiums, in brief, therefore not providing a superior performance. Ultimately, the smart beta world remains a fast growing one which is here to stay and the decision to adopt it or not will perhaps depend on the motivating investment philosophy.

2.2.6. Mean Reversion.

The main idea in statistical arbitrage is to exploit short-term deviations in returns from a long-term equilibrium across different strategies such as exposure to these anomalies (factors). This kind of strategy heavily relies on the assumption of mean-reversion of idiosyncratic returns, i.e., reverting to a long-term mean after some time. Effectively, the returns of such series oscillate around their mean (stationary) and when they frequently cross their mean line, they exhibit a tendency to revert to it. In this sense, we say that the shocks affecting a stationary series have only temporary effects.

The realization that the assumption of stock returns being unpredictable is uncertain, has gradually grown therefore, financial researchers have been in search of theories that could help them to predict the stock behaviour and one of the oldest propositions is the efficient market hypothesis, which states that, stock prices reflect all the information available. Fama (1965) presents strong evidence supporting the efficient market hypothesis (EMH). This implies that prices of securities reflect their intrinsic value and consequently prices are unpredictable and follow a random walk. If the efficient market hypothesis is true, then it should not be possible to devise trading rules which allow abnormal profits to be made. The most basic form of the EMH is the weak form which states that a market is weak form efficient if past prices alone cannot be used to earn abnormal returns.

This theory that financial asset markets are efficient is a staple of contemporary finance. While the stylized version of this theory maintains that financial information is disseminated efficiently and, consequently, stock prices are not predictable, there has been a growing body of literature that questions the universality of such a theory. The growth of discontentment with this theory prompted work on other theories that could help to explain the market phenomenon. The proposition that was suggested in financial economics (by keeping in view the idea that “what goes up must come. down”), is the theory of mean reversion. It is effectively the tendency of the market returns to come back to historic values pulled by a gravitational force. DeBondt and Thaler (1985) show weak forms of market inefficiencies when they documented that stock returns tend to be mean reverting. Other researchers have found evidence of negative autocorrelation, or “mean reversion,” in stock returns over long intervals (such as Poterba and Summers (1988), and more recently, Bessembinder et al. (1995), Balvers et al. (2000), and Gropp (2004 (a)).

The validity of this evidence has since been questioned. For example, Lamoureux and Zhou (1996) criticize the properties of the tests used to detect predictability in stock prices. Fama and French (1988) discuss two competing economic explanations of mean reversion in stock returns. Mean reversion can occur due to mispricing in an irrational market in which prices take long temporary swings away from fundamental values. Alternatively, mean reversion may be caused by the predictable movement over time in the security risk premia.

However, Gangopadhyay and Reinganum (1996) were able to show in their work that only under certain assumptions could these explanations be accepted. And, more generally, Gropp (2004 (b)) addressed previous criticisms of tests for stock/portfolio price predictability and finds strong evidence of mean reversion in portfolio prices. His findings imply a significantly positive speed of reversion with a half-life of approximately four and a half to eight years. He stated possible reasons for previous lack of evidence include, but are not limited to, a lack of statistical power to reject the null hypothesis of a random walk, limited availability of long-term data, a need to identify an asset’s fundamental, or simply that asset prices do not exhibit mean reversion. He employed the Balvers et al. (2000) mean reversion model which bears close resemblance to the Augmented Dickey Fuller model in his analysis.

Overtime now, this theory has not only been used to analyse the features of returns, but it has been exploited as a strategy on its own. For example, Dias and Marques (2005) find that the mean reversion and OLS estimators can together be used as a measure of persistence, while Balvers and Wu (2006) studied the combined effect of the momentum and reversion strategies together and find that the excess return in the joined momentum-mean reversion model is higher than the excess returns found in either of the separate momentum or mean reversion models. Many investigations have found the existence of mean reversion in various market indices.

2.3. Factor Investing and Retirement Portfolio Strategies.

One of the issues with factor investing is to define which factors really matter. For a long time, the standard model was the four-factor model of Carhart (1997). Based on the model developed by Fama and French (1992), which is based on the size and value factors, Carhart (1997) proposed to add the momentum factor found by Jegadeesh and Titman (1993). More recently, other factors have emerged such as the volatility, low beta, quality or liquidity factors, to name just a few. However, the existence of more and more factors does not help. In fact, Cochrane (2011) has recently refers to a “zoo of new factors”. For instance, Harvey et al. (2014) counts over 300 in various academic papers, and their number has been increasing exponentially. In such situations, the investor may be lost in front of the factor proliferation. In fact, Harvey et al. (2014) proposes to adapt the usual t-statistic for testing the significance of a newly discovered risk factor. However, according to Cazalet and Roncalli (2014), from a professional point of view, only a few risk factors and anomalies are reliable, and they find that these includes the size and book to market ratio.

Factor-investing strategies have become increasingly popular. With transparent investment processes, low management fees, and the potential for above-average performance, these strategies have diverted a large amount of assets from traditional active management. According to data obtained by Li et al (2019), assets under management (AUM) in factor-investing exchange-traded funds (ETFs) and mutual funds across global markets increased from just below US$75 billion in 2005 to more than US$800 billion by the end of 2016. Furthermore, this figure probably understates the size of this space because it does not include strategies pursued by institutional investors. The trend in AUM growth is likely to persist because factor investing is a hot topic in industry and academic journals and is commonly covered at industry conferences. Large investment consulting firms also recommend that clients diversify their passive allocations to include factor-investing strategies (Kahn and Lemmon 2016).

With the substantial increase in AUM, however, come risks related to factor investing that demand attention. Implementation is another aspect of factor investing. Until recently, these factors were available by investing in mutual funds. Now, both active management and passive management propose investment vehicles and give access to these factors. The development of factor indexes has certainly helped to rationalize the asset management industry. However, one question remains. How to transform these academic risk factors into investible portfolios? This implies studying the capacity, turnover and transaction costs of these factor portfolios and also the impact of the constraints on such investments such as long-only restrictions. According to Cazalet and Roncalli (2014), there are different issues to take into account in order to obtain a factor-mimicking portfolio. They concern the definition of the asset universe, the weighting scheme and the transaction costs. In their paper, they show that these choices have a big impact on the design of factor indexes.

Berk and Green (2004) demonstrates that fund size is inversely related to performance. It stands to reason, then, that excess returns grow scarcer as AUM rises: managers must buy more of the stocks in their opportunity set, creating upward price pressure that inexorably lowers expected return. Conversely, when the managers exit existing positions, their trading generally pushes prices down, reducing realized return.

Factor-investing indexes are not immune to the return-dampening effect of trading costs, even though these costs are not easily observable. Novy-Marx and Velikov (2016) confirm that most anomalies with low turnover continue to generate statistically significant alpha. They also observe that the asset universe and weighting scheme have an impact on profitability. For instance, they estimate that transaction costs for equally weighted risk factors are two to three times higher than those for value-weighted risk factors.

When designing portfolios for the withdrawal phase of retirement planning, certain parameters for consideration are important. These include identifying sources of excess return, risk minimization and portfolio durability against shocks. Despite the constraints associated with the factor investing strategy, it still presents a viable source of successful investing as evident by the numbers sighted earlier.

Whilst the decision of how much is a sustainable amount to withdraw from a retirement portfolio is an important one, an even more important question is what investment strategy should be adopted to potentially maximize the initial issue of sustainable amount. As Merton, (2014) puts it, the construction of investment portfolios for the retirement journey is the 'known unknowns'. One of the objectives of this study is to explore how factor investing strategy may provide a better "fixed-real" withdrawal experience relative to the market.

There are a number of retirement portfolio investment strategies targeting either the contents (asset) of the portfolio or the mechanism of investment, and whilst there has been some contributions to the latter such as Clare et al (2020) who finds that smoothing the returns on individual assets by simple trend following techniques is a potent tool to enhance withdrawal rates, not much has been done on designing content based strategies with the objective of enhancing the decumulation phase of the retirement journey.

In terms of the content/asset-based strategy, the available literature focuses almost exclusively on bond and equity combinations (i.e., various percentage compositions are used to form the portfolios for analysis) however, diversification to other asset classes such as commodities has been shown to dramatically improve the risk-return possibilities for investors for example, Clare et al, (2016) introduce commodities, real estate, and credit and compared the decumulation possibilities with equity/bond portfolios; they found that the former offers a better withdrawal rate experience in general.

The inclusion of bonds in drawdown portfolios are generally to control risk whilst the equity content is the growth/income element of these portfolios. This thesis attempts to explore asset-based strategies that may enhance the drawdown experience by focusing on equities only portfolios to see if certain peculiarities of stock selection (factor equity strategy) will offer this enhanced experience. This equity only portfolio offers the opportunity to focus on the growth element of portfolios as it is an apparent driver of sustainable rates of withdrawal. Furthermore, this approach strips the analysis of any influence (positive or negative as it may be) of any other asset class and focuses on the exclusive features of the asset class to highlight the strategy itself. More so, Estrada (2016) concludes that an all-equity portfolio is a simple and very effective strategy for retirees to implement and Lui et all (2009) finds that, for real withdrawal rates of 4% or less and expected time horizons of 20 years or less, asset allocation is not an important factor in determining the probability of success, since all asset allocation models tested from 100% bonds to 100% stocks succeeded nearly 100% of the time.

2.4. Withdrawal Strategies.

As stated earlier, one of the main considerations for advisers designing retirement strategies is deciding what withdrawal strategy should be adopted during retirement to ensure a sustainable rate of withdrawal. Withdrawals at the drawdown phase of the retirement journey are generally either fixed or variable, nominal or real, and the most attention has generally been given to fixed, real withdrawals since Bengen (1994) show that an initial withdrawal rate of 4%, with annual withdrawals subsequently adjusted by inflation, was 'safe' in the sense that, historically, this strategy never depleted a portfolio in the US in less than 30 years. Pfau, (2010), Blanchett et al, (2016) however show in subsequent research that over the 115 years between 1900 and 2014, a 60% equity-40% bond portfolio of U.S. stocks and bonds had a failure rate[4] of 4.7%, and portfolios with at least 70% in U.S. stocks had an even lower (3.5%) failure rate. In other markets, much higher failure rates are experienced with the 4% rule.

The 4% fixed-real strategy (Bengen, 1994) is where 4% of the initial portfolio value is set at the initial withdrawal value and this value is subsequently adjusted for inflation in subsequent years. Other variations of the 'fixed' strategy include 'floor and ceiling' strategy (Bengen, 2001), where the 4% from the fixed-real approach is subject to an upper and lower bound; 'modified 4%' strategy where Clyatt (2005) specify that the withdrawal amount in any year should be 4% of the portfolio value in that year

[4] *Failure Rate is the number of periods relative to the total number of simulated periods (or periods considered) when the specified withdrawal rate is not sustained through the period.*

rather than the year of drawdown inception; 'constant probability of failure' strategy (see Frank, Mitchell and Blanchett 2011) where the goal is to determine the percentage that can be withdrawn each year based on the idea of maintaining a constant probability of failure through time; 'decision rule' strategy (see Guyton and Klinger 2006) where dynamic rules such as capital preservation and prosperity rules are used to guide withdrawals; 'safe reset' strategy (see Stein and DeMuth 2005), where the withdrawal rate is a function of the retiree's age and adjusted only for inflation for five years before being reset to a new withdrawal rate determined by the expected number of years remaining in the person's retirement.

More recently the perfect withdrawal rate (PWR) was introduced by Suarez et al, (2015), the methodology involves using Monte Carlo methods to create a distribution of the PWR with a view to considering the likelihood of any particular withdrawal rate leading to success/failure over the ensuing decumulation period. Some fixed strategies have also taken advantage of incorporating the use of annuities for example, 'half annuity' strategy (see Updegrave 2007) and 'delayed annuity' strategy (see Clements 2007). Fixed withdrawals are generally easier to understand hence why it is attractive to advisers as it is simple and practical to convey to clients. Also, in the case of fixed real withdrawals, they preserve purchasing power. However, they do not adjust to changing market conditions or life expectancy: this may lead to depletion of a retirement portfolio earlier than desired, with calamitous results.

Meanwhile, variable withdrawals do adjust to changing conditions and hence reduce or eliminate the risk of failure. They encompass a broad set of strategies in which withdrawals are adjusted based on changing life expectancy (Dus et al, 2005), changing market conditions (Estrada, 2016), or both (Stout and Mitchell, 2006). However, they typically are more difficult to understand and implement (see Stout, 2008) and may require a retiree to reduce their real consumption at some point. An optimising retiree might choose to do this depending on their view of longevity risk, as discussed by Milevsky and Huang (2011). In fact, Lui et al (2009) conclude that dynamic and sophisticated decision rules are not viable strategies for the investing public.

By far, the most attention (both in practice and literature) has been given to the 4% fixed-real strategy by Bengen (1994) however, it is not uncommon to find a blend of both fixed and variable withdrawal strategies in practice and in fact, some of these have been prominent in existing literature. Following the examination of available literature by Spitzer, Jeffrey and Sandeep (2007), if attention is confined to real withdrawals from a portfolio over 30 years, the literature appears to contain conflicting results. Withdrawal rates considered safe or sustainable vary from 3 percent to more than 6 percent, while optimal asset allocations range from 50 percent to 100 percent stock. The results are all plausible because the outcome depends on the subjective definition of sustainable and safe.

2.5. Investigating Sustainable Withdrawal Rates.

There are generally 2 approaches adopted to investigate sustainable withdrawal rates. One set of studies often use rolling historical periods, such as 30-year periods from 1951 to 1980, 1952 to 1981, and so on. Therefore, for example, given a 50-year data, sub data of 30-year blocks are obtained in a rolling (overlapping) period fashion. If the results indicate that all portfolios survived at least 30 years for withdrawal rates of 4 percent for example, these studies conclude that a 4 percent withdrawal rate is "sustainable." This approach clearly requires a sizeable data in order to obtain substantial amounts of sub data. Pioneer studies such as Bengen (1994 (and subsequent updates), Blanchett (2007 and 2008), Pfau (2010), Cooley, Hubbard, and Walz (1998) are some examples of many that employ this approach in their study.

The second approach employs the use of simulations (Monte-Carlo and Bootstrap are the common techniques) where for example, various simulations of a 30-year returns are obtained from an original 30-year period data; this approach is usually employed where there are data constraints (size). Guyton and Klinger (2006), Milevsky, Ho and Robinson (1997), Spitzer, Jeffrey and Sandeep (2007), Pye (2000) and Tezel (2004) are also some examples of this method. Clearly, these 2 approaches in their structure attempt to consider the effect of sequential risk (sequence of returns) and presenting the results in the form of probability estimates of success or failures.

An important question therefore is whether the choice of method used to represent the future affects estimates of the sustainability of a retirement portfolio. Cooley, Hubbard, and Walz (2003) use both methods to calculate portfolio success rates (the percentage of retirement experiences during which the retirement portfolio provided planned withdrawals and finished the period with a positive value) for a range of withdrawal rates, portfolio compositions, and pay-out periods. Their results showed that both models were generally consistent; a 4 percent withdrawal rate could be sustainable over a 30-year period with a balanced portfolio, but to ensure a 90 percent success rate, a larger allocation to equity and withdrawal rates of less than 4 percent were necessary.

Other less common methods have also been introduced. Ragsdale, Seila, and Little (1994) provide a mathematical algorithm that uses discounted cash flows to determine the optimal withdrawal rate from tax-deferred retirement portfolios and in a new approach, Milevsky and Robinson (2005) introduce the concept of a stochastic present value, which addresses the withdrawal issue from an actuarial perspective. Spitzer, Jeffrey and Sandeep (2007) argue that the use of the simulation method provides more extensive examination as it creates thousands of different combinations of sub-periods. They pointed that this is more suited for the probability-based conclusion of sustainability.

2.6. Summing – Up

This chapter introduces the two main types of pension arrangement in the UK and then reviews prior studies into the various factors of excess return other than the beta established by the CAPM theory and how investing in these factors have produced this result both in local and international markets. The chapter further reviewed studies that have been carried out on the various withdrawal strategies retirees adopt at the point of withdrawing their pensions and how prior research have investigated a sustainable withdrawal rate. Based on this review, a significant gap was identified; how these various factors of excess return may be adopted in constructing retirement portfolios primarily for the purpose of sustainable withdrawals. Whilst these studies have established that these factors can be adopted as investment strategies generating various return premiums, not much (if any) has been done on using these factor strategies for withdrawal purposes. The purpose of this thesis is to provide a starting point aimed at filling this gap. The remaining empirical chapters of this thesis examines using stocks within the FTSE350 market to construct portfolios based on some of these established factors and then studying how they perform under various withdrawal scenarios using the simulation approach of studying sustainable withdrawal rates. An ad-hock statistical measure aimed at providing similar information on the stability of these portfolios for withdrawal purpose was also proposed and studied.

3 Factors of Excess Return

3.1 Introduction

This chapter focuses on identifying relevant risk factors (other than that stipulated by the Capital Asset Pricing Theory) and assesses the presence of a reversion to mean of the returns from the exposure to each factor. The existing literature indicates that size, book to market ratio and profitability are established factors that explains excess return. This chapter makes the contribution of studying the UK FTSE 350 universe as an attempt to exclude the long tail of illiquid stocks and also explores the presence and mean reversion of the volatility anomaly in the UK.

Based on the definition of these factors described in the previous chapter, each year, the constituent stocks of the FTSE 350 were grouped accordingly so for example, for the size factor, in year 1996, the FTSE 350 is grouped into small and big stocks then the average monthly capitalised weighted total return is obtained; this is then repeated for subsequent years under consideration. This process is again carried out for the book to market ratio (where value and growth stock are grouped), profitability (where low and high profitability stocks are grouped) and volatility factors (where low and high volatility stocks are grouped). The resulting return series for each portfolio group is then assessed for mean reversion. The finding in this chapter shows that the returns of all the portfolios formed based on these factors are mean reverting with various speeds thereby indicating their potential to be explored as an investment strategy.

Generally, the results from this study show that by observing the returns of the portfolios, small sized portfolios (lower MEs) and high book to market ratio portfolios (value stock) have historically performed better than their respective counterparts (large sized and low BE/ME stocks) in absolute terms. In risk adjusted terms, the CAPM confirmed the size effect, but the value effect was not confirmed. However, in contrast to recent studies which finds that the size and value outperformance have disappeared in recent time (such as Van Dijk (2011)), this study finds that the observed returns of small and value stocks have continued to perform better even in recent times. The observed results of the profitability factor conform largely to existing literature in that high profitable stocks have better average returns compared to low profitable stocks. In risk adjusted terms, the profitability effect was identified. The findings for the volatility factor confirmed the anomaly gaining traction that low volatile stocks produce higher average returns than high volatile stocks. However, although low volatile stocks produced lower observed absolute total returns compared to the high volatile stocks, the risk adjusted returns confirmed the growing consensus of the existence of a volatility anomaly.

In addition, based on observation, compared with the market portfolio, the size and book to market ratio factors produce better returns but the profitability factor has a below market performance. The high volatility stock portfolio produces a higher than market absolute return, but the low volatility portfolio does not. However, in risk adjusted terms, only the low volatility portfolio produced significant returns in excess of the market.

Furthermore, while small stock portfolios have more persistent returns (as they revert slower to their means), the value portfolio returns are less persistent (in comparison to their counterparts). The returns of the low volatile stocks are slightly more persistent compared to the high volatile stocks.

The rest of this chapter is organized as follows: Section 3.2 describes the data and methodology; section 3.3 presents the results and section 3.4 concludes.

3.2 Data and Methodology

Data in this chapter was collected from DataStream, Office of National Statistics and Dimensional Fund Managers Returns web (which combines data from DataStream, UK Debt Management Office and FTSE Tradeweb). The sample period covered ranges from 1985 through 2019 due to constraints of data availability. A universe of FTSE 350 and not the FTSE all share index was considered mainly because, according to Alan, Rajesh and Angela (2009), UK stock market exhibits a large "tail" of small and illiquid stocks, which are almost certainly not part of the tradable universe of the major institutional investors.

Financial companies have not been excluded because pension investment funds will usually include this sector in the portfolio.

Following the logic in Agarwal and Taffler (2008), who notes that 22% of UK firms have March year ends, whereas 37% of firms having December year ends, I have used December-year-ending accounting data. The portfolios are reconstructed at the beginning of January every year. The Fama French construction of market capitalisation (ME) and book to market ratio (BE/ME) has been used. ME is basically the market capitalisation which is commonly defined as the share price multiplied by the number of shares outstanding. Book equity (BE) is the common shareholders' equity in a company and is calculated as the difference between Total Assets and Total Liabilities. The monthly data used for ME spanned from 1985 through 2019 but data for BE only commenced from 1996 in Datastream[5]. Furthermore, in order to account for the effect of reinvesting dividend income, the monthly total return index for the individual stocks is used to proxy individual stock return with the base date of 1964[6]; reinvesting dividends has been shown to have a significant influence in assessments of performance. Transaction cost was also not considered for the purpose of simplicity.

The earnings before interest and tax (EBIT) often used interchangeably with operating profit is the difference between a company's revenue and its costs and expenditures arising directly out of a company's regular operations. It is calculated before any deductions in the income owing to non-operating activities such as interest expense, corporate tax payments, material gains or losses arising from changes in accounting policy and the like and excludes any income derived from outside the firm's regular activities. In line with the Novy-Marx (2013) preference for the use of gross profit, the EBIT data has been used as the measure of profitability and the profitability factor is obtained as EBIT/BE (as in Fama and French (1992)). The data used spanned from 1996 through 2019.

[5] *Data for market capitalisation (ME) is available from 1985 but data for book equity (BE) only commenced from 1996. Therefore, for the purpose of assessing excess returns of the individual factors, these different start dates are considered. For other assessments, dates are harmonised to commence from 1996.*

[6] *The total return index for the FTSE All Share had a more distant base date compared to that of the FTSE350 therefore, the total return of the FTSE All Share was estimated and the total return of the FTSE350 constituents was extracted from this.*

As mentioned earlier, the measure of volatility which is most relevant to the decumulation phase of investment is the total variance hence, the standard deviation of the monthly return of stocks was used. Volatility is estimated as the standard deviation of the preceding 12months. The risk adjusted return was estimated using the Sharpe ratio (Sharpe 1994) $(r_p - r_f)/\sigma_p)$ where, r_p is the return on the portfolio, r_f is the risk-free rate of return and σ_p is the standard deviation of the portfolio returns. The risk-free rate used is the UK 1 month Treasury Bill obtained from the Dimensional Fund Managers Returns Web.

Survivorship bias which is the tendency to ignore data changes over time has been shown to have significant impact on results. With respect to this research, this bias becomes important due to the fact that stocks migrate between indices over time so that in a year, a stock may belong to the FTSE 100 and in another year FTSE 250. The movements between these indices are not necessarily relevant within the context of this research, but the movement outside these (for example, movements into the FTSE Small Cap) is important hence why the index (as well as other indices) are reviewed periodically. In order to account for this potential bias, the data used was collected for each individual year which reflects the annual updates to the index that takes place every September. Therefore, the data reflects the movements (outside the FTSE 100 and FTSE 250) that occurred during the period collected. Following this, the market capitalisation, book equity, earnings before interest and tax and total return index data which is available on Data Stream was collected for the constituent stocks in each of these years. Monthly return is estimated as: (Return index in month 2 - Return index in month 1) / Return index in month 1. Volatility for each month is estimated as the standard deviation of the returns for the preceding 12months. Once the portfolios based on the various factors have been created (as described earlier) on a yearly basis, the monthly return and market capitalisation of the constituent stock of each portfolio (each year) is collected to estimate the portfolio cap weighted monthly return. Following this, the average monthly return for that year (and other years) is estimated to generate average monthly return for each factor portfolio from 1996 through 2019.

3.2.1 Estimation of Factors

In the following, I explain how portfolio for each factor is constructed. The ME factor was defined using the same approach by Fama French (1992) and subsequent similar research. As suggested by Alan, Rajesh and Angela (2009), the FTSE 350 break point is used. Following the collection of the FTSE 350 constituent stocks, the ME (market capitalisation) of each constituent was then obtained. Thereafter, the median of the ranked market capitalization was obtained, and stocks ranked above this value constituted the large stock portfolio while those ranked below form the small stock portfolio. This exercise is repeated yearly to form the '*S*' (small) and '*B*' (big) portfolios for each year in my data range.

The book to market factor was estimated as the ratio of book equity over market capitalisation (BE/ME). Staying with the Fama-French approach, this ratio was obtained for each constituent stock and then ranked (descending order). The 30^{th} and 70^{th} percentiles break points are used. Stocks within the 30^{th} percentile form the value portfolio and stocks within the 70th percentile form the growth portfolio. This process is again repeated for each year to reconstruct updated portfolios. Data for BE commences in 1996 and negative BE firms are excluded in the portfolio formation just as in Fama and French (1992) The '*H*' (value) and '*L*' (Growth) portfolios are formed as described.

The $EBIT_{(t)}/BE_{(t-1)}$ ratio was used to define the profitability factor as it represents a reasonable proxy for economic income in year *t* with the book equity in year (*t-1*). The same 30^{th} and 70^{th} percentile approach is then used after ranking (in descending order) the ratio for each constituent stock to establish high profitability and low profitability portfolios respectively. Data used spans from 1996 through 2019. The '*Hp*' (high profitability) and '*Lp*' (low profitability) portfolios are formed here.

The total volatility measure is employed to analyse the volatility factor. For each constituent stock, the volatility at the beginning of each year was estimated as the standard deviation for the previous

12months. This is then ranked and the 30th and 70th percentile approach is adopted to create high and low volatility portfolios respectively. The '*Hv*' (high volatility) and '*Lv*' (low volatility) portfolios are formed here.

Following this, the total monthly return for each constituent stock in each portfolio in every year is then obtained. For every year, the market cap weighted total monthly return is then obtained for each portfolio [7]. This is obtained for each year by dividing the market capitalisation of each constituent stock by the total market capitalisation of the portfolio and then multiplying the resultant weighting factor by the individual monthly return. Hence, producing a series of monthly cap weighted returns for each portfolio for every year.

3.2.2 Estimation of Mean Reversion.

One of the key trading concepts in the quantitative toolbox is that of mean reversion. This process refers to a time series that displays a tendency to revert to its historical mean value. An important property of stationary series is that they frequently cross their mean line and exhibit a tendency to revert to it. In this sense, we say that the shocks affecting stationary series have only temporary effects. Mathematically, such a (continuous) time series is referred to as an Ornstein-Uhlenbeck process. This is in contrast to a random walk (Brownian motion), which has no "memory" of where it has been at each particular instance of time. The mean-reverting property of a time series can be exploited in order to produce profitable trading strategies. Effectively, this feature has predictive implications that can be used to justify the use of a strategy. The measure of the rate of mean reversion can then be used to infer how quickly a series would revert to its historical mean following a shock.

[7] *The data (particularly for BE) has the issue of missing data. Therefore, each year the missing data is excluded, and the available data forms the universe from which the factor portfolios are formed.*

It is generally accepted that a stationary process displays mean reversion. The statistical tests necessary to identify mean reversion are therefore broadly contained in the concept of stationarity. There are many tests which can be used to check if a data series follows a random walk process (or otherwise). One of which is the Dickey – Fuller unit root test (DF) (Dickey and Fuller, 1979). The DF test consists of estimating the regression coefficient φ of series X_t on X_{t-1}. If this coefficient is significantly below 1, it means that the process is mean reverting: if it is close to 1, the process is a random walk (Geman, 2007: p 233)

Given the auto regression AR (1) process of:

$$\hat{X}_t = c + \varphi(\hat{X}_{t-1}) + \varepsilon_t \qquad (1)$$

where c is the intercept and ε_t the error term.

A weakly stationary AR (1) process notably implies:

$$E[\hat{X}_t] = \mu = constant$$

for all t.

This property may be used (simply by taking the expectations on both sides of the price process above) to find that the stationary mean μ computes as:

$$\mu = \frac{c}{1 - \varphi}$$

or

$$\mu = -\frac{c}{\varphi - 1}$$

Which allows writing the process as (by replacing c given the identity above):

$$\hat{X}_t - \mu = \varphi\left(\hat{X}_{t-1} - \mu\right) + \varepsilon_t \qquad (2.1)$$

or,

$$X_t = \varphi(X_{t-1}) + \varepsilon_t \qquad (2.2)$$

Where, $X_t = \hat{X}_t - E(\hat{X}_t)$ may be interpreted as the distance to the stationary mean.

By modelling the monthly returns of a portfolio R_t, the standard Dickey-Fuller test for on an AR (1) process will take the regression form of equation 2.2 above:

$$R_t = \varphi(R_{t-1}) + \varepsilon_t$$

$$-1 < \varphi < 1 \text{ and } |\varphi| < 1 \text{ for a reverting process}$$

The test for stationarity (unit root) here is whether $\varphi = 1$

The unit root tests described above is valid if the time series R_t is well characterized by an AR (1) process with white noise errors. Many financial time series, however, have a more complicated dynamic structure than is captured by a simple AR (1) model. Said and Dickey (1984) augment the basic autoregressive unit root test to accommodate general ARMA (Auto Regressive Moving Average) models with unknown orders and their test is referred to as the augmented Dickey Fuller (ADF) model.

Subtracting R_{t-1} on both sides of (2.2) gives:

$$R_t - R_{t-1} = \varphi R_{t-1} - R_{t-1} + \varepsilon_t$$

giving:

$$R_t - R_{t-1} = (\varphi - 1)\, R_{t-1} + \varepsilon_t$$

thus, the expression can be written as:

$$\Delta Rt = \beta Rt{-}1 + \varepsilon t \qquad (3)$$

where,

$\beta = \varphi{-}1$

Rt is returns at time t

$\Delta Rt = Rt - Rt{-}1$ *and* $\mathcal{E}t \sim N(0, \sigma 2)$

This is the Augmented Dickey Fuller (ADF) model motivated by the Ornstein-Uhlenbeck stochastic differential equation { $dx_t = \theta(\mu - x_t)dt + \sigma dW_t$ } where θ is the rate of reversion to the mean, μ is the mean value of the process, σ is the variance of the process and W_t is a Wiener Process or Brownian Motion. There will be additional lag regressors to equation 3 for higher order AR processes under the ADF approach.

From the ADF model, a negative β will imply that the series exhibit the tendency to revert to its mean and the reversion to the long-term mean μ (mean of the observations) is proportionate to β. While $|\beta|$ is a measure of speed of mean reversion, φ is a measure of persistence (i.e., Persistence $\varphi = \beta$+1) and by implication, there will be an inverse relationship between these measures (higher persistence implies low mean reversion and vice-versa). Other measures of persistence such as the one suggested by Dias and Marques (2010) tend to produce similar results for AR(1) processes. The regressions under consideration in this section are effectively AR(1)s. It is easy to see that there is a monotonic relationship between speed of mean reversion and |φ|.

A scalar measure of persistence (half-life; the time taken for the process to revert halfway to the mean) for an AR (1) process can also be shown to be:

$$hl = \frac{In\ (0.50)}{In\ |\varphi|}$$

Therefore, from the standard Dickey Fuller test for AR (1) based on (2.2) above, since $\beta = \varphi - 1$, the null hypothesis test becomes $1+\beta = 1$ and the boundaries are $-1 < 1+\beta < 1$. Accordingly, the boundaries for the ADF test based on equation (3) will be $-2 < \beta < 0$ and hence, the null hypothesis is $\beta = 0$. Akter and Majumder (2013) as well as Naznin, Paul and Majumder (2014), used this boundary restrictions in their work on unit root tests.

There are variations of the Augmented Dickey Fuller model above. Firstly, the above version (equation 3) tests for the presence of a pure random walk with no trend or intercept.

The second form is as follows:

$$\Delta R_t = \alpha_0 + \beta R_{t-1} + \varepsilon_t \qquad (4)$$

It tests for a random walk with a drift term, where α_0 is the drift term.

Lastly, the following equation:

$$\Delta R_t = \alpha_0 + \beta R_{t-1} + \alpha_1 t + \varepsilon_t \qquad (5)$$

This tests for a random walk with both a drift, α_0 and a linear trend α_1 . The difference between the three regressions concerns the presence of deterministic elements α_0 and $\alpha_1 t$. The parameter of interest in all the regression equation is $\beta, if\ \beta = 0$, the $\{R_t\}$ sequence contains a unit root (Enders 2004).

Mathematically, the ADF model is based on the idea of testing for the presence of a unit root in an autoregressive time series sample. It makes use of the fact that if a return series possesses mean reversion, then the next return level will be proportional to the current return level. Usually by running a simple linear trend model $r_t = \rho + \gamma(t) + \varepsilon_t$ (where γ is a measure of the average change in r_t from one period to the other, ρ is a constant term and ε_t is the error term), we can decide which of the ADF variations to run. If $\gamma(t)$ is significant (i.e., the *p value)*, this will indicate the presence of a trend stationary process. It is important to include the intercept in the choice of the ADF variation as it shows the response R_t when the explanatory variables take on the value of zero.

The role of the ADF hypothesis test is to consider the null hypothesis that $\beta = 0$, which would indicate (with $\alpha_0 = \alpha_1 = 0$.) that the process is a random walk and thus non mean reverting. If the hypothesis that $\beta = 0$ can be rejected, then the following movement of the return series is proportional to the current return and thus it is unlikely to be a random walk.

To estimate the ADF, the first task is to calculate the test statistic (ADF_τ), which is given by the sample proportionality constant (i.e., the least squares estimate) $\beta = \varphi - 1$ divided by the standard error of the sample proportionality constant:

$$ADF_\tau = \frac{\beta}{SE(\beta)}$$

Dickey and Fuller have previously calculated the distribution of this test statistic, which allows us to determine the rejection of the hypothesis for any chosen percentage critical value. The test statistic is a negative number and thus in order to be significant beyond the critical values, the number must be more negative than these values, i.e., less than the critical values.

After running the simple linear trend model stated earlier, there was no need to include a trend in the regression for the ADF test ($\gamma(t)$ was insignificant). The hypothesis for the ADF test used are: H_0: $\beta = 0$ (presence of unit root) versus the alternative hypothesis of H_1: $\beta < 0$ (stationarity).

The commonly presented "rule of thumb" that spurious regressions are accompanied by low values of DW (Durbin Watson Statistic) statistics and high adjusted R^2 was confirmed by Baumöhl and Lyócsa (2009) using their simulation and they find that the differences are very significant. Generally according to Granger (2008), various statistics can be used to describe the quality (spuriousness) of a regression, including R^2, *t-statistics* for β, and Durbin–Watson statistic d which relates to any autocorrelation in the residuals. A good fitting model should have R^2 quite near one and d near 2. All the regressions run returned DW with roughly the value of 2, suggesting that the models presented in this chapter do not have a spurious regression problem. Furthermore, the Schwarz info criterion was used to establish the optimal length of the ADF (Augmented Dickey Fuller) models run.

Table 1: Summary Statistics Table of Factors (Data harmonised to commence from 1996)

Portfolio	Mean (Dependent Variable)	Standard Deviation (Dependent Variable)	Standard Error of Coefficient	R^2	Durbin Watson Stat.	No. of Observation
Small Portfolio	1.29E-04	5.66%	0.0435	0.4105	1.96	287
Big Portfolio	9.78E-05	5.63%	0.0404	0.4862	1.99	287
Value Portfolio	1.22E-04	7.10%	0.0500	0.4497	1.98	287
Growth Portfolio	-1.50E-05	5.70%	0.0400	0.4910	2.00	287
High Profitability Portfolio	1.15E-04	5.67%	0.0405	0.4927	1.99	287
Low Profitability Portfolio	5.18E-05	7.36%	0.0590	0.4570	1.98	287
High Volatility Portfolio	-1.43E-05	11.00%	0.0811	0.4578	1.98	287
Low Volatility Portfolio	9.36E-05	3.83%	0.0283	0.4555	2.00	287
All of Market	8.02E-05	5.61%	0.0401	0.4900	1.99	287

3.3 Results

Table 2: Summary of Returns of Factors

The FTSE 350 universe was considered. After sorting into the individual portfolios based on the various factors, the weighted monthly return of each constituent is obtained as the stock return times stocks weighted capitalisation relative to the total portfolio capitalisation. The sum of this monthly weighted return for all the portfolio constituents forms the portfolios total monthly return. Then the average monthly returns through the data period (and the last 10yrs till date) was calculated. The risk-free rate is the UK 1month Treasury Bill and it is an average of 0.41% per month for the period of 1986 through 2019. Sharpe Ratio is estimated as :(average monthly portfolio returns – average monthly risk-free rate)/standard deviation of portfolio returns. The standard deviation of the FTSE 350 is 4.36%; 3.26% for the low volatility portfolio and 8.0% for the high volatility portfolios.

	Small Portfolio	Big Portfolio	Value Portfolio	Growth Portfolio	High Profitability Portfolio	Low Profitability Portfolio	High Volatility Portfolio	Low Volatility Portfolio	All of Market		
Average Monthly Total Return (%)	1.398 [1]	1.357 [1]	1.23 [2]	1.22 [2]	1.12 [2]	1.00 [2]	1.495 [3]	1.267 [3]	1.354 [1]	1.168 [2]	1.327 [3]
Average Monthly Return of Last Decade (%)	1.254	1.068	1.423	1.053	0.934	1.171	1.049	0.998		1.071	
Sharpe Ratio	0.21 [1]	0.22 [1]	0.19 [2]	0.24 [2]	0.22 [2]	0.14 [2]	0.14 [3]	0.26 [3]			
Excess Return	0.044	0.003	0.062	0.052	-0.048	-0.168	0.168	-0.06			
Data harmonised from 1996											
CAPM Beta	0.89***	1.00***	1.04***	0.97***	0.98***	1.10***	1.77***	0.53***			
|*t*|-*Stat*	23.67	180.45	21.2	57.85	78.17	23.36	30.62	19.22			
Alpha	0.18%	0.00%	0.03%	0.08%	-0.03%	-0.26%	--0.48%**	0.29%***			
|*t*|-*Stat*	1.18	0.16	0.14	1.09	0.58	1.33	2.00	2.59			
Maximum Drawdown	-67.37%	-22.20%	-61.84%	-19.69%	-21.39%	-58.18%	-52.18%	-21.87%		-22.98%	

1 - Start date 1985

2 - Start date 1996

3 - Start date 1986

**** = 99% confidence level; ** = 95% Confidence level; * = 90% Confidence level*

Over the period of 1985 through 2019, the small portfolio had an observed monthly average total return of 1.398% while the big portfolio returned an average of 1.357%. The average monthly *SMB* premium (small minus big) was about 4 basis points or about 48 basis points per year. Furthermore, the small portfolio outperformed the big portfolio 53% of the time considered and after breaking the period into decades, the small stock has outperformed over the recent 2 decades but underperforms in the earlier periods. Contrary to what current literature suggests, within the universe considered, small stocks have sustained a better performance compared to big stocks over the last 10yrs and generally, this result conforms to historical observations showing small stock's better performance. The CAPM results show that the small portfolio is at least able to produce the market return (less risk-free rate) with a lower beta relative to the big portfolio hence confirming the size anomaly.

Big and Small Portfolio Robust Check

Factor returns obtained from characteristic-based sorting are based on their signal, but also on the procedure and choices of the factor construction method. Since there is no consensus on which choices to make when constructing factors, researchers face a number of degrees of freedom. The approach used to construct the portfolios thus far largely conform to what is contained in existing literature where the small/big portfolios are created based on a median breakpoint and other characteristic factors follow various breakpoint approach, commonly used is the 70th/30th and 80th/20th percentile breakpoints; the factor characteristic breakpoint used in this work is the former. The use of this approach has traditionally been to control for the market cap in the factor portfolios and avoid the inadvertent mega and micro-cap classification.

The robust check in this section aims to observe any impact if any, of sorting the small/big breakpoints with the 70th/ 30 percentile approach rather than the generally adopted median. Table 2a below shows the outcome of using this approach. Over the period of 1985 through 2019, using the simple CAPM model, small portfolio produced a positive risk adjusted return in excess of the market however, this is not statistically significant. This outperformance is also evident in the last 2 decades but only the outperformance of the last decade was significant. The big portfolio did not show any additional risk adjusted return in excess of the market. In addition, the size anomaly is confirmed; over the period 1985 – 2019, the market return (less risk-free rate) could have been achieved by the small portfolio with a lower beta relative to the big portfolio as the small portfolio had a lower (and significant) CAPM beta relative to the big portfolio. Thus, over this period, the outcome of the big and small sort portfolio remains the same irrespective of the sorting approach. Amar and Van Vliet (2022) conclude however, that factor returns should not be compared except the construction methodology is the same. In light of this and in order to compare the outcome of this size effect with literature, the median breakpoint approach earlier presented which shows the presence of this anomaly has been adopted.

Table 3: Summary of Returns of Small and Big Factor Portfolios with $70^{th}/30^{th}$ Breakpoints

The portfolios are created as in table 2a except that the big portfolio is now the top 30% stocks by capitalization and the small portfolio is now the bottom 30% stocks by capitalization. As in table 2a, market beta is given as the coefficient of regressing excess return of a portfolio over risk free rate by the excess return of the market over risk free rate. The alpha is the intercept of this regression. 95% confidence level.

	Small Portfolio	Big Portfolio	All of Market
Average Monthly Return (1985 - 2019)	1.25%	1.35%	1.35%
Std. Dev.	4.54%	4.35%	4.35%
Sharpe Ratio	0.19	0.22	0.21
CAPM Beta	0.82*	0.99*	1.00
$\|t\|$-*Stat*	25.84	209.84	
Alpha	0.06%	0.00%	
$\|t\|$-*Stat*	0.40	0.02	
(Risk-Free rate: 0.43%)			
Average Monthly Return (2010 - 2019)	1.17%	1.04%	1.07%
Std. Dev.	3.14%	3.46%	3.39%
Sharpe Ratio	0.36	0.29	0.3
CAPM Beta	0.77*	1.01*	1.00
$\|t\|$-*Stat*	16.47	92.28	
Alpha	0.33%*	-0.04%	
$\|t\|$-*Stat*	2.01	1.14	
(Risk-Free rate: 0.04%)			
Average Monthly Return (2000 - 2009)	0.99%	0.72%	0.81%
Std. Dev.	5.66%	4.55%	4.61%
Sharpe Ratio	0.11	0.08	0.1
CAPM Beta	0.96*	0.98*	1.00
t-Stat	13.44	114.87	
Alpha	0.20%	-0.08%	
t-Stat	0.61	0.05	
(Risk-Free rate: 0.36%)			

The average total monthly return of the growth portfolio for the period considered was 1.22% while that of the value was 1.23% with an average monthly HML (high minus low) premium of 1 basis points per month or 12 basis points per year. By breaking the period into decades (2 decades and 4yrs), the value portfolio outperformed in the most recent 2 decades (1.423% Vs 1.053% and 1.138 Vs 0.661). However, the 4yrs of 1996 – 1999 saw a very skewing contrast where the growth stock outperformed (3.03% Vs 0.98%). Within the recent 2 decades where value outperformed, the value portfolio outperformed the growth portfolio 52% of the time however, in the 4yrs of 1996 – 1999 where growth outperformed, it did so 73% of the time. This result also contrasts with recent comments about value investing losing its edge over a growth focused strategy (at least in the UK investible market) however, the lack of time series (for book equity) implies that this observation should be taken with a bit of caution. Growth stocks, in general, have the potential to perform better when interest rates are falling and company earnings are rising while value stocks (often stocks of cyclical industries), may do well early in an economic recovery but are typically more likely to lag in a sustained bull market. However, the CAPM result does suggest that this better value return may well be explained in part by the higher beta displayed relative to the growth portfolio, both of which did not produce any significant risk adjusted excess return.

Over the period considered (1996 – 2019), the average total monthly return of the high profitable portfolio was 1.12% while that of the low profitability was 1.00% hence, an average premium of 12 basis points per month or about 1.45% per year. By breaking it into decades (2 decades and 4yrs) as in the size and value analysis, the low profitable stock only outperformed in the most recent decade (1.17% Vs 0.93%). However, high profitability outperformed 49% of the times considered. The returns result conforms to previous and current literature as companies with high operating profit will usually indicate healthy earning which will eventually filter into its market price. In terms of the reverse, it may be the case that companies with low operating profit will pay (or avoid) relatively low dividends in a downturn and investors will tend to negatively price this in and therefore create a low price that soon becomes recognised by investors. This anomaly is further strengthened with the lower beta value of the high profitability portfolio relative to the low profitability portfolio, both of which did not produce any significant risk adjusted excess return.

The average observed total monthly return of the low volatile portfolio for the period was 1.267% while that of the high volatile portfolio was 1.495%. Hence, average monthly premium was 23 basis point or about 2.74% per year. However, the low volatility portfolio outperformed the high volatility one 51% of the time considered (1986 – 2019). By breaking the period into decades, the high volatile portfolio consistently outperformed the low volatile sort in the past 3 decades. Whilst this absolute return does not indicate the presence of the volatility anomaly, the number of times the low volatility portfolio outperformed the high volatility portfolio gives a good reason to suspect that there may be certain factors responsible. A study by Riley (2014) revealed that the anomaly strengthens as size (and book-to-market) decreases, and amongst large stocks, the anomaly was weak by many measures. Vliet, Blitz and Van der Grient (2011), find that the empirical relation between historical volatility and expected returns is negative, with an average quintile return spread of -3.7%. The relation becomes 2% less negative when small caps are excluded.

Most of the available work in the literature have been done using US data (NYSE) consisting of over 2,400 listed shares with a fair portion of small caps. Other indices that have been used include the MSCI UK All Cap which has about 789 constituents and MSCI World which has about 1,586 constituents. The FTSE 350 used in this study not only actively excludes small caps but only comprises of the 350 of the over 2000 stocks traded on the London Stock Exchange and about 58% of the number of shares on the FTSE All Share. All of these together may explain the absence of this volatility anomaly in absolute return terms. However, the result from table 2 shows that the Sharpe ratio of the low volatile stocks is almost double that of the high volatile stock indicating that there is a higher excess return per unit of volatility with the low volatility portfolio. This suspicion is confirmed from the CAPM result which showed that the low volatility portfolio produced a significant risk adjusted excess return with a lower than market beta whilst the high volatility portfolio did not produce any excess return. Therefore, while the FTSE 350 universe may not reveal this anomaly in observed absolute returns terms (potentially for reasons stated above), it shows that this anomaly is vibrant in risk adjusted terms.

The results of table 3 also shows the observed performance of the various factor portfolios against the FTSE 350. The size and book to market ratio factors produces returns in excess of the market i.e., investing in small, big, value or growth stocks would have been more profitable than holding the market (with annual premiums of 53 basis points, 4 basis points, 74 basis points and 62 basis points respectively). However, only the small and growth portfolios showed better risk adjusted performance relative to the market (lower than market betas). The profitability factor (high profitability and low profitability portfolios) underperformed the market, but the high profitability portfolio produced a better risk adjusted performance relative to the market (lower beta). Only the high volatility subset of the volatility factor outperformed the market (with an annual premium of about 202 basis points) however, this is not farfetched due to the nature of the portfolio. However, in terms of returns relative to risk (as stipulated by the CAPM), the table also shows the return per unit of risk for the volatility factor relative to the market. The results indicate the volatility anomaly as the low volatility portfolio produced a significant risk adjusted excess return relative to the market.

Table 3 also reports on the Sharpe ratio analysis of the portfolios; although the Sharpe ratio shows the inherent portfolio performance, it is nonetheless another measure of risk adjusted return. The idea behind the ratio is that if risk free return is with relatively no risk, then the risk in a portfolio must be generating the returns in excess of the risk-free return. The CAPM beta generally presents a standard reference (i.e., the market risk) from which to assess portfolios but the Sharpe ratio gives an assessment of a portfolio's performance relative to the total risk which is a summation of both the market and specific risk. The results show that the low volatility and high profitability both produced a better excess performance per unit of risk taken relative to the high volatility and low profitability portfolios (0.26 Vs 0.14 and 0.22 Vs 0.14 respectively). This tends to conform to the conclusion drawn from the CAPM betas generated showing that these portfolios simply produced better risk adjusted return relative to the market and specific risk. The value portfolio however, showed a lower Sharpe ratio relative to the growth portfolio (0.19 Vs 0.24); this again suggests that the better performance generally observed in the value sector is explained with the excess relative risk within. This result also tends to suggest the same conclusion of the CAPM beta for these portfolios and again unable to confirm the value effect. The big portfolio produced a slightly better ratio compared to the small portfolio (0.22 Vs 0.21) which seems to suggest that whilst the small portfolio may produce a better return relative to market risk, the big portfolio appears to be more risk efficient relative to its specific risk.

Transaction Cost

Holding a portfolio inevitably comes with cost just like most investments which consequently acts as a drag on the returns. Often times, this element is usually omitted in the analysis of performance in order to keep illustrations simple because the issue of transaction cost could be complex, more so, when it is historical. Still, it is important to have a view on what the impact of transaction cost could have on estimated returns.

Clare, Motson and Thomas (2013) provide a methodology and statistic that give a reasonable indication of the scale of the likely costs of replicating an index which can be used in observing if the potential cost of transaction will make the replication worthwhile. They illustrate this statistic with the below:

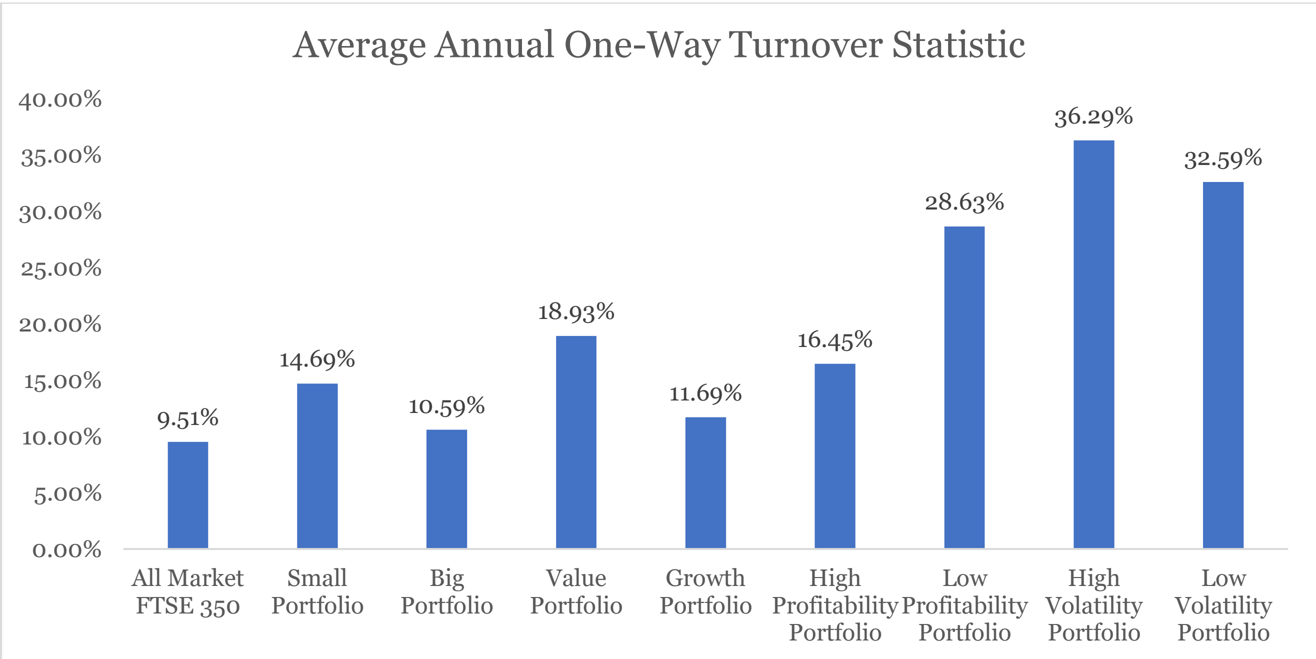
Example of Turnover Calculation

	Stock A	Stock B	Stock C	Stock D	Stock E	
Year 1	50%	25%	15%	10%	0%	
Year 2	0%	30%	20%	10%	40%	
Turnover	50%	5%	5%	0%	40%	50%

The illustration in this table shows the turnover of a portfolio containing stocks A to E between year 1 and year 2. Simply put, the change in the weightings of each stock within this period is summed up and then divided by 2 to get an annual turnover of 50% in this example. For a longer period of time as in this thesis, the average turnover between the years is estimated as above so, turnover between 1996 and 1997, 1997 and 1998, 1999 and 2000 etc. This estimate for the entire period is then averaged to get the annual turnover statistic. The data for low profitability portfolio (for example) is shown below:

Low Profitability Portfolio	
Period	Turnover
1996-1997	39.43%
1997-1998	16.03%
1998-1999	13.77%
1999-2000	34.98%
2000-2001	52.34%
2001-2002	18.30%
2002-2003	11.97%
2003-2004	16.46%
2004-2005	16.85%
2005-2006	16.08%
2006-2007	12.34%
2007-2008	34.28%
2008-2009	25.06%
2009-2010	40.17%
2010-2011	29.13%
2011-2012	40.64%
2012-2013	32.97%
2013-2014	21.36%
2014-2015	54.24%
2015-2016	34.70%
2016-2017	25.27%
2017-2018	20.16%
2018-2019	35.29%
Average Annual Turnover	**27.91%**

The estimate for the portfolios is summarized below:



The figure shows that investing in the market would have generated the lowest one-way average annual turnover of about 9.5% and expectedly the lowest cost of transaction. The big and growth portfolios were the next lowest turnover portfolios with an average annual statistic of 10.59% and 11.69% respectively. The small, high profitability and value portfolios have turnover statistics of 14.69%, 16.45% and 18.93% respectively. The portfolios with the volatility factor show the highest annual turnover rates with turnover rates of 36.29% and 32.59% respectively for the high and low volatility portfolios indicating that activities within these portfolios are relatively more vibrant hence, likely to be more expensive holding these portfolios. The low profitability portfolio showed the next highest annual turnover with a statistic of 27.90%.

Although transaction cost will clearly have a downward effect on any outperformance, it may not necessarily eliminate outperformance. For example, Clare, Motson and Thomas (2013) argue that if an alternative index outperformed a completely costless Market-cap index by 2.0% per year with turnover of 50% per year, then the average bid ask spread on the stocks would need to be 4.0%, which is high, to eliminate the performance difference.

Table 4: Summary of Performance and Turnover Analysis

Portfolio	Average Annual Return	Annual Outperformance	Average Annual Turnover	Turnover Cost Based on 7.5bp	Net Annual Return	Net Annual Outperformance	Additional Turnover	Required Bid-Offer Spread
All Market FTSE 350	14.04%		9.51%	0.71%	13.33%			
Small Portfolio	15.12%	1.08%	14.69%	1.10%	14.02%	0.69%	5.18%	20.85%
Big Portfolio	14.04%	0.00%	10.59%	0.79%	13.25%	-0.08%	1.08%	0.00%
Value Portfolio	14.76%	0.72%	18.93%	1.42%	13.34%	0.01%	9.42%	7.64%
Growth Portfolio	14.64%	0.60%	11.69%	0.88%	13.76%	0.44%	2.18%	27.52%
High Profitability Portfolio	13.44%	-0.60%	16.45%	1.23%	12.21%	-1.12%	6.94%	-8.65%
Low Profitability Portfolio	12.00%	-2.04%	27.90%	2.09%	9.91%	-3.42%	18.39%	-11.09%
High Volatility Portfolio	16.80%	2.76%	36.29%	2.72%	14.08%	0.75%	26.78%	10.31%
Low Volatility Portfolio	12.36%	-1.68%	32.59%	2.44%	9.92%	-3.41%	23.08%	-7.28%

To illustrate this further using the table 4 above where performance has been harmonized to commence in 1996 through 2019, if we consider the small portfolio outperforming the market portfolio by 1.08% annually and the additional annual turnover relative to the market is 5.18%, then the annual bid ask spread will have to be about 20.85% averagely over the sample period for the 1.08% annual outperformance to be eliminated and this is quite unlikely. In the same way, the bid-ask spread annual average would have to be about 8.65% over the sample period for the underperformance of the high profitability portfolio to be eliminated. According to the FCA hosted magazine, Insight (2016), the annual bid-ask spread for equities was about 7.9% in 2015; also, for

ETFs (exchange traded funds) on the London Stock Exchange, the spread is broadly regulated to be between 5% and 10%. Therefore, if we adopt an average annual spread of 7.5bp and estimate the net return of the various portfolios, we can observe that the initial outperformance/underperformance outcome remains unchanged (by comparing the annual performance and net annual performance columns) with the exception of the big portfolio which had about the same performance with the market in the first place; albeit, there is a downward/upward impact on outperformance.

Table 5: Summary of Mean Reversion

The FTSE 350 universe was considered and for uniformity, data from 1996 was used across all portfolios. The market equity (ME) of the index constituent stock was ranked in descending order and the median was obtained. Stocks above the median are classed as large stock and that bellow are small stocks. The book equity (BE) of the index constituents was also obtained and then the ratio BE/ME was ranked in descending order. The top 30th percentile is classed as value stock while the bottom 30th percentile are the growth stock. The ratio EBIT/BE (earnings before interest and tax (EBIT)) was used to obtain high profit and low profit portfolios using the same top and bottom 30th percentile approach. The standard deviation of the monthly total returns of the stocks was also obtained and with the percentile approach, high and low volatility portfolios were obtained. Each portfolio is reconstructed every January. The monthly total return (from the total return index) of the constituent stocks of the portfolios together with the market capitalisation (ME) was then used to establish the market capitalized weighted return of the portfolios. Using the AR(1) i.e. autoregression was run for each portfolios and the Dickey Fuller test statistic was also obtained from the result of the regression to test for stationarity and hence infer mean reversion. Half-life is estimated as: $hl = \frac{In\ (0.50)}{In\ |\varphi|}$

	Small Portfolio	Big Portfolio	Value Portfolio	Growth Portfolio	High Profitability Portfolio	Low Profitability Portfolio	High Volatility Portfolio	Low Volatility Portfolio	All Market
α	0.010 *	0.011 *	0.011 *	0.012 *	0.011 *	0.009 *	0.013 *	0.009 *	0.011 *
(t-stat)	(3.87)	(4.58)	(3.46)	(4.75)	(4.48)	(2.82)	(2.607)	(5.32)	(4.64)
β	-0.823*	-0.973 *	-0.90 *	-0.986 *	-0.986 *	-0.915 *	-0.915 *	-0.911 *	-0.981 *
(t-stat)	(-14.09)	(-16.42)	(-15.26)	(-16.658)	(-16.64)	(-15.49)	(-15.512)	(-15.440)	(-16.55)
Half-life (hl)	0.40	0.19	0.30	0.16	0.16	0.28	0.28	0.29	0.17
ADF Test Stat.	-14.088 (*,**,***)	-16.421 (*,**,***)	-15.261 (*,**,***)	-16.658 (*,**,***)	-16.638 (*,**,***)	-15.494 (*,**,***)	-15.512 (*,**,***)	-15.440 (*,**,***)	-16.547 (*,**,***)

** = Significant at 5% critical level*
*** = Significant at 10% critical level*
**** = Significant at 1% critical level*

Table 5 shows that both big and small portfolios are mean reverting at all critical levels (Augmented Dickey Fuller Test) however, considering the coefficient of the explanatory variable for the small and big portfolios (-0.82% Vs -0.97%), it appears the big portfolio reverts faster and therefore has a weaker persistence of returns. The half-life estimates also conforms to this result as the big sort portfolios have a shorter half-life (0.19 Vs 0.40).

Table 5 also shows that that while both value and growth portfolio returns are mean reverting; the growth portfolio appears to be reverting faster (less persistent) with the coefficient of -0.99 Vs -0.90 and a shorter half-life (0.16 Vs 0.30). The faster rate of reversion of the growth portfolio may also be due to the same reasoning stated for the size effect above. Both the high and low profitability portfolio returns are mean reverting; the high profitable portfolio appears to have a weaker persistence hence reverting faster with the coefficient of -0.99 Vs -0.92 and half-life of 0.16 Vs 0.28. The high and low volatility portfolio returns also revert to their means but, the low volatility portfolio is showing a stronger persistence of returns in comparison which means it is reverting slower than the high volatile portfolio with the coefficient of -0.91 Vs -0.92 and a longer half-life of 0.29 Vs 0.28.

The persistence / reversion outcome reported in table 5 is further strengthened when the performance behaviour of these portfolios during recession is observed; the idea being that more persistent portfolios will show smaller relative changes in their performance level prior to a recession after a recession. To do this I have taken the 2008 global recession as a point of reference to illustrate this. The 2008 financial crises remain one of the most historic financial crises events causing over 50% drop in global market performance. This crisis was due primarily to exposure of securities of packaged subprime loans and credit default swaps issued to insure these loans and their issuers. This rapidly developed into a global crisis resulting in a number of bank failures in Europe and sharp reductions in the value of equities and commodities worldwide. According to the Forbes Magazine[8], this crisis started at about the start of 2008 and although officially ended in June 2009, markets did not fully recover until 2014. Therefore, in the table below, I have estimated the average monthly return of the portfolios up till 2009 (1996 – 2009) and also estimated the average monthly return till 2013 (1996 – 2013). The difference in

8 Forbes Magazine October 2022 "How Long did the Great Recession Last in 2008?

both performance period is then obtained. Generally, if a return series is showing less relative persistence, the series has a higher relative speed of reversion and therefore should have a smaller difference between returns pre and post-recession; a smaller in the difference should expectedly have a smaller persistence measure.

Table 6: Persistence During Crisis.

	Big	Small	Value	Growth	High Profitability	Low Profitability	High Volatility	Low Volatility
Avg. Monthly Return through Crisis (1996 - 2009)	1.24%	1.27%	1.09%	1.34%	1.26%	0.89%	1.66%	1.06%
Avg. Monthly Return Post Crisis (1996 - 2013)	1.24%	1.33%	1.28%	1.33%	1.23%	1.00%	1.52%	1.10%
Difference	0.00%	0.07%	0.19%	-0.01%	-0.03%	0.12%	-0.14%	0.04%
Persistence (Speed of reversion + 1)	0.027	0.177	0.100	0.014	0.014	0.085	0.0846	0.0893

Table 6 shows that the small, big portfolio had a 0% change between this period compared with the small portfolio which had 0.07% difference. In addition, the big portfolio has a smaller persistence measure hence a faster speed of reversion showing that it stays closer to its mean. This observation is seen across board between value and growth portfolios and high profitability low profitability portfolios. However, the volatility sort does not show this outcome as the low volatility has a smaller difference between these periods but a stronger level of persistence.

Observing the outcomes of tables 2 and 5 appear to show a reasonable relationship. With the exception of the small / big and the low volatility / high volatility portfolio sorts, the more persistent comparative portfolio seems to be riskier relative to the market and this risk is not compensated enough (relatively) with performance. The value portfolio as shown in table 5 is more persistent with a lower reversion to mean speed compared to the growth portfolio (0.90 Vs 0.986). The CAPM beta for this value portfolio is also higher relative to the growth portfolio (1.04 Vs 0,97) indicating it is taking more risk relative to the market however, the Sharpe ratio for the value portfolio is lower than that of the growth portfolio (0.19 Vs 0.24) showing a lower rate of excess return relative to risk. This is the same outcome with the high and low profitability sort where the low profitability portfolio is more persistent (speed of reversion: 0.915 Vs 0.986), riskier (CAPM beta: 1.10 Vs 0.98) and lower risk efficiency (Sharpe ratio: 0.14 Vs 0.22). The high volatility and small portfolio had stronger persistence relative to the high volatility portfolio and big portfolios (speed of reversion: 0.911 Vs 0.915 and 0.823 Vs 0.973) however, these portfolios are taking relatively less risk (CAPM Beta: 0.53 Vs 1.77 and 0.89 Vs 1.0). Only the low volatility portfolio displays a better risk efficiency relative to the high volatility portfolio (Sharpe ratio: 0.26 Vs 0.14); the small portfolio is slightly less risk efficient.

Intuitively, a relatively more persistent portfolio will be expected to be relatively less risky however, only the small and low volatility portfolio were able to show this deduction from the analysis above. That said, there may be more at play that what has been explained here because the measure of risk used is more of a systematic risk and excludes idiosyncratic risk.

3.4 Conclusion.

The increasing popularity in DC (defined contribution) pension savings poses two main considerations for advisers designing retirement strategies one of which is what portfolio strategy should be adopted during retirement to ensure a sustainable rate of withdrawal? Hence, this first empirical chapter explores various factors that may be used in a factor-based strategy for this purpose.

The chapter starts by identifying relevant risk factors (other sources of return) and then assessing the presence of a reversion to mean of the returns from the exposure to each factor. The presence of mean reversion shows that returns are likely going to revert to its historical mean at a certain rate. Therefore, there is a potential to use this feature for predictive purposes when designing a portfolio based on exposure to factors. The outcome of the analysis in this chapter shows that the FTSE 350 world (which has not been used as a universe in previous similar works), which stands as a reasonable proxy for the UK investible market, shows evidence appearing to support the various sources of excess return (other than the CAPM beta) explored in existing literature (particularly Fama and French (1992, 2006, 2014, 2017)).

The size effect (small stocks outperforming large stocks) is observed with a premium of about 4 basis point every month however, there seem to be no indication that this trend has changed as some parts of recent literature suggest. In fact, contrary to this, small cap stocks have outperformed large cap stocks in the last 2 decades till date. In risk adjusted terms (using the CAPM process), the small size portfolios have a lower beta compared to the big size portfolios (and the market) showing that the small size portfolios are more risk efficient hence indicating the size anomaly. The value and profitability effect are also evident with premiums of 1 and 12 basis points monthly respectively. The literature also suggests that these factors (size, book to market ratio and profitability) are the most significant factors of the five-factor model by Fama and French (2014). However, whilst the CAPM was able to show the risk adjusted profitability anomaly (high profitability portfolios displaying lower betas relative to low profitability portfolios), the value effect could not be confirmed as it appears the better performance of the value portfolio is due (at least in part) to the higher beta displayed. The much talked about volatility effect (low volatility stocks outperforming high volatility stocks) was also assessed in the FTSE 350 universe and using the total volatility measure, although this effect was not observed in absolute return terms, it was observed in risk adjusted return measure (from the Sharpe ratio and CAPM assessments). Possible explanations for the absence in absolute terms include the exclusion of small cap stocks and the relative population size of the FTSE 350 used.

The performance of the factor portfolios created against FTSE 350 market was also observed. The size and book to market ratio factors showed observed superior performance compared to the market whilst the profitability factor did not outperform the market. In addition, only the high volatility sect of the volatility factor outperformed the market. This volatility factor result may seem to conform with the CAPM stipulation that the excess return is due to higher risk however, looking at the returns relative to risk (which more broadly defines the CAPM principle), then an anomaly emerges. The low volatility stocks produced a higher and significant returns relative to the market. With the high volatility portfolio not producing any excess return, the volatility anomaly is therefore present.

In addition to this, the chapter shows that the sorting approach used for the big and small portfolio sorting (mean sorting of 70/30 sorting) will not create a different outcome in terms of the outperformance observed. Furthermore, the chapter also demonstrates using an indicator of transaction cost that the cost of transaction will not alter the outperformance outcomes estimated albeit, it will reduce the level of outperformance.

This analysis also investigates whether these factor portfolios are viable as an investment strategy, and it does this by examining if the series of returns will revert to their mean overtime. The intuition is that, if the returns revert to its long run mean, then advisers may be able to tell the behaviour of a portfolio strategy (identified by its speed of reversion) following drawdown shocks. The results show that the returns of the portfolios formed from these factors are mean reverting however, the big and growth stocks portfolios are reverting faster in comparison to the small and value stock. A possible explanation for the faster reversion is that big stocks are predominantly growth stocks i.e., relatively high market price therefore, in the event of an upturn in the market, these stocks will relatively have lower returns in comparison to the small stocks which have relatively lower market price. The implication of this is that this small stock higher return attracts more buying investors in comparison and invariable, holders of big stocks will offload. This buy-sell action soon drives down the relative price of the small stocks perhaps after the big stocks have reverted to their mean hence, a relative persistence of small stock portfolio returns at peak prices. The same argument may apply when there is a downturn (where the returns on the small stocks may have fallen much lower than the big portfolio) in which case investors will seek relatively more stable stocks and this will result in an uplift of buy actions hence driving up big stock prices. Generally, the high profitable stocks reverted to their mean faster than the low profitable stocks. The returns of the high volatility stocks reverted slightly faster compared to the low volatility stocks. This could be because of investors sentiments in recognising opportunities when high volatile stocks peak and trough in their prices. In general, the FTSE 350 reverted to its mean faster than all the factor portfolios with the exception of the profitability factor portfolios.

The mean reversion results also implicitly give information on the persistence of the portfolio returns. The results show that the small, value, low profitability and low volatility portfolios have performance that are more persistent relative to the big, growth, high profitability and high volatility portfolios. It goes further to demonstrate this persistence during the global financial crisis of 2008. Intuitively, a relatively less persistent portfolio will be expected to have a post crisis return closer to its pre-crisis return however, all the portfolio sorts were able to show this deduction with the exception of the volatility sort. That said, there may be more at play that what has been explained here because the measure of risk used is more of a systematic risk and excludes idiosyncratic risk.

From the results of this chapter which shows sources of excess return and the mean reversion of these returns, the next chapter broadens the investigation of this sources of excess return by examining portfolios formed by exposure to more than one factor. In addition, the chapter attempts to use the mean reversion tendency of the portfolios (speed of reversion) to create a measure that could infer the recovery behaviour of the portfolios following shocks such as the one that will be created by a withdrawal event.

4 Factor Based Investment Strategies and Relative Stability

4.1 Introduction

Identifying the sources of investment return is perhaps the most fundamental objective of every growth-seeking investor. Therefore, any mechanism that will provide information on whether to buy, sell or hold as well as information on the sources of returns in excess of the market, remains a valuable tool to have. While the mean reversion potential of a strategy may provide information on the former, the factor investing principle has been observed to be an indicator for excess returns.

From a portfolio strategy / trading perspective, the mean reversion feature of a returns series creates the option of either choosing faster 'reverters' who will follow a less sticky path to their long term mean in the event of a downturn or those who will persist at higher-than-average returns in the event of an upturn. Therefore, considering that the drawdown action is effectively a downturn event for any strategy, the preference for faster reversion may be justified. In general, choices will usually be made relative to the expected returns of a strategy as well as volatility.

This research focuses on identifying portfolio strategies with qualities that will potentially indicate their suitability for the decumulation phase of an investor's journey through their ability to provide preferential sustainable drawdown rates. To achieve this, the level of excess return and their potential ability to recover quickly from drawdown shocks was explored. Following on from the previous chapter where 4 factors were identified as sources of excess return (largely inspired by literature), in this chapter, these factors were combined to form two and three sort portfolios as previous studies have shown that one factor is unlikely to capture all the available excess return. I find that out of the 44 considered combined factor portfolios, 23 of them had observed returns better than that of the market and no dominant factor was observed. However, in risk adjusted terms, 7 of these portfolios produced significant returns in excess of the market whilst displaying lower than market betas; 5 of these 7 portfolios also had observed returns better than the market. Notably is the fact that these 7 portfolios were constructed with the low volatility factor. This seems to indicate that portfolio construction combining these factors is a viable strategy to capture returns in excess of the market.

This chapter also tested the portfolios for the presence of mean reversion and whilst all the portfolios formed are mean reverting, some are reverting faster than others. The proportionality factor of reversion (β – speed of reversion), effectively indicates how quickly a series (strategy) will revert to its mean when it wanders away from it, and in the context of a drawdown, it will be when it falls below its mean. However, using this measure alone may be incomplete as it ignores the magnitude of deviation. Hence, not just how fast but how far the series must revert.

Therefore, this chapter proposed the use of a measure that considers the average deviation from the mean; $\frac{|\beta|}{\hat{\sigma}}$. where $\hat{\sigma}$ is one unit deviation of the shock (error term) and this is proportionate to the standard deviation of the series from its mean. Based on this, the low volatility factor appeared in all of the 9 combined factor portfolios that have the measure of $\frac{|\beta|}{\hat{\sigma}}$ that is higher than that of the market and 2 portfolios of these also produced excess return in addition to the higher measure.

The chapter concludes that the 4 excess return factors can combine as a strategy to produce portfolios that not only offer higher than market returns (both in absolute and risk adjusted terms) but also display the potential to be more stable for the purpose of drawing down a portfolio during retirement and the low volatility factor is a prominent driver of this relative stability.

The rest of this chapter is organized as follows: Section 4.2 describes the methodology; section 4.3 presents the results and section 4.4 concludes.

4.1.1. Diversification and Number of Stocks

This advice of diversification was formalised in an investment context by Markowitz (1959) and others in the 1950s. By constructing portfolios with assets that were imperfectly correlated with one another, Markowitz demonstrated that the risk inherent in the portfolio would decline as successive assets were added to it, until eventually the volatility of the portfolio would equate to the average covariance of the assets comprising the portfolio. This work therefore highlighted the importance of the covariance of returns between assets, and drew a distinction between undiversifiable and diversifiable risk, where the latter could be progressively eliminated by adding more and more assets to the portfolio. The work therefore explained how investors could take advantage of one of the few free lunches available in economics.

But how many stocks would produce a portfolio with only undiversifiable risk? This question was first addressed by Evans and Archer (1968). Randomly drawing equities from a pool of assets to construct a large number of n-asset portfolios, their results indicated that most of the diversifiable risk could be eliminated by forming portfolios of eight to ten randomly selected stocks. Since that time many other researchers have addressed the same question. In much later work Statman (1987) concluded that the number was closer to thirty or forty stocks. However, even in the event that the number is double Statman's estimate many mutual fund investors could still be said to be overdiversified, that is, holding portfolios of more assets than are required to reduce diversifiable risk to zero, effectively paying higher transactions charges to manage more assets than they need to hold.

LHabitant and Learned (2002) pose the same question but with respect to hedge funds. For investors looking to invest in hedge fund of funds, how many hedge funds should be included in this basket? Using broadly the same approach and techniques, drawing individual hedge funds randomly, they conclude that in terms of naive diversification, that most of the diversification benefits are achieved by forming fund of funds comprising just five to ten individual hedge funds.

Clare and Motson (2008) state that increased diversification significantly decreases the time series standard deviation of the portfolios, but that the marginal decrease from adding more alternatives decreases rapidly. This implies that if an investor is only concerned with this element of risk, they should hold a portfolio of between eight and ten alternatives. However, for investors with long-term investment horizons the dispersion of terminal wealth outcomes might be a more appropriate measure. Here they find that holding more than the number of alternatives suggested by considering the time series standard deviation might be preferable. In their work, they specify a pool of possible alternative investments that include all those strategies and classes that UK pension funds have invested in already, and/or comprise the alternative pooled vehicles that are currently available to them. From this pool they randomly create portfolios consisting of 2 to 23 of these asset classes, using 100,000 draws for each set of n-asset portfolios. Their results indicate that 40% of the time series risk can be eliminated by combining 8 asset classes, but only a further 4% from combining 12. They also find that an investor could reduce 60% of the dispersion in terminal wealth – which is arguably what investors should really be concerned with – by combining 6 of these less conventional approaches to investment, but only a further 20% by combining 15.

4.2 Methodology

In addition to the single factor portfolios created in the previous chapter, intersects based on 2 and 3 factor sorts were also constructed. Once stocks belonging to individual factors have been grouped, then various combination of 2 factor groups are considered and stocks belonging to the 2 factor groups considered form the 2-factor portfolio; 24 two-factor portfolios were constructed. This process is then repeated for various combination of 3 factors and 34 three-factor portfolios were constructed.

Not surprisingly (and particularly for the 3-factor portfolios), there were certain years where the portfolio had no stocks (inactive periods) i.e., stocks qualifying for the 3 factors could not be found. In order to ensure the practicality of the portfolios considered, only portfolios with active periods (availability of stocks in all the 24 years considered) were assessed; these portfolios had a minimum of 3 stocks on average each year (over 85% of these portfolios had more than 8 stocks on average) with the best performing portfolio (*H,Lv*) having an average of 9 stocks per year. There were fourteen 3-factor portfolios with one or more inactive periods and hence were not assessed further. However, their summary statistics and performance are presented. Two approaches were used to estimate their returns; one where a 0% return is assumed for these inactive periods and then the average return over the entire 24yrs is estimated and another where the average return over only the active period is estimated. Both estimates are presented.

The process of constructing the portfolios based on 1, 2 and 3 combination of factors (1, 2 and 3 portfolio sorts) was repeated every January (hence rebalanced) and the market cap weighted total monthly return was then obtained for each portfolio[9]. This was to examine the individual and combined effects of using these factors to create strategies. Although in the previous chapter, in an attempt to assess the returns of the initial 4 individual factors, data for different time periods were considered due to data constraints, in this chapter data has been harmonised to commence from 1996.

Tables 7: Summary Statistics Table of Portfolios

Data has been harmonised to commence from 1996. 24 periods of data (1996 – 2019) has been considered with a monthly frequency (288 data points). The market equity (ME) of the constituent stock was ranked in descending order and the median was obtained. Stocks above the median are classed as large stock and that bellow are small stocks. The book equity (BE) of the index constituents was also obtained and then the ratio BE/ME was ranked. The top 30th percentile is classed as value stock while the bottom 70th percentile are the growth stock. The ratio EBIT/BE (earnings before interest and tax (EBIT)) was used to obtain high profit and low profit portfolios using the same top and bottom 30th and 70th percentile approach. The standard deviation of the preceding 12month of total returns of the stocks was also obtained and with the percentile approach, high and low volatility portfolios were obtained. Portfolios were first created based on the stocks that fit in the criteria of the 4 main factors (size, book to price, profitability, and volatility). Intersects (stocks qualifying for more than one factor) were then created based on 2 and 3 factor intersects. The colour coded portfolios do not have active portfolios in all the periods considered hence, a zero-portfolio return has been entered for such periods. This may however appear to skew their results. S = small stocks, B = big stocks, L = low book to market ratio stocks (growth stocks), H = high book to market ratio stocks (value stocks), Hp = high profitability stocks, Lp = low profitability stocks, Hv = high volatility stocks, Lv = low volatility stocks

1 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Mean (Dependent Variable)	Standard Deviation (Dependant Variable)	Standard Error of Coefficient	R^2	Durbin Watson Stat.	No. of Observation
Small Portfolio	77	24	1.29E-04	0.0566	0.0435	0.4105	1.96	287
Big Portfolio	77	24	9.78E-05	0.0563	0.0404	0.4862	1.99	287
Value Portfolio	59	24	1.22E-04	0.0710	0.0500	0.4497	1.979	287
Growth Portfolio	59	24	-1.50E-05	0.0570	0.0400	0.4910	2.00	287
High Profitability Portfolio	60	24	1.15E-04	0.0567	0.0405	0.4927	1.99	287
Low Profitability Portfolio	60	24	5.18E-05	0.0736	0.0590	0.4570	1.98	287
High Volatility Portfolio	77	24	-1.43E-05	0.1100	0.0811	0.4578	1.98	287
Low Volatility Portfolio	77	24	9.36E-05	0.0383	0.0283	0.4555	2.00	287

9 *The data (particularly for BE) had the issue of missing data. Therefore, each year the missing data is excluded, and the available data forms the universe from which the factor portfolios are formed.*

2 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Mean (Dependent Variable)	Standard Deviation (Dependant Variable)	Standard Error of Coefficient	R^2	Durbin Watson Stat.	No. of Observation
S, L	6	24	2.04E-04	0.0940	0.0722	0.4106	2.00	287
S, H	42	24	7.50E-05	0.0643	0.0501	0.3921	1.98	287
S, Lp	37	24	6.62E-05	0.0674	0.0525	0.3928	1.98	287
S, Hv	41	24	2.31E-04	0.0877	0.0655	0.4440	1.97	287
S, Lv	41	24	1.51E-05	0.0379	0.0289	0.4214	1.99	287
B, L	53	24	-1.53E-05	0.0571	0.0407	0.4941	1.99	287
B, H	16	24	1.14E-04	0.0793	0.0564	0.5004	2.00	284
B, Hp	50	24	-2.91E-05	0.0585	0.0415	0.4981	1.99	287
B, Lp	22	24	5.70E-05	0.0773	0.0572	0.4516	1.99	287
B, Hv	37	24	5.87E-05	0.1053	0.0747	0.4990	2.00	287
B , Lv	35	24	9.57E-05	0.0399	0.0295	0.4569	1.99	287
L, Hp	44	24	-2.31E-05	0.0574	0.0409	0.4956	1.99	287
L, Lp	6	24	-4.08E-06	0.0882	0.0655	0.4494	1.99	287
L, Hv	17	24	1.99E-04	0.1207	0.0891	0.4580	1.98	287
L, Lv	18	24	5.30E-05	0.0403	0.0298	0.4544	2.00	287
H, Lp	39	24	9.44E-05	0.0783	0.0586	0.4393	1.98	287
H, Hv	23	24	1.17E-04	0.1126	0.0836	0.4512	1.98	287
H, Lv	9	24	2.51E-06	0.0479	0.0351	0.4652	2.00	287
Hp, Hv	15	24	2.28E-04	0.1196	0.0875	0.4667	1.99	287
Hp, Lv	16	24	8.35E-05	0.0468	0.0335	0.4903	2.00	287
Lp, Hv	25	24	-9.90E-06	0.1218	0.0894	0.4629	1.99	287
Lp, Lv	10	24	3.31E-05	0.0529	0.0392	0.4539	1.99	287

3 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Mean (Dependent Variable)	Standard Deviation (Dependant Variable)	Standard Error of Coefficient	R^2	Durbin Watson Stat.	No. of Observation
B, L, Hp	41	24	-3.62E-05	0.0570	0.0410	0.4930	1.99	287
B, L, Lp	5	24	-4.08E-06	0.0884	0.0650	0.4603	1.99	287
B, H, Lp	9	24	4.37E-05	0.0967	0.0667	0.5306	1.97	284
B, Lp, Hv	8	24	-8.99E-05	0.1262	0.0925	0.4653	1.99	287
B, Hp, Hv	11	24	4.09E-05	0.0471	0.0338	0.4877	1.99	287
B, L, Hv	13	24	2.08E-04	0.1199	0.0887	0.4540	1.99	287
B, H, Hv	6	24	8.17E-05	0.1384	0.0994	0.4864	1.99	287
B, Lp, Lv	4	24	3.29E-05	0.0566	0.0415	0.4648	2.00	287
B, Hp, Lv	15	24	4.09E-04	0.0471	0.0338	0.4877	1.99	287
B, L, Lv	15	24	4.73E-05	0.1262	0.0925	0.4653	1.99	287
L, Hp, Hv	10	24	2.55E-04	0.1208	0.0890	0.4594	1.98	287
L, Hp, Lv	13	24	4.46E-05	0.0445	0.0323	0.4727	2.00	287
S, L, Hp	3	24	4.80E-05	0.0975	0.0719	0.4588	2.01	287
S, H, Lp	29	24	2.52E-05	0.0692	0.0525	0.4270	1.97	287
H, Lp, Hv	16	24	-9.20E-06	0.1222	0.0884	0.4789	1.99	287
H, Lp, Lv	6	24	-2.47E-05	0.0479	0.0351	0.4652	2.00	287
S, Lp, Hv	16	24	1.72E-04	0.1031	0.0780	0.4299	1.97	287
S, H, Hv	18	24	1.54E-04	0.1002	0.0752	0.4379	1.98	287
S, Lp, Lv	6	24	-8.12E-05	0.0429	0.0336	0.3871	2.02	287
S, H, Lv	6	24	-1.20E-05	0.0398	0.0307	0.4092	2.01	287
All Market	265	24	8.02E-05	0.0561	0.0401	0.4900	1.99	287
B, H, Lv	3	23	8.95E-05	0.0599	0.0421	0.5068	2.00	287
S, Hp, Lv	2	14	1.52E-04	0.0462	0.0343	0.4500	2.00	287
S, L, Hv	3	21	4.07E-04	0.1176	0.0876	0.4474	1.99	287
S, Hp, Hv	4	21	3.55E-04	0.1231	0.0909	0.4560	1.99	287
S, L, Lv	0.417	8	5.11E-20	0.3573	0.0270	0.4325	1.97	287
H, Hp, Lv	0.417	9	0.00E+00	0.0435	0.0322	0.4551	2.00	287
H, Hp, Hv	0.417	10	6.56E-04	0.1273	0.0918	0.4912	1.94	282
S, H, Hp	1	14	3.60E-05	0.1150	0.0850	0.4650	1.99	282
S, L, Lp	1	13	5.65E-19	0.1420	0.1020	0.4880	1.99	287
L, Lp, Lv	1	21	6.55E-05	0.0558	0.0411	0.4577	1.99	287
L, Lp, Hv	2.7	16	-6.55E-05	0.0117	0.0852	0.4738	2.00	287
B, H, Hp	0.625	7	3.45E-05	0.0780	0.0550	0.5100	1.99	287
S, Hp	9	23	3.94E-05	0.0678	0.0539	0.3683	2.00	287
H, Hp	2	19	3.99E-04	0.9548	0.0698	0.4655	1.99	287

4.2.1 Mean Reversion.

Following from the previous chapter, given the auto regression AR (1) process of:

$$\hat{X}_t = c + \varphi(\hat{X}_{t-1}) + \varepsilon_t \qquad (1)$$

where c is the intercept and ε_t the error term.

A weakly stationary AR (1) process notably implies:

$$E[\hat{X}_t] = \mu = constant$$

for all t.

This property may be used (simply by taking the expectations on both sides of the price process above) to find that the stationary mean μ computes as:

$$\mu = \frac{c}{1 - \varphi}$$

or

$$\mu = -\frac{c}{\varphi - 1}$$

Which allows writing the price process as (by replacing c given the identity above):

$$\hat{X}_t - \mu = \varphi\left(\hat{X}_{t-1} - \mu\right) + \varepsilon_t \qquad (2.1)$$

or,

$$X_t = \varphi(X_{t-1}) + \varepsilon_t \qquad (2.2)$$

Where, $X_t = \hat{X}_t - E(\hat{X}_t)$ may be interpreted as the distance to the stationary mean.

By modelling the monthly returns of a portfolio R_t,the standard Dickey-Fuller test for on an AR (1) process will take the regression form of equation 2.2 above:

$$R_t = \varphi(R_{t-1}) + \varepsilon_t$$

$$-1 < \varphi < 1 \text{ and } |\varphi| < 1 \text{ for a reverting process}$$

The test for stationarity (unit root) here is whether $\varphi = 1$

The unit root tests described above is valid if the time series R_t is well characterized by an AR (1) process with white noise errors. Many financial time series, however, have a more complicated dynamic structure than is captured by a simple AR (1) model. Said and Dickey (1984) augment the basic autoregressive unit root test to accommodate general ARMA (Auto Regressive Moving Average) models with unknown orders and their test is referred to as the augmented Dickey Fuller (ADF) model.

Subtracting R_{t-1} on both sides of (2.2) gives:

$$R_t - R_{t-1} = \varphi R_{t-1} - R_{t-1} + \varepsilon_t$$

giving:

$$R_t - R_{t-1} = (\varphi - 1)\, R_{t-1} + \varepsilon_t$$

thus, the expression can be written as:

$$\Delta R_t = \beta R_{t-1} + \varepsilon_t \qquad (3)$$

where,

$$\beta = \varphi - 1$$

R_t is returns at time t

$$\Delta R_t = R_t - R_{t-1} \; and \; \varepsilon_t \sim N(0, \sigma^2)$$

The ADF test statistic (ADF_τ) as described in the previous chapter is given as:

$$ADF_\tau = \frac{\beta}{SE(\beta)}$$

After running the simple linear trend model stated earlier in the previous chapter, there was no need to include a trend in the regression for the ADF test. The hypothesis for the ADF test used are: H_0: $\beta = 0$ (presence of unit root) hence non-reverting versus the alternative hypothesis of H_1: $\beta < 0$ (stationarity) implying mean reversion.

4.2.2. Relative Stability Measure

"What goes up must come down." This folksy wisdom has turned out to be a highly non-trivial fact about stock market. In the late 1980's DeBondt and Thaler documented the phenomenon that so-called contrarian strategies outperform stock market. These are strategies where portfolios are selected according to past performance. "Contrarian" means that portfolios of former "losers" yielded substantially higher returns than portfolios of former "winners". This was interpreted as evidence of mean reversion, that is, a force that drives prices back to a certain medium level after they went above or below it.

Based on this description, the concept of persistence bears a close relationship with mean reversion. Willis (2003) defines persistence as the speed of return to baseline after a shock while Marques (2004) modified this definition to be the speed of convergence to equilibrium after a shock. Several scalar statistics have been proposed in the literature to measure persistence. These include the "sum of the autoregressive coefficients" the "spectrum at zero frequency", the "largest autoregressive root" and the "half-life".

Andrews and Chen (1994) present a good discussion of the first three of these measures. They basically argue that the cumulative impulse response function (CIRF) is generally a good way of summarizing the information contained in the impulse response function (IRF) and as such a good scalar measure of persistence. They show that the cumulative impulse response of a series of AR model with p-th order is given as CIR = 1/ (1- φ) where in their case, φ is the sum of the AR coefficients. Equivalently, φ is the coefficient on the lagged variable in a Dickey-Fuller regression. As there is a monotonic relation between the CIR and φ it follows that one can simply rely on the "sum of the autoregressive coefficients" as a measure of persistence. The "spectrum at zero frequency", is a well-known measure of the low frequency autocovariance of the series and, there is a simple correspondence between this concept, the CIR and φ, and so they can be seen as equivalent measures of persistence. The "largest autoregressive root" has also been used in literature as a measure of persistence (see, for instance Stock, 2001) however, the use of this statistic as a measure of persistence is criticised both in Andrews and Chen (1994) and in Pivetta and Reis (2001). The main point against this statistic is that it is a very poor summary measure of the impulse response function (IRF).

Finally, the "half-life" is a very popular measure of persistence especially in the literature that tries to evaluate the persistence of deviations from an equilibrium state. The "half-life" is defined as the number of periods for which the effect of a unit shock remains above 0.5. In the case of an AR(1) process, it is easy to show that the half-life may be computed as: hl = $\frac{In\ (0.50)}{In\ |\varphi|}$. The use of the "half-life" has been criticised on several grounds (see, for instance Pivetta and Reis, 2001). First, if the IRF is oscillating the half-life can understate the persistence of the process. Second, even for monotonically decaying processes this measure will not be adequate to compare two different series if one exhibits a faster initial decrease and then a subsequent slower decrease in the IRF than the other. Third, it may also be argued that for highly persistent processes the half-life is always very large and thus makes it difficult to distinguish changes in persistence over time. On the positive side, the half-life has the attractive feature that persistence is measured in units of time, which is not the case of any of the other three above mentioned measures of inflation persistence, and thus may be preferable for communication purposes. This probably explains why, despite the above criticisms, it still remains the most popular measure in the literature that investigates the persistence of deviations as well as speed of mean reversion. The measure of persistence introduced by Marques (2004); $\gamma = 1 - \frac{n}{T}$ (where n stands for the number of times the series crosses the mean during a time interval with T+1 observations), has the advantage of not requiring the researcher to specify and estimate a model for the inflation process.

One identifying characteristic of any stationary time series is that it must exhibit the mean-reversion property. Just as in Marques (2004), in equation (3) the presence of mean reversion is reflected in the term β . This implies that if in period (t −1) the return series R is above (below) the mean, the deviation R_{t-1} - μ (stationary process) will contribute as a "driving force" to a negative (positive) change of the series in the following period, through the coefficient $\beta = \varphi - 1$, thus bringing it closer to the mean. Of course, mean reversion is stronger the larger (in absolute terms) the coefficient $\beta = \varphi - 1$. With the measure persistence by φ and mean reversion by $\beta = \varphi - 1$ established, we conclude that mean reversion and persistence are inversely related: high persistence implies low mean reversion and vice-versa.

The motivation of this section is to explore a measure that describes how quickly a mean reverting series converges to his long run mean relative to the size of shocks to the disturbance term. While β can be used as a measure of the speed of reversion following shocks, it does not give any information on the dispersion of the return series around its stable mean state, hence, it does not tell how far from the series reverts. If ε_t can be considered as disturbances within the model, and if β shows the speed of reversion following dispersions (shocks) from the model mean state, then a measure that will reflect the magnitude of deviation from the mean as well as the speed of reversion may be a useful tool.

If ε_t is a measure of what is not explained by the modelled series (in its stable state), then deviations from ε_t should be a reflection of endogenous strains. Therefore, $\hat{\sigma}$, which is 1 standard deviation of shock ε_t, given as $\sqrt{\sigma^2/n - a}$ (where n is the number of adjusted observations and a is the number of regressors), would give a unit measure of deviation from the 'stable' system.

Looking at this in another way, from equation 2.2.,

$$R_t = \varphi(R_{t-1}) + \varepsilon_t$$

Variance of $R_t = \frac{\sigma^2}{1-\varphi^2}$

Hence, the deviation / dispersion of returns around its mean is proportionate to σ.

As a suggestion, this chapter proposes an *ad hoc* measure $\frac{|\beta|}{\hat{\sigma}}$ where $\hat{\sigma}$ is one standard deviation of the disturbance term ε_t. This can be viewed as a measure of the speed of mean reversion per unit deviation. Generally, the higher this value (in absolute terms), the greater the speed of mean reversion relative to the volatility of returns and therefore the more stable the portfolio and the more preferred the portfolio should be for the consideration of withdrawals. As mentioned earlier, ε_t is endogenous strain and the assumption here is that the model's behaviour to both endogenous and exogeneous shocks (such as withdrawals) will be similar. Therefore, this measure may be viewed as an "*ad hoc"* indicator of relative stability around the long-term mean.

4.3 Results

Tables 8: Summary of Excess Return

*The FTSE 350 universe was considered. The market equity (ME) of the index constituent stock was ranked in descending order and the median was obtained. Stocks above the median are classed as large stock and that bellow are small stocks. The book equity (BE) of the index constituents was also obtained and then the ratio BE/ME was ranked. The top 30th percentile is classed as value stock while the bottom 70th percentile are the growth stock. The ratio EBIT/BE (earnings before interest and tax (EBIT)) was used to obtain high profit and low profit portfolios using the same top and bottom 30th percentile approach. The standard deviation of the monthly total returns of the stocks was also obtained (deviation of the 12months preceding) and with the percentile approach, high and low volatility portfolios were obtained. Portfolios were first created based on the stocks that fit in the criteria of the 4 main factors (size, book to price, profitability, and volatility). Intersects (stocks qualifying for more than one factor) were then created based on 2 and 3 factor intersects. After sorting into the individual portfolios based on the various factors, the weighted monthly total returns (total returns times weighted market cap of the portfolio) of the constituent stocks for each year was obtained. Then the average monthly returns through the data period was calculated. The red coloured coded portfolios do not have active portfolios for the entire period; therefore, the bottom return entry is the average return for the active period only whilst the average return through the entire period (assuming a zero return for inactive periods) was calculated and represents the upper return entry. * = Significant at 90% confidence level; ** = Significant at 95% confidence level; *** = Significant at 99% confidence level*

Portfolio	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Average Monthly Returns
All Market	268	24	1.17%

1 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Average Monthly Returns	Average Monthly Excess Returns	CAPM Beta (***)	$\lvert t \rvert$-*Stat*	Alpha	$\lvert t \rvert$-*Stat*	Maximum Drawdown
Small Portfolio	134	24	1.26%	**0.09%**	0.89	23.67	0.18%	1.18	-67.37%
Big Portfolio	134	24	1.17%	0.00%	1.00	180.45	0.00%	0.16	-22.20%
Value Portfolio	59	24	1.23%	**0.06%**	1.04	21.20	0.03%	0.14	-61.84%
Growth Portfolio	59	24	1.22%	**0.05%**	0.97	57.85	0.08%	1.09	-19.69%
High Profitability Portfolio	60	24	1.12%	-0.05%	0.98	78.17	-0.03%	0.58	-21.39%
Low Profitability Portfolio	60	24	1.00%	-0.17%	1.10	23.36	-0.26%	1.33	-58.18%
High Volatility Portfolio	77	24	1.40%	**0.23%**	1.77	30.62	-0.48%	2.00	-52.18%
Low Volatility Portfolio	77	24	1.03%	-0.14%	0.53	19.22	0.29%	2.59	-21.87%

2 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Average Monthly Returns	Average Monthly Excess Returns	CAPM Beta	$\|t\|$-*Stat*	Alpha	$\|t\|$-*Stat*	Maximum Drawdown
S, L	6	24	1.14%	-0.03%	0.99	11.01	-0.02%	0.05	-283.01%
S, H	42	24	1.23%	**0.06%**	0.92	18.44	0.14%	0.67	-104.19%
S, Lp	37	24	1.41%	**0.24%**	0.94	17.90	0.29%	1.35	-163.42%
S, Hv	41	24	1.22%	**0.05%**	1.10	15.22	-0.03%	0.12	-80.48%
S, Lv	41	24	1.08%	-0.09%	0.56	19.77	**0.32%*****	2.74	-16.14%
B, L	53	24	1.22%	**0.05%**	0.97	53.91	0.08%	1.13	-19.18%
B, H	16	24	1.11%	-0.06%	1.08	19.72	-0.13%	0.59	-57.80%
B, Hp	50	24	1.16%	-0.01%	0.97	48.38	-0.02%	0.23	-21.09%
B, Lp	22	24	0.97%	-0.20%	1.10	20.93	-0.29%	1.36	-58.09%
B, Hv	37	24	1.23%	**0.06%**	1.45	21.09	-0.35%	1.25	-55.40%
B , Lv	35	24	1.05%	-0.12%	0.50	15.53	0.34%	2.61	-20.04%
L, Hp	44	24	1.18%	**0.01%**	0.96	49.13	0.05%	0.59	-19.80%
L, Lp	6	24	1.00%	-0.17%	1.03	13.98	-0.20%	0.67	-100.00%
L, Hv	17	24	1.25%	**0.08%**	1.85	25.48	-0.70%	2.53	-124.79%
L, Lv	18	24	0.95%	-0.22%	0.51	15.75	0.24%	1.80	-372.19%
H, Lp	39	24	1.34%	**0.17%**	1.07	18.78	0.11%	0.46	-75.05%
H, Hv	23	24	1.05%	-0.12%	1.53	18.16	-0.61%	1.75	-185.43%
H, Lv	9	24	1.41%	**0.24%**	0.49	11.36	**0.71%*****	4.00	-21.75%
Hp, Hv	15	24	1.42%	**0.25%**	1.81	25.39	-0.49%	1.68	-663.73%
Hp, Lv	16	24	0.84%	-0.33%	0.52	13.48	0.11%	0.74	-31.47%
Lp, Hv	25	24	1.04%	-0.13%	1.66	18.86	-0.73%	2.03	-74.10%
Lp, Lv	10	24	1.18%	**0.01%**	0.44	8.37	**0.53%***	2.50	-37.66%

3 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Average Monthly Returns	Average Monthly Excess Returns	CAPM Beta	|*t*|-*Stat*	Alpha	|*t*|-*Stat*	Maximum Drawdown
B, L, Hp	41	24	1.18%	**0.01%**	0.96	50.60	0.05%	0.63	-20.44%
B, L, Lp	5	24	1.00%	-0.17%	1.03	13.83	-0.19%	0.64	-100.00%
B, H, Lp	9	24	1.12%	-0.05%	1.13	14.94	-0.17%	0.55	-75.40%
B, Lp, Hv	8	24	0.97%	-0.20%	1.63	16.89	-0.78%	1.96	-57.00%
B, Hp, Hv	11	24	1.47%	**0.30%**	1.76	23.07	-0.40%	1.28	-2259.00%
B, L, Hv	13	24	1.25%	**0.08%**	1.79	23.30	-0.65%	2.04	-128.80%
B, H, Hv	6	24	0.92%	-0.25%	1.74	16.58	-0.92%	2.15	-115.69%
B, Lp, Lv	4	24	1.14%	-0.03%	0.44	7.96	**0.48%**	2.11	-42.65%
B, Hp, Lv	15	24	0.83%	-0.34%	0.52	13.29	0.12%	0.64	-32.12%
B, L, Lv	15	24	0.95%	-0.22%	0.51	15.52	0.24%	1.77	-372.19%
L, Hp, Hv	10	24	1.36%	**0.19%**	1.81	23.79	-0.55%	1.77	-1083.00%
L, Hp, Lv	13	24	0.75%	-0.42%	0.52	14.45	0.02%	0.15	-372.19%
S, L, Hp	3	24	1.19%	**0.02%**	0.84	8.95	0.16%	0.42	-78.71%
S, H, Lp	29	24	1.26%	**0.09%**	0.97	18.36	0.12%	0.53	-133.40%
H, Lp, Hv	16	24	1.25%	**0.08%**	1.50	15.66	-0.38%	0.97	-120.58%
H, Lp, Lv	6	24	1.34%	**0.17%**	0.50	9.95	**0.63%**	3.04	-38.16%
S, Lp, Hv	16	24	1.46%	**0.29%**	1.34	15.69	-0.01%	0.04	-5461.38%
S, H, Hv	18	24	1.21%	**0.04%**	1.31	16.36	-0.25%	0.75	-5461.38%
S, Lp, Lv	6	24	1.28%	**0.11%**	0.40	7.95	**0.69%**	3.60	-25.97%
S, H, Lv	6	24	1.25%	**0.08%**	0.37	9.12	**0.66%**	3.95	-33.69%
B, H, Lv	3	21	1.27% (1.45%)						
S, Hp, Lv	2	14	0.69% (1.18%)						
S, L, Hv	3	21	0.22% (0.26%)						
S, Hp, Hv	4	21	0.84% (0.96%)						
S, L, Lv	0.417	8	0.18% (0.54%)						
H, Hp, Lv	0.417	9	0.37% (0.81%)						
H, Hp, Hv	0.417	10	(-0.64%) (-1.56%)						
S, H, Hp	1	14	(-0.16%) (-0.10%)						
S, L, Lp	1	13	1.39% (0.76%)						
L, Lp, Lv	1	21	0.87% (0.99%)						
L, Lp, Hv	2.7	16	0.78% (1.16%)						
B, H, Hp	0.625	7	(-0.03%) (-0.10%)						
S, Hp	9	23	0.92% (0.96%)						
H, Hp	2	19	0.55% (0.69%)						

The returns result presented above shows that the more factors are combined to form a strategy, the rarer it is to find valid stocks. That said, for the portfolios that have active stock through the entire period, most of them have a reasonable number of stocks per year.

The 1 sort returns show that the size, value and profitability premium is present with the profitability premium being the largest at 12 basis points and the value premium being the smallest at 1 basis point every month. Although, the absolute total monthly returns presented does not confirm the volatility anomaly, the previous chapter has shown this anomaly in risk adjusted returns using the Sharpe ratio and CAPM assessment. Furthermore, the result shows that investing in either small, value, growth and high volatility stocks would have individually provided better returns in comparison to holding the market. High volatility stock investment strategy provided the best return during this period. It is important to recall that in risk adjusted terms, only the low volatility portfolio produced significant returns in excess of the market from this group.

However, as previous research has shown, the available excess return to be captured is not explained by only 1 factor. The result of the 2 sort portfolio observed return show that 11 of the 22 portfolios considered outperformed the market (*S,H; S,Lp; S,Hv; B,L; B,Hv; L,Hp; L,Hv; H,Lp; H,Lv; Hp,Hv; and Lp,Lv*) with the high profitability – high volatility portfolio returning the highest return (25 basis point over the market). The 3 sort portfolios produced 20 active portfolios throughout the entire sample period. 12 of these portfolios were observed to have outperformed the market (*B,L,Hp; B,Hp,Hv; B,L,Hv; L,Hp,Hv; S,L,Hp; S,H,Lp; H,Lp,Hv; H,Lp,Lv; S,Lp,Hv; SLpLv and S,H,Lv; and S,H,Hv) with the B,Hp,Hv* producing the best return within this sort and the entire sorts (30 basis points over the market returns). In risk adjusted terms however, only the S*,Lv; H,Lv; Lp,Lv; B,Lp,Lv; H,Lp,Lv; S,LpLv; and S,H,Lv* produced returns in excess of the market that are significant and this was achieved with smaller than beta value of 1. These results indicate that combining these factors as a type of portfolio strategy provides returns in excess of the market.

Within the 2 and 3 sort portfolios constructed, there were 14 portfolios with at least 1 inactive period and only a few stocks averagely per year. These are the *B,H,Lv; S,Hp,Lv; S,L,Hv; S,Hp,Hv; S,L,Lv; H,Hp,Lv; H,Hp,Hv; S,H,Hp; S,L,Lp; L,Lp,Lv; L,Lp,Hv; B,H,Hp; S,Hp and H,Hp*. Seven of these portfolios had an estimated average of between 0.41 and 1 stock per year and another five had between

2 and 3 stocks on average. When the entire time period is considered (with a zero return for inactive periods), the B,H,Lv portfolio produced the highest return with 28 basis points premium per month. However, it appears that stocks to include for this portfolio (and others within this group) are quite hard to come by hence, these portfolios were not considered any further. In light of Clare and Motson (2008) as well as Evans and Archer (1968), these are highly undiversified portfolios with implicit high levels of idiosyncratic risk. This is not entirely an unusual occurrence in practice and one way to manage such portfolios to make them more practicable will be to hold the qualifying stocks when available (adequately) and hold the portfolio in cash (or money market securities) when there are no adequate stocks available. However, for the purpose of this analysis which aims to establish portfolio strategies for sustainable withdrawals, this solution has not been considered nonetheless, considering such portfolios will be an area subsequent research could shed more light on.

The results also suggest that the driver of observed excess return seems to cut across the various factor combinations. However, significant risk adjusted excess return seems to be attributed to the low volatility factor as all the 7 outperforming portfolios (in risk adjusted terms) were constructed with this factor. This outperformance is a confirmation of the inadequacies of the CAPM as seen in literature.

Robust Check of Big and Small Portfolio Combinations

Following on from the robust check in the previous chapter, I have constructed all the small and big combination portfolios (all portfolios with multiple features that includes small and big stocks) using the 70th / 30th percentile approach for the big and small portfolio sorts to see if there may be the possibility of better outcomes as a result of this portfolio construction approach. The table below shows the result of this.

Portfolio	Avg. No. of Stocks (Years of Active Portfolio out of 24yrs)	Average Monthly Return	Monthly Standard Deviation
Small Stock Returns	81 (24yrs)	1.08%	4.43%
Big Stock Returns	81 (24yrs)	1.15%	4.03%
S,H	26.63 (24yrs)	1.21%	5.13%
S,Hv	28.12 (24yrs)	1.17%	7.42%
S,Lp	25.04 (24yrs)	1.24%	5.28%
S,Lv	29.86 (24yrs)	1.01%	3.10%
S,H,Lp	18.21 (24yrs)	1.20%	5.42%
S,Lp,Hv	11.54 (24yrs)	1.12%	8.57%
S,H,Hv	12.33 (24yrs)	1.32%	8.16%
B,L	4.29 (21yrs)	0.66%	5.13%
B,H (No Stocks)	NA	NA	NA
B,Hp	3.86 (21yrs)	0.85%	5.70%
B,Hv	2.17 (18yrs)	0.22%	8.36%
B,Lv	2.1 (18yrs)	0.69%	4.03%
B,Lp	1.36 (11yrs)	0.87%	4.33%
S,Hp	5.5 (22yrs)	0.85%	7.30%
S,L	2.35 (20yrs)	1.20%	7.34%
B,L,Hp	3.38 (21yrs)	0.75%	5.32%
B,L,Lp	1.33 (9yrs)	0.45%	4.36%
B,H,Lp (No Stocks)	NA	NA	NA
B,H,Hp (No Stocks)	NA	NA	NA
B,Lp,Hv	1.4 (5yrs)	0.11%	5.07%
B,Hp,Hv	2 (12yrs)	-0.04%	9.30%
B,H,Hv (No Stocks)	NA	NA	NA
B,L,Hv	1.8 (15yrs)	0.03%	9.97%
B,Hp,Lv	2 (16yrs)	0.61%	3.36%
B,Lp,Lv	1.25 (5yrs)	0.19%	1.40%
B,H,Lv (No Stocks)	NA	NA	NA
B,L,Lv	2.13 (16yrs)	0.62%	3.31%
S,L,Hp	1.56 (16ys)	0.55%	7.61%
S,L,Lp	1.5 (8yrs)	0.76%	5.14%
S,L,Hv	3.61 (18yrs)	0.59%	8.89%
S,Lp,Lv	4.32 (22yrs)	1.27%	3.77%
S,Hp,Lv	2.33 (18yrs)	0.39%	2.73%
S,H,Lv	4.04 (23yrs)	1.44%	3.28%
S,L,Lv	1.82 (17yrs)	0.60%	2.97%
S,Hp,Hv	3.63 (19yrs)	0.37%	6.60%
S,H,Hp	1.54 (13yrs)	0.00%	8.91%

The lower segment of the table contains portfolios with one or more inactive periods. The segment contains all the small and big combination portfolios in table 5 inactive periods segment as well as some new entrants. This is not surprising considering that the construction approach will implicitly imply a lesser pool of stocks available. In keeping with the idea of only considering portfolios with active stocks through the period considered, these portfolios cannot be considered any further.

The upper segment of the table includes portfolios which originally have active stocks (in table 5) through the entire period considered. However, whilst they still have a significant amount of average stock/year albeit a lesser amount relative to the median approach, the portfolios have produced a lower average monthly return at a relatively higher risk. This is an indication of less efficient portfolios and therefore have not been considered as a potential for better outcome.

Summary of Mean Reversion

The FTSE 350 universe was considered and for uniformity, data from 1996 was used across all portfolios. The market equity (ME) of the index constituent stock was ranked in descending order and the median was obtained. Stocks above the median are classed as large stock and that bellow are small stocks. The book equity (BE) of the index constituents was also obtained and then the ratio BE/ME was ranked. The top 30th percentile is classed as value stock while the bottom 30th percentile are the growth stock. The ratio EBIT/BE (earnings before interest and tax (EBIT)) was used to obtain high profit and low profit portfolios using the same top and bottom 30th percentile approach. The standard deviation of the monthly total returns of the stocks was also obtained and with the percentile approach, high and low volatility portfolios were obtained. Portfolios were first created based on the stocks that fit in the criteria of the 4 main factors (size, book to price, profitability, and volatility). Intersects (stocks qualifying for more than one factor) were then created based on 2 and 3 factor intersects. Each portfolio is rebalanced every January. The monthly total return (from the total return index) of the constituent stocks of the portfolios together with the market capitalisation (ME) was then used to establish the market capitalized weighted return of the portfolios (from which the average monthly return for the period under review was obtained). Using the AR(1) i.e. autoregression for each portfolio and the Dickey Fuller (ADF_τ) test statistic was also obtained from the result of the regression to test for stationarity and hence infer mean reversion.

β = Speed of mean reversion

n-a = Degree of freedom (number of adjusted observations - number of regressors)

1 S.D of Shock = Square root of (sum of squared residual/n-a)

** = Significant at 5% critical level*

*** = Significant at 10% critical level*

**** = Significant at 1% critical level*

Portfolio	α (t-stat)	Value of Coefficient β	ADF_τ	Sum of Squared Residuals σ^2	1 S.D of Shock σ	Value of $\frac{\lvert\beta\rvert}{\hat{\sigma}}$
All Market	0.011 (4.638)*	-0.981	16.548 (***)	0.459	0.040	24.49

Table 9: Summary of Mean Reversion (1 sort portfolios)

Portfolio	α (t-stat)	Value of Coefficient β	ADF_{τ}		Sum of Squared Residuals σ^2	1 S.D of Shock σ	Value of $\frac{\lvert\beta\rvert}{\hat{\sigma}}$
1 Factor Portfolios							
Small Portfolio	0.010 (3.877)*	-0.823	-14.088	(***)	0.540	0.043	18.94
Big Portfolio	0.011 (4.576)*	-0.973	16.421	(***)	0.465	0.040	24.13
Value Portfolio	0.011 (3.463)	-0.900	-15.262	(***)	0.793	0.053	17.09
Growth Portfolio	0.012 (4.751)*	-0.986	-16.658	(***)	0.473	0.041	24.25
High Profitability Portfolio	0.011 (4.476)*	-0.986	-16.638	(***)	0.467	0.040	24.40
Low Profitability Portfolio	0.009 (2.818)*	-0.915	-15.494	(***)	0.840	0.054	16.88
High Volatility Portfolio	0.013 (2.607)	-0.9154	-15.512	(***)	1.875	0.081	11.31
Low Volatility Portfolio	0.0095 (5.320)*	-0.9107	-15.440	(***)	0.229	0.028	**32.18**

Table 10: Summary of Mean Reversion (2 sort portfolios)

Portfolio	α (t-stat)	Value of Coefficient β	ADF_τ		Sum of Squared Residuals σ^2	1 S.D of Shock σ	Value of $\frac{\lvert\beta\rvert}{\hat{\sigma}}$
2 Factor Portfolios							
S, L	0.010 (2.223)	-0.844	-14.385	(***)	1.465	0.072	11.79
S, H	0.010 (3.411)*	-0.833	-14.246	(***)	0.690	0.049	16.96
S, Lp	0.012 (3.766)*	-0.844	-14.412	(***)	0.752	0.051	16.46
S, Hv	0.011 (2.770)	-0.8915	-15.098	(***)	1.221	0.065	13.64
S, Lv	0.009 (4.962)*	-0.8433	-14.407	(***)	0.237	0.029	**29.29**
B, L	0.012 (4.770)*	-0.988	-16.684	(***)	0.471	0.041	24.35
B, H	0.009 (2.513)	-0.799	-6.749	(***)	0.870	0.056	14.28
B, Hp	0.011 (4.509)*	-0.996	-16.817	(***)	0.491	0.041	24.04
B, Lp	0.009 (2.662)	-0.930	-15.737	(***)	0.914	0.057	16.45
B, Hv	0.012 (2.692)	-0.9974	-16.848	(***)	1.588	0.075	13.39
B , Lv	0.010 (5.229)*	-0.9134	-15.449	(***)	0.247	0.029	**31.08**
L, Hp	0.012 (4.613)*	-0.991	-16.735	(***)	0.476	0.041	24.29
L, Lp	0.009 (2.373)	-0.920	-15.583	(***)	1.202	0.065	14.19
L, Hv	0.012 (2.168)	-0.9165	-15.518	(***)	2.260	0.089	10.31
L, Lv	0.009 (4.712)*	-0.9088	-15.406	(***)	0.253	0.030	**30.56**
H, Lp	0.012 (3.568)*	-0.927	-15.683	(***)	0.941	0.057	16.16
H, Hv	0.005 (1.129)	-0.921	-15.590	(***)	1.989	0.083	11.04
H, Lv	0.013 (5.852)*	-0.9304	-15.746	(***)	0.351	0.035	**26.56**
Hp, Hv	0.013 (2.552)	-0.9341	-15.793	(***)	2.180	0.087	10.70
Hp, Lv	0.008 (4.019)*	-0.98	-16.556	(***)	0.320	0.033	**29.30**
Lp, Hv	0.010 (1.801)	-0.9259	-15.673	(***)	2.278	0.089	10.37
Lp, Lv	0.011 (4.418)*	-0.9075	-15.391	(***)	0.438	0.039	23.19

Table 11: Summary of Mean Reversion (3 sort portfolios)

Portfolio	α (t-stat)	Value of Coefficient β	ADF_{τ}		Sum of Squared Residuals σ^2	1 S.D of Shock σ	Value of $\frac{\lvert\beta\rvert}{\hat{\sigma}}$
3 Factor Portfolios							
B, L, Hp	0.012 (4.609)*	-0.9861	-16.660	(***)	0.471	0.041	24.30
B, L, Lp	0.009 (2.378)	-0.9206	-15.591	(***)	1.205	0.065	14.18
B, H, Lp	0.009 (2.222)	-0.8306	-6.805	(***)	1.241	0.067	12.40
B, Lp, Hv	0.009 (1.623)	-0.9303	-15.747	(***)	2.437	0.092	10.08
B, Hp, Hv	0.014 (2.6012)	-0.9241	-15.637	(***)	2.184	0.087	10.57
B, L, Hv	0.011 (2.163)	-0.9087	-15.399	(***)	2.243	0.089	10.26
B, H, Hv	0.009 (1.500)	-0.9732	-16.428	(***)	2.815	0.099	9.81
B, Lp, Lv	0.011 (4.139)*	-0.9295	-15.734	(***)	0.490	0.041	22.46
B, Hp, Lv	0.008 (3.901)*	-0.9749	-16.473	(***)	0.325	0.034	**28.92**
B, L, Lv	0.009 (4.655)*	-0.9055	-15.354	(***)	0.257	0.030	**30.21**
L, Hp, Hv	0.013 (2.364)	-0.9196	-15.564	(***)	2.257	0.089	10.35
L, Hp, Lv	0.007 (3.644)*	-0.9456	-15.985	(***)	0.298	0.032	**29.29**
S, L, Hp	0.011 (2.478)	-0.9185	-15.545	(***)	1.473	0.072	12.80
S, H, Lp	0.011 (3.354)*	-0.8542	-14.572	(***)	0.784	0.052	16.31
H, Lp, Hv	0.012 (2.246)	-0.958	-16.185	(***)	2.227	0.088	10.86
H, Lp, Lv	0.0171 (4.493)*	-0.815	-14.003	(***)	0.426	0.039	21.12
S, Lp, Hv	0.013 (2.689)	-0.8606	-14.659	(***)	1.735	0.078	11.05
S, H, Hv	0.011 (2.356)	-0.8767	-14.902	(***)	1.614	0.075	11.67
S, Lp, Lv	0.010 (4.566)*	-0.7708	-13.417	(***)	0.322	0.034	22.97
S, H, Lv	0.010 (5.184)*	-0.8178	-14.050	(***)	0.268	0.031	**26.72**
B, H, Lv	0.013 (4.938)*	-1.0136	-17.114	(***)	0.505	0.042	24.12
S, Hp, Lv	0.014 (4.167)*	-0.9772	-16.493	(***)	0.336	0.034	28.51
S, L, Hv	0.002 (0.386)	-0.8981	-15.190	(***)	2.186	0.087	10.27
S, Hp, Hv	0.006 (3.045)*	-0.9019	-15.269	(***)	2.357	0.091	9.93
S, L, Lv	0.002 (0.980)	-0.8649	-14.737	(***)	0.207	0.027	32.15
H, Hp, Lv	0.003 (1.762)	-0.9101	-15.427	(***)	0.295	0.032	28.34
H, Hp, Hv	-0.004 (-0.750)	-0.6982	-6.525	(***)	2.317	0.092	7.59
S, H, Hp	0.000 (-0.104)	-0.6243	-6.201	(***)	2.000	0.085	7.40
S, L, Lp	0.007 (1.233)	-0.9767	-16.493	(***)	2.957	0.102	9.58
L, Lp, Lv	0.007 (3.207)*	-0.92	-15.510	(***)	0.482	0.041	22.30
L, Lp, Hv	0.007 (1.461)	-0.9478	-16.021	(***)	2.070	0.085	11.14
B, H, Hp	0.000 (-0.103)	-1.021	-17.240	(***)	0.858	0.055	18.64
S, Hp	0.007 (2.138)	-0.757	-13.169	(***)	0.818	0.053	14.15
H, Hp	0.004 (1.005)	-0.939	-15.816	(***)	1.388	0.070	13.48

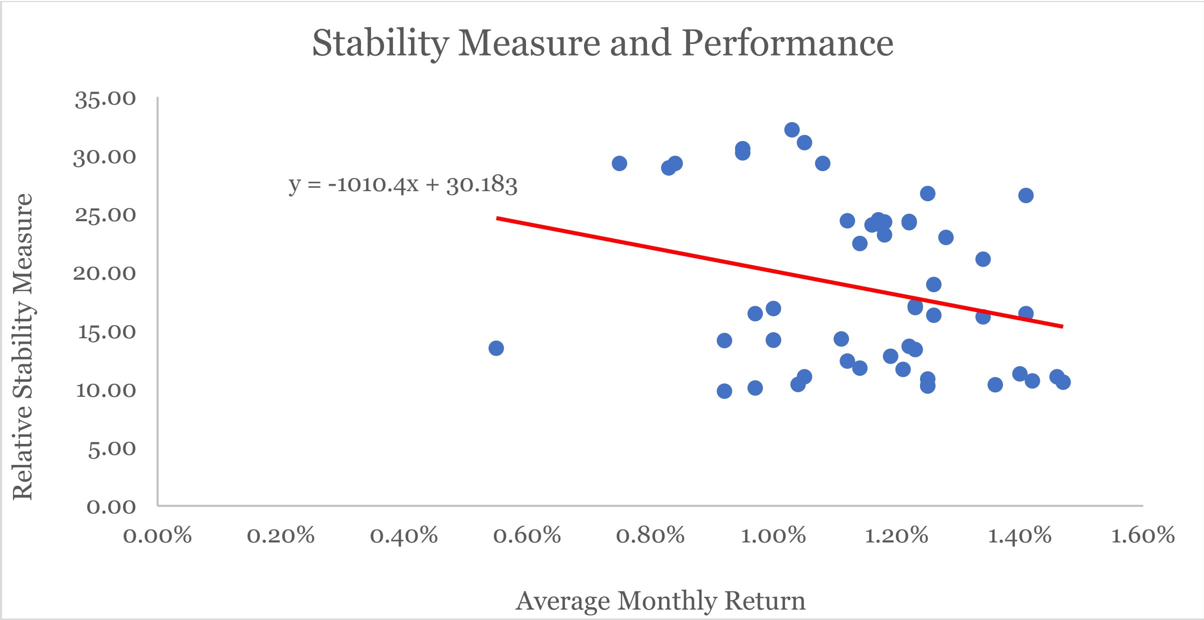
The ADF test result shown in tables 9 to 11 indicates that all the portfolios are mean reverting at 1%, 5% and 10% critical levels and the value of β is within the expected boundary of $-2< \beta<0$. However, due to the small number of average stocks per year and the inactive periods in the 14 portfolios coded in red, their results have been considered skewed and inaccurate and therefore, hasn't been considered further.

Relative to the market, only the *Lv* single factor portfolio produced a better than market ratio. The 2 factor portfolios with higher than market measures are: *B,Lv; L,Lv; H,Lv; Hp,Lv;* and *SLv* while the 3 factor portfolios with a greater than market measure are: *B,L,Lv; L,Hp,Lv; B,Hp,Lv; and S,H,Lv*. Furthermore, the *H,Lv, S,H,Lv,* portfolios not only have higher than market measures but have also outperformed the market in terms of average returns and also have relatively lower volatility.

This result appears to suggest in the first instance that exposure to low volatility stocks have a positive impact on portfolio stability. Considering that the low volatility anomaly describes stocks that provide relatively better returns with less risk, this impact on stability is perhaps understandable. Flavin, Morley and Panopoulu (2014) sight that the low volatility portfolios earn a duration premium because the greater stability in their cash flows tends to lend them a bond-like characteristic and this is a new finding in literature which helps to reconcile the sizeable outperformance of low volatility strategies. On another hand, the "value" qualities of low volatility stock portfolios which are also "value" in nature tends to provide relative superior returns for such portfolio as seen in Basu (1977), Lakonishok, Shleifer, and Vishny (1994), Fama French (1992) (amongst others) who show that there is a premium in value investing.

The result also shows that low volatility investing is not primarily a risk management strategy because it has historically produced relatively high returns which is partly attributed to the behavioural patterns where low volatility stocks are often undervalued relative to high volatility stocks. For portfolios for a long-term investor (such as a pensioner), the question is whether a low volatility portfolio has a place in the strategic asset allocation. Some investment consultants have suggested allocating to low volatility developed market equities to free up a portion of the risk budget for more allocations to emerging market equities; others have recommended replacing traditional equity exposure with low volatility strategies to reduce the volatility of the standard 60/40 stock/bond portfolio without sacrificing returns. The more aggressive pension plans have considered increasing the total equity allocation through low volatility equity strategies which would maintain plan level volatility while increasing returns (Flavin, Morley and Panopoulu (2014)). What this result tends to show is that in addition to these possibilities, a portfolio stable for withdrawal purpose may be achieved alongside.

The estimates of the relative stability shown in tables 9-11 indicates that there is somewhat an inconsistent relationship between the measure of stability and performance as seen in tables 8. This tends to conform to the general understanding that a good performing portfolio in itself should not suggest stability as other factors including volatility and persistence must be taken into account. However, it is important to note some outcomes; the high volatility 1 sort portfolio has the highest return within the 1 sort portfolios but also the lowest stability measure. Also, all the portfolios which outperformed the market (with the exception of H,Lv and S,H,Lv) have a stability measure lower than that of the market. This outcome tends to suggest that portfolios that have produced better than market returns have been more persistent relative to the market. Put differently, it seems to show that better than market performance tends to come with a slower 'risk adjusted' reversion to long run mean; the higher the long run return relative to the market, the slower it is likely to revert to this return following a unit deviation from this return. The scatter diagram below as well as the correlation table generally summarises the point made above; whilst there is no consistent pattern between the stability measure and performance, the relationship between both appears to be negative.



	Average Monthly Returns	Stability Measure
Average Monthly Returns	1	
Stability Measure	-0.26	1

As mentioned earlier, the exception to this outcome (market outperforming portfolios having lower relative stability measure) is with portfolios that consist of stocks that are value and low volatility in nature. These portfolios produced better than market returns and revert to their long run means relatively quicker. This seeming uniqueness of low volatility portfolios was highlighted by Chow et al (2014) who states that low volatility portfolios earn a duration premium because the greater stability in their cash flows tends to lend them a bond-like characteristic. They mention that this is a new finding in the literature which helps reconcile the sizeable outperformance of low volatility strategies. Dutt and Mark (2013) also show that low volatility stocks have higher operating returns, and this might explain why low volatility stocks earn higher stock returns. The rationale is that companies with strong operating results might be more stable and predictable; and thus, also have lower volatilities.

4.4 Conclusion

The growing preference for flexible forms of retirement has thrown the spotlight on the question of designing portfolio strategies for the purpose of withdrawals. Having identified a number of dominant factor exposures which are expected to produce returns in excess of the market in the previous chapter, this chapter focuses on combining these factors (size, book to market value, profitability and volatility) to form a strategies that are expected to mob up the available excess return in the market as it has been shown in literature that the CAPM beta factor does not in itself create the exposure for excess return neither will any other single factor do this satisfactorily.

The constituent stocks of the portfolios from each of these groups exhibiting more than one factor feature were combined in sorts of 2 (22 portfolios were considered) and 3 (20 portfolios were considered) and using the total return index data from 1996 through 2019, their returns were assessed against the market. The FTSE 350 world has been employed to represent the investible space of the UK market and avoid long tails of illiquid stocks. The 3 sort portfolios produced 12 portfolios that outperform the market while the 2-sort produced 11 and the driver of these returns' cuts across various factor combinations.

The chapter also investigates how much of a strategy these portfolio formations represents and it does so by examining the tendency of the returns to revert to its historical mean, hence revealing its predictive tendencies. This was done with the Augmented Dickey Fuller model for stationarity described in chapter 2 (motivated by the Ornstein-Uhlenbeck stochastic differential equation). The results show that all the portfolios created have the tendency to revert to their historical mean.

The objective of this chapter is not only to explore excess return potential from the combination of factor exposure strategies but to also identify portfolios that possess features of stability which will indicate their preference for use as a withdrawal (drawdown) portfolio that will produce growth and sustain withdrawal rates. The mean reversion speed indicates how quickly a portfolio will revert to its mean and this feature becomes important in the context of drawing down a portfolio. The drawdown action is effectively a downward shock on the portfolio (al be it exogenous) and therefore, the ability of a portfolio to revert to its mean will indicate its durability for this purpose. However, it will seem incomplete to only consider how quickly a portfolio reverts without considering how far it deviates from the mean hence, not just how fast, but how far. This chapter proposed the measure of reversion per unit deviation of shock $\frac{|\beta|}{\hat{\sigma}}$ as an *ad hoc* solution under the assumption that returns will have similar behaviour to endogenous and exogenous shocks. This measure is expected to give a reversion to mean speed measure relative to its volatility. This is quite important because designing a portfolio for retirement drawdown purpose should effectively factor in volatility within its structure.

Ten (10) portfolios had a measure that implied better stability than that of the market and all of these were exposed to the low volatility factor. Relative to the market, only the *Lv* single factor portfolio produced a better than market ratio. The 2 factor portfolios with higher than market measures are: *B,Lv; L,Lv; H,Lv; Hp,Lv;* and *SLv* while the 3 factor portfolios with a greater than market measure are: *B,L,Lv; L,Hp,Lv; B,Hp,Lv; and S,H,Lv*. Furthermore, the *H,Lv, S,H,Lv,* portfolios not only have higher than market measures but have also outperformed the market with lower volatility.

The stability measure proposed is effectively a one-dimensional measure; it takes into account a rate of change (speed of reversion) and the magnitude of change from an average (deviation). Therefore, it is clear to see that the measure does not account for the level of performance. Whilst performance in itself does not indicate stability, it has a bearing on how a portfolio will absorb a specified shock such as withdrawals hence, this measure may be ignoring an important component (performance) in the context of specified drawdown shocks. This situation means it may be difficult to achieve a consistent correlation in the predictions of this measure and the outcomes of empirical tests when assessing the stability of a portfolio to specified shocks. Some form of standardisation to the measure may therefore be required to enhance consistency with empirical outcomes. In addition to this, since the measure considers rate of change and the size of deviation of shock, there is a tendency for the observed standard deviation of the portfolio returns to appear as a proxy in terms of predicting stability. This is because the size of deviation of shock is generally seen to be close in magnitude to the observed portfolio standard deviation.

The main conclusion here is that the 4 excess return factors earlier identified can combine as a strategy to produce portfolios that not only offer higher than market returns but also display the potential to be more stable (relative to the market) for the purpose of withdrawals during retirement. In addition to this, the low volatility factor appears to be a prominent driver of this stability.

5 Portfolio Strategies and Sustainable Withdrawals

5.1 Introduction

As stated earlier, the introduction of the pension's freedom legislation in April 2015 significantly changed the pension investment landscape especially with the abolition of the need to annuitize at age 75. The need to annuitize at age 75 basically ensured that the retiree generated a sustained retirement income up till death. The abolition of this need effectively created the need to offer solutions providing sustainable income through retirement with the use of the DC plan. So far, the preceding chapters have identified various factors which if adopted as a strategy to create withdrawal portfolios will generate returns in excess of the market. It has also shown that combining these factors as a strategy will also produce portfolios with better than market returns. In addition to this, a measure of relative stability which is expected to give an indication of how stable a portfolio is during withdrawals was proposed.

The objective of this chapter is to establish the performance of these portfolios during withdrawals in comparison to the market and to assess if the proposed measure of stability effectively indicates stability/sustainability since it can be expected that a more stable portfolio will have a better level of success in sustaining withdrawals. Due to the constraint of long-term data, the analysis has been carried out using a simulation analysis to establish the success rates of the portfolios at different rates of withdrawal. The success rate is defined as the number of times out of the total simulation when the portfolio balance remains positive throughout the full period in a simulation. In addition to this, the chapter also assesses the correlation of the success rates with the relative stability measure to test whether it can be used as a proxy for portfolio withdrawal stability.

The chapter also finds that diversified portfolios constructed based on some individual factors such as size, book to market, profitability and volatility offer the potential to be more successful in sustaining a withdrawal rate of up to 8%, compared to the FTSE350 market. When portfolios are constructed based on a combination of these factors, some can even sustain a withdrawal rate at 10% with high levels of success. In general, four particular portfolios were identified as the most successful; *H,Lv* (value and low volatility stocks), *S,H,Lv* (small, value and low volatility stocks), *S,Lp,Lv* (small, low profitability and low volatility stocks) and *H,Lp,Lv* (value, low profitability and low volatility stocks)). The result of this chapter indicates that the low volatility factor *Lv* as well as the value factor *H* appears to be drivers of sustainable withdrawals.

Furthermore, the chapter finds that these successful portfolios also produce good bequest funds (residual portfolio fund usually left as death benefit for beneficiaries), up to 57% of the largest bequest fund from all portfolios constructed. In addition to this, conditional on the failure simulations, these portfolios sustained withdrawals for a considerable amount of time before failing. In addition, this chapter also finds that the proposed relative stability measure provides a good indication of stability conditional on failure, yet it is not as good in indicating the success rate of portfolio. A possible explanation for this is that the success rate is a function of both average returns and deviation (2 dimensional) whilst the stability measure is a ratio of change and deviation (one dimensional).

The rest of this chapter is organized as follows: Section 5.2 describes the methodology; section 5.3 presents the results and section 5.4 concludes.

5.2 Methodology.

In this chapter, a success rate analysis was carried out on the portfolios using the Monte-Carlo simulation method. Monte Carlo simulations are often used when the problem at hand has a probabilistic component. An expected value of that probabilistic component can be studied using Monte Carlo due to the law of large numbers (as the number of identically distributed, randomly generated variables increases, their sample mean (average) approaches their theoretical mean).

The traditional Monte Carlo methodology assumes the returns an investor is able to achieve are equally likely over the entire retirement period. However, in order to ensure that the returns series generated by the Monte-Carlo technique conforms to the model stipulation for mean reversion/persistence, certain conditions were imposed on the Monte-Carlo specification. An autocorrelation coefficient was specified so that portfolio mean, and standard deviation can be held constant. Recalling equations 3 and 4 from chapter 3,

$$\Delta R_t = \beta R_{t-1} + \varepsilon_t \qquad (3)$$

where,

$$\beta = \varphi - 1$$

and,

R_t is returns at time t

$$\Delta R_t = R_t - R_{t-1} \; and \; \varepsilon_t \sim N(0, \sigma^2)$$

$$\Delta R_t = \alpha_0 + \beta R_{t-1} + \varepsilon_t \qquad (4)$$

where α_0 is the drift term.

The model specification for the Monte-Carlo returns series can be written as:

$$R_t = \alpha_0 + (1 + \beta) R_{t-1} + \varepsilon_t$$

Where, $1 + \beta$ is basically persistence.

The Monte-Carlo technique is explained in appendix A but to summarize, first, a series of ε_t was generated by specifying a normal distribution with a mean of 0 and 1 standard deviation of shock of the portfolio. Following this, the first return in the simulated series R_t is specified as

$$R_1 = \alpha_0 + (1 + \beta) R_0 + \varepsilon_1$$

Where R_0 is the mean return of the portfolio.

Therefore, the second return of the simulated series is specified as:

$$R_2 = \alpha_0 + (1 + \beta) R_1 + \varepsilon_2$$

This returns data simulation continues till the 288-return data in the set i.e.,

$$R_{288} = \alpha_0 + (1+\beta)R_{287} + \varepsilon_{288}$$

10,000 simulation paths (sets) were then generated for each portfolio so that each portfolio has 10,000 paths with each path containing a return series with 288 return points.

Following this, an initial pot of £100,000 is then used to calculate the value of portfolio at each time point in the path. To be precise, the value of the portfolio is updated every month based on the simulated monthly rate of return, and at the year-end a withdrawal is paid out, reducing the value of the pot. The rates of withdrawal assessed are 4%, 6%, 8% and 10%. The first withdrawal is taken at the end of the first month of investment and then subsequently at the end of each year (so there will be 2 withdrawals in the 1st year). This is because generally, for a retail client, the demand for a drawdown portfolio is usually triggered following a notice for withdrawal. I have assumed the switch into a drawdown portfolio has been done 1 month before the first release. This withdrawal sum is then increased yearly by a rate of inflation. Annual CPI inflation rates for the corresponding periods (from 1996 to 2019) of the data was obtained from the Bank of England Library (available online[10]) to create a pool of 24 annual inflation rates. The rate used to annually increase the withdrawal sum was then selected randomly from this pool.

The continuous compounding takes the form below where, P_0 is the starting pot, P_1 is the portfolio value at the end of month 1, P_n is the final portfolio value (which is effectively P_{288}), r is the portfolio month return, w is the withdrawal rate taken every 12months (w_1 will be the rate of withdrawal multiplied by the starting accumulated pot), i is the inflation rate.

$P_1 = (P_0 - w_1)(1 + r_1)$

[10] *Available at https://www.ons.gov.uk/economy/inflationandpriceindices/timeseries/d7g7/mm23*

$$P_2 = P_1\,(1 + r_2) = (P_0 - w_1)\,(1 + r_1)\,(1 + r_2)$$

$$P_3 = P_2\,(1 + r_3) = (P_0 - w_1)(1 + r_1)\,(1 + r_2)\,(1 + r_3)$$

.....

.....

$$P_{12} = [P_{11}\,(1 + r_{12})] - w_2 = [(P_0 - w_1)\,(1 + r_1)\,(1 + r_2)(1 + r_3) \dots\dots(1 + r_{11})(1 + r_{12})] - (1 + i_1)w_1$$

.....

....

$$P_{24} = [P_{23}\,(1 + r_{24})] - w_3 = \{[(P_0 - w_1)\,(1 + r_1)\,(1 + r_2)(1 + r_3) \dots\dots(1 + r_{11})(1 + r_{12})] - (1 + i_1)w_1\}[(1 + r_{13})(1 + r_{14}) \dots\dots(1 + r_{23})(1 + r_{24})] - (1 + i_2)w_2$$

So that,

$$P_n = [P_{n-1}(1 + r_n)] - w_{\left(\frac{n}{12}\right)+1}$$

and a success for a particular path is defined as $P_n \geq £0$

and the success rate for a portfolio is defined as $\frac{Number\ of\ times\ P_n \geq £0}{Total\ Number\ of\ Simulated\ Paths\ for\ the\ Portfolio}$

So, for example, for a 4% withdrawal, if a portfolio made 2% return in the 1st month following a switch into the drawdown portfolio, then there will be £98,000 to invest in the 2nd month {(£100,000 x 1.02) - £4,000} and if 5% return was made in month 2, then there will be £102,900 to invest in month 3 (£98,000 x 1.05) and if 3% is made in month 3 then there will be £105,987 to invest in month 4 (£102,900 x 1.03). Assuming there was £120,000 at the end of the first 12 months (year 1) and the inflation rate has been 2%, then in month 13 (start of the 2nd year) the client will have £115,920 to invest {(£120,000-(£4,000 x 1.02)}.

For each of the withdrawal rates, the final portfolio balance at the end of the period for each portfolio for 10,000 simulations are then obtained. So, for example, portfolio 'A' will have 10,000 portfolio balances for 4% withdrawals and another 10,000-portfolio balance for 6% withdrawals and the same at 8% and 10% withdrawals. At each withdrawal rate, the success rate is defined as the number of times (out of the 10,000) a portfolio is having a ending balance greater than or equal to £0. The average balance of the successful portfolios (for each withdrawal rate) is obtained and serves as the average bequest value for the withdrawal rate considered. Also, for the simulated portfolios where the balance is less than £0 (i.e., conditional on failure), the average time period when the portfolios become less than £0 is also obtained and serves as the average failure point.

To determine the reliability of the relative stability measure, a correlation analysis of the success rates and relative stability measure is also analysed.

5.3 Results.

This section examines the results obtained from the Monte-Carlo simulation of the 50 portfolios considered to identify how successful the portfolios are at the various withdrawal rates. It also shows the result of the failure point assessment which identifies how long on average the portfolios sustained withdrawals before failing. Furthermore, the average portfolio residual value for the portfolios is also presented. In addition to these, the proposed relative stability measure for the portfolios is tested for its correlation with the success rate and failure point.

Table 12: Summary of Success Rate Analysis for 1 Sort Portfolios

The FTSE 350 universe was considered. Portfolios based on the factors of size, book to market ratio, profitability and volatility were formed. Hence, primary portfolios of 'S' (small stocks), 'B' (large stocks), 'H' (value stocks), 'L' (Growth stocks), 'Hp' (high profitability), 'Lp' (low profitability), 'Hv' (high volatility) and 'Lv' (low volatility) were formed. After sorting into the individual portfolios based on the various factors, the weighted monthly total returns (total returns times weighted market cap of the portfolio) of the constituent stocks for each year was obtained. Then the average monthly returns through the data period was calculated. Withdrawal rates of 4%, 6%, 8% and 10% of the initial portfolio value was considered. A 10,000 Monte-Carlo simulation was carried out on each portfolio for each withdrawal rate. This assumed a normal distribution with replacements. For each simulation, a final portfolio value was established. This was done by assuming a starting portfolio value of £100,000 and this was compounded with the monthly simulated returns. 2 withdrawals are made in the first year (January and December) and subsequently every December. The sum withdrawn was increased by the inflation rate randomly selected from a pool of annual inflation rate for the period under consideration (1996 – 2019). The inflation rate was obtained from the Bank of England online library. Success is defined as an ending portfolio balance greater than or equal to £0

Success Rates

Portfolio	Average Monthly Return	Observed Standard Deviation	Success @ Rate 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
FTSE 350	1.17%	4.00%	99.98%	97.99%	86.45%	62.77%
Small Portfolio	1.26%	4.41%	99.68%	96.29%	85.31%	48.47%
Big Portfolios	1.17%	4.03%	99.95%	97.83%	86.99%	62.46%
Value Portfolio	1.23%	5.28%	98.47%	91.52%	87.00%	63.11%
Growth	1.22%	4.06%	99.93%	98.45%	88.62%	66.26%
High Profitability	1.12%	4.03%	99.89%	97.14%	84.48%	57.98%
Low Profitability	1.00%	5.43%	94.94%	80.01%	58.16%	36.91%
High Volatility	1.40%	8.12%	90.90%	77.06%	63.02%	49.16%
Low Volatility	1.03%	2.83%	100.00%	99.04%	88.01%	54.64%

The table 12 above shows the result of investing in a diversified portfolio based on the individual factors. In addition to the FTSE350 market, all the individual factor portfolios have success rates at the 4% withdrawal rate ranging from 90%[11] (high volatility portfolio) to 100% (low volatility portfolio), most of which (including the FTSE350 market) achieved a 99% success rate. The portfolios are slightly less successful at the 6% withdrawal rate; With the exception of the low profitability and high volatility portfolios, the success rates range from 91% (value portfolios) to 99% (low volatility portfolios); the market has about 98% success. At the 8% withdrawal rate, more significant levels of failure (1-success rate) are observed. At this withdrawal, only growth and low volatility portfolios have an 88% success rate; the market has 86% while low profitability one has only 58% (the lowest). Others have success rates ranging from 63% (high volatility) to 87% (value). At 10% withdrawal rate, it is fair to say all the portfolios fail as success rates range only from 36% (low profitability portfolios) to 66% (growth portfolios).

[11] *A 92% success rate implies that 1800 simulated 24yr periods out of the total 10000 simulations, the portfolio value at the end of the period was less than £0*

Table 13: Summary of Success Rate Analysis for 2 Sort Portfolios

The FTSE 350 universe was considered. Portfolios based on the factors of size, book to market ratio, profitability and volatility were formed. Hence, primary portfolios of 'S' (small stocks), 'B' (large stocks), 'H' (value stocks), 'L' (Growth stocks), 'Hp' (high profitability), 'Lp' (low profitability), 'Hv' (high volatility) and 'Lv' (low volatility) were formed. Intersects (stocks qualifying for more than one factor) were then created based on 2 factor intersects. After sorting into the individual portfolios based on the various factors, the weighted monthly total returns (total returns times weighted market cap of the portfolio) of the constituent stocks for each year was obtained. Then the average monthly returns through the data period was calculated. Withdrawal rates of 4%, 6%, 8% and 10%of the initial portfolio value was considered. A 10,000 Monte-Carlo simulation was carried out on each portfolio for each withdrawal rate. This assumed a normal distribution with replacements. For each simulation, a final portfolio value was established. This was done by assuming a starting portfolio value of £100,000 and this was compounded with the monthly simulated returns. 2 withdrawals are made in the first year (January and December) and subsequently every December. The sum withdrawn was increased by the inflation rate randomly selected from a pool of annual inflation rate for the period under consideration (1996 – 2019). The inflation rate was obtained from the Bank of England online library. Success is defined as an ending portfolio balance greater than or equal to £0.

			Success Rates			
Portfolio	Average Monthly Return	Observed Standard Deviation	Success @ Rate 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
S, L	1.14%	7.24%	85.30%	69.98%	53.02%	38.44%
S, H	1.23%	4.98%	98.79%	91.76%	77.13%	57.93%
S, Hp	0.92%	5.51%	87.64%	68.73%	48.51%	31.35%
S, Lp	1.41%	5.18%	99.57%	95.36%	85.81%	70.89%
S, Hv	1.22%	6.56%	94.25%	81.78%	64.73%	48.73%
S, Lv	1.08%	2.92%	99.99%	98.60%	86.67%	55.78%
B, L	1.22%	4.06%	99.97%	98.32%	88.70%	66.66%
B, H	1.11%	5.60%	94.52%	81.61%	63.78%	45.34%
B, Hp	1.16%	4.14%	99.93%	97.44%	84.93%	60.40%
B, Lp	0.97%	5.66%	92.51%	75.01%	52.61%	33.44%
B, Hv	1.23%	7.45%	91.25%	76.43%	58.74%	42.89%
B , Lv	1.05%	2.95%	100.00%	99.16%	88.33%	57.66%
L, Hp	1.18%	4.08%	99.97%	97.92%	86.39%	63.05%
L, Lp	1.00%	6.49%	88.26%	69.79%	49.82%	33.49%
L, Hv	1.25%	8.91%	81.36%	65.44%	50.55%	38.57%
L, Lv	0.95%	2.98%	99.98%	96.72%	75.94%	40.43%
H, Hp	0.55%	7.83%	48.33%	26.05%	13.04%	6.64%
H, Lp	1.34%	5.74%	98.82%	93.07%	81.10%	63.53%
H, Hv	1.05%	8.37%	75.39%	57.19%	41.57%	27.60%
H, Lv	**1.41%**	**3.51%**	**100.00%**	**99.85%**	**98.38%**	**89.21%**
Hp, Hv	1.42%	8.73%	88.71%	76.42%	62.65%	48.40%
Hp, Lv	0.84%	3.34%	99.68%	89.54%	57.96%	21.43%
Lp, Hv	1.04%	8.93%	71.66%	53.80%	39.46%	26.60%
Lp, Lv	1.18%	3.92%	99.92%	97.44%	87.02%	63.72%

Table 13 shows the success rate results for the 2 sort portfolios. There are a total of 24 portfolios created. At the 4%, only the *H,Lv* portfolios have a 100% success rate. Eight other portfolios have a success rate of 99%; the *H,Hp* portfolio has the worst success rate of 48%. At 6% withdrawal rate, *B,Lv* and *H,Lv* portfolios have the highest success rate (99%) however, only 9 portfolios have success rates in excess of 90% (ranging from 91% to 98%). The *H,Hp* portfolios has the lowest success rate of 26%. At the 8% rate of withdrawal, the *H,Lv* portfolio is the most successful with a success rate of 98%. The *B,L and B,Lv* were the next most successful portfolios with over 88% success rates while the *H,Hp* remained the worst performing portfolio with 13% success rate. Only the *H,Lv* portfolio has about 90% success rate at the 10% rate of withdrawal whilst the S,Lp and B,L portfolios were the next most successful portfolios with 70% and 66% success rates respectively.

Table 14: Summary of Success Rate Analysis for 3 Sort Portfolios

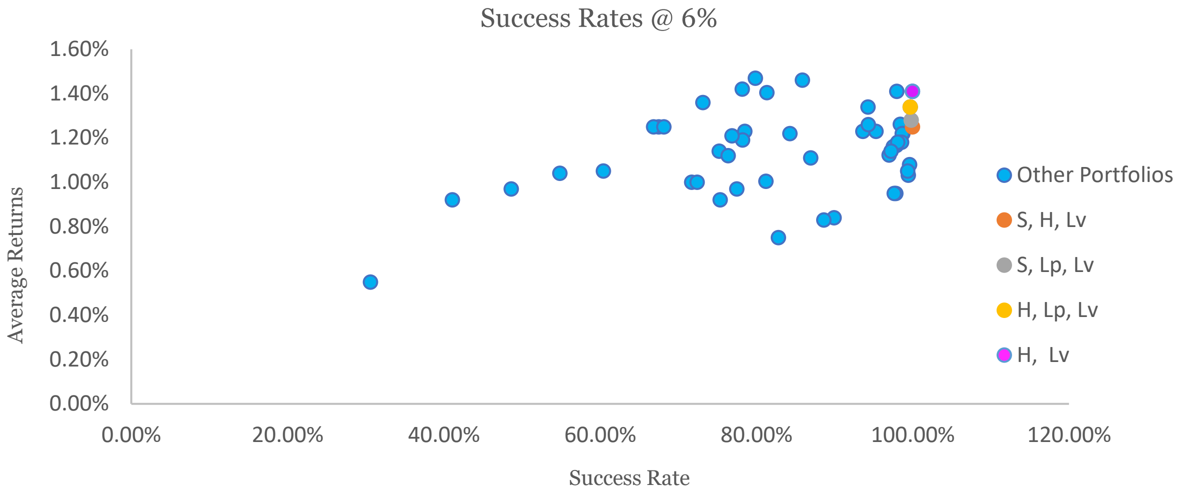
The FTSE 350 universe was considered. Portfolios based on the factors of size, book to market ratio, profitability and volatility were formed. Hence, primary portfolios of 'S' (small stocks), 'B' (large stocks), 'H' (value stocks), 'L' (Growth stocks), 'Hp' (high profitability), 'Lp' (low profitability), 'Hv' (high volatility) and 'Lv' (low volatility) were formed. Intersects (stocks qualifying for more than one factor) were then created based on 3 factor intersects. After sorting into the individual portfolios based on the various factors, the weighted monthly total returns (total returns times weighted market cap of the portfolio) of the constituent stocks for each year was obtained. Then the average monthly returns through the data period was calculated. Withdrawal rates of 4%, 6%, 8% and 10%of the initial portfolio value was considered. A 10,000 Monte-Carlo simulation was carried out on each portfolio for each withdrawal rate. This assumed a normal distribution with replacements. For each simulation, a final portfolio value was established. This was done by assuming a starting portfolio value of £100,000 and this was compounded with the monthly simulated returns. 2 withdrawals are made in the first year (January and December) and subsequently every December. The sum withdrawn was increased by the inflation rate randomly selected from a pool of annual inflation rate for the period under consideration (1996 – 2019). The inflation rate was obtained from the Bank of England online library. Success is defined as an ending portfolio balance greater than or equal to £0.

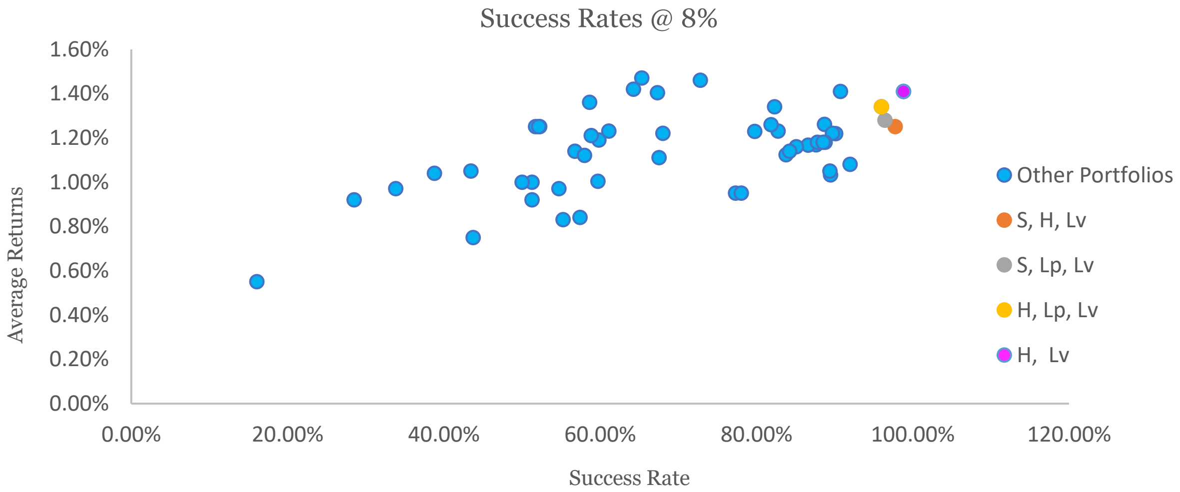
			Success Rates			
Portfolio	Average Monthly Return	Observed Standard Deviation	Success @ Rate 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
B, L, Hp	1.18%	4.00%	99.89%	97.62%	86.50%	61.78%
B, L, Lp	1.00%	6.50%	88.36%	68.86%	50.24%	33.75%
B, H, Lp	1.12%	6.80%	89.29%	72.23%	56.52%	41.26%
B, Lp, Hv	0.97%	9.24%	65.30%	46.86%	33.45%	23.64%
B, Hp, Hv	1.47%	8.75%	90.41%	76.93%	64.29%	51.31%
B, L, Hv	1.25%	8.88%	81.18%	64.62%	50.85%	39.05%
B, H, Hv	0.92%	9.91%	58.49%	40.48%	27.99%	19.29%
B, Lp, Lv	1.14%	4.10%	99.83%	95.86%	81.93%	56.98%
B, Hp, Lv	0.83%	3.40%	99.43%	87.25%	53.15%	21.39%
B, L, Lv	0.95%	3.00%	99.98%	96.81%	76.36%	40.83%
L, Hp, Hv	1.36%	8.90%	85.52%	71.69%	57.22%	43.47%
L, Hp, Lv	0.75%	3.23%	98.99%	82.17%	43.39%	13.80%
S, L, Hp	1.19%	7.20%	89.34%	74.72%	57.04%	40.75%
S, H, Lp	1.26%	5.30%	98.61%	91.87%	77.16%	58.87%
H, Lp, Hv	1.25%	8.82%	82.42%	66.54%	50.88%	37.41%
H, Lp, Lv	**1.34%**	**3.92%**	**99.94%**	**98.44%**	**91.19%**	**75.04%**
S, Lp, Hv	1.46%	7.85%	92.58%	81.97%	68.16%	54.83%
S, H, Hv	1.21%	7.56%	87.92%	71.69%	56.99%	42.76%
S, Lp, Lv	**1.28%**	**3.46%**	**99.94%**	**98.66%**	**90.76%**	**73.09%**
S, H, Lv	**1.25%**	**3.12%**	**100.00%**	**99.33%**	**94.08%**	**75.97%**

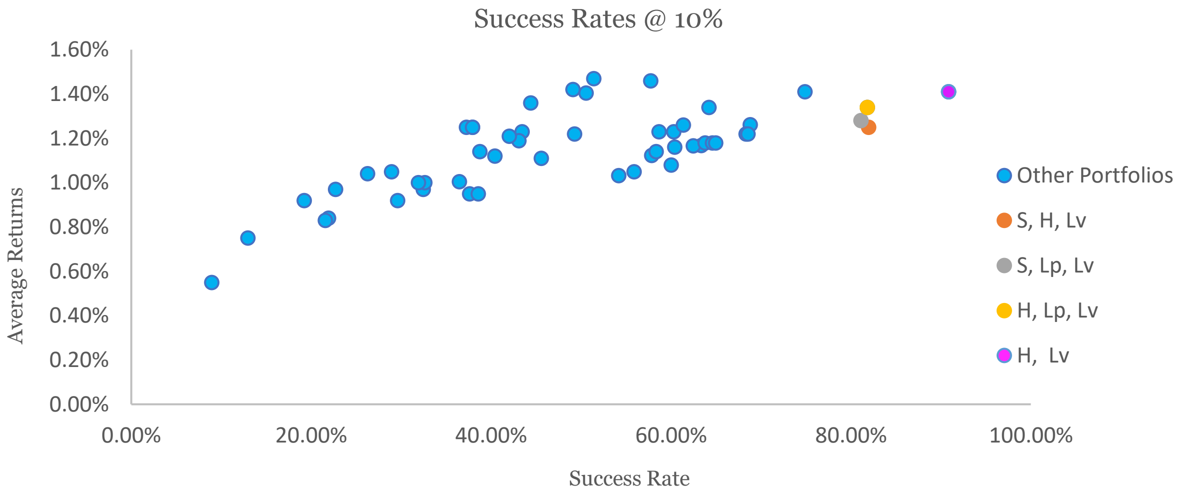
Table 14 shows the result of success rates for the 3 sort portfolios (20 portfolios were considered). At the 4% withdrawal rate, 6 portfolios including *S,Lp,Lv* and *H,Lp,Lv* have 99% success rate whilst the

S,H,Lv achieved 100% success. The *B,H,Hv* portfolio was the least successful with 58% success rate. At 6% rate of withdrawal, *H,Lp,Lv; S,Lp,Lv* and *S,H,Lv* are the most successful portfolios with 98%, 98% and 99% success rates respectively and only 4 other portfolios have a success rate of over 90% (between 91% and 97%). At the 8% rate of withdrawal, *H,Lp,Lv; S,Lp,Lv* and *S,H,Lv* have respective success rates of 91%, 90% and 94%; these are the only above 90% success rates. The remaining portfolios produce success rates ranging from 27% *(B,H,Hv)* to 86%. The *H,Lp,Lv*; *S,Lp,Lv* and *S,H,Lv* have 75%, 73% and 75% success rates at the 10% withdrawal rate whilst the remaining portfolios have success rates ranging from 13% to 61%.

Success Rate Scatter Plots (All Portfolios)


Success Rates @ 6%
Average Returns
Success Rate
1.60%
1.40%
1.20%
1.00%
0.80%
0.60%
0.40%
0.20%
0.00%
0.00%
20.00%
40.00%
60.00%
80.00%
100.00%
120.00%
Other Portfolios
S, H, Lv
S, Lp, Lv
H, Lp, Lv
H, Lv


Success Rates @ 8%
Average Returns
Success Rate
1.60%
1.40%
1.20%
1.00%
0.80%
0.60%
0.40%
0.20%
0.00%
0.00%
20.00%
40.00%
60.00%
80.00%
100.00%
120.00%
Other Portfolios
S, H, Lv
S, Lp, Lv
H, Lp, Lv
H, Lv


Success Rates @ 10%
Average Returns
Success Rate
1.60%
1.40%
1.20%
1.00%
0.80%
0.60%
0.40%
0.20%
0.00%
0.00%
20.00%
40.00%
60.00%
80.00%
100.00%
Other Portfolios
S, H, Lv
S, Lp, Lv
H, Lp, Lv
H, Lv

In summary, 53 portfolios are considered and at the 4% withdrawal rate, the market and 28 other portfolios sustain this rate of withdrawal with a success rate of between 99%-100%. Another 16 portfolios are able to sustain this rate of withdrawal with at least a 90% success. The worst performing portfolio is the *H,Hp* with a success rate of 52%. None of the portfolios (including the market) have a 100% success rate at 6% withdrawal but 25 of the portfolios can still reach at least 90%, with portfolios *S,H,Lv* and *H,Lv* having the highest success rate of 99%.

At 8% withdrawal rate, the *H,Lv* portfolio is the most successful with 98% success. The *S,H,Lv* portfolio has a 97% success rate. Two other portfolios are also as successful; the *H,Lp,Lv* portfolio has a success rate of 96% and the *S,Lp,Lv* portfolio has a success rate of 96%. The worst performing portfolio (*H,Hp*) has a success rate of 16%. At the 10% withdrawal rate, only the *H,Lv* portfolio has a decent success rate of about 90%. The *S,H,Lv*, *H,Lp,Lv* and *S,Lp,Lv* have a success rate of 81% respectively. These are the best performing portfolios. The remaining portfolios have success rates ranging from 8% to 74%. The results tend to show that combining these factors as a strategy helps to create more stable portfolios for withdrawal purpose as the most successful portfolios are found in the combined factor sorts.

Furthermore, these results tend to align with some of the observations from Athavale and Goebel (2011), who note that firstly, a scenario where a portfolios average return exceed the withdrawal rate does not necessarily imply that such portfolio will be relatively more successful (for example, this can be seen at the 10% withdrawal rate where most of the portfolios had very low success rates but their annual average returns are in excess of 12%). Secondly, relatively lower (higher) returns do not necessarily imply relatively worse (better) stability. Taken together, they conclude that although larger returns and smaller standard deviations contribute to portfolio success, these are not sufficient conditions to ensure success, and other factors including the timing of returns and the occurrence of negative or positive runs are also important. This conclusion appears to describe the 4 most successful portfolios identified as they produced relatively better returns with relatively lower standard deviation. Notably also is that the 4 most successful portfolios have the low volatility factor in their construction and 3 of these have the value factor in addition. As it seems, the combined effect of these two factors tends to enhance sustainability.

However, although a successful rate can help us predict the probability of a failure (Failure rate = 1 – Success rate), it does not provide information on the timing of failure. A strategy that sustained a retiree's withdrawal plan halfway through his retirement is very different from another that carried him 90% of the way. In the work by Estrada (2017), two variables are proposed; one measure how long before the end of the retirement period a strategy fails and the other measures what proportion of the retirement period a strategy sustains a retiree's withdrawals. The work concludes that these variables, together with the failure rate provide a better picture of the main risk retirees have to bear during retirement.

Estrada's work points out that although 2 strategies may be exposed to the same level of risk using the failure/success rate assessment, the timing of failure can tell a different and/or more informative story on strategy's stability.

Table 15: Summary of Failure Point (1 sort portfolios)

Following on from the details of table 9, the default periods (failure point) were obtained. This is the month (the particular month during the 288 months of each simulation) when the failure occurred. The minimum, maximum and average failure periods are presented in the table. The empty cells indicate a 100% success rate (no failure).

	Default Period *(Years)*											
	4% Withdrawal Rate			**6% Withdrawal Rate**			**8% Withdrawal Rate**			**10% Withdrawal Rate**		
	Min Period	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**
FTSE 350	17	21	24	10	19	24	7	17	24	7	15	24
Small Portfolio	13	19	24	7	17	24	6	16	24	5	14	24
Big Portfolios	16	20	23	11	19	24	7	17	24	6	15	24
Value Portfolio	9	19	24	8	16	24	7	17	24	6	15	24
Growth	15	19	23	10	19	24	8	17	24	6	15	24
High Profitability	17	21	24	10	19	24	7	17	24	6	16	24
Low Profitability	10	19	24	7	17	24	5	15	24	4	13	24
High Volatility	5	16	24	4	14	24	4	13	24	3	11	24
Low Volatility	-	-	-	13	21	24	9	19	24	7	16	24

Table 16: Summary of Failure Point (2 sort portfolios)

Following on from the details of table 10, the default periods (failure point) were obtained. This is the month (the particular month during the 288 months of each simulation) when the failure occurred. The minimum, maximum and average failure periods are presented in the table. The empty cells indicate a 100% success rate (no failure).

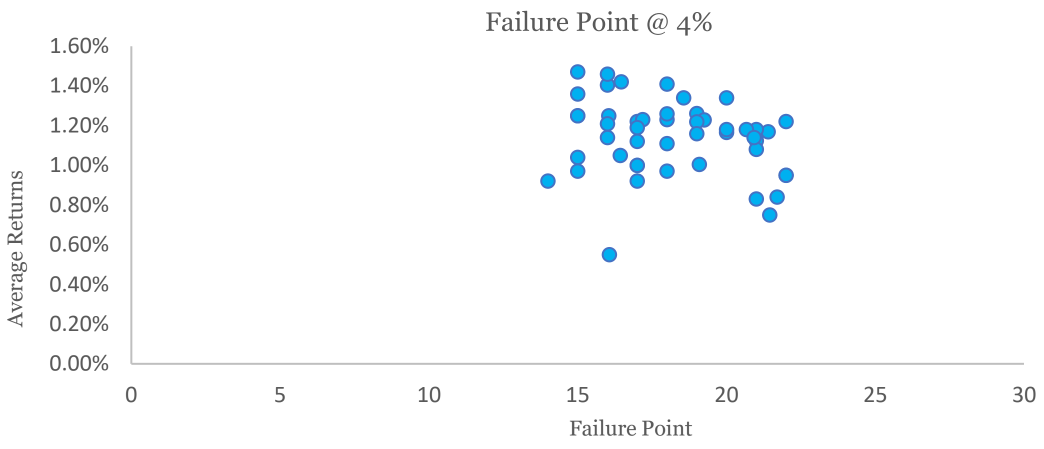
	Default Period *(Years)*											
	4% Withdrawal Rate			**6% Withdrawal Rate**			**8% Withdrawal Rate**			**10% Withdrawal Rate**		
	Min Period	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**
S, L	6	16	24	4	14	24	4	13	24	3	11	24
S, H	9	18	24	6	17	24	5	15	24	5	14	24
S, Hp	7	17	24	5	15	24	4	14	24	3	12	24
S, Lp	10	18	24	7	17	24	5	15	24	4	14	24
S, Hv	7	17	24	4	15	24	4	14	24	4	12	24
S, Lv	21	21	21	13	20	24	8	18	24	7	16	24
B, L	20	22	24	10	19	24	8	17	24	6	15	24
B, H	7	18	24	6	16	24	5	14	24	4	12	24
B, Hp	14	19	24	10	18	24	7	17	24	6	15	24
B, Lp	8	18	24	6	16	24	5	14	24	4	12	24
B, Hv	7	17	24	4	15	24	4	13	24	3	12	24
B , Lv	-	-	-	12	20	24	9	18	24	7	16	24
L, Hp	17	21	24	9	19	24	7	17	24	6	15	24
L, Lp	7	17	24	4	15	24	4	14	24	3	12	24
L, Hv	4	15	24	3	13	24	3	12	24	2	11	24
L, Lv	19	22	23	11	20	24	8	18	24	6	15	24
H, Hp	5	16	24	4	13	24	4	11	24	3	10	24
H, Lp	9	19	24	7	16	24	5	15	24	4	13	24
H, Hv	5	16	24	4	14	24	3	12	24	3	11	24
H, Lv	**-**	**-**	**-**	**18**	**21**	**24**	**8**	**18**	**24**	**7**	**16**	**24**
Hp, Hv	5	16	24	4	14	24	3	12	24	3	11	24
Hp, Lv	15	22	24	11	19	24	7	17	24	6	14	24
Lp, Hv	4	15	24	3	13	24	3	12	24	2	10	24
Lp, Lv	15	20	24	11	18	24	7	17	24	6	15	24

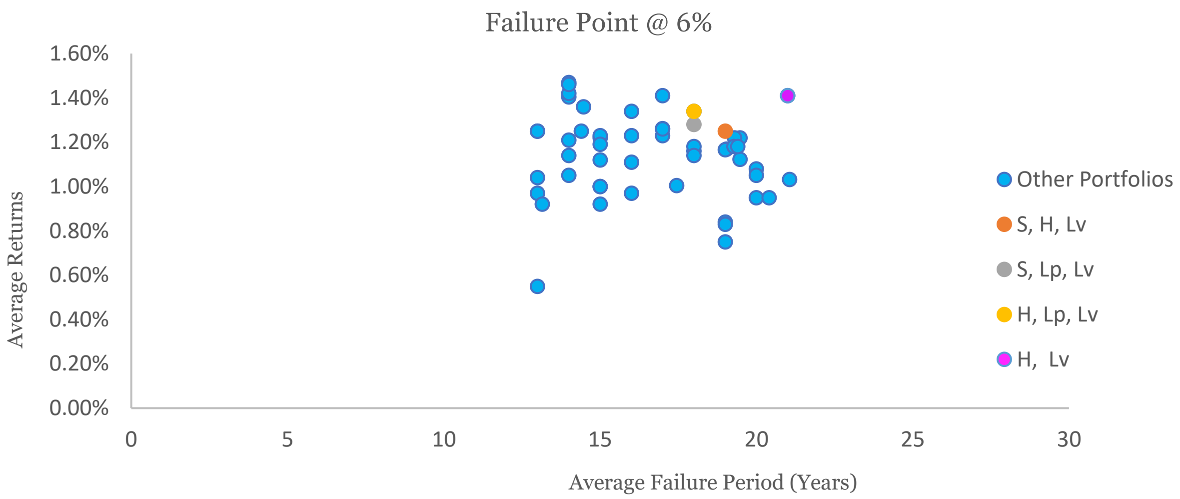
Table 17: Summary of Failure Point (3 sort portfolios)

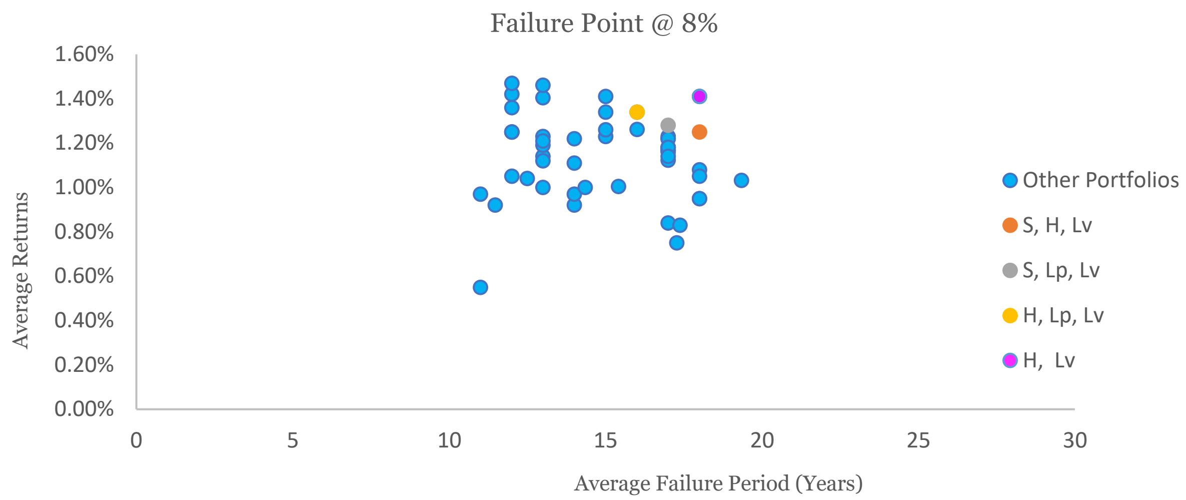
Following on from the details of table 11, the default periods (failure point) were obtained. This is the month (the particular month during the 288 months of each simulation) when the failure occurred. The minimum, maximum and average failure periods are presented in the table. The empty cells indicate a 100% success rate (no failure).

	Default Period *(Years)*											
	4% Withdrawal Rate			**6% Withdrawal Rate**			**8% Withdrawal Rate**			**10% Withdrawal Rate**		
	Min Period	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**	**Min Period**	**Average Period**	**Max Period**
B, L, Hp	17	21	24	10	19	24	7	17	24	6	15	24
B, L, Lp	6	17	24	5	15	24	4	13	24	3	12	24
B, H, Lp	6	17	24	4	15	24	4	13	24	3	12	24
B, Lp, Hv	5	15	24	3	13	24	3	11	24	2	10	24
B, Hp, Hv	5	15	24	4	14	24	3	12	24	2	11	24
B, L, Hv	5	15	24	3	13	24	3	12	24	2	11	24
B, H, Hv	4	14	24	3	13	24	3	11	24	2	10	24
B, Lp, Lv	13	21	24	10	18	24	6	17	24	6	15	24
B, Hp, Lv	15	21	24	9	19	24	7	17	24	6	14	24
B, L, Lv	19	22	24	11	20	24	8	18	24	6	15	24
L, Hp, Hv	5	15	24	4	14	24	3	12	24	3	11	24
L, Hp, Lv	15	21	24	10	19	24	6	17	24	6	14	24
S, L, Hp	7	17	24	5	15	24	4	13	24	3	12	24
S, H, Lp	10	18	24	7	17	24	5	15	24	4	13	24
H, Lp, Hv	5	16	24	4	14	24	3	12	24	3	11	24
H, Lp, Lv	**16**	**20**	**24**	**10**	**18**	**24**	**6**	**16**	**24**	**6**	**15**	**24**
S, Lp, Hv	6	16	24	5	14	24	4	13	24	3	11	24
S, H, Hv	6	16	24	4	14	24	3	13	24	3	11	24
S, Lp, Lv	**17**	**20**	**23**	**10**	**18**	**24**	**8**	**17**	**24**	**6**	**15**	**24**
S, H, Lv	**-**	**-**	**-**	**14**	**19**	**24**	**8**	**18**	**24**	**7**	**16**	**24**

Failure Point Scatter Plots (All Portfolios)







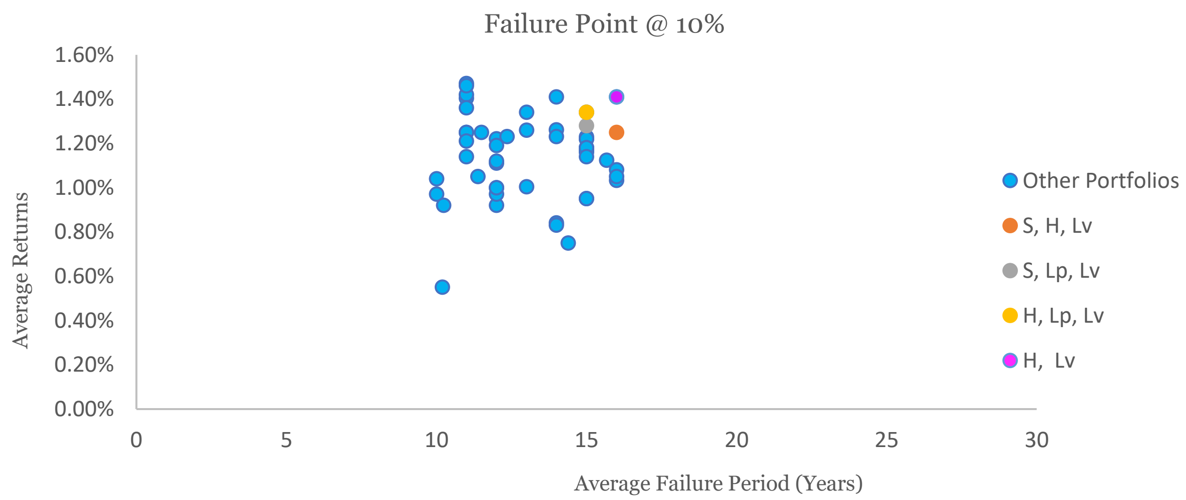


Table 15 show the summary of the failure points for the 1 sort set of portfolios. Conditional on failure, the FTSE350 and high profitability portfolios sustained the 4% rate of withdrawal up to the 21st (out of 24) year on average. This was the furthest failure point in this category at this withdrawal rate. The low volatility portfolio had no failed periods at this withdrawal rate. At 6%, 8% and 10% withdrawal rates, the low volatility portfolio was the most sable.

Table 16 shows the summary of the failure point for the 2 sort portfolios. At 4% withdrawal rate, only the *B,L; L,Lv*; and *Hp,Lv* had better stability compared to the FTSE 350 conditional on failure at year 22 average failure point, whilst the *B,Lv* and *H,Lv* portfolios did not have any failed period. At 6% withdrawal rate, only 4 portfolios had a better failure point on average compared to the market (year 19) with *H,Lv* having the furthest period at 21 years. At the 8% and 10% withdrawal rates, fewer portfolios (4 and 3 respectively) had a better failure point compared to the market.

Table 17 shows the summary of the failure point for the 3 sort portfolios. At 4% withdrawal rate, on average and conditional on failure, most of the portfolios did not sustain this withdrawal as long as the FTSE350 did (year 21) however, whilst the *S,H,Lv* portfolios did not have any failed period. At the 8% withdrawal rate, the *B,L,Lv* portfolio sustained withdrawals for the longest period (year 20) and this is 1year more than that of the market (year 19). Again the difference in the success rate and failure point implications can be spotted; whilst the *B,L,Lv* has a success rate of 78% at this 8% withdrawal rate compared to the *B,L,Hp* portfolio with 88% success rate; it sustained withdrawals for an additional 1 year on average during failed periods. Also, the *L,Hp,Hv* and the *S,H,Hv* both have similar success rate of 58% however, the *S,H,Hv* portfolio sustained withdrawals for an additional year. At the 10% withdrawal rate, only the *S,H,Lv* portfolios had an average failure point further than that of the market (year 16).

In summary tables 15 through 17 show that the 4 most successful portfolios (*H,Lv*, *H,Lp,Lv*, *S,Lp,Lv* and *H,Lp,Lv*) identified earlier either did not fail or sustained withdrawals till year 20 when they failed through all the simulated periods at the 4% withdrawal rate. At 6% withdrawal rate, *H,Lv* and *S,H,Lv* had similar failure rates (1-success rate) but *S,H,Lv* sustained this withdrawal rate one year more year on average during failure periods. At the 6% and 8% withdrawal rate, *H,Lp,Lv* and *S,Lp,Lv* had similar failure points (*S,Lp,Lv* had a marginally better failure point in both cases). At 8% withdrawal rate *H,Lv* had a similar failure point with the *S,H,Lv* portfolios but the *H,Lv* portfolio sustained withdrawals for an additional 2 years on average at the 6% withdrawal rate. At the 10% withdrawal rate however they both had the same failure point on average during failure periods.

These results generally agree with Estrada (2017) as it indicates that the failure point which sometimes gives a different but deeper information compared to that of the failure/success rate can be used as an added layer for assessing the sustainability of portfolios. In addition to this, the result also show that the 4 most successful portfolios provide competitive average failure points as they produce either the furthest average failure point or very close to the furthest achieved average failure point through all the withdrawal rates.

Table 18: Summary of Residual Value (1 sort portfolios)

Following on from the details of table 9, the residual value presented shows the minimum, average and maximum residual portfolio values during successful periods.

	Residual Value (£m)											
	4% Withdrawal Rate			**6% Withdrawal Rate**			**8% Withdrawal Rate**			**10% Withdrawal Rate**		
	Min Value	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**
FTSE 350	£0.00	£1.89	£26.50	£0.00	£1.43	£30.00	£0.00	£1.06	£18.60	£0.00	£0.91	£14.20
Small Portfolio	£0.00	£2.95	£72.80	£0.00	£2.39	£91.00	£0.00	£1.91	£52.90	£0.00	£0.72	£122.00
Big Portfolios	£0.00	£1.94	£17.10	£0.00	£1.47	£22.30	£0.00	£1.11	£22.70	£0.00	£0.87	£13.90
Value Portfolio	£0.00	£2.61	£47.20	£0.00	£2.14	£72.60	£0.00	£1.13	£21.70	£0.00	£0.93	£12.50
Growth	£0.03	£2.13	£27.20	£0.00	£1.66	£28.00	£0.00	£1.27	£27.20	£0.00	£1.00	£20.60
High Profitability	£0.00	£1.69	£33.90	£0.00	£1.24	£15.30	£0.00	£0.95	£20.40	£0.00	£0.78	£15.00
Low Profitability	£0.00	£1.29	£58.40	£0.00	£1.05	£19.60	£0.00	£0.97	£23.80	£0.00	£0.96	£23.40
High Volatility	£0.00	£4.80	£347.00	£0.00	£4.87	£708.0	£0.00	£4.93	£377.00	£0.00	£4.46	£177.00
Low Volatility	£0.05	£1.30	£8.66	£0.00	£0.92	£8.53	£0.00	£0.63	£6.07	£0.00	£0.46	£5.05

Table 19: Summary of Residual Value (2 sort portfolios)

Following on from the details of table 10, the residual value presented shows the minimum, average and maximum residual portfolio values during successful periods

	Residual Value											
	4%			**6% Withdrawal Rate**			**8% Withdrawal Rate**			**10% Withdrawal Rate**		
	Min Value	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**
S, L	£0.00	£2.66	£22.40	£0.00	£2.59	£60.90	£0.00	£2.59	£185.00	£0.00	£2.46	£108.00
S, H	£0.00	£2.66	£20.20	£0.00	£2.22	£86.90	£0.00	£1.89	£43.70	£0.00	£1.33	£41.40
S, Hp	£0.00	£1.29	£117.00	£0.00	£1.17	£68.90	£0.00	£1.16	£55.10	£0.00	£1.14	£51.50
S, Lp	£0.00	£4.64	£153.00	£0.00	£3.86	£98.50	£0.00	£3.12	£152.00	£0.00	£2.96	£163.00
S, Hv	£0.00	£2.87	£156.00	£0.00	£2.44	£153.00	£0.00	£2.43	£118.00	£0.00	£2.35	£297.00
S, Lv	£0.02	£1.41	£13.10	£0.00	£1.04	£12.80	£0.00	£0.72	£7.71	£0.00	£0.56	£7.50
B, L	£0.00	£2.14	£27.60	£0.00	£1.63	£20.30	£0.00	£1.28	£23.50	£0.00	£1.02	£14.70
B, H	£0.00	£2.19	£84.70	£0.00	£1.99	£128.00	£0.00	£1.70	£80.50	£0.00	£1.76	£50.40
B, Hp	£0.00	£1.76	£27.60	£0.00	£1.38	£28.80	£0.00	£1.03	£17.50	£0.00	£0.85	£16.00
B, Lp	£0.00	£1.16	£46.50	£0.00	£1.02	£61.10	£0.00	£0.91	£33.30	£0.00	£0.97	£24.10
B, Hv	£0.00	£2.37	£161.00	£0.00	£2.22	£235.00	£0.00	£2.02	£122.00	£0.00	£2.20	£87.10
B , Lv	£0.06	£1.37	£11.90	£0.00	£0.99	£12.40	£0.00	£0.66	£7.98	£0.00	£0.51	£5.96
L, Hp	£0.01	£1.92	£27.90	£0.00	£1.47	£31.80	£0.00	£1.10	£19.70	£0.00	£0.94	£15.00
L, Lp	£0.00	£1.42	£81.80	£0.00	£1.33	£116.00	£0.00	£1.22	£59.90	£0.00	£1.34	£42.30
L, Hv	£0.00	£3.93	£548.00	£0.00	£4.15	£940.00	£0.00	£3.85	£429.00	£0.00	£4.53	£286.00
L, Lv	£0.01	£0.97	£9.47	£0.00	£0.67	£9.86	£0.00	£0.46	£6.19	£0.00	£0.38	£4.54
H, Hp	£0.00	£0.34	£18.90	£0.00	£0.38	£26.70	£0.00	£0.39	£13.00	£0.00	£0.47	£8.89
H, Lp	£0.00	£3.35	£69.60	£0.00	£2.94	£109.00	£0.00	£2.72	£136.00	£0.00	£2.40	£83.20
H, Hv	£0.00	£1.86	£82.20	£0.00	£1.80	£165.00	£0.00	£2.22	£241.00	£0.00	£2.29	£133.00
H, Lv	£0.10	**£4.14**	£29.70	£0.00	**£3.32**	£37.60	£0.00	**£2.61**	£40.20	£0.00	**£1.95**	£26.40
Hp, Hv	£0.00	£5.50	£285.00	£0.00	£5.09	£576.00	£0.00	£5.90	£863.00	£0.00	£5.65	£502.00
Hp, Lv	£0.00	£0.62	£4.33	£0.00	£0.41	£5.18	£0.00	£0.31	£5.02	£0.00	£0.27	£2.54
Lp, Hv	£0.00	£2.09	£104.00	£0.00	£2.06	£220.00	£0.00	£2.63	£332.00	£0.00	£2.71	£182.00
Lp, Lv	£0.00	£2.05	£19.60	£0.00	£1.58	£26.10	£0.00	£1.26	£28.50	£0.00	£1.01	£17.50

Table 20: Summary of Residual Value (3 sort portfolios)

Following on from the details of table 11, the residual value presented shows the minimum, average and maximum residual portfolio values during successful periods.

	Residual Value (£m)											
	4%			**6% Withdrawal Rate**			**8% Withdrawal Rate**			**10% Withdrawal Rate**		
	Min Value	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**	**Min Value**	**Average Value**	**Max Value**
B, L, Hp	£0.00	£1.93	£40.80	£0.00	£1.43	£18.30	£0.00	£1.11	£24.70	£0.00	£0.91	£18.20
B, L, Lp	£0.00	£1.46	£160.00	£0.00	£1.27	£47.60	£0.00	£1.27	£86.30	£0.00	£1.22	£59.90
B, H, Lp	£0.00	£2.58	£426.00	£0.00	£2.26	£115.00	£0.00	£2.30	£220.00	£0.00	£2.20	£154.00
B, Lp, Hv	£0.00	£2.05	£659.00	£0.00	£1.85	£131.00	£0.00	£2.15	£313.00	£0.00	£2.09	£197.00
B, Hp, Hv	£0.00	**£7.16**	£2,260.00	£0.00	**£6.16**	£515.00	£0.00	**£6.47**	£1,120.00	£0.00	**£6.12**	£739.00
B, L, Hv	£0.00	£4.39	£1,530.00	£0.00	£3.85	£325.00	£0.00	£4.23	£741.00	£0.00	£4.02	£480.00
B, H, Hv	£0.00	£1.74	£590.00	£0.00	£1.58	£110.00	£0.00	£1.90	£277.00	£0.00	£1.87	£168.00
B, Lp, Lv	£0.00	£1.74	£17.70	£0.00	£1.33	£23.80	£0.00	£1.08	£26.10	£0.00	£0.88	£15.70
B, Hp, Lv	£0.00	£0.59	£6.91	£0.00	£0.40	£7.19	£0.00	£0.29	£4.38	£0.00	£0.26	£3.14
B, L, Lv	£0.01	£0.98	£9.64	£0.00	£0.68	£10.10	£0.00	£0.47	£6.32	£0.00	£0.38	£4.63
L, Hp, Hv	£0.00	£5.37	£1,040.00	£0.00	£4.60	£233.00	£0.00	£5.13	£369.00	£0.00	£5.51	£439.00
L, Hp, Lv	£0.00	£0.46	£6.05	£0.00	£0.30	£2.70	£0.00	£0.23	£2.51	£0.00	£0.20	£1.70
S, L, Hp	£0.00	£2.49	£250.00	£0.00	£2.13	£67.80	£0.00	£2.19	£93.80	£0.00	£2.27	£104.00
S, H, Lp	£0.00	£2.95	£140.00	£0.00	£2.38	£47.30	£0.00	£2.13	£59.20	£0.00	£1.98	£59.80
H, Lp, Hv	£0.00	£3.25	£513.00	£0.00	£2.83	£123.00	£0.00	£3.14	£188.00	£0.00	£3.35	£218.00
H, Lp, Lv	**£0.00**	**£3.50**	**£82.60**	**£0.00**	**£2.76**	**£33.70**	**£0.00**	**£2.22**	**£39.20**	**£0.00**	**£1.87**	**£36.80**
S, Lp, Hv	£0.00	**£7.22**	£1,170.00	£0.00	**£6.09**	£282.00	£0.00	**£6.41**	£421.00	£0.00	£6.54	£494.00
S, H, Hv	£0.00	£3.40	£731.00	£0.00	£2.98	£184.00	£0.00	£3.09	£372.00	£0.00	£2.96	£254.00
S, Lp, Lv	**£0.03**	**£2.80**	**£35.00**	**£0.00**	**£2.21**	**£38.90**	**£0.00**	**£1.73**	**£23.70**	**£0.00**	**£1.40**	**£29.60**
S, H, Lv	**£0.00**	**£2.51**	**£18.60**	**£0.00**	**£1.93**	**£23.70**	**£0.00**	**£1.48**	**£25.20**	**£0.00**	**£1.11**	**£15.70**

Table 18 shows the summary of the residual values for the 1 sort portfolios. At the 4% withdrawal rate the high volatility stocks produced on average the largest residual value (£4.8m). This is considerably higher than what the market produced on average (£1.9m). Small, value and growth portfolios were the other portfolios that produced higher than market average residual value. Although, it is tempting to assume that the scale of the residual value is entirely informed by the volatility of portfolios especially when one considers that the high volatility portfolios have the highest level of deviation (8%); however, although the value portfolio has a larger deviation compared to the small portfolio (5% Vs 4%), the small portfolio produced a larger average residual value (£2.95m Vs £2.61m). At the 6%, 8% and 10%, the same comparative performance was observed.

Table 19 shows the summary of the residual values for the 2 sort portfolios. At 4% withdrawal rate, a total of 14 portfolios out of the 22 considered produced a better than market average residual value. The *Hp,Hv* (high profitability and high volatility sort portfolios) portfolios produced the highest average residual value. Again, the association between the average residual value and the standard deviation described earlier was observed as the *Hp,Hv* portfolio though had the highest average residual value, it did not produce the highest standard deviation amongst this sort. At the 6% withdrawal rate, a total of 15 portfolios produced a better than market average residual value and there were 17 and 16 portfolios that produced a better than market average residual value at the 8% and also at the 10% rates of withdrawal respectively.

Table 20 shows the summary of the residual values for the 3 sort portfolios and 20 portfolios were considered in this category. There were 14 portfolios that produced a better than market average residual value at the 4% withdrawal rate, 14 at the 6% withdrawal rate, 17 at the 8% withdrawal rates and 15 at the 10% withdrawal rate.

In summary, from all the 4 most successful portfolios identified earlier, the *H,Lv* portfolios consistently produced the largest average residual value during successful periods for all the rates of withdrawal (from about £4m at 4% withdrawal to about £1.95m at 10% withdrawal). Next to this performance was the *H,Lp,Lv* portfolio with the same consistency across all the withdrawal rates, followed by the *S,Lp,Lv* and finally the *S,H Lv* portfolio. The *B,Hp,Hv*, portfolio produced the largest residual value at the 6%, 8% and 10% withdrawal rates whilst the *S,Lp,Hv* portfolios produced the largest residual value at the 4% withdrawal rate. (the *B,Hp,Hv* portfolio had success rates of 79%, 65% and 51% respectively at the 6%, 8% and 10% withdrawal rates while the *S,Lp,Hv* portfolio had success rates of 95% at the 4% withdrawal rate).

A retiree's proper management of his nest egg requires a careful balancing of two financial risks. On the one hand, the retiree may spend too much and outlive his savings; on the other hand, the retiree may unnecessarily lower his lifestyle and end up with an unintended bequest. The results of Tables 18 through 20 tend to show that increasing the volatility of a portfolio will have a positive impact on how much bequest is left (although, as mentioned earlier, this does not entirely explain the level of bequest). However, this comes with an increased probability of failure therefore a careful balance of bequest motive and acceptable possibility of failure will have to be carefully assessed during the design of retirement portfolios.

At the 4% withdrawal rate, the 4 most successful portfolios produced between 35% and 57% of the highest average residual value obtained and at the 6% withdrawal rate, they produced between 31% and 54% of the highest average residual value obtained. At 8% withdrawal rate, these portfolios offered between 23% and 40% of the highest average residual value and at 10% withdrawal rate it was between 18% and 32% of the highest average value obtained.

Table 21: Summary of Relative Stability Measure and Success Rate

The table is broadly extracted from Table 1 (success rates) except for column 3 (Relative Stability Measure). The monthly total return (from the total return index) of the constituent stocks of the portfolios together with the market capitalisation (ME) was then used to establish the market capitalized monthly weighted return of the portfolios (from which the average monthly return for the period under review was obtained; column 2). Using the AR(1) i.e. autoregression for each portfolio (Augmented Dickey Fuller model) the regressing coefficient β was obtained. This was then divided by 1 standard deviation of shock (to give the relative stability measure) given as 1 S.D of Shock = Square root of (sum of squared residual/n-a) where n-a = Degree of freedom (number of adjusted observations - number of regressors).

Relative Stability Measure and Success Rate Summary						
Portfolio	Average Monthly Return	Relative Stability Measure	Success @ Rate 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
FTSE 350	1.17%	24.49	99.98%	97.99%	86.45%	62.77%
Small Portfolio	1.26%	18.94	99.68%	96.29%	85.31%	48.47%
Big Portfolios	1.17%	24.13	99.95%	97.83%	86.99%	62.46%
Value Portfolio	1.23%	17.09	98.47%	91.52%	87.00%	63.11%
Growth	1.22%	24.25	99.93%	98.45%	88.62%	66.26%
High Profitability	1.12%	24.4	99.89%	97.14%	84.48%	57.98%
Low Profitability	1.00%	16.88	94.94%	80.01%	58.16%	36.91%
High Volaility	1.40%	11.31	90.90%	77.06%	63.02%	49.16%
Low Volatility	1.03%	32.18	100.00%	99.04%	88.01%	54.64%
S, L	1.14%	11.79	85.30%	69.98%	53.02%	38.44%
S, H	1.23%	16.96	98.79%	91.76%	77.13%	57.93%
S, Hp	0.92%	14.15	87.64%	68.73%	48.51%	31.35%
S, Lp	1.41%	16.46	99.57%	95.36%	85.81%	70.89%
S, Hv	1.22%	13.64	94.25%	81.78%	64.73%	48.73%
S, Lv	1.08%	29.29	99.99%	98.60%	86.67%	55.78%
B, L	1.22%	24.35	99.97%	98.32%	88.70%	66.66%
B, H	1.11%	14.28	94.52%	81.61%	63.78%	45.34%
B, Hp	1.16%	24.04	99.93%	97.44%	84.93%	60.40%
B, Lp	0.97%	16.45	92.51%	75.01%	52.61%	33.44%
B, Hv	1.23%	13.39	91.25%	76.43%	58.74%	42.89%
B , Lv	1.05%	31.08	100.00%	99.16%	88.33%	57.66%
L, Hp	1.18%	24.29	99.97%	97.92%	86.39%	63.05%
L, Lp	1.00%	14.19	88.26%	69.79%	49.82%	33.49%
L, Hv	1.25%	10.31	81.36%	65.44%	50.55%	38.57%
L, Lv	0.95%	30.56	99.98%	96.72%	75.94%	40.43%
H, Hp	0.55%	13.48	48.33%	26.05%	13.04%	6.64%
H, Lp	1.34%	16.16	98.82%	93.07%	81.10%	63.53%
H, Hv	1.05%	11.04	75.39%	57.19%	41.57%	27.60%
H, Lv	1.41%	26.56	**100.00%**	**99.85%**	**98.38%**	**89.21%**
Hp, Hv	1.42%	10.70	88.71%	76.42%	62.65%	48.40%
Hp, Lv	0.84%	29.30	99.68%	89.54%	57.96%	21.43%
Lp, Hv	1.04%	10.37	71.66%	53.80%	39.46%	26.60%
Lp, Lv	1.18%	23.19	99.92%	97.44%	87.02%	63.72%
B, L, Hp	1.18%	24.30	99.89%	97.62%	86.50%	61.78%
B, L, Lp	1.00%	14.18	88.36%	68.86%	50.24%	33.75%
B, H, Lp	1.12%	12.40	89.29%	72.23%	56.52%	41.26%
B, Lp, Hv	0.97%	10.08	65.30%	46.86%	33.45%	23.64%
B, Hp, Hv	1.47%	10.57	90.41%	76.93%	64.29%	51.31%
B, L, Hv	1.25%	10.26	81.18%	64.62%	50.85%	39.05%
B, H, Hv	0.92%	9.81	58.49%	40.48%	27.99%	19.29%
B, Lp, Lv	1.14%	22.46	99.83%	95.86%	81.93%	56.98%
B, Hp, Lv	0.83%	28.92	99.43%	87.25%	53.15%	21.39%
B, L, Lv	0.95%	30.21	99.98%	96.81%	76.36%	40.83%
L, Hp, Hv	1.36%	10.35	85.52%	71.69%	57.22%	43.47%
L, Hp, Lv	0.75%	29.29	98.99%	82.17%	43.39%	13.80%
S, L, Hp	1.19%	12.80	89.34%	74.72%	57.04%	40.75%
S, H, Lp	1.26%	16.31	98.61%	91.87%	77.16%	58.87%
H, Lp, Hv	1.25%	10.86	82.42%	66.54%	50.88%	37.41%
H, Lp, Lv	1.34%	21.12	**99.94%**	**98.44%**	**91.19%**	**75.04%**
S, Lp, Hv	1.46%	11.05	92.58%	81.97%	68.16%	54.83%
S, H, Hv	1.21%	11.67	87.92%	71.69%	56.99%	42.76%
S, Lp, Lv	1.28%	22.97	**99.94%**	**98.66%**	**90.76%**	**73.09%**
S, H, Lv	1.25%	26.72	**100.00%**	**99.33%**	**94.08%**	**75.97%**

Table 22: Correlation summary of Relative Stability Measure and Success Rate

The table shows the correlation between the relative stability measure (described above) with the results of the success rates.

	Relative Stability Measure
Relative Stability Measure	1.00
Success Rate @ 4%	0.665018591
Success Rate @ 6%	0.740408715
Success Rate @ 8%	0.6230075
Success Rate @ 10%	0.342608703

Table 23: Correlation summary of Relative Stability Measure and Failure Point

The table shows the correlation between the relative stability measure (described above) with the results of the failure point.

	Relative Stability Measure
Relative Stability Measure	1.00
Failure Point @ 4%	0.950806053
Failure Point @ 6%	0.958088827
Failure Point @ 8%	0.947660442
Failure Point @ 10%	0.906039241

In the previous chapter, the relative stability measure is proposed as a *ad hock* measure of a portfolio's stability. The reasoning was that if the speed of mean reversion implies the rate at which a portfolio is expected to return to its long run mean following a deviation from this mean, and if this speed is represented by the regressing coefficient β from an AR(1) process of an augmented Dickey-Fuller model, then dividing this measure by 1 standard deviation of shock of the process will imply the rate at which the portfolio returns is expected to return to its long run mean following a unit deviation of shock. It is proposed that with this reasoning, this measure will give a theoretical framework for indicating how stable (ability to recover) a portfolio is.

A measure of this sort will certainly be useful in assessing the suitability of a strategy for drawdown purposes. Even more useful will be the ability to use this measure to imply the level of sustainability indicated by the success rate. This is understandable since one would expect that a more stable portfolio should implicitly have a better success rate. With this in mind, table 21 summarises the respective measure of relative stability (of which, the higher the value the better the stability expected) with the various success rates. A quick glance at the table clearly shows an inconsistency between this measure and the success rates as confirmed in the correlation results in table 22.

The reason for this inconsistency is perhaps not farfetched. The relative stability measure is effectively a 1-dimensional measure based on standard deviation and a rate of change, but the success rate is 2 dimensional as it is a product of the combined effect of average returns and standard deviation. So, for example, the growth and high profitability portfolios have similar standard deviations but potentially due to the difference in their average monthly return, they produce different levels of success rate. In the same way, portfolios *B,L,Hv* and *S,H,Lv* have the same average monthly return but different standard deviations hence, producing different levels of success. Clearly, what this tends to show is that on the surface, stability does not necessarily imply better success during withdrawals.

One of the measures introduced by Estrada (2017) mentioned earlier and shown in tables 15 through 17 (failure point) was shown to give an even deeper information on sustainability. Again, from the reasoning of what the relative stability measure represents, one would expect that all else being equal, a more stable portfolio will fail at a later point of the retirement period. Table 23 shows that conditional on failure, the correlation between the failure point and relative stability measure is much stronger. This result tends to indicate there may be some level of reliability of the relative stability measure after all. One potential reason for the difference in this correlation result compared to that of the success rate is that the failure rate may be providing a more rounded information about sustainability that is less about the returns but more about stability.

Table 24: Summary of Returns Grouped Relative Stability Measure and Success Rate

Portfolio	Average Monthly Return	Observed Standard Deviation	Relative Stability Measure	Success Rate @ 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
FTSE 350	1.17%	4.00%	24.49	99.98%	97.99%	86.45%	62.77%
Big Portfolios	1.17%	4.03%	24.13	99.95%	97.83%	86.99%	62.46%
Small Portfolio	1.26%	4.41%	18.94	99.68%	96.29%	85.31%	48.47%
S, H, Lp	1.26%	5.30%	16.31	98.61%	91.87%	77.16%	58.87%
Value Portfolio	1.23%	5.28%	17.09	98.47%	91.52%	87.00%	63.11%
B, Hv	1.23%	7.45%	13.39	91.25%	76.43%	58.74%	42.89%
Growth	1.22%	4.06%	24.25	99.93%	98.45%	88.62%	66.26%
B, L	1.22%	4.06%	24.35	99.97%	98.32%	88.70%	66.66%
High Profitability	1.12%	4.03%	24.4	99.89%	97.14%	84.48%	57.98%
B, H, Lp	1.12%	6.80%	12.40	89.29%	72.23%	56.52%	41.26%
Low Profitability	1.00%	5.43%	16.88	94.94%	80.01%	58.16%	36.91%
L, Lp	1.00%	6.49%	14.19	88.26%	69.79%	49.82%	33.49%
B, L, Lp	1.00%	6.50%	14.18	88.36%	68.86%	50.24%	33.75%
S, L	1.14%	7.24%	11.79	85.30%	69.98%	53.02%	38.44%
B, Lp, Lv	1.14%	4.10%	22.46	99.83%	95.86%	81.93%	56.98%
S, Hp	0.92%	5.51%	14.15	87.64%	68.73%	48.51%	31.35%
B, H, Hv	0.92%	9.91%	9.81	58.49%	40.48%	27.99%	19.29%
S, Lp	1.41%	5.18%	16.46	99.57%	95.36%	85.81%	70.89%
H, Lv	1.41%	3.51%	26.56	100.00%	99.85%	98.38%	89.21%
B, Lp	0.97%	5.66%	16.45	92.51%	75.01%	52.61%	33.44%
B, Lp, Hv	0.97%	9.24%	10.08	65.30%	46.86%	33.45%	23.64%
B , Lv	1.05%	2.95%	31.08	100.00%	99.16%	88.33%	57.66%
H, Hv	1.05%	8.37%	11.04	75.39%	57.19%	41.57%	27.60%
L, Hp	1.18%	4.08%	24.29	99.97%	97.92%	86.39%	63.05%
B, L, Hp	1.18%	4.00%	24.30	99.89%	97.62%	86.50%	61.78%
L, Hv	1.25%	8.91%	10.31	81.36%	65.44%	50.55%	38.57%
B, L, Hv	1.25%	8.88%	10.26	81.18%	64.62%	50.85%	39.05%
S, H, Lv	1.25%	3.12%	26.72	100.00%	99.33%	94.08%	75.97%
L, Lv	0.95%	2.98%	30.56	99.98%	96.72%	75.94%	40.43%
B, L, Lv	0.95%	3.00%	30.21	99.98%	96.81%	76.36%	40.83%
H, Lp	1.34%	5.74%	16.16	98.82%	93.07%	81.10%	63.53%
H, Lp, Lv	1.34%	3.92%	21.12	99.94%	98.44%	91.19%	75.04%

Furthermore, looking closely at the results in table 24, the relative stability measure gives a reasonably consistent indication of the level of success when 2 portfolios with similar returns are considered. Table 24 shows the grouping of portfolios with similar returns. This observation tends to indicate that the measure contains much of the information provided by the standard deviation as the more stable portfolio of each grouping above generally have lower standard deviation. However, it does seem also that the measure provides more indication of stability compared to the standard deviation; for example, portfolios *L,Hv* and *B,L,Hv* both have similar returns but *L,Hv* has a higher standard deviation, higher relative stability measure and is generally more successful. Both portfolios have similar failure points failure point generally. From the 4 most successful portfolios, *H,Lv* and *S,H,Lv* had better relative stability measure and produced higher levels of success relative to the market with *H,Lv* showing the better stability (relative to the market). In general, this result seems to suggest that all else being equal, the relative stability measure is reasonably consistent with the success rate of portfolios during withdrawals.

With the various degree of inconsistencies observed between the relative stability measure and success of portfolios during withdrawals, it will be helpful to see the relative importance of the mean reverting property and the size of the shock in affecting the success rate of the portfolios. To show this, I have selected the *H,Lv* portfolio (value-low volatility portfolio) at the 6% withdrawal rate. In addition, the measure of persistence of returns, $1+\beta$, has been used rather than β which relates to changes. Persistence $1+\beta$ (φ) has a range of -1 to 1 where 0 to 1 is a mirror of 0 to -1; I have chosen persistence ranging from 0.2 to 0.5 in addition to the actual portfolio persistence of 0.07 (half the range length of persistence) to access the importance of mean reversion during different levels of shock in returns as shown below:

Success Rate at 6% Withdrawal Rate (*H,Lv* Portfolio)

		σ				
		2%	3%	Actual (3.51%)	4%	5%
1+β	Actual (0.0696)	100.00%	100.00%	99.91%	99.62%	97.63%
	0.2	100.00%	100.00%	99.98%	99.79%	98.39%
	0.3	100.00%	100.00%	99.97%	99.83%	98.74%
	0.4	100.00%	100.00%	99.98%	99.89%	99.01%
	0.5	100.00%	100.00%	99.98%	99.93%	99.25%

Average Residual Value (millions) at 6% Withdrawal Rate (*H,Lv* Portfolio)

		σ				
		2%	3%	Actual (3.51%)	4%	5%
1+β	Actual (0.0696)	£3.12	£3.24	£3.29	£3.31	£3.53
	0.2	£6.93	£7.13	£7.32	£7.71	£8.41
	0.3	£14.60	£15.70	£16.20	£16.80	£19.40
	0.4	£38.20	£42.80	£45.70	£50.80	£63.00
	0.5	£145.00	£176.00	£200.00	£234.00	£319.00

Average Failure Period (years) at 6% Withdrawal Rate (*H,Lv* Portfolio)

		σ				
		2%	3%	Actual (3.51%)	4%	5%
1+β	Actual (0.0696)	0	0	20	18	17
	0.2	0	0	19	17	16
	0.3	0	0	16	19	15
	0.4	0	0	18	15	14
	0.5	0	0	9	12	11

Looking at the success rate, it appears that when persistence is held constant, the rate of success decreases as 1 standard deviation of shock increases, and the rate of success increases with persistence when 1 standard deviation of shock increases. Hence, all else being equal, higher persistence (lower mean reversion) tends to produce better success and lower deviation of shock tends to produce better success. Therefore, portfolios having a combination of relatively low reversion (high persistence) and low deviation of shock are expected to be more successful during withdrawals.

However, higher persistence (lower reversion to mean) and deviation of shock tends to produce lower failure points and larger residual value. Hence, all else being equal, investors with a preference for high residual value portfolios will need high risk and high persistent portfolios. Although, failure point generally should not be considered outside the corresponding success rate, it does appear that all else being equal, portfolios with low persistence (high mean reversion) and low deviation of shock sustain withdrawals for a relatively longer period conditional on failure.

5.4 Conclusion.

This chapter brings together the results from the previous chapters. It assesses how these various factor-based portfolio strategies performed as a drawdown solution. Essentially, it estimates how successful they perform at drawdown rates of 4%, 6%, 8% and 10%. This is done using the Monte-Carlo simulation approach; 10,000 simulations of returns (assuming normal distribution) were obtained for each portfolio. A £100,000 starting portfolio value was assumed and then compounded with the simulated monthly returns. An annual fixed rate withdrawal strategy adjusted for inflation was adopted to estimate how many simulated portfolios (out of 10,000) will end up with a balance equal to or greater than £0 at the end of the period.

Out of the 53 portfolios considered, the market and 28 other portfolios sustained the 4% rate of withdrawal with a success rate of between 99%-100%. Another 16 portfolios were able to sustain this rate of withdrawal with at least a 90% success. The worst performing portfolio was the *H,Hp* (value high profitability stocks) with a success rate of 52%. None of the portfolios (including the market) had a 100% success rate at 6% withdrawal but 25 portfolios had a success rate of at least 90% with portfolios *S,H,Lv* (Small, value, low volatility stocks) and *H,Lv* (value low volatility stocks) having the highest success rate of 99%. The worst success rate was 30%.

At 8% withdrawal rate, the *H,Lv* portfolio was the most successful with 98% success. The *S,H,Lv* portfolio had a 94% success rate. Two other portfolios were also as successful; the *H,Lp,Lv* portfolio (value, low profitability and low volatility stocks) had a success rate of 91% (this portfolio also had a success rate of 99% and 98% at the 4% and 6% withdrawal rates respectively) and the *S,Lp,Lv* portfolio (small, low profitability and low volatility stocks) had a success rate of 90% (this portfolio also had a success rate of 99% and 98% at the 4% and 6% withdrawal rates respectively). At the 10% withdrawal rate, only the *H,Lv* portfolio had a decent success rate of about 90%. The *S,H,Lv*, *H,Lp,Lv* and *S,Lp,Lv* had a success rate of 75%, 75% and 73% respectively. These results appear to show that this factor-based approach to portfolio construction is reasonably broadly effective in sustaining withdrawals up to 8% whilst the star portfolio will sustain up to 10% withdrawal rates.

In line with Estrada (2017), this chapter further examines the failure points of these portfolios i.e., conditional on failure, the particular period (average) when failure occurred. It has been argued that the result of this analysis sheds a deeper light into the performance of a portfolio during withdrawals. The results revealed that making conclusions based solely on the failure rates (1 – success rate) could be misleading for example, the failure rates could imply 2 portfolios have similar risk exposures, but the failure point may suggest otherwise because it gives a deeper layer of information. However, whilst this possible dissimilarity of the 2 measures was noted, the 4 most successful portfolios provided competitive average failure points as they produced either the best average failure point or very close to the best average failure point through all the withdrawal rates. The *H,Lv* portfolio was comparatively better than that of the market across all withdrawal rates.

Furthermore, a fundamental objective of some retirees is to leave a sizeable fund as a death benefit to beneficiaries (bequest) and therefore, an additional layer of analysis was carried out to see how well these portfolios could satisfy this objective. The results indicated that increasing the volatility of a portfolio will have a positive impact on how much bequest is left although, this alone does not explain a larger bequest value; the sequence of returns will also explain this. However, this comes with an increased probability of failure therefore a careful balance of bequest motive and acceptable possibility of failure will have to be carefully assessed during the design of retirement portfolios. At the 4% withdrawal rate, the 4 most successful portfolios produced up to 57% of the highest average residual value obtained and up to 54% at the 6% withdrawal rate. They produced up to 40% and 32% at the 8% and 10% withdrawal rates.

As stated earlier, the stability of a drawdown portfolio is an important consideration and whilst the analysis thus far could generally be employed to shed some light on this, an *ad hoc* measure (relative stability measure) expected to capture the same inference was proposed in chapter 4 and its reliability for this inference was assessed in this chapter. The results show that this measure was reasonably consistent with the failure point assessment, but low correlations were observed when compared with the success rate. An explanation for this is the difference in the dimensions making up the relative stability measure and the success rate (the relative stability measure is basically made up of measures of deviation whilst the success rate reflects the combined effect of returns and standard deviation). An inference based on this analysis is that stability does not necessarily imply success. It was however observed that when portfolios with similar returns are considered, the measure is considerably consistent in indicating the more successful one.

With these inconsistencies observed, the chapter therefore explored how mean reversion and size of shock affects the success of the portfolios during withdrawals. The results show that all else being equal, a portfolio that is more persistent (lower relative speed of reversion) and with smaller deviation of shock will likely be more successful during withdrawals. In addition, conditional on failure and all else being equal, portfolios with low persistence (high mean reversion) and low deviation of shock sustain withdrawals for a relatively longer period however, this conclusion should generally not be considered in isolation of the corresponding success rate. Furthermore, retirees who are interested in having a sizeable balance in their portfolios as a death benefit will need portfolios that have high persistence and high deviation of shock.

Appendix for Chapter 5: Stata Coding for Success Rate Analysis

program define mytest61, rclass

(To define the program in rclass)

drop _all

(To remove all existing variables or observation from the dataset in the memory)

import excel "C:\Users\Olu\Desktop\PHD Docs\Chapter One Analysis\NEW DATA FOR ANALYSIS\FTSE 350 ANALYSIS\Annual WIthdrawal Rate Mtr.xlsx", sheet("Sheet1") firstrow

(Imports the withdrawal rate series to be used)

set obs 288

(Sets the observation to equal the number of months under consideration)

gen error = rnormal(0,0.089)

(generates error ε_t which is normally distributed with mean 0 and standard deviation 8.9%)

gen double return = 0.0126 + (0.0804*0.0136)+ error

(generates return $R_1 = \alpha_0 + (1+\beta)R_0 + \varepsilon_1$ where drift is 0.0126; persistence $1+\beta$ is 0.0804 and ε_1 is error as above)

replace return = 0.0126 + (0.0804*return[_n-1])+error if _n > 1

(generates R_2 to R_{228})

gen double PortRetn = ((1+return)*100000)-WR4

(Generates the variable PortRetn (portfolio return) which is the starting value of the portfolio (£100,000) multiplied by the monthly portfolio return less the withdrawal for the month)

replace PortRetn = (PortRetn[_n-1]*(1+return))-WR4 if _n > 1

(Goes back to PortRetn to replace the capital of £100,000 with the previous month portfolio balance and this starts from month 2)

return scalar Final_Balance = PortRetn[288]

(Returns the final portfolio balance)

end

simulate PortBal = r(Final_Balance) , reps(10000):mytest61

(This simulates (generates) a variable called PortBal which performs the command above but does this 10,000 times)

sum PortBal

(Summarises key stats for the simulated 10,000 PortBal)

sum PortBal if PortBal >= 0

(Summarises key stats for total PortBal greater than or equal to £0)

Appendix B. Stata Coding for Default Period Analysis

program define mytest86, rclass

(To define the program in rclass)

drop _all

(To remove all existing variables or observation from the dataset in the memory)

import excel "C:\Users\Olu\Desktop\PHD Docs\Chapter One Analysis\NEW DATA FOR ANALYSIS\FTSE 350 ANALYSIS\Annual WIthdrawal Rate Mtr.xlsx", sheet("Sheet1") firstrow

(Imports the withdrawal rate series to be used)

set obs 288

(Sets the observation to equal the number of months under consideration)

gen error = rnormal(0,0.089)

(generates error ε_t which is normally distributed with mean 0 and standard deviation 8.9%)

gen double return = 0.0126 + (0.0804*0.0136)+ error

(*generates return $R_1 = \alpha_0 + (1+\beta)R_0 + \varepsilon_1$ where drift is 0.0126; persistence $1+\beta$ is 0.0804 and ε_1 is error as above)*

replace return = 0.0126 + (0.0804*return[_n-1])+error if _n > 1

(generates R_2 to R_{228})

gen double PortRetn = ((1+return)*100000)-WR4

(Generates the variable PortRetn (portfolio return) which is the starting value of the portfolio (£100,000) multiplied by the monthly portfolio return less the withdrawal for the month)

replace PortRetn = (PortRetn[_n-1]*(1+return))-WR4 if _n > 1

(Goes back to PortRetn to replace the capital of £100,000 with the previous month portfolio balance and this starts from month 2)

gen BT = [_n] if PortRetn <= 0

(Generates a variable BT which contains the month number (and other months thereafter) when the portfolio balance (PortRetn) is £0)

sum BT

(Summarises key stats of BT)

return scalar Default_Period = r(min)

(Returns the minimum value of BT which is effectively the month from which the portfolio becomes £0 (fails))

end

simulate Avg_Def_Prd = r(Default_Period) , reps(1000):mytest86

(This simulates (generates) a variable called average default period which performs the command above but does this 10,000 times)

sum Avg_Def_Prd

(Provides key stats for the average default period)

6. Thesis Conclusion

6.1 Empirical Findings

One of the fundamental implications of the UK pension legislation in 2015 was that it placed the responsibility of ensuring the sustainability of defined contribution plans solely in the hands of holders of such plans. Constructing a retirement portfolio for the withdrawal phase (drawdown) of a retiree's retirement journey requires certain important considerations: generating excess return within the portfolio, deciding on a preferred withdrawal strategy including a withdrawal rate, and ensuring the portfolio is stable enough to sustain the chosen withdrawal rate. This research has focused on exploring sources of excess returns and withdrawal stability as they are the most factual. The choice of withdrawal rates is quite subjective, hence, a range of withdrawal rates including the current industry standard of 4% as well as 6%, 8% and 10% withdrawal rates were analysed. That way the reader can see the range of possible outcomes by varying the selected withdrawal rate.

Chapter 3 of this study examined portfolios constructed from 4 factors, three of which are known and established in literature (size, book to value and profitability) to generate excess returns other than the acclaimed beta from the CAPM theory. The fourth factor, volatility, is increasingly becoming accepted as another anomaly in the factor investing space. Using the FTSE 350 world (which implicitly excludes small caps), the results showed that constructing and investing in small sized stocks will produce observed absolute returns in excess of the market and that of big stocks while the returns of investing in big stocks will produce about the same absolute return as investing in the market. However, in risk adjusted terms, neither the small or big stock portfolios produce a significant returns in excess of the market although the small stock portfolios showed better risk efficient returns based on its beta value. Furthermore, investing in stocks with either high or low book to market ratio (value or growth stocks) will also produce absolute returns in excess of the market but the returns of the value portfolio was less risk efficient based on its beta value thus failing to confirm the value anomaly by indicating (in risk adjusted terms) that the better performance is due to higher risk. These results broadly align with the various results shown in literature regarding the size effect but to a lesser degree the value effect. The recent trends reported which describes a reversal of these outperformance was not observed. Although the profitability factor did not produce returns in excess of the market, investing in high profitable stocks still produced returns in excess of that of low profitable stocks both in absolute and risk adjusted terms and this result also aligns with literature confirming this as an anomaly since the high profitable stocks have lower CAPM beta.

The inclusion of the volatility factor is justified naturally because of the importance of volatility in the broad considerations for designing portfolios at the withdrawal stage of retirement planning. The volatility anomaly confirms that low volatile stocks tend to produce higher absolute and/or risk adjusted returns relative to high volatile stocks and this clearly does not conform to the fundamentals of finance. Chapter 3 showed that the low volatility classification of stocks formed from the volatility factor did not outperform the market or the high volatility classed stocks in absolute return terms but in risk adjusted terms, it outperformed both. One reason for this was noted as the nature of the universe used as not including small cap stocks. The high volatility classed stocks outperformed the market in absolute terms. Chapter 3 further assessed if the returns from these portfolios reverted to their mean. The intuition is that, if the returns revert to its long run mean, then advisers may be able to tell the behaviour of a portfolio strategy (identified by its speed of reversion) following drawdown shocks. The results showed that the returns of the various portfolios were all reverting to their long term mean at various rates.

Chapter 4 went further by creating more portfolios based on overlapping factors into 2 and 3 sorts. 44 additional portfolios were considered and 23 of them produced absolute returns in excess of the market while 7 of them produced significant risk adjusted returns in excess of the market all of which were formed with the low volatility factor. All the portfolio returns were also mean reverting. Furthermore, the chapter proposed a ad hock measure to assess the stability of portfolios; this measure (intended to create a theoretical framework to assess stability) was based on the ratio of the speed of reversion to one standard deviation of shock. This assessment of stability is necessary (to the retiree) because in identifying a factor strategy for the purpose of withdrawals it is important that such a strategy will 'hold form' during withdrawals. The objective is that this measure can be used to identify such strategies from those which have a better than market return. The results showed that 10 portfolios generally had a better (in magnitude) measure compared to the market all of which were exposed to the low volatility factor and 2 of these (value/low volatility and small/value/low volatility portfolios) also performed better than the market and with lower risk.

Chapter 5 brings the preceding outcomes together. Using the Monte-Carlo simulation approach, it assessed how these portfolios performed during withdrawals and also compared the outcome of this assessment with the ad hock measure of stability. The idea is to see that both a stochastic and statistical representation of stability tell the same story. The Monte-Carlo analysis was used to assess the success rate of the portfolios at 4%, 6%, 8% and 10% fixed real rate of withdrawal; how long the withdrawal was sustained for even when failure occurred and the residual portfolio value during successful periods.

The result showed that higher returns or lower volatility does not necessarily imply a better success rate and lower returns does not necessarily imply higher a failure rate. Four portfolios were identified as the most successful considering the blend of success rate, failure point and residual value. Furthermore, the stability measure was found to be inconsistent with the success rate result but more consistent with the failure point assessment. Basically, the success rate reflects a combined effect of returns and volatility whilst the stability measure does not take into account returns in absolute terms only its rate of change therefore, within this context, stability does not necessarily imply success. However, when the success rate of portfolios with similar returns were compared, the rates of success was found to be significantly consistent with the measure. This prompted an exploration of how the speed of reversion (or level of persistence) and size of shock affect the performance of the portfolios during withdrawals. Higher levels of success are observed as persistence increased (slowing speed of reversion) and when size of shock is lower. However, conditional on failure, low persistence (higher speed of reversion) and smaller shocks tends to produce longer sustainability. Furthermore, high persistence and large shock size produced larger residual values.

6.2 Key Summary of Conclusion.

- This work makes an initial contribution of considering the FTSE 350 market in the analysis of portfolio construction for withdrawal purposes.
- 4 factors (size, book to market ratio, profitability, and volatility) were identified as potential sources of return in excess of the market.
- For the period considered (1996 – 2019), in absolute terms, the profitability factor did not outperform the market, but the size and value factors did; the portfolios in this group did not produce any significant risk adjusted returns in excess of the market. In addition, with respect to the CAPM theory, the size, profitability and volatility anomaly was observed.
- The low volatility factor (a subset of the volatility factor) only had a better risk adjusted return compared to the market. Based on this, the low volatility anomaly (presently gaining traction) that low volatility stocks generally produce higher returns/risk adjusted returns was confirmed. In addition, 7 other portfolios with the low volatility combination produced significant risk adjusted excess return.
- Most of the portfolios formed with the factors considered will sustain a 4% fixed real rate of withdrawal.
- Compared to the market and the entire portfolios formed, the *H,Lv*, *S,H,Lv*, *H,Lp,Lv* and *S,Lp,Lv* were identified as the most successful portfolios as they sustained up to 8% withdrawal with at least 90% success rate. The *H,Lv* sustained 10% withdrawal with about 90% success. These portfolios also produced significant risk adjusted returns in excess of the market.
- Hence, this research further makes the contribution that these factor strategies are successful equity strategies that can be used to generate returns in excess of the market and at the same time sustain higher than the generally adopted 4% fixed real withdrawal rate. With respect to the residual values obtained, the portfolios produced up to 57% of the highest residual value observed.
- Conditional on failure, these portfolios sustained withdrawals for competitively long periods relative to the market and other portfolios.

- As a further contribution, the low volatility factor '*Lv*' (as well as the value factor *'H'*) appears to be a driver of stability / sustainability. This tends to align with the low volatility anomaly as stocks offering relatively high returns at low risk.
- As an additional contribution, the relative stability *ad hoc* measure proposed (which measures the speed of reversion per unit deviation of shock) whilst it may not be consistently reflective of the success rates (because, unlike the relatively stability measure, the success rate is influenced by the level of returns), it can be used as an indicator of better stability and success when the comparative portfolios have similar return.
- The relative stability measure has strong correlation with sustainability conditional on failure.
- All else being equal, portfolios with high persistence (slower speed f reversion) and smaller deviation of shock tends to produce better success during withdrawals.
- All else being equal, portfolios with low persistence (faster speed of reversion) and smaller deviation of shock tends to produce better sustainability conditional on failure.

The findings of this thesis have important implications for retirees (and their advisers) who are entering the withdrawal phase of their retirement journey. Traditional investment strategies have not been designed with this phase of their investment at mind therefore, the outcome of this thesis offers at least a starting point in the construction of retirement portfolios with the objective of sustaining withdrawals. The availability of active stocks through the entire period considered indicates that these are available strategies that can be explored to give the right balance of likelihood of success and availability of bequest; two important requirements for these retirees.

6.3 Areas of Further Research.

Considering that this is a relatively un-researched aspect of retirement planning, there are a number of extensions to this thesis that is suggested.

- Transaction cost has not been directly included in this thesis for the purpose of simplicity hence examining the effect of this on the use of these strategies will be important in shedding more light on the practicality of these strategies.
- The break points used in categorising most of the portfolios (30^{th} and 70^{th} percentiles) means that there is a large population of intermediate stocks. It will be beneficial to see if portfolios formed from this pool offer better results.
- As mentioned earlier, at the withdrawal phase, the 2 main asset classes used for portfolio construction are equities (stocks) and fixed interest (bonds). Whilst the equity content is the main driver of growth (hence the focus on all equity-based portfolios in this thesis), the bond content is predominantly to create some stability in the portfolios hence there are largely included to reduce risk. It will be beneficial to see if including bonds into the portfolios formed will change the outcome of the results seen in this thesis.
- The measure of stability proposed may also be enhanced (perhaps with some form of standardisation) to reflect the returns dimension of the portfolios and therefore make it possible to compare portfolios not only with similar returns.
- The data used in this thesis ended in 2019 however, with the recent series of unprecedented events (e.g., COVID 19 pandemic), it will be useful to update this research with updated data as examine if the results will be different in light of these events.
- The returns of the portfolios constructed were done as caps (value) weighted; it will be insightful to see if the results are affected by other weighting regimes such as equal weighted.
- There is currently a plethora of factors been proposed as other sources of excess return. Using more factors for this analysis will be interesting to see if they produce similar or different results to that of this thesis.

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