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Machining dynamics and chatters in micro-milling: A critical review on the state-of-the-art and future perspectives



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Chatters suppression;
Digital twin

Abstract Micro-milling technology is widely applied in micro manufacturing, particularly for the fabrication of miniature and micro components. However, the chatters and machining dynamics related issues in micro-milling are often the main challenges restricting its machining quality and productivity. Many research works have rendered that the machining dynamics and chatters in micro-milling are more complex compared with the conventional macro-milling process, likely because of the size effect and rigidity of the micro-milling system including the tooling, workpiece, process variables, materials involved, and the high-speed milling machines, and further their collective dynamic effects. Therefore, in this paper, the state of the art focusing on micro-milling chatters and dynamics related issues over the past years are comprehensively and critically reviewed to provide some insights for potential researchers and practitioners. Firstly, typical applications and the problems caused by the machining dynamics and chatters in micro-milling have been put forward in this paper. Then, the research on the underlying micro-cutting mechanics and dynamics, stability analysis, chatters detection, and chatter suppression are summarized critically. Furthermore, the underlying scientific and technological challenges are discussed particularly against typical precision engineering applications. Finally, the possible future directions and trends in research and development of micro-milling have been discussed.

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1. Introduction

As a critical enabling technology for micro-devices and systems, micro-manufacturing technology has a significant impact on the performance of the final product. Among the various micro-manufacturing techniques, micro-milling, micro-electrical discharge machining (micro-EDM), LIGA, and deep reactive-ion etching (DRIE) are frequently employed. The

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characteristics of these techniques may present various advantages and limitations.

As illustrated in Table 1, micro-milling technology possesses several advantages over other micro-fabrication methods, including the ability to machine a wide range of materials, high machining efficiency, high accuracy, and the ability to process complex 3D structures.^{1–8} Especially regarding surface quality, in the micro-milling process, selecting appropriate processing equipment, parameters, and cutting tools can reduce the surface roughness to tens of nanometres.⁹ This is difficult to achieve in other micro-machining methods, which is why micro-milling holds a distinct advantage in applications where high surface quality is required.¹⁰ These advantages have led to its extensive application in various fields, such as the fabrication of terahertz slow-wave structures,^{11,12} micro nozzles,¹³ microturbines,¹⁴ microelectrodes, microfluidic chips,^{15–17} and micro molds,^{18–20} as depicted in Fig. 1.

In contrast to conventional macro-milling, micro-milling technology is commonly used as the final machining procedure for microstructures, with a direct and significant impact on machining quality and service performance.^{21–23} Typical application scenarios of micro-milling technology reveal that it is primarily used for the manufacturing of small-sized and high-precision parts. The key requirements for micro-milling can be summarized as follows.

- (1) Surface Integrity. Surface quality is a critical factor in a variety of applications. In terahertz slow-wave structures, for example, achieving a high level of conductivity requires controlling surface roughness to within tens of nanometres.²⁴ In aerospace micro-nozzle fabrication, the performance of the micro-propulsion system is highly dependent on surface roughness, while surface defects on micro-molds will inevitably affect the workpiece, underscoring the need to ensure surface integrity for optimal product quality. Apart from surface roughness, the fractal dimension and surface anisotropy are also critical indicators for evaluating surface integrity in the fabrication of thin-walled micro parts utilized for MEMS accelerometers.^{25,26} Furthermore, it is important to improve the residual stress state in the sub-surface of components in micro-milling, as residual stresses can impact both the frictional wear behaviour and the characteristics of micro-crack propagation.²⁷ This holds crucial importance for the fatigue life and service performance of the components because sub-surface compressive stress can suppress the generation of micro-cracks and other defects.²⁸

- (2) Dimensional accuracy. In the terahertz slow-wave structures, the dimensional consistency must be controlled strictly within a few microns. Dimensional error is a critical criterion for evaluating the machining quality of micro impellers, and microelectrodes, as they all feature thin-walled structures that are prone to deformation under the periodic force of micro-milling.²⁹
- (3) Burr suppression. Burr suppression is another significant challenge in the micro-milling process, due to the presence of several types of burrs in microfluidic chips, namely top, side, and exit burrs, as shown in Fig. 1(e). The size of micro burrs is usually in the micrometer range, which is close to the dimensions of microstructures themselves. The existence of micro burrs can negatively impact the positioning and assembly of microstructures. Nevertheless, due to the small size, deburring is challenging in the micro-milling process.^{30,31}
- (4) Machining efficiency. In addition to ensuring machining quality, machining efficiency must also be guaranteed in the industrial production process. Material removal rate is a typical indicator of machining efficiency. However, an increase in the material removal rate can lead to regenerative chatter, which, in turn, deteriorates machining quality.^{32,33} Thus, the relationship between machining quality and efficiency must be carefully balanced.

To improve product performance from a manufacturing perspective, deeper explorations are necessary regarding the main quality problems that exist in micro-milling and their underlying causes. Micro-milling, as a material removal processing technology, originates from conventional macro-milling, but possesses many unique features, such as weak stiffness of the micro-milling tool, an ultra-high rotational speed of the spindle, and changing material removal mechanism.³⁴ Research has shown that regenerative chatter is a primary cause of decreased workpiece surface integrity, geometric accuracy, and tool life in micro-milling.³⁵ Regenerative chatter is also the main challenge that restricts the improvement of machining efficiency.³⁶

The performance of micro-milling under stable and unstable conditions is compared in Fig. 2.^{37–40} The time-domain distribution of micro-milling forces under stable and unstable cutting processes is shown in Fig. 2(a). The cutting force amplitude increases and fluctuates severely where chatter occurs, due to the irregular changes in the cutting thickness caused by regenerative chatter. This can deteriorate surface

Table 1 Comparison of machining performance of various micro-manufacturing methods.

Methods	Micro-milling ^{1,2}	Micro EDM ^{3,4}	LIGA ^{5,6}	DIRE ^{7,8}
Material capability	Metals, polymers, glass, ceramics, KDP crystals, etc.	Mainly conductive materials	Metals, plastics, polymers, glass, ceramics, etc.	Mainly silicon materials
Surface quality	10–100 nm	0.8–10 μm	0.4–6 μm	4–20 μm
Processing efficiency	High	Medium	Low	Low
Other characteristics	Ability for machining complex 3D structures	Severe electrode loss, poor surface quality	Complex and expensive equipment	Mainly used for processing 2D structures

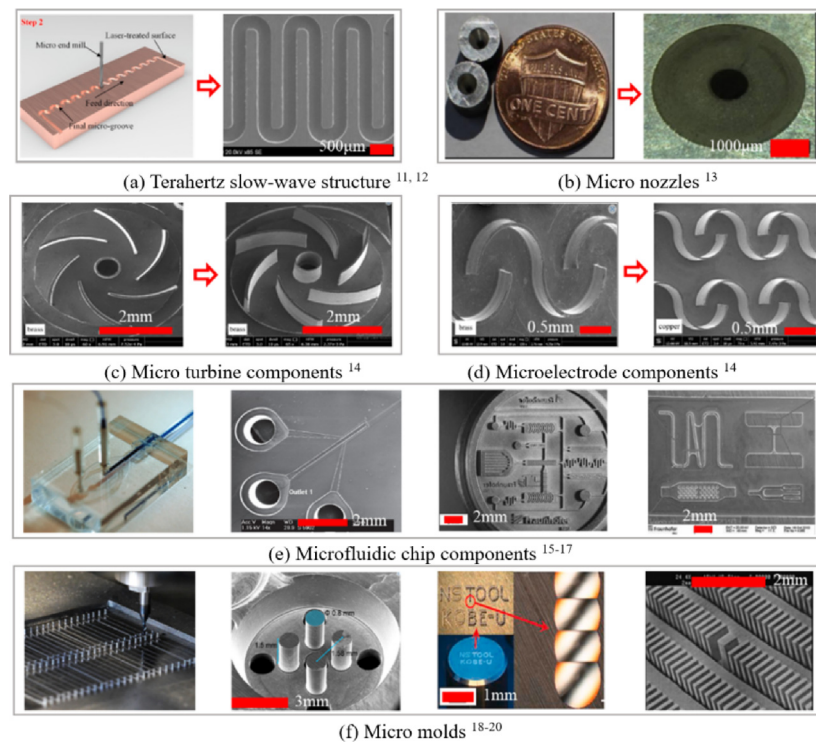


Fig. 1 Application fields of micro milling.

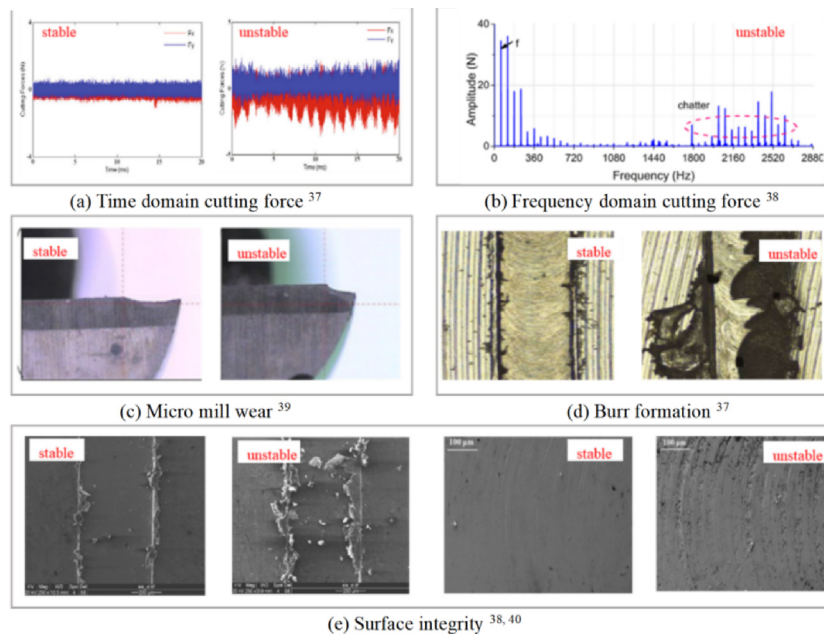


Fig. 2 Micro-milling performance under stable and unstable conditions.

quality, tool life, and dimensional accuracy, and generate burrs. Fig. 2(b) shows the frequency domain variations of the micro-milling force under unstable cutting states. Results indicate that obvious high-order frequencies emerge when chatter vibration occurs. However, the main frequencies of stable cutting forces are distributed around the spindle rotation frequency and the cutter passing frequency. Additionally, severe changes in cutting force can increase microtool wear,

and the rounded cutting edge can further aggravate the friction and ploughing effect between micro cutters and workpiece material. This is also a negative factor for machinability, as depicted in Fig. 2(c)–(e).

As mentioned earlier, due to the scale effect and high spindle speed, regenerative chatter is the primary problem that restricts machinability and machining efficiency in micro-milling.

Therefore, special attention must be paid to the chatter mechanism, prediction, detection, and suppression methodologies in the micro-milling process. Numerous scholars have conducted in-depth research on this issue.^{41–43} Since 2009, around 100 pieces of relevant literature indexed by the Web of Science have focused on chatter in micro-milling, using the search keywords “chatter + micro-milling”. Among these publications, more than 60 percent concentrate on the prediction of micro-milling chatter, with only a few aimed at chatter detection and suppression, as shown in Fig. 3.

In traditional macro-milling research, extensive review articles on chatter issues have been published and updated.^{44–46} The review article by Balázs et al. summarized the research works on micro-milling stability.⁴⁷ These studies are of great significance for the further development of micro-milling technology. Building upon these foundations, this paper further delves into the unique features of micro-milling compared to traditional milling in terms of stability-related issues. Due to unique natures, there are significant differences in the chatter mechanism between micro-milling and macro-milling processes. The existing research on regenerative chatter in macro-milling cannot be applied directly to micro-milling. Therefore, this paper thoroughly reviews the state of the art on micro-milling regenerative chatter to establish a foundation for improving machining quality and efficiency. In addition to summarizing and analyzing recent research on stability prediction, chatter monitoring, and chatter suppression in micro-milling, this paper also focuses on dissecting the unique technical characteristics of micro-milling as distinct from traditional milling and their impact on stability. Moreover, it makes some reasonable predictions about key areas for future research.

This paper’s structure is illustrated in Fig. 4. The introduction section explains the motivation for reviewing micro-milling chatter studies. The second part discusses the differences between micro-milling and macro-milling chatter mechanisms. Sections 3 and 4 summarize and critically analyze the state-of-the-art on chatter prediction, detection, and suppression in micro-milling. Finally, we propose a data-knowledge-driven digital twin (DT) model for chatter suppression based on the remaining issues. Throughout this paper, the term “chatter” refers to regenerative chatter, unless otherwise stated.

2. Uniqueness of chatter mechanism in micro-milling process

Micro-milling and conventional macro-milling share kinematic similarities, but micro-milling differs significantly in several other aspects. Fig. 5 illustrates the process uniqueness of micro-milling compared to macro-milling.

- (1) Relatively weak stiffness. The micro-milling tool diameter and stiffness are lower those of macro-milling tools. However, the cutting forces in micro-milling are also smaller than those in macro milling. Therefore, to illustrate the impact of tool deflection under cutting forces on micro-milling stability, the concept of ‘relatively weak stiffness’ is adopted. As shown in Fig. 6, Zhang et al. simplified the deflection of micro-milling tools as a cantilever beam model.⁴⁸

The deflection of the tool at the axial depth z can be expressed as

$$d(z) = \frac{F(L-a)^2}{6EI}(2L+a-3z) \quad (1)$$

where F is the total force, L is the tool overhang length, a is the distance between the tool tip and the concentrated force, E is the modulus of elasticity, and I is the moment of inertia.

To facilitate a comparative analysis of tool deflection between macro-milling and micro-milling, it is assumed that both a and z are set to 0. Under this condition, Eq. (1) simplifies to:

$$d(z) = \frac{FL^3}{3EI} \quad (2)$$

Under the assumption of constant tool overhang length and elastic modulus, $d(z)$ is directly proportional to F/I . For circular tool cross-section, the moment of inertia can be expressed as:

$$I = \frac{\pi}{4} \cdot R^4 \quad (3)$$

where R represents the tool radius.

Traditional milling forces and traditional milling tool diameters are 5–100 times larger than those in micro-milling. Therefore, it can be inferred that the effect of cutting forces on

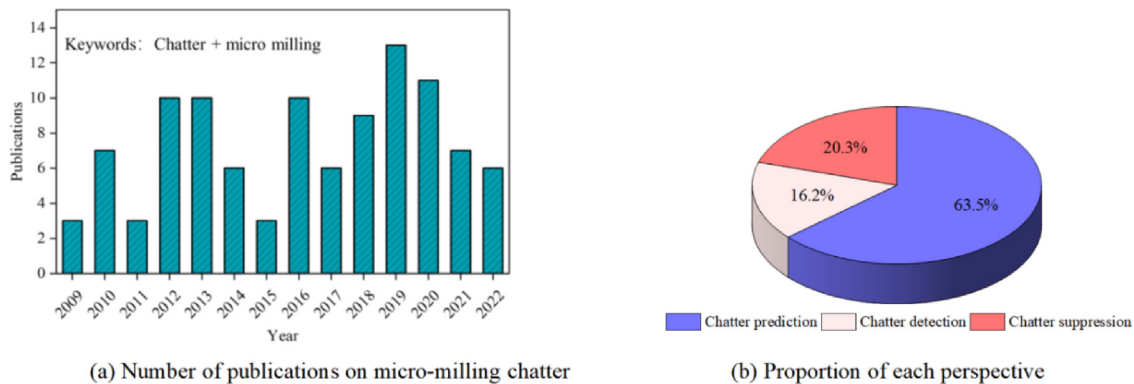


Fig. 3 Number and proportion of publications on micro-milling chatter.

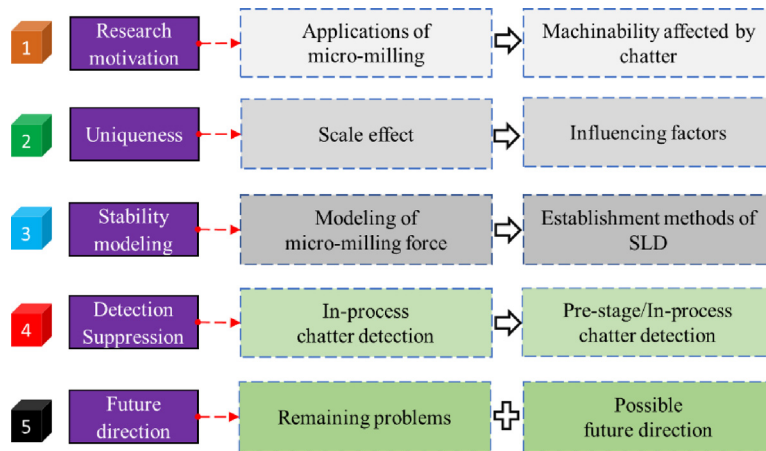


Fig. 4 Structure of this paper.

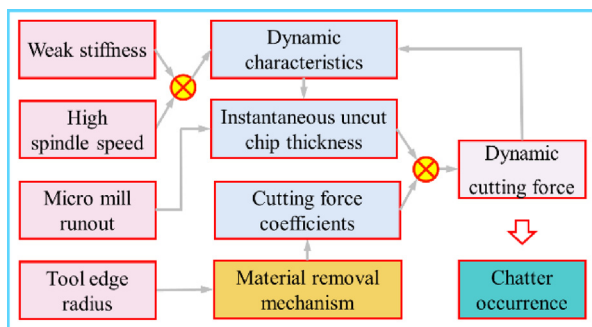


Fig. 5 Features causing chatter vibration in micro-milling process.

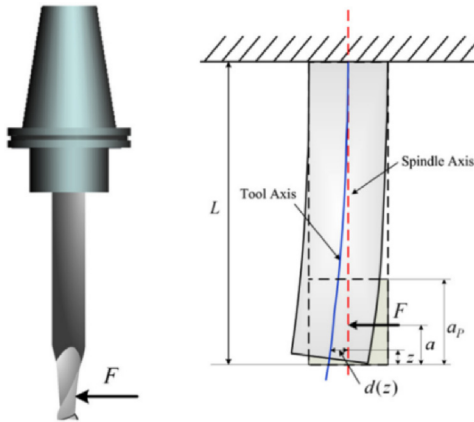


Fig. 6 Tool deflection model: simplified cantilever beam.⁴⁸

micro-milling tool deflection is greater, indicating a ‘relatively weak stiffness’.^{49,50} As a result, the uneven variations in micro-milling cutting thickness and forces cannot be ignored.

The micro-milling tools can be modelled as spring-damping elements⁵¹ or beam models depending on the studies, actually, as illustrated in Fig. 7.

(2) High spindle speed. In micro-milling, spindle speeds can reach between 10,000 to 400,000 r/min, significantly higher than those in macro-milling. Consequently, the

centrifugal force and gyroscopic effect, which are typically disregarded in macro-milling, can significantly impact the dynamic characteristics of the micro-milling tool.^{52–56}

- (3) Radial error motions. The diameter of micro-milling tools is small. Therefore, even small radial error motions can have significant impacts on the motion trajectory of micro-milling tools. The quality of the machined parts depends on the accuracy of the tool-tip trajectory as the spindle rotates. The main sources of error in radial motions include the installation error of the tool with respect to the axis of rotation, the spindle error motions, the tool runout, etc., which can be referred to as the dynamic runout. The radial error motions cause the cutting-edge trajectory to deviate from the ideal one, significantly affecting the surface quality and dimensional accuracy, as well as the micro-milling forces and stability.^{57–59}
- (4) Size effects. During micro-milling, the cutting thickness and cutting-edge radius are on a micrometer scale, which results in there are three material removal mechanisms in micro-milling processes, as shown in Fig. 8.

The material removal mechanism depends on the instantaneous cutting thickness. When the instantaneous cutting thickness is smaller than the critical chip thickness, the micro-milling process is dominated by the ploughing in which no chips format.

When the instantaneous cutting thickness is larger than the minimum chip thickness, the shearing is dominated in the micro-milling process. When the instantaneous cutting thickness is between the critical chip thickness and the minimum chip thickness, elastic and plastic deformation occurs at the same time.^{60–62}

The size effect is closely related to micro-milling performance, including cutting force, chip formation, burr formation, process stability, etc., as shown in Fig. 9. In summary, it is important to recognize that micro-milling technology cannot be considered a smaller version of conventional milling. Without a comprehensive understanding of the distinctive characteristics that compromise stability during the micro-milling process, it is difficult to eliminate regenerative chatter and enhance machining quality and efficiency.⁶³

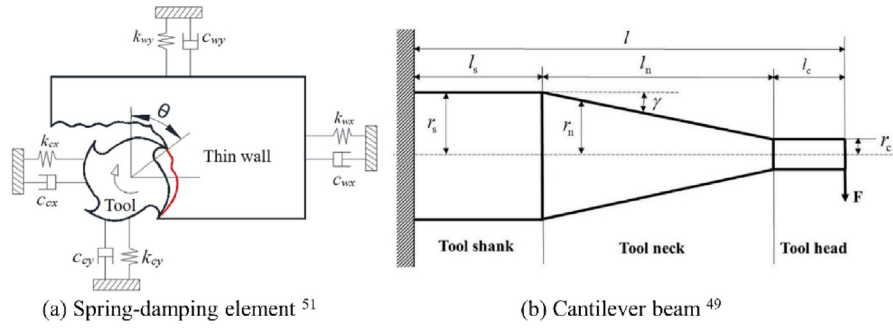


Fig. 7 Simplified structure model of micro-milling tool.

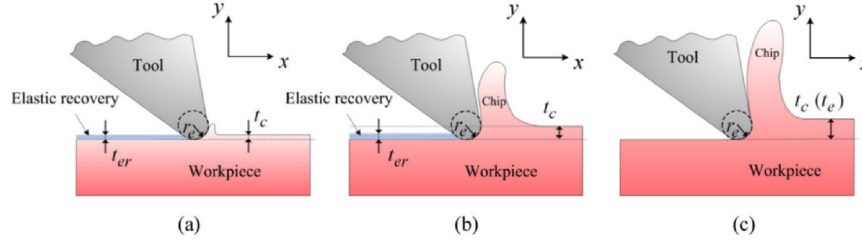


Fig. 8 Different material removal mechanisms in micro-milling: (a) Elastic deformation region, (b) Elastic-plastic deformation region, (c) Shearing region.⁴⁸

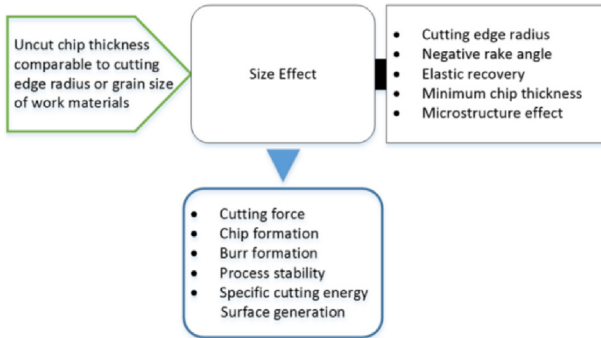


Fig. 9 Size effect in micro-milling.¹

3. Stability prediction in the micro-milling process

To achieve optimal machining quality and minimize unnecessary noise, micro-milling parameters are typically chosen from the stable machining region. The stable machining region can be identified using a stability lobe diagram (SLD), which shows the maximum cutting depth that can be achieved at specific spindle speeds.⁶⁴ The SLD is generally obtained by solving a time-delay differential equation that describes self-excited vibration (as shown in Eq. (4)). Therefore, accurately modeling and solving the dynamic equations is crucial for ensuring the precision of the SLD.⁶⁵

$$\begin{bmatrix} m_x & 0 \\ 0 & m_y \end{bmatrix} \begin{bmatrix} x'' \\ y'' \end{bmatrix} + \begin{bmatrix} c_x & 0 \\ 0 & c_y \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} F_x(t) \\ F_y(t) \end{bmatrix} \quad (4)$$

where m_x , m_y , c_x , c_y , k_x , k_y are the mass, damping and stiffness in the feed (x), and normal (y) directions, respectively; $F_x(t)$

and $F_y(t)$ are the cutting forces in the feed (x) and normal (y) directions, respectively.

Eq. (4) reveals three essential aspects required for SLD acquisition, namely, micro-milling force, dynamic parameters of the system, and solution methods for time-delay differential equations.

3.1. Micro-milling force modeling

As previously mentioned, due to the influence of scale effects, there are three modes of material removal in micro-milling, and the modeling methods for cutting forces differ for each mode.

3.1.1. Modeling of cutting forces in the shearing region

The mechanical model is a semi-analytical method based on the empirical model, which is widely used in micro-milling force prediction, as shown in Eq. (5).

$$\begin{cases} dF_{ij} = [K_{te}t_e + K_{re}]dz \\ dF_{rj} = [K_{re}t_e + K_{te}]dz \end{cases} \quad (5)$$

where dF_{ij} and dF_{rj} represent the tangential and radial micro-element cutting forces, respectively; K_{te} , K_{re} represent the tangential and radial cutting force coefficients, respectively; K_{te} and K_{re} represent the edge force coefficients, respectively; t_e represents the instantaneous cutting thickness (ICT); dz is micro-element thickness.

By performing integration of the micro-element cutting force, Eq. (6) illustrates the determination of the total cutting force in both the x and y directions.

$$\begin{aligned} \begin{cases} F_x \\ F_y \end{cases} &= \sum_{j=1}^N \int_0^{a_p} \begin{bmatrix} dF_{xj} \\ dF_{yj} \end{bmatrix} dz \\ &= \sum_{j=1}^N \int_0^{a_p} \begin{bmatrix} \cos \theta_j & \sin \theta_j \\ -\sin \theta_j & \cos \theta_j \end{bmatrix} \begin{bmatrix} dF_{ij} \\ dF_{rj} \end{bmatrix} dz \end{aligned} \quad (6)$$

where F_x and F_y are the cutting forces in the feed (x) and normal (y) directions, respectively; dF_{xj} and dF_{yj} represent the micro-element cutting forces in the feed (x) and normal (y) directions, respectively; θ represents the tool position angle; a_p represents the axial cutting depth.

The modeling of ICT is crucial for enhancing the accuracy of cutting force prediction. To this end, numerous scholars have dedicated their efforts to this area of research. The ICT model was proposed for the milling process, which found the extensive use in conventional milling force prediction, as demonstrated in Eq. (7).⁶⁶

$$t_e = f_t \sin \theta \quad (7)$$

where f_t represents the feed per tooth.

The classic cutting thickness model assumes a circular tool path and neglects tool runout and deformation. However, this simplification method is not applicable for micro-milling due to the scale effect.^{67–69} The performance of micro-milling strongly depends on the tool path model, which is influenced by various factors such as tool trochoidal trajectories, tool runout, and tool deformation. The generic ICT model was proposed, as shown in Eq. (8), which takes the influences of trochoidal trajectory and tool runout into account.

$$h = f_t \sin \theta - \frac{f_t M \delta}{2\pi} \sin \theta + \frac{f_t^2 \cos^2 \theta}{2r} + \sqrt{r^2 + r_0^2 + 2rr_0 \cos \left(\gamma + \frac{2\pi m}{M} \right)} - \sqrt{r^2 + r_0^2 + 2rr_0 \cos \left(\gamma + \frac{2\pi(m-1)}{M} \right)} \quad (8)$$

where r_0 is the tool runout, m is the ordinal number of tool teeth, M is the total number of tool teeth, r is the tool radius, γ is the runout angle.

The cutting forces obtained using this prediction model closely match the experimental values compared with the previous model,⁷⁰ which only considers the influence of tool runout on cutting forces. A micro-milling force model had been proposed to consider both the influences of tool runout and the deflection of micro-tools. The ICT cannot be expressed as analytical expressions, while numerical iterations are applied based on the coordinates of cutting edges, as shown in Eq. (9), where tool deformation is modeled as beam deflection under a concentrated load.

$$\begin{cases} x_a(\theta(t_i)) = x_n(\theta(t_i)) + x_\rho(\theta(t_i), \rho, \alpha, \tau, \phi) + x_d(\theta(t_i)) \\ y_a(\theta(t_i)) = y_n(\theta(t_i)) + y_\rho(\theta(t_i), \rho, \alpha, \tau, \phi) + y_d(\theta(t_i)) \end{cases} \quad (9)$$

where the subscripts a, n, ρ , and d are the actual, nominal, runout-caused, and deflection-caused positions of the cutting edges, respectively.

When comparing with cutting force models that do not account for micro milling tool deformation, the accuracy of micro milling force prediction has been significantly improved, as shown in Fig. 10.

A micro-milling force model based on the influence of tool runout and tooltip dynamics has been developed, in which the ICT can be derived from the coordinates of cutting edges, as shown in Eq. (10).

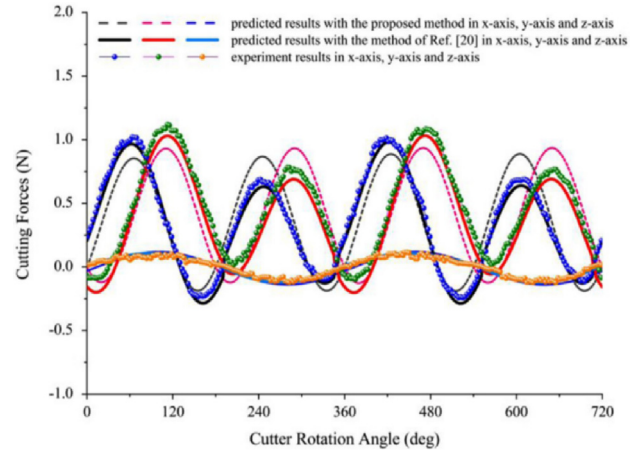


Fig. 10 Simulated and experimentally determined cutting forces.⁴⁸

$$\begin{cases} X_{C_i} = f_t N \frac{\Delta}{60} t_i + r_o \sin \theta_i + d_{x_i} \\ Y_{C_i} = r_o \cos \theta_i + d_{y_i} \\ d_{x_i} = \int_0^t F_x(\xi) g_{11}(t - \xi) d\xi \\ d_{y_i} = \int_0^t F_y(\xi) g_{11}(t - \xi) d\xi \end{cases} \quad (10)$$

where N is the number of flutes, Δ is the rpm of the spindle, and d_x and d_y are the dynamic tool deflections as a result of cutting forces in the x and y directions, respectively; g_{11} is the impulse response of the micro-tool.

It can be observed that the deflections of micro-tool are modelled as dynamic deformation, which is consistent with the actual micro-milling processes. At the same time, the expression of ICT becomes increasingly complex as modeling accuracy improves.

3.1.2. Modeling of cutting forces in the ploughing and mixed region

The micro-milling force in the ploughing region is proportional to the volume between the workpiece and cutting tool. The tangential and radial elemental ploughing force can be expressed as

$$\begin{cases} dF_{tpj}(\theta) = [K_{tp} A_p + K_{te}] dz \\ dF_{rpj}(\theta) = [K_{rp} A_p + K_{re}] dz \end{cases} \quad (11)$$

where K_{tp} and K_{rp} are the ploughing coefficients in the tangential and radial directions, respectively; A_p is the ploughed area.

Similarly, the micro-milling force in the mixed (elastic-plastic) region can be expressed as

$$\begin{cases} dF_{tpj}(\theta) = [K_{te} t_e + K_{tp} A_p + K_{te}] dz \\ dF_{rpj}(\theta) = [K_{re} t_e + K_{rp} A_p + K_{re}] dz \end{cases} \quad (12)$$

As shown in Fig. 11, once A_p is determined, the micro-milling forces for the two aforementioned stages can be obtained. According to the geometric simplified model, the ploughed area A_p can be expressed as Eq. (13).

$$A_p = \frac{1}{2} r_e^2 (\alpha_p + \psi_2 - \sin(\alpha_p + \psi_2)) \quad (13)$$

where angle ψ_2 , α_p can be calculated according the geometric relation.

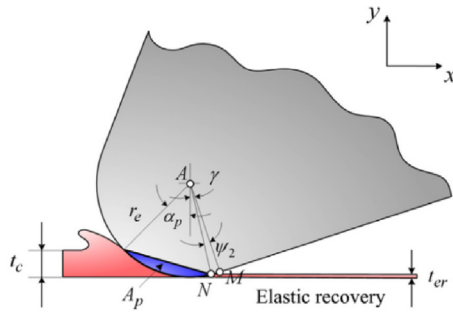


Fig. 11 Ploughing area and elastic recovery.⁴⁸

3.2. Dynamic parameters of the micro-milling system

Dynamic parameters of the micro-milling system play a crucial role in chatter modeling. Several studies have focused on improving the performance of machine tools by optimizing their structures and dynamic parameters.⁷¹ In macro-milling systems, dynamic parameter acquisition is usually accomplished through hammer tests. However, the small size of micro-milling tools can make them vulnerable to damage or deformation from direct impact, and sensor installation can also be challenging.

To generate excitation on the micro-milling cutter, smaller impact hammers are used, and tool-tip displacement is recorded using a laser displacement sensor.^{72–74} Furthermore, some researchers use a piezoelectric actuator to input harmonic excitation, and the vibration output of the micro-milling tool is obtained using a Doppler laser vibrometer.⁷⁵ As a result, special methods are adopted to obtain dynamic characteristics in the micro-milling system, which can be classified as direct methods and indirect methods, as shown in Table 2.^{76–84}

However, these methods bear their own limitations. Only dynamic parameters while no machining is performed can be obtained, which leads to the influence of centrifugal force and gyroscopic effect cannot be included when the spindle speed is high, and the impact force of the hammer is hard to control. To address these problems, a repeatable impact excitation system (IES) was developed to obtain the tool-tip dynamics of micro-machining spindles, as shown in Fig. 12.

The IES can be used to excite the spindle dynamics in x and y directions, repeatedly, and the impact force can be controlled within a small range for protecting the fragile micro-tools. The radial displacements of the micro-milling cutter can be measured by the laser Doppler vibrometers. The results showed that spindle speed has strong effects on both the damping

ratios and the natural frequencies. The compressed air can be used to drive the bearing ball as the input source, and the dynamic characteristics can be obtained under various spindle speeds. Finite element analysis (FEA) has been applied to acquire dynamic characteristics, in which the machine tool and cutting tool are carefully simulated, as shown in Fig. 13. The main advantage of the FEA method is that it can reduce experimental costs, but its reliability depends largely on the modeling accuracy. Given that the machine tool is a complex electro-mechanical-hydraulic system, the modeling difficulty is very high and the modeling procedures are very complicated.⁸⁵ The contact characteristics of the bearings and bolt joints have great impacts on the dynamics of machine tools in FEA.

Compared with the direct method, the response coupling method (RC) is an indirect method for determining the dynamic characteristics of the micro-milling system. As shown in Fig. 14, the micro tool and machine tool are divided into two parts in RC method. The dynamic parameters of the two parts are solved separately and coupled by Eq. (14). Part B is generally obtained directly through hammer testing, while Part A can be solved using the FEA method or theoretical modeling. The Timoshenko and Euler beam theories are commonly used theoretical methods for determining the dynamic characteristics of micro tools.

$$G_{11} = \frac{X_1}{F_1} = H_{11} - H_{12}(H_{22} + H_{33})^{-1}H_{21} \quad (14)$$

where G and H denote the assembled and substructure dynamics, respectively.

However, Shekhar et al. pointed out that considerable deviations can be caused in tool-tip dynamics prediction by the RC method, because it is not suitable for high-speed machining processes with over 5 kHz frequency, as shown in Fig. 15.⁵⁴

When the frequencies up to 4 kHz, the predicted results by RC method show significant inaccuracies compared with the experimental results.

3.3. SLD solution methodology in the micro-milling process

Predicting the stable cutting region provides an essential theoretical basis for selecting cutting parameters, which plays a critical role in avoiding chatter in the pre-stage. The dynamic model of the micro-milling process can be described by a set of time-delay differential equations. However, these equations have an infinite-dimensional state space, making them difficult to solve directly. To address this issue, various methods have

Table 2 Micro-milling dynamics acquisition methods.

Classifications	Specific methods
Direct methods	Small-size hammer test ^{53,54,72–74,58} Bearing ball impact ⁷⁵ Integrated FEM ^{39,76,77} Actuator impact test ^{41,78}
Indirect methods (RC)	Hammer test + FEM ^{38,79} Hammer test + Timoshenko beam theory ^{80,81} Hammer test + Euler beam theory ^{82–84}

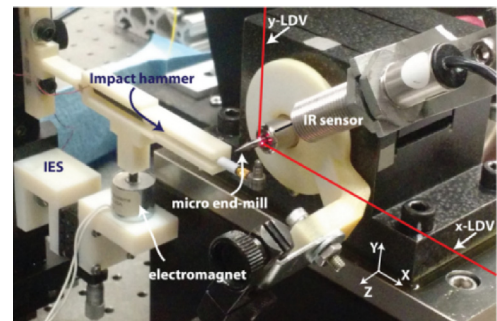


Fig. 12 Impact excitation system.⁵⁸

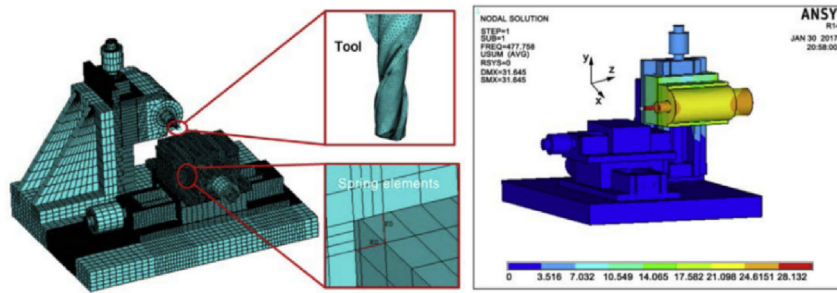


Fig. 13 Dynamic characteristics by FEM.⁷⁶

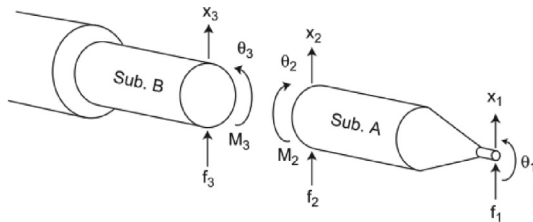


Fig. 14 Dynamic characteristics by RC method.⁶⁷

Table 3 SLD acquisition methodology.

Classifications	Specific methods
Frequency domain method	ZOA ^{41,72,87}
	Nyquist criterion ^{57,88}
	Robust prediction theorem ^{75,89,93}
Time domain method (NI + SC)	Tool tip displacement variance ^{64,90}
	Ratio of DCT to SCT ⁸⁶
	LMI condition ⁷⁹
	Floquet theory ^{91,92}

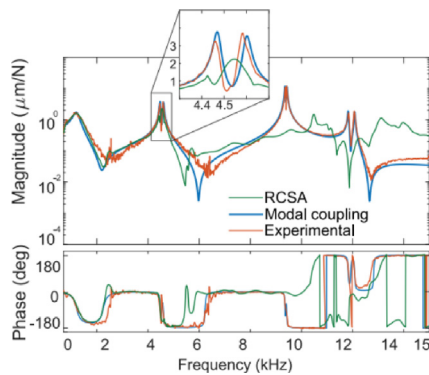


Fig. 15 Accuracy comparisons of the dynamics obtained by different methods.⁵⁴

been proposed by researchers, which can be divided into the frequency domain method and time domain method, as shown in Table 3.⁸⁶⁻⁹³

In frequency domain methods, the zero-order approximation method (ZOA) has been used for predicting the stable cutting region in macro-milling for a long time, and it was also thought as one of the earliest methods applied to micro-milling. ZOA transforms the calculation of critical cutting depth into the characteristic root solution problem of a closed-loop system. To simplify the solution process, only the zero-order term of the cutting force coefficient is considered, while the cross-transfer function of the milling cutter-workpiece contact area is ignored. ZOA has extremely high computational efficiency, but when the radial depth of cut is small, the prediction results have a large deviation from experimental results. Therefore, ZOA is not recommended for micro-milling stable cutting region prediction.

Singh obtained the stable cutting region in micro-milling of titanium alloys using the Nyquist criterion. Only the Nyquist diagram needs to be checked during the process. Cao found

that the prediction results of the robust prediction theorem are conservative but significantly more accurate than those of ZOA, as shown in Fig. 16.⁹³

Time-domain methods can comprehensively consider factors such as tool runout, deformation, and wear, making them more accurate than frequency-domain methods. However, the numerical integration process in time-domain methods is computationally inefficient, and the stability criterion for these methods has not been unified. In certain condition, tool displacement variance can be chosen as the stability criterion, assuming that chatter occurs when the variance value exceeds 1 µm. The ratio of dynamic cutting thickness to static cutting thickness can be used as the stability criterion, while Biermann adopted the tool point trace on the Poincaré section.⁹⁴

To reduce the computational burden, the linear matrix inequality (LMI) condition has been employed in time-

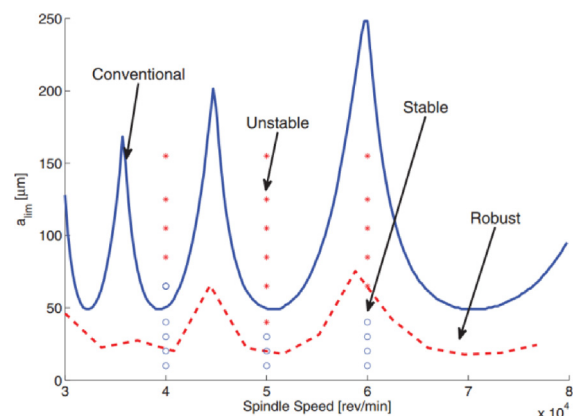


Fig. 16 Robust method for SLD solution.⁹³

domain methods. As shown in Fig. 17, LMI has better prediction accuracy than the frequency domain method but is relatively conservative⁷⁹.

To balance computational accuracy and efficiency, the semi-discrete method (SDM) proposed by Insperger has been employed in predicting the stable cutting region.^{95,96} SDM simplifies the time-delay differential equations that describe micro-milling dynamics into a set of ordinary differential equations. The cutting period is discretized into finite time intervals, and coefficient terms are averaged in each interval, thereby reducing computational complexity. After obtaining the transfer matrix of two adjacent milling cycles, the spectral radius of the transfer matrix is calculated based on Floquet theory. When the spectral radius is less than 1, the micro-milling process is unstable.

3.4. Influencing factors of SLD

The unique features of SLD in micro-milling are size effects and high spindle speeds in comparison with traditional milling. This results in differing impacts of the same influencing factors in both processes.

Understanding the unique aspects of micro-milling SLD is crucial for accurately determining the stable machining zone and making rational selections of process parameters. This section focuses on comparing SLD in micro-milling and traditional milling in terms of size effects, process damping, tool deflection, and gyroscopic effects, allowing for a more in-depth understanding of the fundamental nature of stability in micro-milling operations.⁹⁷

Size effects are one of the most fundamental characteristics of micro-milling operations, directly impacting the material removal form and the coefficients of cutting force. Zhao et al. considered the impact of size effects when modelling micro-milling force.⁹⁸ Based on this model, the SLD is established taking into account the regenerative effects in both the shear and mixed domains. Under the same processing conditions, using the same solving methods, and within the same machining time, the SLD obtained through the proposed method is closer to the experimental results compared with other methods, as shown in Fig. 18.

This is due to the latter's neglect of the enhanced size effect, the impact on machining stability from the transition zones gradually dominates as the cutting-edge radius wears down during the machining process. Therefore, using SLD modeling methods in traditional milling for stability analysis and the parameters optimization in micro-milling will lead to inaccura-

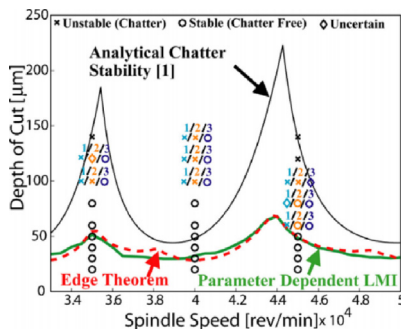


Fig. 17 Comparison of different SLD solution methods.⁷⁹

cies. Similar conclusions can also be drawn from Ding et al.'s research.⁹⁹

The impact of process damping in micro-milling SLD also differs from that in traditional milling. Process damping can enlarge the stable machining zone of SLD in traditional milling. In micro-milling, there exists a critical speed: when the spindle speed is below this critical value, process damping can improve machining stability. However, once the spindle speed exceeds this critical threshold, the effect of process damping is significantly weakened, as shown in the Fig. 19.¹⁰⁰ This is because process damping is influenced by two factors: tool wear and spindle speed, as indicated by Eq. (15).

$$\begin{cases} F_{pd} = K_{pd} \frac{\dot{r}}{R\omega_{spindle}} \\ K_{pd} = 2C_p a \{E[r(t)]\}^2 \left[1 - \frac{h}{E[r(t)]}\right] \left[1 - \frac{h_e}{E[r(t)]}\right] \end{cases} \quad (15)$$

where F_{pd} represents the process damping force, K_{pd} denotes the process damping force coefficient, \dot{r} indicates the tool wear rate, R refers to the tool radius, C_p is the plowing force coefficient, a is the axial cutting depth, h is the cutting thickness, h_e is the elastic recovery height, and $E[r(t)]$ is the tool tip radius taking wear into account.¹⁰¹

It can be observed that when the spindle speed is relatively low, the process damping coefficient increases gradually as the tool wears, leading to an increase in process damping force and thus enhancing machining stability. However, as the spindle speed continues to increase, the process damping force will significantly decrease. At this point, process damping can be disregarded, simplifying the calculations without compromising accuracy and also improving computational efficiency.¹⁰² The critical speed depends on the characteristics of the machining tools and materials.

Tool runout has impacts on SLD in both micro-milling and traditional milling. Due to the size differences in tool diameter, runouts lower than 5 µm do not lead to boundary changes of traditional milling's SLD.¹⁰³ As tool runout gradually increases, the stable region of traditional milling's SLD will expand, as illustrated in Fig. 20(a).¹⁰⁴ However, this is not the case for micro-milling. As micro-tool runout gradually increases from 0.5 to 1 and then to 2, the cutting depth limits of SLD will decrease, as shown in Fig. 20(b).¹⁰⁵

The gyroscopic effect has a minor impact on the SLD in traditional milling processes, as shown in the Fig. 21.¹⁰⁶ This is because the gyroscopic effect influences stability by affecting the dynamic characteristics of the machining system, which is not very noticeable at lower spindle speeds, as illustrated in the Fig. 22.¹⁰⁶

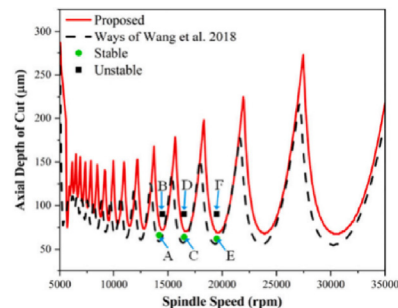


Fig. 18 Influence of size effects on SLD.⁹⁸

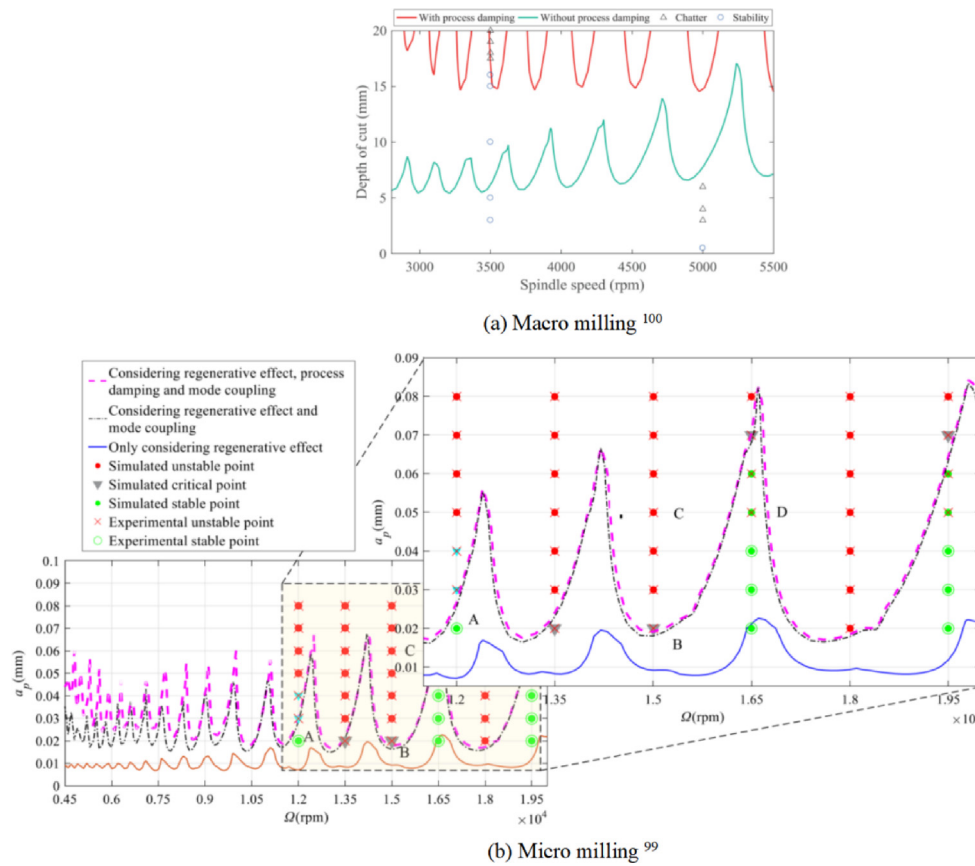


Fig. 19 Influence of process damping on SLD.

However, as the spindle speed increases to higher levels, the gyroscopic effect becomes pronounced. When the spindle speed in micro-milling exceeds 80,000 rpm, the stability boundary of the SLD will decrease, as shown in Fig. 23.¹⁰⁷

In summary, the accuracy of SLD is speed-dependent and material removal mechanism-dependent. It is essential to consider the impact of both factors comprehensively in order to obtain accurate results in micro-milling. Only by doing so can we truly differentiate micro-milling from a scaled-down version of traditional milling.

4. Chatter detection and suppression strategies in micro-milling

Chatter mechanism analysis and chatter detection in micro-milling are the basis for chatter suppression. Although conservative selection of cutting parameters can avoid the occurrence of chatter, industrial production requires not only machining quality but also efficiency and profits.¹⁰⁸ Therefore, cutting parameters cannot be selected too small. As discussed in Section 3, there are many factors affecting the stability of micro-milling, and even if cutting parameters are chosen strictly according to SLD, chatter cannot be completely avoided due to limitations in modeling accuracy and computational efficiency. Therefore, research on chatter suppression in micro-milling is of particular importance.

Chatter suppression strategies can be divided into two categories: active and passive chatter suppression. Active chatter suppression compensates for chatter by driving actuators

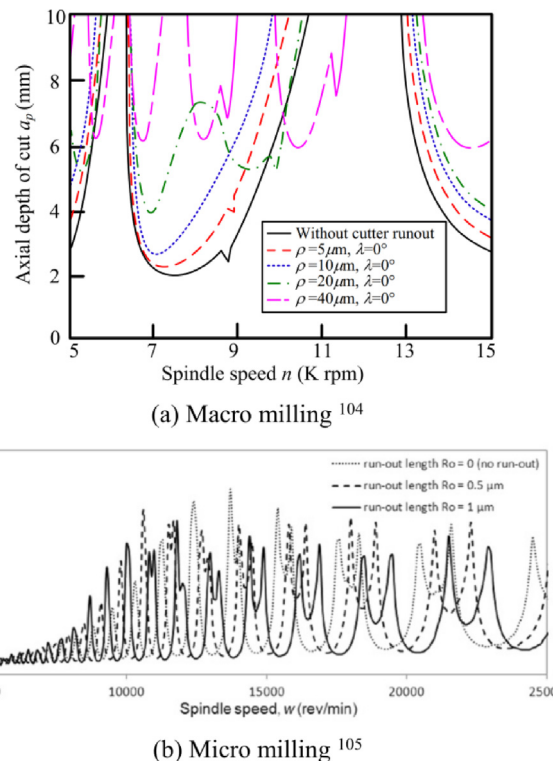


Fig. 20 Influence of process damping on SLD.

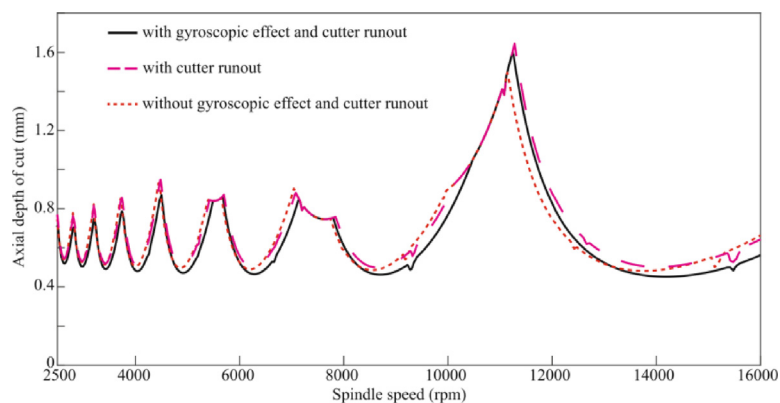


Fig. 21 Influence of gyroscopic effect on SLD in macro milling.¹⁰⁶

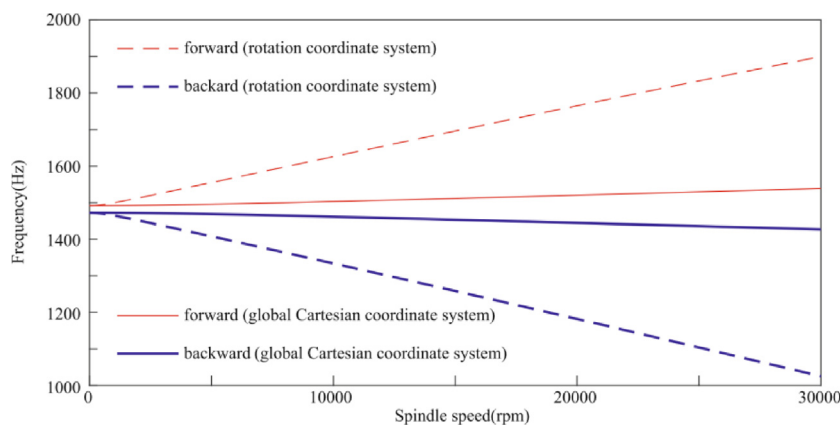


Fig. 22 Forward and backward frequencies of the bending mode.¹⁰⁶

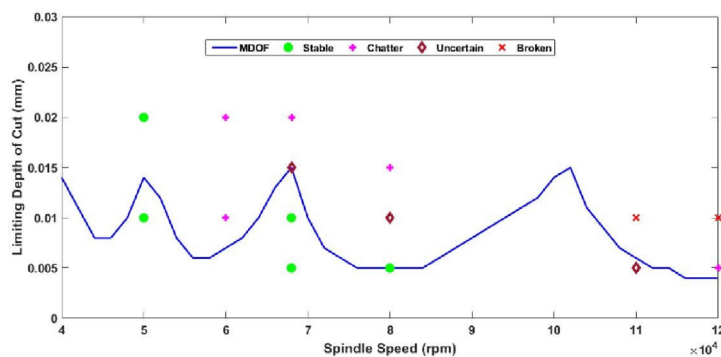


Fig. 23 SLD of ultra-high-speed micro-milling.¹⁰⁷

based on chatter detection results. Passive chatter suppression expands the stable cutting boundary by reducing cutting force and changing the dynamic characteristics of the micro-milling system.

4.1. Chatter detection and active chatter suppression strategies

Chatter detection is the foundation for active chatter suppression, which includes feature extraction of machining signals and classification of machining states.

4.1.1. Machining data acquisition

Before collecting machining signals, sensors must be installed. Special attention needs to be paid to ensure that the sensors do not have a negative impact on the dynamics of the micro-milling system and do not interfere with the micro-milling process.^{109,110} Sensors must have sufficient bandwidth and noise resistance to accurately collect machining data.¹¹¹

The accurate acquisition of micro-milling process data is essential for detecting and suppressing chatter. To achieve this, signals that precisely reflect the micro-milling process must be

selected. Among them, the micro-milling force stands out as a critical physical quantity that influences cutting heat generation, tool wear, and workpiece surface integrity.¹¹² Therefore, it is commonly used as a feature signal for optimizing cutting parameters and detecting chatter.¹¹³ However, due to limited bandwidth, the dynamometer may not detect high-frequency chatters. In such cases, acceleration signals generated during machining can be useful for detecting chatter as they closely relate to self-excited vibrations.^{114,115} As material is removed during cutting, internal stresses in the workpiece will be redistributed, releasing mechanical energy in the form of acoustic emission (AE) waves. Acoustic emission signals are closely associated with machining states and are often used for determining the current state of the process.¹¹⁶ In the micro-milling of carbon steel with varying grain sizes, Ribeiro demonstrated the sensitivity of the AE signal to chatter occurrence and established its relationship with micro-milling stability.¹¹⁷ Displacement sensors with a bandwidth of 10 kHz can capture chatter in high-speed micromachining. However, low signal-to-noise ratio caused by environmental noise and vibration is a major challenge for noise and acceleration signals.¹¹⁸ To solve the installation problem of laser displacement sensors, a measurement system for micro-milling chatter developed that includes a set of multi-angle adjustable brackets. Meanwhile, it is suggested that the machined surface topography provides the best index for detecting micro-milling chatter, as it can clearly reflect the occurrence and severity of chatter.^{119–123} Commonly used signals for chatter detection in micro-milling include cutting forces, vibrations, displacements, acoustic emission (AE) signals, and noises, as shown in Table 4.

In addition, new cutting force characterization methods have been proposed and can be used for real-time machining state monitoring in the micromachining process. Previous cutting force modeling methods primarily focused on accurately predicting cutting force values. However, in ultra-precision and micro-machining processes, micro-milling force absolute values typically do not exceed 10 Newton, making measurement and analysis challenging. Consequently, Niu and other scholars proposed a new cutting force modeling and analysis concept to gain a deeper understanding of the micromachining mechanism, improve micromachining quality, and facilitate industrially feasible micro-cutting force analysis.^{124–126} This method calculates and analyzes micro-cutting force at the unit length, unit area, and unit volume, respectively.

Results indicate that machining performance closely relates to these three indicators. The cutting force at the unit length can predict burr formation, while the cutting force at the unit area is closely linked to the workpiece material's Young's modulus and can predict chip formation and breakage, laying a

foundation for surface formation and tool wear analysis. The cutting force at the unit volume has its physical meaning, representing the heat partition on the workpiece, chip, and cutting tool, and providing a quantitative basis for tool wear analysis. This innovative cutting force modeling method enhances scientific understanding of the micromachining process and holds significant implications for burr suppression, chip formation, surface generation, and tool wear. From a multiscale mechanism perspective, this method holds greater industrial application value than traditional cutting force modeling.

4.1.2. Feature extraction and chatter identification

During the machining process, noise can have a detrimental effect on chatter detection accuracy, making the collected signals unsuitable for machining states identification.¹²⁷ When the signal-to-noise ratio (SNR) is low, there is a higher likelihood of deviations in determining the machining states. Selecting appropriate strategies to eliminate interfering components from the raw data and extracting chatter information is crucial.¹²⁸

In micro-milling, the signal amplitudes are low due to scale effects and limitations of small process parameters. Therefore, the SNR is lower in micro-milling compared to conventional milling, as shown in Fig. 24. This means environmental noise has greater impacts on machining state detection in micro-milling.^{129,130} In order to accurately identify machining states in micro-milling, the feature extraction methods for signals must be capable of effectively filtering out the influence of environmental noise.

Feature extraction methods can be categorized into intelligent methods and traditional methods, including time domain methods, frequency domain methods, time–frequency domain methods, as summarized in Table 5. Regardless of the chosen method, reasonable thresholds must be set to identify the severity and position of chatter.

In traditional methods, the chatter threshold is often determined based on researchers' knowledge and experience, leading to the inability to fully utilize extracted feature information, and limit the subdividing and further evaluation of machining states. Therefore, several intelligent chatter detection methods based on machine learning have been proposed for micro-milling in recent years.

Numerous studies have utilized traditional feature extraction methods. According to Singh, unstable machining commences when acceleration amplitudes exceed 20 m/s². Chen, for instance, proposed a chatter detection method based on peak value variations of the time-domain micro-milling force. Singh suggested that micro-milling chatter can be detected by applying fast Fourier transformation (FFT) analysis. The proportion of high-frequency components increases significantly in the presence of chatter. The ratio between the peak-to-peak values can be used as the chatter indicator, as shown in Eq. (16).

$$\eta = \frac{\max(\text{RMS}_m) - \min(\text{RMS}_m)}{\text{mean}(\text{RMS}_m)} \quad (16)$$

where RMS_m is the average value of a number of RMS points.

When the indicator η exceeds 1.2, the cutting process is unstable, when its value is below 0.8, the cutting process is stable. On this basis, a chatter indicator ζ has been developed, as shown in Eq. (17).

Table 4 Signals used for chatter detection and suppression.

Signals	Literature
Micro-milling force	Chen, et al. ⁷⁷ ; Wang, et al. ¹¹² ; Afazov, et al. ¹⁰⁸
Vibration signal	Li, et al. ¹¹⁴ ; Singh, et al. ⁸⁸ ; Yuan, et al. ¹¹⁵
Displacement signal	Afazov, et al. ¹¹¹ ; Lu, et al. ¹¹⁶
AE signal	Sestito, et al. ⁴² ; Ribeiro, et al. ¹¹⁷
Noise	Wang, et al. ¹¹⁸
Surface topography	Afazov, et al. ⁹⁰

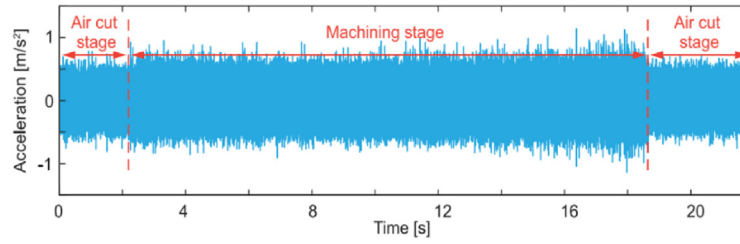
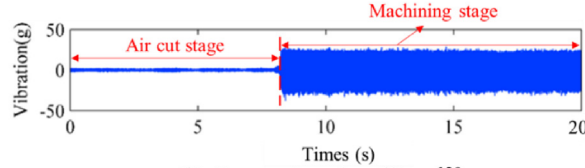
(a) Micro-milling ¹²⁹(b) Conventional milling ¹³⁰

Fig. 24 Influences of environment noises.

Table 5 Chatter detection methodology in micro-milling.

Classifications	Specific methods
Time-domain method	Peak-to-peak cutting force values ⁷¹ RMS of AE signal ¹¹⁷ Statistical variances of the cutting tool displacements ¹¹⁴
Frequency-domain method	Chatter frequency detection ^{108,113,120} PSD of AE signal and surface topography ^{88,117} VMD ¹¹²
Time-frequency domain method	WPT ¹¹⁴ Wavelet coherence functions ¹¹⁵
Intelligent chatter detection method	SVM classification ^{42,109,112,130} ANN based classifier ⁴² Neural network ¹¹⁶

$$\zeta = \frac{\sum_n \max(AE_{rms}) + \sum_n \min(AE_{rms})}{n} \quad (17)$$

where n is a positive integer that represents the number of maximum height pair; AE_{rms} is the root mean square values of AE signals.

For different workpiece materials, the upper and lower thresholds are 0.14 and 0.09, which means the stable cutting processes occur with ζ above 0.14, and chatter-free cutting occurs with ζ below 0.09. A new model for micro-milling chatter detection has been proposed, where chatter is identified when variances values of cutting tool displacements in the x and y directions surpass the set thresholds. The statistical variances are given as Eq. (18).

$$S_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}, S_y^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1} \quad (18)$$

where x_i and y_i are the displacements in the x and y directions, respectively; n is the number of time increments; \bar{x} and \bar{y} are the averaged displacements in the x and y directions.

When the statistical variances exceed $1 \mu\text{m}^2$, the micro-milling cutting is considered unstable. A machine learning-

based chatter detection method was proposed. After filtering the micro-milling force, multi-scale entropy features were extracted as input vectors for the support vector machine (SVM). The trained SVM model could accurately classify machining states into stable cutting, slight chatter cutting, and severe chatter cutting with high precision. The selection of multi-scale entropy also impacted the identification accuracy, with multi-scale sampling entropy outperforming multi-scale fuzzy entropy. Additionally, the performance of different machine learning algorithms for chatter detection was compared, revealing that SVM-based classifiers are superior to Perceptron-based classifiers in terms of the trade-off between the complexity of input vectors and the model's convergence. SVM-based classifiers can detect chatter adequately with fewer features and faster speed. A chatter detection method for micro-milling based on the variable forgetting factor recursive least-squares (VFF-RLS) algorithm is proposed, in which the influences of environmental noise can be filtered out and only components related to chatter will be retained. The flowchart of the proposed method is shown as Fig. 25.¹²⁹ After processing the data with this algorithm, the chatter identification accuracy based on SVM reaches over 90%.

4.1.3. Active chatter suppression strategies

When chatter is detected in micro-milling processes, suppression measures are required. Active chatter suppression involves taking actions, such as applying acceleration, velocity, displacement, or other measures opposite to the chatter. Piezoelectric actuators are commonly utilized due to their robustness and fast response characteristics.

Liu employed two piezoelectric actuators in perpendicular directions to compensate for chatter during micro-milling. Adaptive control strategies drove the piezoelectric actuators, effectively eliminating self-excited vibration of the micro-milling tool and improving machining quality and efficiency.¹³¹ Additionally, the Lyapunov-Krasovskii function can handle the time-delayed effect in micro-milling, playing a crucial role in addressing the hysteresis effect of piezoelectric actuators. Tool vibration can be suppressed in less than 4 ms before machining quality deteriorates.¹³²

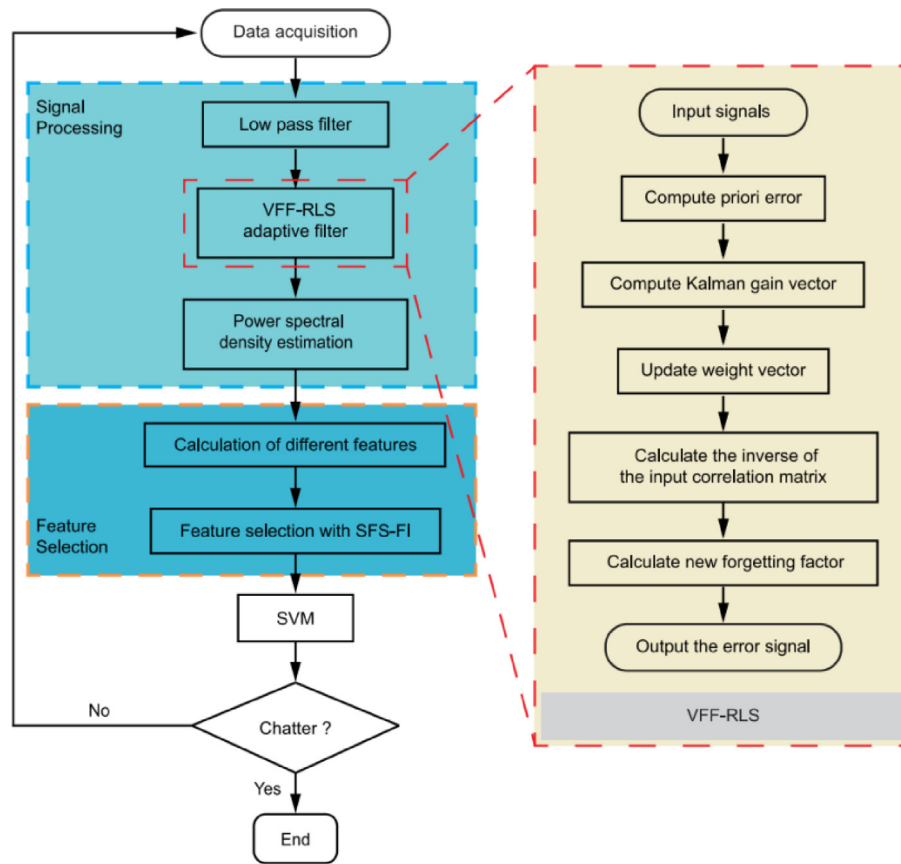


Fig. 25 Flowchart of the proposed micro milling chatter detection method based on VFF-RLS.¹²⁹

4.2. Passive chatter suppression strategies

Passive suppression strategies aim to expand the stable cutting region in SLD (stability lobe diagram). This can be achieved by increasing system damping, reducing cutting force,^{133–135} optimizing tool structure,^{136,137} and implementing auxiliary methods, as summarized in Table 6.

To enhance the stability of the micro-milling system and increase the critical cutting depth, Ma attached a 2-degree-of-freedom tuned mass damper to the shank of the micro-milling cutter.⁴³ The parameters of the damper were determined by maximizing the critical cutting depth. The results showed that the tuned mass damper increased the stable cutting depth by 13 times, as illustrated in Fig. 26.⁴³ Shakeri also employed a damper to suppress chatter during micro-milling, finding that nonlinear dampers outperform linear dampers for chatter suppression.¹³³

When micro-milling hard-to-machine materials, instability can be detected with the increasing in cutting force. Thus, reducing friction and wear between the tool and workpiece material is an effective method for chatter suppression. Sahoo investigated the stability of micro-milling die steel P20 with coated cutters, finding that TiAlN coating can reduce cutting force and improve machining stability by 20.71% compared to uncoated WC cutters.¹¹⁹ Additionally, the use of cutting fluid can decrease cutting forces. Mittal identified a critical speed for the lubrication effect in micro-milling.¹³⁵ When the spindle speed exceeds the critical value (47000 r/min), the

Table 6 Passive chatter suppression methods.

Method	Literature
Decrease cutting force	Sahoo, et al. ^{119,134} ; Mittal, et al. ¹³⁵
Optimize tool structure and toolpath	Mokhtari, et al. ¹³⁶ ; Mayor, et al. ¹³⁷
Ultrasonic Assisted Machining Method	Wan, et al. ⁹²
Increase damping	Ma, et al. ⁴³ ; Shakeri, et al. ¹³³

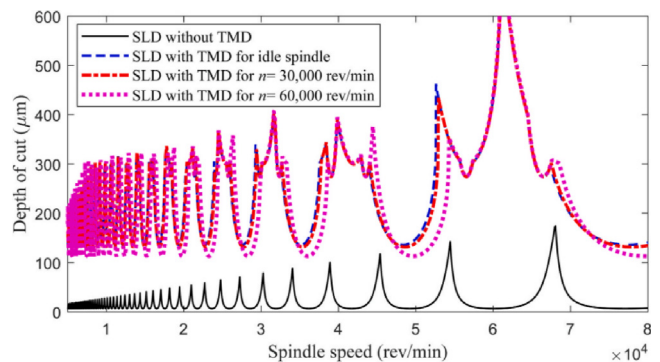


Fig. 26 TMD effects on SLD.⁴³

cutting fluid becomes effective for chatter suppression, and when the spindle speed exceeds 100,000 r/min, the use of cutting fluid improved micro-milling stability by 20%.

Optimizing the tool structure is another chatter suppression method that disrupts self-excited vibrations during machining. Mokhtari developed a tool structure optimization method based on the genetic algorithm, whereby the maximum stable cutting depth serves as the penalty function. After optimizing the cutter teeth number and the tool shank geometry, the stable machining area can be expanded by 1.9 times.

Ultrasonic-assisted machining enables cutting fluid to enter the machining area effectively through the continuous periodic separation between the tool and workpiece. This reduces the cutting force and heat generation significantly, as reported in.¹³⁸ Moreover, in vibration-assisted machining, Ko found that the effective rake angle can be increased, promoting stability in micro-milling.¹³⁹ However, Wan noted that ultrasonic-assisted machining can shift the SLD position, but there is no clear evidence of an increase in the critical cutting depth.

5. Possible future directions

5.1. Remaining problems in micro-milling chatter

In industrial production, the pursuit of high machinability and productivity can be at odds. Achieving good machining quality with high efficiency is a major challenge faced by both consumers and researchers, particularly in micro-milling systems with high spindle speeds and weak stiffness. While chatter has long been recognized as a critical hindrance to machining quality and efficiency in conventional macro-milling processes, further exploration is needed in micro-milling processes, especially with regards to chatter detection and suppression. Therefore, additional research is required in this area to address these challenges. In studies related to micro-milling chatter, several issues remain, including:

(1) The accuracy of SLD

As previously mentioned, micro-milling parameters are typically determined by the stability lobes diagram (SLD) to avoid the detrimental effects of regenerative chatter. Consequently, the accuracy of the SLD is critical to the machining quality. Numerous factors can impact the modeling and solving accuracy of the dynamic equation of micro-milling systems, including cutting force coefficients, dynamic characteristics, micro-tool runout, tool wear, process damping, and gyroscopic effects. Hence, acquiring accurate values for these parameters is of significant importance but can be challenging. The universality and simplification of the experimental equipment should be further taken into consideration. Additionally, researchers have observed that the dynamic parameters of the micro-milling process system change with the material removal process and spindle speed. Therefore, the primary function of the stability lobes diagram (SLD) in micro-milling is to provide a range of initial machining parameter selections. As such, it is unreliable to rely solely on SLD for chatter suppression unless the dynamic characteristics of the micro-milling system can be determined in real-time.

(2) Intelligent chatter detection and suppression strategies

As previously noted, chatter cannot be entirely avoided even if the machining parameters fall within the stability lobes diagram (SLD) due to the system's time-varying dynamics. Thus, passive chatter suppression methods are employed as open-loop control systems that cannot monitor the state of the machining process in real-time. Consequently, workpiece quality can only be assessed through offline instruments following the machining process, resulting in an inevitable time lag. Therefore, achieving online chatter detection and suppression is a critical endeavor. Currently, active chatter detection faces significant obstacles related to feature extraction and identification. Traditional feature extraction methods typically yield non-digitized data that introduce randomness and uncertainty into the identification process, which impedes in-process chatter suppression. In contrast, machine learning algorithms, such as support vector machines (SVMs), decision trees, and neural networks, have been used to develop smart chatter detection and suppression in conventional milling processes. However, the application of such techniques in the micro-milling process is still in its nascent stages.

(3) Thin-walled micro parts fabrication

Thin-walled micro parts, which are microstructures with an aspect ratio greater than 5 and a width less than 100 μm , are in high demand in various fields, including micro-electromechanical systems (MEMS), terahertz (THz) slow wave structures, and microfluidic chips. However, the fabrication of thin-walled micro parts presents more significant challenges than typical micro-milling processes. In addition to the micro tool, the structures of workpieces themselves often have weak stiffness, exacerbating the impact of regenerative chatter on machining deformation.^{140,141} As such, particular attention should be paid to this issue. Ensuring machining quality, particularly dimension consistency, is challenging,^{142,143} as illustrated in Fig. 27.

5.2. Possible directions in future research

Currently, the predominant focus of micro-milling chatter research is on in-advance chatter avoidance and passive chatter suppression methods. Nevertheless, numerous factors affect the stability of micro-milling processes, and the inhomogeneity of workpiece materials and randomness of external vibration sources cannot be entirely eliminated. To balance machining quality and productivity, online chatter detection and effective chatter suppression strategies must be implemented during the initial stages of micro-milling processes. Consequently, the concept of a smart manufacturing system for chatter suppression in micro-milling has been proposed, as illustrated in Fig. 28.

A smart manufacturing system for micro-milling processes should possess real-time, accurate, and reliable characteristics. The digital twin (DT) method has shown extraordinary potential in smart manufacturing and possesses these critical advantages. Research on the application of the DT method in chatter suppression during micro-milling processes needs further exploration in the following areas:

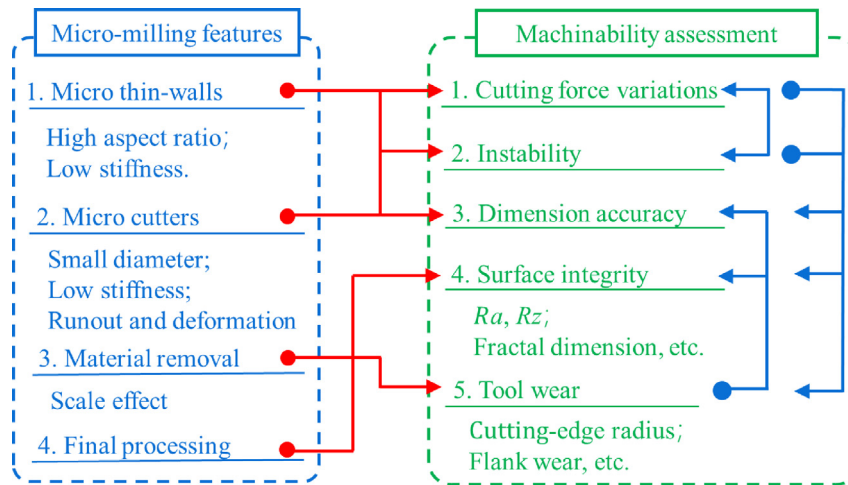


Fig. 27 Main challenges of micro-milling thin-walled parts.

(1) Enhancement and optimization of the DT method:

Given the outstanding outcomes from applying the DT method across various fields, it is imperative that future development should focus on enhancing its capacity to digitize physical entities for analyzing, optimizing, and evaluating product quality in the data space.^{144–148} To leverage the full potential of the DT method, the fidelity between the model and the physical space must be meticulously maintained.¹⁴⁹ This can ensure accurate mapping of the physical behavior, an area warranting concentrated research and innovation to overcome the challenges posed by the inherent lack of physical meaning in data. Further advancements in this method should consider the integration of knowledge space to act as a “translator” between the physical-data space, particularly for applications like chatter suppression in micro-milling.^{150,151}

(2) Advancement in integration and communication technologies:

The three distinctive spaces involved in micro-milling are illustrated in Fig. 29: the physical space, which consists of

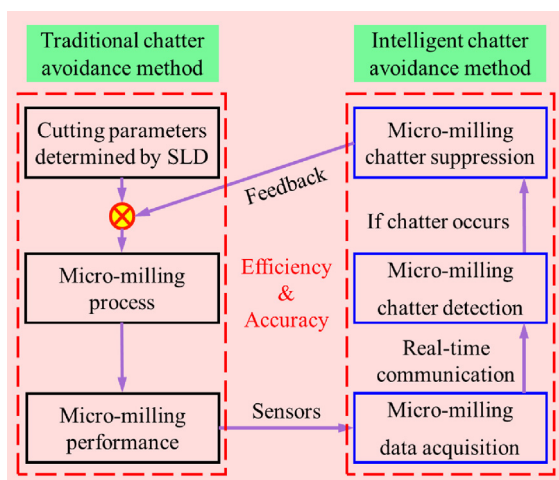


Fig. 28 A smart manufacturing configuration for micro-milling chatter suppression.

the micro-milling machining center, micro-milling cutters, and workpieces; the data space, which encapsulates big data pertinent to the micro-milling process; and the knowledge space, focused on chatter prediction, detection, and suppression models. As sensors mature and become more integral in facilitating data acquisition, it becomes imperative to intensify research and development efforts aimed at bolstering communication between the physical and digital realms, with a specialized focus on advancing real-time data transmission capabilities. The emergence of 5G and other pioneering wireless signal transmission technologies has ushered in unprecedented possibilities in this domain. Harnessing these advancements is pivotal for making significant strides in real-time data transmission capabilities and fully realizing the potential of integration and communication technologies in the context of micro-milling and beyond.

(3) Innovation in signal processing and chatter suppression measures:

In the future, more refined signal feature extraction methods and algorithms should be developed to accurately determine the machining states from the obtained time-domain signals. Advancements in features in the frequency and time-frequency domains, including discrete Fourier transform (DFT), power spectral density (PSD), short-time Fourier transform (STFT), wavelet transform (WT), wavelet packet transform (WPT), and Hilbert-Huang transform (HHT), are crucial.^{152–156} Moreover, the performance of empirical mode decomposition (EMD), ensemble empirical mode decomposition (EEMD), variational mode decomposition (VMD), and entropy methods needs to be expanded and optimized.^{157–160} Once signal feature extraction in micro-milling processes is refined, the application of more sophisticated artificial intelligence (AI) methods can be explored to subdivide the severity of chatter more accurately, such as through enhanced neural networks, support vector machines, and decision trees.^{161–164} Besides, future endeavors should prioritize developing immediate and effective chatter suppression measures when chatter occurs, based on the actual conditions.^{165–172} This includes researching and innovating simple methods like changing the spindle speed and applying auxiliary cutting techniques, using cutting fluid or vibration-assisted machining, and developing

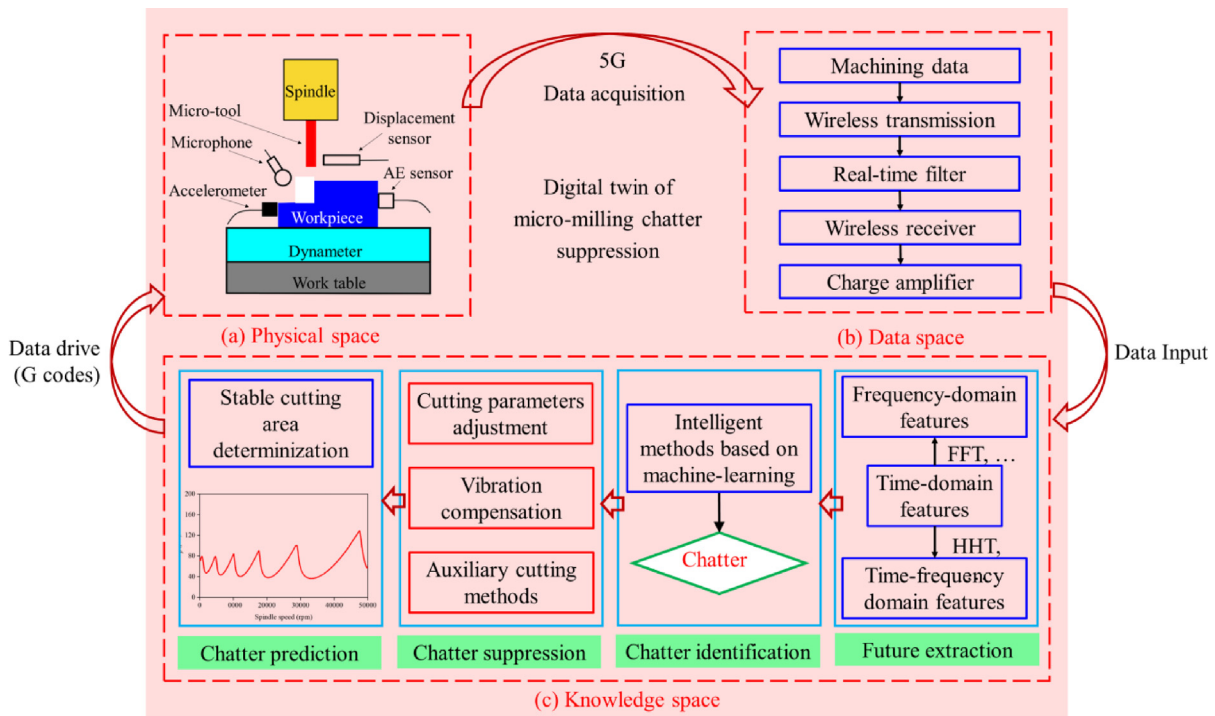


Fig. 29 Digital twin model for micro-milling chatter suppression.

active methods where feedback signals can drive piezoelectric actuators to compensate for vibration and ensure machining quality and efficiency.^{173–177} In this way, the digital twin method can be optimized and fully realized for its vast applications in the field of smart manufacturing and beyond.

6. Conclusions

- (1) This paper has provided a comprehensive and critical review of recent progress in micro-milling chatter, detailing mechanism analysis, prediction, detection, and suppression strategies. Overall, micro-milling chatter is more complex than macro-milling due to the scale effect and high spindle speed.
- (2) Current micro-milling chatter research primarily focuses on SLD modeling and solutions, in-depth research on real-time chatter detection and suppression strategies remains necessary.
- (3) A knowledge- and data-driven DT approach has been proposed to address current issues in in-process chatter suppression. The DT method's integration of physical space, digital space, and knowledge space aligns with the intelligent and green manufacturing development trend. Promoting micro-milling quality and efficiency requires interdisciplinary research in multiple fields.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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