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Modelling and Forecasting of Exchange Rate Pairs Using the Kalman Filter

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ABSTRACT

Developing and employing practically useful and easy to calibrate models for prediction of exchange rates remains a challenging task, especially for highly volatile emerging market currencies. In this paper, we propose a novel approach for joint prediction of correlated exchange rates for two different currencies with respect to the same base currency. For this purpose, we reformulate a generalized version of a bivariate ARMA model into a state space model and use the Kalman filter for estimation and forecasting of the underlying exchange rates as latent variables. With extensive numerical experiments spanning 18 different exchange rates (across both emerging markets, developing and developed economies), we demonstrate that our approach consistently outperforms univariate ARMA models as well as the random walk model in short term out-of-sample prediction for various exchange rate pairs. Our study fills a gap in the empirical finance literature in terms of robust, explainable, accurate, and easy to calibrate models for forecasting correlated exchange rates. The proposed methodology has applications in exchange rate risk management as well as pricing of financial derivatives based on two exchange rates.

1 | Introduction

Predicting exchange rates has been the subject of an ongoing complex debate for many years. Decades ago, Meese and Rogoff (1983) established that the conventional economic models of exchange rates, such as the purchase power parity, interest rate parity models, and the monetary models among others, were incapable of forecasting exchange rates as accurately as the random walk model, commonly referred as the naïve model. This finding instigated a variety of research on this subject as an attempt to explain and overcome the poor performance of the traditional models while also aiming to obtain an improved forecasting method for exchange rates. Broadly speaking, on one hand, a substantial amount of the literature considers the predictive ability of exchange rates to be largely dependent on the choice of predictors. On the other hand, numerous studies emphasize strongly on the importance of model specifications. Rossi (2013) who provides an extensive discussion of the existing literature up to the last decade finds that, though there have been some significant developments and arenas in which this subject has been addressed, the consistent effectiveness of random walk to predict out-of-sample exchange rates is unbeatable.

Relating to the strand of literature focused on the choice of predictors, studies have continuously assessed a variety of macroeconomic and financial indicators relating to exchange rate theories to compare to the random walk process. Models, such as the purchasing power parity, interest rate parity, and the monetary models among others, have most often been claimed to be weak at out-of-sample forecasting (see, e.g., Cheung, Chinn, and Pascual 2005), while Taylor's rule model, based on inflation rate and output gap, is generally known to be a reliable forecasting method in many scenarios as in studies by Molodtsova and Papell (2009) and Byrne, Korobilis, and Ribeiro (2016). More recently, Bulut (2018) re-evaluates the forecasting ability of the same fundamentals by drawing a distinction between real-time data derived from Google Trends and the official realized data for 11 OECD countries' currencies. This study finds real-time data to be an important contributing factor to forecast exchange rates as it is shown that the same economic models are able to outperform random walks when using real data, although robust to only 5 out of 11 currencies.

Further, Jamali and Yamani (2019) examine the out-of-sample forecast accuracy models with a range of fundamentals and technical predictors for 14 emerging market currencies. Among all the macroeconomic indicators used, they only find Taylor's method to be superior to random walk forecasts, not only confirming the previous consensus on Taylor's rule but also demonstrating yet again the poor predictability of the remaining models for exchange rates. Additionally, You and Liu (2020) found Taylor's fundamentals to be a reliable predictor even when forecasting exchange rate volatility for six major currencies, specifically the British Pound Sterling, the Canadian Dollar, the Australian Dollar, the Japanese Yen, the German Deutsche Mark, and the Euro. More recently, Chang and Matsuki (2022) examine their forecasting ability with and without smoothening method for the United Kingdom, Australia, Canada, New Zealand, and the United States. They find that this method is robust in outdoing the random walk process only in the case of Australia and New Zealand, while inducing mixed outcome for the rest of the sample based on the forecasting criteria. Such finding relates to the main criticism of the Taylor rule-based forecasts whereby their predictive ability is often found to be questionable due to inconsistencies and lack of robustness across samples (Rossi 2013). This statement is further supported by Cheung et al. (2019) who investigate exchange rate forecasting for the United States, Canada, the United Kingdom, Japan, Germany, and Switzerland and find mixed and weak evidence of Taylor rule across different periods. Similarly, Jaworski (2021), who compares the effects of country specific macroeconomic fundamentals against global ones to forecast exchange rates, finds the predictions of the former outperforms random walk, although at varying horizons only in the long run, while insignificant in the short run.

This leads to the second strand of the literature that considers the predictive ability of exchange rates to be of an empirical issue. Researchers have investigated the forecasting performance of various linear, nonlinear, univariate, and multivariate time series methods to cope with the Meese-Rogoff dilemma. The most commonly applied linear and nonlinear time series methods in exchange rate forecasting are autoregressive integrated moving average (ARIMA) models and artificial neural network (ANN) models as in Khashei, Rafiei, and Bijari (2013), Khashei, Montazeri, and Bijari (2015), and Galeshchuk (2016) among others. Recent papers have also proposed different variations of machine learning techniques. For example, Amat, Michalski, and Stoltz (2018) adopt a standard machine learning setting with various fundamentals as weights to capture the driving elements of currency predictions of 12 major industrial economies and find compelling evidence in favor of forecasting exchange rates. This method is later revisited by Pfahler (2021) who assesses the predictive power of macroeconomic fundamentals using an advanced machine learning technique, aiming to capture nonlinear aspects that cannot be detected in simple machine learning modelling. Their study, however, is unable to provide any statistical contribution to forecast exchange rates compared to random walks as the latter is found to prevail. As such, these outcomes reiterate the review of Rossi (2013), whereby linear settings in general succeed at predicting currencies better than nonlinear ones.

This statement is nonetheless challenged by Wang, Morley, and Stamatogiannis (2019) who demonstrate that nonlinear specifications can be advantageous. The use of smooth transition regressions for the United Kingdom, Sweden, and Australian currencies is found to improve the out-of-sample forecasting accuracy of the Taylor-rule model and lead them to outperform random walk, unlike other nonlinear studies. The authors argue that findings on this subject differ as nonlinearities vary significantly across countries as they tend to stem from the latter's macroeconomic characteristics, such as, interest rates in their case. This analysis is consistent with the findings of Boero and Marrocu (2002), where nonlinear models were found to also have a higher predictive power than linear models in out-of-sample forecasting. In this case, however, the authors highlight the role of the assessment criteria as a key differentiating factor. Alternatively, other authors resort to the use of hybrids methods to investigate both linear and nonlinear aspects altogether and find some evidence of good predictive ability. For example, Sreeram and Sayed (2024) employ various methods, consisting of ARIMA, ANN, and variations of exponential smoothening, for BRIC currencies and find that these hybrids outperform their respective individual tests, including the nonlinear ones. Though the authors state that these specifications are ideal for complex financial indicators, especially exchange rates, their finding is not robust to all hybrid variations and countries in their sample. Similarly, Khashei and Sharif (2020) proposes a novel Kalman filter-based hybrid ARIMA and ANN model to overcome the limitations of traditional hybrid models as they process their initial data through the filter before forecasting. They find strong evidence of better predictions with the hybrids when compared to their respective individual models, though the extent to which they compare to random walk is unaddressed.

While the majority of these studies is univariate in nature, a relatively small but growing body of the literature perpetuates the advantages of multivariate modelling. For example, Crespo Cuaresma, Fortin, and Hlouskova (2018) use a set of individual and composite forecasts with a wide variation of vector autoregressive (VAR) and vector error correction (VEC) techniques to include currencies and macroeconomic fundamentals for the euro against the USD, GBP, and JPY. They demonstrate the ability of an optimized portfolio using these models to outperform any benchmark portfolio based on the random walk process, although this outperformance is only significant in the short term. Also using combined forecasts, Ren, Liang, and Wang (2021) suggest that the poor predictive performance of economic models is due to the lack of information in single models and proposes this method in addition to panel modelling with fixed effects to also account for any possible country characteristics. Using a combination of eight fundamental-based methods for eleven currencies, they successfully show that their method outperforms random walks and are robust to different time horizons and variation of their sample, suggesting that information coverage improves the predictability of exchange rates and aids to reduces inconsistency and inadequacy associated with currency forecasting across the literature. While these studies consider multivariate modelling by accounting for the economic theories of exchange rates, Carriero, Kapetanios, and Marcellino (2009) exhibit the importance of accounting for dynamic co-movement in exchange rates instead. Using a panel of 33 exchange rates against the US dollar, the authors find significant evidence of increased forecasting accuracy as opposed to random walks. They underline that the latter being the strongest benchmark in the literature, it is key to consider a similar process when aiming to forecast exchange rates while taking advantage of potential aspects of co-moving currencies, leading to the motivation of this study.

Taken together, the variation in the findings reported and the applications used in the recent literature illustrate the complexity of exchange rate forecasting. Although some authors have undoubtedly overcome what is known as the toughest benchmark in the literature, that is, the random walk process, most of them fail to provide evidence that is sustained across their whole sample or other studies consistently, except those on the multivariate side. We learn from Carriero, Kapetanios, and Marcellino (2009)'s study that there are significant benefits to accounting for co-moving patterns between currencies, and yet it is impossible to find more evidence or discussion on this specific subject in the existing literature.

In the context of the research outlined above, this study aims to investigate exchange rate models for correlated exchange rate pairs, with the aim to improve short term out-of-sample forecasting for both the exchange rates. Unlike previous studies who have adopted multivariate modelling in a panel format or using aggregate currency measures to obtain predictions (see, e.g., Carriero, Kapetanios, and Marcellino 2009; Crespo Cuaresma, Fortin, and Hlouskova 2018; Ren, Liang, and Wang 2021), our study assesses the predictive ability of individual co-moving currencies. In this way, we are able to make concrete predictions on the movement of the linear and nonlinear Kalman filtering, a well-established method for reliable short-term forecasting in economics and in engineering. To our knowledge, multivariate linear or extended Kalman filtering has not been utilized for forecasting exchange rates in the literature. With the linear modelling, we contribute by introducing co-moving currencies in the dynamics, mainly to account for additional correlation information and regional as well as trade related exogenous factors that may affect the predictions. Further, we introduce the use of forward rates in the nonlinear modelling segment to observe the extent to which they can add to exchange rate forecasting.[1] Lastly, we perform the calibration and out of sample forecasting tests on several currency pairs to assess the robustness and the consistency of the methods across different countries. Table 1 outlines the contribution of our paper in the context of existing literature.

The rest of this paper is organized as follows. Section 2 describes the linear and the nonlinear methods to be used in the empirical

TABLE 1 | Highlight of this study's contribution to the existing literature.

Research paper	Uses multivariate modelling	Uses co-moving currencies	Uses Kalman filter	Predicts single spot rates	Uses forward rates for prediction
Carriero et al. (2009)	Yes	Yes	No	No	No
Narayan and Sharma (2015)	Yes	No	No	No	Yes
Crespo Cuaresma et al. (2018)	Yes	No	No	No	No
Narayan et al. (2020)	No	No	No	Yes	Yes
Sreeram and Sayed (2024)	No	No	No	Yes	No
Jaworski (2021)	No	No	No	Yes	No
Pfahler (2021)	No	No	No	Yes	No
Ren et al. (2021)	Yes	No	No	No	No
This paper	Yes	Yes	Yes	Yes	Yes

analysis. Section 3 then covers numerical implementation, including data, methodology, and empirical findings.

2 | Model Specification

This section provides a description of the empirical methods used in the paper. Several models are considered, which can be categorized into linear and nonlinear models. The two benchmark linear models are the random walk and the univariate autoregressive moving average (ARMA) models, and the proposed model to be compared with these benchmarks is the state space form of a bivariate ARMA model, implemented in the form of a Kalman filter. In the nonlinear modelling subsection, we look into the second proposed model relating to a multivariate state space model which is implemented using an extended Kalman filter.

2.1 | Linear Modelling

2.1.1 | The Random Walk Process

In the existing literature, the random walk model to date is considered as the most reliable model and the toughest benchmark to forecast currencies (Meese and Rogoff 1983). Hence, we include this method in our analysis as one of the benchmark models to compare its performance in terms of short term out-of-sample forecasting with our proposed model. By definition, the random walk process implies that the time series' forecast values is composed of the last period's values and it is expressed as follows:

$$s_t = s_{t-1} + w_t, \tag{1}$$

where s_t is the respective country's spot rate, w_t are zero mean, i.i.d. random variables, and t is the time dimension.

2.1.2 | Univariate ARMA Model

Another method that has been widely employed and highlighted in the exchange rates forecasting literature is the ARMA model. Previous authors claim this method to be an efficient forecasting tool and it has often been shown to be as reliable as the random walk process across various studies (Khashei, Rafiei, and Bijari 2013). We thus include this method as a second benchmark model in our analysis to provide further comparison of forecasting performance alongside our proposed models. In the ARMA process, the future value of a time series is defined as a linear function of a combination of past observations known as the autoregressive (AR) terms and past random errors represented by the moving average (MA) terms formulated as follows:

$$s_t = \sum_{i=1}^{n} \alpha_i s_{t-i} + \sum_{i=1}^{m} \beta_i w_{t-i} + w_t, \tag{2}$$

wher α_i, β_i are model parameters, n defines the order of autoregressive terms, and m represents the order of moving average. Our comparisons will be based on $n = m = 1$, since our numerical experience indicated that further lags do not add any forecasting ability while complicating reliable parameter estimation.

2.1.3 | Bivariate ARMA Model With the Kalman Filter

Given our interest in evaluating the predictive ability of co-moving exchange rates, our first proposed methodology relates to the application of Kalman filtering using a bivariate ARMA model. In this method, we consider two exchange rates, each driven by an ARMA(1,1) model as follows:

$$s_t^i = \alpha_i s_{t-1}^i + \beta_i w_{t-1}^i + w_t^i, \tag{3}$$

where s_t^i, $i = 1{,}2$ represent the individual spot rates and w_t^i refer to their respective noise terms with $w_t^i \sim N(0, \sigma_i)$. These noise terms are correlated and are modelled as follows:

$$w_t^1 = \sigma_1 \sqrt{1-\rho^2} v_t^1 + \rho \sigma_1 v_t^2, \tag{4}$$

$$w_t^2 = \sigma_2 v_t^2, \tag{5}$$

where the correlation between the two currencies is introduced by ρ and $v_t^i \sim N(0,1)$ i.i.d., $i = 1{,}2$. We assume that the exchange rate is observed in noise, which acts as a proxy for error potentially induced by model mis-specification. This problem can be formulated into a state space model in the following form:

$$\textbf{The transition equation: } x_t = A x_{t-1} + H v_t, \tag{6}$$

$$\textbf{The measurement equation: } y_t = C x_t + G u_t, \tag{7}$$

which, taking into account Equations (3), (4), and (5), can be expressed as

$$\overbrace{\begin{bmatrix} s_t^1 \\ s_t^2 \\ w_t^1 \\ w_t^2 \end{bmatrix}}^{x_t} = \overbrace{\begin{bmatrix} \alpha_1 & 0 & \beta_1 & 0 \\ 0 & \alpha_2 & 0 & \beta_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} s_{t-1}^1 \\ s_{t-1}^1 \\ w_{t-1}^1 \\ w_{t-1}^2 \end{bmatrix}}^{x_{t-1}} + \overbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sigma_1\sqrt{1-\rho^2} & \rho\sigma_1 \\ 0 & \sigma_2 \end{bmatrix}}^{H} \begin{bmatrix} v_t^1 \\ v_t^2 \end{bmatrix}, \tag{8}$$

$$y_t = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{C} x_t + \underbrace{\begin{bmatrix} \zeta_1 & 0 \\ 0 & \zeta_2 \end{bmatrix}}_{G} \begin{bmatrix} u_t^1 \\ u_t^2 \end{bmatrix}. \tag{9}$$

In Equations (6) and (7), x_t is the state vector, A is the state transition matrix, HH^T is the system noise covariance, C is the observation matrix, GG^T is the observation noise covariance, and $v_t^i, u_t^i \sim \mathcal{N}(0,1)$, $i = 1,2$, are independent noise sequences. ζ_i represents potential effect of model mis-specification.

The above state space model allows us to analyze the evolution of the states using the observed data, which in this case are the spot prices of two respective countries. Once in the state space form, we use the Kalman filter to proceed with the predictions. The Kalman filter, introduced by Kalman (1960), is a method following a recursive statistical algorithm on the observed data to obtain the true estimates of the hidden states (Durbin and Koopman 2012).

Typically, given information about the measurements and state estimates at time $t-1$, the Kalman filter (KF) operates in two main steps: (i) the prediction step and (ii) the correction step, as shown in Table 2. In the prediction step, the KF uses the state estimate to predict the state at time t using the model and updates it using a linear function of the measurement at time t once it becomes available. In the correction step, the filter corrects the state estimate and the covariance through a linear combination of the predicted estimate and a weighted difference between the predicted and the observed output. The weighting matrix minimizes the conditional variance of the estimate and is called the Kalman gain. The two steps are performed at each step to produce the one step ahead forecasts.

The algorithm from Table 2 requires the parameters from the state space model to be known to produce the one-step ahead predictions. From the above formulation, we treat α_1, α_2 β_1, β_2, σ_1,σ_2,ρ and ζ_i as unknown parameters grouped into a new vector

$$\theta_1 = \begin{bmatrix} \alpha_1 & \alpha_2 & \beta_1 & \beta_2 & \sigma_1 & \sigma_2 & \rho & \zeta_1 & \zeta_2 \end{bmatrix}.$$

To obtain the parameter estimates, we proceed to calibrate the model using the maximum likelihood method explained in detail in Section 2.3. First, we look at using forward rates to possibly enhance the prediction of spot exchange rates, using the Extended Kalman filter.

TABLE 2 | Implementation of the Kalman filter for the bivariate ARMA state space model.

Kalman filter algorithm
Prediction:
1. Predict state: $\hat{x}_t^- = A\hat{x}_{t-1}$
2. Predict covariance: $P_t^- = AP_{t-1}A^\top + Q$
Correction:
3. Compute Kalman gain: $K_t = P_t^- C^\top (CP_t^- C^\top + R)^{-1}$
4. Corrected conditional state: $\hat{x}_t = \hat{x}_t^- + K_t \underbrace{(y_t - C\hat{x}_t^-)}_{e_t}$
5. Corrected conditional covariance: $P_t = P_t^- + K_t C P_t^- C^\top K_t^\top + K_t R K_t^\top - P_t^- C^\top K_t^\top - K_t C P_t^-$

2.2 | Nonlinear Modelling

2.2.1 | Multivariate State Space Model With Extended Kalman Filter

To further advance our empirical analysis, we consider the potential of information carried by forward rates by introducing them alongside co-moving currencies as added observations in the spot rate prediction model. For this purpose, we propose a multivariate state space model in a nonlinear form which can be estimated with the Extended Kalman filter. This method is derived from the traditional association between spot rates and forward rates as follows[2]:

$$f_t^j = \ln(1 + d_t^j) + s_t^j, \tag{10}$$

where $d_t^j = r_t^j - q_t$ and represents the interest rate differential, q_t is the interest rate for the United States in this case, and r_t^j is emerging market interest rate for the same maturity and,

$$d_t^i = \gamma_i d_{t-1}^i + \delta_i + \eta_i \epsilon_t^i, i = 1,2. \tag{11}$$

which is an affine discrete time model for the interest rate differential, which can be arrived at by discretizing an affine stochastic differential equation for the differential spot rate, such as the Vasicek model. Here, γ_i, δ_i, and η_i are constants. With the introduction of forward rates, the initial state space Equations (8) and (9) can be modified and formulated as the following:

$$\overbrace{\begin{bmatrix} s_t^1 \\ s_t^2 \\ w_t^1 \\ w_t^2 \\ d_t^1 \\ d_t^2 \end{bmatrix}}^{x_t} = \overbrace{\begin{bmatrix} \alpha_1 & 0 & \beta_1 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & \beta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_2 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} s_{t-1}^1 \\ s_{t-1}^1 \\ w_{t-1}^1 \\ w_{t-1}^2 \\ d_{t-1}^1 \\ d_{t-1}^2 \end{bmatrix}}^{x_{t-1}} + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \delta_1 \\ \delta_2 \end{bmatrix}}^{b} \tag{12}$$

$$+ \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sigma_1\sqrt{1-\rho^2} & \rho\sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \eta_1 & 0 \\ 0 & 0 & 0 & \eta_2 \end{bmatrix}}_{\Phi} \begin{bmatrix} v_t^1 \\ v_t^2 \\ \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix}, \tag{13}$$

$$y_t = \underbrace{\begin{bmatrix} s_t^1 \\ s_t^2 \\ \ln(1+d_t^1)+s_t^1 \\ \ln(1+d_t^2)+s_t^2 \end{bmatrix}}_{\lambda} + \underbrace{\begin{bmatrix} \zeta_1 & 0 & 0 & 0 \\ 0 & \zeta_2 & 0 & 0 \\ 0 & 0 & \zeta_3 & 0 \\ 0 & 0 & 0 & \zeta_4 \end{bmatrix}}_{\Psi} \begin{bmatrix} u_t^1 \\ u_t^2 \\ u_t^3 \\ u_t^4 \end{bmatrix}, \tag{14}$$

where v_t^i, ϵ_t^i, and u_t^i are the zero mean, identity, and independent noise terms. In this case, the state vector x_t is modified to include the interest rates d_t^1 and d_t^2, and the observation vector y_t is modified to account for the forward rates with $\ln(1+d_t^1)+s_t^1$ relating to f_t^1 and $\ln(1+d_t^2)+s_t^2$ representing f_t^2.

This formulation introduces a mild nonlinearity in the procedure, which can be estimated using the extended Kalman filter (EKF) methodology. The EKF is a recursive algorithm consisting of the prediction and correction steps as in Kalman filter. The entire algorithm is shown in Table 3. In this case, a key function of the EKF is that it implements the Kalman filter for a system through the linearization of the initial nonlinear filter using Jacobian matrices at each time step (Bhaumik and Date 2019). The Jacobians are the first partial derivative of the observation matrix:

$$\Gamma_t = \frac{\partial \lambda_{i,t}}{\partial y_t},$$

where i represents each element of the corresponding vector at t time step. The time-varying matrix Γ_t replaces the time invariant matrix C in the initial Kalman filter algorithm. For the EKF, all the parameters of the model are grouped into a vector as

$$\theta_2 = \begin{bmatrix} \alpha_1 & \alpha_2 & \beta_1 & \beta_2 & \gamma_1 & \gamma_2 & \delta_1 & \delta_2 & \sigma_1 & \sigma_2 \\ \rho & \eta_1 & \eta_2 & \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 \end{bmatrix}.$$

θ_2 needs to be known to proceed with the predictions as in the EKF, which in reality has to be estimated. The same holds true for θ_1 in the KF. The procedure for parameter calibration using the maximum likelihood method is described in the next section.

TABLE 3 | Implementation of the extended Kalman filter for the multivariate state space model.

Extended Kalman filter algorithm
Prediction:
1. Predict state: $\hat{x}_t^- = A\hat{x}_{t-1} + b$
2. Predict covariance: $P_t^- = AP_{t-1}A^\top + \Phi\Phi^T$
Correction:
3. Compute the Jacobian matrix: $\Gamma_t^- = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \frac{1}{1+\hat{d}_t^{1-}} & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{1+\hat{d}_t^{1-}} \end{bmatrix}$
4. Compute Kalman gain: $K_t = P_t^-(\Gamma_t^-)^\top(\Gamma_t^- P_t^-(\Gamma_t^-)^\top + \Psi\Psi^T)^{-1}$
5. Corrected conditional state: $\hat{x}_t = \hat{x}_t^- + K_t e_t \{ \underbrace{\left(y_t - \begin{bmatrix} \widehat{s}_t^{1-} \\ \widehat{s}_t^{2-} \\ \ln(1+\widehat{d}_t^{1-})+\widehat{s}_t^{1-} \\ \ln(1+\widehat{d}_t^{2-})+\widehat{s}_t^{2-} \end{bmatrix} \right)}$
6. Corrected conditional covariance: $P_t = P_t^- + K_t\Gamma_t^- P_t^-(\Gamma_t^-)^\top K_t^\top + K_t\Psi\Psi^T K_t^\top - P_t^-(\Gamma_t^-)^\top K_t^\top - K_t(\Gamma_t^-)P_t^-$

2.3 | Parameter Calibration

In this section, we provide a description of the method to estimate all unknown parameters from time series data, which is commonly referred to as model calibration. In both the KF and the EKF based models proposed in this paper, the filtering process require the knowledge of θ_1 and θ_2 to fit the matrices A, C, H, G of the state space models in Equations (8) and (9) and $A, b, \Phi, \lambda, \Psi$ in Equations (12) and (13). They are typically obtained using the maximum likelihood method that maximizes the likelihood of observations with the log likelihood expressed as follows:

$$\log L(y;\theta) = -\frac{1}{2}\sum_{i=1}^{N}(\log|P_i| + e_i P_i^{-1} e_i), \quad (15)$$

where $\theta = \theta_1$ and $\theta = \theta_2$ for the KF and the EKF respectively, and where P_i, e_i are as defined in Tables 2 and 3. In general, parameter estimation using the maximum likelihood methodology is sensitive to the number of parameters and the choice of initial values. To simplify our experiments, we treat all model error constants, ζ_i, equal to each other as one constant. To obtain the parameter estimates, we assign various initial values to the unknown parameters of θ_1 and θ_2 and conduct experiments until we find estimates that are robust to the choice of initial values.[3]

3 | Numerical Implementation

3.1 | Data

To conduct the empirical tests in this paper, we utilize data from the Refinitiv database. Daily data of the opening prices of 18 spot rates are collected for the period of January 1, 2019, to June 30, 2023. The 18 spot rates are paired together as per their location and trading partners individually and make up to a total of 21 pairs.[4] Regarding the 1-month forward rates, we collect data for two currency pairs for the period of January 1, 2019, to December 31, 2021.[5]

All observations are split with a proportion of 80% and 20% into training datasets for calibration and into testing datasets to perform the out-of-sample forecasts respectively. In terms of the spot rates, 3 main datasets (6 datasets when split for calibration and model validation) have been constructed from the overall sample data. The different datasets, time period, and number of observations for each training and testing dataset are shown in Table 4.

First, we have dataset 1, which covers data from January 1, 2019, to December 31, 2021, and consists of 7 regional currency pairs to account for any perceived geographic or geopolitical correlation. The geographic location is particularly of interest as a pairing factor as currencies in close regional proximity are generally

TABLE 4 | Summary of data analysis and estimation.

	Dataset 1 (initial dataset)	Dataset 2	Dataset 3
Total sample period	January 2019–December 2021	January 2019–June 2023	July 2021–June 2023
In-sample period	January 2019–November 2020 (500 observations)	January 2019–December 2021 (500 observations)	July 2021–December 2022 (400 observations)
Out-of-sample period	December 2020–December 2021 (284 observations)	January 2022–June 2023 (374 observations)	January 2023–June 2023 (123 observations)
Currency pairing	Regional	Regional trading partners	Regional trading partners
Findings	Table 11	Tables 12 and 13	Tables 14 and 15

known to experience similar influences if a neighboring country's currency is impacted, suggesting that there is a degree of correlation between the currencies in each pair. Overall they relate to exchange rates for 12 emerging market economies and 2 advanced economies vis a vis the US dollar. The emerging market economy pairs are Argentina–Brazil, China–India, Czech Republic–Poland, Indonesia–Malaysia, Nigeria–South Africa, and Singapore–South Korea. The advanced economy pair is British pound sterling–euro.

The second dataset, named dataset 2, is based on the sample period of January 1, 2019, to June 30, 2023, and consists of the same 7 regional currency pairs as in dataset 1 and an additional 14 currency pairs based on trade linkages. The latter is also an important pairing factor due to how strongly currencies are influenced among trading partners. When a country's currency is negatively impacted, their trading partners often lose competitiveness and are especially targeted by speculators, causing them to also bear any related consequences. Hence, this paper considers such association to account for any trade based correlation among the countries used in this sample. In this case, we have broaden the sample to include currencies around the world, which have been paired according their leading trading partner. The 14 currency pairs are listed as follows: Argentina–China, Brazil–China, Singapore–China, Malaysia–China, South Korea–China, Indonesia–China, South Africa–Euro, Nigeria–Euro, Poland-Euro, Czech Republic–Euro, China–Euro, Switzerland–Euro, Australia–New Zealand, and Nepal–India.[6]

Lastly, we have dataset 3, which mirrors dataset 2 in terms of the currency pairs, that is, consisting of the same regional and trade pairs, but covers a slightly different sample period starting from July 1, 2021, to June 30, 2023. This particular sample data were chosen to perform further tests to ensure that our forecasts are not biased due to overlapped data. The training data have some minor overlapping when compared to dataset 2 but cover non-overlapping time period in terms of model validation, hence allowing us to provide a more robust insight.

The descriptive statistics of the overall sample data for each country are presented in Table 5. In general, all currencies are shown to be quite stable with fairly balanced minimum and maximum values when compared to their respective mean, except for Argentina. The latter is portrayed with a relatively low minimum and high maximum value, reflecting the volatility and instability that is known to be associated with Argentina's exchange rate.

3.2 | Methodology

Following the model specifications outlined in the Section 2, we evaluate and compare the predictive performance of each model: the random walk process, the univariate ARMA model, the bivariate ARMA model with the Kalman filter (refer to Section 2.1), and the multivariate state space model with the extended Kalman filter (see Section 2.2). More specifically, we compare the out-of-sample forecasts of each model using two standard performance criteria: the root mean square error (RMSE) and the mean absolute error (MAE).

The RMSE is the standard deviation of the prediction errors and is defined as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(c_i - \hat{c}_i)^2} \tag{16}$$

Additionally, the MAE is the average of all absolute errors computed as follows:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|c_i - \hat{c}_i| \tag{17}$$

In both the cases, c_i and $\hat{c}_i$, respectively, represent the true or the measured value of the exchange rate and one step ahead prediction of the exchange rate, respectively, and n is the out of sample data length. We first provide the comparison of performance of all the linear models for the different datasets in Tables 11–15 and then compare the Kalman filter estimation with that of the extended Kalman filter in Table 16. The outcomes are discussed in the next subsection.

3.3 | Empirical Findings

This section provides a discussion of all estimated models using the different datasets. Tables 6–10 provide the parameter values for the Kalman filter based models for all datasets

TABLE 5 | Summary of statistics of daily spot rates of all currencies over the whole sample period—January 1, 2019, to June 30, 2023.

	Mean	Median	Min.	Max.	St. dev.	Skewness
Argentine peso	99.90	91.68	36.86	256.70	50.90	1.16
Brazilian real	4.93	5.16	3.65	5.92	0.61	−0.77
Indian rupee	75.12	74.41	68.40	82.98	4.03	0.54
Chinese renminbi	6.77	6.78	6.31	7.32	0.26	−0.12
Singapore dollar	1.36	1.36	1.31	1.46	0.03	0.79
Malaysian ringgit	4.25	4.19	4.00	4.74	0.16	1.07
Korean won	1206	1189	1084	1445	77.42	0.89
Indonesian rupiah	14504	14355	13570	16500	506	1.09
S. African rand	15.81	15.29	13.25	19.82	1.50	0.58
Nigerian naira	383.70	380.70	305.70	763.00	61.89	1.35
Polish zloty	4.04	3.93	3.62	5.00	0.31	1.05
Czech koruna	22.68	22.62	20.74	25.98	1.05	0.69
British pound sterling	1.29	1.29	1.07	1.42	0.07	−0.21
Euro	1.12	1.12	0.96	1.23	0.06	−0.29
Swiss franc	0.95	0.94	0.88	1.02	0.04	0.18
Australian dollar	0.70	0.70	0.57	0.80	0.04	−0.01
New Zealand dollar	0.69	0.66	0.56	0.74	0.04	−0.14
Nepalese rupee	120.2	119.1	109.3	132.8	6.46	0.54

TABLE 6 | Parameter estimates using maximum likelihood method with Kalman filter—dataset 1.

		Optimal						
Parameters	**Initial**	**Pair 1**	**Pair 2**	**Pair 3**	**Pair 4**	**Pair 5**	**Pair 6**	**Pair 7**
α_1	0.3	0.306	0.378	0.329	0.300	0.304	0.317	0.331
α_2	0.4	0.389	0.399	0.457	0.325	0.480	0.461	0.338
β_1	0.6	0.645	0.618	0.536	0.688	0.532	0.531	0.638
β_2	0.5	0.498	0.522	0.547	0.516	0.580	0.561	0.569
σ_1	0.2	0.259	0.268	0.132	0.235	0.225	0.141	0.200
σ_2	0.2	0.224	0.173	0.277	0.278	0.290	0.261	0.175
ρ	0.6	0.584	0.643	0.647	0.632	0.641	0.671	0.672
ζ	10^{-6}	0	0	0	0	0	0	0

including the chosen initial values for reference and reproducibility. The models are calibrated using the procedure outlined in Section 2.3.

The results for the out-of-sample forecasts of all linear models using all datasets are presented in Tables 11–15. Regarding dataset 1, Table 11 shows that when comparing the three models altogether, that is, the proposed joint Kalman filter with two benchmark models, the ARMA(1,1) and random walk processes, we find the proposed model to consistently outperform the benchmark models across all currency pairs.[7] For the RMSE, the joint Kalman filter shows a lower value with all currencies, denoting a relatively strong improvement in the out-of-sample predictions for most currencies, with an approximate of 66% and 99% for Argentina and Brazil, 19% and 98% for India and China, 99% and 98% for Singapore and Malaysia, 95% and 99% for Korea

TABLE 7 | Parameter estimates using maximum likelihood method with Kalman filter—dataset 2 (regional).

		Optimal						
Parameters	**Initial**	**Pair 1**	**Pair 2**	**Pair 3**	**Pair 4**	**Pair 5**	**Pair 6**	**Pair 7**
α_1	0.3	0.293	0.315	0.309	0.316	0.316	0.302	0.317
α_2	0.4	0.394	0.400	0.419	0.407	0.407	0.399	0.390
β_1	0.6	0.600	0.605	0.583	0.579	0.579	0.605	0.617
β_2	0.5	0.515	0.514	0.519	0.506	0.506	0.486	0.514
σ_1	0.2	0.230	0.255	0.127	0.260	0.260	0.191	0.277
σ_2	0.2	0.201	0.188	0.279	0.236	0.236	0.243	0.140
ρ	0.6	0.612	0.643	0.609	0.600	0.600	0.654	0.617
ζ	10^{-6}	0.061	0.045	0.039	0.077	0.077	0.094	0.037

TABLE 8 | Parameter estimates using maximum likelihood method with Kalman filter—dataset 2 (trading partners).

		Optimal						
Parameters	**Initial**	**Pair 1**	**Pair 2**	**Pair 3**	**Pair 4**	**Pair 5**	**Pair 6**	**Pair 7**
α_1	0.3	0.355	0.346	0.290	0.305	0.396	0.396	0.319
α_2	0.4	0.400	0.419	0.407	0.409	0.395	0.395	0.398
β_1	0.6	0.625	0.616	0.614	0.605	0.636	0.586	0.609
β_2	0.5	0.484	0.467	0.517	0.499	0.475	0.505	0.473
σ_1	0.2	0.265	0.256	0.127	0.140	0.276	0.266	0.249
σ_2	0.2	0.140	0.136	0.277	0.275	0.244	0.244	0.164
ρ	0.6	0.603	0.626	0.637	0.605	0.602	0.602	0.649
ζ	10^{-6}	0.045	0.036	0.037	0.035	0.056	0.056	0.039
		Optimal						
Parameters	**Initial**	**Pair 8**	**Pair 9**	**Pair 10**	**Pair 11**	**Pair 12**	**Pair 13**	**Pair 14**
α_1	0.3	0.356	0.348	0.316	0.316	0.303	0.346	0.303
α_2	0.4	0.385	0.410	0.405	0.419	0.403	0.439	0.417
β_1	0.6	0.626	0.618	0.616	0.616	0.592	0.616	0.613
β_2	0.5	0.495	0.483	0.511	0.498	0.543	0.467	0.510
σ_1	0.2	0.266	0.118	0.276	0.276	0.121	0.126	0.253
σ_2	0.2	0.244	0.268	0.138	0.138	0.273	0.266	0.197
ρ	0.6	0.602	0.628	0.616	0.616	0.603	0.626	0.594
ζ	10^{-6}	0.056	0.038	0.036	0.036	0.033	0.036	0.084

and Indonesia, 94% and 97% for South Africa and Nigeria, and lastly 97% and 99% for Poland and Czech Republic. In terms of the MAE criterion, we find similar results where the errors are significantly lower across all currencies except for India. In this case the highest improvement is identified with Nigeria, China, Czech Republic, Indonesia, and Malaysia with an approximate of 71%, 47%, 46%, 46%, and 45%, respectively. Instances where the Kalman filter leads to poorer error metrics are shown in boldface in Tables 11–15.

Overall, these findings provide a strong ground in favor of the joint Kalman filter method to provide reliable out-of-sample forecasts for exchange rates, even in the cases where the exchange rates are relatively volatile over the sample period, such

TABLE 9 | Parameter estimates using maximum likelihood method with Kalman filter—dataset 3 (regional).

Parameters	Initial	Optimal						
		Pair 1	Pair 2	Pair 3	Pair 4	Pair 5	Pair 6	Pair 7
α_1	0.3	0.306	0.315	0.309	0.312	0.302	0.297	0.305
α_2	0.4	0.410	0.400	0.410	0.386	0.409	0.412	0.419
β_1	0.6	0.609	0.595	0.592	0.606	0.598	0.589	0.605
β_2	0.5	0.483	0.504	0.530	0.501	0.516	0.532	0.499
σ_1	0.2	0.231	0.245	0.125	0.180	0.232	0.128	0.265
σ_2	0.2	0.222	0.170	0.280	0.277	0.232	0.282	0.130
ρ	0.6	0.593	0.643	0.603	0.605	0.609	0.612	0.605
ζ	10^{-6}	0.080	0.045	0.040	0.067	0.072	0.042	0.035

TABLE 10 | Parameter estimates using maximum likelihood method with Kalman filter—dataset 3 (trading partners).

Parameters	Initial	Optimal						
		Pair 1	Pair 2	Pair 3	Pair 4	Pair 5	Pair 6	Pair 7
α_1	0.3	0.331	0.307	0.305	0.294	0.306	0.306	0.302
α_2	0.4	0.393	0.419	0.409	0.408	0.400	0.413	0.416
β_1	0.6	0.611	0.587	0.605	0.585	0.607	0.607	0.598
β_2	0.5	0.491	0.486	0.499	0.499	0.513	0.490	0.499
σ_1	0.2	0.271	0.237	0.140	0.215	0.253	0.243	0.222
σ_2	0.2	0.220	0.162	0.265	0.229	0.175	0.240	0.192
ρ	0.6	0.575	0.647	0.605	0.609	0.585	0.585	0.609
ζ	10^{-6}	0.061	0.037	0.035	0.049	0.093	0.093	0.072
Parameters	**Initial**	**Optimal**						
		Pair 8	Pair 9	Pair 10	Pair 11	Pair 12	Pair 13	Pair 14
α_1	0.3	0.316	0.304	0.303	0.308	0.305	0.301	0.302
α_2	0.4	0.412	0.408	0.405	0.405	0.419	0.401	0.405
β_1	0.6	0.611	0.597	0.603	0.586	0.605	0.591	0.611
β_2	0.5	0.506	0.496	0.513	0.518	0.499	0.511	0.530
σ_1	0.2	0.217	0.241	0.223	0.278	0.130	0.126	0.195
σ_2	0.2	0.130	0.131	0.178	0.119	0.265	0.261	0.180
ρ	0.6	0.605	0.611	0.593	0.608	0.605	0.591	0.597
ζ	10^{-6}	0.067	0.051	0.033	0.058	0.035	0.021	0.091

as with Argentina and Brazil. As such, our results provide significant evidence to support the inclusion of co-moving currencies to improve exchange rate forecasting, as stated by Carriero, Kapetanios, and Marcellino (2009).

This outcome is further supported by the results obtained using dataset 2 and dataset 3. The out-of sample errors for the two datasets are reported in Tables 12–15, respectively. Similarly, we compare our proposed model with the benchmark models using the RMSE and the MAE measurements, and we can see that in general, the inferences made from dataset 1 are still valid as we find the lowest error values with the Kalman filter in most cases. Overall, we assess the performance across 49 currency pairs over the 3 out-of-sample datasets (7 regional,

TABLE 11 | Comparison of out-of-sample forecasting results on daily spot rates—dataset 1.

Currency	RMSE			MAE		
	Joint Kalman filter	Single ARMA	Single random walk	Joint Kalman filter	Single ARMA	Single random walk
Pair 1						
Argentine peso	0.0531	0.1606	0.1606	0.0897	0.1168	0.1168
Brazilian real	0.0006	0.8827	0.8827	0.1164	0.1907	0.1904
Pair 2						
Indian rupee	0.1651	0.2047	0.2048	**0.2606**	0.1497	0.1497
Chinese renminbi	0.0004	0.0123	0.0123	0.0046	0.0086	0.0086
Pair 3						
Singapore dollar	0.00003	0.0030	0.0030	0.0016	0.0024	0.0024
Malaysian ringgit	0.00020	0.0087	0.0087	0.0035	0.0064	0.0064
Pair 4						
Korean won	0.2326	4.6845	4.6854	2.8732	3.7357	3.7365
Indonesian rupiah	0.3965	37.1138	37.0821	14.5916	26.7800	26.6965
Pair 5						
S. African rand	0.0016	0.1295	0.1295	0.07862	0.1040	0.1040
Nigerian naira	0.0555	1.7532	1.7532	0.07862	0.2741	0.2743
Pair 6						
Polish zloty	0.0007	0.0204	0.0204	0.0108	0.0159	0.0159
Czech koruna	0.0001	0.1023	0.1023	0.0430	0.0797	0.0796
Pair 7						
British pound sterling	0.0043	0.0057	0.0057	0.0034	0.0045	0.0045
Euro	0.0032	0.0042	0.0042	0.0025	0.0033	0.0033

21 regional and trading partners, and the same 21 regional and trading partners for a different time period) using two different error metrics (RMSE and MAE). In total, we obtain 588 error measurements (84, 252, and 252 measurements for each dataset respectively). Across all the estimated errors, the Kalman filter was found to be inferior to the benchmark models only 17 times out 588 times. More specifically, 1 out of 84 errors in dataset 1, 7 out of 252 errors in dataset 2, and 9 out of 252 errors in dataset 3 showed a poorer performance for the Kalman filter.

When doing an exchange rate prediction on a pair of countries with a strong difference in economic strength (GDP per capita, strength of currency etc.), we observed that the performance of the Kalman filter is inferior to benchmark models when predicting the stronger of the two currencies. This is found in the currency pairs of Korea–China, Indonesia–China, Nigeria–Euro, and Czech Republic–Euro. This outcome, as shown in Tables 13 and 15, indicates that any correlation with the weaker currency does not add information in prediction of the USD exchange rate of the stronger currency when there is a significant difference in the strength of the two economies, which is consistent with economic intuition.

Similarly, when currencies from more developed economies are paired together, the proposed model is able to perform better due to the potent information which seem to contribute to the prediction both currencies. This is shown with the pairs of GBP–Euro and Australian–New Zealand dollar among others, where the proposed model yielded better predictions that the benchmark models for both countries. This outcome provides a particularly interesting perspective when aiming to efficiently forecast exchange rates using correlated spot rates. It shows that information contained in currencies have a significant role to play, and aside from a reliable forecasting method, it is equally important to consider what may be more effective for a given country in order to obtain more accurate predictions.

Next, in addition to correlated spot rates, as mentioned in the previous section we take the analysis further to test whether spot rates can be better predicted by including information about the 30-day forward rates in the method. Hence in this

TABLE 12 | Comparison of out-of-sample forecasting results on daily spot rates—dataset 2 (regional).

Currency	RMSE			MAE		
	Joint Kalman filter	Single ARMA	Single random walk	Joint Kalman filter	Single ARMA	Single random walk
Pair 1						
Argentine peso	0.3078	0.5889	0.5889	0.3394	0.3978	0.3978
Brazilian real	0.0184	0.0538	0.0538	0.0384	0.0411	0.0411
Pair 2						
Indian rupee	0.0120	4.2509	4.2509	0.1402	0.3954	0.3953
Chinese renminbi	0.0007	0.0258	0.0258	0.0166	0.0177	0.0177
Pair 3						
Singapore dollar	0.0000001	0.0702	0.0702	0.0026	0.0070	0.0070
Malaysian ringgit	0.0008	0.2421	0.2421	0.0057	0.0217	0.0217
Pair 4						
Korean won	0.2062	68.9353	68.9353	4.8203	10.3247	10.3247
Indonesian rupiah	1.2734	776.1584	776.1601	26.6928	76.3433	76.3285
Pair 5						
S. African rand	0.0211	0.1611	0.1611	**0.1331**	0.1271	0.1271
Nigerian naira	0.6281	8.7720	8.7720	0.8279	1.1487	1.1487
Pair 6						
Polish zloty	0.0002	0.0394	0.0394	0.0197	0.0296	0.0296
Czech koruna	0.0007	0.1667	0.1667	0.0975	0.1278	0.1278
Pair 7						
British pound sterling	0.0002	0.0085	0.0085	0.0044	0.0064	0.0064
Euro	0.00001	0.0062	0.0062	0.0034	0.0048	0.0048

case, we compare the proposed joint Kalman filter and the extended Kalman filter and the findings are illustrated in Table 16. Table 17 lists the values of the relevant parameters. Interestingly with the inclusion of the forward rates, we find the error values to deteriorate significantly as shown with both the RMSE and the MAE values, suggesting that the 30-day forward rates tend to worsen the spot rate predictions.

The significant change is seen with both currency pairs tested, suggesting that forward rates in general do not appear to contribute to the predictions. This outcome may be due to the fact that forward rates carry information of where the market thinks the spot rates are going to be in 30 days' time, which may be significantly different from one day ahead expectation. It implies that these currencies are more likely to have instant reactions and adjustments to new information as stated by Narayan et al. (2020), who found no out-of-sample predictability with forward rates for some currencies in their sample. This is particularly relevant for exchange rates which are renowned for their fickle nature as they tend to fluctuate easily throughout the day. Hence, the findings are once again shown to be in favor of the linear joint Kalman filter, highlighting the significance and usefulness of information that are carried through in day-to-day correlating currencies to produce reliable forecasts.

Overall, what we find through our empirical analysis is that our first proposed model can significantly improve forecasting ability of exchange rates consistently for emerging market, developing and developed economies, and this is applicable even when we observe a higher level of instability in the currencies, such as with Argentina and Brazil which exhibit a higher level of volatility throughout the overall sample period as opposed to the remaining currencies. This is particularly favorable as in general predicting highly volatile currencies is known to be a challenging task, hence obtaining improved forecasts in such cases shows the reliability of the proposed model. Moreover, the fact that the latter is relatively easy to calibrate and practical also makes it superior to more complex models employed in the literature. As such, this research contributes significantly to the literature by providing an innovative and reliable method to forecast exchange rates. This information can be useful for financial investors looking to

TABLE 13 | Comparison of out-of-sample forecasting results on daily spot rates—dataset 2 (trading partners).

	RMSE			MAE		
Currency	Joint Kalman filter	Single ARMA	Single random walk	Joint Kalman filter	Single ARMA	Single random walk
Pair 1						
Argentine peso	0.2719	0.5889	0.5889	0.2992	0.3978	0.3978
Chinese renminbi	0.0096	0.0258	0.0258	0.0171	0.0177	0.0177
Pair 2						
Brazilian real	0.0011	0.0538	0.0538	0.0278	0.0411	0.0412
Chinese renminbi	0.0016	0.0258	0.0258	0.0128	0.0177	0.0177
Pair 3						
Singapore dollar	0.00002	0.0702	0.0702	0.0027	0.0070	0.0070
Chinese renminbi	0.0015	0.0258	0.0258	0.0111	0.0177	0.0177
Pair 4						
Malaysian ringgit	0.0009	0.2421	0.2421	0.0069	0.0217	0.0217
Chinese renminbi	0.0014	0.0258	0.0258	0.0110	0.0177	0.0177
Pair 5						
Korean won	0.1890	68.9352	68.9352	5.1275	10.3251	10.3247
Chinese renminbi	0.0084	0.0258	0.0258	**0.2355**	0.0177	0.0177
Pair 6						
Indonesian rupiah	1.2047	776.1584	776.1601	24.1927	76.3433	76.3285
Chinese renminbi	**0.0423**	0.0258	0.0258	**1.3873**	0.0177	0.0177
Pair 7						
S. African rand	0.0070	0.1611	0.1611	0.0906	0.1271	0.1271
Euro	0.0004	0.0062	0.0062	**0.0071**	0.0048	0.0048
Pair 8						
Nigerian naira	0.8645	8.7720	8.7719	1.0716	1.1487	1.1488
Euro	0.0032	0.0062	0.0062	**0.0096**	0.0048	0.0048
Pair 9						
Polish zloty	0.0002	0.0394	0.0394	0.0244	0.0296	0.0296
Euro	0.00004	0.0062	0.0062	0.0043	0.0048	0.0048
Pair 10						
Czech koruna	0.0008	0.1667	0.1667	0.1009	0.1278	0.1278
Euro	0.0001	0.0062	0.0062	**0.0065**	0.0048	0.0048
Pair 11						
Chinese renminbi	0.0016	0.0258	0.0258	0.0136	0.0177	0.0177
Euro	0.0001	0.0062	0.0061	0.0038	0.0048	0.0048
Pair 12						
Swiss franc	0.00002	0.0055	0.0055	0.0032	0.0041	0.0041

(Continues)

TABLE 13 | (Continued)

Currency	RMSE			MAE		
	Joint Kalman filter	Single ARMA	Single random walk	Joint Kalman filter	Single ARMA	Single random walk
Euro	0.00008	0.0062	0.0062	0.0031	0.0048	0.0048
Pair 13						
Australian dollar	0.0001	0.0055	0.0055	0.0034	0.0043	0.0043
New Zealand dollar	0.0001	0.0048	0.0048	0.0026	0.0038	0.0038
Pair 14						
Nepalese rupee	0.0241	6.7956	6.7956	0.2021	0.6179	0.6179
Indian rupee	0.0134	4.2509	4.2509	0.1256	0.3954	0.3954

TABLE 14 | Comparison of out-of-sample forecasting results on daily spot rates—dataset 3 (regional).

Currency	RMSE			MAE		
	Joint Kalman filter	Single ARMA	Single random walk	Joint Kalman filter	Single ARMA	Single random walk
Pair 1						
Argentine peso	0.4799	0.7487	0.7488	0.5155	0.5216	0.5224
Brazilian real	0.0353	0.0392	0.0392	0.0428	0.0314	0.0314
Pair 2						
Indian rupee	0.0034	0.1991	0.1992	0.1025	0.1455	0.1456
Chinese renminbi	0.0027	0.0218	0.0219	0.0135	0.0161	0.0162
Pair 3						
Singapore dollar	0.0001	0.0038	0.0038	0.0023	0.0030	0.0030
Malaysian ringgit	0.0015	0.0150	0.0150	0.0073	0.0117	0.0117
Pair 4						
Korean won	0.6503	8.1333	8.1307	4.8413	6.3417	6.3400
Indonesian rupiah	3.3070	776.1584	776.1601	30.2989	76.3433	76.3285
Pair 5						
S. African rand	0.0640	0.1609	0.1610	**0.2094**	0.1259	0.1259
Nigerian naira	1.7013	15.2647	15.2734	2.1724	3.0473	2.9959
Pair 6						
Polish zloty	0.0017	0.0394	0.0394	0.0151	0.0296	0.0296
Czech koruna	0.0023	0.1330	0.1330	0.0628	0.1001	0.1001
Pair 7						
British pound sterling	0.0003	0.0064	0.0064	0.0036	0.0051	0.0051
Euro	0.00001	0.0051	0.0051	0.0028	0.0041	0.0041

maximize their hedging potential and also paves way to the possibility of identifying the optimal hedge ratio by assessing whether information from co-moving derivatives can improve hedging effectiveness given the impact it has on exchange rate predictions. This is however a separate investigation which we leave for future research.

TABLE 15 | Comparison of out-of-sample forecasting results on daily spot rates—dataset 3 (trading partners).

	RMSE			MAE		
Currency	**Joint Kalman filter**	**Single ARMA**	**Single random walk**	**Joint Kalman filter**	**Single ARMA**	**Single random walk**
Pair 1						
Argentine peso	0.4425	0.7487	0.7488	0.4745	0.5216	0.5224
Chinese renminbi	0.0169	0.0218	0.0219	0.0245	0.0161	0.0162
Pair 2						
Brazilian real	0.0021	0.0392	0.0392	0.0223	0.0314	0.0314
Chinese renminbi	0.0027	0.0218	0.0219	0.0114	0.0161	0.0162
Pair 3						
Singapore dollar	0.00009	0.0038	0.0038	0.0023	0.0030	0.0030
Chinese renminbi	0.0025	0.0218	0.0219	0.0102	0.0161	0.0162
Pair 4						
Malaysian ringgit	0.0018	0.0150	0.0150	0.0087	0.0117	0.0117
Chinese renminbi	0.0026	0.0218	0.0219	0.0108	0.0161	0.0162
Pair 5						
Korean won	0.5175	8.1333	8.1307	5.0411	6.3417	6.3400
Chinese renminbi	**0.0350**	0.0218	0.0219	**0.3609**	0.0161	0.0162
Pair 6						
Indonesian rupiah	3.6926	776.1584	776.1601	34.2339	76.3433	76.3285
Chinese renminbi	**0.2381**	0.0218	0.0219	**2.5390**	0.0161	0.0162
Pair 7						
S. African rand	0.0107	0.1602	0.1602	0.0984	0.1251	0.1251
Euro	0.0008	0.0051	0.0051	**0.0093**	0.0041	0.0041
Pair 8						
Nigerian naira	1.8443	15.2660	15.2734	2.2657	3.0400	2.9959
Euro	**0.1353**	0.0051	0.0051	**0.1670**	0.0041	0.0041
Pair 9						
Polish zloty	0.0017	0.0394	0.0394	0.0162	0.0296	0.0296
Euro	0.0002	0.0051	0.0051	0.0041	0.0041	0.0041
Pair 10						
Czech koruna	0.0028	0.1330	0.1330	0.0727	0.1001	0.1001
Euro	0.0001	0.0051	0.0051	**0.0052**	0.0041	0.0041
Pair 11						
Chinese renminbi	0.0028	0.0215	0.0215	0.0118	0.0160	0.0160
Euro	0.0001	0.0051	0.0051	0.0034	0.0041	0.0041
Pair 12						
Swiss franc	0.0001	0.0048	0.0048	0.0029	0.0037	0.0037

(Continues)

TABLE 15 | (Continued)

Currency	RMSE			MAE		
	Joint Kalman filter	Single ARMA	Single random walk	Joint Kalman filter	Single ARMA	Single random walk
Euro	0.0001	0.0051	0.0051	0.0026	0.0041	0.0041
Pair 13						
Australian dollar	0.0002	0.0047	0.0047	0.0027	0.0037	0.0037
New Zealand dollar	0.0001	0.0043	0.0043	0.0020	0.0033	0.0033
Pair 14						
Nepalese rupee	0.0036	0.3043	0.3046	0.1745	0.2263	0.2265
Indian rupee	0.0036	0.1991	0.1991	0.1045	0.1456	0.1456

TABLE 16 | Comparison of out-of-sample forecasting results of Kalman filter with 1-month forward rates: sample results.

	RMSE		MAE	
	Joint Kalman filter	Extended Kalman filter	Joint Kalman filter	Extended Kalman filter
Pair 1				
Argentine peso	0.0531	1.681	0.0897	4.951
Brazilian real	0.0006	2.538	0.1164	7.284
Pair 3				
Singapore dollar	0.00003	1.2801	0.0016	5.1933
Malaysian ringgit	0.00020	2.3116	0.0035	8.2964

TABLE 17 | Parameter estimates using maximum likelihood method with extended Kalman filter.

Parameters	Initial	Optimal	
		Pair 1	Pair 3
α_1	0.3	0.213	0.228
α_2	0.4	0.475	0.324
β_1	0.6	0.602	0.531
β_2	0.5	0.893	0.549
γ_1	0.1	0.104	0.101
γ_2	0.3	0.207	0.248
δ_1	0.2	0.189	0.132
δ_1	0.2	0.128	0.145
σ_1	0.2	0.455	0.186
σ_2	0.2	0.112	0.176
ρ	0.6	0.453	0.604
η_1	0.3	0.134	0.312
η_2	0.1	0.119	0.118
ζ	10^{-6}	0	0

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Data Availability Statement

The data that support the findings of this study is available from the corresponding author for non-commercial use. Meta-data provided (including dates for training/validation data and the initial values for optimization) is adequate to reproduce the results, if a commercial database such as Refinitiv is accessible.

Endnotes

[1] There is a small literature on this specific subject. See, for example, Wang and Jones (2003), Narayan and Sharma (2015), and Narayan et al. (2020). In general, the findings about forward rates to forecast spot rates are mixed and even stated to result in incorrect predictions in some cases. Although among the papers cited, only Narayan et al. (2020) compare their out-of-sample forecasts to the random walk process and find significant evidence in favor of forward rates for 12 out of 16 currencies.

[2] See Jabbour (1994) for further information on this association.

[3] This procedure is conducted using the function *optim* in R.

[4] More details on each pair are provided in the next few paragraphs.

[5] This sample period is chosen based on the relevance of the time during which this research was being conducted.

[6] India–China and GBP–Euro also fall under the trade currency pairs as China is the leading trading partner for India and so is the European region for the United Kingdom. However, since both pairs are also regional and have already been used in the model in that category, they are excluded from the trading partners list to avoid repetition.

[7] We have also compared it to univariate models of higher order for two currency pairs, such as the ARMA(1,2) and ARMA(2,1), to verify the robustness of the results and the suitability of model selection. We found that the ARMA(1,1) model yields the lowest values in most cases, with extremely minor advantage for higher order models in a few cases. Hence, our proposed model is still superior as in the reported findings.

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