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Observer-Based Fuzzy PID Control for Nonlinear Systems With Degraded Measurements: Dealing With Randomly Perturbed Sampling Periods

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***Abstract*—This paper addresses the problem of observer-based fuzzy proportional-integral-derivative (PID) control for a class of nonlinear systems subject to degraded measurements and randomly perturbed sampling periods (RPSPs). In the existing results, the degraded measurements and RPSPs are handled separately, where the sampling of different sensors is usually assumed to be synchronous. In our work, a comprehensive model is built to reflect the joint effects of degraded measurements and RPSPs by using a series of stochastic variable sequences and a set of Markov processes. In this model, the sampling periods of each sensor are allowed to be diverse, time-varying and randomly perturbed, thereby fully capturing the environmental effects and device constraints. Different from the existing literature that uses proportional type controllers, an observer-based fuzzy PID controller with a modified structure is proposed which fully utilizes the system information. To overcome the difficulties of the incomplete measurement information, some auxiliary variables related to the sampling periods are introduced under which the measurement output is transformed into a form delayed with stochastic delays. Subsequently, by using the special variable separation and inequality technique, sufficient conditions are derived to ensure the exponentially ultimate boundedness of the closed-loop system in the mean-square sense. The desired gains for the observer and PID controller are obtained through the solution of an optimization problem. Lastly, the effectiveness of the developed approach is demonstrated through simulation examples.**

***Index Terms*—Takagi-Sugeno fuzzy system, observer-based control, randomly perturbed sampling periods, proportional-integral-derivative control.**

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I. Introduction

The Takagi-Sugeno (T-S) fuzzy systems, recognized for their effective handling of complex nonlinear functions, have been a focus of extensive research. These systems utilize fuzzy sets and fuzzy membership functions to "blend" multiple linear submodels, thus representing a smooth nonlinear function in the form of a T-S fuzzy model. Such an approach seamlessly integrates local linearity with global nonlinearity, resulting in a model that is not only structurally concise but also easy to understand, and this clarity benefits the analysis and synthesis of various nonlinear systems. With the evolution of fuzzy control theory, the problem of T-S fuzzy-model-based control has been tackled for an array of nonlinear systems such as networked control systems, stochastic systems, cyber-physical systems, impulsive systems, among others. The substantial volume of literature on these topics [1]–[7] stands as a testament to the ongoing interest in this field.

In recent decades, a variety of fuzzy control strategies have been developed for T-S fuzzy models to handle tasks such as stabilization, bounded control, H_∞ control, and l_2-l_∞ control. These strategies typically involve state-feedback controllers, output-feedback controllers, observer-based controllers, sliding mode controllers, and reset controllers, which are all of the proportional-type (P-type), where the control laws are highly dependent on current system information. In contrast, proportional-integral-derivative (PID) controllers utilize a blend of current information, historical data, and signal change trends. PID controllers are well known for their clear physical interpretation, high fault tolerance, and robustness [8]–[11]. For example, a novel PID control method has been proposed in [12] to deal with input saturations. Due to their effectiveness and reliability, fuzzy PID controllers have garnered special attention as they combine the advantages of both fuzzy control and PID control.

In the past few years, initial developments in T-S fuzzy PID control have emerged. For example, a switching PID controller has been proposed in [13] for networked nonlinear systems by specifically considering the scheduling of the FlexRay communication protocol. In [14], the fuzzy PID control method has been applied to a two-degree-of-freedom helicopter system with event-triggered mechanisms in order to achieve effective flight control. Other notable works in this area can be found in [15]–[18]. Within the existing literature, controllers have been developed based on either system outputs or system states (assuming they are available). While state feedback controllers offer greater design flexibility, their reliance on measuring all system states can be impractical. In contrast, observer-based controllers, which are more desired as they leverage system information more effectively, have gained particular attention. For example, in [19], the observer-based control problem has been first addressed for a flexible spacecraft by proposing an effective approach. In [20], a novel observer has been developed for boundary control targeted at robotic manipulators which has aroused much attention. Nevertheless, such schemes remain underexplored in fuzzy PID control despite their theoretical promise.

With advancements in digital communication and signal processing technology, digital controllers and estimators have become prevalent in engineering practice [21]–[28]. Accordingly, there has been an increased research interest in sampled-data systems with a substantial body of literature available [29]–[38]. In sampled-data systems, non-uniform sampling is a common occurrence, which often results from sampler faults or device capacity limitations. This non-uniformity introduces multi-rate characteristics with varying time scales, thereby posing significant challenges in system analysis and synthesis. In the literature, there are mainly two methodologies for tackling the non-uniformity, i.e., the deterministic approach [31], [39] and the stochastic approach [40], [41], where the former employs known bounds to define the range of sampling periods, and the latter characterizes the sampling process through probability distributions.

Compared to its deterministic counterpart, the stochastic approach has been increasingly favored for modeling aperiodic sampling where the occurrences are probabilistic. This approach's effectiveness is highlighted in instances such as [41], where a Markov chain has been used to characterize the randomly perturbed sampling periods (RPSPs). It has been shown that the model of RPSPs captures the time-varying, stochastic and uncertain features of perturbed sampling, which is effective for describing a wealth of aperiodic sampling processes such as paper machine systems and brushless DC servo systems. The existence of RPSPs makes the traditional digital control method no longer applicable as it introduces multi-time scales, especially for multi-sensor systems with diverse sampling mechanisms.

Since the original work [41], RPSPs have generated extensive research interest. For example, the results in [41] have been extended to neural networks in [40] by incorporating partly unknown transition probabilities. Further illustrating this approach, the secure sliding control problem has been addressed in [42] for linear systems. In [43], the fault detection problem has been considered for multi-rate systems subject to RPSPs, dynamic quantization, and missing measurements. In [44], the T-S fuzzy model has been used to deal with the RPSPs, where the measurements of all sensors have been assumed to be sampled simultaneously. In a recent work [45], the authors have discussed the non-uniformly sampled neural networks with inaccessible sampling intervals where the hidden information has been utilized to design estimators. The exploration of the effects of RPSPs on digital controller design, as evidenced by these studies, is not only of theoretical interest but also of practical significance, underlining one of the key motivations in this field.

The phenomenon of measurement degradation has recently become a focal point in research owing to causes like sensor aging, device failures, and channel fadings. This degradation typically manifests as a random decrease in the amplitude of sensed signals, which leads to a shortfall in system information. In response to this challenge, a substantial amount of research has been conducted on controller/estimator design under conditions of degraded measurements. Examples of this research include studies on fading-induced degradation [46]–[48] and fault-induced degradation [49], [50]. In most of literature, the sensors have been assumed to be sampled simultaneously. For the case of sensors with different sampling periods, it is rather complex to establish a unified model describing both sampling uncertainties and amplitude degradation. In this case, the RPSPs of sensors would introduce different time scales and make the system to be stochastic, thus bringing new challenges and difficulties in the system analysis. Despite these advancements, there appears to be a lack of investigation into the bounded PID control problem for general nonlinear systems simultaneously experiencing RPSPs and measurement degradation, and the main purpose of this paper is to shorten such a gap.

In summarizing the discussions thus far, our focus is on the observer-based fuzzy PID control problem for nonlinear systems subjected to RPSPs and degraded measurements. Addressing this control design involves several challenges: 1) the construction of an appropriate model that encapsulates the effects of both RPSPs and degraded measurements in a unified framework; 2) the analysis of system performance under the constraints of RPSPs, measurement degradation, and unknown-but-bounded (UBB) external noises; and 3) the development of a controller design method that ensures the exponentially ultimate boundedness of the closed-loop system in the mean-square sense.

In response to the identified challenges, the key contributions of our study are summarized as follows.

1) A unified framework is built for dealing with the new control problem subject to RPSPs, sensor degradation, and transmission noises. Compared with the existing works (such as [39], [43], [44]) where sensors are assumed to have the same sampling period, the sampling periods of sensors in this paper are allowed to be different. Thus, the considered problem is more general which can reflect more complex engineering phenomena with general nonlinearities and sampling uncertainties.

2) An observer-based fuzzy PID controller is proposed which utilizes incomplete measurements and can deal with immeasurable premise variables. The designed controller has a flexible structure which is more general than the widely-used observer-based output feedback controller [51], [52].

3) By resorting to the special inequality technique, the

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controller gains are solved in terms of certain strict linear matrix inequalities (LMIs). The proposed method doesn't need extra rank assumptions on input matrices [53] and avoids the multiple optimization process in the cone complementarity linearization approach [51].

The remainder of this article is structured as follows. Section II formulates the control problem, introducing the discrete-time T-S fuzzy model with multiple sensors, the characterization of degraded sensors with RPSPs, the observer-based fuzzy PID controller, and the performance index of boundedness. Section III presents two theorems dedicated to analyzing system performance and designing controller gains, where an algorithm is provided to explain the implementation details of the proposed method. In Section IV, a simulation example is provided, accompanied by a comparative analysis to demonstrate the efficacy of the proposed control method. Finally, conclusions are drawn in Section V.

Notations

$\mathbb{R}^o$	The o-dimensional Euclidean space
A^T	The transposition of a matrix A
A^{-1}	The inverse of a matrix A
$\text{diag}\{\cdots\}$	A diagonal matrix
$\lambda_{\min}(A)$	The minimum eigenvalue of a matrix A
$A-B<0$	Matrix $A-B$ is negative definite
$\lvert a\rvert$	The absolute value of a scalar a
$\mathbb{E}\{\alpha\}$	The mathematical expectation of a stochastic variable α
$\mathbf{Pr}\{U\}$	The occurrence probability of an event U
$\delta(b)$	The delta function that equals 1 if $b=0$ and equals 0 otherwise

II. Problem Statement and Preliminaries

A. Nonlinear Plant

Consider a type of nonlinear plant modeled by the following discrete-time T-S fuzzy system:

$$\begin{cases} x(T_{k+1}) = \sum_{n=1}^{\bar{n}} \eta_n(\rho(T_k))\big(\mathcal{A}_n x(T_k) + \mathcal{B}_n u(T_k) \\ \qquad + \mathcal{E}_n \omega(T_k)\big) \\ z(T_k) = \sum_{n=1}^{\bar{n}} \eta_n(\rho(T_k))\mathcal{M}_n x(T_k) \end{cases} \tag{1}$$

where $0 = T_0 < T_1 < \cdots < T_k < \cdots$ is the time instant sequence of the system state updating; $n \in \{1, 2, \cdots, \bar{n}\}$ and $\bar{n}$ is a known integer referring to the total number of fuzzy rules; $T_{k+1} - T_k = \epsilon$ is the updating interval with $\epsilon > 0$ being a known scalar; $x(T_k) \in \mathbb{R}^{o_x}$ is the internal system state; $z(T_k) \in \mathbb{R}^{o_z}$ is the variable controlled to satisfy certain performance requirements; $\omega(T_k) \in \mathbb{R}^{o_\omega}$ is the noise which satisfies $\omega^T(T_k)\omega(T_k) \le \bar{\omega}$ with $\bar{\omega} > 0$ being a known scalar; $u(T_k) \in \mathbb{R}^{o_u}$ is the control signal received by the actuator; the matrices $\mathcal{A}_n$, $\mathcal{B}_n$, $\mathcal{E}_n$ and $\mathcal{M}_n$ are known system parameters with proper dimensions; $\rho(T_k)$ is the premise variable vector and is assumed to be a function of the immeasurable system states; and $\eta_n(\rho(T_k))$ is the membership function with the following properties:

$$\sum_{n=1}^{\bar{n}} \eta_n(\rho(T_k)) = 1, \quad \eta_n(\rho(T_k)) \ge 0, \quad n = 1, 2, \cdots, \bar{n}.$$

Define the sampling instant sequence of the i-th ($i \in \{1, 2, \cdots, o_y\}$) sensor as $0 = s_i^{(0)} < s_i^{(1)} < \cdots < s_i^{(\tau)} < \cdots$ where the superscript τ means the τ-th sampling instant and o_y is the total number of sensors. By considering the sensor degradation phenomenon, the output of the i-th sensor is modeled by

$$y_i\left(s_i^{(\tau)}\right) = \beta_i\left(s_i^{(\tau)}\right)\mathcal{C}_i x\left(s_i^{(\tau)}\right) + \mathcal{D}_i \omega\left(s_i^{(\tau)}\right) \tag{2}$$

where $\mathcal{C}_i$ and $\mathcal{D}_i$ are known real matrices and $\beta_i(\cdot)$ is an independent stochastic variable sequence taking values in $[0,1]$ with

$$\mathbb{E}\left\{\beta_i\left(s_i^{(\tau)}\right)\right\} = \bar{\beta}_i,$$
$$\mathbb{E}\left\{\left[\beta_i\left(s_i^{(\tau)}\right) - \bar{\beta}_i\right]\left[\beta_i\left(s_i^{(\tau)}\right) - \bar{\beta}_i\right]\right\} = \beta_i^*$$

where the known scalar $\bar{\beta}_i > 0$ is the mathematical expectation and $\beta_i^* \ge 0$ is the variance.

As discussed in the Introduction, the sensors are subject to RPSPs. To be more specific, the sampling periods of sensors are considered to be time-varying, random and integer multiple of ϵ, that is, $s_i^{(\tau+1)} - s_i^{(\tau)} = p_i^{(\tau)}\epsilon$. In this paper, we assume that $p_i^{(\tau)}$ takes values in a finite set $\{1, 2, \cdots, \bar{p}_i\}$ where $\bar{p}_i$ is a known positive scalar and $\bar{p}_i\epsilon$ reflects the maximum sampling interval of the i-th sensor.

We define an auxiliary variable related to the sampling interval:

$$\alpha_i(T_k) \triangleq \frac{T_k - s_i^{(\tau)}}{\epsilon}, \quad s_i^{(\tau)} \le T_k < s_i^{(\tau+1)}. \tag{3}$$

Here, $\alpha_i(T_k)$ reflects the time length from the current instant T_k to the last sampling instant $s_i^{(\tau)}$ of the i-th sensor. It is easy to see that $\alpha_i(T_k) \in \{0, 1, 2, \cdots, \bar{p}_i - 1\}$. Then, the following assumption is given to capture the time-varying and random features of sensor sampling periods.

Assumption 1: [42] $\{\alpha_i(T_k),\ T_k \ge 0\}$ is a Markov process that takes values in the finite set $\mathbb{P}_i \triangleq \{0, 1, 2, \cdots, \bar{p}_i - 1\}$ and is independent of $\beta_i(\cdot)$ ($i \in \{1, 2, \cdots, o_y\}$). The corresponding transition probability matrix is $\Theta_i \triangleq [\theta^{(i)}_{a_i+1, b_i+1}]_{\bar{p}_i \times \bar{p}_i}$ with the conditional transition probability given by

$$\mathbf{Pr}\{\alpha_i(T_{k+1}) = b_i | \alpha_i(T_k) = a_i\} \triangleq \theta^{(i)}_{a_i+1, b_i+1},$$
$$\forall a_i, b_i \in \mathbb{P}_i.$$

For $s_i^{(\tau)} \le T_k < s_i^{(\tau+1)}$, one can easily obtain that $s_i^{(\tau)} < T_{k+1} \le s_i^{(\tau+1)}$. Then, the value of $\alpha_i(T_{k+1})$ is calculated in the following two cases.

Case 1: The data is unsuccessfully sampled at T_{k+1}, i.e., $s_i^{(\tau)} < T_{k+1} < s_i^{(\tau+1)}$. It follows from the definition of $\alpha_i(T_k)$ in (3) that

$$\alpha_i(T_{k+1}) = \frac{T_{k+1} - s_i^{(\tau)}}{\epsilon} = \frac{T_k + \epsilon - s_i^{(\tau)}}{\epsilon} = \alpha_i(T_k) + 1. \tag{4}$$

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Case 2: The data is successfully sampled at T_{k+1}, i.e., $T_{k+1} = s_i^{(\tau+1)}$. In this case, one has

$$\alpha_i(T_{k+1}) = \frac{T_{k+1} - s_i^{(\tau+1)}}{\epsilon} = 0. \tag{5}$$

By taking **Case 1** and **Case 2** into account, for $\alpha_i(T_k) = a_i$ ($a_i \in \mathbb{P}_i$), one obtains

$$\alpha_i(T_{k+1}) = \begin{cases} a_i + 1, & s_i^{(\tau)} < T_{k+1} < s_i^{(\tau+1)} \\ 0, & T_{k+1} = s_i^{(\tau+1)}. \end{cases} \tag{6}$$

Based on the above analysis, the specific form of the transition probability for $\alpha_i(T_k) \in \{0, 1, \cdots, \bar{p}_i - 2\}$ is given as follows:

$$\begin{aligned} \mathbf{Pr}\{\alpha_i(T_{k+1}) = 0 | \alpha_i(T_k) = a_i\} &\triangleq \theta^{(i)}_{a_i+1,1}, \\ \mathbf{Pr}\{\alpha_i(T_{k+1}) = a_i + 1 | \alpha_i(T_k) = a_i\} &\triangleq \theta^{(i)}_{a_i+1,a_i+2}. \end{aligned}$$

It is easy to see

$$\theta^{(i)}_{a_i+1,a_i+2} = 1 - \theta^{(i)}_{a_i+1,1}. \tag{7}$$

For $\alpha_i(T_k) = \bar{p}_i - 1$, it is calculated that $\alpha_i(T_{k+1}) = 0$ under the constraint of the maximum sampling interval, which implies

$$\mathbf{Pr}\{\alpha_i(T_{k+1}) = 0 | \alpha_i(T_k) = \bar{p}_i - 1\} \triangleq \theta^{(i)}_{\bar{p}_i,1} = 1. \tag{8}$$

Thus, the transition probability matrix of $\alpha_i(T_k)$ is written as follows:

$$\Theta_i \triangleq \begin{bmatrix} \theta^{(i)}_{1,1} & 1 - \theta^{(i)}_{1,1} & 0 & \cdots & 0 \\ \theta^{(i)}_{2,1} & 0 & 1 - \theta^{(i)}_{2,1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta^{(i)}_{\bar{p}_i-1,1} & 0 & 0 & \cdots & 1 - \theta^{(i)}_{\bar{p}_i-1,1} \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Due to the potential variation in the sampling periods across different sensors, a total of o_y ($o_y > 1$) Markovian chains are utilized to characterize the sensor sampling. To aid in the subsequent system description and analysis, a new variable $\alpha(T_k)$ is introduced. Along with this introduction, a lemma is provided to establish a one-to-one mapping between $\alpha(T_k)$ and $\alpha_i(T_k)$ $(i = 1, 2, \cdots, o_y)$.

Lemma 1: [42] The multiple Markovian chains $\alpha_i(T_k)$ can be mapped into a new Markovian chain $\alpha(T_k)$ $(\alpha(T_k) \in \mathbb{P} \triangleq \{0, 1, 2, \cdots, \bar{p}\}$ where $\bar{p} \triangleq \prod_{i=1}^{o_y} \bar{p}_i - 1)$ according to the mapping rule $f(\cdot)$:

$$\begin{aligned} \alpha(T_k) &= f\left(\alpha_1(T_k), \alpha_2(T_k), \cdots, \alpha_{o_y}(T_k)\right) \\ &= \alpha_1(T_k) + \sum_{i=2}^{o_y} \left(\alpha_i(T_k) \prod_{t=1}^{i-1} \bar{p}_t \right). \end{aligned}$$

Furthermore, if the value of $\alpha(T_k)$ is known, then $\alpha_i(T_k)$ can be calculated based on the mapping rule $\phi_i(\cdot)$ that

$$\begin{cases} \alpha_{o_y}(T_k) = \left\lfloor \dfrac{\alpha(T_k)}{\prod_{i=1}^{o_y-1} \bar{p}_i} \right\rfloor \triangleq \phi_{o_y}(\alpha(T_k)) \\ \quad \vdots \\ \alpha_i(T_k) = \left\lfloor \dfrac{\alpha(T_k) - \sum_{l=i+1}^{o_y} \left(\alpha_l(T_k) \prod_{t=1}^{l-1} \bar{p}_t \right)}{\prod_{t=1}^{i-1} \bar{p}_t} \right\rfloor \\ \qquad \triangleq \phi_i(\alpha(T_k)), \quad 1 < i < o_y \\ \alpha_1(T_k) = \alpha(T_k) - \displaystyle\sum_{i=2}^{o_y} \left(\alpha_i(T_k) \prod_{t=1}^{i-1} \bar{p}_t \right) \triangleq \phi_1(\alpha(T_k)). \end{cases}$$

In terms of the definition of $\phi_i(\alpha(T_k))$ $(i = 1, 2, \cdots, o_y)$, the transition probability of stochastic variable $\alpha(T_k)$ is given as follows:

$$\begin{aligned} \theta_{a+1,b+1} &\triangleq \mathbf{Pr}\left\{\alpha(T_{k+1}) = b | \alpha(T_k) = a\right\} \\ &= \prod_{i=1}^{o_y} \mathbf{Pr}\left\{\alpha_i(T_{k+1}) = \phi_i(b) | \alpha_i(T_k) = \phi_i(a)\right\} \\ &= \prod_{i=1}^{o_y} \theta^{(i)}_{\phi_i(a)+1, \phi_i(b)+1}. \end{aligned}$$

B. Signal Transmissions

After introducing the considered system model, this subsection describes the complex transmission process. As shown in Fig. 1, the degraded measurements of sensors are transmitted to the controller via a noise-affected channel. Since the sampling interval of sensors is randomly perturbed, the transmissions from sensors to the controller would be intermittent. To facilitate the generation of the desired control laws, the zero-order holder (ZOH) is applied such that the last measurements can be held until the new data comes.

Define $\bar{y}_i(T_k) \in \mathbb{R}$ as the output of the ZOH related to the i-th sensor, one has

$$\bar{y}_i(T_k) = y_i\left(s_i^{(\tau)}\right) + v_i\left(s_i^{(\tau)}\right), \quad s_i^{(\tau)} \le T_k < s_i^{(\tau+1)} \tag{9}$$

where $v_i(\cdot) \in \mathbb{R}$ denotes the bounded transmission noise which satisfies $v_i^2(\cdot) \le \bar{v}_i$ with $\bar{v}_i > 0$ being a known scalar.

By recalling the definition of $\alpha_i(T_k)$, one derives that

$$\begin{aligned} \bar{y}_i(T_k) &= y_i\left(s_i^{(\tau)}\right) + v_i\left(s_i^{(\tau)}\right) \\ &= y_i\left(T_k - \alpha_i(T_k)\epsilon\right) + v_i\left(T_k - \alpha_i(T_k)\epsilon\right) \\ &= \beta_i\left(T_k - \alpha_i(T_k)\epsilon\right) \mathcal{C}_i x\left(T_k - \alpha_i(T_k)\epsilon\right) \\ &\quad + \mathcal{D}_i \omega\left(T_k - \alpha_i(T_k)\epsilon\right) + v_i\left(T_k - \alpha_i(T_k)\epsilon\right). \end{aligned} \tag{10}$$

Defining the overall output of the ZOH as $\bar{y}(T_k) \triangleq \begin{bmatrix} \bar{y}_1(T_k) & \bar{y}_2(T_k) & \cdots & \bar{y}_{o_y}(T_k) \end{bmatrix}^T$, one further obtains

$$\begin{aligned} \bar{y}(T_k) = &\sum_{i=1}^{o_y} \beta_i\left(T_k - \alpha_i(T_k)\epsilon\right) C_i x\left(T_k - \alpha_i(T_k)\epsilon\right) \\ &+ D\breve{\omega}\left(T_k - \alpha(T_k)\epsilon\right) + \breve{v}\left(T_k - \alpha(T_k)\epsilon\right) \end{aligned} \tag{11}$$

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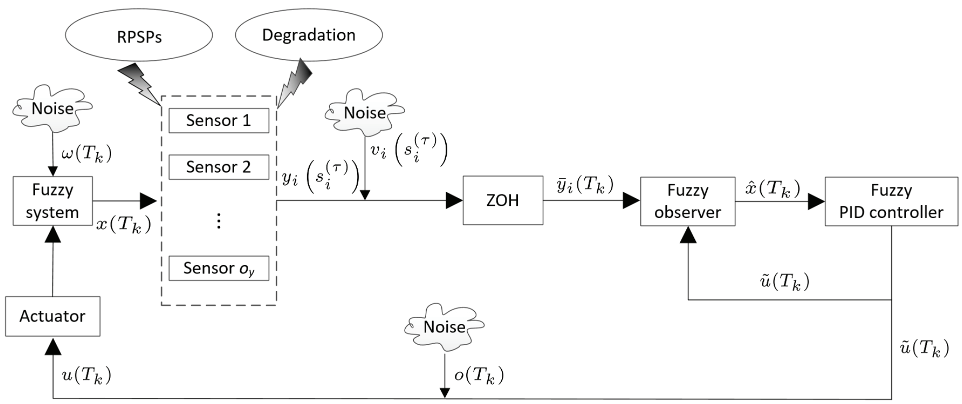

Fig. 1: Observer-based fuzzy PID control system with incomplete measurement information

where

$$C_i \triangleq \begin{bmatrix} 0 & \cdots & 0 & \underbrace{\mathcal{C}_i^T}_{\text{the } 1\times i \text{ th block}} & 0 & \cdots & 0 \end{bmatrix}^T,$$

$$D \triangleq \text{diag}\left\{\mathcal{D}_1, \mathcal{D}_2, \cdots, \mathcal{D}_{o_y}\right\},$$

$$\breve{\omega}(T_k - \alpha(T_k)\epsilon) \triangleq \begin{bmatrix} \omega\left(T_k - \alpha_1(T_k)\epsilon\right) \\ \omega\left(T_k - \alpha_2(T_k)\epsilon\right) \\ \vdots \\ \omega\left(T_k - \alpha_{o_y}(T_k)\epsilon\right) \end{bmatrix},$$

$$\breve{v}(T_k - \alpha(T_k)\epsilon) \triangleq \begin{bmatrix} v_1\left(T_k - \alpha_1(T_k)\epsilon\right) \\ v_2\left(T_k - \alpha_2(T_k)\epsilon\right) \\ \vdots \\ v_{o_y}\left(T_k - \alpha_{o_y}(T_k)\epsilon\right) \end{bmatrix}.$$

Remark 1: An auxiliary variable $\alpha_i(T_k)$ is defined in (3) to reflect the sampling results of each sensor node. Note that $\alpha_i(T_k)$ is well defined which includes all considered sampling cases, where the normal case (i.e., successive sampling) is denoted by $\alpha_i(T_k) = 0$. We first calculate the transition probability matrix of $\alpha_i(T_k)$, and then rewrite the degraded outputs with RPSPs as a form of distributed stochastic delays. The introduction of $\alpha_i(T_k)$ facilitates the description of degraded outputs, and contributes to the application of the mature Markov jump system technique and delay methods, thereby overcoming the challenges caused by the incomplete information.

C. Observer-Based Fuzzy PID Controller

By utilizing noise-affected degradation measurements, we adopt the following observer-based fuzzy PID controller:

$$\begin{cases} \hat{x}(T_{k+1}) = \sum_{m=1}^{\bar{n}} \eta_m(\hat{\rho}(T_k))\Big(\mathcal{A}_m\hat{x}(T_k) + \mathcal{B}_m\tilde{u}(T_k) \\ \qquad + L_m\left(\bar{y}(T_k) - \Lambda\mathcal{C}\hat{x}(T_k)\right)\Big) \\ \tilde{u}(T_k) = \sum_{\pi=1}^{\bar{n}} \eta_\pi(\hat{\rho}(T_k))\Bigg(K_\pi^P\hat{x}(T_k) + K_\pi^I\sum_{\sigma=1}^{\bar{\sigma}}\hat{x}(T_{k-\sigma}) \\ \qquad + K_\pi^D\left(\hat{x}(T_k) - \hat{x}(T_{k-1})\right)\Bigg) \end{cases} \tag{12}$$

where L_m is the observer gain; K_π^P, K_π^I and K_π^D are proportional, integral and derivative gains, respectively; $\bar{\sigma} > 0$ is a given integer representing the data length used in the integral-loop; $\hat{\rho}(T_k)$ is the estimated premise variable vector that would be different with $\rho(T_k)$; $\hat{x}(T_k)$ is the estimate of $x(T_k)$; $\tilde{u}(T_k)$ is the control law generated by the PID controller; and

$$\mathcal{C} \triangleq \begin{bmatrix} \mathcal{C}_1^T & \mathcal{C}_2^T & \cdots & \mathcal{C}_{o_y}^T \end{bmatrix}^T,$$

$$\Lambda \triangleq \text{diag}\left\{\bar{\beta}_1,\ \bar{\beta}_2,\ \cdots,\ \bar{\beta}_{o_y}\right\}.$$

Remark 2: The advantages and features of the designed controller are summarized as follows. 1) The RPSPs would lead to discontinuous signal transmissions, and it is rather difficult to obtain the accurate value of $\alpha_i(T_k)$ timely by the noise-affected measurements. Thus, the controller is designed to be independent of the sampling modes $\alpha_i(T_k)$; 2) Different from the traditional PID controller, the integral term of the designed controller is based on past information with finite length. Such a modified structure could reduce computational burden and weaken the side effects of accumulated errors; 3) By utilizing the observed states, the controller has more design degrees of freedom compared with the output-based controller, and is thus more flexible; and 4) Since our attention is paid to the bounded control problem, the reference signal of the PID controller is set to be zero [13].

After the control signal is generated, it will be transmitted to the actuator which would be disturbed by the channel noise as illustrated in Fig. 1. The available signal for the actuator is modeled by

$$u(T_k) = \tilde{u}(T_k) + o(T_k) \tag{13}$$

where $o(T_k) \in \mathbb{R}^{o_u}$ denotes the bounded channel noise satisfying $o^T(T_k)o(T_k) \leq \bar{o}$ with the known upper bound $\bar{o} > 0$.

Define the state estimation error and the augmentation state vector, respectively, as follows

$$e(T_k) \triangleq x(T_k) - \hat{x}(T_k), \quad \zeta(T_k) \triangleq \begin{bmatrix} x(T_k) \\ e(T_k) \end{bmatrix}.$$

By considering (1), (11), (12) and (13) simultaneously and defining $k \triangleq T_k$, $\alpha \triangleq \alpha(T_k)$ and $\alpha_i \triangleq \alpha_i(T_k)$ for notation

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convenience, we derive the following closed-loop fuzzy system:

$$\begin{aligned}\zeta(k+1) = &\sum_{n=1}^{\bar{n}}\sum_{m=1}^{\bar{n}}\sum_{\pi=1}^{\bar{n}}\eta_n(\rho(k))\eta_m(\hat{\rho}(k))\eta_\pi(\hat{\rho}(k))\\ &\times\Big(A_{n,m,\pi}\zeta(k)+B_{n,m,\pi}\zeta(k-1)+\bar{B}_{n,m,\pi}\\ &\times\sum_{\sigma=2}^{\bar{\sigma}}\zeta(k-\sigma)+\sum_{i=1}^{o_y}\Big(\bar{L}_{i,m}+\tilde{\beta}_i(k-\alpha_i)\\ &\times\tilde{L}_{i,m}\Big)\zeta(k-\alpha_i)+\bar{D}_m\breve{\omega}(k-\alpha)+\bar{E}_n\omega(k)\\ &+\tilde{I}_m\breve{v}(k-\alpha)+\vec{B}_n o(k)\Big)\end{aligned}\tag{14}$$

and

$$z(k)=\sum_{n=1}^{\bar{n}}\eta_n(\rho(k))M_n\zeta(k)\tag{15}$$

where

$$\begin{aligned}
A_{n,m,\pi} &\triangleq \begin{bmatrix} A^{(1,1)}_{n,\pi} & A^{(1,2)}_{n,\pi}\\ A^{(2,1)}_{n,m,\pi} & A^{(2,2)}_{n,m,\pi}\end{bmatrix},\\
B_{n,m,\pi} &\triangleq \begin{bmatrix} B^{(1,1)}_{n,\pi} & B^{(1,2)}_{n,\pi}\\ B^{(2,1)}_{n,m,\pi} & B^{(2,2)}_{n,m,\pi}\end{bmatrix},\\
A^{(1,1)}_{n,\pi} &\triangleq \mathcal{A}_n+\mathcal{B}_nK^P_\pi+\mathcal{B}_nK^D_\pi,\\
A^{(1,2)}_{n,\pi} &\triangleq -\mathcal{B}_nK^P_\pi-\mathcal{B}_nK^D_\pi,\\
A^{(2,1)}_{n,m,\pi} &\triangleq \mathcal{A}_n+\mathcal{B}_nK^P_\pi+\mathcal{B}_nK^D_\pi-\mathcal{A}_m-\mathcal{B}_mK^P_\pi\\
&\quad -\mathcal{B}_mK^D_\pi+L_m\Lambda C,\\
A^{(2,2)}_{n,m,\pi} &\triangleq \mathcal{A}_m+\mathcal{B}_mK^P_\pi+\mathcal{B}_mK^D_\pi-L_m\Lambda C\\
&\quad -\mathcal{B}_nK^P_\pi-\mathcal{B}_nK^D_\pi,\\
B^{(1,1)}_{n,\pi} &\triangleq \mathcal{B}_nK^I_\pi-\mathcal{B}_nK^D_\pi,\ B^{(1,2)}_{n,\pi}\triangleq\mathcal{B}_nK^D_\pi-\mathcal{B}_nK^I_\pi,\\
B^{(2,1)}_{n,m,\pi} &\triangleq \mathcal{B}_nK^I_\pi-\mathcal{B}_nK^D_\pi-\mathcal{B}_mK^I_\pi+\mathcal{B}_mK^D_\pi,\\
B^{(2,2)}_{n,m,\pi} &\triangleq \mathcal{B}_mK^I_\pi-\mathcal{B}_mK^D_\pi-\mathcal{B}_nK^I_\pi+\mathcal{B}_nK^D_\pi,\\
\bar{B}_{n,m,\pi} &\triangleq \begin{bmatrix}\mathcal{B}_nK^I_\pi & -\mathcal{B}_nK^I_\pi\\ \mathcal{B}_nK^I_\pi-\mathcal{B}_mK^I_\pi & \mathcal{B}_mK^I_\pi-\mathcal{B}_nK^I_\pi\end{bmatrix},\\
\bar{L}_{i,m} &\triangleq \begin{bmatrix}0 & 0\\ -\bar{\beta}_iL_mC_i & 0\end{bmatrix},\ \tilde{L}_{i,m}\triangleq\begin{bmatrix}0 & 0\\ -L_mC_i & 0\end{bmatrix},\\
\bar{D}_m &\triangleq \begin{bmatrix}0\\ -L_mD\end{bmatrix},\ \tilde{I}_m\triangleq\begin{bmatrix}0\\ -L_m\end{bmatrix},\ M_n\triangleq\begin{bmatrix}\mathcal{M}_n & 0\end{bmatrix},\\
\bar{E}_n &\triangleq \begin{bmatrix}E_n\\ E_n\end{bmatrix},\quad \vec{B}_n\triangleq\begin{bmatrix}B_n\\ B_n\end{bmatrix},\quad \tilde{\beta}_i(k)\triangleq\beta_i(k)-\bar{\beta}_i.
\end{aligned}$$

Definition 1: [54] The augmentation system (14) is said to be exponentially ultimately bounded in the mean-square sense if, there are scalars $0\le\mu_1<1$, $\mu_2>0$ and $\mu_3>0$ such that the following inequality holds:

$$\mathbb{E}\left\{\zeta^T(k)\zeta(k)\right\}\le\mu_1^k\mu_2+\mu_3\tag{16}$$

where μ_3 is called the asymptotic upper bound of $\mathbb{E}\{\zeta^T(k)\zeta(k)\}$.

The aim of this work is to obtain conditions that guarantee mean-square boundedness of system (14) in terms of Definition 1, and then design the observer-based T-S fuzzy PID controller to minimize the asymptotic upper bound of the controlled output $\mathbb{E}\{z^T(k)z(k)\}$. The main theoretical results will be given in the next section.

III. Main Results

In this section, the conditions that guarantee mean-square boundedness of the closed-loop system are presented based on the stochastic analysis technique.

In the following theorem, we discuss the effects of degraded measurements and the perturbed sampling on system performance.

Theorem 1: Consider the closed-loop system (14) with the observer-based PID controller (12). Assume that parameters K^P_π, K^I_π, K^D_π, L_m ($\pi,m\in\{1,2,\cdots,\bar{n}\}$) and a scalar $\kappa\in(0,1)$ are given. Then, the closed-loop system (14) is exponentially ultimately bounded in the mean-square if, for $n,m,\pi\in\{1,2,\cdots,\bar{n}\}$, $l\in\{1,2,\cdots,\bar{\sigma}\}$, $a\in\mathbb{P}$ and $c\in\{1,2\cdots,\bar{p}\}$, there exist matrices $P_a>0$, $Q_{c,a}>0$, $X_l>0$ and a scalar $h>0$ such that

$$\left(\Gamma^{n,m}_{\pi,a}\right)^T\bar{\mathcal{P}}_a\Gamma^{n,m}_{\pi,a}+\Pi_a<0\tag{17}$$

where

$$\begin{aligned}
\bar{Q}_{c,a} &\triangleq \sum_{j=0}^{\bar{p}}\theta_{a+1,j+1}Q_{c,j},\\
\bar{X} &\triangleq \mathrm{diag}\left\{\kappa X_1,\kappa^2X_2,\cdots,\kappa^{\bar{\sigma}}X_{\bar{\sigma}}\right\},\\
\tilde{Q}_a &\triangleq \mathrm{diag}\left\{\bar{Q}_{2,a}-\kappa Q_{1,a},\cdots,-\kappa Q_{\bar{p},a}\right\},\\
\Gamma^{n,m}_{\pi,a} &\triangleq \begin{bmatrix}\bar{A}^{n,m}_{\pi,a} & \tilde{B}_{n,m,\pi} & \acute{L}_{m,a} & \vec{E}_{n,m}\\ 0 & 0 & \grave{L}_{m,a} & 0\\ L^*_{0,m,a} & 0 & 0 & 0\end{bmatrix},\\
\Pi_a &\triangleq \mathrm{diag}\left\{-\kappa P_a+\bar{Q}_{1,a}+\sum_{l=1}^{\bar{\sigma}}X_l,-\bar{X},\tilde{Q}_a,-hI\right\},\\
\bar{A}^{n,m}_{\pi,a} &\triangleq A_{n,m,\pi}+\check{L}_{0,m,a},\ \acute{L}_{m,a}\triangleq\begin{bmatrix}\check{L}_{1,m,a} & \cdots & \check{L}_{\bar{p},m,a}\end{bmatrix},\\
\tilde{B}_{n,m,\pi} &\triangleq \Big[B_{n,m,\pi}\ \underbrace{\bar{B}_{n,m,\pi}\ \cdots\ \bar{B}_{n,m,\pi}}_{\bar{\sigma}-1}\Big],\\
\check{L}_{\iota,m,a} &\triangleq \begin{bmatrix}0 & 0\\ L_m\Omega_{\iota,a}C & 0\end{bmatrix},\ L^*_{\iota,m,a}\triangleq\begin{bmatrix}0 & 0\\ L_m\bar{\Omega}_{\iota,a} & 0\end{bmatrix},\\
\Omega_{\iota,a} &\triangleq \mathrm{diag}\{-\bar{\beta}_1\delta(\iota-\phi_1(a)),\cdots,-\bar{\beta}_{o_y}\delta(\iota-\phi_{o_y}(a))\},\\
\grave{L}_{m,a} &\triangleq \mathrm{diag}\{L^*_{1,m,a},L^*_{2,m,a},\cdots,L^*_{\bar{p},m,a}\},\\
\bar{\Omega}_{\iota,a} &\triangleq \mathrm{diag}\Big\{-\sqrt{\beta^*_1}\delta(\iota-\phi_1(a)),\cdots,\\
&\qquad -\sqrt{\beta^*_{o_y}}\delta(\iota-\phi_{o_y}(a))\Big\}C,\\
\vec{E}_{n,m} &\triangleq \begin{bmatrix}\bar{D}_m & \tilde{I}_m & \bar{E}_n & \vec{B}_n\end{bmatrix},\\
\mathcal{P}_a &\triangleq \mathrm{diag}\{\underbrace{\bar{P}_a,\cdots,\bar{P}_a}_{\bar{p}}\},\quad \bar{P}_a\triangleq\sum_{j=0}^{\bar{p}}\theta_{a+1,j+1}P_j,\\
\bar{\mathcal{P}}_a &\triangleq \mathrm{diag}\{\underbrace{\bar{P}_a,\cdots,\bar{P}_a}_{\bar{p}+2}\},\ C\triangleq\begin{bmatrix}\mathcal{C}_1^T & \mathcal{C}_2^T & \cdots & \mathcal{C}_{o_y}^T\end{bmatrix}^T.
\end{aligned}$$

Proof: We choose the following Lyapunov functional candidate:

$$V(k,\alpha(k))\triangleq V_1(k,\alpha(k))+V_2(k,\alpha(k))+V_3(k)\tag{18}$$

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where

$$V_1(k,\alpha(k)) \triangleq \zeta^T(k)P_{\alpha(k)}\zeta(k),$$
$$V_2(k,\alpha(k)) \triangleq \sum_{c=1}^{\bar{p}} \zeta^T(k-c)Q_{c,\alpha(k)}\zeta(k-c),$$
$$V_3(k) \triangleq \sum_{l=1}^{\bar{\sigma}} \sum_{q=k-l}^{k-1} \kappa^{k-q-1}\zeta^T(q)X_l\zeta(q).$$

By defining the following augmentation vector:

$$\xi(k) \triangleq \begin{bmatrix}\zeta^T(0) & \zeta^T(1) & \cdots & \zeta^T(k)\end{bmatrix}^T,$$

the conditional mathematical expectation is calculated by

$$\begin{aligned}&\mathbb{E}\{V_1(k+1,\alpha(k+1)) - \kappa V_1(k,\alpha(k))|\xi(k),\alpha(k)\}\\ &= \mathbb{E}\{V_1(k+1,\alpha(k+1))|\xi(k),\alpha(k)\} - \kappa V_1(k,\alpha(k))\\ &= \mathbb{E}\{\zeta^T(k+1)P_{\alpha(k+1)}\zeta(k+1)|\xi(k),\alpha(k)\}\\ &\quad - \kappa\zeta^T(k)P_{\alpha(k)}\zeta(k). \end{aligned} \tag{19}$$

Letting $\alpha(k) = a$, $\alpha(k+1) = b$ $(a, b \in \mathbb{P})$ and using Lemma 1, one has that

$$\begin{aligned}&\mathbb{E}\{V_1(k+1,\alpha(k+1))|\xi(k),\alpha(k)\} - \kappa V_1(k,\alpha(k))\\ &= \sum_{b=0}^{\bar{p}} \theta_{a+1,b+1}\mathbb{E}\{\zeta^T(k+1)P_b\zeta(k+1)|\xi(k),a\}\\ &\quad - \kappa\zeta^T(k)P_a\zeta(k)\\ &\le \sum_{n=1}^{\bar{n}}\sum_{m=1}^{\bar{n}}\sum_{\pi=1}^{\bar{n}} \eta_n(\rho(k))\eta_m(\hat{\rho}(k))\eta_\pi(\hat{\rho}(k))\\ &\quad \times \mathbb{E}\Bigg\{\Bigg(A_{n,m,\pi}\zeta(k) + \tilde{B}_{n,m,\pi}\bar{\zeta}(k) + \vec{E}_{n,m}\vec{\omega}(k)\\ &\quad + \sum_{i=1}^{o_y}\Big(\bar{L}_{i,m} + \tilde{\beta}_i(k-\alpha_i)\tilde{L}_{i,m}\Big)\zeta(k-\alpha_i)\Bigg)^T \bar{P}_a\\ &\quad \times \Bigg(\sum_{i=1}^{o_y}\Big(\bar{L}_{i,m} + \tilde{\beta}_i(k-\alpha_i)\tilde{L}_{i,m}\Big)\zeta(k-\alpha_i)\\ &\quad + A_{n,m,\pi}\zeta(k) + \tilde{B}_{n,m,\pi}\bar{\zeta}(k) + \vec{E}_{n,m}\vec{\omega}(k)\Bigg)|\xi(k),a\Bigg\}\\ &\quad - \kappa\zeta^T(k)P_a\zeta(k)\\ &= \sum_{n=1}^{\bar{n}}\sum_{m=1}^{\bar{n}}\sum_{\pi=1}^{\bar{n}} \eta_n(\rho(k))\eta_m(\hat{\rho}(k))\eta_\pi(\hat{\rho}(k))\\ &\quad \times \mathbb{E}\Bigg\{\zeta^T(k)A_{n,m,\pi}^T\bar{P}_aA_{n,m,\pi}\zeta(k) + \vec{\omega}^T(k)\vec{E}_{n,m}^T\bar{P}_a\\ &\quad \times \vec{E}_{n,m}\vec{\omega}(k) + \bar{\zeta}^T(k)\tilde{B}_{n,m,\pi}^T\bar{P}_a\tilde{B}_{n,m,\pi}\bar{\zeta}(k)\\ &\quad + \sum_{i=1}^{o_y}\zeta^T(k-\alpha_i)\vec{L}_{i,m,a}^T(k)\bar{P}_a\sum_{i=1}^{o_y}\vec{L}_{i,m,a}(k)\zeta(k-\alpha_i)\\ &\quad + 2\big(A_{n,m,\pi}\zeta(k)\big)^T\bar{P}_a\tilde{B}_{n,m,\pi}\bar{\zeta}(k) + 2\zeta^T(k)A_{n,m,\pi}^T\\ &\quad \times \bar{P}_a\vec{E}_{n,m}\vec{\omega}(k) + 2\zeta^T(k)A_{n,m,\pi}^T\bar{P}_a\sum_{i=1}^{o_y}\vec{L}_{i,m,a}(k)\\ &\quad \times \zeta(k-\alpha_i) + 2\sum_{i=1}^{o_y}\zeta^T(k-\alpha_i)\vec{L}_{i,m,a}^T(k)\bar{P}_a\Big(\tilde{B}_{n,m,\pi}\bar{\zeta}(k)\\ &\quad + \vec{E}_{n,m}\vec{\omega}(k)\Big) + 2\bar{\zeta}^T(k)\tilde{B}_{n,m,\pi}^T\bar{P}_a\vec{E}_{n,m}\vec{\omega}(k)|\xi(k),a\Bigg\}\\ &\quad - \kappa\zeta^T(k)P_a\zeta(k) \end{aligned} \tag{20}$$

where

$$\begin{aligned}&\vec{L}_{i,m,a}(k) \triangleq \bar{L}_{i,m} + \tilde{\beta}_i(k-\phi_i(a))\tilde{L}_{i,m},\\ &\quad \vec{\omega}(k) \triangleq \begin{bmatrix}\check{\omega}^T(k-\alpha) & \check{v}^T(k-\alpha) & \omega^T(k) & o^T(k)\end{bmatrix}^T,\\ &\quad \bar{\zeta}(k) \triangleq \begin{bmatrix}\zeta^T(k-1) & \zeta^T(k-2) & \cdots & \zeta^T(k-\bar{\sigma})\end{bmatrix}^T,\end{aligned}$$

and $\phi_i(a)$ is defined in Lemma 1.

From the assumption of $\beta_i(k)$ $(i = 1, 2, \cdots, o_y)$, one derives that

$$\begin{aligned}&\mathbb{E}\left\{\tilde{\beta}_i(k)\right\} = 0, \quad \mathbb{E}\left\{\tilde{\beta}_i(k)\tilde{\beta}_i(k)\right\} = \beta_i^*,\\ &\mathbb{E}\left\{\tilde{\beta}_i(k)\tilde{\beta}_{\bar{i}}(k)\right\} = 0, \quad \forall i \neq \bar{i}.\end{aligned}$$

Therefore, one has from (20) that

$$\begin{aligned}&\mathbb{E}\{V_1(k+1,\alpha(k+1))|\xi(k),\alpha(k)\} - \kappa V_1(k,\alpha(k))\\ &\le \sum_{n=1}^{\bar{n}}\sum_{m=1}^{\bar{n}}\sum_{\pi=1}^{\bar{n}} \eta_n(\rho(k))\eta_m(\hat{\rho}(k))\eta_\pi(\hat{\rho}(k))\\ &\quad \Bigg(A_{n,m,\pi}\zeta(k) + \sum_{i=1}^{o_y}\bar{L}_{i,m}\zeta(k-\phi_i(a)) + \tilde{B}_{n,m,\pi}\bar{\zeta}(k)\\ &\quad + \vec{E}_{n,m}\vec{\omega}(k)\Bigg)^T\bar{P}_a\Bigg(A_{n,m,\pi}\zeta(k) + \tilde{B}_{n,m,\pi}\bar{\zeta}(k)\\ &\quad + \vec{E}_{n,m}\vec{\omega}(k) + \sum_{i=1}^{o_y}\bar{L}_{i,m}\zeta(k-\phi_i(a))\Bigg)\\ &\quad + \sum_{i=1}^{o_y}\Big(\sqrt{\beta_i^*}\zeta^T(k-\phi_i(a))\tilde{L}_{i,m}^T\bar{P}_a\\ &\quad \times \sqrt{\beta_i^*}\tilde{L}_{i,m}\zeta(k-\phi_i(a))\Big) - \kappa\zeta^T(k)P_a\zeta(k). \end{aligned} \tag{21}$$

By utilizing the following relations:

$$\begin{aligned}&\sum_{i=1}^{o_y}\bar{L}_{i,m}\zeta(k-\phi_i(a))\\ &\quad = \sum_{c=0}^{\bar{p}}\check{L}_{c,m,a}\zeta(k-c) = \check{L}_{0,m,a}\zeta(k) + \acute{L}_{m,a}\acute{\zeta}(k),\\ &\sum_{i=1}^{o_y}\Big(\sqrt{\beta_i^*}\zeta^T(k-\phi_i(a))\tilde{L}_{i,m}^T\bar{P}_a\sqrt{\beta_i^*}\tilde{L}_{i,m}\zeta(k-\phi_i(a))\Big)\\ &\quad = \sum_{c=0}^{\bar{p}}\Big(L_{c,m,a}^*\zeta(k-c)\Big)^T\bar{P}_aL_{c,m,a}^*\zeta(k-c)\\ &\quad = \big(L_{0,m,a}^*\zeta(k)\big)^T\bar{P}_aL_{0,m,a}^*\zeta(k) + \acute{\zeta}^T(k)\grave{L}_{m,a}^T\mathcal{P}_a\grave{L}_{m,a}\acute{\zeta}(k)\end{aligned}$$

where

$$\acute{\zeta}(k) \triangleq \begin{bmatrix}\zeta^T(k-1) & \zeta^T(k-2) & \cdots & \zeta^T(k-\bar{p})\end{bmatrix}^T,$$

one further obtains

$$\begin{aligned}&\mathbb{E}\{V_1(k+1,\alpha(k+1))|\xi(k),\alpha(k)\} - \kappa V_1(k,\alpha(k))\\ &\le \sum_{n=1}^{\bar{n}}\sum_{m=1}^{\bar{n}}\sum_{\pi=1}^{\bar{n}} \eta_n(\rho(k))\eta_m(\hat{\rho}(k))\eta_\pi(\hat{\rho}(k))\end{aligned}$$

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$$\times \Xi^T(k)\left(\left(\Gamma_{\pi,a}^{n,m}\right)^T \bar{\mathcal{P}}_a \Gamma_{\pi,a}^{n,m} + \bar{\Pi}_a\right)\Xi(k) \tag{22}$$

where

$$\bar{\Pi}_a \triangleq \mathrm{diag}\left\{-\kappa P_a, 0, 0, 0\right\},$$
$$\Xi(k) \triangleq \begin{bmatrix}\zeta^T(k) & \bar{\zeta}^T(k) & \acute{\zeta}^T(k) & \vec{\omega}^T(k)\end{bmatrix}^T.$$

The difference of $V_2(k,\alpha(k))$ is calculated as follows:

$$\begin{aligned}
&\mathbb{E}\left\{V_2(k+1,\alpha(k+1))|\xi(k),\alpha(k)\right\} - \kappa V_2(k,\alpha(k)) \\
=&\sum_{c=1}^{\bar{p}}\big(\zeta^T(k+1-c)\bar{Q}_{c,a}\zeta(k+1-c) \\
&- \kappa\zeta^T(k-c)Q_{c,a}\zeta(k-c)\big) \\
=&\zeta^T(k)\bar{Q}_{1,a}\zeta(k) + \zeta^T(k-1)\big(\bar{Q}_{2,a} - \kappa Q_{1,a}\big)\zeta(k-1) \\
&+ \zeta^T(k-2)\big(\bar{Q}_{3,a} - \kappa Q_{2,a}\big)\zeta(k-2) \\
&+ \cdots \\
&+ \zeta^T(k-\bar{p}+1)\big(\bar{Q}_{\bar{p},a} - \kappa Q_{\bar{p}-1,a}\big)\zeta(k-\bar{p}+1) \\
&- \zeta^T(k-\bar{p})\kappa Q_{\bar{p},a}\zeta(k-\bar{p}).
\end{aligned} \tag{23}$$

Similarly, it is easy to obtain that

$$\begin{aligned}
&\mathbb{E}\left\{V_3(k+1)|\xi(k),\alpha(k)\right\} - \kappa V_3(k) \\
=&\sum_{l=1}^{\bar{\sigma}}\left(\sum_{q=k+1-l}^{k}\kappa^{k-q}\zeta^T(q)X_l\zeta(q)\right. \\
&\left.- \sum_{q=k-l}^{k-1}\kappa^{k-q}\zeta^T(q)X_l\zeta(q)\right) \\
=&\sum_{l=1}^{\bar{\sigma}}\big(\zeta^T(k)X_l\zeta(k) - \kappa^l\zeta^T(k-l)X_l\zeta(k-l)\big) \\
=&\zeta^T(k)\sum_{l=1}^{\bar{\sigma}}X_l\zeta(k) - \sum_{l=1}^{\bar{\sigma}}\kappa^l\zeta^T(k-l)X_l\zeta(k-l).
\end{aligned} \tag{24}$$

From the definition of $\vec{\omega}(k)$, one knows that the following inequality holds:

$$h\left(\gamma - \vec{\omega}^T(k)\vec{\omega}(k)\right) \geq 0 \tag{25}$$

where

$$\gamma \triangleq (1+o_y)\bar{\omega} + \sum_{i=1}^{o_y}\bar{v}_i + \bar{o}.$$

Taking (22)-(25) into account, one gets

$$\begin{aligned}
&\mathbb{E}\left\{V(k+1,\alpha(k+1)) - \kappa V(k,\alpha(k))|\xi(k),\alpha(k)\right\} \\
\leq&\sum_{n=1}^{\bar{n}}\sum_{m=1}^{\bar{n}}\sum_{\pi=1}^{\bar{n}}\eta_n(\rho(k))\eta_m(\hat{\rho}(k))\eta_\pi(\hat{\rho}(k)) \\
&\times \Xi^T(k)\left(\left(\Gamma_{\pi,a}^{n,m}\right)^T \bar{\mathcal{P}}_a \Gamma_{\pi,a}^{n,m} + \Pi_a\right)\Xi(k) + h\gamma.
\end{aligned} \tag{26}$$

The condition (17) leads to

$$\begin{aligned}
&\mathbb{E}\left\{V(k+1,\alpha(k+1))|\xi(k),\alpha(k)\right\} \\
\leq&\mathbb{E}\left\{\kappa V(k,\alpha(k))|\xi(k),\alpha(k)\right\} + h\gamma.
\end{aligned}$$

Taking the mathematical expectation on both sides of the above inequality, one obtains

$$\mathbb{E}\left\{V(k+1,\alpha(k+1))\right\} \leq \kappa\mathbb{E}\left\{V(k,\alpha(k))\right\} + h\gamma. \tag{27}$$

Then, one infers that

$$\begin{aligned}
\mathbb{E}\left\{V(k,\alpha(k))\right\} \leq&\kappa\mathbb{E}\left\{V(k-1,\alpha(k-1))\right\} + h\gamma \\
\leq&\kappa^2\mathbb{E}\left\{V(k-2,\alpha(k-2))\right\} + \kappa h\gamma + h\gamma \\
\leq&\cdots \\
\leq&\kappa^k\mathbb{E}\left\{V(0,\alpha(0))\right\} + \frac{1-\kappa^k}{1-\kappa}h\gamma.
\end{aligned} \tag{28}$$

By defining $\vartheta \triangleq \min\{\lambda(P_a)\}$ $(a \in \mathbb{P})$, one has that

$$\begin{aligned}
\mathbb{E}\left\{\zeta^T(k)\zeta(k)\right\} \leq&\mathbb{E}\left\{\frac{V_1(k,\alpha(k))}{\vartheta}\right\} \\
\leq&\mathbb{E}\left\{\frac{V(k,\alpha(k))}{\vartheta}\right\} \\
\leq&\kappa^k\mathbb{E}\left\{\frac{V(0,\alpha(0))}{\vartheta}\right\} + \frac{1-\kappa^k}{(1-\kappa)\vartheta}h\gamma.
\end{aligned} \tag{29}$$

Due to $\kappa \in (0,1)$, it is concluded that the closed-loop system (14) is exponentially ultimately bounded in the mean-square sense. The proof is complete. ■

Remark 3: In Theorem 1, the conservatism mainly comes from the selected common Lyapunov functional and the applied delay-independent approach. To further reduce the conservatism, we can choose more complex Lyapunov functionals (such as piecewise and fuzzy ones) by utilizing more system information at the cost of increasing the computational burden. Furthermore, our results are established based on systems with accurate models. If there are unmodeled dynamics or uncertainties in both systems and transmission processes, how to deal with the RPSPs needs further research.

Remark 4: The dimension of the matrix inequality (17) is closely related to the system information and complexities such as state variables, sensor numbers, external noises, control inputs and the maximum sampling interval. Recall $x(T_k) \in \mathbb{R}^{o_x}$, $u(T_k) \in \mathbb{R}^{o_u}$, $\vec{\omega}(T_k) \in \mathbb{R}^{(1+o_y)o_\omega+o_y+o_u}$, the integral window length $\bar{\sigma}$ and the total mode number $\bar{p}$. The dimensions of the term $\bar{\mathcal{P}}_a$, $\Gamma_{\pi,a}^{n,m}$ and Π_a in inequality (17) can be calculated as $2(\bar{p}+2)o_x \times 2(\bar{p}+2)o_x$, $2(\bar{p}+2)o_x \times \big((2+2\bar{\sigma}+2\bar{p})o_x + (1+o_y)o_\omega + o_y + o_u\big)$ and $\big((2+2\bar{\sigma}+2\bar{p})o_x + (1+o_y)o_\omega + o_y + o_u\big) \times \big((2+2\bar{\sigma}+2\bar{p})o_x + (1+o_y)o_\omega + o_y + o_u\big)$, respectively.

In light of the conditions obtained in Theorem 1, we will present the calculation procedure of the desired observer and controller gains. The details are given in the following theorem.

Theorem 2: Consider the closed-loop system (14) with the observer-based PID controller (12). Assume that a scalar $\kappa \in (0,1)$ is given. Then, the closed-loop system (14) is exponentially ultimately bounded in the mean-square if, for $n,m,\pi \in \{1,2,\cdots,\bar{n}\}$, $l \in \{1,2,\cdots,\bar{\sigma}\}$, $a \in \mathbb{P}$ and $c \in \{1,2\cdots,\bar{p}\}$, there exist matrices K_π^P, K_π^I, K_π^D, L_m, $P_a > 0$, $Q_{c,a} > 0$, $X_l > 0$ and a scalar $h > 0$ such that

$$\begin{bmatrix}\Pi_a & * \\ \hat{\Gamma}_{\pi,a}^{n,m} & \bar{\mathcal{P}}_a - 2I\end{bmatrix} < 0 \tag{30}$$

$$M_n^T M_n < P_a \tag{31}$$

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where

$$\hat{\Gamma}_{\pi,a}^{n,m} \triangleq \begin{bmatrix} \hat{A}_{\pi,a}^{n,m} & \hat{B}_{n,m,\pi} & \hat{L}_{m,a}^{(1)} & \vec{E}_{n,m} \\ 0 & 0 & \hat{L}_{m,a}^{(2)} & 0 \\ \hat{L}_{m,a}^{(3)} & 0 & 0 & 0 \end{bmatrix}, \ \bar{I}_1 \triangleq \begin{bmatrix} I \\ 0 \end{bmatrix},$$

$$\begin{aligned} \hat{A}_{\pi,a}^{n,m} \triangleq & \hat{A}_{n,m}^{(1)} + \bar{I}_1\big(\mathcal{B}_n K_\pi^P + \mathcal{B}_n K_\pi^D\big)\bar{I}_1^T - \bar{I}_1\big(\mathcal{B}_n K_\pi^P \\ & + \mathcal{B}_n K_\pi^D\big)\bar{I}_2^T + \bar{I}_2\big(\mathcal{B}_n K_\pi^P + \mathcal{B}_n K_\pi^D - \mathcal{B}_m K_\pi^P \\ & - \mathcal{B}_m K_\pi^D + L_m \Lambda C\big)\bar{I}_1^T + \bar{I}_2\big(\mathcal{B}_m K_\pi^P \\ & + \mathcal{B}_m K_\pi^D - L_m \Lambda C - \mathcal{B}_n K_\pi^P - \mathcal{B}_n K_\pi^D\big)\bar{I}_2^T \\ & + \bar{I}_2 L_m \Omega_{0,a} C \bar{I}_1^T, \end{aligned}$$

$$\hat{A}_{n,m}^{(1)} \triangleq \begin{bmatrix} \mathcal{A}_n & 0 \\ \mathcal{A}_n - \mathcal{A}_m & \mathcal{A}_m \end{bmatrix}, \ \hat{L}_{m,a}^{(3)} \triangleq \bar{I}_2 L_m \bar{\Omega}_{0,a} \bar{I}_1^T,$$

$$\begin{aligned} \hat{B}_{n,m,\pi} \triangleq & \bar{I}_1 \mathcal{B}_n K_\pi^I \tilde{I}_1 + \bar{I}_1 \mathcal{B}_n K_\pi^D \tilde{I}_2 + \bar{I}_2 \mathcal{B}_n K_\pi^I \tilde{I}_1 \\ & + \bar{I}_2 \mathcal{B}_n K_\pi^D \tilde{I}_2 - \bar{I}_2 \mathcal{B}_m K_\pi^I \tilde{I}_1 - \bar{I}_2 \mathcal{B}_m K_\pi^D \tilde{I}_2, \end{aligned}$$

$$\tilde{I}_1 \triangleq \big[\underbrace{I \ \ -I \ \ I \ \ -I \ \ \cdots \ \ I \ \ -I}_{2\bar{\sigma}}\big],$$

$$\tilde{I}_2 \triangleq \big[-I \ \ I \ \ \underbrace{0 \ \ 0 \ \ \cdots \ \ 0 \ \ 0}_{2(\bar{\sigma}-1)}\big], \ \bar{I}_2 \triangleq \begin{bmatrix} 0 \\ I \end{bmatrix},$$

$$\hat{L}_{m,a}^{(1)} \triangleq \bar{I}_2 L_m \hat{\Omega}_a^{(1)}, \quad \hat{L}_{m,a}^{(2)} \triangleq \sum_{c=1}^{\bar{p}} \hat{I}_c L_m \hat{\Omega}_{c,a}^{(2)},$$

$$\hat{\Omega}_a^{(1)} \triangleq \big[\Omega_{1,a} C \quad 0 \quad \cdots \quad \Omega_{\bar{p},a} C \quad 0\big],$$

$$\hat{\Omega}_{c,a}^{(2)} \triangleq \Big[0 \ \cdots \ 0 \qquad \underbrace{\bar{\Omega}_{c,a}}_{\text{the } 1\times(2c-1)\text{th block}} \qquad 0 \ \cdots \ 0\Big],$$

$$\hat{I}_c \triangleq \Big[0 \ \cdots \ 0 \qquad \underbrace{I}_{\text{the } 1\times 2c \text{ block}} \qquad 0 \ \cdots \ 0\Big]^T.$$

The minimum of the asymptotic upper bound of $\mathbb{E}\{z^T(k)z(k)\}$ can be obtained by solving the following optimization problem:

$$\min h \tag{32}$$

subject to constraints (30) and (31). Furthermore, if the optimization problem (32) is solvable, then the obtained matrices K_π^P, K_π^I and K_π^D are the desired controller gains and matrices L_m are observer gains.

Proof: By using the Schur Complement Lemma, the inequality (17) in Theorem 1 holds if and only if the following inequality holds:

$$\begin{bmatrix} \Pi_a & * \\ \Gamma_{\pi,a}^{n,m} & -\bar{\mathcal{P}}_a^{-1} \end{bmatrix} < 0. \tag{33}$$

The inequality of

$$\left(\bar{\mathcal{P}}_a - I\right)^T \bar{\mathcal{P}}_a^{-1} \left(\bar{\mathcal{P}}_a - I\right) \geq 0 \tag{34}$$

implies

$$\bar{\mathcal{P}}_a - 2I \geq -\bar{\mathcal{P}}_a^{-1}. \tag{35}$$

From the definition of $\Gamma_{\pi,a}^{n,m}$ (in Theorem 1) and $\hat{\Gamma}_{\pi,a}^{n,m}$ (in Theorem 2), it is easy to see that $\hat{\Gamma}_{\pi,a}^{n,m} = \Gamma_{\pi,a}^{n,m}$. Then, it can be concluded from the above discussions and (30) that

$$\begin{bmatrix} \Pi_a & * \\ \Gamma_{\pi,a}^{n,m} & -\bar{\mathcal{P}}_a^{-1} \end{bmatrix} \leq \begin{bmatrix} \Pi_a & * \\ \hat{\Gamma}_{\pi,a}^{n,m} & \bar{\mathcal{P}}_a - 2I \end{bmatrix} < 0. \tag{36}$$

Thus, we know that the closed-loop system (14) is exponentially ultimately bounded in the mean-square.

By recalling the definition of $V(k,\alpha(k))$ and considering (31), one can infer that

$$\begin{aligned} \mathbb{E}\left\{z^T(k)z(k)\right\} = & \mathbb{E}\Bigg\{\sum_{n=1}^{\bar{n}}\sum_{d=1}^{\bar{n}} \eta_n(\rho(k))\eta_d(\rho(k))\zeta^T(k)M_n^T \\ & \times M_d \zeta(k)\Bigg\} \\ \leq & \mathbb{E}\left\{\sum_{n=1}^{\bar{n}} \eta_n(\rho(k))\zeta^T(k)M_n^T M_n \zeta(k)\right\} \\ \leq & \mathbb{E}\left\{\zeta^T(k)P_{\alpha(k)}\zeta(k)\right\} \\ \leq & \mathbb{E}\left\{V(k,\alpha(k))\right\} \\ \leq & \kappa^k \mathbb{E}\left\{V(0,\alpha(0))\right\} + \frac{1-\kappa^k}{1-\kappa} h\gamma. \end{aligned} \tag{37}$$

It is clear that the asymptotic upper bound of $\mathbb{E}\{z^T(k)z(k)\}$ can be obtained by solving (32) which completes the proof. ■

Based on Theorem 2, Algorithm 1 is given to show the implementation details of the proposed control method.

Algorithm 1: Observer-based fuzzy PID control	
Step 1.	Given a nonlinear plant, select the premise variables and construct fuzzy rules to obtain a T-S fuzzy model in the form of (1).
Step 2.	In terms of the sampling features of sensors, determine Θ_i and $\bar{p}_i$ for sensor i. Then, apply Lemma 1 to calculate $\theta_{a+1,b+1}$ $(a,b \in \mathbb{P})$ and $\bar{p}$ based on Θ_i and $\bar{p}_i$.
Step 3.	Construct the observer-based fuzzy PID controller (12) and select the size of the integral window by determining $\bar{\sigma}$.
Step 4.	Give a scalar $\kappa \in (0,1)$. Solve the minimization problem (32) to obtain control gains K_π^P, K_π^I, K_π^D and observer gains L_m.
Step 5.	Apply the generated control law (12) to the system (1).

Remark 5: The obtained main results are based on the LMI technique that is essential a convex optimization algorithm and has polynomial time complexity. To be more specific, the computational complexity is proportional to the total row size $\mathcal{M}$ of the LMIs, and the total number $\mathcal{N}$ of scalar decision variables [55]. For system (1) with RPSPs and degradation measurements, the variable information can be seen from $x(T_k) \in \mathbb{R}^{o_x}$, $u(T_k) \in \mathbb{R}^{o_u}$, $\vec{\omega}(T_k) \in \mathbb{R}^{(1+o_y)o_\omega + o_y + o_u}$, the integral window length $\bar{\sigma}$ and the total mode number $\bar{p}$. Here, $\mathcal{M}$ and $\mathcal{N}$ can be calculated as follows:

$$\begin{aligned} \mathcal{M} = & \left(\bar{n}^3(\bar{p}+1)\right) \times \left((2+2\bar{\sigma}+2\bar{p})o_x + (1+o_y)o_\omega + o_y \right. \\ & \left. + o_u\right) + \Big(\big(2(2+\bar{p})\bar{n} + 2 + 2\bar{p}\big)(\bar{p}+1) + 2\bar{\sigma}\Big)o_x + 1, \\ \mathcal{N} = & \left((1+\bar{p})^2 + \bar{\sigma}\right)o_x(2o_x+1) + 3\bar{n}o_u o_x + \bar{n}o_x o_y + 1. \end{aligned}$$

Compared with the P-type control method [55], the proposed fuzzy PID control has more design degrees of freedom for improving performance, and thereby has higher computational complexity. In addition, compared with the genetic algorithm [56] and the two-step method [57], our proposed method can be implemented directly which avoids multiple optimization processes, and thus has a relatively low computational cost.

Remark 6: So far, we have dealt with the mean-square bounded control problem for a class of discrete-time nonlinear systems using an observer-based fuzzy PID control approach.

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A detailed mathematical model has been built to encapsulate the nuances of incomplete measurements as characterized by amplitude degradation and RPSPs. Such a model is effective for capturing the time-varying and stochastic features of the sampling process of sensors. Note that, the mathematical derivation has been conducted in the discrete-time framework, where the effects of RPSPs have been handled using the stochastic delay method and the Markov jump system theory. The conditions specified in Theorems 1-2 encompass a wide range of complexities such as system parameters and delay factors stemming from RPSPs.

Remark 7: This paper distinguishes itself from existing literature on sampled-data systems in two primary aspects.

1) *Introduction of a New Control Problem*: This study pioneers the exploration of the exponentially ultimately bounded control problem for general nonlinear systems impacted by degraded sensors, RPSPs, and UBB transmission noises. This particular problem has not been addressed in existing research due primarily to the complexity involved in its analysis.
2) *Development of an Observer-Based Fuzzy PID Controller*: The proposed observer-based fuzzy PID controller features a computation-efficient and straightforward structure. Such a controller design specifically targets the mitigation of the adverse effects caused by incomplete measurement information, which not only enhances the control system's effectiveness but also simplifies its implementation.

The effectiveness and advantages of the proposed fuzzy control method will be verified in the next section through simulation examples.

IV. Simulation Examples

In this section, one numerical example and one application-motivated example are presented to show the control performance in the presence of measurement degradation and RPSPs.

A. Example 1

Consider a discrete-time T-S fuzzy system in the form of (1) with two rules and the following parameter matrices:

$$\mathcal{A}_1 = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.3 & 0.2 & 0.2 \\ 0.3 & 0 & 0.99 \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0 & 0.2 \\ 0 & 0.2 & 1 \end{bmatrix},$$
$$\mathcal{B}_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.7 \end{bmatrix}, \quad \mathcal{B}_2 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.7 \end{bmatrix}, \quad \mathcal{C}_1 = \begin{bmatrix} 0.5 \\ 0.6 \\ 0.7 \end{bmatrix}^T,$$
$$\mathcal{C}_2 = \begin{bmatrix} 0.3 \\ -0.2 \\ 0.5 \end{bmatrix}^T, \quad \mathcal{E}_1 = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.2 \end{bmatrix}, \quad \mathcal{E}_2 = \begin{bmatrix} 0.3 \\ 0.1 \\ 0.1 \end{bmatrix},$$
$$\mathcal{D}_1 = 0.3, \quad \mathcal{D}_2 = 0.5, \quad \mathcal{M}_1 = \begin{bmatrix} 0.3 & 0.4 & 0.5 \end{bmatrix},$$
$$\mathcal{M}_2 = \begin{bmatrix} 0.1 & 0.2 & 0.8 \end{bmatrix}.$$

The fuzzy membership functions are assumed to be

$$\eta_1(\rho(k)) = \sin^2(x_1(k)), \quad \eta_2(\rho(k)) = 1 - \sin^2(x_1(k))$$

where $x_1(k)$ is the first element of the state variable.

The state updating interval is $T_{k+1} - T_k = \epsilon = 1$ second. Note that two sensors are considered in this example whose sampling periods $p_1\left(s_1^{(\tau)}\right)$ and $p_2\left(s_2^{(\tau)}\right)$ take values in $\{\epsilon, 2\epsilon, 3\epsilon\}$ randomly. The stochastic variables $\beta_1(k)$ and $\beta_2(k)$ used to represent the measurement degradation have the following statistical property:

$$\mathbb{E}\{\beta_1(\cdot)\} = 0.6, \ \mathbb{E}\{(\beta_1(\cdot) - 0.6)^2\} = 0.01,$$
$$\mathbb{E}\{\beta_2(\cdot)\} = 0.7, \ \mathbb{E}\{(\beta_2(\cdot) - 0.6)^2\} = 0.01.$$

We assume that the transition probability matrix of $\alpha(k)$ is of the following form:

$$\Theta = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.9 & 0 & 0.1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Let the system noise and transmission noises be $\omega(T_k) = 0.3\cos(k)$, $v_1(k) = 0.1\sin(k)$, $v_2(k) = 0.2\sin(k)$ and $o(k) = 0.2\sin(k)$. In this example, the noises that the system suffers from are all bounded and only the upper bound of noises is required to be known by the designer.

Upon solving the optimization problem (32), the gains for the observer and the PID controller are determined as follows:

$$K_1^P = \begin{bmatrix} -0.437220 & -0.242516 & -0.724263 \end{bmatrix},$$
$$K_2^P = \begin{bmatrix} -0.437112 & -0.242485 & -0.724145 \end{bmatrix},$$
$$K_1^I = \begin{bmatrix} 0.001139 & 0.001192 & 0.001156 \end{bmatrix},$$
$$K_2^I = \begin{bmatrix} 0.001102 & 0.001156 & 0.001120 \end{bmatrix},$$
$$K_1^D = \begin{bmatrix} 0.001597 & 0.001652 & 0.001616 \end{bmatrix},$$
$$K_2^D = \begin{bmatrix} 0.001545 & 0.001601 & 0.001564 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 0.059474 & 0.020007 \\ 0.411134 & -0.199015 \\ 0.916034 & 0.748723 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 0.211396 & -0.301644 \\ 0.169176 & 0.155749 \\ 1.019763 & 0.542132 \end{bmatrix}.$$

The simulation is conducted with a length of $k_{\max} = 100$, and the results are illustrated in Figs. 2–7. Fig. 2 demonstrates the state evolution of the fuzzy system without any control strategy, revealing that the original nonlinear system is unstable, with its state deviating rapidly from the equilibrium point $x = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$. When the designed observer-based PID controller (12) is applied, the state evolution of the closed-loop system is depicted in Fig. 3. It can be seen that the controlled states remain bounded around the equilibrium point. The estimate for three state components is depicted in Figs. 4–6. The state estimation error is exhibited in Fig. 7 which is shown to be bounded.

To display the effects of measurement degradation, we define the accumulative controlled output as follows

$$\tilde{z} \triangleq \sum_{k=0}^{k_{\max}} z^T(k) z(k).$$

Then, some comparison results under different degradation levels ($\underline{\beta} \triangleq 1 - \bar{\beta}_1 = 1 - \bar{\beta}_2$) are given in Table I. Here, $\tilde{z}$

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reflects the degree that the system deviates from equilibrium caused by external noises and incomplete information. Clearly, a smaller $\tilde{z}$ means a better control performance. From Table I, it can be seen that a higher degradation level leads to a worse system performance. Such a result is rather reasonable as the more serious the degradation, the less useful information can be used for the controller.

TABLE I: Accumulative output under different degradation levels

degradation level $\underline{\beta}$	0.45	0.4	0.3	0.1
output $\tilde{z}$	33.9565	31.7320	30.9408	29.8561

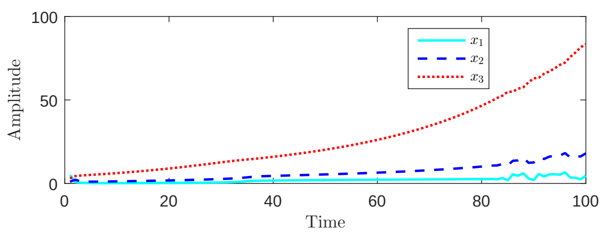

Fig. 2: State evolution of the open-loop system without any control law (unstable)

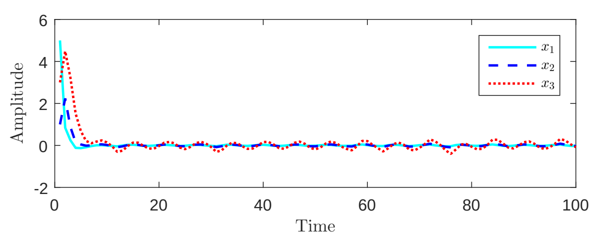

Fig. 3: State evolution of the closed-loop system under the proposed fuzzy PID control (bounded)

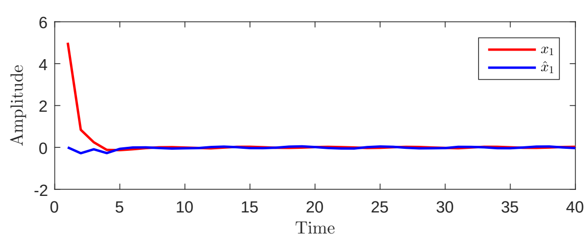

Fig. 4: State x_1 and its estimate using the observer in (12)

To demonstrate the superiority of the proposed control method over the existing P-type control approach [19], [20], we list some comparison results in Table II that shows the obtained $\tilde{z}$ under different degradation levels. Since the devised observer-based PID controller processes a flexible structure and extra design parameters, the $\tilde{z}$ obtained using our method is less than that using the existing method in all cases. All simulation examples validate the effectiveness of the proposed control approach in dealing with the incomplete information.

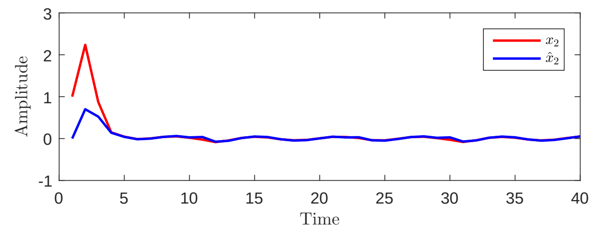

Fig. 5: State x_2 and its estimate using the observer in (12)

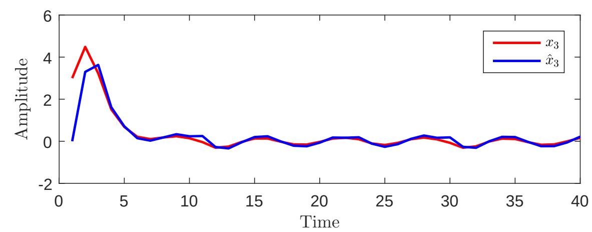

Fig. 6: State x_3 and its estimate using the observer in (12)

B. Example 2

In this subsection, we consider a truck-trailer control system modeled as follows [58]:

$$\begin{cases} x_1(k+1) = \left(1 - \dfrac{\gamma\epsilon}{W}\right) x_1(k) + \dfrac{\gamma\epsilon}{w} u(k) + 0.1\omega(k) \\ x_2(k+1) = \dfrac{\gamma\epsilon}{W} x_1(k) + x_2(k) + 0.1\omega(k) \\ x_3(k+1) = \gamma\epsilon \sin\left(\dfrac{\gamma\epsilon}{2W} x_1(k) + x_2(k)\right) + x_3(k) \end{cases} \tag{38}$$

where $x_1(k)$ is the angle difference between the truck and the trailer; $x_2(k)$ is the angle of the trailer; x_3 is the vertical position of the rear end of the trailer; $u(k)$ is the steering angle; $\omega(k)$ is the external disturbance; $w = 2.8m$ is the length of the truck; $W = 5.5m$ is the length of the trailer; $\epsilon = 1s$ is the sampling time; and $\gamma = -0.5m/s$ is the constant speed of backing up.

From the structure of model (38), it can be seen that the control task for (38) is more difficult than the model used in Example 1. To apply the proposed fuzzy PID control

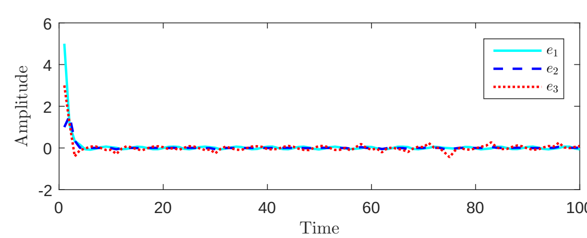

Fig. 7: State estimation error $e \triangleq x - \hat{x}$ under the RPSPs (bounded)

TABLE II: Control performance comparison under different degradation levels

degradation level $\underline{\beta}$	0.45	0.4	0.3	0.1
output $\tilde{z}$ (our method)	33.9565	31.7320	30.9408	29.8561
output $\tilde{z}$ (P-type method)	35.1035	31.8925	31.6254	29.9983

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method, we first establish a T-S fuzzy model according to the nonlinear term $\sin\left(\frac{\gamma\epsilon}{2W}x_1(k)+x_2(k)\right)$. When establishing a T-S fuzzy model for nonlinear systems, the selection of fuzzy membership functions is not sole which depends on the specific form of nonlinear terms of the system. Generally speaking, the fuzzy membership functions should fully reflect the essential features of nonlinearities such as their value ranges and variation trend, such that the obtained fuzzy model has ideal approximation capability for the nonlinear plant. In terms of the key points 0 rad, $\pm\frac{\pi}{6}$ rad, $\pm\pi$ rad and by using the standard fuzzy modeling technique, we can obtain the following discrete-time T-S fuzzy model:

$$x(k+1)=\sum_{n=1}^{3}\eta_n(\rho(k))\Big(\mathcal{A}_i x(k)+\mathcal{B}u(k)+\mathcal{E}\omega(k)\Big) \quad (39)$$

where

$$\mathcal{A}_1 \triangleq \begin{bmatrix} 1-\frac{\gamma\epsilon}{W} & 0 & 0 \\ \frac{\gamma\epsilon}{W} & 1 & 0 \\ \frac{\gamma^2\epsilon^2}{2W} & \gamma\epsilon & 1 \end{bmatrix}, \mathcal{A}_2 \triangleq \begin{bmatrix} 1-\frac{\gamma\epsilon}{W} & 0 & 0 \\ \frac{\gamma\epsilon}{W} & 1 & 0 \\ \frac{3\gamma^2\epsilon^2}{2\pi N} & \frac{3\gamma\epsilon}{\pi} & 1 \end{bmatrix},$$

$$\mathcal{A}_3 \triangleq \begin{bmatrix} 1-\frac{\gamma\epsilon}{W} & 0 & 0 \\ \frac{\gamma\epsilon}{W} & 1 & 0 \\ \frac{\gamma^2 T}{200W^2} & \frac{\gamma}{100W} & 1 \end{bmatrix}, \quad x(k) \triangleq \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix},$$

$$\mathcal{B} \triangleq \begin{bmatrix} \frac{\gamma\epsilon}{w} \\ 0 \\ 0 \end{bmatrix}, \quad \rho(k) \triangleq \frac{\gamma\epsilon}{2W}x_1(k)+x_2(k), \;\; \mathcal{E} \triangleq \begin{bmatrix} 0.1 \\ 0.1 \\ 0 \end{bmatrix}.$$

It is assumed that there are two sensors with parameters $\mathcal{C}_1 \triangleq \begin{bmatrix}5 & -2 & 1\end{bmatrix}$, $\mathcal{C}_2 \triangleq \begin{bmatrix}3 & -1 & 0.2\end{bmatrix}$, $\mathcal{M}_1=\mathcal{M}_2=\mathcal{M}_3=\begin{bmatrix}0.1 & 0.1 & 0.1\end{bmatrix}$. The fuzzy membership functions of plant (38) are plotted in Fig. 8. The system noises are set to be $\omega(k)=0.1\sin(k)$, $v_1(k)=0.1\sin(k)$, $v_2(k)=0$ and $o(k)=0.1\cos(k)$. The information about the sensor degradation is given as follows:

$$\mathbb{E}\{\beta_1(\cdot)\}=0.6,\;\; \mathbb{E}\{(\beta_1(\cdot)-0.6)^2\}=0.01,$$
$$\mathbb{E}\{\beta_2(\cdot)\}=0.7,\;\; \mathbb{E}\{(\beta_2(\cdot)-0.6)^2\}=0.01.$$

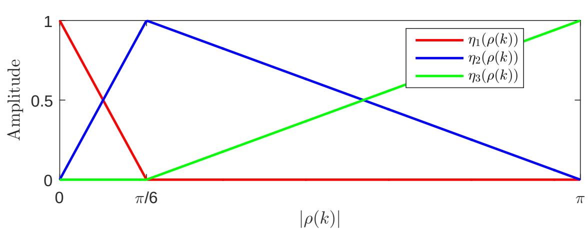

Fig. 8: Fuzzy membership functions of the plant

The aim of this example is to design control laws such that the variables of truck-trailer system (38) are bounded. Set the simulation length to be $k_{\max}=400$. By applying the observer-based fuzzy PID controller (12) combined with the utilized technique in [58], simulation results are presented in Figs. 9–10. We can conclude from these figures that the proposed controller performs well for the truck-trailer system, as the controlled variables remain bounded in the presence of the incomplete information.

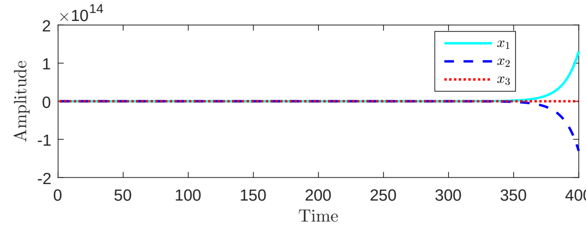

Fig. 9: State evolution of the open-loop truck-trailer system without any control law (unstable)

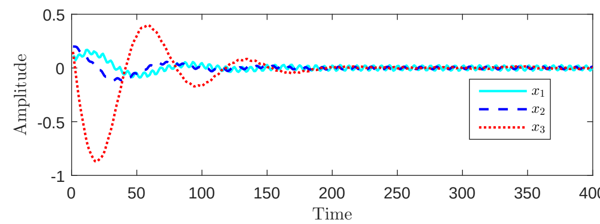

Fig. 10: State evolution of the closed-loop truck-trailer system under the proposed fuzzy PID control (bounded)

V. Conclusion

This paper has addressed the observer-based fuzzy PID control problem for T-S fuzzy systems experiencing degraded measurements and RPSPs. The amplitude degradation of outputs has been characterized by using a series of independent stochastic variables with known mean and variance. The sampling periods of each sensor, which are time-varying and random, are governed by a set of Markov chains. An observer-based fuzzy PID controller with an appropriate structure has been proposed by considering degraded measurements, RPSPs, and transmission noises. Sufficient conditions have been derived to ensure the mean-square boundedness of the controlled output of T-S fuzzy systems. Based on these conditions, controller parameters have been determined by solving an optimization problem. To demonstrate the efficacy of the theoretical framework, simulation examples have been provided to showcase the practical applicability of the proposed control strategy in handling complex real-world scenarios.

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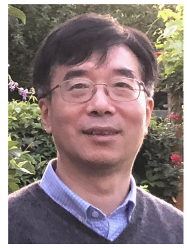

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