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Privacy Preserving Control for Two-Dimensional Systems with Guaranteed Probability

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***Abstract*—This article addresses the privacy preserving control issue for two-dimensional systems with probabilistic constraints. According to the exclusive or logical operation and the dynamic coding-decoding rule, a privacy preserving mechanism (PPM) is developed, under which the transmitted data is efficiently compressed and encrypted into a ciphertext with finite bits. A PPM-based controller is designed that simultaneously guarantees a prescribed probabilistic constraint, mean-square boundedness, and privacy performance. Mathematical techniques, including mathematical induction, Chebyshev inequality, and matrix analysis, are employed to establish sufficient conditions for the presence of the desired controller gains. Additionally, the privacy and secrecy performance of the PPM is analyzed and simulation examples are presented to showcase the efficacy of the proposed controller design method.**

***Index Terms*—Privacy preserving mechanism, guaranteed probability, dynamic coding-decoding rule, two-dimensional systems.**

Abbreviations and Notations

NCS	Networked control system
PPM	Privacy preserving mechanism
XOR	Exclusive or
2-D	Two-dimensional
$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{R}^{m\times n}$	The set of all $m \times n$ real matrices
$\lvert * \rvert$	The absolute value of "$*$"
$\lVert * \rVert$	The Euclidean norm of "$*$"
$\oplus$	The XOR logical operator
$\mathbb{E}\{\cdot\}$	The expectation operator
$\mathbf{Pr}\{A\}$	The occurrence probability of an event A
$\bigcap_{i=0}^{l} E_i$	All events E_i $(i = 1, 2, \cdots, l)$ occur
$\bigcup_{i=0}^{l} E_i$	At least one event E_i $(i = 1, 2, \cdots, l)$ occurs
$\mathrm{Tr}(Q)$	The trace of a matrix Q

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I. Introduction

A networked control system (NCS) is a complex system in which system components (such as sensors, controllers, and actuators) are interconnected via a shared communication network. This kind of system has several advantages, including easy maintenance, low cost, and high reliability, as evidenced in recent studies [4], [11], [24], [25], [27], [29], [43]. Unlike conventional point-to-point control systems, the data in NCSs is typically transferred through the communication network after the process of sampling, quantization, coding and decoding [16], [26], [33]. In view of the above advantages and features, NCSs are the backbone of several infrastructures (e.g., smart grid), but the *openness* of the communication network would lead to security hidden risks [6], [17], [40]. The system data, when sent to the remote controller via the public network, is likely to be eavesdropped on by hackers who can further utilize this data to degrade the system's performance [5], [7], [51]. Accordingly, it is essential to construct a *privacy preserving mechanism* (PPM) for NCSs to guarantee the security and availability of the data.

Recently, the PPM of NCSs has received considerable attention and achieved rapid development [9]. Specifically speaking, the privacy preserving method can be divided into two categories from technical, namely, 1) the data-perturbation-based approach and 2) the cryptography-based approach. For the first approach, one representative method is called the differential privacy, which protects the security of systems by adding designed random noises (such as Laplace noise) to the data [12], [18], [44], [48]. Note that it might be difficult for the differential privacy method to play adequate trade-off between the system control performance and the privacy level. For the cryptography-based method, the data is encrypted through specific forms of algebraic operations (e.g., homomorphic encryption), and then the ciphertext is decrypted by the carefully constructed decryption algorithm [32], [34], [49].

It should be noted that research on PPM is still in its early stages, with only a few primary results available in the literature [14], [28], [37]–[39]. For instance, differential privacy filtering has been investigated for linear discrete-time systems in [23], and a differentially private consensus algorithm has been designed for multi-agent systems to safeguard the initial state information of all agents using the

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differential privacy approach [41]. On the other hand, to guarantee the computational privacy and achieve the cloud computing, the homomorphic encryption approach has been utilized for cyber-physical systems in [22]. Unfortunately, it seems that the resource consumption (including computational time and communication burden) is high under the above-mentioned methods, which is not applicable to NCSs with limited communication bandwidth. Therefore, in this paper, we propose a simple yet efficient PPM for NCSs to ensure both transmission efficiency and data privacy under limited bandwidth communication channels.

In practical engineering, because of limitations of actual physical devices and safety requirements, it is meaningful to design a state-related constraint index for systems [1], [10], [13], [21], [42], [46]. For example, the robot's range of motion is subject to given constraints for the sake of preventing the potential faults or dangers [20]. Unfortunately, the stochastic noise (which is widely encountered in NCSs) may cause the control problem subject to hard constraints difficult or unable to solve. In response to the previously mentioned difficulty, the probabilistic constraint method is proposed, which requires that the constraint is satisfied with given probability [8], [15], [35]. Nowadays, the research on probabilistic constraint problem mainly aims at steady-state performance for systems subject to individual probabilistic constraints, in spite of the fact that the transient performance and the *probabilistic constraint* makes practical sense, and this provides another motivation for our research.

In this paper, we focus on the PPM-based control issue for networked two-dimensional (2-D) systems with guaranteed probability and limited communication bandwidths. The *challenges* are outlined as listed below.

1) How to propose a suitable PPM that takes the data transmission efficiency and data privacy preserving into account?
2) How to develop a control scheme that can conquer the difficulties caused by PPM, probabilistic constraints, and stochastic noises?
3) How to guarantee the secrecy of the proposed PPM?

Regarding the challenges that have been identified, this paper's *contributions* are summarized as follows.

1) By means of the one-time pad method, exclusive or logical operation, and dynamic coding-decoding method, the desired PPM is, for the first time, constructed for NCSs with limited communication bandwidth.
2) Sufficient conditions to guarantee the probabilistic constraints and mean-square boundedness are derived and the approach to compute the corresponding controller parameters is given.
3) A PPM-based control scheme (which prevents the data leakage problem) is developed for systems with probabilistic constraints, and the secrecy performance is rigorously analyzed.

The subsequent sections of this paper are structured as follows. Section II introduces the probabilistic constraint, PPM, and the corresponding PPM-based controller. In Section III, a PPM-based control scheme is proposed for systems under guaranteed probability, and the privacy performance of the proposed PPM is analyzed. The validity of the results is verified in Section IV via two simulation examples. Lastly, the conclusion is presented in Section V.

II. System Description

A. The System Model

A 2-D system with additive stochastic noises is described as follows [19]:

$$\begin{aligned} x_{k+1,h+1} = {} & A^{[1]}x_{k,h+1} + A^{[2]}x_{k+1,h} + B^{[1]}u_{k,h+1} \\ & + B^{[2]}u_{k+1,h} + E^{[1]}w_{k,h+1} + E^{[2]}w_{k+1,h} \end{aligned} \quad (1)$$

where k, h are, respectively, integer-valued horizontal and vertical coordinates; $x_{k,h} \in \mathbb{R}^{n_x}$ represents the state variable; $u_{k,h} \in \mathbb{R}^{n_u}$ denotes the control input; $w_{k,h} \in \mathbb{R}^{n_w}$ is the truncated zero-mean white noise with variance $W_{k,h}$; and $A^{[1]}$, $A^{[2]}$, $B^{[1]}$, $B^{[2]}$, $E^{[1]}$ and $E^{[2]}$ are known matrices. In addition, it is assumed that $W_{k,h} \leq \bar{W}$ with $\bar{W}$ being symmetric positive-definite.

The system state is subject to the following *probabilistic constraint*:

$$\mathbf{Pr}\left\{\bigcap_{h=0}^{H}\bigcap_{k=0}^{K} x_{k,h}^{\mathrm{T}}\Phi_{k,h}^{-1}x_{k,h} \leq 1\right\} \geq 1-\varepsilon \quad (2)$$

where K and H are positive integers, $\varepsilon \in (0,1]$ denotes the maximum probability value of violating the probabilistic constraint, and $\Phi_{k,h}$ is the positive-definite matrix.

Remark 1: In recent years, 2-D systems have garnered significant attention due to their profound theoretical significance and wide-ranging industrial applications [30], [31], [47], [50]. It should be noted that the dynamics in 2-D systems undergo independent evolution along two directions, namely, the horizon direction and the vertical direction. Based on this feature, numerous complex industrial systems can be modeled by the 2-D system model with examples including vehicle platoon and chemical process. In the applications above, the variable that varies with different spatial and temporal positions is denoted by h and k, respectively.

Remark 2: In real-world applications, it is unavoidable for constraints to be violated due to various factors including unpredictable stochastic noises. In this case, it would be really challenging (if not impossible) to impose the hard constraints on system states of 2-D systems. On the other hand, it is often practically acceptable to ensure the constraints to be met with satisfactory probability, that is, the constraints are allowed to be occasionally violated as long as the overall engineering performance index is achieved in the statistical sense. As such, it is sensible to introduce the probabilistic constraint that serves as a reasonable relaxation of the traditional hard constraint. It is worth noting that our efforts represent a pioneering initiative aimed at systematically investigating the issue of probabilistic constraints for 2-D systems.

B. The Privacy Preserving Mechanism

In the sensors to the remote controller communication channel, a privacy preserving mechanism (PPM) is utilized

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to protect the security and privacy of the transmitted data (see Fig. 1). The PPM is implemented by four procedures, namely, coding, encryption, decryption, and decoding. The specific methods are described in the following.

Coding and Encryption. This process includes two steps: 1) converting the system signal into the 0-1 binary bit strings by using the dynamic coding rule; and 2) encrypting the binary sequences by applying the exclusive or (XOR) operation and the one-time pad method.

First, the dynamic coding rule is given as

$$\begin{cases} \eta_{k,h}^{(m)} = \mathfrak{a}_{k-1,h}^{(m,1)}\eta_{k-1,h}^{(m)} + \mathfrak{a}_{k,h-1}^{(m,2)}\eta_{k,h-1}^{(m)} + \mathfrak{b}_{k,h}^{(m)}\nu_{k,h}^{(m)} \\ \nu_{k,h}^{(m)} = \mathrm{Q}\left(\check{\eta}_{k,h}^{(m)}, \mathfrak{b}_{k,h}^{(m)}\right) \end{cases} \tag{3}$$

for $m = 1, 2, \ldots, n_x$, where

$$\check{\eta}_{k,h}^{(m)} \triangleq x_{k,h}^{(m)} - \mathfrak{a}_{k-1,h}^{(m,1)}\eta_{k-1,h}^{(m)} - \mathfrak{a}_{k,h-1}^{(m,2)}\eta_{k,h-1}^{(m)}.$$

Here, $x_{k,h}^{(m)} \in \mathbb{R}$, $\eta_{k,h}^{(m)} \in \mathbb{R}$ and $\nu_{k,h}^{(m)} \in \mathbb{R}$ are, respectively, the mth entry of the state variable $x_{k,h}$, the internal dynamic variable $\eta_{k,h}$ and the coded signal $\nu_{k,h}$; and $\mathfrak{a}_{k,h}^{(m,1)} \in \mathbb{R}$, $\mathfrak{a}_{k,h}^{(m,2)} \in \mathbb{R}$ and $\mathfrak{b}_{k,h}^{(m)} \in \mathbb{R}$ are the known scaling parameters that are norm-bounded. The initial condition is $\eta_{0,h}^{(m)} = \eta_{k,0}^{(m)} = \wp^{(m)}$ with $\wp^{(m)} \in \mathbb{R}$ being the known scalar.

In the coding process, the quantization rule $\mathrm{Q}(\cdot,\cdot)$ is

$$\mathrm{Q}\left(\check{\eta}_{k,h}^{(m)}, \mathfrak{b}_{k,h}^{(m)}\right) = \begin{cases} -r + \frac{r}{2^l-1}, & \frac{\check{\eta}_{k,h}^{(m)}}{\mathfrak{b}_{k,h}^{(m)}} \in \mathscr{S}_1 \\ -r + \frac{(2\mathfrak{n}-1)r}{2^l-1}, & \frac{\check{\eta}_{k,h}^{(m)}}{\mathfrak{b}_{k,h}^{(m)}} \in \mathscr{S}_{\mathfrak{n}} \end{cases} \tag{4}$$

where

$$\mathscr{S}_1 \triangleq \left[-r, -r + \frac{2r}{2^l-1}\right]$$
$$\mathscr{S}_{\mathfrak{n}} \triangleq \left(-r + \frac{2(\mathfrak{n}-1)r}{2^l-1}, -r + \frac{2\mathfrak{n}r}{2^l-1}\right].$$

Here, $\mathfrak{n} \in \{2, 3, \ldots, 2^l - 1\}$, l is the number of 0-1 binary bits, and $[-r, r]$ is the quantization range with $r > 0$. It what follows, for the purpose of encryption, the coded value $\nu_{k,h}^{(m)}$ is converted into the l-bit binary sequence, i.e., $\vec{\nu}_{k,h}^{(m)}$.

Next, in the encryption step, the XOR operation is used for encrypting the binary data, which is realized by

$$c_{k,h}^{(m)} = \mathbf{Enc}\left(\vec{\nu}_{k,h}^{(m)}, \kappa_{k,h}^{(m)}\right) = \vec{\nu}_{k,h}^{(m)} \oplus \kappa_{k,h}^{(m)} \tag{5}$$

where $\mathbf{Enc}(\cdot,\cdot)$ is the encryption function; $\oplus$ is the XOR logical operator; $\vec{\nu}_{k,h}^{(m)}$ is the plaintext; $\kappa_{k,h}^{(m)}$ is the key sequence; and $c_{k,h}^{(m)}$ is the ciphertext to be transmitted through the communication channel.

Decryption and Decoding. This process consists of two steps: 1) decrypting the ciphertext by applying the XOR operation; and 2) restoring the binary data (i.e., plaintext) to the decimal data (i.e., decoded value) according to the decoding rule.

The decryption rule is described by

$$\check{\nu}_{k,h}^{(m)} = \mathbf{Dec}\left(c_{k,h}^{(m)}, \kappa_{k,h}^{(m)}\right) = c_{k,h}^{(m)} \oplus \kappa_{k,h}^{(m)} \tag{6}$$

for $m = 1, 2, \ldots, n_x$, where $\mathbf{Dec}(\cdot,\cdot)$ is the decryption function and $\check{\nu}_{k,h}^{(m)}$ is the plaintext. According to the rule of the XOR logical operation, one has

$$\check{\nu}_{k,h}^{(m)} = \vec{\nu}_{k,h}^{(m)} \oplus \kappa_{k,h}^{(m)} \oplus \kappa_{k,h}^{(m)} = \vec{\nu}_{k,h}^{(m)}. \tag{7}$$

Then, the binary bit string $\check{\nu}_{k,h}^{(m)}$ is converted to the decimal signal $\nu_{k,h}^{(m)}$.

In order to recover the system signal, the decoding rule is given as

$$\vec{x}_{k,h}^{(m)} = \mathfrak{a}_{k-1,h}^{(m,1)}\vec{x}_{k-1,h}^{(m)} + \mathfrak{a}_{k,h-1}^{(m,2)}\vec{x}_{k,h-1}^{(m)} + \mathfrak{b}_{k,h}^{(m)}\nu_{k,h}^{(m)} \tag{8}$$

where $\vec{x}_{k,h}^{(m)} \in \mathbb{R}$ is the mth entry of the decoder output $\vec{x}_{k,h}$ and the initial condition is given as $\vec{x}_{0,h}^{(m)} = \vec{x}_{k,0}^{(m)} = \wp^{(m)}$.

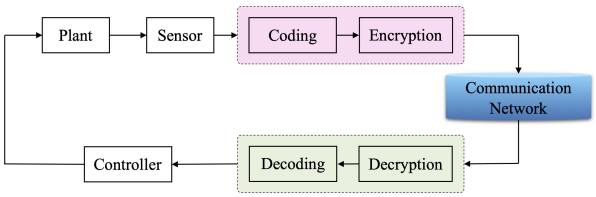

Fig. 1: Architecture of PPM-based control for NCSs.

The controller based on *PPM* is designed in this paper as

$$u_{k,h} = F_{k,h}\vec{x}_{k,h} \tag{9}$$

where $F_{k,h}$ is the controller gain to be determined. Then, by defining

$$e_{k,h} \triangleq \vec{x}_{k,h} - x_{k,h} \tag{10}$$

as the decoding error and substituting (9)–(10) into (1), we arrive at the following closed-loop system:

$$\begin{aligned} x_{k+1,h+1} = \vec{A}_{k,h+1}^{[1]}x_{k,h+1} + \vec{A}_{k+1,h}^{[2]}x_{k+1,h} + \vec{B}_{k,h+1}^{[1]}e_{k,h+1} \\ \vec{B}_{k+1,h}^{[2]}e_{k+1,h} + E^{[1]}w_{k,h+1} + E^{[2]}w_{k+1,h} \end{aligned} \tag{11}$$

where, for $o = 1, 2$,

$$\vec{A}_{k,h}^{[o]} \triangleq A^{[o]} + \vec{B}_{k,h}^{[o]}, \quad \vec{B}_{k,h}^{[o]} \triangleq B^{[o]}F_{k,h}.$$

Remark 3: The proposed PPM exhibits the following characteristics. 1) The integration of dynamic coding-decoding technology within the PPM framework effectively mitigates the burden associated with network communication; and 2) The incorporation of XOR logical operation and OTP method in PPM ensures data privacy preservation.

Definition 1: System (11) is said to be bounded in mean-square over a finite horizon, if, for $0 \le k \le K$ and $0 \le h \le H$, there exists a positive scalar $\beta_{k,h}$ such that

$$\mathbb{E}\left\{x_{k,h}^{\mathrm{T}}x_{k,h}\right\} \le \beta_{k,h}. \tag{12}$$

The objectives of this paper are outlined as follows.

1) Design a desired PPM-based controller for 2-D systems by tackling the optimization problem:

$$\begin{aligned} \mathbf{OP}: \ & \min_{F_{k,h}} \ \beta_{k,h} \\ & \text{s.t.} \ \ (1), (2) \text{ and } (12) \end{aligned} \tag{13}$$

such that the probabilistic constraint is satisfied and the mean-square boundedness is guaranteed.

2) Examine the secrecy behavior of the proposed PPM.

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III. Main Results

The subsequent lemmas play a vital role in deriving the main results.

Lemma 1: (*Matrix Inverse Lemma* [36]) For given matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ with proper dimensions, assume that $\mathcal{A}$, $\mathcal{B}$ and $\vec{\mathcal{B}}$ are invertible. Then,

$$(\mathcal{A}+\mathcal{C}\mathcal{B}\mathcal{D})^{-1}=\mathcal{A}^{-1}-\mathcal{A}^{-1}\mathcal{C}\vec{\mathcal{B}}^{-1}\mathcal{D}\mathcal{A}^{-1} \tag{14}$$

where

$$\vec{\mathcal{B}}\triangleq\mathcal{B}^{-1}+\mathcal{D}\mathcal{A}^{-1}\mathcal{C}.$$

Lemma 2: (*Chebyshev Inequality* [3]) Let $\delta\in\mathbb{R}^{n_\delta}$ be a random vector whose covariance is $\Delta>0$. For any $0<\varepsilon\leq 1$, one has

$$\mathbf{Pr}\left\{(\delta-\mathbb{E}\{\delta\})^{\mathrm{T}}\Delta^{-1}(\delta-\mathbb{E}\{\delta\})\leq\frac{n_\delta}{\varepsilon}\right\}\geq 1-\varepsilon. \tag{15}$$

Lemma 3: Consider the decoding error $e_{k,h}$ in (10). Then, one has

$$e_{k,h}^{\mathrm{T}}e_{k,h}\leq\sum_{m=1}^{n_x}\left(\frac{\mathfrak{b}_{k,h}^{(m)}r}{2^l-1}\right)^2. \tag{16}$$

Proof: See Appendix. ∎

A. The Probabilistic Constraint

The subsequent theorem offers a sufficient condition to ensure that the probabilistic constraint (2) is met.

Theorem 1: Let the scalar $\varepsilon>0$ and the matrices $W_{k,h}>0$, $\Phi_{k,h}>0$ be given. The probability constraint (2) is fulfilled if, for $0\leq k\leq K$, $0\leq h\leq H$, and $\epsilon\delta_{k,h}\delta_{k,h}^{\mathrm{T}}\leq n_x\vec{W}_{k,h}$, there exist positive scalars $\tau_{k,h}$ and matrices $F_{k,h}$ such that

$$\begin{bmatrix}-\Phi_{k+1,h+1} & * & * \\ \Psi_{k,h}^{\mathrm{T}} & -\vec{\varepsilon}_{k,h}I & * \\ \vec{W}_{k,h}^{\frac{1}{2}} & 0 & -\tau_{k,h}I\end{bmatrix}\leq 0 \tag{17}$$

holds, where

$$\begin{aligned}
\Psi_{k,h}&\triangleq\begin{bmatrix}\vec{A}_{k,h+1}^{[1]} & \vec{A}_{k+1,h}^{[2]} & -A^{[1]} & -A^{[2]}\end{bmatrix}\\
\vec{\varepsilon}_{k,h}&\triangleq\frac{1}{\rho_{k,h}}(1-\tau_{k,h}\frac{n_x}{\epsilon}),\quad \epsilon\triangleq\frac{\varepsilon}{(K+1)(H+1)}\\
\rho_{k,h}&\triangleq\vec{x}_{k,h+1}^{\mathrm{T}}\vec{x}_{k,h+1}+\vec{x}_{k+1,h}^{\mathrm{T}}\vec{x}_{k+1,h}+\ell_{k,h}\\
\ell_{k,h}&\triangleq\sum_{m=1}^{n_x}\left(\left(\frac{\mathfrak{b}_{k,h+1}^{(m)}r}{2^l}\right)^2+\left(\frac{\mathfrak{b}_{k+1,h}^{(m)}r}{2^l}\right)^2\right)\\
\vec{W}_{k,h}&\triangleq\mathscr{E}^{\mathrm{T}}\mathscr{W}_{k,h}\mathscr{E},\quad \mathscr{E}\triangleq\begin{bmatrix}(E^{[1]})^{\mathrm{T}}\\(E^{[2]})^{\mathrm{T}}\end{bmatrix}\\
\mathscr{W}_{k,h}&\triangleq\mathrm{diag}\left\{W_{k,h+1},W_{k+1,h}\right\}.
\end{aligned}$$

Proof: First, we rewrite the closed-loop system (11) as

$$x_{k+1,h+1}=\bar{x}_{k+1,h+1}+E^{[1]}w_{k,h+1}+E^{[2]}w_{k+1,h} \tag{18}$$

where

$$\begin{aligned}
\bar{x}_{k+1,h+1}&\triangleq\Psi_{k,h}\zeta_{k,h}\\
\zeta_{k,h}&\triangleq\mathrm{col}\{\vec{x}_{k,h+1},\vec{x}_{k+1,h},e_{k,h+1},e_{k+1,h}\}.
\end{aligned}$$

Letting

$$\delta_{k+1,h+1}\triangleq x_{k+1,h+1}-\bar{x}_{k+1,h+1},$$

we calculate that

$$\begin{aligned}
&\mathbb{E}\{\delta_{k+1,h+1}\}=0\\
&\mathbb{E}\{\delta_{k+1,h+1}\delta_{k+1,h+1}^{\mathrm{T}}\}=\vec{W}_{k,h}.
\end{aligned}$$

Next, we define a function as follows:

$$\begin{aligned}
\mathscr{H}_{k+1,h+1}\triangleq 1-x_{k+1,h+1}^{\mathrm{T}}\Phi_{k+1,h+1}^{-1}x_{k+1,h+1}-\tau_{k,h}\Big(\frac{n_x}{\epsilon}\\
-\delta_{k+1,h+1}^{\mathrm{T}}\vec{W}_{k,h}^{-1}\delta_{k+1,h+1}\Big).
\end{aligned} \tag{19}$$

By utilizing the completing-the-square technique with respect to $x_{k+1,h+1}$, (19) is rewritten as

$$\begin{aligned}
\mathscr{H}_{k+1,h+1}=(x_{k+1,h+1}-x_{k+1,h+1}^{\star})^{\mathrm{T}}\mathcal{M}_{k,h}(x_{k+1,h+1}\\
-x_{k+1,h+1}^{\star})-\acute{x}_{k+1,h+1}^{\mathrm{T}}(\mathcal{M}_{k,h})^{-1}\acute{x}_{k+1,h+1}\\
+z_{k+1,h+1}
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
x_{k+1,h+1}^{\star}&\triangleq(\mathcal{M}_{k,h})^{-1}\acute{x}_{k+1,h+1}\\
\mathcal{M}_{k,h}&\triangleq\tau_{k,h}\vec{W}_{k,h}^{-1}-\Phi_{k+1,h+1}^{-1}\\
\acute{x}_{k+1,h+1}&\triangleq\tau_{k,h}\vec{W}_{k,h}^{-1}\bar{x}_{k+1,h+1}\\
z_{k+1,h+1}&\triangleq 1+\tau_{k,h}\bar{x}_{k+1,h+1}^{\mathrm{T}}\vec{W}_{k,h}^{-1}\bar{x}_{k+1,h+1}-\tau_{k,h}\frac{n_x}{\epsilon}.
\end{aligned}$$

Applying Lemma 1 (Matrix Inverse Lemma) to the term $\mathcal{M}_{k,h}$, one has

$$\begin{aligned}
\mathcal{M}_{k,h}^{-1}&=(\tau_{k,h}\vec{W}_{k,h}^{-1}-\Phi_{k+1,h+1}^{-1})^{-1}\\
&=\tau_{k,h}^{-1}\vec{W}_{k,h}+\tau_{k,h}^{-1}\vec{W}_{k,h}\vec{\Phi}_{k+1,h+1}^{-1}\tau_{k,h}^{-1}\vec{W}_{k,h}
\end{aligned} \tag{21}$$

where

$$\vec{\Phi}_{k+1,h+1}\triangleq\Phi_{k+1,h+1}-\tau_{k,h}^{-1}\vec{W}_{k,h}.$$

Then, by substituting (21) into (20), one has

$$\begin{aligned}
&-\acute{x}_{k+1,h+1}^{\mathrm{T}}(\mathcal{M}_{k,h})^{-1}\acute{x}_{k+1,h+1}+z_{k+1,h+1}\\
&=1-\tau_{k,h}\frac{n_x}{\epsilon}+\tau_{k,h}\bar{x}_{k+1,h+1}^{\mathrm{T}}\vec{W}_{k,h}^{-1}\bar{x}_{k+1,h+1}-(\tau_{k,h}\\
&\quad\times\vec{W}_{k,h}^{-1}\bar{x}_{k+1,h+1})^{\mathrm{T}}(\tau_{k,h}^{-1}\vec{W}_{k,h}+\tau_{k,h}^{-1}\vec{W}_{k,h}\vec{\Phi}_{k+1,h+1}^{-1}\\
&\quad\times\tau_{k,h}^{-1}\vec{W}_{k,h})(\tau_{k,h}\vec{W}_{k,h}^{-1}\bar{x}_{k+1,h+1})\\
&=1-\tau_{k,h}\frac{n_x}{\epsilon}+\tau_{k,h}\bar{x}_{k+1,h+1}^{\mathrm{T}}\vec{W}_{k,h}^{-1}\bar{x}_{k+1,h+1}\\
&\quad-\bar{x}_{k+1,h+1}^{\mathrm{T}}(\tau_{k,h}\vec{W}_{k,h}^{-1}+\vec{\Phi}_{k+1,h+1}^{-1})\bar{x}_{k+1,h+1}\\
&=1-\tau_{k,h}\frac{n_x}{\epsilon}-\bar{x}_{k+1,h+1}^{\mathrm{T}}\vec{\Phi}_{k+1,h+1}^{-1}\bar{x}_{k+1,h+1}.
\end{aligned} \tag{22}$$

By further substituting (22) into (20), we obtain

$$\begin{aligned}
\mathscr{H}_{k+1,h+1}=(x_{k+1,h+1}-x_{k+1,h+1}^{\star})^{\mathrm{T}}\mathcal{M}_{k,h}(x_{k+1,h+1}\\
-x_{k+1,h+1}^{\star})+1-\tau_{k,h}\frac{n_x}{\epsilon}-\bar{x}_{k+1,h+1}^{\mathrm{T}}\\
\times\vec{\Phi}_{k+1,h+1}^{-1}\bar{x}_{k+1,h+1}.
\end{aligned} \tag{23}$$

With the aid of Schur Complement Lemma [2], we acquire from (17) that

$$\Phi_{k+1,h+1}-\frac{\Psi_{k,h}\Psi_{k,h}^{\mathrm{T}}}{\vec{\varepsilon}_{k,h}}-\frac{\vec{W}_{k,h}}{\tau_{k,h}}\geq 0. \tag{24}$$

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By leveraging Lemma 3, we have

$$\zeta_{k,h}^{\mathrm{T}}\zeta_{k,h} \leq \rho_{k,h} \tag{25}$$

which further implies that

$$\zeta_{k,h}\zeta_{k,h}^{\mathrm{T}} \leq \rho_{k,h} I. \tag{26}$$

Then, one has

$$\begin{aligned}\bar{x}_{k+1,h+1}\bar{x}_{k+1,h+1}^{\mathrm{T}} &= \Psi_{k,h}\zeta_{k,h}\zeta_{k,h}^{\mathrm{T}}\Psi_{k,h}^{\mathrm{T}} \\ &\leq \rho_{k,h}\Psi_{k,h}\Psi_{k,h}^{\mathrm{T}}. \end{aligned}\tag{27}$$

Combining (24) and (27), one has

$$\Phi_{k+1,h+1} - \frac{\bar{x}_{k+1,h+1}\bar{x}_{k+1,h+1}^{\mathrm{T}}}{1-\tau_{k,h}\frac{n_x}{\epsilon}} - \frac{\vec{W}_{k,h}}{\tau_{k,h}} \geq 0. \tag{28}$$

Utilizing Schur Complement Lemma again, we obtain from (28) that

$$\begin{bmatrix} 1-\tau_{k,h}\frac{n_x}{\epsilon} & * \\ \bar{x}_{k+1,h+1} & \Phi_{k+1,h+1} - \tau_{k,h}^{-1}\vec{W}_{k,h} \end{bmatrix} \geq 0 \tag{29}$$

and, similarly, we rewrite (29) as

$$1 - \tau_{k,h}\frac{n_x}{\epsilon} - \bar{x}_{k+1,h+1}^{\mathrm{T}}\vec{\Phi}_{k+1,h+1}^{-1}\bar{x}_{k+1,h+1} \geq 0 \tag{30}$$

with $\vec{\Phi}_{k+1,h+1} \triangleq \Phi_{k+1,h+1} - \tau_{k,h}^{-1}\vec{W}_{k,h}$.

Combining (23) and (30) together, it is obvious that

$$\mathscr{H}_{k+1,h+1} \geq 0. \tag{31}$$

By means of an S-procedure Lemma [2], it follows from (19) and (31) that

$$\begin{aligned}&\frac{n_x}{\epsilon} - \delta_{k+1,h+1}^{\mathrm{T}}\vec{W}_{k,h}^{-1}\delta_{k+1,h+1} \geq 0 \\ \Rightarrow\ & 1 - x_{k+1,h+1}^{\mathrm{T}}\Phi_{k+1,h+1}^{-1}x_{k+1,h+1} \geq 0 \end{aligned}\tag{32}$$

which further implies

$$\begin{aligned}&\mathbf{Pr}\left\{x_{k+1,h+1}^{\mathrm{T}}\Phi_{k+1,h+1}^{-1}x_{k+1,h+1} \leq 1\right\} \\ \geq\ &\mathbf{Pr}\left\{\delta_{k+1,h+1}^{\mathrm{T}}\vec{W}_{k,h}^{-1}\delta_{k+1,h+1} \leq \frac{n_x}{\epsilon}\right\}. \end{aligned}\tag{33}$$

According to Lemma 2 (Chebyshev Inequality), one has

$$\mathbf{Pr}\left\{\delta_{k+1,h+1}^{\mathrm{T}}\vec{W}_{k,h}^{-1}\delta_{k+1,h+1} \leq \frac{n_x}{\epsilon}\right\} \geq 1-\epsilon. \tag{34}$$

Then, from (33)–(34), it is easy to obtain

$$\mathbf{Pr}\left\{x_{k+1,h+1}^{\mathrm{T}}\Phi_{k+1,h+1}^{-1}x_{k+1,h+1} \leq 1\right\} \geq 1-\epsilon \tag{35}$$

which is equivalent to

$$\mathbf{Pr}\left\{x_{k+1,h+1}^{\mathrm{T}}\Phi_{k+1,h+1}^{-1}x_{k+1,h+1} > 1\right\} < \epsilon. \tag{36}$$

With the aid of Boole's inequality technique and combining (36), we have

$$\begin{aligned}&\mathbf{Pr}\left\{\bigcup_{h=0}^{H}\bigcup_{k=0}^{K} x_{k,h}^{\mathrm{T}}\Phi_{k,h}^{-1}x_{k,h} > 1\right\} \\ \leq\ &\sum_{h=0}^{H}\sum_{k=0}^{K}\mathbf{Pr}\left\{x_{k,h}^{\mathrm{T}}\Phi_{k,h}^{-1}x_{k,h} > 1\right\} \\ <\ &\sum_{h=0}^{H}\sum_{k=0}^{K}\epsilon \leq \varepsilon \end{aligned}\tag{37}$$

which leads to

$$\mathbf{Pr}\left\{\bigcap_{h=0}^{H}\bigcap_{k=0}^{K} x_{k,h}^{\mathrm{T}}\Phi_{k,h}^{-1}x_{k,h} \leq 1\right\} \geq 1-\varepsilon. \tag{38}$$

Thus, the probabilistic constraint (2) is satisfied, which completes the proof. ■

B. The Mean-Square Boundedness

The mean-square boundedness of the system is guaranteed by the following theorem.

Theorem 2: Let the positive-definite matrix $W_{k,h}$ be given. Suppose that there exist positive scalars $\lambda_{k,h}$, $\pi_{k,h}$, $\varpi_{k,h}$, $\phi_{k,h}$, positive-definite matrices $P_{k,h}$, $Q_{k,h}$, and matrices F_k such that

$$\begin{bmatrix} -\varpi_{k,h} I & * \\ \vec{\mathscr{E}} & -\vec{\mathscr{Q}}_{k,h} \end{bmatrix} \leq 0 \tag{39}$$

$$\begin{bmatrix} -\vec{\lambda}_{k,h}\mathscr{Q}_{k,h} & * & * \\ 0 & -\pi_{k,h} I & * \\ \vec{\mathscr{A}}_{k,h} & \vec{\mathscr{B}}_{k,h} & -\vec{\mathscr{Q}}_{k,h} \end{bmatrix} \leq 0 \tag{40}$$

$$\begin{bmatrix} -\lambda_{k,h}\mathscr{Q}_{k,h} & * & * \\ 0 & -\vec{\phi}_{k,h} I & * \\ \mathscr{A}_{k,h} & \mathscr{B}_{k,h} & -I \end{bmatrix} \leq 0 \tag{41}$$

$$-(\phi_{k,h} - \varpi_{k,h})I + \mathscr{E}\mathscr{E}^{\mathrm{T}} \leq 0 \tag{42}$$

where

$$\begin{aligned}&\mathscr{A}_{k,h} \triangleq \begin{bmatrix} \vec{A}_{k,h+1}^{[1]} & \vec{A}_{k+1,h}^{[2]} \end{bmatrix},\ \mathscr{B}_{k,h} \triangleq \begin{bmatrix} \vec{B}_{k,h+1}^{[1]} & \vec{B}_{k+1,h}^{[2]} \end{bmatrix} \\ &\vec{\mathscr{A}}_{k,h} \triangleq \begin{bmatrix} \mathscr{A}_{k,h} \\ \mathscr{A}_{k,h} \end{bmatrix},\ \vec{\mathscr{B}}_{k,h} \triangleq \begin{bmatrix} \mathscr{B}_{k,h} \\ \mathscr{B}_{k,h} \end{bmatrix},\ \vec{\mathscr{E}} \triangleq \begin{bmatrix} \mathscr{E} \\ \mathscr{E} \end{bmatrix} \\ &\mathscr{Q}_{k,h} \triangleq \mathrm{diag}\{P_{k,h+1}, Q_{k+1,h}\},\ \ \vec{\phi}_{k,h} \triangleq \phi_{k,h} - \pi_{k,h} \\ &\vec{\mathscr{Q}}_{k,h} \triangleq \mathrm{diag}\{P_{k+1,h+1}^{-1}, Q_{k+1,h+1}^{-1}\},\ \ \vec{\lambda}_{k,h} \triangleq 1 - \lambda_{k,h}. \end{aligned}$$

Then, the system (11) is mean-square bounded with the upper bound as

$$\phi_{k,h}(\ell_{k,h} + \mathrm{Tr}(\mathscr{W}_{k,h})). \tag{43}$$

Proof: First, we define a function of the following form:

$$\begin{aligned}\mathscr{J}_{k+1,h+1} \triangleq\ & V_{k+1,h+1} - \vec{\lambda}_{k,h}\left(V_{k,h+1}^{[1]} + V_{k+1,h}^{[2]}\right) \\ &- \pi_{k,h}\vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} - \varpi_{k,h}\vec{w}_{k,h}^{\mathrm{T}}\vec{w}_{k,h} \end{aligned}\tag{44}$$

where

$$\begin{aligned}V_{k,h} &\triangleq V_{k,h}^{[1]} + V_{k,h}^{[2]} \\ V_{k,h}^{[1]} &\triangleq x_{k,h}^{\mathrm{T}}P_{k,h}x_{k,h} \\ V_{k,h}^{[2]} &\triangleq x_{k,h}^{\mathrm{T}}Q_{k,h}x_{k,h} \\ \vec{e}_{k,h} &\triangleq \mathrm{col}\left\{e_{k,h+1}, e_{k+1,h}\right\} \\ \vec{w}_{k,h} &\triangleq \mathrm{col}\left\{w_{k,h+1}, w_{k+1,h}\right\}. \end{aligned}$$

It follows from the mathematical expectation of (44) that

$$\begin{aligned}&\mathbb{E}\left\{\mathscr{J}_{k+1,h+1}\right\} \\ =\ &\mathbb{E}\left\{V_{k+1,h+1} - \vec{\lambda}_{k,h}\left(V_{k,h+1}^{[1]} + V_{k+1,h}^{[2]}\right)\right\} \end{aligned}$$

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$$
\begin{aligned}
&- \pi_{k,h}\vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} - \varpi_{k,h}\mathbb{E}\{\vec{w}_{k,h}^{\mathrm{T}}\vec{w}_{k,h}\} \\
&= \vartheta_{k,h}^{\mathrm{T}}\Gamma_{k,h}^{\mathrm{T}}(P_{k+1,h+1}+Q_{k+1,h+1})\Gamma_{k,h}\vartheta_{k,h} \\
&\quad + \mathrm{Tr}\left(\vec{W}_{k,h}(P_{k+1,h+1}+Q_{k+1,h+1})\right) - \vec{\lambda}_{k,h}x_{k,h+1}^{\mathrm{T}} \\
&\quad \times P_{k,h+1}x_{k,h+1} - \vec{\lambda}_{k,h}x_{k+1,h}^{\mathrm{T}}Q_{k+1,h}x_{k+1,h} \\
&\quad - \pi_{k,h}\vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} - \varpi_{k,h}\mathrm{Tr}(\mathscr{W}_{k,h}) \\
&= \vartheta_{k,h}^{\mathrm{T}}\vec{\Gamma}_{k,h}\vartheta_{k,h} + \mathrm{Tr}\Big(\vec{W}_{k,h}(P_{k+1,h+1}+Q_{k+1,h+1})\Big) \\
&\quad - \mathrm{Tr}\Big(\varpi_{k,h}\mathscr{W}_{k,h}\Big) \qquad (45)
\end{aligned}
$$

where

$$
\begin{aligned}
&\vartheta_{k,h} \triangleq \mathrm{col}\{x_{k,h+1}, x_{k+1,h}, e_{k,h+1}, e_{k+1,h}\} \\
&\Gamma_{k,h} \triangleq \begin{bmatrix} \vec{A}_{k,h+1}^{[1]} & \vec{A}_{k+1,h}^{[2]} & \vec{B}_{k,h+1}^{[1]} & \vec{B}_{k+1,h}^{[2]} \end{bmatrix} \\
&\vec{\Gamma}_{k,h} \triangleq \Gamma_{k,h}^{\mathrm{T}}(P_{k+1,h+1}+Q_{k+1,h+1})\Gamma_{k,h} \\
&\qquad - \mathrm{diag}\left\{\vec{\lambda}_{k,h}P_{k,h+1}, \vec{\lambda}_{k,h}Q_{k+1,h}, \pi_{k,h}I, \pi_{k,h}I\right\}.
\end{aligned}
$$

By leveraging Schur Complement Lemma, it follows from (39), (40) and (45) that

$$\mathbb{E}\left\{\mathscr{J}_{k+1,h+1}\right\} \leq 0 \qquad (46)$$

which is further derived that

$$
\begin{aligned}
\mathbb{E}\left\{\Delta V_{k,h}\right\} \leq &- \lambda_{k,h}\mathbb{E}\left\{V_{k,h+1}^{[1]} + V_{k+1,h}^{[2]}\right\} \\
&+ \pi_{k,h}\vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} + \varpi_{k,h}\mathrm{Tr}(\mathscr{W}_{k,h}) \qquad (47)
\end{aligned}
$$

where

$$\Delta V_{k,h} \triangleq V_{k+1,h+1} - V_{k,h+1}^{[1]} - V_{k+1,h}^{[2]}.$$

For the case

$$\mathbb{E}\left\{V_{k,h+1}^{[1]} + V_{k+1,h}^{[2]}\right\} > \lambda_{k,h}^{-1}\left(\pi_{k,h}\ell_{k,h} + \varpi_{k,h}\mathrm{Tr}(\mathscr{W}_{k,h})\right),$$

according to Lemma 3 and (47), we have

$$
\begin{aligned}
\mathbb{E}\left\{\Delta V_{k,h}\right\} < &- \pi_{k,h}\ell_{k,h} - \varpi_{k,h}\mathrm{Tr}(\mathscr{W}_{k,h}) \\
&+ \pi_{k,h}\vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} + \varpi_{k,h}\mathrm{Tr}(\mathscr{W}_{k,h}) \\
&< 0. \qquad (48)
\end{aligned}
$$

Then, as per (48), it is easy to verify that

$$\mathbb{E}\left\{V_{k,h+1}^{[1]} + V_{k+1,h}^{[2]}\right\} \leq \lambda_{k,h}^{-1}(\pi_{k,h}\ell_{k,h} + \varpi_{k,h}\mathrm{Tr}(\mathscr{W}_{k,h})) \qquad (49)$$

is satisfied.

To further analyze the upper bound of the mean-square boundedness, we define a function as follows:

$$
\begin{aligned}
\mathscr{L}_{k+1,h+1} \triangleq &\, x_{k+1,h+1}^{\mathrm{T}}x_{k+1,h+1} - \lambda_{k,h}\Big(V_{k,h+1}^{[1]} \\
&+ V_{k+1,h}^{[2]}\Big) - (\phi_{k,h}-\pi_{k,h})\vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} \\
&- (\phi_{k,h}-\varpi_{k,h})\vec{w}_{k,h}^{\mathrm{T}}\vec{w}_{k,h}. \qquad (50)
\end{aligned}
$$

Then, through the use of the stochastic analysis technique, one gets

$$
\begin{aligned}
&\mathbb{E}\{\mathscr{L}_{k+1,h+1}\} \\
&= \mathbb{E}\{x_{k+1,h+1}^{\mathrm{T}}x_{k+1,h+1}\} - \lambda_{k,h}\mathbb{E}\left\{V_{k,h+1}^{[1]} + V_{k+1,h}^{[2]}\right\} \\
&\quad - (\phi_{k,h}-\pi_{k,h})\vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} - (\phi_{k,h}-\varpi_{k,h})\mathbb{E}\{\vec{w}_{k,h}^{\mathrm{T}}\vec{w}_{k,h}\} \\
&= \vartheta_{k,h}^{\mathrm{T}}\Gamma_{k,h}^{\mathrm{T}}\Gamma_{k,h}\vartheta_{k,h} + \mathrm{Tr}(\vec{W}_{k,h}) - \lambda_{k,h}x_{k,h+1}^{\mathrm{T}}P_{k,h+1} \\
&\quad \times x_{k,h+1} - \lambda_{k,h}x_{k+1,h}^{\mathrm{T}}Q_{k+1,h}x_{k+1,h} - (\phi_{k,h}-\pi_{k,h}) \\
&\quad \times \vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} - (\phi_{k,h}-\varpi_{k,h})\mathrm{Tr}(\mathscr{W}_{k,h}) \\
&= \vartheta_{k,h}^{\mathrm{T}}\tilde{\Gamma}_{k,h}\vartheta_{k,h} + \mathrm{Tr}(\vec{W}_{k,h}) - (\phi_{k,h}-\varpi_{k,h})\mathrm{Tr}(\mathscr{W}_{k,h}) \qquad (51)
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{\Gamma}_{k,h} \triangleq &\, \Gamma_{k,h}^{\mathrm{T}}\Gamma_{k,h} - \mathrm{diag}\{\lambda_{k,h}P_{k,h+1}, \lambda_{k,h}Q_{k+1,h}, \\
&(\phi_{k,h}-\pi_{k,h})I, (\phi_{k,h}-\pi_{k,h})I\}.
\end{aligned}
$$

From (41)–(42), it is easy to deduce that

$$\mathbb{E}\left\{\mathscr{L}_{k+1,h+1}\right\} \leq 0. \qquad (52)$$

Then, it follows from (49)–(50) and (52) that

$$
\begin{aligned}
&\mathbb{E}\left\{x_{k+1,h+1}^{\mathrm{T}}x_{k+1,h+1}\right\} \\
&\leq \lambda_{k,h}\mathbb{E}\left\{V_{k,h+1}^{[1]} + V_{k+1,h}^{[2]}\right\} + (\phi_{k,h}-\pi_{k,h})\vec{e}_{k,h}^{\mathrm{T}}\vec{e}_{k,h} \\
&\quad + (\phi_{k,h}-\varpi_{k,h})\mathbb{E}\{\vec{w}_{k,h}^{\mathrm{T}}\vec{w}_{k,h}\} \\
&\leq \pi_{k,h}\ell_{k,h} + \varpi_{k,h}\mathrm{Tr}(\mathscr{W}_{k,h}) + (\phi_{k,h}-\pi_{k,h})\ell_{k,h} \\
&\quad + (\phi_{k,h}-\varpi_{k,h})\mathrm{Tr}(\mathscr{W}_{k,h}) \\
&= \phi_{k,h}(\ell_{k,h} + \mathrm{Tr}(\mathscr{W}_{k,h})) \qquad (53)
\end{aligned}
$$

which implies that (11) is indeed bounded in mean-square. ■

Theorem 3: Let the positive scalar ε and the positive-definite matrices $W_{k,h}$, $\Phi_{k,h}$ be given. For $0 \leq k \leq H$ and $0 \leq h \leq H$, suppose that there exist matrices F_k such that the optimization problem

$$
\begin{aligned}
\textbf{OP}: \ &\min_{F_{k,h}} \ \beta_{k,h} \\
&\text{s.t.} \ \ (1), (17), (39)\text{–}(42) \qquad (54)
\end{aligned}
$$

is feasible, where

$$\beta_{k,h} \triangleq \phi_{k,h}(\ell_{k,h} + \mathrm{Tr}(\mathscr{W}_{k,h})).$$

Then, the probabilistic constraint and the mean-square boundedness are guaranteed.

Proof: The proof is straightforward and hence omitted for brevity. ■

Algorithm 1 PPM-based control algorithm

1: **Input**. $\mathfrak{a}_{k,h}^{(m,1)}$, $\mathfrak{a}_{k,h}^{(m,2)}$, $\mathfrak{b}_{k,h}^{(m)}$, $\kappa_{k,h}^{(m)}$
2: **Output**. $F_{k,h}$
3: *Step 1*. Utilizing PPM (3)–(8) to process system signals.
4: *Step 2*. Calculate the controller gain by solving the **OP** in Theorem 3.
5: **Return**. The new system state signal.

Remark 4: In Theorems 1 and 2, sufficient conditions for the validity of the probabilistic constraint and the mean-square boundedness problems have been derived through successfully navigating the challenges posed by the PPM and the stochastic noises. Through the use of contemporary techniques such as matrix analysis technique (e.g., matrix inverse lemma), and stochastic analysis technique (e.g., Chebyshev inequality and Boole's inequality), the procedure to obtain the controller gains has been deduced.

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C. Analysis of the Privacy Performance

When system data is transmitted through the digital communication channel from the sensor side to the controller side, the eavesdropper may intercept the data and then obtain the system state information. The detailed decipherment process is described as follows.

Eavesdropper: The encryption-decryption method (except the key sequence $\kappa_{k,h}^{(m)}$), the coding-decoding rule, and the parameters (i.e., $\wp^{(m)}$, $\mathfrak{a}_{k,h}^{(m,1)}$, $\mathfrak{a}_{k,h}^{(m,2)}$, and $\mathfrak{b}_{k,h}^{(m)}$, for $m = 1, 2, \ldots, n_x$) are *public* knowledge which can be obtained by the eavesdropper. To obtain the system state information, the eavesdropper first deciphers the intercepted data by using the following decryption rule:

$$\hat{\boldsymbol{\nu}}_{k,h}^{(m)} = \mathbf{Dec}\left(c_{k,h}^{(m)}, \hat{\boldsymbol{\kappa}}_{k,h}^{(m)}\right) = c_{k,h}^{(m)} \oplus \hat{\boldsymbol{\kappa}}_{k,h}^{(m)} \tag{55}$$

for $m = 1, 2, \ldots, n_x$, where $c_{k,h}^{(m)}$ is the ciphertext intercepted by the eavesdropper, $\hat{\boldsymbol{\kappa}}_{k,h}^{(m)}$ is the key sequence used by the eavesdropper, and $\hat{\boldsymbol{\nu}}_{k,h}^{(m)}$ is the plaintext deciphered by the eavesdropper. Then, for subsequent decoding operation, the eavesdropper converts the binary bit string $\hat{\boldsymbol{\nu}}_{k,h}^{(m)}$ to the decimal data $\tilde{\boldsymbol{\nu}}_{k,h}^{(m)}$.

Next, the eavesdropper aims to decode the system state information by applying the following rule:

$$\hat{x}_{k,h}^{(m)} = \mathfrak{a}_{k-1,h}^{(m,1)}\hat{x}_{k-1,h}^{(m)} + \mathfrak{a}_{k,h-1}^{(m,2)}\hat{x}_{k,h-1}^{(m)} + \mathfrak{b}_{k,h}^{(m)}\tilde{\boldsymbol{\nu}}_{k,h}^{(m)} \tag{56}$$

where $\hat{x}_{k,h}^{(m)} \in \mathbb{R}$ is the mth entry of the eavesdropper's decoder output $\hat{x}_{k,h}$, and the initial condition is given as $\hat{x}_{0,h}^{(m)} = \hat{x}_{k,0}^{(m)} = \wp^{(m)}$. Here, $\hat{x}_{k,h}$ is system state information deciphered by the eavesdropper.

Clearly, the above-mentioned data leakage (or eavesdropping) problem would jeopardize the security of the system. Accordingly, the PPM is proposed in section II-B for guaranteeing the confidentiality and security of data transmission. In what follows, we endeavor to analyze the *secrecy* performance of the proposed PPM.

Before continuing, we first provide the following definition.

Definition 2: The PPM (3)–(8) is said to achieve the *secrecy* if the system dynamics deciphered by the eavesdropper satisfies

$$\lim_{\substack{k\to\infty \\ h\to\infty}} \|\hat{x}_{k,h}\| = \infty. \tag{57}$$

Theorem 4: Let $\mathfrak{a}_{k,h}^{(m,1)} > 1$, $\mathfrak{a}_{k,h}^{(m,2)} > 1$ and $\mathfrak{b}_{k,h}^{(m)} > 0$ be given parameters. If there exist a $m \in \{1, 2, \ldots, n_x\}$ and $k, h \in \mathbb{N}$ such that

$$\hat{\boldsymbol{\kappa}}_{k,h}^{(m)} \oplus \kappa_{k,h}^{(m)} \neq 0, \tag{58}$$

then the secrecy performance is achieved.

Proof: First, it follows from (6) and (55) that

$$\begin{aligned}\hat{\boldsymbol{\nu}}_{k,h}^{(m)} &= \mathbf{Dec}\left(c_{k,h}^{(m)}, \hat{\boldsymbol{\kappa}}_{k,h}^{(m)}\right) = c_{k,h}^{(m)} \oplus \hat{\boldsymbol{\kappa}}_{k,h}^{(m)} \\ &= c_{k,h}^{(m)} \oplus \hat{\boldsymbol{\kappa}}_{k,h}^{(m)} \oplus \kappa_{k,h}^{(m)} \oplus \kappa_{k,h}^{(m)} \\ &= \check{\nu}_{k,h}^{(m)} \oplus \hat{\boldsymbol{\kappa}}_{k,h}^{(m)} \oplus \kappa_{k,h}^{(m)}.\end{aligned} \tag{59}$$

Then, according to (58), it is obvious that the

$$\tilde{\boldsymbol{\nu}}_{k,h}^{(m)} \neq \nu_{k,h}^{(m)}. \tag{60}$$

That is, the decimal plaintext deciphered by the eavesdropper is not equal to the real decimal plaintext.

By defining $\psi_{k,h}^{(m)} \triangleq \tilde{\boldsymbol{\nu}}_{k,h}^{(m)} - \nu_{k,h}^{(m)}$ and $\xi_{k,h}^{(m)} \triangleq \hat{x}_{k,h}^{(m)} - \vec{x}_{k,h}^{(m)}$, we have

$$\xi_{k,h}^{(m)} = \mathfrak{a}_{k-1,h}^{(m,1)}\xi_{k-1,h}^{(m)} + \mathfrak{a}_{k,h-1}^{(m,2)}\xi_{k,h-1}^{(m)} + \mathfrak{b}_{k,h}^{(m)}\psi_{k,h}^{(m)}. \tag{61}$$

It follows from (8), (56), (58), and (61) that

$$\begin{cases} \xi_{k+1,h}^{(m)} = \mathfrak{a}_{k,h}^{(m,1)}\mathfrak{b}_{k,h}^{(m)}\psi_{k,h}^{(m)} \\ \xi_{k+2,h}^{(m)} = \mathfrak{a}_{k+1,h}^{(m,1)}\mathfrak{a}_{k,h}^{(m,1)}\mathfrak{b}_{k,h}^{(m)}\psi_{k,h}^{(m)} \\ \quad\vdots \\ \xi_{k+\mathcal{K},h}^{(m)} = \left(\prod_{\imath=0}^{\mathcal{K}-1}\mathfrak{a}_{k+\imath,h}^{(m,1)}\right)\mathfrak{b}_{k,h}^{(m)}\psi_{k,h}^{(m)} \end{cases} \tag{62}$$

and

$$\begin{cases} \xi_{k,h+1}^{(m)} = \mathfrak{a}_{k,h}^{(m,2)}\mathfrak{b}_{k,h}^{(m)}\psi_{k,h}^{(m)} \\ \xi_{k,h+1}^{(m)} = \mathfrak{a}_{k,h+1}^{(m,2)}\mathfrak{a}_{k,h}^{(m,2)}\mathfrak{b}_{k,h}^{(m)}\psi_{k,h}^{(m)} \\ \quad\vdots \\ \xi_{k,h+\mathcal{H}}^{(m)} = \left(\prod_{\jmath=0}^{\mathcal{H}-1}\mathfrak{a}_{k,h+\jmath}^{(m,2)}\right)\mathfrak{b}_{k,h}^{(m)}\psi_{k,h}^{(m)} \end{cases} \tag{63}$$

where $\mathcal{K}$ and $\mathcal{H}$ are positive integers. Then, combining with (61)–(63), one has

$$\begin{aligned}&\xi_{k+\mathcal{K},h+\mathcal{H}}^{(m)} \\ &= \mathfrak{a}_{k+\mathcal{K}-1,h+\mathcal{H}}^{(m,1)}\xi_{k+\mathcal{K}-1,h+\mathcal{H}}^{(m)} + \mathfrak{a}_{k+\mathcal{K},h+\mathcal{H}-1}^{(m,2)}\xi_{k+\mathcal{K},h+\mathcal{H}-1}^{(m)} \\ &\quad + \mathfrak{b}_{k+\mathcal{K},h+\mathcal{H}}^{(m)}\psi_{k+\mathcal{K},h+\mathcal{H}}^{(m)} \\ &= \mathfrak{a}_{k+\mathcal{K}-1,h+\mathcal{H}}^{(m,1)}\mathfrak{a}_{k+\mathcal{K}-2,h+\mathcal{H}}^{(m,1)}\xi_{k+\mathcal{K}-2,h+\mathcal{H}}^{(m)} + \mathfrak{a}_{k+\mathcal{K}-1,h+\mathcal{H}}^{(m,1)} \\ &\quad \times \mathfrak{a}_{k+\mathcal{K}-1,h+\mathcal{H}-1}^{(m,1)}\xi_{k+\mathcal{K}-1,h+\mathcal{H}-1}^{(m)} + \mathfrak{a}_{k+\mathcal{K}-1,h+\mathcal{H}}^{(m,1)} \\ &\quad \times \mathfrak{b}_{k+\mathcal{K}-1,h+\mathcal{H}}^{(m)}\psi_{k+\mathcal{K}-1,h+\mathcal{H}}^{(m)} + \mathfrak{a}_{k+\mathcal{K},h+\mathcal{H}-1}^{(m,2)} \\ &\quad \times \mathfrak{a}_{k+\mathcal{K}-1,h+\mathcal{H}-1}^{(m,2)}\xi_{k+\mathcal{K}-1,h+\mathcal{H}-1}^{(m)} + \mathfrak{a}_{k+\mathcal{K},h+\mathcal{H}-1}^{(m,2)} \\ &\quad \times \mathfrak{a}_{k+\mathcal{K},h+\mathcal{H}-2}^{(m,2)}\xi_{k+\mathcal{K},h+\mathcal{H}-2}^{(m)} + \mathfrak{a}_{k+\mathcal{K},h+\mathcal{H}-1}^{(m,2)} \\ &\quad \times \mathfrak{b}_{k+\mathcal{K},h+\mathcal{H}-1}^{(m)}\psi_{k+\mathcal{K},h+\mathcal{H}-1}^{(m)} \\ &\quad\vdots \\ &= \sum_{\imath=k+1}^{k+\mathcal{K}}\Theta_{k+\mathcal{K},h+\mathcal{H}}^{(m)}(k+\mathcal{K}-\imath, h+\mathcal{H}-1)\mathfrak{a}_{\imath,h}^{(m,1)}\xi_{\imath,h}^{(m)} \\ &\quad + \sum_{\imath=k+1}^{k+\mathcal{K}}\Theta_{k+\mathcal{K},h+\mathcal{H}}^{(m)}(k+\mathcal{K}-1, h+\mathcal{H}-\jmath)\mathfrak{a}_{k,\jmath}^{(m,2)}\xi_{k,\jmath}^{(m)}\end{aligned} \tag{64}$$

where

$$\begin{aligned}&\Theta_{0,0}^{(m)}(\imath,\jmath) = I, \quad \imath \geq 0, \quad \jmath \geq 0 \\ &\Theta_{k,h}^{(m)}(\imath,\jmath) = 0, \quad k < 0,\ h < 0,\ \imath < 0, \text{ or } \jmath < 0 \\ &\Theta_{k,h}^{(m)}(\imath,\jmath) = \mathfrak{a}_{k-1,h}^{(m,1)}\Theta_{k-1,h}^{(m)}(\imath-1,\jmath) \\ &\qquad\qquad + \mathfrak{a}_{k,h-1}^{(m,2)}\Theta_{k,h-1}^{(m)}(\imath,\jmath-1).\end{aligned}$$

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Next, we analyze the dynamics of $\xi^{(m)}_{k+\mathcal{K},h+\mathcal{H}}$ from the following two cases: i) $\psi^{(m)}_{k,h} > 0$ and ii) $\psi^{(m)}_{k,h} < 0$.

For the first case (i.e., $\psi^{(m)}_{k,h} > 0$), we have

$$\begin{cases} \xi^{(m)}_{k+\imath,h} > 0, & \imath = 1,2,\ldots,\mathcal{K} \\ \xi^{(m)}_{k,h+\jmath} > 0, & \jmath = 1,2,\ldots,\mathcal{H}. \end{cases} \tag{65}$$

Then, it is deduced from (64)–(65) that

$$\begin{aligned} &\xi^{(m)}_{k+\mathcal{K},h+\mathcal{H}} \\ &= \sum_{\imath=k+1}^{k+\mathcal{K}} \Theta^{(m)}_{k+\mathcal{K},h+\mathcal{H}}(k+\mathcal{K}-\imath, h+\mathcal{H}-1)\mathfrak{a}^{(m,1)}_{\imath,h}\xi^{(m)}_{\imath,h} \\ &\quad + \sum_{\imath=k+1}^{k+\mathcal{K}} \Theta^{(m)}_{k+\mathcal{K},h+\mathcal{H}}(k+\mathcal{K}-1, h+\mathcal{H}-\jmath)\mathfrak{a}^{(m,2)}_{k,\jmath}\xi^{(m)}_{k,\jmath} \\ &> \Theta^{(m)}_{k+\mathcal{K},h+\mathcal{H}}(0, h+\mathcal{H}-1)\mathfrak{a}^{(m,1)}_{k+\mathcal{K},h}\xi^{(m)}_{k+\mathcal{K},h} \\ &\quad + \Theta^{(m)}_{k+\mathcal{K},h+\mathcal{H}}(k+\mathcal{K}-1, 0)\mathfrak{a}^{(m,2)}_{k,h+\mathcal{H}}\xi^{(m)}_{k,h+\mathcal{H}} \\ &> \mathfrak{a}^{(m,1)}_{k+\mathcal{K},h}\xi^{(m)}_{k+\mathcal{K},h} + \mathfrak{a}^{(m,2)}_{k,h+\mathcal{H}}\xi^{(m)}_{k,h+\mathcal{H}} \\ &= \left(\prod_{\imath=0}^{\mathcal{K}} \mathfrak{a}^{(m,1)}_{k+\imath,h} + \prod_{\jmath=0}^{\mathcal{H}} \mathfrak{a}^{(m,2)}_{k,h+\jmath}\right) \mathfrak{b}^{(m)}_{k,h}\psi^{(m)}_{k,h}. \end{aligned} \tag{66}$$

By considering

$$\mathfrak{a}^{(m,1)}_{k+\imath,h} > 1,\ \mathfrak{a}^{(m,2)}_{k,h+\jmath} > 1,\ \mathfrak{b}^{(m)}_{k,h} > 0,$$

it is easy to derive that

$$\lim_{\substack{\mathcal{K}\to\infty \\ \mathcal{H}\to\infty}} \xi^{(m)}_{k+\mathcal{K},h+\mathcal{H}} = +\infty. \tag{67}$$

Similarly, for the second case (i.e., $\psi^{(m)}_{k,h} < 0$), we have

$$\lim_{\substack{\mathcal{K}\to\infty \\ \mathcal{H}\to\infty}} \xi^{(m)}_{k+\mathcal{K},h+\mathcal{H}} = -\infty. \tag{68}$$

On the other hand, according to Theorem 2, it is apparent to see

$$\mathbb{E}\left\{\|x_{k,h}\|\right\} \le \sqrt{\bar{\beta}} \tag{69}$$

where

$$\bar{\beta} \triangleq 2\phi\left(n_x\left(\frac{\bar{\mathfrak{b}}r}{2^l}\right)^2 + \mathrm{Tr}(\bar{W})\right).$$

Together with (67)–(69), one has

$$\begin{aligned} \lim_{\substack{k\to\infty \\ h\to\infty}} \hat{x}^{(m)}_{k,h} &= \lim_{\substack{k\to\infty \\ h\to\infty}} \left(\xi^{(m)}_{k,h} + \vec{x}^{(m)}_{k,h}\right) \\ &= \lim_{\substack{k\to\infty \\ h\to\infty}} \left(\xi^{(m)}_{k,h} + x^{(m)}_{k,h} + e^{(m)}_{k,h}\right) \\ &= \infty. \end{aligned} \tag{70}$$

Then, it is deduced that

$$\lim_{\substack{k\to\infty \\ h\to\infty}} \|\hat{x}_{k,h}\| = \infty, \tag{71}$$

which ends the proof. ∎

Remark 5: Thus far, the PPM-based controller design issue has been handled for the considered NCSs with guaranteed probability. Here are the summary of the *characteristics* of our results: 1) the addressed privacy preserving control problem is new, which takes probabilistic constraints, bandwidth constraints, and privacy preserving into consideration; 2) the proposed PPM is new, which is able to preserve the data privacy and alleviate the network communication burden; and 3) the privacy performance analysis is new, through which the feasibility of privacy preserving of the proposed PPM is ensured.

Remark 6: In this article, the PPM-based control problem has been systematically studied for networked systems subject to probabilistic constraints and communication bandwidth constraints. The probabilistic constraint index, mean-square boundedness of the states, and privacy performance of the PPM have been analyzed and guaranteed simultaneously. With the aid of some up-to-date techniques, the corresponding PPM-based controller gains have been characterized by solving an optimization problem.

IV. Illustrative Example

Two simulation examples are presented in this section to demonstrate the effectiveness of the proposed control algorithm based on PPM.

A. Example 1

The parameters of system (1) are

$$A^{[1]} = \begin{bmatrix} 0.6 & 0 \\ -0.2 & 0.1 \end{bmatrix},\quad A^{[2]} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$$
$$B^{[1]} = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix},\ B^{[2]} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix},\ E^{[1]} = \begin{bmatrix} 0.15 \\ 0.1 \end{bmatrix},\ E^{[2]} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

For the probabilistic constraint, the maximum probability value of violating the index (2) is $\varepsilon = 0.1$ and the positive-definite matrices $\Phi_{k,h}$ are given as

$$\Phi_{k,h} = \begin{cases} \mathrm{diag}\{2.25, 4\}, & (0,h) \text{ and } (k,0), k \ge 0, h \ge 0 \\ \mathrm{diag}\{1.44, 2.25\}, & (1,h) \text{ and } (k,1), k \ge 1, h \ge 1 \\ \mathrm{diag}\{1, 1\}, & (2,h) \text{ and } (k,2), k \ge 2, h \ge 2 \\ \mathrm{diag}\{0.25, 0.25\}, & (3,h) \text{ and } (k,3), k \ge 3, h \ge 3 \\ \mathrm{diag}\{0.3^2, 0.25^2\}, & \text{others.} \end{cases}$$

The variance of the additive stochastic noise $w_{k,h}$ is $0.5I$. In the PPM, the parameters are, respectively, set as $\wp^{(m)} = \begin{bmatrix} 2 & 1.2 \end{bmatrix}^{\mathrm{T}}$, $\mathfrak{a}^{(m,1)}_{k,h} = \mathfrak{a}^{(m,2)}_{k,h} = 1.5$, $\mathfrak{b}^{(m)}_{k,h} = 1$, and $l = 8$.

Figs. 2–8 present the simulation results. Fig. 2 clearly shows that the open-loop 2-D system is unstable with the specified parameters. State trajectories of the closed-loop 2-D system are shown in Fig.3 indicating that the proposed controller achieves the desired performance. Additionally, to demonstrate the effectiveness of the proposed probabilistic constraint, we conducted 100 rounds of simulations and plotted the state trajectories for $k = 4$ in Figs. 4–5.

Fig. 6 shows the amplitude changes of the system signal $x_{k,h}$, the coder output $\nu_{k,h}$, and the decoder output $\vec{x}_{k,h}$. It is clear that the amplitude of the coder output is smaller than the system signal. In this case, one can use less bits to code the transmitted information (i.e., $\nu_{k,h}$), which means that

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the proposed PPM can mitigate the burden associated with network communication.

In this simulation, the eavesdropper uses a false key sequence $\hat{\boldsymbol{\kappa}}_{k,h}^{(m)}$ at the instant $(14, 14)$, i.e., $\hat{\boldsymbol{\kappa}}_{14,14}^{(m)} \oplus \kappa_{14,14}^{(m)} \neq 0$ $(m = 1, 2)$. From Figs. 7–8, it is clear to know that the state dynamic behaviors deciphered by the eavesdropper are unbounded, but the actual state dynamic behaviors of the system are bounded, which means that the secrecy performance of the proposed PPM is ensured. In addition, the average runtime of the PPM at every instant is only 0.002 seconds on a standard 2.7 GHz Intel Core i7 processor, which does not incur a long time-delay. Thus, the proposed PPM is a computationally efficient approach.

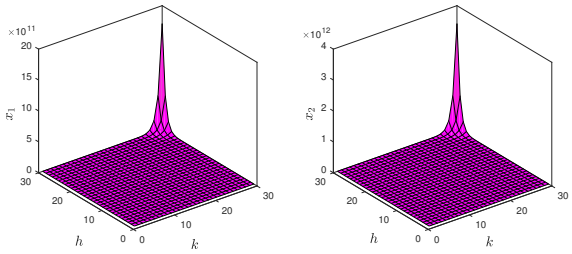

Fig. 2: State variable x of the open-loop system.

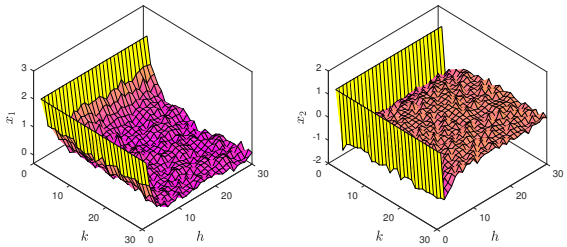

Fig. 3: State variable x of the closed-loop system.

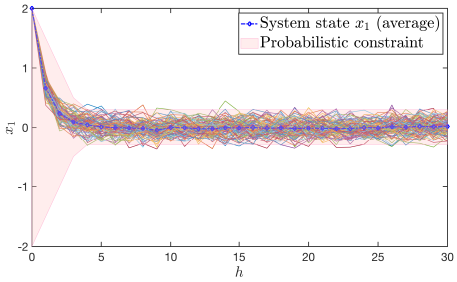

Fig. 4: System state x_1 under probabilistic constraint in 100 rounds of stochastic simulations for $k = 4$.

B. Example 2 (Metal Rolling Process)

In this example, a practical example called metal rolling process (shown in Fig. 9) is used to illustrate the efficacy of the proposed control algorithm.

The model of the metal rolling process is

$$\ddot{\chi}_{k,t} + \frac{\mathbf{c}}{M}\chi_{k,t} = \frac{\mathbf{c}}{\mathbf{s}}\ddot{\chi}_{k-1,t} + \frac{\mathbf{c}}{M}\chi_{k-1,t} - \frac{\mathbf{c}}{M\mathbf{h}}F_M$$

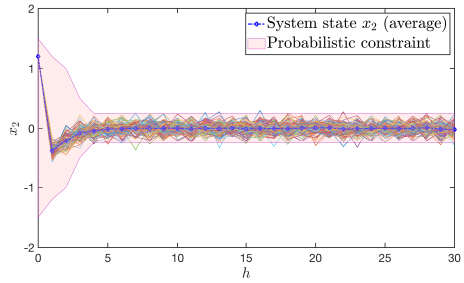

Fig. 5: System state x_2 under probabilistic constraint in 100 rounds of stochastic simulations for $k = 4$.

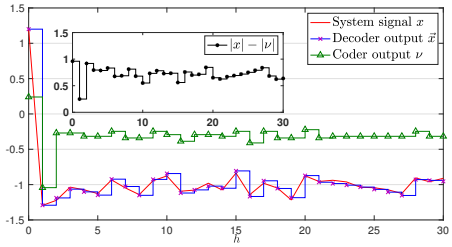

Fig. 6: System signal, decoder output and coder output.

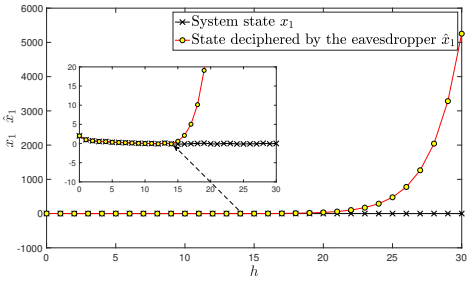

Fig. 7: System state x_1 and state information $\hat{x}_1$ deciphered by the eavesdropper.

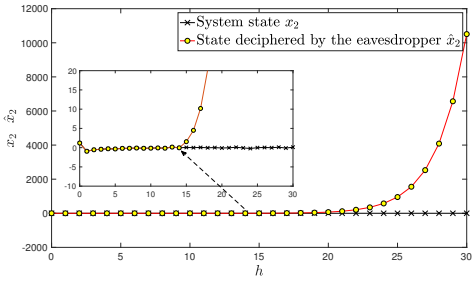

Fig. 8: System state x_2 and state information $\hat{x}_2$ deciphered by the eavesdropper.

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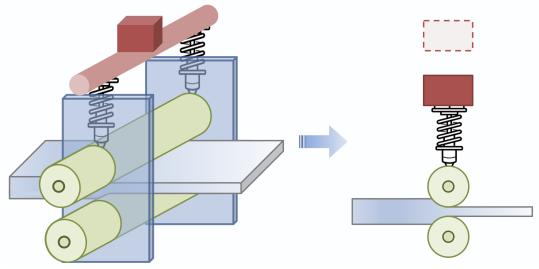

Fig. 9: Metal rolling process.

where $\chi_{k,t}$ denotes the kth roll-gap thickness at time instant t and F_M is the force of the motor. The meanings and values of the other variables are specified in Table I.

Variable	Meaning	Value
M	mass of the roll-gap adjusting mechanism	50kg
$\mathbf{s}$	stiffness of the adjusting mechanism spring	3000N/mm
$\mathbf{h}$	hardness of the metal strip	500N/mm
$\mathbf{c}$	composite stiffness ($\frac{\mathbf{sh}}{\mathbf{s+h}}$)	$\frac{3000}{7}$N/mm

TABLE I: Variables in the metal rolling process.

By leveraging the backward difference technique, the differential equation can be converted into the 2-D system, the parameters are

$$A^{[1]} = \begin{bmatrix} 1 & -0.0299 & 0.2090 & -0.1045 & 0.0149 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^{[2]} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -0.0299 & 0.2090 & -0.1045 & 0.0149 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B^{[1]} = \begin{bmatrix} -0.0018 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$B^{[2]} = \begin{bmatrix} 0 & 0 & -0.0018 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$E^{[1]} = \begin{bmatrix} -0.02 & 0 & 0.03 & 0 & 0.1 \end{bmatrix}^{\mathrm{T}}$$

$$E^{[2]} = \begin{bmatrix} 0 & 0.1 & 0.05 & 0 & 0.1 \end{bmatrix}^{\mathrm{T}}.$$

For the probabilistic constraint, the maximum probability value of violating the index (2) is $\varepsilon = 0.05$ and the positive-definite matrices $\Phi_{k,h}$ are given as

$$\Phi_{k,h} = \begin{cases} \mathrm{diag}\{4,4\}, & (0,h) \text{ and } (k,0), k \le 5, h \le 5 \\ \mathrm{diag}\{0.2,0.2\}, & \text{others.} \end{cases}$$

In addition, the variance of the additive stochastic noise $w_{k,h}$ is $0.35I$. In the PPM, the parameters are, respectively, set as $\wp^{(m)} = \begin{bmatrix} 1 & 1.2 & -1 & 0.5 & -0.5 \end{bmatrix}^{\mathrm{T}}$, $\mathfrak{a}_{k,h}^{(m,1)} = 1.2$, $\mathfrak{a}_{k,h}^{(m,2)} = 1.8$, $\mathfrak{b}_{k,h}^{(m)} = 4$, and $l = 8$.

By utilizing the controller gains calculated through Theorem 3, the control results of the meta rolling process are displayed in Figs. 10–11, which shows that the proposed control algorithm achieves a satisfactory level of performance under the effects of stochastic noises and PPM. Fig. 12 shows that the proposed PPM can reduce the network communication burden.

In this metal rolling process, the eavesdropper uses a false key sequence $\hat{\boldsymbol{\kappa}}_{k,h}^{(m)}$ at the instant $(9,10)$, i.e., $\hat{\boldsymbol{\kappa}}_{9,10}^{(m)} \oplus \kappa_{9,10}^{(m)} \neq 0$ $(m = 1,2,\ldots,5)$. Fig. 13 shows that the state dynamic behaviors deciphered by the eavesdropper are unbounded, but the actual state dynamic behaviors of the system are bounded, which means that the secrecy performance of the proposed PPM is ensured.

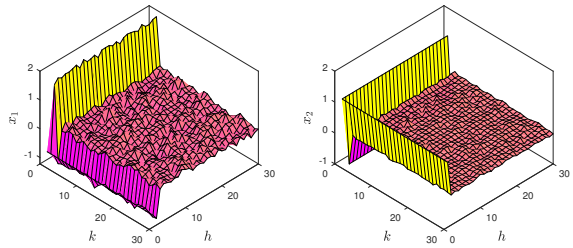

Fig. 10: State variable x of the closed-loop system.

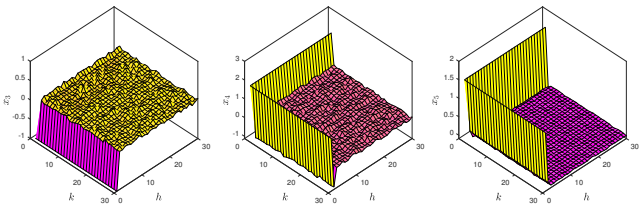

Fig. 11: State variable x of the closed-loop system.

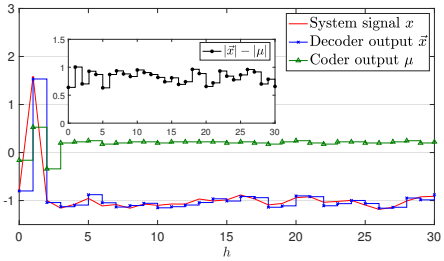

Fig. 12: System signal, decoder output and coder output.

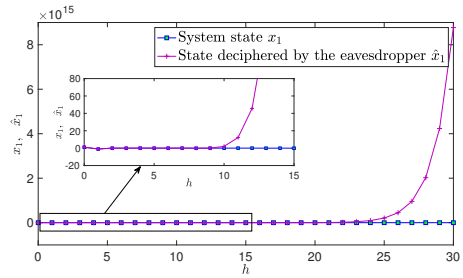

Fig. 13: System state x_1 and state information $\hat{x}_1$ deciphered by the eavesdropper.

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V. Conclusion

This paper has proposed a PPM-based control strategy for 2-D systems with probabilistic constraints, considering privacy preservation and communication efficiency for NCSs. The proposed PPM is novel, which ensures mean-square boundedness and privacy performance under the probabilistic constraints. The control scheme has been designed to guarantee probabilistic constraints, mean-square boundedness and privacy performance. Sufficient conditions have been presented to compute the required controller parameters, and simulation results have demonstrated the effectiveness of the proposed control scheme. In practical engineering, there remain several complex and significant challenges, such as nonlinear systems and distributed systems, that have yet to be explored within a unified framework of 2-D systems. Furthermore, additional control approaches, such as the proportional-integral-derivative control technique, warrant further investigation.

Appendix
The Proof of Lemma 3

Proof: This lemma is proved by leveraging the mathematical induction technique, which includes the following two steps, namely, initial step and inductive step.

Step 1: It follows from the initial conditions in (3) and (8) that $\eta_{0,h}^{(m)} = \vec{x}_{0,h}^{(m)}$ and $\eta_{k,0}^{(m)} = \vec{x}_{k,0}^{(m)}$.

Step 2: Suppose that $\eta_{k-1,h}^{(m)} = \vec{x}_{k-1,h}^{(m)}$ and $\eta_{k,h-1}^{(m)} = \vec{x}_{k,h-1}^{(m)}$ are true. Then, we need to show that $\eta_{k,h}^{(m)} = \vec{x}_{k,h}^{(m)}$ is also true. In fact, we obtain

$$\begin{aligned}
&\eta_{k,h}^{(m)} - \vec{x}_{k,h}^{(m)} \\
&= \mathfrak{a}_{k-1,h}^{(m,1)} \eta_{k-1,h}^{(m)} + \mathfrak{a}_{k,h-1}^{(m,2)} \eta_{k,h-1}^{(m)} + \mathfrak{b}_{k,h}^{(m)} \nu_{k,h}^{(m)} \\
&\quad - \mathfrak{a}_{k-1,h}^{(m,1)} \vec{x}_{k-1,h}^{(m)} - \mathfrak{a}_{k,h-1}^{(m,2)} \vec{x}_{k,h-1}^{(m)} - \mathfrak{b}_{k,h}^{(m)} \nu_{k,h}^{(m)} \\
&= 0.
\end{aligned} \tag{72}$$

Thus, according to the mathematical induction, one has

$$\eta_{k,h}^{(m)} = \vec{x}_{k,h}^{(m)}, \quad \forall\, k, h \in \mathbb{N}, \quad m = 1, 2, \ldots, n_x. \tag{73}$$

Next, it follows from (3), (8) and (73) that

$$\begin{aligned}
e_{k,h}^{(m)} &= \vec{x}_{k,h}^{(m)} - x_{k,h}^{(m)} \\
&= \mathfrak{a}_{k-1,h}^{(m,1)} \vec{x}_{k-1,h}^{(m)} + \mathfrak{a}_{k,h-1}^{(m,2)} \vec{x}_{k,h-1}^{(m)} + \mathfrak{b}_{k,h}^{(m)} \nu_{k,h}^{(m)} - x_{k,h}^{(m)} \\
&= \mathfrak{a}_{k-1,h}^{(m,1)} \vec{x}_{k-1,h}^{(m)} + \mathfrak{a}_{k,h-1}^{(m,2)} \vec{x}_{k,h-1}^{(m)} + x_{k,h}^{(m)} - \mathfrak{a}_{k-1,h}^{(m,1)} \eta_{k-1,h}^{(m)} \\
&\quad - \mathfrak{a}_{k,h-1}^{(m,2)} \eta_{k,h-1}^{(m)} + \mathfrak{b}_{k,h}^{(m)} e_{k,h}^{(m,q)} - x_{k,h}^{(m)}
\end{aligned} \tag{74}$$

which indicates that

$$\left| e_{k,h}^{(m)} \right| \leq \frac{\mathfrak{b}_{k,h}^{(m)} r}{2^l - 1} \tag{75}$$

with $e_{k,h}^{(m,q)}$ being the quantization error of quantizer (4). Then, it is obvious that

$$e_{k,h}^{\mathrm{T}} e_{k,h} \leq \sum_{m=1}^{n_x} \left(\frac{\mathfrak{b}_{k,h}^{(m)} r}{2^l - 1} \right)^2. \tag{76}$$

Therefore, the proof is complete. ■

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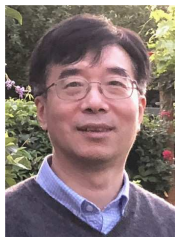

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