This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



# Local Design of Distributed State Estimators for Linear Discrete Time-Varying Systems Over Binary Sensor Networks: A Set-Membership Approach

Fei Han, *Member, IEEE*, Zidong Wang, *Fellow, IEEE*, Hongjian Liu, Hongli Dong, *Senior Member, IEEE*, and Guoping Lu

***Abstract*—This paper is concerned with the distributed set-membership estimation problem for a class of discrete time-varying systems over binary sensor networks. For binary sensors, the cases of fixed and time-varying thresholds are considered. In both cases, the information useful for state estimation purposes is extracted by utilizing the crossings of binary measurements at two adjacent time instants, and then distributed estimators are constructed for each sensor node with the aid of the available measurements, where a set of vector saturation functions is introduced to resist the adverse effect of outliers during signal transmission. A novel distributed set-membership performance index is provided by averaging over the ellipsoidal constraints of all sensor nodes, and the local performance analysis method is employed to establish sufficient criteria that guarantee the existence of desired estimators whose parameters are then derived for every node by recursively optimizing certain ellipsoids in the sense of matrix trace. The applicability and feasibility of the distributed set-membership schemes developed in this paper are verified by two illustrative examples.**

***Index Terms*—Binary sensors, sensor networks, binary measurements, distributed set-membership estimation, local performance analysis.**

## I. INTRODUCTION

Over the past two decades, the ever-growing popularity of sensor networks (SNs) has made it possible to collect a huge amount of information through sensors densely deployed in the interested region [2], [5], [6], [21], [23], [25], [30], [39]. Accordingly, a key issue with SNs is about how to acquire robust yet reliable state estimates of the monitored plants through available network measurements. In response to the large scale of SNs, the state estimation algorithms are expected to be scalable and this has demanded the rapid development of *distributed* estimation techniques whose idea is to use each sensor to estimate the system state via local communications among neighboring nodes [42], [45], [48]. In fact, owing to their distinctive advantages in improving computation efficiency and saving communication resources, the distribution estimation techniques have recently received a surge of research attention with various algorithms appeared in the literature, see e.g. distributed Kalman filtering schemes [31], distributed $H_\infty$ estimation strategies [36], distributed fusion estimation [16], [19], distributed moving-horizon estimation methods [3], [4], and distributed set-membership estimation techniques [20], [24], [29].

Binary sensors have been widely utilized in nowadays SN mainly because of their cost-effectiveness [33], [38] and small overhead in terms of energy and bandwidth in communication [2]. Up to now, binary SNs have found many successful applications in engineering practice including consensus control [38], source localization [2], tracking control [13], system identification [33], [37], [37], privacy protection [34], etc. For more applications of binary sensor, the readers are referred to [37] and the references therein. As a binary sensor can only provide *one bit* of measurement output, it appears to be especially challenging as how to extract useful/beneficial information from such extremely coarse measurements. To date, some initial efforts have been devoted to extract *functional information* from binary measurements (BMs) which could help capture the dynamics involved in the sensor outputs, see [3], [15] via the switching of BMs in case of deterministic noises and [38], [40] via a certain distributed function in case of random noises [38], [40]. Also, much research interest has recently been focused on many BM-related dynamics analysis problems which include, but are not limited to, consensus control [38], source localization [2], system identification [40], [41], and parameter and state estimation problems [3], [4], [15], [49].

In the context of binary sensors, the thresholds determining the binary outputs have played a vitally important role in the related dynamics analysis problems. Up to now, most existing results concerning BMs have been obtained based on *fixed* thresholds for analysis convenience and easy implementation [3], [4], [15], [33], [49], [52]. For binary sensors with such fixed thresholds, the corresponding measurement outputs contain very limited useful information that can be utilized for

This work was supported in part by the National Natural Science Foundation of China under Grants 62073070, U21A2019, 61933007, and 62273005, in part by the Hainan Province Science and Technology Special Fund of China under Grant ZDYF2022SHFZ105, and in part by the Alexander Von Humboldt Foundation of Germany. *(Corresponding author: Hongli Dong.)*

Fei Han and Hongli Dong are with the State Key Laboratory of Continental Shale Oil, Northeast Petroleum University, Daqing 163318, China, also with the Sanya Offshore Oil & Gas Research Institute, Northeast Petroleum University, Sanya 572025, China, also with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing, 163318, China, and also with the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Northeast Petroleum University, Daqing 163318, China. (Emails: tomcumt@126.com, shiningdhl@vip.126.com)

Zidong Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk)

Hongjian Liu is with the Key Laboratory of Advanced Perception and Intelligent Control of High-end Equipment, Ministry of Education, Anhui Polytechnic University, Wuhu 241000, China, and also with the School of Mathematics and Physics, Anhui Polytechnic University, Wuhu 241000, China. (Email: hjliu1980@gmail.com)

Guoping Lu is with the School of Electrical Engineering, Nantong University, Nantong 226019, China. (Email: lu.gp@ntu.edu.cn)

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



state estimation purposes. In this case, it makes both practical and theoretical sense to look into *time-varying* thresholds so as to facilitate the extraction of more functional information with the increasing crossing times of BMs. In this regard, some pioneering results have been published on a number of dynamics analysis issues, see e.g. distributed Kalman filtering [31], parameter estimation [47], system identification [41], [53], tracking control [13], and recursive estimation [7]. In particular, the threshold has been designed in [41] based on the estimated parameter at the current instant.

When the underlying system is subjected to deterministic norm-bounded noises, the set-membership estimation is regarded to be an efficient approach whose aim is to use the available measurements to recursively calculate a bounding ellipsoid containing the accurate states, see [50] and the references therein. In relation to SNs, the distributed set-membership estimation issue has recently attracted a surge of research interest [20], [28], [29], [44], and many excellent algorithms have been developed for systems undergoing various imperfect measurements that might be induced by coding-decoding communications [24], cyber-attacks [25], [46], and measurement saturations [43]. However, when it comes to the coarsest measurements such as BMs, the corresponding results have been really scattered, and this gives rise to another motivation for the current investigation.

Outliers, as a kind of abnormalities that are significantly deviated from their normal value, are often encountered in industrial applications as a result of deception attacks, sensor failures, environmental sudden changes, impulsive noises with heavy tails, etc. [1]. As is well known, the distributed estimation is realized via the information exchange among neighboring sensor nodes and, if the transmitted information is contaminated by outliers, the performance of the distributed estimators might be seriously impaired [2], [35]. As such, it is critically important to reduce the adverse outlier-induced impact and, along this line, an effective way is to introduce certain delicately designed functions (e.g. the Huber function [35] and the saturation function [10], [11], [51]) to restrain the effect of possible outliers. Following this idea, this paper aims to propose a novel saturation function to accomplish the outlier-resistant design of the BM-based distributed estimators.

Summarizing the discussions made thus far, we conclude that it is of great theoretical and practical significance to conduct a major study on examining the impact of the BMs on the desired performance of the distributed set-membership estimators. In doing so, we are faced with the fundamental issues as follows: 1) how to extract the information from BMs and reduce the conservatism stemming from the fixed threshold in terms of estimation accuracy? 2) how to resist the harmful effect of the possible outliers during the information transmission? 3) how to establish an adequate performance index that accounts for the distributed set-membership estimation over the entire SN? and 4) how to further develop local conditions for every sensor node to deal with distributed set-membership estimation issues with BMs? Accordingly, the main objective of this paper is to tackle these identified issues.

The main novelties of this paper are highlighted as follows: *1) the distributed set-membership estimation problem is, for the first time, investigated for a class of discrete time-varying systems over binary SNs; 2) a new time-varying threshold for binary sensors is purposely designed to improve the estimation accuracy; 3) a novel performance index regarding the distributed set-membership estimation is proposed in the average sense over all the sensor nodes; and 4) indicator-variable-dependent conditions are derived for each sensor node to guarantee the average performance of the distributed set-membership estimation in both cases of fixed and time-varying thresholds.*

The structure of this paper is outlined as follows. In Section II, the underlying system and the BMs are formulated, and the time-varying threshold strategy is proposed. Furthermore, a new method is developed to extract the information from BMs, and a novel average performance index is put forward to reflect the overall distributed set-membership estimation scheme. The main results under two kinds of thresholds are presented in Sections III and IV, respectively. Two numerical simulation examples are provided in Section V to demonstrate the effectiveness of distributed set-membership estimation schemes developed in this paper. Section VI concludes this paper by pointing out some future research directions.

## II. Problem Formulation

A digraph $\mathcal{G}(\mathcal{V},\mathcal{S},\mathcal{A})$ is utilized to describe the communication topology of the SN considered in this paper. Specifically, $\mathcal{V} \triangleq \{1,2,\cdots,N\}$ denotes the sensor node set, $\mathcal{S} \triangleq \{(i,j) : i,j \in \mathcal{V}\}$ indicates the edge set, and $\mathcal{A} \triangleq [a_{ij}]_{N\times N}$ refers to the adjacency matrix. Moreover, $(i,j) \in \mathcal{S}$ if and only if $a_{ij} > 0$. If a directed edge $(i,j) \in \mathcal{S}$, then $j$ is called a neighbor of $i$ that has access to the information from sensor node $j$. For sensor $i$, all its neighbors are denoted as $\mathcal{N}_i \triangleq \{j \in \mathcal{V} : (i,j) \in \mathcal{S}, j \neq i\}$. To be specific, $\mathcal{N}_i \triangleq \{j_{i_1}, j_{i_2}, \ldots, j_{i_{p_i}}\}$ with $p_i \triangleq \sum_{j=1}^{N} a_{ij}$ and $q_i \triangleq \sum_{j=1}^{N} a_{ji}$ being the in-degree and out-degree of sensor node $i$, respectively. Assume that the digraph is weakly connected, which means that there exists an undirected path from every node to other node. For more details about weakly connected digraph, the readers are referred to [36].

Consider a class of linear discrete time-varying plants:

$$\begin{cases} x_{s+1} = A_s x_s + B_s w_s, \\ y_{i,s} = C_{i,s} x_s + D_{i,s} v_{i,s} \end{cases} \tag{1}$$

where $x_s \in \mathbb{R}^{n_x}$ represents the system state to be estimated; $y_{i,s} \in \mathbb{R}$ denotes the input to binary sensor node $i$ that cannot be observed directly; $w_s \in \mathbb{R}^{n_w}$ and $v_{i,s} \in \mathbb{R}^{n_v}$ are unknown but bounded noises; $t \in \mathbb{Z}^+$ refers to time instant; and $A_s$, $B_s$, $C_{i,s}$ and $D_{i,s}$ are known time-varying matrices with appropriate dimensions.

*Assumption 1:* The noise sequences $\{w_s\}_{t\in\mathbb{Z}^+}$ and $\{v_{i,s}\}_{\in\mathbb{Z}^+}$ are, respectively, confined to the following ellipsoidal sets:

$$\begin{aligned} \mathscr{W}_s &\triangleq \left\{ w_s^T T_s^{-1} w_s \leq 1 \right\}, \\ \mathscr{V}_{i,s} &\triangleq \left\{ v_{i,s}^T R_{i,s}^{-1} v_{i,s} \leq 1 \right\} \end{aligned} \tag{2}$$

where $\{T_s\}_{t\in\mathbb{Z}^+}$ and $\{R_{i,s}\}_{t\in\mathbb{Z}^+}$ are sequences of known positive definite matrices with compatible dimensions.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



### A. Binary Measurement with Fixed Threshold

The measurement output of binary sensor $i$ $(i \in \mathcal{V})$ with a fixed threshold is given as follows:

$$\bar{z}_{i,s} = \hbar_i(y_{i,s}) = \begin{cases} 1, & \text{if } y_{i,s} \geq \kappa_i; \\ 0, & \text{if } y_{i,s} < \kappa_i \end{cases} \tag{3}$$

where $\bar{z}_{i,s} \in \mathbb{R}$ is the output of the BM and $\kappa_i \in \mathbb{R}$ is a fixed constant.

The distributed estimator under BM with a fixed threshold is given as follows:

$$\begin{aligned} \bar{x}_{i,s+1} =& A_t \bar{x}_{i,s} + \sum_{j \in \mathcal{N}_i} \bar{H}_{ij,s} \bar{\sigma}_{i,s}(\bar{x}_{i,s} - \bar{x}_{j,s}) \\ &+ \pi_{i,s} \bar{K}_{i,s}(\kappa_i - 0.5(C_{i,s+1}A_s + C_{i,s})\bar{x}_{i,s}) \end{aligned} \tag{4}$$

where

$$\pi_{i,s} \triangleq |\bar{z}_{i,s+1} - \bar{z}_{i,s}|, \tag{5}$$

$$\kappa_i = 0.5(1 - \varsigma_{i,s})y_{i,s} + 0.5(1 + \varsigma_{i,s})y_{i,s+1}, \text{ if } \pi_{i,s} = 1. \tag{6}$$

Here, $\bar{x}_{i,s}$ is the estimate of system on sensor node $i$; $\varsigma_{i,s} \in [-1, 1]$ is an uncertain variable; and $\bar{K}_{i,s}$ and $\bar{H}_{ij,s}$ are the estimator gains to be determined later.

*Remark 1:* From (3), the BM of sensor node $i$ provides only one bit of information at each time instant, and this is insufficient for state estimation purposes. As such, we like to extract as much useful information as possible from the switchings of the BM. Clearly, at the crossing time instants of the BMs, we have $\pi_{i,s} = 1$ and then (6) (i.e., $\kappa_i$ falls in between $y_{i,s}$ and $y_{i,s+1}$), and such functional information can be utilized to construct the desired estimator.

The vector saturation function $\bar{\sigma}_{i,s}(\cdot)$: $\mathbb{R}^{n_x} \mapsto \mathbb{R}^{n_x}$ is defined by:

$$\bar{\sigma}_{i,s}(\psi_{ij,s}) \triangleq \begin{bmatrix} \bar{\sigma}^{(1)}_{i,s}(\psi^{(1)}_{ij,s}) & \bar{\sigma}^{(2)}_{i,s}(\psi^{(2)}_{ij,s}) & \cdots & \bar{\sigma}^{(n_x)}_{i,s}(\psi^{(n_x)}_{ij,s}) \end{bmatrix}^T$$

with

$$\bar{\sigma}^{(l)}_{i,s}(\psi^{(l)}_{ij,s}) \triangleq \text{sign}(\psi^{(l)}_{ij,s}) \min\{\varpi^{(l)}_{i,s}, |\psi^{(l)}_{ij,s}|\} \quad (l = 1, 2, \ldots, n_x)$$

where $\varpi^{(l)}_{i,s} > 0$ is a known constant; and sign$(\cdot)$ and $|\cdot|$ are, respectively, the signum function and the absolute value function.

Now, we introduce two indicator variables $\bar{\Phi}^{(l)}_{ij,s}$ and $\bar{\Psi}^{(l)}_{ij,s}$ as follows:

$$\bar{\Phi}^{(l)}_{ij,s} \triangleq \begin{cases} 1, & \text{if } |\psi^{(l)}_{ij,s}| < \varpi^{(l)}_{i,s}; \\ 0, & \text{otherwise}, \end{cases}$$

$$\bar{\Psi}^{(l)}_{ij,s} \triangleq \begin{cases} 1, & \text{if } \psi^{(l)}_{ij,s} > -\varpi^{(l)}_{i,s}; \\ 0, & \text{otherwise}, \end{cases}$$

and then obtain the following new form of the saturation function:

$$\bar{\sigma}^{(l)}_{i,s}(\psi^{(l)}_{ij,s}) = \bar{\Phi}^{(l)}_{ij,s}\psi^{(l)}_{ij,s} + (1 - \bar{\Phi}^{(l)}_{ij,s})(2\bar{\Psi}^{(l)}_{ij,s} - 1)\varpi^{(l)}_{i,s}.$$

Next, we obtain the following compact form:

$$\bar{\sigma}_i(\psi_{ij,s}) = \bar{\Phi}_{ij,s}\psi_{ij,s} + (I - \bar{\Phi}_{ij,s})(2\bar{\Psi}_{ij,s} - I)\varpi_{i,s} \tag{7}$$

where

$$\varpi_{i,s} \triangleq \left[\varpi^{(1)}_{i,s}, \ldots, \varpi^{(n_x)}_{i,s}\right]^T, \quad \psi_{ij,s} \triangleq \left[\psi^{(1)}_{ij,s}, \ldots, \psi^{(n_x)}_{ij,s}\right]^T,$$

$$\bar{\Phi}_{ij,s} \triangleq \text{diag}\left\{\bar{\Phi}^{(1)}_{ij,s}, \bar{\Phi}^{(2)}_{ij,s}, \ldots, \bar{\Phi}^{(n_x)}_{ij,s}\right\},$$

$$\bar{\Psi}_{ij,s} \triangleq \text{diag}\left\{\bar{\Psi}^{(1)}_{ij,s}, \bar{\Psi}^{(2)}_{ij,s}, \ldots, \bar{\Psi}^{(n_x)}_{ij,s}\right\}.$$

*Remark 2:* It is a common method to use the saturation functions for suppressing the adverse outlier-induced impact [10], [11], [35], [43]. The focus is how to deal with the saturation function during the analysis. In [35], the Huber function (it is also a saturation function) has been directly used to prove the desirable properties of the estimator. Differently, the sector-bounded constraint has been commonly used to address the vector saturation function [10], [11], [43]. Nevertheless, those two new matrices in the formulation of sector-bounded constraints could lead to unnecessary conservatism. In this paper, we tackle this issue by developing an equivalent form (7) by introducing two indicator variables. Also, we can make full use of the information of $\psi_{ij,s}$ via (7).

Defining the estimation error $\bar{e}_{i,s} \triangleq x_s - \bar{x}_{i,s}$, one calculates from (1), (4), (5), (6), and (7) that

$$\begin{aligned} &\bar{e}_{i,s+1} \\ =& A_s\bar{e}_{i,s} + B_s w_s - \sum_{j \in \mathcal{N}_i} \bar{H}_{ij,s}\bar{\Phi}_{ij,s}(\bar{e}_{j,s} - \bar{e}_{i,s}) \\ &- \sum_{j \in \mathcal{N}_i} \bar{H}_{ij,s}(I - \bar{\Phi}_{ij,s})(2\bar{\Psi}_{ij,s} - I)\varpi_{i,s} \\ &- 0.5\pi_{i,s}\bar{K}_{i,s}(C_{i,s+1}A_s + C_{i,s})\bar{e}_{i,s} \\ &- 0.5\pi_{i,s}\varsigma_{i,s}\bar{K}_{i,s}(C_{i,s+1}A_s - C_{i,s})x_s \\ &- 0.5\pi_{i,s}\bar{K}_{i,s}D_{i,s}v_{i,s} + 0.5\pi_{i,s}\varsigma_{i,s}\bar{K}_{i,s}D_{i,s}v_{i,s} \\ &- 0.5\pi_{i,s}\bar{K}_{i,s}(C_{i,s+1}B_t w_t + D_{i,s+1}v_{i,s+1}) \\ &- 0.5\pi_{i,s}\varsigma_{i,s}\bar{K}_{i,s}(C_{i,s+1}B_s w_t + D_{i,s+1}v_{i,s+1}). \end{aligned} \tag{8}$$

Letting $\eta_{i,s} \triangleq \begin{bmatrix} x^T_s & \bar{e}^T_{i,s} \end{bmatrix}^T$, $\hat{\varpi}_{i,s} \triangleq \begin{bmatrix} \varpi^T_{i,s} & \varpi^T_{i,s} \end{bmatrix}^T$, and $\xi_{i,s} \triangleq \begin{bmatrix} w^T_s & v^T_{i,s} & v^T_{i,s+1} \end{bmatrix}^T$, one has

$$\begin{aligned} \eta_{i,s+1} =& (\mathcal{A}_s - 0.5\pi_{i,s}\mathcal{K}_{i,s}\mathcal{C}_{i,s} + \sum_{j \in \mathcal{N}_i} \mathcal{H}_{ij,s}\hat{\Phi}_{ij,s})\eta_{i,s} \\ &+ \sum_{j \in \mathcal{N}_i} \mathcal{H}_{ij,s}\hat{\Phi}_{ij,s}\eta_{j,s} + (\mathcal{B}_s - 0.5\pi_{i,s}\mathcal{K}_{i,s}\mathcal{D}_{i,s})\xi_{i,s} \\ &- \sum_{j \in \mathcal{N}_i} \mathcal{H}_{ij,s}(I - \hat{\Phi}_{ij,s})(2\hat{\Psi}_{ij,s} - I)\hat{\varpi}_{i,s} \end{aligned} \tag{9}$$

where

$$\mathcal{A}_s \triangleq \text{diag}\{A_s, A_s\}, \quad \mathcal{K}_{i,s} \triangleq \text{diag}\{0, \bar{K}_{i,s}\},$$

$$\mathcal{C}_{i,s} \triangleq \begin{bmatrix} 0 & 0 \\ \varsigma_{i,s}(C_{i,s+1}A_s - C_{i,s}) & (C_{i,s+1}A_s + C_{i,s}) \end{bmatrix},$$

$$\mathcal{H}_{ij,s} \triangleq \text{diag}\{0, \bar{H}_{ij,s}\}, \quad \hat{\Phi}_{ij,s} \triangleq \text{diag}\{0, \bar{\Phi}_{ij,s}\},$$

$$\mathcal{B}_s \triangleq \begin{bmatrix} B_s & 0 & 0 \\ B_s & 0 & 0 \end{bmatrix}, \quad \mathcal{D}_{i,s} \triangleq \begin{bmatrix} 0 & 0 & 0 \\ \mathcal{D}^{(21)}_{i,s} & \mathcal{D}^{(22)}_{i,s} & \mathcal{D}^{(23)}_{i,s} \end{bmatrix},$$

$$\mathcal{D}^{(21)}_{i,s} \triangleq C_{i,s+1}B_s + \varsigma_{i,s}C_{i,s+1}B_s, \quad \hat{\Psi}_{ij,s} \triangleq \text{diag}\{0, \bar{\Psi}_{ij,s}\},$$

$$\mathcal{D}^{(22)}_{i,s} \triangleq D_{i,s} - \varsigma_{i,s}D_{i,s}, \quad \mathcal{D}^{(23)}_{i,s} \triangleq D_{i,s+1} + \varsigma_{i,s}D_{i,s+1}.$$

*Assumption 2:* For a node $i \in \mathcal{V}$, the initial value $\eta_{i,0}$ satisfies

$$\frac{1}{N}\sum_{i=1}^{N} \eta^T_{i,0} Q^{-1}_{i,0} \eta_{i,0} \leq 1 \tag{10}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



where $Q_{i,0} > 0$ is a given positive definite matrix.

For the given digraph $\mathcal{G}$, consider the linear discrete time-varying system (1) with BMs (3) and the distributed estimator (4). In the case of BMs with fixed thresholds, our objectives are stated as follows:

- for a given positive definite matrix sequence $\{Q_{i,s}\}_{s\in\mathbb{Z}^+}$, design the sequences of estimator gains $\{\mathcal{K}_{i,s}\}_{s\in\mathbb{Z}^+}$ and $\{\mathcal{H}_{ij,s}\}_{s\in\mathbb{Z}^+}$ such that the following constraint holds for any $s \in \mathbb{Z}^+$:

$$\frac{1}{N}\sum_{i=1}^{N}\eta_{i,s}^T Q_{i,s}^{-1}\eta_{i,s} \leq 1; \tag{11}$$

- minimize $\mathrm{Tr}(Q_{i,s})$ at each time instant for sensor node $i$ $(i \in \mathcal{V})$ in a fully distributed manner.

*Remark 3:* It should be noted that (11) is actually an average performance constraint on all sensor nodes, which is different from that of the existing distributed set-membership estimation in [24]. For example, in [24], the desired performance constraint has been given for a sensor node $i$ as follows:

$$\eta_{i,s}^T Q_{i,s}^{-1}\eta_{i,s} \leq 1, \quad i \in \mathcal{V}.$$

Nevertheless, given the large scale of SNs, a natural idea would be to set the average performance over all sensor nodes (rather than on an individual node), and this gives rise to the motivation for us to propose the distributed *average* set-membership performance constraint (11).

*Remark 4:* Note that (11) can be converted into an ellipsoidal constraint by means of the augmented method. To be specific, by letting $\eta_s \triangleq \begin{bmatrix}\eta_{1,s}^T & \eta_{2,s}^T & \cdots & \eta_{N,s}^T\end{bmatrix}^T$, (11) can be rewritten as

$$\eta_s^T Q_s^{-1}\eta_s \leq 1 \tag{12}$$

where $Q_s^{-1} \triangleq \frac{1}{N}\mathrm{diag}\{Q_{1,s}, Q_{2,s}, \ldots, Q_{N,s}\}$.

It is worthwhile noting that the introduction of a fixed threshold would yield a certain conservatism on the desired performance. First, (11) implies that a constraint is posed on the system state $x_s$. More specifically, letting $Q_{i,s} \triangleq \{Q_{i1,s}, Q_{i2,s}\}$, it follows from (11) that

$$\frac{1}{N}\sum_{i=1}^{N}x_s^T Q_{i1,s}^{-1}x_s + \frac{1}{N}\sum_{i=1}^{N}\bar{e}_{i,s}^T Q_{i2,s}^{-1}\bar{e}_{i,s} \leq 1, s \geq 0,$$

which means that $\frac{1}{N}\sum_{i=1}^{N}x_s^T Q_{i1,s}^{-1}x_s \leq 1$. Next, the fourth term of the right-hand side in (8) (i.e., $0.5\pi_{i,s}\varsigma_{i,s}\bar{K}_{i,s}(C_{i,s+1}A_s - C_{i,s})x_s$) can be regarded as the multiplicative uncertainty [14], which could be a source for undesired estimation performance degradation. In view of these two observations, in the next subsection, we propose a *time-varying* threshold strategy regarding the BM with the hope to mitigate the induced conservatism.

### *B. Binary Measurement with Time-Varying Threshold*

As opposed to its fixed (or time-invariant) counterpart, the time-varying threshold could increase the crossing times of the BMs, thereby helping extract more functional information from the measurement dynamics and improving the estimation accuracy, see [7], [41], [53] for some representative results.

Let us consider the measurement output of the $i$-th binary sensor with a time-varying threshold as follows:

$$z_{i,s} = h_i(y_{i,s}) = \begin{cases} 1, & \text{if } y_{i,s} \geq \tau_{i,s}, \\ 0, & \text{if } y_{i,s} < \tau_{i,s} \end{cases} \tag{13}$$

where $z_{i,s} \in \mathbb{R}$ is the BM and $\tau_{i,s} \in \mathbb{R}$ is a time-varying threshold. Furthermore, let $\tau_{i,s} \triangleq \tau_i + \varphi_{i,s}$ with $\tau_i$ being a fixed constant and $\varphi_{i,s}$ being a time-varying variable.

Next, design the distributed estimator for sensor node $i$ with the following form:

$$\hat{x}_{i,s+1} = A_s\hat{x}_{i,s} + \theta_{i,s}K_{i,s}\varrho_{i,s} + \sum_{j\in\mathcal{N}_i}H_{ij,s}\sigma_{i,s}(\zeta_{ij,s}), \tag{14}$$

$$\theta_{i,s} = |z_{i,s+1} - z_{i,s}| \tag{15}$$

where $\hat{x}_{i,s}$ is the state estimate from sensor node $i$, $\varrho_{i,s} \triangleq \tau_{i,s} - C_{i,s}\hat{x}_{i,s}$, $\zeta_{ij,s} \triangleq \hat{x}_{i,s} - \hat{x}_{j,s}$, and $K_{i,s}$ and $H_{ij,s}$ are the desired estimator gains to be determined later.

Clearly, if $\theta_{i,s} = 1$, then $\tau_i$ falls in the interval between $y_{i,s} - \varphi_{i,s}$ and $y_{i,s+1} - \varphi_{i,s+1}$, which can be modelled as

$$\begin{aligned}\tau_{i,s} =& 0.5((y_{i,s+1} - \varphi_{i,s+1}) + (y_{i,s} - \varphi_{i,s})) \\ &+ 0.5\delta_{i,s}((y_{i,s+1} - \varphi_{i,s+1}) - (y_{i,s} - \varphi_{i,s}))\end{aligned} \tag{16}$$

where $\delta_{i,s} \in [-1, 1]$ is an uncertain variable, and $\varphi_{i,s} \in \mathbb{R}$ evolves according to the dynamic equation as follows:

$$\varphi_{i,s+1} = (1 - 2h_{i,s})\varphi_{i,s} + (C_{i,s+1}A_s - C_{i,s})\hat{x}_{i,s} \tag{17}$$

where $h_{i,s}$ is a known time-varying real number belonging to the interval $[0, 0.5]$.

In the following, we begin to discuss the boundedness of $\varphi_{i,s}$. For the convenience of expression, denote

$$h_i(s) \triangleq \min_{0\leq t\leq s} h_{i,t}, \quad \hbar_i(s) \triangleq 1 - 2h_i(s),$$

$$\bar{\varphi}_i(s) \triangleq \max_{0\leq t\leq s} |\bar{\varphi}_{i,t}|, \quad \bar{\varphi}_{i,s} \triangleq (C_{i,s+1}A_s - C_{i,s})\hat{x}_{i,s}.$$

Lemma 1 considers two special cases over the time horizon $0 \leq t \leq s$: 1) $h_{i,t} \neq 0$; 2) $h_{i,t} \equiv 0, \forall t \geq 0$.

*Lemma 1:* $\varphi_{i,s}$ is confined to the following ellipsoidal set:

$$\mathscr{R}_s \triangleq \{\varphi_{i,s}^2\bar{r}_{i,s}^{-1} \leq 1\}$$

where

$$\bar{r}_{i,s} \triangleq \begin{cases} \left(\varphi_i(0) + \sum_{t=0}^{s}|\bar{\varphi}_{i,t}|\right)^2, & \text{if } h_{i,t} \equiv 0; \\ \left((\hbar_i(s))^s\varphi_i(0) + \frac{(1-\hbar_i(s))^s\bar{\varphi}_i(s)}{1-\hbar_i(s)}\right)^2, & \text{if } h_{i,t} \neq 0. \end{cases}$$

Now, we present a recursive formula of $\bar{r}_{i,s}$.

*Lemma 2:* If there exists a positive real number $\bar{r}_{i,s}$ such that $\varphi_{i,s}^2 \leq \bar{r}_{i,s}$ holds, then one has

$$\varphi_{i,s+1}^2\bar{r}_{i,s+1}^{-1} \leq 1$$

where

$$\bar{r}_{i,s+1} \triangleq \begin{cases} \left(\sqrt{\bar{r}_{i,s}} + |\bar{\varphi}_{i,s}|\right)^2, & \text{if } h_{i,s} = 0; \\ \left((1 - 2h_{i,s})\sqrt{\bar{r}_{i,s}} + |\bar{\varphi}_{i,s}|\right)^2, & \text{if } h_{i,s} \neq 0. \end{cases}$$

The proofs of Lemmas 1 and 2 are derived via (17) and thus are omitted due to the space-saving.

*Remark 5:* The time-varying threshold proposed in this paper has such a structure: $\tau_{i,s} \triangleq \tau_i + \varphi_{i,s}$, where $\varphi_{i,s}$ is

Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



introduced to adjust the fixed threshold $\tau_i$ for reducing the conservatism. Actually, according to Lemma 1, $\varphi_{i,s}$ could be seen as a bounded disturbance of the fixed threshold. In addition, from the dynamic equation (17) of $\varphi_{i,s}$, the introduction of $h_{i,s}$ is helpful to increase the design freedom. Due to the introduction of time-varying threshold, we need to calculate the threshold $\varphi_{i,s}$ and weight $\bar{r}_{i,s}$ according to (17) and Lemmas 1 and 2, respectively. It should be noted that the additional burden of node $i$ at every time instant is $O(1)$ through the analysis of computational complexity.

The vector saturation function $\sigma_{i,s}(\cdot)$: $\mathbb{R}^{n_x} \mapsto \mathbb{R}^{n_x}$ is defined by:

$$\sigma_{i,s}(\zeta_{ij,s}) \triangleq \begin{bmatrix}\sigma_{i,s}^{(1)}(\zeta_{ij,s}^{(1)}) & \sigma_{i,s}^{(2)}(\zeta_{ij,s}^{(2)}) & \cdots & \sigma_{i,s}^{(n_x)}(\zeta_{ij,s}^{(n_x)})\end{bmatrix}^T$$

with

$$\sigma_{i,s}^{(l)}(\zeta_{ij,s}^{(l)}) = \Phi_{ij,s}^{(l)}\zeta_{ij,s}^{(l)} + (1-\Phi_{ij,s}^{(l)})(2\Psi_{ij,s}^{(l)}-1)\rho_{i,s}^{(l)}$$

where

$$\Phi_{ij,s}^{(l)} \triangleq \begin{cases} 1, & \text{if} \quad |\zeta_{ij,s}^{(l)}| < \rho_{i,s}^{(l)}; \\ 0, & \text{otherwise}, \end{cases}$$

$$\Psi_{ij,s}^{(l)} \triangleq \begin{cases} 1, & \text{if} \quad \zeta_{ij,s}^{(l)} > -\rho_{i,s}^{(l)}; \\ 0, & \text{otherwise}. \end{cases}$$

By denoting

$$\begin{aligned}
\Phi_{ij,s} \triangleq& \mathrm{diag}\left\{\Phi_{ij,s}^{(1)}, \Phi_{ij,s}^{(2)}, \ldots, \Phi_{ij,s}^{(n_x)}\right\}, \\
\Psi_{ij,s} \triangleq& \mathrm{diag}\left\{\Psi_{ij,s}^{(1)}, \Psi_{ij,s}^{(2)}, \ldots, \Psi_{ij,s}^{(n_x)}\right\}, \\
\rho_{i,s} \triangleq& \left[\rho_{i,s}^{(1)}, \ldots, \rho_{i,s}^{(n_x)}\right]^T, \quad \zeta_{ij,s} \triangleq \left[\zeta_{ij,s}^{(1)}, \ldots, \zeta_{ij,s}^{(n_x)}\right]^T,
\end{aligned}$$

one further has

$$\sigma_i(\zeta_{ij,s}) = \Phi_{ij,s}\zeta_{ij,s} + (I-\Phi_{ij,s})(2\Psi_{ij,s}-I)\rho_{i,s}.$$

Denoting $e_{i,s} \triangleq x_s - \hat{x}_{i,s}$, the estimation error dynamics of sensor node $i$ under the case of time-varying threshold is calculated as follows:

$$\begin{aligned}
e_{i,s+1} =& A_s e_{i,s} + B_s w_s - \sum_{j\in\mathcal{N}_i} H_{ij,s}\Phi_{ij,s}(e_{j,s}-e_{i,s}) \\
&- \sum_{j\in\mathcal{N}_i} H_{ij,s}(I-\Phi_{ij,s})(2\Psi_{ij,s}-I)\rho_{i,s} \\
&- 0.5\theta_{i,s}K_{i,s}(C_{i,s+1}A_s + C_{i,s})e_{i,s} \\
&- 0.5\delta_{i,s}\theta_{i,s}K_{i,s}(C_{i,s+1}A_s - C_{i,s})e_{i,s} \\
&- 0.5\theta_{i,s}(1+\delta_{i,s})K_{i,s}C_{i,s+1}B_s w_s \\
&- 0.5\theta_{i,s}(1-\delta_{i,s})K_{i,s}D_{i,s}v_{i,s} \\
&- 0.5\theta_{i,s}(1+\delta_{i,s})K_{i,s}D_{i,s+1}v_{i,s+1} \\
&+ \theta_{i,s}(1-h_{i,s}-h_{i,s}\delta_{i,s})K_{i,s}\varphi_{i,s}.
\end{aligned} \tag{18}$$

*Assumption 3:* For a sensor node $i \in \mathcal{V}$, the initial estimation error $e_{i,0}$ satisfies the following constraint

$$\frac{1}{N}\sum_{i=1}^{N} e_{i,0}^T P_{i,0}^{-1} e_{i,0} \leq 1 \tag{19}$$

where $P_{i,0} > 0$ is a given positive definite matrix.

Consider the linear discrete time-varying system (1) with BMs (13) and the distributed estimator (14). Let the digraph $\mathcal{G}$ and the positive definite matrix sequence $\{P_{i,s}\}_{s\in\mathbb{Z}^+}$ be given. The desired objective of this subsection is twofold:

1) design the sequences of estimator gains $\{K_{i,s}\}_{s\in\mathbb{Z}^+}$ and $\{H_{ij,s}\}_{s\in\mathbb{Z}^+}$ such that the following constraint holds for any $s \in \mathbb{Z}^+$:

$$\frac{1}{N}\sum_{i=1}^{N} e_{i,s}^T P_{i,s}^{-1} e_{i,s} \leq 1; \tag{20}$$

2) minimize $\{\mathrm{Tr}(P_{i,s})\}_{s\in\mathbb{Z}^+}$ at each time instant.

Inspired by the idea of the ellipsoid fusion in [32], (20) can be seen as the fusion of $N$ ellipsoids with the weighted coefficients $\frac{1}{N}$. After some simple algebraic operations, (20) can be rewritten as

$$(x_s - \hat{x}_s^*)^T P_s^* (x_s - \hat{x}_s^*) \leq 1 - \nu_s^* \tag{21}$$

where

$$\begin{aligned}
\hat{x}_s^* \triangleq& \frac{1}{N}(P_s^*)^{-1}\sum_{i=1}^{N} P_{i,s}^{-1}\hat{x}_{i,s}, \quad P_t^* \triangleq \frac{1}{N}\sum_{i=1}^{N} P_{i,s}^{-1}, \\
\nu_s^* \triangleq& \frac{1}{N}\sum_{i=1}^{N} \hat{x}_{i,s}^T P_{i,s}^{-1}\hat{x}_{i,s} - (\hat{x}_s^*)^T P_s^* \hat{x}_s^*.
\end{aligned}$$

*Lemma 3:* For given a set of estimates $\hat{x}_{i,s}$ ($i \in \mathcal{V}$ and $s \geq 0$), there always exists a set of positive definite matrices $P_{i,s} > 0$ such that $\nu_s^* < 1$.

*Proof:* See Appendix VII-A. ∎

By means of Lemma 2, one knows that (21) corresponds to an ellipsoid constraint, where $\hat{x}_s^*$ can be regarded as a fusion estimate of $\hat{x}_{i,s}$ with weighted matrix $\frac{1}{N}(P_s^*)^{-1}P_{i,s}^{-1}$. In addition, (20) implies that the ellipsoid constraint $(x_s - \hat{x}_{i,s})^T P_{i,s}^{-1}(x_s - \hat{x}_{i,s}) \leq 1$ holds for any sensor node $i \in \mathcal{V}$. The intersection of these ellipsoid constraints is characterized by (21) since it is equivalent to (20).

*Remark 6:* It is noted that the uncertain term involving $\delta_{i,s}$ would yield a certain conservatism for the design of distributed estimation scheme. In this context, it makes sense to reduce the adverse effect of such an uncertain term. According to (16), if

$$\varphi_{i,s+1} - y_{i,s+1} = \varphi_{i,s} - y_{i,s}, \tag{22}$$

then the uncertainties can be completely eliminated but, unfortunately, (22) cannot be true due to the existence of noises. Under the time-varying threshold proposed in this paper, one knows from (18) that $\delta_{i,s}$ is not related to the system state, which implies that the negative effect of the uncertainty can be mitigated to some extent.

## III. Main results regarding fixed thresholds

In this section, by means of the local performance analysis (LPA) method on the basis of the vector dissipativity theory, local sufficient conditions (with respect to any sensor node $i \in \mathcal{V}$) are first established to ensure the error dynamics (9) to satisfy the desired performance criterion (11). Next, the matrices $Q_{i,s}$ ($i \in \mathcal{V}$) are minimized in a fully distributed manner to obtain the estimator gains. Here, a fully distributed manner implies that a node $i$ can minimize its own $Q_{i,s}$ subject to the constraints associated with its neighboring nodes to calculate the estimator gains.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



The vector dissipation is defined as follows to facilitate the subsequent analysis.

*Definition 1:* The system dynamics (9) is vector dissipative regarding the vector supply rate function $\mathbf{S}_s \triangleq \begin{bmatrix} S_{1,s} & \cdots & S_{N,s} \end{bmatrix}^T$ if there exist a vector of nonnegative definite storage functions $\mathbf{V}_s \triangleq \begin{bmatrix} V_{1,s} & \cdots & V_{N,s} \end{bmatrix}^T$ (with $\mathbf{V}_0 = 0$) and a sequence of nonsingular column sub-stochastic dissipation matrices $W_s \in \mathbb{R}^{N\times N}$ such that the following vector dissipation inequality is satisfied for all $s \in \mathbb{Z}^+$:

$$\mathbf{V}_{s+1} \leq\leq W_s \mathbf{V}_s + \mathbf{S}_s. \tag{23}$$

Here, for any two vectors $c = \begin{bmatrix} c_1 & c_2 & \cdots & c_N \end{bmatrix}^T$ and $d = \begin{bmatrix} d_1 & d_2 & \cdots & d_N \end{bmatrix}^T \in \mathbb{R}^N$, $c \leq\leq d$ means that $c_i \leq d_i$ $(\forall i = 1, 2, \ldots, N)$.

Before conducting the performance analysis, an interval-valued function of the out-degree $q_i$ is defined by:

$$\mathscr{I}_{q_i} \triangleq \begin{cases} \left(0, \frac{1+q_i}{2q_i}\right), & \text{if } q_i \neq 0; \\ (0, 1], & \text{if } q_i = 0, \end{cases} \tag{24}$$

and the supply rate function is chosen as follows:

$$\overline{S}_{i,s} \triangleq \frac{N\alpha_{i,s}}{1+q_i}.$$

For the fixed threshold case, by means of the LPA method, a sufficient criterion is provided in the subsequent theorem to guarantee the vector dissipation of system (9) regarding the supply rate function $\overline{S}_{i,s}$, which further deduces the desired performance criterion (11) for the system dynamics (9).

*Theorem 1:* Consider a sensor node $i \in \mathcal{V}$ with its neighboring sensor node $j \in \mathcal{N}_i$. Let a sequence of positive definite matrices $\{Q_{i,s}\}_{s\in\mathbb{Z}^+}$ and a sequence of scalars $\{\alpha_{i,s}\}_{s\in\mathbb{Z}^+} \in \mathscr{I}_{q_i}$ be given. Then, the system dynamics (9) satisfies the $Q_{i,s}$-dependent constraint condition (11) if there exist real-valued matrix sequences $\{\mathcal{K}_{i,s}\}_{s\in\mathbb{Z}^+}$, $\{\mathcal{H}_{ij,s}\}_{s\in\mathbb{Z}^+}$, and scalar sequence $\{\phi_{i1,s}\}_{s\in\mathbb{Z}^+}$ satisfying the following $\pi_{i,s}$-dependent matrix inequality:

$$\Omega_{i1,s}^T Q_{i,s+1}^{-1} \Omega_{i1,s} + \Omega_{i0,s} \leq 0 \tag{25}$$

where

$$\begin{aligned}
\Omega_{i0,s} \triangleq & \mathrm{diag}\left\{\Omega_{i0,s}^{(1)}, \Omega_{i0,s}^{(2)}, \Omega_{i0,s}^{(3)}, \Omega_{i0,s}^{(4)}\right\}, \\
\Omega_{i0,s}^{(1)} \triangleq & \phi_{i1,s} - \frac{N\alpha_{i,s}}{1+q_i}, \quad \Omega_{i0,s}^{(2)} \triangleq -(1-\alpha_{i,s})Q_{i,s}^{-1}, \\
\Omega_{i0,s}^{(3)} \triangleq & -\mathrm{diag}_{p_i}\left\{\frac{\alpha_{j,s}}{1+q_j} Q_{j,s}^{-1}\right\}, \\
\Omega_{i0,s}^{(4)} \triangleq & -\frac{1}{3}\mathrm{diag}\left\{T_s^{-1}, R_{i,s}^{-1}, R_{i,s+1}^{-1}\right\}, \\
\Omega_{i1,s} \triangleq & \begin{bmatrix} \Omega_{i1,s}^{(1)} & \Omega_{i1,s}^{(2)} & \Omega_{i1,s}^{(3)} & \Omega_{i1,s}^{(4)} \end{bmatrix}, \\
\Omega_{i1,s}^{(1)} \triangleq & -\sum_{j\in\mathcal{N}_i} \mathcal{H}_{ij,s}(I - \hat{\Phi}_{ij,s})(2\hat{\Psi}_{ij,s} - I)\hat{\varpi}_{i,s}, \\
\Omega_{i1,s}^{(2)} \triangleq & \mathcal{A}_s - 0.5\pi_{i,s}\mathcal{K}_{i,s}\mathcal{C}_{i,s} + \sum_{j\in\mathcal{N}_i} \bar{\mathcal{H}}_{ij,s}\hat{\Phi}_{ij,s}, \\
\Omega_{i1,s}^{(3)} \triangleq & \begin{bmatrix} \mathcal{H}_{ij_1,s}\hat{\Phi}_{ij_1,s} & \ldots & \mathcal{H}_{ij_{p_i},s}\hat{\Phi}_{ij_{p_i},s} \end{bmatrix}, \\
\Omega_{i1,s}^{(4)} \triangleq & \mathcal{B}_s - 0.5\pi_{i,s}\mathcal{K}_{i,s}\mathcal{D}_{i,s}.
\end{aligned}$$

*Proof:* See Appendix VII-B. ■

Up to now, the performance analysis is carried out via the LPA method in Theorem 1. Next, we begin to deal with the uncertain terms in (25) and design the desired estimator gains.

*Theorem 2:* Consider a sensor node $i \in \mathcal{V}$ with its neighboring sensor node $j \in \mathcal{N}_i$. Let a positive definite matrix sequence $\{Q_{i,s}\}_{s\in\mathbb{Z}^+}$ and a scalar sequence $\{\alpha_{i,s}\}_{s\in\mathbb{Z}^+} \in \mathscr{I}_{q_i}$ be given. The system dynamics (9) satisfies the $Q_{i,s}$-dependent constraint condition (11) if there exist real-valued matrix sequences $\{\mathcal{K}_{i,s}\}_{s\in\mathbb{Z}^+}$, $\{\mathcal{H}_{ij,s}\}_{s\in\mathbb{Z}^+}$, and scalar sequences $\{\phi_{it,s}\}_{s\in\mathbb{Z}^+}$ $(t = 1, 2)$ satisfying the following $\pi_{i,s}$-dependent recursive linear matrix inequality (RLMI):

$$\begin{bmatrix} \bar{\Omega}_{i0,s} & * & * \\ \bar{\Omega}_{i1,s} & -Q_{i,s+1} & * \\ 0 & \mathcal{M}_{i,s}^T & -\phi_{i2,s} \end{bmatrix} \leq 0 \tag{26}$$

where

$$\begin{aligned}
\bar{\Omega}_{i0,s} \triangleq & \begin{bmatrix} \bar{\Omega}_{i0,s}^{(1)} & * & * & * \\ 0 & \bar{\Omega}_{i0,s}^{(2)} & * & * \\ 0 & 0 & \bar{\Omega}_{i0,s}^{(3)} & * \\ 0 & \tilde{\mathcal{D}}_{i,s}^T \tilde{\mathcal{C}}_{i,s} & 0 & \bar{\Omega}_{i0,s}^{(4)} \end{bmatrix}, \\
\bar{\Omega}_{i0,s}^{(1)} \triangleq & \phi_{i1,s} - \frac{N\alpha_{i,s}}{1+q_i}, \quad \bar{\Omega}_{i0,s}^{(3)} \triangleq -\mathrm{diag}_{p_i}\{\frac{\alpha_{j,s}}{1+q_j} Q_{j,s}^{-1}\}, \\
\bar{\Omega}_{i0,s}^{(2)} \triangleq & -(1-\alpha_{i,s})Q_{i,s}^{-1} + 0.25\phi_{i2,s}\tilde{\mathcal{C}}_{i,s}^T\tilde{\mathcal{C}}_{i,s}, \\
\bar{\Omega}_{i0,s}^{(4)} \triangleq & -\frac{\phi_{i1,s}}{3}\mathrm{diag}\{T_t^{-1}, R_{i,s}^{-1}, R_{i,s+1}^{-1}\} + 0.25\phi_{i2,s}\tilde{\mathcal{D}}_{i,s}^T\tilde{\mathcal{D}}_{i,s}, \\
\bar{\Omega}_{i1,s} \triangleq & \begin{bmatrix} \bar{\Omega}_{i1,s}^{(1)} & \bar{\Omega}_{i1,s}^{(2)} & \bar{\Omega}_{i1,s}^{(3)} & \bar{\Omega}_{i1,s}^{(4)} \end{bmatrix}, \\
\bar{\Omega}_{i1,s}^{(1)} \triangleq & -\sum_{j\in\mathcal{N}_i} \mathcal{H}_{ij,s}(I - \hat{\Phi}_{ij,s})(2\hat{\Psi}_{ij,s} - I)\hat{\varpi}_{i,s}, \\
\bar{\Omega}_{i1,s}^{(2)} \triangleq & \mathcal{A}_s - 0.5\pi_{i,s}\mathcal{K}_{i,s}\bar{\mathcal{C}}_{i,s} + \sum_{j\in\mathcal{N}_i} \bar{\mathcal{H}}_{ij,s}\hat{\Phi}_{ij,s}, \\
\bar{\Omega}_{i1,s}^{(3)} \triangleq & \begin{bmatrix} \mathcal{H}_{ij_1,s}\hat{\Phi}_{ij_1,s} & \ldots & \mathcal{H}_{ij_{p_i},s}\hat{\Phi}_{ij_{p_i},s} \end{bmatrix}, \\
\bar{\Omega}_{i1,s}^{(4)} \triangleq & \mathcal{B}_s - 0.5\hat{\theta}_{i,s}\mathcal{K}_{i,s}\bar{\mathcal{D}}_{i,s}, \quad \mathcal{M}_{i,s} \triangleq \pi_{i,s}\mathcal{K}_{i,s}, \\
\bar{\mathcal{C}}_{i,s} \triangleq & \mathrm{diag}\{0, C_{i,s+1}A_s + C_{i,s}\}, \\
\tilde{\mathcal{C}}_{i,s} \triangleq & \begin{bmatrix} 0 & 0 \\ C_{i,s+1}A_s - C_{i,s} & 0 \end{bmatrix}, \\
\bar{\mathcal{D}}_{i,s} \triangleq & \begin{bmatrix} 0 & 0 & 0 \\ C_{i,s+1}B_s & D_{i,s} & D_{i,s+1} \end{bmatrix}, \\
\tilde{\mathcal{D}}_{i,s} \triangleq & \begin{bmatrix} 0 & 0 & 0 \\ C_{i,s+1}B_s & -D_{i,s} & D_{i,s+1} \end{bmatrix}.
\end{aligned}$$

The proof of Theorem 2 is achieved by Schur complement lemma, and is thus omitted.

In Theorem 2, conditions are given to guarantee the existence of the ellipsoidal set containing all possible state values. In the following, such an ellipsoid is optimized by exerting the convex optimization approach. Specifically, the optimal matrix $Q_{i,s+1}$ can be obtained by minimizing the sequences of matrices $\{Q_{i,s+1}\}_{s\in\mathbb{Z}^+}$ $(i \in \mathcal{V})$ to guarantee the locally optimal estimation performance.

*Corollary 1:* Consider a sensor node $i \in \mathcal{V}$ with its neighboring sensor node $j \in \mathcal{N}_i$. Let a scalar sequence $\{\alpha_{i,s}\}_{s\in\mathbb{Z}^+} \in \mathscr{I}_{q_i}$ be given. The scalar sequence

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



$\{\mathrm{Tr}(Q_{i,s+1})\}_{s\in\mathbb{Z}^+}$ is minimized if there exist real-valued matrix sequences $\{\mathcal{K}_{i,s}\}_{s\in\mathbb{Z}^+}$, $\{\mathcal{H}_{ij,s}\}_{t\in\mathbb{Z}^+}$, and positive scalar sequences $\{\phi_{it,s}\}_{s\in\mathbb{Z}^+}$ $(t=1,2)$ that can solve the following optimization problem

$$\min_{\{Q_{i,s+1},\mathcal{K}_{i,s},\mathcal{H}_{ij,s},\phi_{it,s}\}} \quad \mathrm{Tr}(Q_{i,s+1}) \tag{27}$$

subject to (26).

## IV. Main Results under Time-Varying thresholds

First, we choose the supply rate function as follows:

$$S_{i,s} \triangleq \frac{N\alpha_{i,s}}{1+q_i}.$$

Following the similar line in Section III, the local sufficient criterion is established for each sensor node to 1) guarantee the vector dissipativity of the system (18) regarding the supply rate function $S_{i,s}$, and 2) further deduce the desired performance constraint (20) for the system dynamics (18).

*Theorem 3:* Consider a sensor node $i\in\mathcal{V}$ with its neighboring sensor node $j\in\mathcal{N}_i$. Let a positive definite matrix sequence $\{P_{i,s}\}_{s\in\mathbb{Z}^+}$ and a scalar sequence $\{\alpha_{i,s}\}_{s\in\mathbb{Z}^+}\in\mathscr{I}_{q_i}$ be given. Then, the system dynamics (18) satisfies the $P_{i,s}$-dependent constraint condition (20) if there exist real-valued matrix sequences $\{K_{i,s}\}_{s\in\mathbb{Z}^+}$, $\{H_{ij,s}\}_{s\in\mathbb{Z}^+}$, and scalar sequences $\{\mu_{it,s}\}_{s\in\mathbb{Z}^+}$ $(t=1,2,3,4)$ such that the following $\theta_{i,s}$-dependent matrix inequality holds:

$$\Pi_{i1,s}^T P_{i,s+1}^{-1}\Pi_{i1,s}+\Pi_{i0,s}\le 0 \tag{28}$$

where

$$\begin{aligned}
\Pi_{i0,s} \triangleq&\mathrm{diag}\left\{\Pi_{i0,s}^{(1)},\Pi_{i0,s}^{(2)},\Pi_{i0,s}^{(3)},\Pi_{i0,s}^{(4)},\Pi_{i0,s}^{(5)},\Pi_{i0,s}^{(6)},\Pi_{i0,s}^{(7)}\right\},\\
\Pi_{i0,s}^{(1)} \triangleq&\sum_{s=1}^{4}\mu_{is,s}-\frac{N\alpha_{i,s}}{1+q_i},\quad \Pi_{i0,s}^{(2)}\triangleq-(1-\alpha_{i,s})P_{i,s}^{-1},\\
\Pi_{i0,s}^{(3)} \triangleq&-\mathrm{diag}_{p_i}\left\{\frac{\alpha_{j,s}}{1+q_j}P_{j,s}^{-1}\right\},\quad \Pi_{i0,s}^{(4)}\triangleq-\mu_{i1,s}T_s^{-1},\\
\Pi_{i0,s}^{(5)} \triangleq&-\mu_{i2,s}R_{i,s}^{-1},\Pi_{i0,s}^{(6)}\triangleq-\mu_{i3,s}R_{i,s+1}^{-1},\Pi_{i0,s}^{(7)}\triangleq-\mu_{i4,s}\bar{r}_{i,s}^{-1},\\
\Pi_{i1,s} \triangleq&\begin{bmatrix}\Pi_{i1,s}^{(1)} & \Pi_{i1,s}^{(2)} & \Pi_{i1,s}^{(3)} & \Pi_{i1,s}^{(4)} & \Pi_{i1,s}^{(5)} & \Pi_{i1,s}^{(6)} & \Pi_{i1,s}^{(7)}\end{bmatrix},\\
\Pi_{i1,s}^{(1)} \triangleq&-\sum_{j\in\mathcal{N}_i}H_{ij,s}(I-\Phi_{ij,s})(2\Psi_{ij,s}-I)\rho_{i,s},\\
\Pi_{i1,s}^{(2)} \triangleq&\sum_{j\in\mathcal{N}_i}H_{ij,s}\Phi_{ij,s}-0.5\theta_{i,s}K_{i,s}(C_{i,s+1}A_s+C_{i,s}),\\
&+A_s-0.5\delta_{i,s}\theta_{i,s}K_{i,s}(C_{i,s+1}A_s-C_{i,s}),\\
\Pi_{i1,s}^{(3)} \triangleq&-\begin{bmatrix}H_{ij_1,s}\Phi_{ij_1,s} & \cdots & H_{ij_{p_i},s}\Phi_{ij_{p_i},s}\end{bmatrix},\\
\Pi_{i1,s}^{(4)} \triangleq&B_s-0.5\theta_{i,s}(1+\delta_{i,s})K_{i,s}C_{i,s+1}B_s,\\
\Pi_{i1,s}^{(5)} \triangleq&-0.5\theta_{i,s}(1-\delta_{i,s})K_{i,s}D_{i,s},\\
\Pi_{i1,s}^{(6)} \triangleq&-0.5\theta_{i,s}(1+\delta_{i,s})K_{i,s}D_{i,s+1},\\
\Pi_{i1,s}^{(7)} \triangleq&\theta_{i,s}(1-h_{i,s}-h_{i,s}\delta_{i,s})K_{i,s}.
\end{aligned}$$

*Theorem 4:* Consider a sensor node $i\in\mathcal{V}$ with its neighboring sensor node $j\in\mathcal{N}_i$. Let a positive definite matrix sequence $\{P_{i,s}\}_{s\in\mathbb{Z}^+}$ and a scalar sequence $\{\alpha_{i,s}\}_{s\in\mathbb{Z}^+}\in\mathscr{I}_{q_i}$ be given. Then, the system dynamics (18) satisfies the $P_{i,s}$-dependent constraint condition (20) if there exist real-valued matrix sequences $\{K_{i,s}\}_{s\in\mathbb{Z}^+}$, $\{H_{ij,s}\}_{s\in\mathbb{Z}^+}$, and scalar sequences $\{\mu_{it,s}\}_{s\in\mathbb{Z}^+}$ $(t=1,2,3,4,5)$ such that the following $\theta_{i,s}$-dependent RLMI holds:

$$\begin{bmatrix}\bar{\Pi}_{i0,s} & * & * \\ \bar{\Pi}_{i1,s} & -P_{i,s+1} & * \\ 0 & M_{i,s}^T & -\mu_{i5,s}\end{bmatrix}\le 0 \tag{29}$$

where

$$\begin{aligned}
\bar{\Pi}_{i0,s} \triangleq&\mathrm{diag}\left\{\bar{\Pi}_{i0,s}^{(1)},\bar{\Pi}_{i0,s}^{(2)},\bar{\Pi}_{i0,s}^{(3)},\bar{\Pi}_{i0,s}^{(4)},\bar{\Pi}_{i0,s}^{(5)},\bar{\Pi}_{i0,s}^{(6)},\bar{\Pi}_{i0,s}^{(7)}\right\},\\
\bar{\Pi}_{i0,s}^{(1)} \triangleq&\sum_{s=1}^{4}\mu_{is,s}-\frac{N\alpha_{i,s}}{1+q_i},\quad \bar{\Pi}_{i0,s}^{(3)}\triangleq-\mathrm{diag}_{p_i}\left\{\frac{\alpha_{j,s}}{1+q_j}P_{j,s}^{-1}\right\},\\
\bar{\Pi}_{i0,s}^{(2)} \triangleq&\mu_{i5,s}(C_{i,s+1}A_s-C_{i,s})^T(C_{i,s+1}A_s-C_{i,s})\\
&-(1-\alpha_{i,s})P_{i,s}^{-1},\quad \bar{\Pi}_{i0,s}^{(7)}\triangleq-\mu_{i4,s}\bar{r}_{i,s}^{-1}+\mu_{i5,s}h_{i,s}^2,\\
\bar{\Pi}_{i0,s}^{(4)} \triangleq&-\mu_{i1,s}T_s^{-1}+\mu_{i5,s}(C_{i,s+1}B_s)^T(C_{i,s+1}B_s),\\
\bar{\Pi}_{i0,s}^{(5)} \triangleq&-\mu_{i2,s}R_{i,s}^{-1}+\mu_{i5,s}D_{i,s}^TD_{i,s},\\
\bar{\Pi}_{i0,s}^{(6)} \triangleq&-\mu_{i3,s}R_{i,s+1}^{-1}+\mu_{i5,s}D_{i,s+1}^TD_{i,s+1},\\
\bar{\Pi}_{i1,s} \triangleq&\begin{bmatrix}\bar{\Pi}_{i1,s}^{(1)} & \bar{\Pi}_{i1,s}^{(2)} & \bar{\Pi}_{i1,s}^{(3)} & \bar{\Pi}_{i1,s}^{(4)} & \bar{\Pi}_{i1,s}^{(5)} & \bar{\Pi}_{i1,s}^{(6)} & \bar{\Pi}_{i1,s}^{(7)}\end{bmatrix},\\
\bar{\Pi}_{i1,s}^{(1)} \triangleq&-\sum_{j\in\mathcal{N}_i}H_{ij,s}(I-\Phi_{ij,s})(2\Psi_{ij,s}-I)\rho_{i,s},\\
\bar{\Pi}_{i1,s}^{(2)} \triangleq&A_s-0.5\theta_{i,s}K_{i,s}(C_{i,s+1}A_s+C_{i,s})+\sum_{j\in\mathcal{N}_i}H_{ij,s}\Phi_{ij,s},\\
\bar{\Pi}_{i1,s}^{(3)} \triangleq&-\begin{bmatrix}H_{ij_1,s}\Phi_{ij_1,s} & \cdots & H_{ij_{p_i},s}\Phi_{ij_{p_i},s}\end{bmatrix},\\
\bar{\Pi}_{i1,s}^{(4)} \triangleq&B_s-0.5\theta_{i,s}K_{i,s}C_{i,s+1}B_s,\quad \bar{\Pi}_{i1,s}^{(5)}\triangleq-0.5\theta_{i,s}K_{i,s}D_{i,s}\\
\bar{\Pi}_{i1,s}^{(6)} \triangleq&-0.5\theta_{i,s}K_{i,s}D_{i,s+1},\quad \bar{\Pi}_{i1,s}^{(7)}\triangleq\theta_{i,s}(1-h_{i,s})K_{i,s},\\
M_{i,s} \triangleq&-0.5\theta_{i,s}\begin{bmatrix}0 & K_{i,s} & 0 & K_{i,s} & -K_{i,s} & K_{i,s} & 2K_{i,s}\end{bmatrix}.
\end{aligned}$$

The proofs of Theorems 3 and 4 are similar to those of Theorems 1 and 2, respectively, and are thus omitted.

Finally, the optimal matrix $P_{i,s+1}$ can be obtained by minimizing the matrix sequence $\{P_{i,s+1}\}_{s\ge 0}$ for every sensor node $i$ $(i\in\mathcal{V})$ to guarantee the optimal estimation performance.

*Corollary 2:* Consider a sensor node $i\in\mathcal{V}$ with its neighboring sensor node $j\in\mathcal{N}_i$. Let a scalar sequence $\{\alpha_{i,s}\}_{s\in\mathbb{Z}^+}\in\mathscr{I}_{q_i}$ be given. The scalar sequence $\{\mathrm{Tr}(P_{i,s+1})\}_{s\in\mathbb{Z}^+}$ is minimized if there exist real-valued matrix sequences $\{K_{i,s}\}_{s\in\mathbb{Z}^+}$ and $\{H_{ij,s}\}_{s\in\mathbb{Z}^+}$, and positive scalar sequences $\{\mu_{it,s}\}_{s\in\mathbb{Z}^+}$ $(t=1,2,3,4,5)$ solving the optimization problem as follows:

$$\min_{\{P_{i,s+1},K_{i,s},H_{ij,s},\mu_{it,s}\}} \quad \mathrm{Tr}(P_{i,s+1}) \tag{30}$$

subject to (29).

*Remark 7:* It can be observed that Theorems 1-4 are dependent on two indicator variables $\pi_{i,s}$ and $\theta_{i,s}$, respectively. Such a formulation is helpful to reflect the impact of the information extraction of the BMs on the main results and cover different cases for the convenience of expressing the main results.

*Remark 8:* The distributed filtering algorithms under the fixed threshold and time-varying threshold are compared in terms of the time complexities. In this paper, the proposed schemes are presented for every node in terms of a set of RLMIs from Corollaries 1 and 2. The standard

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



LMI system has a polynomial time complexity bounded by $O(\mathcal{M}\mathcal{N}^3\log(\mathcal{V}/\varepsilon))$, where $\mathcal{M}$ is the total row size of the LMI system, $\mathcal{N}$ is the total number of scalar decision variables, $\mathcal{V}$ is a data-dependent scaling factor, and $\varepsilon$ is the relative accuracy set for the algorithm. Here, we assume that $\mathcal{V}$ and $\varepsilon$ are fixed, which are neglected in the following analysis. As for the case of fixed threshold, it can be calculated from Corollary 1 that $\mathcal{M}_i^f = 2(p_i+2)n_x + n_w + 2n_v + 2$ and $\mathcal{N}_i^f = (p_i+2)n_x^2 + 2n_x + 2$, which implies that the time complexity for node $i$ at every time instant is represented as $O((p_i+2)^4 n_x^7 + (p_i+2)^3(n_w+2n_v+2)n_x^6)$. Also, it can be calculated from Corollary 2 that $\mathcal{M}_i^t = (2+p_i)n_x + n_w + 2n_v + 3$ and $\mathcal{N}_i^t = (p_i+\frac{1}{2})n_x^2 + \frac{3}{2}n_x + 5$, which shows that the time complexity of the algorithm for node $i$ at every time instant under the case of time-varying threshold can be represented as $O((p_i+\frac{1}{2})^3(p_i+2)n_x^7 + (p_i+\frac{1}{2})^3(n_w+2n_v+3)n_x^6)$. It can be concluded that there exist no significant difference in the time complexities of distributed filtering schemes between the time-varying threshold and the fixed threshold.

*Remark 9:* In comparison with the case of the fixed threshold, the introduction of the time-varying threshold developed in this paper would help reduce conservatism. First, the multiplicative uncertainties can be eliminated to improve the desired estimation performance. Next, the estimation error system can be directly analyzed without imposing any requirements on the system. Nevertheless, the utilization of the time-varying threshold would lead to a slightly increased computational burden as the estimate at the current instant is needed to calculate the time-varying threshold at the next time instant.

*Remark 10:* This paper investigates the distributed set-membership estimation problem under BMs with fixed and time-varying thresholds, respectively. Compared with the existing results, this paper exhibits the following characteristics: 1) the addressed problem is new in that the BMs are, for the first time, considered in a framework of distributed set-membership estimation via the LPA method; 2) based on the analysis on the fixed threshold case, a novel time-varying threshold strategy is proposed to further improve the estimation accuracy; 3) a distributed average set-membership performance index is proposed over all sensor nodes, which is more reasonable than the existing performance indices on individual sensor node; and 4) for each sensor node, local sufficient conditions are established via the LPA method and the estimator gains are derived by recursively solving the optimization problems on each sensor node in a fully distributed manner.

## V. Illustrative Numerical Examples

This section provides two illustrative numerical examples to demonstrate the effectiveness and applicability of distributed set-membership estimate schemes with BMs under the fixed and time-varying thresholds developed in this paper.

The communication topology of SN is described by the digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with the set of sensor nodes $\mathcal{V} = \{1,2,3,4,5,6\}$, the set of edges $\mathcal{E} = \{(1,3),(2,4),(3,5),(4,6),(5,1),(1,2)\}$ and the adjacency matrix is given as

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Next, consider an industrial nonisothermal continuous stirred tank reactor (CSTR), which is used to investigate distributed $H_\infty$ consensus filtering problems over sensor networks (not BSNs) in [12]. In the following, the state matrix of the discretized and linearized state-space model of the CSTR is borrowed from [12]. Meanwhile, we take into account the time-varying effect. As such, the parameters of the system (1) is set as follows:

$$A_s = \begin{bmatrix} 0.9719 & -0.0013 - 0.01\sin(s) \\ -0.0340 & 0.8628 \end{bmatrix}, \quad B_t = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$C_{i,s} = \begin{bmatrix} -0.2 & 0.1 + 0.01\sin(s) \end{bmatrix}, \quad D_{i,s} = 0.1.$$

The matrices $T_s$ and $R_{i,s}$ are chosen as $T_s = 0.25I$ and $R_{i,s} = 0.25$, respectively. The bounded noises are selected as $w_s = 0.3\sin(3s)$ and $v_{i,s} = 0.5\sin(3s)$, respectively. When $\hat{x}_{4,s}$ is transmitted to Node 6, the outliers occur at time instants $s = 10, 20, 25$ with the abnormal amplitude $10\hat{x}_{4,s}$. The parameters of the LPA method are set as $\alpha_{i,s} = 0.5$. The initial system state $x_0$ and estimator values $\bar{x}_{i,0}$ are set to be $x_0 = \begin{bmatrix} 2.0 & -1.0 \end{bmatrix}^T$, $\bar{x}_{1,0} = \begin{bmatrix} 1.2 & 1.2 \end{bmatrix}^T$, $\bar{x}_{2,0} = \begin{bmatrix} 0.9 & 1.1 \end{bmatrix}^T$, $\bar{x}_{3,0} = \begin{bmatrix} 0.8 & 1.1 \end{bmatrix}^T$, $\bar{x}_{4,0} = \begin{bmatrix} 1.1 & 0.9 \end{bmatrix}^T$, $\bar{x}_{5,0} = \begin{bmatrix} 0.8 & 1.1 \end{bmatrix}^T$, and $\bar{x}_{6,0} = \begin{bmatrix} 1.0 & 1.0 \end{bmatrix}^T$, respectively.

**Example 1:** For the case of fixed threshold, the threshold parameters of binary sensor nodes are set as $\tau_i = -0.3$ $(i = 1, 2, \ldots, 6)$. The initial values of $Q_{i,s}$ are given as $Q_{i,0} = 10I$. This simulation example considers two cases, i.e., Case I: no outlier-resistant design (i.e., the saturation levels are given as $\varpi_{i,s} = \begin{bmatrix} 50 & 50 \end{bmatrix}^T$); Case II: outlier-resistant design (i.e., the saturation levels are set as $\varpi_{i,s} = \begin{bmatrix} 0.25 & 0.25 \end{bmatrix}^T$). Then, all the estimator parameters can be obtained in terms of Corollary 1 with Theorem 2 by means of MATLAB software with the YALMIP toolbox. Under such two cases, all the simulation results are displayed in Figs. 1-5, where Fig. 1 describes the time instants of extracting BMs for every sensor node, Figs. 2 and 3 plot $\|x_s - \bar{x}_{i,s}\|^2$ to measure the estimation errors of every sensor node, and Figs. 4-5 draw $\sum_{j\in\mathcal{N}_i}\|\bar{x}_{i,s} - \bar{x}_{j,s}\|^2$ to measure the consensus errors between every sensor node and its neighboring nodes, respectively.

It is worth noting that in two cases, the time instants of extracting BMs are the same due to the fixed threshold. By comparing Fig. 2 with Fig. 3, and Fig. 4 with Fig. 5, it follows that the adverse impact of the outliers would be significantly reduced due to the introduction of saturation function.

**Example 2:** With the same parameters in Example 1, we will verify the advantage of the time-varying threshold over the fixed threshold. Choose $P_{i,0} = 10I$, $h_{i,s} \equiv 0.4$, and $\varphi_{i,0} = 0$. As for the time-varying thresholds, this simulation example considers two cases with outlier-resistant design (i.e., the saturation levels are given as $\rho_{i,s} = \begin{bmatrix} 0.25 & 0.25 \end{bmatrix}^T$), i.e.,

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



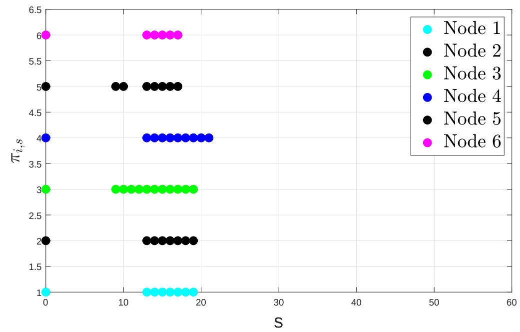



Fig. 1: The information extraction instants under Cases I and II.

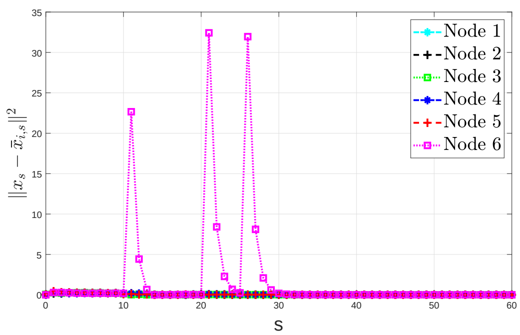



Fig. 2: $\|x_s - \bar{x}_{i,s}\|^2$ under Case I.

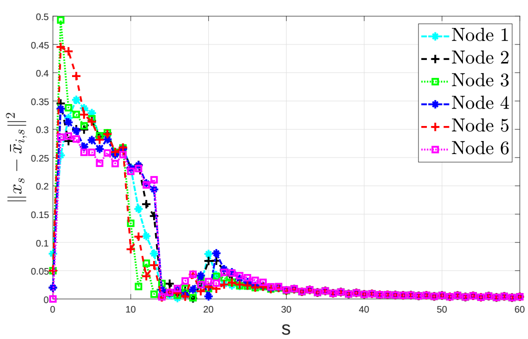



Fig. 3: $\|x_s - \bar{x}_{i,s}\|^2$ under Case II.

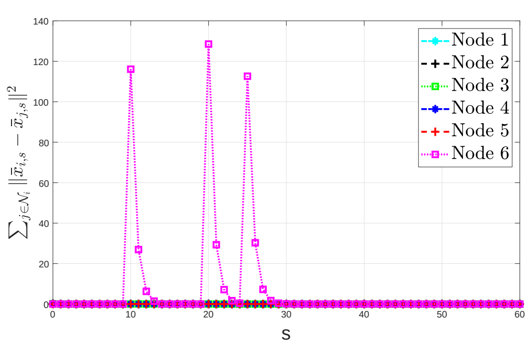



Fig. 4: $\sum_{j\in\mathcal{N}_i}\|\bar{x}_{i,s} - \bar{x}_{j,s}\|^2$ under Case I.

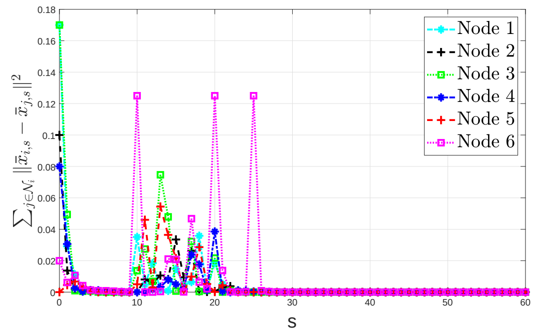



Fig. 5: $\sum_{j\in\mathcal{N}_i}\|\bar{x}_{i,s} - \bar{x}_{j,s}\|^2$ under Case II.

Case III: $\tau_i = -0.3$, and Case IV: $\tau_i = -0.36$. Under Cases III and IV, all the estimator parameters can be obtained in terms of Corollary 2 with Theorem 4 by means of MATLAB software with the YALMIP toolbox. The simulation results under these two cases are displayed in Figs. 6-11, where Figs. 6 and 7 plot the time instants of extracting BMs for every sensor node, Figs. 8 and 9 describe $\|x_s - \hat{x}_{i,s}\|^2$ to measure the estimation errors of every sensor node, Figs. 10 and 11 plot $\sum_{j\in\mathcal{N}_i}\|\hat{x}_{i,s} - \hat{x}_{j,s}\|^2$ to measure the consensus errors between every sensor node and its neighboring nodes, and Figs. 12 and 13 draw the time-varying threshold $\tau_{1,s}$ of sensor node 1, respectively.

It can be seen from the comparison of Figs. 1 and 6 that the time instants of extracting BMs become less due to the introduction of the time-varying threshold. Even then, by comparing Fig. 3 with Fig. 8, and Fig. 5 with Fig. 10, it can be observed that the performance of distributed filtering would be significantly improved, which demonstrates the advantage of time-varying threshold design developed in this paper.

In addition, it follows from Figs. 6 and 7 that the time instants under case III are more than that under Case IV. Next, by comparing Fig. 8 with Fig. 9, and Fig. 10 with Fig. 11, it can be observed that the advantage of the time-varying threshold would be more prominent if choosing a suitable fixed threshold $\tau_i$ due to $\tau_{i,s} = \tau_i + \varphi_{i,s}$. Therefore, all the presented simulation results demonstrate that the distributed set-membership estimation algorithms under BMs with the fixed and time-varying thresholds developed in this paper are indeed effective.

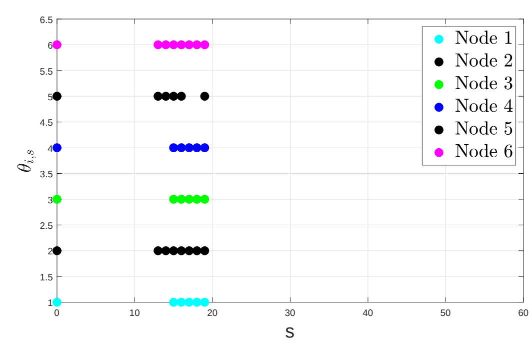



Fig. 6: The information extraction time instants under Case III.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems

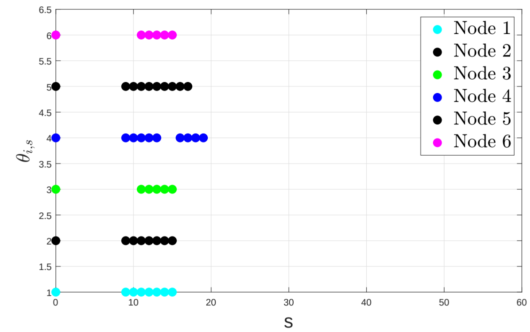




Fig. 7: The information extraction time instants under Case IV.

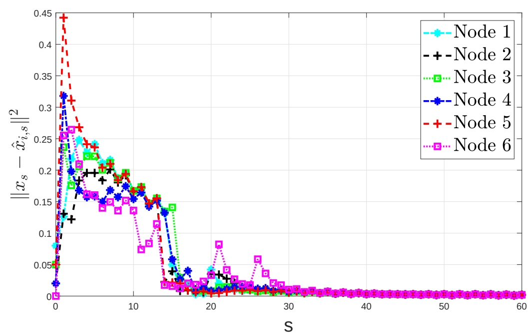



Fig. 8: $\|x_s - \hat{x}_{i,s}\|^2$ under Case III.

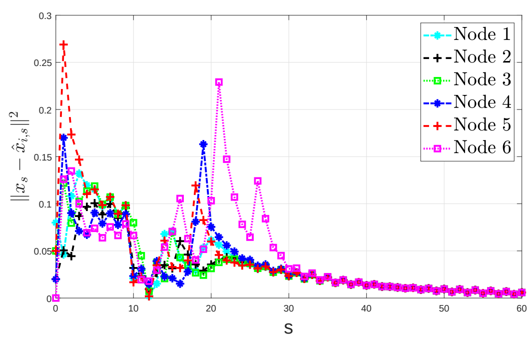



Fig. 9: $\|x_s - \hat{x}_{i,s}\|^2$ under Case IV.

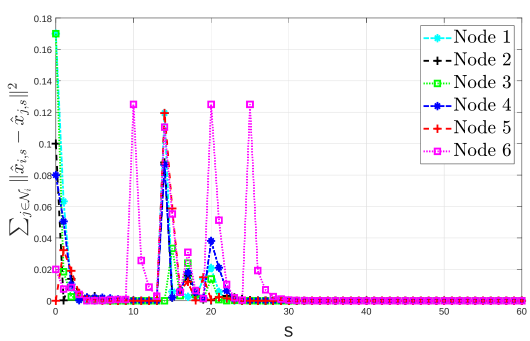



Fig. 10: $\sum_{j\in\mathcal{N}_i}\|\hat{x}_{i,s} - \hat{x}_{j,s}\|^2$ under Case III.

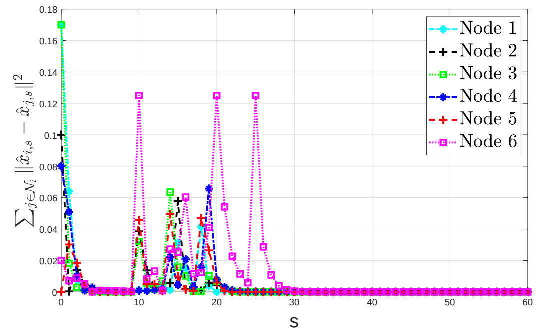



Fig. 11: $\sum_{j\in\mathcal{N}_i}\|\hat{x}_{i,s} - \hat{x}_{j,s}\|^2$ under Case IV.

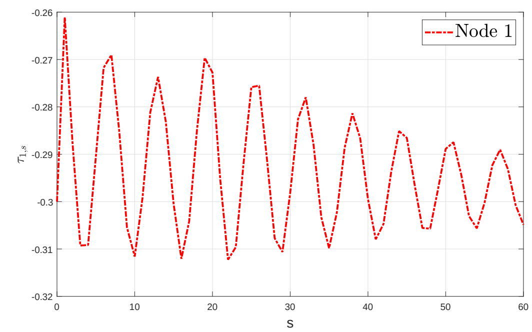



Fig. 12: $\tau_{1,s}$ under Case III.

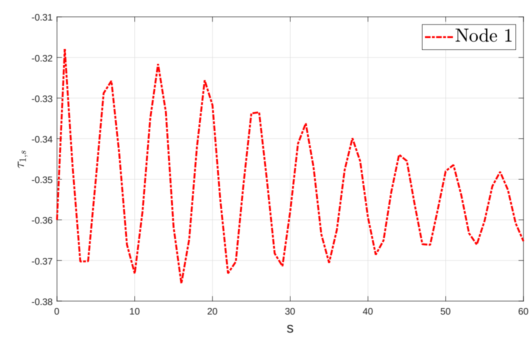



Fig. 13: $\tau_{1,s}$ under Case IV.

## VI. Conclusions

The distributed set-membership estimation problem has been investigated for a class of linear discrete time-varying systems under BMs with fixed and time-varying thresholds, respectively. The useful information has been extracted at the crossing instants of BMs. The time-varying threshold strategy has been proposed to reduce conservatism in the case of fixed thresholds. The distributed estimator with BMs under two cases has been proposed based on the extracted information from BMs as well as from the plant and its neighboring sensor nodes. Then, the distributed average set-membership performances over all sensor nodes have been proposed, which has less conservative than the existing performance indices. Subsequently, the local sufficient criteria have been established for each sensor node via the LPA method, and the estimator gains have been calculated by solving an optimization problem on each sensor node in a fully distributed manner. Finally, two illustrative simulation examples have been given to demonstrate the applicability and effectiveness of distributed

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



set-membership estimation schemes proposed in this paper.

For further improving the estimation accuracy under BMs, first research direction is to seek some efficient strategies to derive more nonzero measurements. Another is to reduce the uncertainties in the extracted information as far as possible by means of some methods, such as machine learning. In the end, the potential research interests would be to study the distributed set-membership estimation for more complicated systems (e.g., the switching system [27], hybrid model [18], and the complicated relationship [17]) and topology (e.g., the sequentially connected topology [8]).

## VII. Appendix

### A. Proof of Lemma 3

*Proof:* It is easily verified that Lemma 3 always holds for the case of $N = 1$, and the rest is to prove that Lemma 3 is true when $N \geq 2$. Next, assume that $\|\hat{x}_{1,s}\| \leq \|\hat{x}_{2,s}\| \leq \cdots \leq \|\hat{x}_{N,s}\|$ for a given time instant $s \in \mathbb{Z}^+$. According to (21), one immediately has

$$(\hat{x}_s^*)^T P_s^* \hat{x}_s^* \geq \frac{1}{N} \sum_{i=1}^{N} \hat{x}_{1,s}^T P_{i,s}^{-1} \hat{x}_{1,s}. \tag{31}$$

From (31), it follows that

$$\begin{aligned}
&\frac{1}{N} \sum_{i=1}^{N} \hat{x}_{i,s}^T P_{i,s}^{-1} \hat{x}_{i,s} - (\hat{x}_s^*)^T P_s^* \hat{x}_s^* \\
\leq& \frac{1}{N} \sum_{i=2}^{N} \left( (\hat{x}_{1,s} + \lambda_s)^T P_{i,s}^{-1} (\hat{x}_{1,s} + \lambda_s) - \hat{x}_{1,s}^T P_{i,s}^{-1} \hat{x}_{1,s} \right) \\
=& \frac{1}{N} \sum_{i=2}^{N} \left( \mathrm{Tr}(P_{i,s}^{-1} \Lambda_s) + 2\mathrm{Tr}(P_{i,s}^{-1} \lambda_s \hat{x}_{1,s}^T) \right)
\end{aligned} \tag{32}$$

where

$$\begin{aligned}
\iota_s &\triangleq \arg \max_{2 \leq i \leq N} \|\hat{x}_{i,s} - \hat{x}_{1,s}\|, \\
\lambda_s &\triangleq \hat{x}_{\iota_s,s} - \hat{x}_{1,s}, \quad \Lambda_s \triangleq \lambda_s \lambda_s^T.
\end{aligned}$$

For given a set of estimates $\hat{x}_{i,s}$ $(i = 2, \ldots, N)$, one always finds a positive definite matrix $P_{i,s} > 0$ such that the following inequalities simultaneously hold

$$P_{i,s}^{-1} \Lambda_s < \frac{1}{3(N-1)} I, \quad P_{i,s}^{-1} \lambda_s \hat{x}_{1,s}^T < \frac{1}{3(N-1)} I. \tag{33}$$

The proof is complete by substituting (33) into (32). ■

### B. Proof of Theorem 1

*Proof:* The mathematical induction method is employed to perform the proof. First, from Assumption 2, it is straightforward to have $\frac{1}{N} \sum_{i=1}^{N} \eta_{i,0}^T Q_{i,0}^{-1} \eta_{i,0} \leq 1$. Next, at time instant $s$, assume $\frac{1}{N} \sum_{i=1}^{N} \eta_{i,s}^T Q_{i,s}^{-1} \eta_{i,s} \leq 1$. Then, we shall prove $\frac{1}{N} \sum_{i=1}^{N} \eta_{i,s+1}^T Q_{i,s+1}^{-1} \eta_{i,s+1} \leq 1$ holds. For this purpose, according to Definition 1, we choose the scalar storage function as follows:

$$\overline{V}_{i,s} \triangleq \eta_{i,s}^T Q_{i,s}^{-1} \eta_{i,s},$$

and then calculate the scalar storage function along the system dynamics (9) as follows:

$$\begin{aligned}
\overline{V}_{i,s+1} =& \eta_{i,s+1}^T Q_{i,s+1}^{-1} \eta_{i,s+1} \\
\triangleq& \chi_{i,s}^T \Omega_{i1,s}^T Q_{i,s+1}^{-1} \Omega_{i1,s} \chi_{i,s}
\end{aligned} \tag{34}$$

where

$$\begin{aligned}
\chi_{i,s} &\triangleq \begin{bmatrix} 1 & \eta_{i,s}^T & \eta_{\mathcal{N}_i,s}^T & \xi_{i,s}^T \end{bmatrix}^T, \\
\eta_{\mathcal{N}_i,s} &\triangleq \begin{bmatrix} \eta_{j_{i_1},s}^T & \eta_{j_{i_2},s}^T & \cdots & \eta_{j_{i_{p_i}},s}^T \end{bmatrix}^T.
\end{aligned}$$

Noticing the following inequality from Assumption 1:

$$\frac{1}{3} \xi_{i,s}^T \mathrm{diag}\{T_s^{-1}, R_{i,s}^{-1}, R_{i,s+1}^{-1}\} \xi_{i,s} \leq 1, \tag{35}$$

one further has

$$\begin{aligned}
&\overline{V}_{i,s+1} - (1 - \alpha_{i,s}) \overline{V}_{i,s} - \sum_{j \in \mathcal{N}_i} \frac{\alpha_{j,s}}{1 + q_j} \overline{V}_{j,s} - \overline{S}_{i,s} \\
\leq& \chi_{i,s}^T \Omega_{i1,s}^T Q_{i,s+1}^{-1} \Omega_{i1,s} \chi_{i,s} - (1 - \alpha_{i,s}) \eta_{i,s}^T Q_{i,s}^{-1} \eta_{i,s} \\
&- \sum_{j \in \mathcal{N}_i} \frac{\alpha_{j,s}}{1 + q_j} \eta_{j,s}^T Q_{j,s}^{-1} \eta_{j,s} - \overline{S}_{i,s} \\
&- \phi_{i1,s} \left( \frac{1}{3} \xi_{i,s}^T \mathrm{diag}\{T_s^{-1}, R_{i,s}^{-1}, R_{i,s+1}^{-1}\} \xi_{i,s} - 1 \right) \\
\triangleq& \chi_{i,s}^T \Omega_{i1,s}^T Q_{i,s+1}^{-1} \Omega_{i1,s} \chi_{i,s} + \chi_{i,s}^T \Omega_{i0,s} \chi_{i,s}.
\end{aligned} \tag{36}$$

Substituting (25) into (36) yields

$$\overline{V}_{i,s+1} \leq (1 - \alpha_{i,s}) \overline{V}_{i,s} + \sum_{j \in \mathcal{N}_i} \frac{\alpha_{j,s}}{1 + q_j} \overline{V}_{j,s} + \overline{S}_{i,s},$$

which, by introducing some notations, can be updated as

$$\overline{V}_{i,s+1} \leq [W_s \overline{\mathbf{V}}_s]_i + \overline{S}_{i,s} \tag{37}$$

where

$$\begin{aligned}
\overline{\mathbf{V}}_s &\triangleq \begin{bmatrix} \overline{V}_{1,s} & \overline{V}_{2,s} & \cdots & \overline{V}_{N,s} \end{bmatrix}^T, \\
W_s &\triangleq \begin{bmatrix} (1 - \alpha_{1,s}) & a_{12} \frac{\alpha_{2,s}}{1+q_2} & \cdots & a_{1N} \frac{\alpha_{N,s}}{1+q_N} \\ a_{21} \frac{\alpha_{1,s}}{1+q_1} & (1 - \alpha_{2,s}) & \cdots & a_{2N} \frac{\alpha_{N,s}}{1+q_N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} \frac{\alpha_{1,s}}{1+q_1} & a_{N2} \frac{\alpha_{2,s}}{1+q_2} & \cdots & (1 - \alpha_{N,s}) \end{bmatrix}.
\end{aligned}$$

Since (37) holds for any sensor node $i \in \mathcal{V}$, a compact form of all sensor nodes is derived as

$$\overline{\mathbf{V}}_{s+1} \leq\leq W_s \overline{\mathbf{V}}_s + \overline{\mathbf{S}}_s \tag{38}$$

where $\overline{\mathbf{S}}_s \triangleq \begin{bmatrix} \frac{N\alpha_{1,s}}{1+q_1} & \cdots & \frac{N\alpha_{N,s}}{1+q_N} \end{bmatrix}^T$.

According to Definition 1, the system dynamics (9) is vector dissipative regarding the vector supply rate function $\overline{\mathbf{S}}_s$.

Next, left-multiplying $\mathbf{1}^T$ on both sides of (38) yields

$$\mathbf{1}^T \overline{\mathbf{V}}_{s+1} \leq \mathbf{1}^T W_s \overline{\mathbf{V}}_s + \mathbf{1}^T \overline{\mathbf{S}}_s, \tag{39}$$

which further means

$$\sum_{i=1}^{N} \overline{V}_{i,s+1} \leq \sum_{i=1}^{N} \left( 1 - \frac{\alpha_{i,s}}{1 + q_i} \right) \overline{V}_{i,s} + \sum_{i=1}^{N} \frac{N\alpha_{i,s}}{1 + q_i}. \tag{40}$$

Since $1 - \frac{\alpha_{i,s}}{1+q_i} \geq 0$ and $\overline{V}_{i,s} \geq 0$, one immediately has

$$\sum_{i=1}^{N} \left(1 - \frac{\alpha_{i,s}}{1+q_i}\right) \overline{V}_{i,s} \leq N \sum_{j=1}^{N} \left(1 - \frac{\alpha_{j,s}}{1+q_j}\right). \tag{41}$$

Substituting (41) into (40), one obtains

$$\sum_{i=1}^{N} \eta_{i,s+1}^{T} Q_{i,s+1}^{-1} \eta_{i,s+1} \leq N. \tag{42}$$

Consequently, the induction is accomplished. Therefore, the proof of this theorem is completed. ■

## References

[1] C. C. Aggarwal, *Outlier Analysis*, Springer. Cham, Switzerland, 2017.

[2] E.-W. Bai, "Source localization by a binary sensor network in the presence of imperfection, noise and outliers," *IEEE Transactions on Automatic Control*, vol. 63, no. 2, pp. 347–359, Feb. 2018.

[3] G. Battistelli, L. Chisci and S. Gherardini, "Moving horizon estimation for discrete-time linear systems with binary sensors: Algorithms and stability results," *Automatica*, vol. 85, pp. 374–385, Nov. 2017.

[4] G. Battistelli, L. Chisci, N. Forti and S. Gherardini, "MAP moving horizon estimation for threshold measurements with application to field monitoring," *International Journal of Adapt Control and Signal Processing*, vol. 34, no. 6, pp. 796–811, Jun. 2020.

[5] D. Ciuonzo, A. Aubry and V. Carotenuto, "Rician MIMO channel- and jamming-aware decision fusion," *IEEE Transactions on Signal Processing*, vol. 65, no. 15, pp. 3866–3880, Aug. 2017.

[6] R. Caballero-Águilaa, A. Hermoso-Carazob and J. Linares-Pérezb, "Optimal state estimation for networked systems with random parameter matrices, correlated noises and delayed measurements," *International Journal of General Systems*, vol. 44, no. 2, pp. 142–154, Jan. 2015.

[7] B. C. Csáji and E. Weyer, "Recursive estimation of ARX systems using binary sensors with adjustable thresholds," *IFAC Proceedings Volumes*, vol. 45, no. 16, pp. 1185–1190, Jul. 2012.

[8] Y. Cui, Y. Liu, W. Zhang and F. E. Alsaadi, "Event-based consensus for a class of nonlinear multi-agent systems with sequentially connected topology," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 65, no. 10, pp. 3506–3518, Oct. 2018.

[9] J. Dai and H. C. So, "Sparse bayesian learning approach for outlier-resistant direction-of-arrival estimation," *IEEE Transactions on Signal Processing*, vol. 66, no. 3, pp. 744–756, Feb. 2018.

[10] L. Ding and W. Sun, "Predefined time fuzzy adaptive control of switched fractional-order nonlinear systems with input saturation," *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 4, art. no. 100019, Dec. 2023.

[11] H. Gao, Y. Li, L. Yu and H. Yu, "Collaborative-prediction-based recursive filtering for nonlinear systems with sensor saturation under duty cycle scheduling," *Systems Science & Control Engineering*, vol. 11, no. 1, art. no. 2247007, Aug. 2023.

[12] X. Ge, Q.-L. Han, and Z. Wang, "A threshold-parameter-dependent approach to designing distributed event-triggered $H_\infty$ consensus filters over sensor networks," *IEEE Transactions on Cybernetics*, vol. 49, no. 4, pp. 1148–1159, Apr. 2019.

[13] J. Guo, J.-F. Zhang and Y. Zhao, "Adaptive tracking control of a class of first-order systems with binary-valued observations and time-varying thresholds," *IEEE Transactions on Automatic Control*, vol. 56, no. 12, pp. 2991–2996, Dec. 2011.

[14] D. He, C. Xu, J. Zhu and H. Du, "Moving horizon $H_\infty$ estimation of constrained multisensor systems with uncertainties and fading channels," *IEEE Transactions on Instrumentation and Measurement*, vol. 70, art. no. 9511112, Jul. 2021.

[15] Z. Hu, B. Chen, Y. Zhang and L. Yu, "Kalman-like filter under binary sensors," *IEEE Transactions on Instrumentation and Measurement*, vol. 71, art. no. 9503111, Feb. 2022.

[16] J. Hu, J. Li, G.-P. Liu, X. Yi and Z. Wu, "Optimized distributed filtering for time-varying saturated stochastic systems with energy harvesting sensors over sensor networks," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 9, pp. 412-426, Jun. 2023.

[17] G. He, M. Hu and Z. Shen, "Consensus of switched multi-agents system with cooperative and competitive relationship," *Systems Science & Control Engineering*, vol. 11, no. 1, art. no. 2192008, Mar. 2023.

[18] S. Feng, X. Li, S. Zhang, Z. Jian, H. Duan and Z. Wang, "A review: state estimation based on hybrid models of Kalman filter and neural network," *Systems Science & Control Engineering*, vol. 11, no. 1, art. no. 2173682, Feb. 2023.

[19] J. Hu, C. Wang, R. Caballero-Águila and H. Liu, "Distributed optimal fusion filtering for singular systems with random transmission delays and packet dropout compensations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 119, article no. 107093, May 2023.

[20] C. Ierardi, L. Orihuela and I. Jurado, "A distributed set-membership estimator for linear systems with reduced computational requirements," *Automatica*, vol. 132, art. no. 109802, Oct. 2021.

[21] Y. Jin, X. Ma, X. Meng and Y. Chen, "Distributed fusion filtering for cyber-physical systems under Round-Robin protocol: a mixed $H_2/H_\infty$ framework," *International Journal of Systems Science*, vol. 54, no. 8, pp. 1661–1675, Apr. 2023.

[22] K. Knudson, R. Saab and R. Ward, "One-bit compressive sensing with norm estimation," *IEEE Transactions on Information Theory*, vol. 62, no. 5, 2748–2758, May 2016.

[23] W. Li and F. Yang, "Information fusion over network dynamics with unknown correlations: An overview," *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 2, art. no. 100003, Jun. 2023.

[24] L. Liu, L. Ma, J. Guo, J. Zhang and Y. Bo, "Distributed set-membership filtering for time-varying systems: A coding-decoding-based approach," *Automatica*, vol. 129, art. no. 109684, Jul. 2021.

[25] X. Li and D. Ye, "Dynamic event-triggered distributed filtering design for interval type-2 fuzzy systems over sensor networks under deception attacks," *International Journal of Systems Science*, vol. 54, no. 15, pp. 2875–2890, Jan. 2023.

[26] Z. Lu and G. Guo, "Control and communication scheduling co-design for networked control systems: A survey," *International Journal of Systems Science*, vol. 54, no. 1, pp. 189–203, Jul. 2023.

[27] Y. Liu, A. Arumugam, S. Rathinasamy and F. E. Alsaadi, "Event-triggered non-fragile finite-time guaranteed cost control for uncertain switched nonlinear networked systems," *Nonlinear Analysis: Hybrid Systems*, vol. 36, art. no. 100884, May 2020.

[28] E. Mousavinejad, X. Ge, Q. -L. Han, T. J. Lim and L. Vlacic, "An ellipsoidal set-membership approach to distributed joint state and sensor fault estimation of autonomous ground vehicles," *IEEE/CAA Journal of Automatica Sinica*, vol. 8, no. 6, pp. 1107–1118, May 2021.

[29] L. Orihuela, S. Roshany-Yamchi, R. A. García and Pablo Millán, "Distributed set-membership observers for interconnected multi-rate systems," *Automatica*, vol. 85, pp. 221–226, Nov. 2017.

[30] X. Qian and B. Cui, "A mobile sensing approach to distributed consensus filtering of 2D stochastic nonlinear parabolic systems with disturbances," *Systems Science & Control Engineering*, vol. 11, no. 1, art. no. 2167885, Feb. 2023.

[31] A. Ribeiro, G. B. Giannakis and S. I. Roumeliotis, "SOI-KF: Distributed Kalman filtering with low-cost communications using the sign of innovations," *IEEE Transactions on Signal Processing*, vol. 54, no. 12, pp. 4782–4795, Dec. 2006.

[32] L. Ros, A. Sabater and F. Thomas, "An ellipsoidal calculus based on propagation and fusion," *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, vol. 32, no. 4, pp. 430–442, Aug. 2002.

[33] Q. Song, "Recursive identification of systems with binary-valued outputs and with ARMA noises," *Automatica*, vol. 93, pp. 106–113, Jul. 2018.

[34] T.-H. Tan, L. Badarch, W.-X. Zeng, M. Gochoo, F. S. Alnajjar and J.-W. Hsieh, "Binary sensors-based privacy-preserved activity recognition of elderly living alone using an RNN," *Sensors*, vol. 21, no. 16, art. no. 5371, Aug. 2021.

[35] H. Tsaknakis, "Outlier resistant filtering and smoothing," *Information and Computation*, vol. 79, pp. 163–192, Nov. 1988.

[36] V. Ugrinovskii, "Distributed robust filtering with $H_\infty$ consensus of estimates," *Automatica*, vol. 47, pp. 1–13, Jan. 2011.

[37] L. Wang, J.-F. Zhang, G. G. Yin, "System identification using binary sensors," *IEEE Transactions on Automatic Control*, vol. 48, no. 11, pp. 1892–1907, Nov. 2003.

[38] T. Wang, H. Zhang and Y. Zhao, "Consensus of multi-agent systems under binary-valued measurements and recursive projection algorithm," *IEEE Transactions on Automatic Control*, vol. 65, no. 6, pp. 2678–2685, Jun. 2020.

[39] Y. Wang, H.-J. Liu and H.-L. Tan, "An overview of filtering for sampled-data systems under communication constraints," *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 3, art. no. 100011, Sep. 2023.

[40] Y. Wang, Y. Zhao and J.-F. Zhang, "Distributed recursive projection identification with binary-valued observations," *Journal of Systems Science and Complexity*, vol. 34, pp. 2048–2068, Oct. 2021.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



[41] Y. Wang, Y. Zhao, J.-F. Zhang and J. Guo, "A unified identification algorithm of FIR systems based on binary observations with time-varying thresholds," *Automatica*, vol. 135, art. no. 109990, Jan. 2022.

[42] Y.-A. Wang, B. Shen, L. Zou and Q.-L. Han, "A survey on recent advances in distributed filtering over sensor networks subject to communication constraints," *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 2, art. no. 100007, Jun. 2023.

[43] Y. Shui, L. Dong, Y. Zhang and C. Sun, "Event-based adaptive fuzzy tracking control for nonlinear systems with input magnitude and rate saturations," *International Journal of Systems Science*, vol. 54, no. 16, pp. 3045–3058, Sept. 2023.

[44] Y. Xie, S. Ding, F. Yang, L. Wang and X. Xie, "Probabilistic-constrained distributed set-membership estimation over sensor networks: A dynamic periodic event-triggered approach," *IEEE Transactions on Network Science and Engineering*, vol. 9, no. 6, pp. 4444–4457, Nov.-Dec. 2022.

[45] W. Xu, S. Yang and J. Cao, "Fully distributed self-triggered control for second-order consensus of multiagent systems," *IEEE Transactions on System, Man, and Cybernetics: Systems*, vol. 51, no. 6, pp. 3541–3551, Jun. 2021.

[46] X. Yi, H. Yu, Z. Fang and L. Ma, "Probability-guaranteed state estimation for nonlinear delayed systems under mixed attacks," *International Journal of Systems Science*, vol. 54, no. 9, pp. 2059–2071, 2023.

[47] K. You, "Recursive algorithms for parameter estimation with adaptive quantizer," *Automatica*, vol. 52, pp. 192–201, Feb. 2015.

[48] X.-M. Zhang, Q.-L. Han and B.-L. Zhang, "An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems," *IEEE Transactions on Industrial Informatics*, vol. 13, no. 1, pp. 4-16, Feb. 2017.

[49] Y. Zhang, B. Chen and L. Yu, "Fusion estimation under binary sensors," *Automatica*, vol. 115, art. no. 108861, May 2020.

[50] Y. Zhang, N. Xia, Q.-L. Han and F. Yang, "Set-Membership global estimation of networked systems," *IEEE Transactions on Cybernetics*, vol. 52, no. 3, pp. 1454–1464, Mar. 2022.

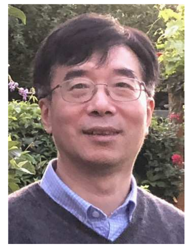
[51] X. Zhao, C. Liu, J. Liu and E. Tian, "Probabilistic-constrained reliable $H_\infty$ tracking control for a class of stochastic nonlinear systems: An outlier-resistant event-triggered scheme," *Journal of the Franklin Institute*, vol. 358, pp. 4741–4760, Jun. 2021.

[52] Y. Zhao, T. Wang and W. Bi, "Consensus protocol for multiagent systems with undirected topologies and binary-valued communications," *IEEE Transactions on Automatic Control*, vol. 64, no. 1, pp. 206–221, Jan. 2019.

[53] W. Zhao, H.-F. Chen, R. Tempo and F. Dabbene, "Recursive nonparametric identification of nonlinear systems with adaptive binary sensors," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 3959–3971, Aug. 2017.

**Fei Han** (M'17) received the Ph.D. degree in system analysis and integration from University of Shanghai for Science and Technology, Shanghai, China, in 2017. From March 2018 to June 2018, he was a Senior Research Assistant with the Department of Electronic Engineering, the City University of Hong Kong. From 2021 to 2022, he was a Visiting Scholar with the Department of Computer Science, Brunel University London, Uxbridge, U.K. He is currently a Professor with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing, China.

His current research interests include distributed control and filtering.



**Zidong Wang** (SM'03-F'14) received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently a Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the U.K. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published a number of papers in international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, the Editor-in-Chief for *Systems Science & Control Engineering*, and an Associate Editor for 12 international journals including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a Member of the Academia Europaea, a Member of the European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a Fellow of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

**Hongjian Liu** received his B.Sc. degree in applied mathematics in 2003 from Anhui University, Hefei, China, and the M.Sc. degree in detection technology and automation equipment in 2009 from Anhui Polytechnic University, Wuhu, China, and the Ph.D. degree in control science and engineering in 2018 from Donghua University, Shanghai, China. In 2016, he was a Research Assistant with the Department of Mathematics, Texas A&M University at Qatar, Doha, Qatar, for two months. From March 2017 to March 2018, he was a Visiting Scholar in the Department of Information Systems and Computing, Brunel University London, UK. He is currently a Professor in the School of Mathematics and Physics, Anhui Polytechnic University, Wuhu, China.

Dr. Liu's current research interests include filtering theory, memristive neural networks, and network communication systems. He is a very active reviewer for many international journals.

**Hongli Dong** (SM'16) received the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2012. From 2009 to 2010, she was a Research Assistant with the Department of Applied Mathematics, City University of Hong Kong, Hong Kong. From 2010 to 2011, she was a Research Assistant with the Department of Mechanical Engineering, The University of Hong Kong, Hong Kong. From 2011 to 2012, she was a Visiting Scholar with the Department of Information Systems and Computing, Brunel University London, London, U.K. From 2012 to 2014, she was an Alexander von Humboldt Research Fellow with the University of Duisburg-Essen, Duisburg, Germany. She is currently a Professor with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing, China. She is also the Director of the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Daqing. Her current research interests include robust control and networked control systems.

Dr. Dong is a very active reviewer for many international journals.

Citation information: DOI: 10.1109/TSMC.2024.3409611, IEEE Transactions on Systems, Man, and Cybernetics: Systems



**Guoping Lu** received the B.S. degree from the Department of Applied Mathematics, Chengdu University of Science and Technology, Chengdu, China, in 1984 and the M.S. and Ph.D. degrees in applied mathematics from the Department of Mathematics, East China Normal University, Shanghai, China, in 1989 and 1998, respectively.

He is currently a Professor with the School of Electrical Engineering, Nantong University, Nantong, China. His current research interests include singular systems, multiagent systems, networked control, and nonlinear signal processing.