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Set-Membership State Estimation for Multi-Rate Nonlinear Complex Networks under FlexRay Protocols: A Neural-Network-Based Approach

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***Abstract*—In this paper, the set-membership state estimation problem is investigated for a class of nonlinear complex networks under the FlexRay protocols. In order to address practical engineering requirements, the multi-rate sampling is taken into account which allows for different sampling periods of the system state and the measurement. On the other hand, the FlexRay protocol is deployed in the communication network from sensors to estimators in order to alleviate the communication burden. The underlying nonlinearity studied in this paper is of a general nature, and an approach based on neural networks is employed to handle the nonlinearity. By utilizing the convex optimization technique, sufficient conditions are established in order to restrain the estimation errors within certain ellipsoidal constraints. Then, the estimator gains and the tuning scalars of the neural network are derived by solving several optimization problems. Finally, a practical simulation is conducted to verify the validity of the developed set-membership estimation scheme.**

***Index Terms*—Multi-rate systems, FlexRay protocols, complex networks, neural networks, set-membership state estimation.**

Notations

FRP	FlexRay protocol
RRP	Round-Robin protocol
NNB	Neural-network-based
WTP	weighted Try-Once-Discard protocol
$\mathbb{R}^n$	The n-dimensional Euclidean space
$\|x\|$	The Euclidean norm of the vector x
$\mathrm{tr}\{M\}$	The trace of matrix M
M^{-1}	The inverse of matrix M
$\|M\|_F$	The Frobenius norm of matrix M
$M > N$	$M-N$ is positive definite
$M \geq N$	$M-N$ is positive semi-definite
$M \otimes N$	The Kronecker product of matrices M and N
$\mathrm{col}\{\cdots\}$	A column vector
$\mathrm{diag}\{\cdots\}$	A block diagonal matrix
$\mathrm{diag}_n\{A_i\}$	A block diagonal matrix $\mathrm{diag}\{A_1, A_2, \cdots, A_n\}$
$\delta(a, b)$	The Kronecker delta function
$\lfloor\cdot\rfloor$	The floor function
$\mathrm{mod}(a, b)$	The unique nonnegative remainder on division of a by b

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I. Introduction

Complex networks have garnered enduring interest from researchers in the field of systems and control due to their remarkable capability in characterizing practical systems such as power grids, traffic networks, and artificial neural networks [9], [14], [16], [21]. In a general sense, a complex network comprises numerous nodes, wherein the dynamics of each node are interconnected through a predefined topology. The evolution of an individual node's dynamics depends on its own past dynamics as well as the dynamics of the interconnected nodes. Due to this distinct feature, the analysis and synthesis of complex networks have gained significant attention, resulting in numerous findings in areas such as synchronization, consensus, and state estimation for complex networks [17], [24], [41], [46], [47], [56]. Notably, state estimation for complex networks has attracted considerable research enthusiasm due to the practical need to acquire information about the network states in various complex network applications [7], [27]. Thus far, various estimation algorithms have been developed for complex networks including the H_∞ scheme [49], the Kalman filtering approach [10], [37], and the set-membership estimation method [8], [32].

Through a literature review, it has been discovered that most of the results on the state estimation problem for complex networks are derived based on an assumption on synchronous sampling of the system state and measurements [19], [25], [33], [59]. However, in practice, this assumption is often

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violated due to the challenges of unifying the sampling rates of the state and measurements [35]. In real-world systems such as aluminium electrolysis cells [42] and power networks [36], different components of the system employ varying sampling rates due to their diverse physical characteristics. Additionally, sampling the measurements at each state update instant is not cost-effective for systems with slow state evolution [42]. Hence, multi-rate sampling is widely adopted, and the estimation of states in multi-rate systems holds significant importance.

In recent times, there has been a growing research focus on the state estimation of multi-rate systems, resulting in various research findings across different types of systems [20], [64]. For instance, in [23], the zonotopic set-membership state estimation problem has been studied for networked systems with varying measurement sampling periods compared to the plant sampling period. In [58], the emphasis has been on distributed fusion estimation for systems where the state, measurement, and estimate are asynchronously sampled. Moreover, in [64], the consideration has been given to moving horizon estimation for networked systems that adopt different sampling periods for two groups of sensors. It is worth noting that, despite the growing number of results for multi-rate systems, there is a lack of adequate attention given to multi-rate complex networks. Therefore, the primary motivation of our ongoing research is to explore state estimation techniques specifically tailored for multi-rate complex networks.

Wireless communication networks have become extensively implemented in modern industrial systems [3]. The adoption of wireless communication networks brings numerous benefits including easy installation, high flexibility, and reliability [50], [51]. However, the limited communication capacity of wireless networks also introduces undesirable phenomena such as transmission delays and missing measurements [5]. To address these challenges, communication protocols are deployed to alleviate the communication burden by scheduling transmission orders based on certain principles [11], [13], [38], [45], [54], [55]. In the literature, various communication protocols have been extensively investigated which include time-triggered protocols (e.g. Round-Robin protocol (RRP)) and event-triggered protocols (e.g. weighted Try-Once-Discard protocol (WTP) [60]). Numerous results have been developed in regard to the estimation problems under communication protocols, see e.g. [15], [18], [63] and the references cited therein.

The FlexRay protocol (FRP), serving as a specialized communication protocol, has received significant interest in fields such as car manufacturing and automotive electronics. Distinct from traditional time- or event-triggered protocols, the FRP is a hybrid protocol that incorporates both time- and event-triggered selection principles [28]. More specifically, the communication cycle in FRP is divided into static and dynamic segments, where time-triggered and event-triggered selection principles are employed, respectively. Compared to purely time- or event-triggered protocols, the FRP provides enhanced flexibility for data communications and has started to stir some initial research attention. For instance, in [48], the observer design problem has been addressed for nonlinear networked control systems under FRPs. Additionally, in [40], the tracking control problem has been examined for cyber-physical systems under the FRP. In this paper, our goal is to further expand the body of knowledge regarding FRP-based estimations by studying set-membership estimation under FRPs.

Nonlinearities are pervasive in practical systems and, in the past few decades, the state estimation and control problems for nonlinear systems have long been prominent research subjects [2], [6], [12], [30], [39]. In general, when dealing with nonlinear systems, the most commonly employed methods involve imposing sector-bounded conditions or linearizing the nonlinear function [34], [43], [44], [53]. However, both of these approaches require knowledge of the nonlinearities, which can be challenging to obtain in practice. As an alternative method, the neural-network-based (NNB) estimation technique has been developed for nonlinear systems, where a neural network is employed to approximate the unknown nonlinear dynamics [4]. Applications of the NNB method can be found in works such as [61] and [31], where it has been utilized for distributed state estimation in networked nonlinear systems and Lipschitz nonlinear systems, respectively. Moreover, the NNB method has also been used in [52] for observer-based security control problem on switched nonlinear systems and in [62] for output-feedback control problem on uncertain nonsmooth nonlinear systems. However, the application of the NNB estimation method to nonlinear complex networks has been rarely addressed in the literature, which is particularly true in the context of multi-rate sampling under FRP.

In response to the aforementioned motivations, our research aims to address the NNB set-membership state estimation for multi-rate nonlinear complex networks under FRPs. The key challenges in designing the estimation scheme are as follows: 1) how can we accurately describe the scheduling of the FRP and incorporate it into the estimator design? and 2) how can we construct an appropriate neural network tuning law that ensures bounded neural network weights and state estimation errors? By effectively addressing these challenges, our research makes the following contributions:

1) we propose a novel NNB set-membership estimation scheme specifically tailored for nonlinear complex networks operating under multi-rate sampling and FRPs,
2) we establish a mathematical model that characterizes the measurements after the scheduling of FRPs and integrate it into the estimator,
3) we develop a suitable neural network tuning law that guarantees desired performance for both state estimation error and neural network weight estimation error, and
4) the developed estimation method allows for a more accurate approximation of the unknown nonlinear dynamics compared to traditional methods.

Through these contributions, our main research objective is to provide an innovative approach for set-membership estimation in nonlinear complex networks by tackling the complexities of multi-rate sampling and FRPs.

The subsequent sections are structured as follows. Section II focuses on the conversion of the underlying multi-rate complex network into a single-rate network. It also introduces the FRP and characterizes its effect on the sensors' scheduling.

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Additionally, a neural network is employed to approximate the nonlinearity present in the complex network. Moving on to Section III, the design of the estimator gains and the neural network tuning parameters is discussed. In Section IV, a practical simulation is presented to demonstrate the effectiveness of the proposed approach. Finally, Section V provides a conclusion to this paper.

II. Problem Formulation

Consider the following complex network with unknown nonlinear dynamics:

$$x_{i,l+1} = A_{i,l}x_{i,l} + f(x_{i,l}) + \sum_{j=1}^{N} \omega_{ij}\Gamma x_{j,l} + B_{i,l}w_{i,l}, \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_{i,l} \in \mathbb{R}^{n_x}$ is the state of the i-th node, $f(\cdot)$ is an unknown smooth nonlinear function on a compact set, and $w_{i,l} \in \mathbb{R}^{n_w}$ is the process noise satisfying

$$\Psi(0, W_{i,l}) \triangleq \{w_{i,l} | w_{i,l}^T W_{i,l}^{-1} w_{i,l} \leq 1\}$$

where $W_{i,l}$ is a known positive-definite matrix and $\Psi(a, X)$ is an ellipsoid set with the center a and the shape matrix $X > 0$. $A_{i,l}$ and $B_{i,l}$ are known matrices with suitable dimensions. $\Gamma \triangleq \mathrm{diag}\{\gamma_1, \gamma_2, \cdots, \gamma_N\}$ is the inner coupling matrix with $\gamma_i \neq 0$ being the linking with the j-th state variable. $\Omega \triangleq \left[\omega_{ij}\right]_{N\times N}$ is the coupled configuration matrix of the network with $\omega_{ij} \geq 0$ $(i \neq j)$ but not all zeros and $\omega_{ii} = -\sum_{j=1,j\neq i}^{N} \omega_{ij}$. The initial condition of $x_{i,l}$ is $x_{i,0}$.

For the i-th node, the measurement output $y_{i,t_k} \in \mathbb{R}^{n_y}$ with a sampling period $b \triangleq t_{k+1} - t_k$ is modeled by

$$y_{i,t_k} = C_{i,t_k}x_{i,t_k} + v_{i,t_k} \tag{2}$$

where $v_{i,t_k} \in \mathbb{R}^{n_v}$ is the measurement noise belonging to

$$\Psi(0, V_{i,t_k}) \triangleq \{v_{i,t_k} | v_{i,t_k}^T V_{i,t_k}^{-1} v_{i,t_k} \leq 1\}$$

with V_{i,t_k} being a known positive-definite matrix. C_{i,t_k} is a known matrix with compatible dimensions.

To address the unknown nonlinear function $f(\cdot)$, a neural network is employed to approximate $f(\cdot)$ by capitalizing on its universal approximation property. The specific approximation of the nonlinear function $f(\cdot)$ is as follows [26]:

$$f(x_{i,l}) = U\sigma(x_{i,l}) + \varrho_{i,l} \tag{3}$$

where $\varrho_{i,l}$ is the approximation error, U is the weight matrix, and $\sigma(\cdot)$ is the activation function. Moreover, we assume that U, $\varrho_{i,l}$, and $\sigma(\cdot)$ satisfy [57]:

$$\|U\|_F \leq \bar{u}, \quad \|\varrho_{i,l}\| \leq \bar{\varrho}_i, \quad \|\sigma(\cdot)\| \leq \bar{\sigma}$$

where $\bar{u}$, $\bar{\varrho}_i$, and $\bar{\sigma}$ are known positive constants.

During data transmissions in communication networks, the limited network bandwidth can result in data congestion, leading to issues such as packet dropouts, transmission delays, and packet disorder. In this paper, the FRP is utilized to manage the scheduling of transmissions from sensors to estimators. With the FRP, only one sensor is granted access to transmit at each time instant, effectively reducing network congestion and mitigating the aforementioned issues.

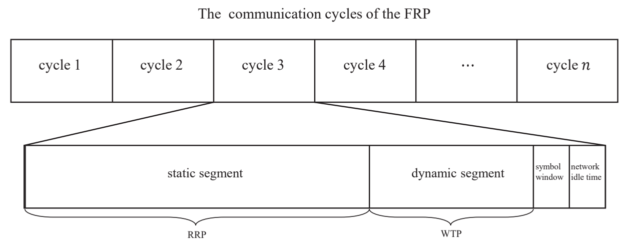

Fig. 1: The illustration of the FRP

Figure 1 illustrates the components of a communication cycle in the FRP, which includes a static segment, a dynamic segment, a symbol window, and a network idle time. It is important to note that the durations of the symbol window and the network idle time are significantly smaller compared to those of the static and dynamic segments. As a result, this paper focuses solely on the static and dynamic segments, while disregarding the symbol window and the network idle time.

In order to facilitate the analysis, we define the time lengths of the communication cycle, the static segment, and the dynamic segment of the FRP as Lb $(L < N)$, L_1b, and L_2b $(L_2 < L_1)$, respectively. It is obvious that $L = L_1 + L_2$. Based on the principles of the FRP, the RRP is active during the static segment, while the WTP is executed during the dynamic segment. Considering the varying real-time requirements of different sensors, we designate the first L_1 sensors as the set $S_1 \triangleq \{1, 2, \ldots, L_1\}$ and schedule them using the RRP. The remaining $N - L_1$ sensors belong to the set $S_2 \triangleq \{L_1 + 1, L_1 + 2, \ldots, N\}$ and are scheduled using the WTP.

Now, let us illustrate the scheduling of the FRP on the sensors. It is known that the RRP and the WTP are active during specific time intervals: $\Upsilon_{1,i} \triangleq [iLb, iLb + L_1b)$ and $\Upsilon_{2,i} \triangleq [iLb + L_1b, (i + 1)Lb)$ $(i = 0, 1, 2, \ldots)$, respectively. We denote ε_{t_k} as the sensor granted transmission access at time instant t_k under the RRP, and ϵ_{t_k} as the sensor granted transmission access at time instant t_k under the WTP. By utilizing the knowledge about the RRP and the WTP, we can deduce the following relationships:

$$\varepsilon_{t_k} = \begin{cases} \mathrm{mod}\left(k - \left\lfloor \frac{k}{L} \right\rfloor, L - 1\right) + 1, & \text{for } t_k \in \Upsilon_{1,i}; \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

$$\epsilon_{t_k} = \begin{cases} \arg\max_{j\in S_2} \tilde{y}_{j,t_k}^T \Pi_j \tilde{y}_{j,t_k}, & \text{for } t_k \in \Upsilon_{2,i}; \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where Π_j are given positive definite matrices and

$$\tilde{y}_{j,t_k} \triangleq y_{j,t_k} - y_{j,t_k}^*$$

with y_{j,t_k}^* being the latest transmitted measurement of sensor j.

After the scheduling of the FRP, the measurement arrived at the i-th estimator is written as

$$\bar{y}_{i,t_k} = \begin{cases} \delta(\varepsilon_{t_k}, i)y_{i,t_k}, & \text{for } i \in S_1; \\ \delta(\epsilon_{t_k}, i)y_{i,t_k}, & \text{for } i \in S_2 \end{cases} \tag{6}$$

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where $\delta(\cdot,\cdot)$ is the Kronecker delta function. Since $\varepsilon_{t_k}=i$ and $\epsilon_{t_k}=i$ cannot be satisfied simultaneously, we further have

$$\bar{y}_{i,t_k}=\delta(\varepsilon_{t_k},i)y_{i,t_k}+\delta(\epsilon_{t_k},i)y_{i,t_k},\quad \text{for } 1\le i\le N. \tag{7}$$

Note that the complex network (1)-(2) is actually a multi-rate system. In order to simplify the subsequent analysis, we will unify the sampling rates of both the state and the measurement. By introducing

$$\zeta_l \triangleq \begin{cases} 1, & \text{if } l/b\in\mathbb{N};\\ 0, & \text{otherwise}\end{cases}$$

and setting $\varepsilon_l=0$, $\epsilon_l=0$ for $l/b\notin\mathbb{N}$, the original measurement y_{i,t_k} is reconstructed as

$$y_{i,l}=\zeta_l C_{i,l}x_{i,l}+\zeta_l v_{i,l} \tag{8}$$

and the measurement $\bar{y}_{i,t_k}$ received by the estimator is reconstructed as

$$\bar{y}_{i,l}=\bar{\delta}_{i,l}y_{i,l},\quad \text{for } 1\le i\le N \tag{9}$$

where

$$\bar{\delta}_{i,l}\triangleq\delta(\varepsilon_l,i)+\delta(\epsilon_l,i).$$

Remark 1: For the considered multi-rate system (1)-(2), due to the existence of the nonlinear function, the conventional lifting technique is no longer applicable. In order to convert the multi-rate system into single-rate one, we introduce an indicator variable ζ_l that equals 1 when $l/b\in\mathbb{N}$ and equals 0 otherwise. With the help of ζ_l, the measurement output (2) is rewritten as (8) whose sampling period is the same as the state update period.

In this paper, the i-th estimator is designed as the following form:

$$\begin{aligned}\hat{x}_{i,l+1}=&A_{i,l}\hat{x}_{i,l}+\hat{U}_{i,l}\sigma(\hat{x}_{i,l})+\sum_{j=1}^{N}\omega_{ij}\Gamma\hat{x}_{j,l}\\ &+K_{i,l}(\bar{y}_{i,l}-\tilde{\delta}_{i,l}C_{i,l}\hat{x}_{i,l})\end{aligned} \tag{10}$$

where $\hat{x}_{i,l}$ and $\hat{U}_{i,l}$ are the estimates of $x_{i,l}$ and U, respectively. $K_{i,l}$ is the gain matrix to be determined and $\tilde{\delta}_{i,l}\triangleq\bar{\delta}_{i,l}\zeta_l$.

Define the cost function as

$$\begin{aligned}\mathcal{J}_{i,l}\triangleq&\frac{1}{2}(\bar{y}_{i,l}-\tilde{\delta}_{i,l}C_{i,l}\hat{x}_{i,l})^T\\ &\times(\bar{y}_{i,l}-\tilde{\delta}_{i,l}C_{i,l}\hat{x}_{i,l}).\end{aligned}$$

Taking the partial derivative of $\mathcal{J}_{i,l+1}$ with respect to $\hat{U}_{i,l}$, we have

$$\begin{aligned}\frac{\partial\mathcal{J}_{i,l+1}}{\partial\hat{U}_{i,l}}=&-\tilde{\delta}_{i,l+1}C_{i,l+1}^T\\ &\times(\bar{y}_{i,l+1}-C_{i,l+1}\hat{x}_{i,l+1})\sigma^T(\hat{x}_{i,l}).\end{aligned}$$

Accordingly, the tuning law for $\hat{U}_{i,l}$ is chosen as

$$\begin{aligned}\hat{U}_{i,l+1}=&\phi_{i,l}\hat{U}_{i,l}+\varphi_{i,l}\tilde{\delta}_{i,l+1}C_{i,l+1}^T\\ &\times(\bar{y}_{i,l+1}-C_{i,l+1}\hat{x}_{i,l+1})\sigma^T(\hat{x}_{i,l})\end{aligned} \tag{11}$$

where $\phi_{i,l}$ and $\varphi_{i,l}$ are the positive tuning scalars to be determined.

Denoting the estimation error as $e_{i,l}\triangleq x_{i,l}-\hat{x}_{i,l}$ and the weight estimation error as $\tilde{U}_{i,l}\triangleq U-\hat{U}_{i,l}$, we have

$$\begin{aligned}\tilde{U}_{i,l+1}=&(1-\phi_{i,l})U+\phi_{i,l}\tilde{U}_{i,l}\\ &-\varphi_{i,l}\tilde{\delta}_{i,l+1}C_{i,l+1}^T\lambda_{i,l+1}\sigma^T(\hat{x}_{i,l})\end{aligned} \tag{12}$$

where

$$\lambda_{i,l}\triangleq\bar{y}_{i,l}-C_{i,l}\hat{x}_{i,l}.$$

Remark 2: The tuning law (11) is developed by using the gradient descent method. In Section III, the tuning parameters $\phi_{i,l}$ and $\varphi_{i,l}$ will be designed along with the estimator gains. With the characterized tuning parameters $\phi_{i,l}$ and $\varphi_{i,l}$, the estimate $\hat{U}_{i,l+1}$ is updated according to (11).

Assumption 1: The initial conditions $e_{i,0}$ and $\tilde{U}_{i,0}$ satisfy

$$\begin{aligned}\mathrm{tr}\left[\tilde{U}_{i,0}^TR_{i1,0}^{-1}\tilde{U}_{i,0}\right]&\le 1,\\ e_{i,0}^TR_{i2,0}^{-1}e_{i,0}&\le 1\end{aligned}$$

with $R_{i1,0}$ and $R_{i2,0}$ being positive definite matrices of suitable dimensions.

Our purpose is to choose appropriate tuning parameters $\phi_{i,l}$, $\varphi_{i,l}$ and estimator gain $K_{i,l}$ such that

$$\mathrm{tr}\left[\tilde{U}_{i,l}^TR_{i1,l}^{-1}\tilde{U}_{i,l}\right]\le 1 \tag{13}$$

and

$$x_{i,l}\in\Psi(\hat{x}_{i,l},R_{i2,l})\triangleq\{x_{i,l}|e_{i,l}^TR_{i2,l}^{-1}e_{i,l}\le 1\} \tag{14}$$

hold with $R_{i1,l}$ and $R_{i2,l}$ being positive definite matrices of suitable dimensions. Moreover, we are going to minimize $R_{i2,l}$ in the matrix trace sense to obtain the optimal performance.

III. Main results

In this section, the main results on the design of state estimators for multi-rate nonlinear complex networks under FRP are presented. First, the tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$ of (11) are designed. Then, the characterization of the desired state estimators is provided. Subsequently, optimal problems are solved to minimize the ellipsoidal constraints on the state estimation errors. Finally, the discussion revolves around the boundedness of the weight estimation error $\tilde{U}_{i,l}$.

The following lemma is introduced which is required for later analysis.

Lemma 1: [1] Let $\chi_i(\mu)\triangleq\mu^TY_i\mu$ be given where μ is a known vector and $Y_i^T=Y_i$. If there exist scalars $\theta_1\ge 0,\ldots,\theta_n\ge 0$ such that $Y_0-\sum_{i=1}^n\theta_iY_i\le 0$, then

$$\chi_1(\cdot)\le 0,\ldots,\chi_n(\cdot)\le 0\Rightarrow\chi_0(\cdot)\le 0.$$

A. The tuning parameter design

In this subsection, a theorem is presented, which provides a sufficient condition for the existence of the tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$.

Theorem 1: Let the positive definite matrix $R_{i1,l}$ be given. For complex networks (1)-(2) with the tuning law (11) under

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the FRP, the requirement (13) is satisfied if there are positive scalars $\beta_{i,1l}$, $\beta_{i,2l}$ and tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$ such that

$$\begin{bmatrix} -\bar{R}_{i1,l+1} & \Xi_{i,l} \\ \Xi_{i,l}^T & \Sigma_{i,l} \end{bmatrix} \le 0 \tag{15}$$

where

$$\begin{aligned}
\Sigma_{i,l} &\triangleq \text{diag}\{\beta_{i,1l}\bar{u}^2 + \beta_{i,2l} - 1, -\beta_{i,2l}I, -\beta_{i,1l}I\}, \\
\Xi_{i,l} &\triangleq \begin{bmatrix} -\varphi_{i,l}\tilde{\delta}_{i,l+1}(\sigma(\hat{x}_{i,l}) \otimes C_{i,l+1}^T)\lambda_{i,l+1} & \bar{\Xi}_{i,l} \end{bmatrix}, \\
\bar{\Xi}_{i,l} &\triangleq \begin{bmatrix} \phi_{i,l}\bar{Q}_{i1,l} & (1-\phi_{i,l})I \end{bmatrix}, \\
\bar{R}_{i1,l} &\triangleq R_{i1,l} \otimes I_{n_x}
\end{aligned}$$

and $\bar{Q}_{i1,l}$ is a factorization of $\bar{R}_{i1,l}$.

Proof: See Appendix A. ■

Under Theorem 1, the weight estimation error $\tilde{U}_{i,l}$ is constrained by a predefined ellipsoidal constraint. Moreover, the tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$ are calculated by solving the matrix inequality (15).

B. The set-membership estimator design

In this subsection, the characterization of the state estimator gain $K_{i,l}$ is achieved by utilizing Theorem 1. A sufficient condition is established that ensures the state estimation error to satisfy (14).

Theorem 2: Let the positive definite matrix $R_{i2,l}$ and the estimator gain $K_{i,l}$ be given. For complex networks (1)-(2) with the tuning law (11) under the FRP, the objective (14) is satisfied if there are positive scalars $\alpha_{i,jl}$ $(j = 1,2,3,4,5,6)$ such that

$$\begin{bmatrix} -R_{i2,l+1} & \Theta_{i,l} \\ \Theta_{i,l}^T & -\bar{\Sigma}_{i,l} \end{bmatrix} \le 0 \tag{16}$$

where

$$\begin{aligned}
\bar{\Sigma}_{i,l} &\triangleq \text{diag}\{\bar{\Sigma}_{i,l}^{(1)}, \bar{\Sigma}_{i,l}^{(2)}, \bar{\Sigma}_{i,l}^{(3)}\}, \\
\vec{\sigma}(\hat{x}_{i,l}) &\triangleq \text{diag}_{n_x}\{\sigma^T(\hat{x}_{i,l})\}, \\
\bar{\Sigma}_{i,l}^{(1)} &\triangleq 1 - \alpha_{i,1l} - 2\alpha_{i,2l}N\bar{u}\bar{\sigma}^2 - \alpha_{i,3l} \\
&\quad - \alpha_{i,4l} - \alpha_{i,5l} - \alpha_{i,6l}\bar{\varrho}_i, \\
\bar{\Sigma}_{i,l}^{(2)} &\triangleq \text{diag}\{\alpha_{i,1l}\mathbf{I}_i, \alpha_{i,2l}I, \alpha_{i,3l}I\}, \\
\bar{\Sigma}_{i,l}^{(3)} &\triangleq \text{diag}\{\alpha_{i,4l}W_{i,l}^{-1}, \alpha_{i,5l}V_{i,l}^{-1}, \alpha_{i,6l}I\}, \\
\Theta_{i,l} &\triangleq \begin{bmatrix} 0 & \Delta_{i,l} & I & \vec{\sigma}(\hat{x}_{i,l})\bar{Q}_{i1,l} & B_{i,l} & -\tilde{\delta}_{i,l}K_{i,l} & I \end{bmatrix}, \\
\bar{\Gamma}_{ij,l} &\triangleq \omega_{ij}\Gamma Q_{j2,l}, \ \bar{K}_{i,l} \triangleq A_{i,l}Q_{i2,l} - \tilde{\delta}_{i,l}K_{i,l}C_{i,l}Q_{i2,l}, \\
\Delta_{i,l} &\triangleq \begin{bmatrix} \bar{\Gamma}_{i1,l} & \cdots & \bar{\Gamma}_{ii-1,l} & \bar{K}_{i,l} + \bar{\Gamma}_{ii,l} & \bar{\Gamma}_{ii+1,l} & \cdots & \bar{\Gamma}_{iN,l} \end{bmatrix}, \\
\mathbf{I}_i &\triangleq \text{diag}\{\underbrace{0, \cdots, 0}_{i-1}, I, 0, \cdots, 0\}
\end{aligned}$$

and $Q_{i2,l}$ is a factorization of $R_{i2,l}$.

Proof: See Appendix B. ■

Next, leveraging Theorems 1-2, we present a sufficient condition for the solvability of the state estimator design problem for multi-rate nonlinear complex networks under FRP. By employing the following theorem, the desired tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$ and estimator gain $K_{i,l}$ can be designed.

Theorem 3: Let the positive definite matrices $R_{i1,l}$ and $R_{i2,l}$ be given. For complex networks (1)-(2) with the tuning law (11) under the FRP, if there exist positive scalars $\beta_{i,1l}$, $\beta_{i,2l}$, $\alpha_{i,jl}$ $(j = 1,2,3,4,5,6)$, tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$, and estimator gain matrix $K_{i,l}$ such that (15) and (16) hold, then (13) and (14) are satisfied simultaneously.

Proof: The proof can be readily obtained by employing Theorems 1-2, and is therefore omitted here. ■

C. Optimization of the ellipsoid

In Subsection III-B, the characterization of the tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$ and estimator gain $K_{i,l}$ is accomplished. However, it should be noted that the estimator gain obtained by solving the matrix inequality (16) may constitute a set. Therefore, in the subsequent discussion, an optimization problem is introduced to determine the optimal estimator gain, which ensures the minimal ellipsoidal constraint on the state estimation error.

Theorem 4: Let the positive definite matrices $R_{i1,l}$ and $R_{i2,0}$ be given. For complex networks (1)-(2) with the tuning law (11) under the FRP, the system state $x_{i,l}$ is constrained within the optimal ellipsoid $\Psi(\hat{x}_{i,l}, R_{i2,l})$ with $R_{i2,l}$ minimized in the matrix trace sense if there are positive scalars $\beta_{i,1l}$, $\beta_{i,2l}$, $\alpha_{i,jl}$ $(j = 1,2,3,4,5,6)$, tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$, and estimator gain matrix $K_{i,l}$ such that

$$\min_{K_{i,l}} \text{tr}(R_{i2,l+1}) \tag{17}$$

$$\text{subject to (15) and (16)}$$

is feasible.

The following algorithm outlines the procedure for characterizing the set-membership state estimator for multi-rate nonlinear complex networks under FRP.

Algorithm 1 NNB set-membership state estimator design algorithm for multi-rate nonlinear complex networks under FRP

Step 1. Calculate the variables ε_{t_k} and ϵ_{t_k} based on (4) and (5), respectively;
Step 2. Set initial conditions $x_{i,0}$, $\hat{x}_{i,0}$, $\hat{U}_{i,0}$ and positive definite matrices $R_{i1,l}$ and $R_{i2,0}$. Choose the activation function $\sigma(\cdot)$ and the maximum time step T;
Step 3. At time instant l, obtain the estimator gain $K_{i,l}$ and the tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$ according to Theorem 4. Then, calculate the state estimate $\hat{x}_{i,l+1}$ based on (10);
Step 4. Calculate the innovation $\lambda_{i,l+1}$. Update the estimate $\hat{U}_{i,l+1}$ of the neural network weight according to (11);
Step 5. If $l < T$, then go to Step 3, else go to Step 6;
Step 6. Stop.

D. Boundedness analysis

In this subsection, we will discuss the ultimate boundedness of $\tilde{U}_{i,l}$.

Assumption 2: There are positive scalars $\bar{c}$, $\bar{r}$, and $\bar{v}$ such that

$$\begin{aligned}
&C_{i,l+1}^T C_{i,l+1} \le \bar{c}I, \\
&\text{tr}\{R_{i2,l+1}\} \le \bar{r}, \ \text{tr}\{V_{i,t_k}\} \le \bar{v}.
\end{aligned}$$

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In the sequel, a sufficient condition is developed such that $\tilde{U}_{i,l}$ is ultimately bounded.

Theorem 5: Under Assumption 2, the weight estimation error $\tilde{U}_{i,l}$ is ultimately bounded if

$$4\phi_{i,l}^2 - 1 < 0. \tag{18}$$

Proof: See Appendix C. ■

Remark 3: By now, the NNB set-membership state estimation problem for multi-rate nonlinear complex networks under FRPs has been addressed. Theorems 1 and 2 establish sufficient conditions for the existence of the desired tuning scalars $\phi_{i,l}$ and $\varphi_{i,l}$, as well as for constraining the state estimation errors within specified ellipsoids. Based on the results of Theorems 1-2, the state estimator gains and tuning scalars are characterized through the optimization problems presented in Theorem 4. Additionally, Theorem 5 analyzes the ultimate boundedness of the weight estimation error. It is important to note that the considered system in this paper accounts for practical engineering complexities including nonlinearities, multi-rate sampling and FRPs, making the results applicable to real-world scenarios. Moreover, the parameters of these complexities are all reflected in Theorem 4 and have influence on the estimation performance.

E. A corollary

In this subsection, we consider the NNB set-membership estimator design for single-rate systems under the FRP. Consider the complex networks (1) with

$$y_{i,l} = C_{i,l}x_{i,l} + v_{i,l}, \quad i = 1, 2, \ldots, N. \tag{19}$$

Obviously, (19) is a single-rate system. With the neural network designed as (3) and considering the FRP, the state estimator is constructed as

$$\begin{aligned}\hat{x}_{i,l+1} =& A_{i,l}\hat{x}_{i,l} + \hat{U}_{i,l}\sigma(\hat{x}_{i,l}) + \sum_{j=1}^{N}\omega_{ij}\Gamma\hat{x}_{j,l} \\ &+ K_{i,l}(\check{y}_{i,l} - \bar{\delta}_{i,l}C_{i,l}\hat{x}_{i,l})\end{aligned}$$

where

$$\check{y}_{i,l} = \bar{\delta}_{i,l}C_{i,l}x_{i,l} + \bar{\delta}_{i,l}v_{i,l}.$$

The tuning law of neural network weight is designed as

$$\begin{aligned}\hat{U}_{i,l+1} = & \phi_{i,l}\hat{U}_{i,l} + \varphi_{i,l}\bar{\delta}_{i,l+1}C_{i,l+1}^T \\ & \times (\check{y}_{i,l+1} - C_{i,l+1}\hat{x}_{i,l+1})\sigma^T(\hat{x}_{i,l}).\end{aligned}$$

The following corollary presents the result of NNB set-membership estimator design for single-rate systems (1) and (19).

Corollary 1: Let the positive definite matrices $R_{i1,l}$ and $R_{i2,l}$ be given. For complex networks (1) with the measurement output (19), if there exist positive scalars $\beta_{i,1l}$, $\beta_{i,2l}$, $\alpha_{i,jl}$ $(j = 1, 2, 3, 4, 5, 6)$, tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$, and estimator gain matrix $K_{i,l}$ such that

$$\begin{bmatrix} -\bar{R}_{i1,l+1} & \Xi_{i,l}^1 \\ (\Xi_{i,l}^1)^T & \Sigma_{i,l} \end{bmatrix} \leq 0$$

and

$$\begin{bmatrix} -R_{i2,l+1} & \Theta_{i,l}^1 \\ (\Theta_{i,l}^1)^T & -\bar{\Sigma}_{i,l} \end{bmatrix} \leq 0$$

hold where

$$\begin{aligned}
\Xi_{i,l}^1 &\triangleq \begin{bmatrix} -\varphi_{i,l}\bar{\delta}_{i,l+1}(\sigma(\hat{x}_{i,l}) \otimes C_{i,l+1}^T)\lambda_{i,l+1} & \bar{\Xi}_{i,l} \end{bmatrix}, \\
\Theta_{i,l}^1 &\triangleq \begin{bmatrix} 0 & \Delta_{i,l}^1 & I & \vec{\sigma}(\hat{x}_{i,l})\bar{Q}_{i1,l} & B_{i,l} & -\bar{\delta}_{i,l}K_{i,l} & I \end{bmatrix}, \\
\bar{K}_{i,l}^1 &\triangleq A_{i,l}Q_{i2,l} - \bar{\delta}_{i,l}K_{i,l}C_{i,l}Q_{i2,l}, \\
\Delta_{i,l}^1 &\triangleq \begin{bmatrix} \bar{\Gamma}_{i1,l} & \cdots & \bar{\Gamma}_{ii-1,l} & \bar{K}_{i,l}^1 + \bar{\Gamma}_{ii,l} & \bar{\Gamma}_{ii+1,l} & \cdots & \bar{\Gamma}_{iN,l} \end{bmatrix},
\end{aligned}$$

then the objectives (13) and (14) are satisfied simultaneously.

Proof: The proof is easily accomplished from Theorem 3 and is omitted here. ■

Remark 4: In this paper, we have addressed the NNB set-membership state estimation problem for multi-rate nonlinear complex networks under FRPs. Our results have several distinguishing features when compared to existing literature:

1) *Novelty*: The estimation problem considered in this paper is new as it takes into account engineering-oriented complexities such as multi-rate sampling and FRPs. These complexities are often present in practical systems but have not been extensively studied in the literature.
2) *Characterization of FRP scheduling*: We have properly characterized and reflected the scheduling effect of the FRP in the developed estimation algorithm, which ensures that the estimation scheme is tailored to the specific communication protocol used in the network.
3) *Utilization of the NNB method*: We have employed the NNB method to handle the nonlinearities present in the system. This approach has significant practical significance, as it allows for a more accurate approximation of the unknown nonlinear dynamics compared to traditional methods.

In the following section, we will provide a practical simulation to verify the effectiveness of the developed estimation algorithm.

IV. A PRACTICAL SIMULATION

In this section, we provide a practical example to validate the effectiveness of Algorithm 1.

Let's consider a complex network consisting of five coupled RLC circuits. The dynamics of the i-th RLC circuit can be described as follows [22]:

$$\begin{aligned}\dot{x}_{i2} &= \frac{1}{L_i}x_{i1}, \\ \dot{x}_{i1} &= -\frac{1}{C_i}x_{i2} - \frac{R_i}{L_i}x_{i1} + u_i\end{aligned}$$

where x_{i2} is the charge in the capacitor and x_{i1} is the flux in the inductance. u_i is the voltage input. L_i, C_i, and R_i are the inductance, the capacitor, and the resistance, respectively.

Denoting $x_i \triangleq \text{col}\{x_{i1}, x_{i2}\}$ and discretizing the obtained state-space model with sampling period $h = 0.5s$, we have [22]

$$x_{i,l+1} = A_i x_{i,l} + F_i u_{i,l}$$

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where $A_i = e^{\bar{A}_i h}$ and $F_i = \int_0^h e^{\bar{A}_i s} ds \bar{F}$ with

$$\bar{A}_i = \begin{bmatrix} -\frac{R_i}{L_i} & -\frac{1}{C_i} \\ \frac{1}{L_i} & 0 \end{bmatrix}, \ \bar{F} = \begin{bmatrix} I \\ 0 \end{bmatrix}.$$

As in [22], designing the voltage input $u_{i,l}$ as $u_{i,l} = \sum_{j=1}^{5} \omega_{ij} x_{j1,l}$ and considering the external noise as well as the environment- or modeling-induced nonlinearities, the dynamic of the i-th RLC circuit is obtained as

$$\begin{aligned} x_{i,l+1} = & A_i x_{i,l} + \sum_{j=1}^{5} \omega_{ij} \tilde{F}_i x_{j,l} \\ & + f(x_{i,l}) + B_{i,l} w_{i,l} \end{aligned}$$

where

$$\begin{aligned} f(x_{i,l}) &= \mathrm{col}\{\cos(x_{i1,l}), 0.1\sin(x_{i2,l})\}, \\ B_{1,l} &= \begin{bmatrix} 0.45 & 0.5 + 0.1\sin(l) \end{bmatrix}^T, \\ B_{2,l} &= \begin{bmatrix} 0.49 & 0.5 + 0.2\cos(l) \end{bmatrix}^T, \\ B_{3,l} &= \begin{bmatrix} 0.81 & 0.5 + 0.1\cos(l) \end{bmatrix}^T, \\ B_{4,l} &= \begin{bmatrix} 0.73 & 0.5 + 0.2\sin(l) \end{bmatrix}^T, \\ B_{5,l} &= \begin{bmatrix} 0.50 & 0.5 + 0.1\sin(l) \end{bmatrix}^T, \\ \tilde{F}_i &\triangleq F_i \begin{bmatrix} I & 0 \end{bmatrix}, \ w_{1,l} = 0.1\cos(0.5l), \\ w_{2,l} &= 0.1\cos(l), \ w_{3,l} = 0.1\cos(0.8l), \\ w_{4,l} &= 0.1\sin(0.2l), \ w_{5,l} = 0.1\cos(0.5l). \end{aligned}$$

It is easily known that the ellipsoidal constraints on the process noises are satisfied with $W_{1,l} = 0.2$, $W_{2,l} = 0.2$, $W_{3,l} = 0.2$, $W_{4,l} = 0.2$, and $W_{5,l} = 0.2$. Choose the parameters as $L_i = 0.5H$, $C_i = 0.5F$, and $R_i = 1\Omega$. The coupling strength is set as $\omega_{ij} = 0.1$ for $i \neq j$ and $\omega_{ij} = -0.4$ for $i = j$.

For the i-th RLC circuit, the measurement output y_{i,t_k} with $b = 2h$ is modeled as

$$y_{i,t_k} = C_{i,t_k} x_{i,t_k} + v_{i,t_k}$$

where

$$\begin{aligned} C_{1,t_k} &= \begin{bmatrix} 0.5 & 0.5 + 0.1\sin(k) \end{bmatrix}, \\ C_{2,t_k} &= \begin{bmatrix} 0.6 & 0.3 + 0.1\sin(k) \end{bmatrix}, \\ C_{3,t_k} &= \begin{bmatrix} 0.6 & 0.4 + 0.1\cos(k) \end{bmatrix}, \\ C_{4,t_k} &= \begin{bmatrix} 0.4 & 0.5 + 0.1\sin(k) \end{bmatrix}, \\ C_{5,t_k} &= \begin{bmatrix} 0.5 & 0.5 + 0.1\cos(k) \end{bmatrix}, \\ v_{1,t_k} &= 0.1\cos(0.5k), \ v_{2,t_k} = 0.1\sin(0.2k), \\ v_{3,t_k} &= 0.1\sin(0.5k), \ v_{4,t_k} = 0.1\cos(0.4k), \\ v_{5,t_k} &= 0.1\cos(0.5k), \end{aligned}$$

Similarly, the ellipsoidal constraints on the measurement noises are satisfied with $V_{1,t_k} = 0.2$, $V_{2,t_k} = 0.2$, $V_{3,t_k} = 0.2$, $V_{4,t_k} = 0.2$, and $V_{5,t_k} = 0.2$.

The activation function $\sigma(\cdot)$ is designed as

$$\sigma(x_{i,l}) = \mathrm{col}\{\tanh(x_{i1,l}), \tanh(x_{i2,l})\}.$$

Moreover, the initial conditions are chosen as $x_{i,0} = \hat{x}_{i,0} = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}^T$, $\hat{U}_{i,0} = 2I$, and $R_{i2,0} = 10I$.

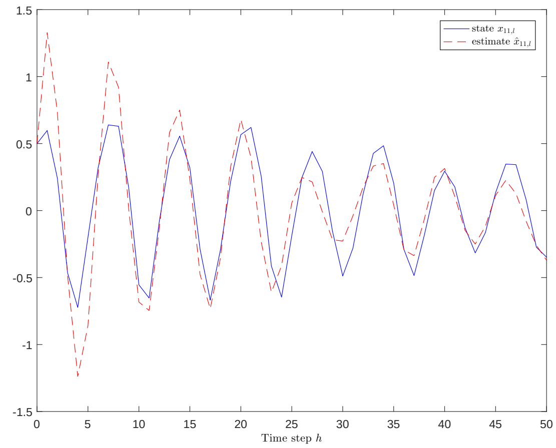

Fig. 2: $x_{11,l}$ and its estimate

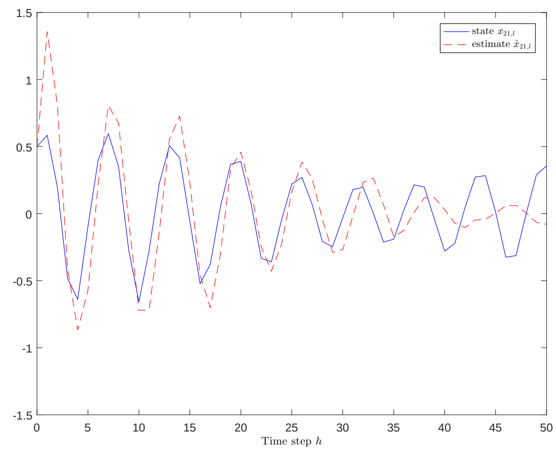

Fig. 3: $x_{21,l}$ and its estimate

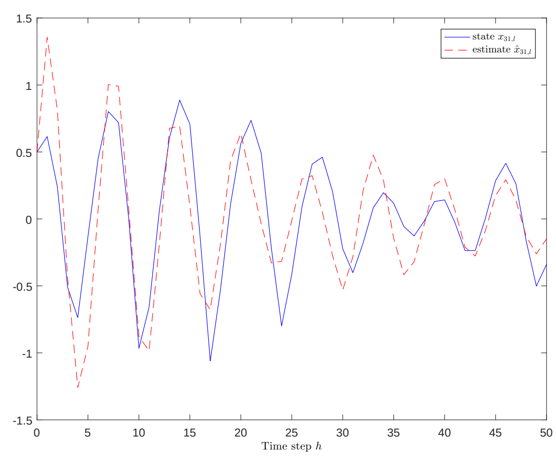

Fig. 4: $x_{31,l}$ and its estimate

By solving the optimization problem (17), the gain matrix $K_{i,l}$ and the tuning scalars $\phi_{i,l}$, $\varphi_{i,l}$ are characterized. The

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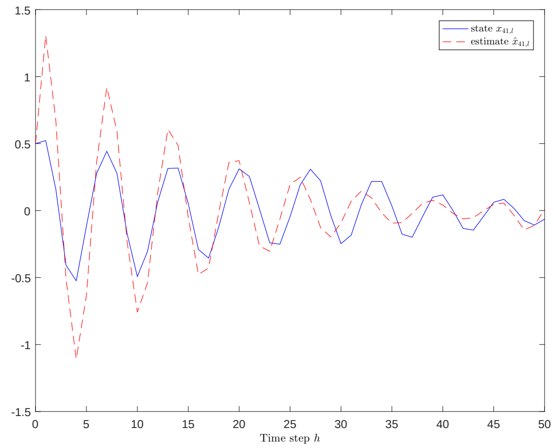

Fig. 5: $x_{41,l}$ and its estimate

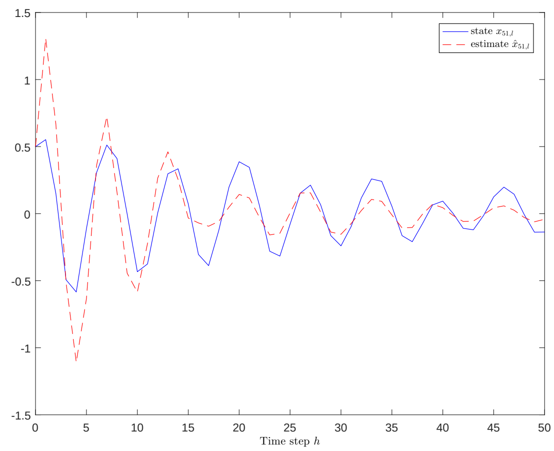

Fig. 6: $x_{51,l}$ and its estimate

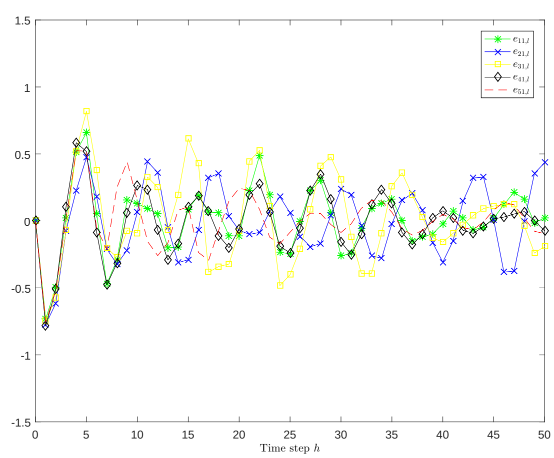

Fig. 7: The estimation errors

simulation results are displayed in Figs. 2-8. The states and the estimates are plotted in Figs. 2-6. In Fig. 7, the estimation

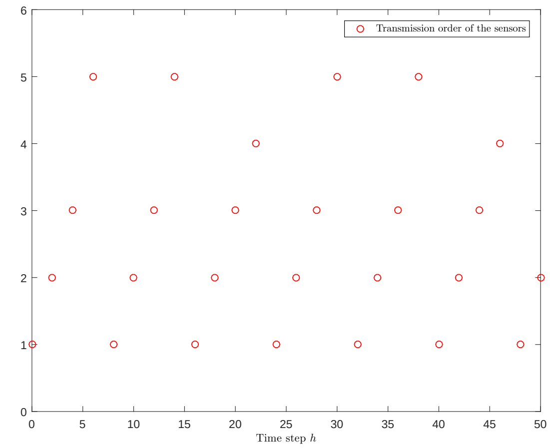

Fig. 8: The transmission access for sensors

errors are shown. The transmission orders of the sensors are given in Fig. 8 where the ordinate represents the sensor number and the abscissa represents the time. From Fig. 8, we can see that the transmission load of the communication network is greatly lightened. From the simulation results, it is seen that, despite the significantly reduction of the available measurement information, the developed NNB set-membership estimation scheme can still effectively estimate the target system state. Therefore, the usefulness of the proposed estimation method is confirmed.

V. Conclusions

This paper has addressed the NNB set-membership state estimation problem for a specific class of multi-rate nonlinear complex networks under FRPs. The considerations of both multi-rate sampling and FRPs are significant as they are commonly employed in engineering practice. To handle the asynchronous sampling rates, an indicator variable has been introduced to unify the sampling rates. Additionally, the scheduling effect of FRPs on the sensors has been characterized based on the FRP mechanism. To handle the general nonlinearity present in the system, the NNB approach has been utilized to approximate the nonlinear dynamics. Sufficient conditions have been derived to ensure that the estimation errors satisfy specific ellipsoidal constraints. Furthermore, the design of both the estimator gains and the neural network tuning parameters has been addressed. Finally, a practical example has been provided to demonstrate the effectiveness of the proposed estimation scheme. In our future research, we plan to apply the NNB approach to other networked systems such as sensor networks [29].

Appendix A
The Proof of Theorem 1

The proof is conducted using the mathematical induction method. We know from Assumption 1 that the initial condition $\mathrm{tr}\left[\tilde{U}_{i,0}^T R_{i1,0}^{-1} \tilde{U}_{i,0}\right] \leq 1$ holds. Supposing that

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$\mathrm{tr}\left[\tilde{U}_{i,l}^T R_{i1,l}^{-1} \tilde{U}_{i,l}\right] \leq 1$ holds, we need to find the condition under which $\mathrm{tr}\left[\tilde{U}_{i,l+1}^T R_{i1,l+1}^{-1} \tilde{U}_{i,l+1}\right] \leq 1$ holds.

Note that the condition $\mathrm{tr}\left[\tilde{U}_{i,l}^T R_{i1,l}^{-1} \tilde{U}_{i,l}\right] \leq 1$ holds. Let

$$\mathrm{vec}(\tilde{U}_{i,l}) \triangleq \begin{bmatrix} \tilde{U}_{i,l}^{(1)} & \tilde{U}_{i,l}^{(2)} & \cdots & \tilde{U}_{i,l}^{(n_x)} \end{bmatrix}^T$$

with $\tilde{U}_{i,l}^{(i)}$ being the i-th row of $\tilde{U}_{i,l}$. Then, one has

$$(\mathrm{vec}(\tilde{U}_{i,l}))^T \bar{R}_{i1,l}^{-1} \mathrm{vec}(\tilde{U}_{i,l}) \leq 1. \tag{20}$$

From (20), it is known that there is a vector $\vartheta_{i,l}$ fulfilling $\vartheta_{i,l}^T \vartheta_{i,l} \leq 1$ such that

$$\mathrm{vec}(\tilde{U}_{i,l}) = \bar{Q}_{i1,l} \vartheta_{i,l}$$

holds.

Based on the weight estimation error dynamics (12), it is derived that

$$\begin{aligned} \mathrm{vec}(\tilde{U}_{i,l+1}) =& (1-\phi_{i,l})\bar{U} + \phi_{i,l}\bar{Q}_{i1,l}\vartheta_{i,l} \\ &- \varphi_{i,l}\tilde{\delta}_{i,l+1}(\sigma(\hat{x}_{i,l}) \otimes C_{i,l+1}^T)\lambda_{i,l+1} \end{aligned}$$

where

$$\bar{U} \triangleq \begin{bmatrix} U^{(1)} & U^{(2)} & \cdots & U^{(n_x)} \end{bmatrix}^T$$

with $U^{(i)}$ being the i-th row of U.

By introducing a vector $\eta_{i,l} \triangleq \mathrm{col}\{1, \vartheta_{i,l}, \bar{U}\}$, we have

$$\mathrm{vec}(\tilde{U}_{i,l+1}) = \Xi_{i,l}\eta_{i,l}.$$

Note that $\mathrm{tr}\left[\tilde{U}_{i,l+1}^T R_{i1,l+1}^{-1} \tilde{U}_{i,l+1}\right] \leq 1$ holds if

$$(\mathrm{vec}(\tilde{U}_{i,l+1}))^T \bar{R}_{i1,l+1}^{-1} \mathrm{vec}(\tilde{U}_{i,l+1}) \leq 1 \tag{21}$$

holds, which is equivalent to

$$\eta_{i,l}^T \Xi_{i,l}^T \bar{R}_{i1,l+1}^{-1} \Xi_{i,l}\eta_{i,l} - \eta_{i,l}^T \mathrm{diag}\{1,0,0\}\eta_{i,l} \leq 0. \tag{22}$$

Now, it remains to prove that (22) is true. It is obvious that $\vartheta_{i,l}^T \vartheta_{i,l} \leq 1$ can be rewritten as

$$\eta_{i,l}^T \mathrm{diag}\{0,I,0\}\eta_{i,l} - \eta_{i,l}^T \mathrm{diag}\{1,0,0\}\eta_{i,l} \leq 0.$$

Moreover, it is known from $\|U\|_F \leq \bar{u}$ that

$$\eta_{i,l}^T \mathrm{diag}\{-\bar{u}^2, 0, I\}\eta_{i,l} \leq 0.$$

Therefore, according to Lemma 1, (22) holds (i.e., (21) holds) if there are positive scalars $\beta_{i,1l}$ and $\beta_{i,2l}$ such that

$$\begin{aligned} &\Xi_{i,l}^T \bar{R}_{i1,l+1}^{-1} \Xi_{i,l} - \mathrm{diag}\{1,0,0\} \\ &\quad - \beta_{i,1l}\mathrm{diag}\{-\bar{u}^2, 0, I\} \\ &\quad - \beta_{i,2l}\mathrm{diag}\{-1, I, 0\} \leq 0 \end{aligned} \tag{23}$$

holds.

By employing the Schur Complement Lemma, it can be concluded that (23) holds if and only if (15) holds. Thus, the proof is complete.

Appendix B
The Proof of Theorem 2

The proof is also conducted using the mathematical induction method. It is derived from Assumption 1 that the initial condition $e_{i,0}^T R_{i2,0}^{-1} e_{i,0} \leq 1$ is true.

Assuming that $e_{i,l}^T R_{i2,l}^{-1} e_{i,l} \leq 1$ holds true, our objective is to establish the condition $e_{i,l+1}^T R_{i2,l+1}^{-1} e_{i,l+1} \leq 1$. Noting $e_{i,l}^T R_{i2,l}^{-1} e_{i,l} \leq 1$, there exists a vector $\varpi_{i,l}$ satisfying $\varpi_{i,l}^T \varpi_{i,l} \leq 1$ such that

$$e_{i,l} = Q_{i2,l}\varpi_{i,l}. \tag{24}$$

It follows from (1), (3) and (10) that

$$\begin{aligned} e_{i,l+1} =& A_{i,l}e_{i,l} + \sum_{j=1}^{N} \omega_{ij}\Gamma e_{j,l} \\ &+ U\bar{\sigma}_{i,l} + \tilde{U}_{i,l}\sigma(\hat{x}_{i,l}) \\ &+ B_{i,l}w_{i,l} + \varrho_{i,l} \\ &- \tilde{\delta}_{i,l}K_{i,l}C_{i,l}e_{i,l} - \tilde{\delta}_{i,l}K_{i,l}v_{i,l} \end{aligned} \tag{25}$$

where

$$\bar{\sigma}_{i,l} \triangleq \sigma(x_{i,l}) - \sigma(\hat{x}_{i,l}).$$

By utilizing matrix operations, it becomes evident that

$$\tilde{U}_{i,l}\sigma(\hat{x}_{i,l}) = \vec{\sigma}(\hat{x}_{i,l})\mathrm{vec}(\tilde{U}_{i,l}).$$

Then, (25) is rewritten as

$$\begin{aligned} e_{i,l+1} =& A_{i,l}Q_{i2,l}\varpi_{i,l} + \sum_{j=1}^{N} \omega_{ij}\Gamma Q_{j2,l}\varpi_{j,l} \\ &+ U\bar{\sigma}_{i,l} + \vec{\sigma}(\hat{x}_{i,l})\bar{Q}_{i1,l}\vartheta_{i,l} \\ &+ B_{i,l}w_{i,l} + \varrho_{i,l} \\ &- \tilde{\delta}_{i,l}K_{i,l}C_{i,l}Q_{i2,l}\varpi_{i,l} - \tilde{\delta}_{i,l}K_{i,l}v_{i,l} \\ =& \Delta_{i,l}\bar{\varpi}_l + U\bar{\sigma}_{i,l} + \vec{\sigma}(\hat{x}_{i,l})\bar{Q}_{i1,l}\vartheta_{i,l} \\ &+ B_{i,l}w_{i,l} + \varrho_{i,l} - \tilde{\delta}_{i,l}K_{i,l}v_{i,l} \end{aligned} \tag{26}$$

where

$$\bar{\varpi}_l \triangleq \mathrm{col}\{\varpi_{1,l}, \varpi_{2,l}, \cdots, \varpi_{N,l}\}.$$

By denoting $\varsigma_{i,l} \triangleq \mathrm{col}\{1, \bar{\varpi}_l, U\bar{\sigma}_{i,l}, \vartheta_{i,l}, w_{i,l}, v_{i,l}, \varrho_{i,l}\}$, one has

$$e_{i,l+1} = \Theta_{i,l}\varsigma_{i,l}. \tag{27}$$

From $\varpi_{i,l}^T \varpi_{i,l} \leq 1$ and $\vartheta_{i,l}^T \vartheta_{i,l} \leq 1$, we have

$$\varsigma_{i,l}^T \mathrm{diag}\{-1, \mathbf{I}_i, 0, 0, 0, 0, 0\}\varsigma_{i,l} \leq 0,$$
$$\varsigma_{i,l}^T \mathrm{diag}\{-1, 0, 0, I, 0, 0, 0\}\varsigma_{i,l} \leq 0.$$

Similarly, it is obtained from the constrains on the noises that

$$\varsigma_{i,l}^T \mathrm{diag}\{-1, 0, 0, 0, W_{i,l}^{-1}, 0, 0\}\varsigma_{i,l} \leq 0,$$
$$\varsigma_{i,l}^T \mathrm{diag}\{-1, 0, 0, 0, 0, V_{i,l}^{-1}, 0\}\varsigma_{i,l} \leq 0.$$

It is known that $\|U\bar{\sigma}_{i,l}\| \leq 2N\bar{u}\bar{\sigma}^2$. Then, we have

$$\varsigma_{i,l}^T \mathrm{diag}\{-2N\bar{u}\bar{\sigma}^2, 0, I, 0, 0, 0, 0\}\varsigma_{i,l} \leq 0,$$
$$\varsigma_{i,l}^T \mathrm{diag}\{-\bar{\varrho}_i, 0, 0, 0, 0, 0, I\}\varsigma_{i,l} \leq 0.$$

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With the help of Lemma 1, we know that $e_{i,l+1}^T R_{i2,l+1}^{-1} e_{i,l+1} \leq 1$ is true if there are positive scalars $\alpha_{i,jl}$ $(j = 1, 2, 3, 4, 5, 6)$ such that

$$\begin{aligned}\Theta_{i,l}^T R_{i2,l+1}^{-1} \Theta_{i,l} &- \alpha_{i,1l} \mathrm{diag}\{-1, \mathbf{I}_i, 0, 0, 0, 0, 0\} \\ &- \alpha_{i,2l} \mathrm{diag}\{-2N\bar{u}\bar{\sigma}^2, 0, I, 0, 0, 0, 0\} \\ &- \alpha_{i,3l} \mathrm{diag}\{-1, 0, 0, I, 0, 0, 0\} \\ &- \alpha_{i,4l} \mathrm{diag}\{-1, 0, 0, 0, W_{i,l}^{-1}, 0, 0\} \\ &- \alpha_{i,5l} \mathrm{diag}\{-1, 0, 0, 0, 0, V_{i,l}^{-1}, 0\} \\ &- \alpha_{i,6l} \mathrm{diag}\{-\bar{\varrho}_i, 0, 0, 0, 0, 0, I\} \\ &- \mathrm{diag}\{1, 0, 0, 0, 0, 0, 0\} \leq 0 \end{aligned} \tag{28}$$

holds.

By resorting to the Schur Complement Lemma, the inequality (28) is true if and only if (16) is true, which completes the proof.

Appendix C
The Proof of Theorem 5

Defining a function as

$$\mathcal{V}_{i,l} \triangleq \mathrm{tr}\{\tilde{U}_{i,l}^T \tilde{U}_{i,l}\},$$

we have from (12) that

$$\begin{aligned}\tilde{U}_{i,l+1} =&(1-\phi_{i,l})U + \phi_{i,l}\tilde{U}_{i,l} \\ &- \varphi_{i,l}\tilde{\delta}_{i,l+1} C_{i,l+1}^T C_{i,l+1} e_{i,l+1} \sigma^T(\hat{x}_{i,l}) \\ &- \varphi_{i,l}\tilde{\delta}_{i,l+1} C_{i,l+1}^T v_{i,l+1} \sigma^T(\hat{x}_{i,l}). \end{aligned} \tag{29}$$

Calculating the difference of $\mathcal{V}_{i,l}$ along the trajectory of (29), one has

$$\begin{aligned}\Delta\mathcal{V}_{i,l} \triangleq& \mathcal{V}_{i,l+1} - \mathcal{V}_{i,l} \\ =& \mathrm{tr}\Big\{\big((1-\phi_{i,l})U^T + \phi_{i,l}\tilde{U}_{i,l}^T \\ &- \varphi_{i,l}\tilde{\delta}_{i,l+1}\sigma(\hat{x}_{i,l}) e_{i,l+1}^T C_{i,l+1}^T C_{i,l+1} \\ &- \varphi_{i,l}\tilde{\delta}_{i,l+1}\sigma(\hat{x}_{i,l}) v_{i,l+1}^T C_{i,l+1}\big) \\ &\times \big((1-\phi_{i,l})U + \phi_{i,l}\tilde{U}_{i,l} \\ &- \varphi_{i,l}\tilde{\delta}_{i,l+1} C_{i,l+1}^T C_{i,l+1} e_{i,l+1}\sigma^T(\hat{x}_{i,l}) \\ &- \varphi_{i,l}\tilde{\delta}_{i,l+1} C_{i,l+1}^T v_{i,l+1}\sigma^T(\hat{x}_{i,l})\big)\Big\} \\ &- \mathrm{tr}\{\tilde{U}_{i,l}^T\tilde{U}_{i,l}\} \\ \leq& \mathrm{tr}\Big\{4(1-\phi_{i,l})^2 U^T U + (4\phi_{i,l}^2 - 1)\tilde{U}_{i,l}^T\tilde{U}_{i,l} \\ &+ 4\varphi_{i,l}^2\tilde{\delta}_{i,l+1}\sigma(\hat{x}_{i,l}) e_{i,l+1}^T C_{i,l+1}^T C_{i,l+1} \\ &\times C_{i,l+1}^T C_{i,l+1} e_{i,l+1}\sigma^T(\hat{x}_{i,l}) \\ &+ 4\varphi_{i,l}^2\tilde{\delta}_{i,l+1}\sigma(\hat{x}_{i,l}) v_{i,l+1}^T C_{i,l+1} \\ &\times C_{i,l+1}^T v_{i,l+1}\sigma^T(\hat{x}_{i,l})\Big\}. \end{aligned}$$

It is known from $\|U\|_F \leq \bar{u}$ that

$$\mathrm{tr}\{U^T U\} \leq \bar{u}^2.$$

Moreover, it is derived from $\|\sigma(\cdot)\| \leq \bar{\sigma}$ and Assumption 2 that

$$\begin{aligned}&\mathrm{tr}\big\{\sigma(\hat{x}_{i,l}) e_{i,l+1}^T C_{i,l+1}^T C_{i,l+1} \\ &\quad\times C_{i,l+1}^T C_{i,l+1} e_{i,l+1}\sigma^T(\hat{x}_{i,l})\big\} \\ &=\mathrm{tr}\big\{C_{i,l+1}^T C_{i,l+1} e_{i,l+1}\sigma^T(\hat{x}_{i,l}) \\ &\quad\times \sigma(\hat{x}_{i,l}) e_{i,l+1}^T C_{i,l+1}^T C_{i,l+1}\big\} \\ &\leq\bar{\sigma}^2\mathrm{tr}\big\{e_{i,l+1}^T C_{i,l+1}^T C_{i,l+1} C_{i,l+1}^T C_{i,l+1} e_{i,l+1}\big\} \\ &\leq\bar{\sigma}^2\bar{c}\mathrm{tr}\big\{e_{i,l+1}^T C_{i,l+1}^T C_{i,l+1} e_{i,l+1}\big\} \\ &\leq\bar{\sigma}^2\bar{c}^2\mathrm{tr}\big\{e_{i,l+1}^T e_{i,l+1}\big\} \\ &\leq\bar{\sigma}^2\bar{c}^2\bar{r} \end{aligned}$$

and

$$\begin{aligned}&\mathrm{tr}\big\{\sigma(\hat{x}_{i,l}) v_{i,l+1}^T C_{i,l+1} C_{i,l+1}^T v_{i,l+1}\sigma^T(\hat{x}_{i,l})\big\} \\ &=\mathrm{tr}\big\{C_{i,l+1}^T v_{i,l+1}\sigma^T(\hat{x}_{i,l})\sigma(\hat{x}_{i,l}) v_{i,l+1}^T C_{i,l+1}\big\} \\ &\leq\bar{\sigma}^2\mathrm{tr}\big\{C_{i,l+1}^T v_{i,l+1} v_{i,l+1}^T C_{i,l+1}\big\} \\ &=\bar{\sigma}^2\mathrm{tr}\big\{v_{i,l+1}^T C_{i,l+1} C_{i,l+1}^T v_{i,l+1}\big\} \\ &\leq\bar{\sigma}^2\bar{c}\mathrm{tr}\big\{v_{i,l+1}^T v_{i,l+1}\big\} \\ &\leq\bar{\sigma}^2\bar{c}\bar{v}. \end{aligned}$$

Then, we have

$$\begin{aligned}\Delta\mathcal{V}_{i,l} \leq& 4(1-\phi_{i,l})^2\mathrm{tr}\{U^T U\} + (4\phi_{i,l}^2 - 1)\mathrm{tr}\{\tilde{U}_{i,l}^T\tilde{U}_{i,l}\} \\ &+ 4\varphi_{i,l}^2\tilde{\delta}_{i,l+1}\mathrm{tr}\big\{\sigma(\hat{x}_{i,l}) e_{i,l+1}^T C_{i,l+1}^T C_{i,l+1} \\ &\times C_{i,l+1}^T C_{i,l+1} e_{i,l+1}\sigma^T(\hat{x}_{i,l})\big\} \\ &+ 4\varphi_{i,l}^2\tilde{\delta}_{i,l+1}\mathrm{tr}\big\{\sigma(\hat{x}_{i,l}) v_{i,l+1}^T C_{i,l+1} \\ &\times C_{i,l+1}^T v_{i,l+1}\sigma^T(\hat{x}_{i,l})\big\} \\ \leq& (4\phi_{i,l}^2 - 1)\mathrm{tr}\{\tilde{U}_{i,l}^T\tilde{U}_{i,l}\} + 4(1-\phi_{i,l})^2\bar{u}^2 \\ &+ 4\varphi_{i,l}^2\tilde{\delta}_{i,l+1}\bar{\sigma}^2\bar{c}^2\bar{r} \\ &+ 4\varphi_{i,l}^2\tilde{\delta}_{i,l+1}\bar{\sigma}^2\bar{c}\bar{v}. \end{aligned}$$

Noting (18), we know that $\tilde{U}_{i,l}$ is ultimately bounded, and the proof is complete.

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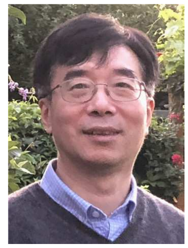

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