Encryption-Decryption-Based Set-Membership Filtering for Two-Dimensional Systems: On Security and Boundedness ⋆

Kaiqun Zhu [a,b], Zidong Wang [c], Derui Ding [a,∗], Hongli Dong [d], Guoliang Wei [e]

[a] Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China.
[b] State Key Laboratory of Internet of Things for Smart City, University of Macau, Macau 999078, China.
[c] Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom.
[d] Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing 163318, China.
[e] Business School, University of Shanghai for Science and Technology, Shanghai 200093, China.

Abstract

In this paper, the encryption-decryption-based set-membership filtering issue is considered for a class of networked two-dimensional systems with unknown-but-bounded noises. In order to preserve data privacy and reduce communication burden, an encryption-decryption mechanism is put forward based on the exclusive or logical operation technique and the one-time pad method, under which the data is encrypted into a ciphertext with finite bits before being transmitted through the networks. The aim of the addressed problem is to develop an encryption-decryption-based set-membership filter (EDSMF) that is capable of generating an ellipsoidal set that contains the true system state and ensuring the security performance. Sufficient conditions are established for the existence of the desired ellipsoidal set, and the corresponding EDSMF gains are obtained by applying the Lagrange multiplier method. Moreover, the uniform boundedness of the ellipsoidal shape-defining matrix is thoroughly studied. Finally, the effectiveness of the proposed filter design method is demonstrated by a practical example of a heat exchanger.

Key words: Encryption-decryption mechanism; Two-dimensional systems; Unknown-but-bounded noises; Set-membership filtering; Uniform boundedness.

1 Introduction

Since it was first proposed in the 1960s, the set-membership filtering method has been widely applied in many practical engineering areas including autonomous ground vehicles and target tracking [1, 16, 28, 30, 46, 50]. The main idea of set-membership filtering method is to construct a compact set that contains the true system state under the influence of unknown-but-bounded (UBB) noises [15, 31, 51]. Compared with the existing point-wise filtering methods (e.g., the Kalman filtering algorithm), the set-membership filtering method owns the following two distinct *advantages*: 1) the statistical property of the noise is no longer required; and 2) the determined ellipsoidal set includes the true system state with 100% confidence.

Until now, there are mainly two approaches to computing the set-membership filter (SMF) gain matrtix, namely, the linear matrix inequality (LMI) technique [7, 19, 24, 25, 54, 56] and the difference equation technique [26, 40, 47]. For the LMI-based approach, the performance index of the SMF is guaranteed by exploiting the Schur complement lemma. For the difference-equation-based approach, the performance index is ensured through solving some recursive equations. So far, most SMF-related results have been concerned with the construction of the ellipsoidal set encompassing the system state, but the *uniform boundedness* of such a set has not been adequately investigated despite its significance in evaluating the filter performance.

With the quick revolution of the communication network, networked systems, especially networked two-dimensional systems, have found many applications in practical areas (e.g., vehicle platoon and grid sensor networks [5, 8, 17, 34, 39]), and the corresponding theoretical research has also attracted much attention in recent years, see e.g. [12, 13, 21, 33, 43, 52, 55]. On the other hand, in the context of general networked systems, the remote filtering problem has been an active research topic for decades [3, 10, 23]. Unfortunately, the signals transmitted via shared networks are likely to be eavesdropped by hackers [2, 4, 14, 22, 27, 48, 49, 53] and

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∗ Corresponding author.

Email addresses: `zkqun@163.com` (Kaiqun Zhu), `Zidong.Wang@brunel.ac.uk` (Zidong Wang), `deruiding2010@usst.edu.cn` (Derui Ding), `shiningdhl@vip.126.com` (Hongli Dong), `guoliang.wei@usst.edu.cn` (Guoliang Wei).

the resulting data eavesdropping problem, if not addressed, would seriously jeopardize the performance of the remote filter. Therefore, it is of great importance to construct an *encryption-decryption mechanism* (EDM) for networked systems with hope to ensure the security of data transmission in the network.

It should be pointed out that the research on EDM is still in its infancy [9,20,44,45]. As a typical method, the data is encrypted by a dynamic model at the encoder side and then decrypted by another dynamic model at the decoder side under the codec-based encryption method. Despite its easy-to-implement feature, the codec-based encryption method cannot guarantee the perfect secrecy. As such, there is a need to further look into EDM in order to improve the security of signal transmissions and efficiency of network communication. For this purpose, in this paper, we adopt the exclusive or (XOR) logical operation and the one-time pad method in the EDM so as to enhance the transmission security over networked systems. To the best of our knowledge, the XOR-based EDM has not been investigated yet, let alone the consideration of the UBB noises, the set-membership filter and the bandwidth constraints.

In this paper, our focus is on the EDM-based set-membership filtering issue for systems with UBB noises. The *challenges* are outlined as follows: 1) how to propose a suitable EDM that can simultaneously preserve data privacy and mitigate communication burden? 2) how to develop a secure set-membership filtering scheme such that the security performance is ensured and the true system state always resides in the ellipsoidal set? 3) how to evaluate the impact of the EDM on the filtering performance by examining the uniform boundedness of the ellipsoidal shape-defining matrix?

The main *contributions* are summarized as follows. 1) Based on the XOR logical operation and the encoding-decoding technology, a dedicated EDM is designed to cater for both the preservation of data privacy and the reduction of communication burden. 2) An encryption-decryption-based set-membership filter (EDSMF) is constructed for systems subject to bandwidth constraints, ensuring the security of data transmission and the effectiveness of filtering algorithms. 3) The security performance is ensured. The decryption error issue is researched by means of the interval analysis technique and the set theory, and the uniform boundedness of resulting ellipsoids is thoroughly examined.

2 System Description and Preliminaries

2.1 The 2-D System Model

For the purpose of problem formulation, we first give the following definition.

Definition 1 *An ellipsoidal set is defined as*

$$\mathscr{X}(c, \tau P) \triangleq \left\{x \mid (x-c)^{\mathrm{T}} P^{-1}(x-c) \leq \tau\right\}$$

where c is the center of the ellipsoidal set, $P > 0$ is the shape-defining matrix that determines the shape of the ellipsoidal set, and τ is the positive scalar.

Consider the following two-dimensional (2-D) system:

$$\begin{aligned} x_{k+1,h+1} = & A^{(1)}_{k+1,h} x_{k+1,h} + A^{(2)}_{k,h+1} x_{k,h+1} \\ & + w_{k+1,h} + w_{k,h+1} \end{aligned} \tag{1}$$

$$y_{k,h} = C_{k,h} x_{k,h} + v_{k,h} \tag{2}$$

where k and h are horizontal and vertical coordinates; $x_{k,h} \in \mathbb{R}^{n_x}$ represents the state variable and $y_{k,h} \in \mathbb{R}^{n_y}$ is the measurement output; $A^{(1)}_{k,h}$, $A^{(2)}_{k,h}$, and $C_{k,h}$ are known matrices; $w_{k,h} \in \mathbb{R}^{n_w}$ represents the UBB process noise; and $v_{k,h} \in \mathbb{R}^{n_v}$ is the UBB measurement noise. Here, for all k and h, the noises are confined to the following ellipsoidal sets:

$$\mathscr{X}(0, Q_{k,h}) = \left\{w_{k,h} \mid w^{\mathrm{T}}_{k,h} Q^{-1}_{k,h} w_{k,h} \leq 1\right\} \tag{3}$$

$$\mathscr{X}(0, R_{k,h}) = \left\{v_{k,h} \mid v^{\mathrm{T}}_{k,h} R^{-1}_{k,h} v_{k,h} \leq 1\right\} \tag{4}$$

where $Q_{k,h} > 0$ and $R_{k,h} > 0$ are known shape-defining matrices.

Remark 1 *It is worth mentioning that the development of 2-D systems primarily arises from the practical requirements to describe the target plant with signals broadcasting in two directions, e.g., grid sensor networks and heat exchangers [8, 34, 39]. In the aforementioned physical dynamic processes, the variable varies with different spatial and temporal positions, are denoted by k and h, respectively. Therefore, the 2-D system studied in this paper has engineering application backgrounds.*

2.2 The XOR-Based EDM

To ensure data security and reduce the utilization of communication resources, the signals are encrypted/decrypted by the XOR-based EDM.

Encrypter: The encrypter includes two parts: 1) converting the measurement output into the 0-1 binary bit sequence; and 2) encrypting the binary bit sequence.

First, the encoding rule is

$$\begin{cases} \chi^{[s]}_{k,h} = \flat^{[s]}_{k,h} \xi^{[s]}_{k,h} + g^{[1,s]}_{k,h-1} + g^{[2,s]}_{k-1,h} \\ \xi^{[s]}_{k,h} = \mathrm{Q}\Big(\dfrac{1}{\flat^{[s]}_{k,h}}\big(y^{[s]}_{k,h} - g^{[1,s]}_{k,h-1} - g^{[2,s]}_{k-1,h}\big)\Big) \end{cases} \tag{5}$$

for $s = 1, 2, \cdots, n_y$, where $\chi^{[s]}_{k,h} \in \mathbb{R}$ and $\xi^{[s]}_{k,h} \in \mathbb{R}$ are, respectively, the sth entry of $\chi_{k,h}$ and $\xi_{k,h}$; $\flat^{[s]}_{k,h} \in \mathbb{R}$ is the non-zero scaling parameter; $\chi^{[s]}_{k,0} = \chi^{[s]}_{0,h} = \imath^{[s]}$; $g^{[1,s]}_{k,h-1} \triangleq \sum^{n_y}_{t=1} H^{[1,t]}_{k,h-1} \chi^{[t]}_{k,h-1} + f_1(y^{[s]}_{k,h})$; $g^{[2,s]}_{k-1,h} \triangleq \sum^{n_y}_{t=1} H^{[2,t]}_{k-1,h} \chi^{[t]}_{k-1,h} + f_2(y^{[s]}_{k,h})$; $H^{[1,t]}_{k,h}$, $H^{[2,t]}_{k,h}$ are weighted matrices; and $f_1(\cdot)$, $f_2(\cdot)$ are known functions. In the encoding rule, the quantizer is given as

$$\mathrm{Q}(\eta^{[s]}) \triangleq \begin{cases} \wp\ell^{[s]}, & \eta^{[s]} \in \big((\wp-1)\ell^{[s]}, \wp\ell^{[s]}\big] \\ 0, & \eta^{[s]} = 0 \\ -\wp\ell^{[s]}, & \eta^{[s]} \in \big[-\wp\ell^{[s]}, -(\wp-1)\ell^{[s]}\big) \end{cases} \tag{6}$$

where $\eta^{[s]}$ is the sth entry of η, $\ell^{[s]}$ is the quantization interval, $\wp \in \{1, 2, \ldots, \mathcal{R}\}$, and $2\mathcal{R} + 1$ is the number of quantization levels. At this stage, the signal $\xi_{k,h}$ is converted into the 0-1 binary bit sequence $\vec{\xi}_{k,h}$.

Next, the encryption process is described by

$$c_{k,h} = \mathrm{Enc}(\vec{\xi}_{k,h}, \varkappa_{k,h}) = \vec{\xi}_{k,h} \boxplus \varkappa_{k,h} \tag{7}$$

where $\mathrm{Enc}(\cdot,\cdot)$ is the encryption function; $\boxplus$ is the exclusive or (XOR) logical operation; $\vec{\xi}_{k,h}$ is the plaintext; $\varkappa_{k,h}$ is the key sequence; and $c_{k,h}$ is the ciphertext to be transmitted via the communication network.

Decrypter: The decrypter includes two parts: 1) decrypting the ciphertext to the plaintext; and 2) converting the plaintext into the decimal signal.

First, the decryption rule is given as

$$\hat{\xi}_{k,h} = \mathrm{Dec}(c_{k,h}, \varkappa_{k,h}) = c_{k,h} \boxplus \varkappa_{k,h} \tag{8}$$

where $\mathrm{Dec}(\cdot,\cdot)$ is the decryption function and $\hat{\xi}_{k,h}$ is the plaintext. According to the rule of the XOR logical operation, we have

$$\hat{\xi}_{k,h} = \vec{\xi}_{k,h} \boxplus \varkappa_{k,h} \boxplus \varkappa_{k,h} = \vec{\xi}_{k,h}. \tag{9}$$

Then, the binary bit string $\hat{\xi}_{k,h}$ is converted to the decimal signal $\xi_{k,h}$.

Next, the decoding rule is described by

$$\mu^{[s]}_{k,h} = \flat^{[s]}_{k,h}\xi^{[s]}_{k,h} + \bar{g}^{[1,s]}_{k,h-1} + \bar{g}^{[2,s]}_{k-1,h} \tag{10}$$

where $\mu^{[s]}_{k,h} \in \mathbb{R}$ is the sth entry of the decoder output $\mu_{k,h}$, $\mu^{[s]}_{k,0} = \mu^{[s]}_{0,h} = \imath^{[s]}$, $\bar{g}^{[1,s]}_{k,h-1} \triangleq \sum_{t=1}^{n_y} H^{[1,t]}_{k,h-1}\mu^{[t]}_{k,h-1} + f_1(\xi^{[s]}_{k,h})$, and $\bar{g}^{[2,s]}_{k-1,h} \triangleq \sum_{t=1}^{n_y} H^{[2,t]}_{k-1,h}\mu^{[t]}_{k-1,h} + f_2(\xi^{[s]}_{k,h})$.

Remark 2 *In this paper, the EDM is designed based on the XOR logical operation, and such an EDM has been widely used for data encryption with applications to image encryption and secure wireless communication [11, 29, 41]. In encryption rule (7), the key $\varkappa_{k,h}$ is called the one-time pad (OTP) if the following conditions are satisfied: 1) the key contains as many bits as the plaintext $\vec{\xi}_{k,h}$; and 2) the key is a random sequence. It is worth pointing out that Claude Shannon has proved that the encryption mechanism with the OTP is of perfect secrecy by utilizing the information theory [32]. As such, the XOR-based EDM exploited in this paper can effectively ensure the security of data transmission.*

Remark 3 *The proposed EDM has the following characteristics: 1) the dynamical encoding and decoding technology used in the EDM can alleviate the burden of network communication; and 2) the XOR logical operation and the OTP method applied in the EDM can preserve data privacy. Note that this paper presents the first time for the XOR-based EDM to be used in the dynamical system for handling the filtering problem.*

Letting $e_{k,h} \triangleq \mu_{k,h} - y_{k,h}$ be the decoding error, one has

$$\left|e^{[s]}_{k,h}\right| \le \flat^{[s]}_{k,h}\ell^{[s]}, \quad s = 1, 2, \ldots, n_y \tag{11}$$

with $|\star|$ being the absolute value of $\star$.

2.3 The EDSMF

In this paper, the EDSMF is designed as

$$\vec{x}_{k+1,h+1} = A^{(1)}_{k+1,h}\hat{x}_{k+1,h} + A^{(2)}_{k,h+1}\hat{x}_{k,h+1} \tag{12}$$

$$\hat{x}_{k+1,h+1} = \vec{x}_{k+1,h+1} + K_{k+1,h+1}(\mu_{k+1,h+1} - C_{k+1,h+1}\vec{x}_{k+1,h+1}) \tag{13}$$

where $\vec{x}_{k,h} \in \mathbb{R}^{n_x}$ is the one-step prediction, $\hat{x}_{k,h} \in \mathbb{R}^{n_x}$ is the estimate of $x_{k,h}$, and $K_{k,h}$ is the filter parameter to be determined.

Assumption 1 *The initial states of (1) satisfy*

$$\begin{cases} x_{k,0} \in \mathscr{X}(\hat{x}_{k,0}, \tau_{k,0}P_{k,0}) \\ x_{0,h} \in \mathscr{X}(\hat{x}_{0,h}, \tau_{0,h}P_{0,h}) \end{cases} \tag{14}$$

where $\hat{\vartheta}_{k,h} \triangleq x_{k,h} - \hat{x}_{k,h}$ and

$$\mathscr{X}(\hat{x}_{k,0}, \tau_{k,0}P_{k,0}) \triangleq \left\{x_{k,0} \mid \hat{\vartheta}^{\mathrm{T}}_{k,0}P^{-1}_{k,0}\hat{\vartheta}_{k,0} \le \tau_{k,0}\right\}$$
$$\mathscr{X}(\hat{x}_{0,h}, \tau_{0,h}P_{0,h}) \triangleq \left\{x_{0,h} \mid \hat{\vartheta}^{\mathrm{T}}_{0,h}P^{-1}_{0,h}\hat{\vartheta}_{0,h} \le \tau_{0,h}\right\}$$

with $\tau_{k,0}$, $\tau_{0,h}$ being known positive scalars and $P_{k,0} > 0$, $P_{0,h} > 0$ being known shape-defining matrices.

Remark 4 *It is worth mentioning that Assumption 1 is a standard assumption for set-membership estimation methods whose purpose is to guarantee the initial feasibility of the designed algorithm [26, 47]. In addition, this assumption is widely used in practical engineering, such as autonomous ground vehicles and simultaneous localization and mapping [16, 28].*

Definition 2 *The Minkowski sum of two ellipsoidal sets $\mathscr{X}_1$ and $\mathscr{X}_2$ is defined as*

$$\mathscr{X}_1 \oplus \mathscr{X}_2 \triangleq \{x_1 + x_2 \mid x_1 \in \mathscr{X}_1,\ x_2 \in \mathscr{X}_2\}.$$

We are now in a position to outline the main objectives of this paper as follows.

1) Develop a set-membership filtering scheme such that, under the influence of the UBB noise and EDM, the system state $x_{k,h}$ is confined to an ellipsoidal set:
$$x_{k,h} \in \mathscr{X}(\hat{x}_{k,h}, \tau_{k,h}P_{k,h}) \tag{15}$$
where $\mathscr{X}(\hat{x}_{k,h}, \tau_{k,h}P_{k,h}) \triangleq \{x_{k,h} \mid \hat{\vartheta}^{\mathrm{T}}_{k,h}P^{-1}_{k,h}\hat{\vartheta}_{k,h} \le \tau_{k,h}\}$ with $\tau_{k,h}$ being the positive scalar and $P_{k,h} > 0$ being the shape-defining matrix.
2) Determine the EDSMF parameter $K_{k,h}$ by minimizing the ellipsoidal set (15) in the sense of matrix trace.
3) Establish the condition under which the ellipsoidal shape-defining matrix $P_{k,h}$ is guaranteed to be uniformly bounded.

3 Main Results

The following lemmas are necessary for deriving our main results.

Lemma 1 *[6] For $\iota = 1, 2, \ldots, n$, let $\mathscr{X}(c_\iota, P_\iota)$ be given ellipsoidal sets. The Minkowski sum of the given ellipsoidal sets can be bounded by the ellipsoidal set $\mathscr{X}(c, P)$:*

$$\mathscr{X}(c_1, P_1) \oplus \mathscr{X}(c_2, P_2) \oplus \cdots \oplus \mathscr{X}(c_n, P_n) \subseteq \mathscr{X}(c, P)$$

where the center c and the shape-defining matrix $P > 0$ of the ellipsoidal set satisfy

$$c = \sum_{\iota=1}^{n} c_\iota, \quad P = \sum_{\iota=1}^{n} \varrho_\iota^{-1} P_\iota, \quad \sum_{\iota=1}^{n} \varrho_\iota = 1, \quad \varrho_\iota > 0.$$

Lemma 2 *Consider the decoding error $e_{k,h}$ in (11). Then, the following condition holds:*

$$e_{k,h} \in \mathscr{X}\left(0, E_{k,h}\right) \tag{16}$$

where

$$E_{k,h}^{[\jmath,s]} \triangleq \begin{cases} \left(\dfrac{\flat_{k,h}^{[s]} \ell^{[s]}}{\sqrt{n_y}}\right)^2, & \jmath = s \\ 0, & \jmath \neq s \end{cases}$$

with $E_{k,h}^{[\jmath,s]}$ being the $(\jmath, s)$ element of the shape-defining matrix $E_{k,h}$.

Proof: According to (11), one has

$$e_{k,h}^{[s]} \in \left[-\flat_{k,h}^{[s]} \ell^{[s]},\ \flat_{k,h}^{[s]} \ell^{[s]}\right]. \tag{17}$$

Then, it is easy to see that $\sum_{s=1}^{n_y} e_{k,h}^{[s]} (E_{k,h}^{[s,s]})^{-1} e_{k,h}^{[s]} \leq 1$, which implies that the decoding error $e_{k,h}$ belongs to the ellipsoidal set in (16). ■

3.1 Design of the EDSMF

The following theorem provides a sufficient condition for the existence of an ellipsoidal set (15) that includes the true state of the system.

Theorem 1 *Consider the system (1), the XOR-based EDM (5)–(10), and the EDSMF (12)–(13). Suppose that*

$$\begin{cases} x_{k+1,h} \in \mathscr{X}\left(\hat{x}_{k+1,h}, \tau_{k+1,h} P_{k+1,h}\right) \\ x_{k,h+1} \in \mathscr{X}\left(\hat{x}_{k,h+1}, \tau_{k,h+1} P_{k,h+1}\right). \end{cases} \tag{18}$$

Let $\varrho_{k+1,h+1}^{(1)}$, $\varrho_{k+1,h+1}^{(2)}$, $\varrho_{k+1,h+1}^{(3)}$, $\varrho_{k+1,h+1}^{(4)}$, $\varrho_{k+1,h+1}^{(5)}$, $\varrho_{k+1,h+1}^{(6)}$, and $\rho_{k+1,h+1}$ be given positive scalars. Calculate the shape-defining matrices $\Phi_{k+1,h+1}$ and $P_{k+1,h+1}$ according to

$$\begin{aligned} \Phi_{k+1,h+1} &= (\varrho_{k+1,h+1}^{(1)})^{-1} \mathcal{P}_{k+1,h}^{(1)} + (\varrho_{k+1,h+1}^{(2)})^{-1} \mathcal{P}_{k,h+1}^{(2)} \\ &\quad + (\varrho_{k+1,h+1}^{(3)})^{-1} Q_{k+1,h} + (\varrho_{k+1,h+1}^{(4)})^{-1} Q_{k,h+1} \end{aligned} \tag{19}$$

$$P_{k+1,h+1} = \bar{\rho}_{k+1,h+1}^{-1} (I - K_{k+1,h+1} C_{k+1,h+1}) \Phi_{k+1,h+1}. \tag{20}$$

Then, the system state is confined to the ellipsoidal set:

$$x_{k+1,h+1} \in \mathscr{X}\left(\hat{x}_{k+1,h+1}, \tau_{k+1,h+1} P_{k+1,h+1}\right). \tag{21}$$

Moreover, the filter gain matrix is given by

$$K_{k+1,h+1} = \bar{\rho}_{k+1,h+1}^{-1} \Phi_{k+1,h+1} C_{k+1,h+1}^{\mathrm{T}} S_{k+1,h+1}^{-1} \tag{22}$$

where $\kappa = 1, 2$, $\iota = 1, 2, 3, 4$, and

$$\tau_{k,h} = 1 - \zeta_{k,h}^{\mathrm{T}} S_{k,h}^{-1} \zeta_{k,h} \tag{23}$$

$$\mathcal{P}_{k,h}^{(\kappa)} \triangleq \tau_{k,h} A_{k,h}^{(\kappa)} P_{k,h} (A_{k,h}^{(\kappa)})^{\mathrm{T}},\ \varrho_{k,h}^{(\iota)} > 0,\ \sum_{\iota=1}^{4} \varrho_{k,h}^{(\iota)} = 1$$

$$\bar{\rho}_{k,h} \triangleq 1 - \rho_{k,h},\ 0 \leq \rho_{k,h} < 1,\ \zeta_{k,h} \triangleq \mu_{k,h} - C_{k,h} \vec{x}_{k,h}$$

$$\varrho_{k,h}^{(5)} > 0,\ \varrho_{k,h}^{(6)} > 0,\ \varrho_{k,h}^{(5)} + \varrho_{k,h}^{(6)} = 1$$

$$\bar{R}_{k,h} \triangleq (\varrho_{k,h}^{(5)})^{-1} R_{k,h} + (\varrho_{k,h}^{(6)})^{-1} E_{k,h}$$

$$S_{k,h} \triangleq \rho_{k,h}^{-1} \bar{R}_{k,h} + \bar{\rho}_{k,h}^{-1} C_{k,h} \Phi_{k,h} C_{k,h}^{\mathrm{T}}.$$

Proof: In this theorem, the filter is designed in two steps: the prediction step and the correction step.

1) *Prediction Step.* According to Definition 2, it is easy to see from (1), (3) and (18) that the system state $x_{k+1,h+1}$ is included in the following convex set:

$$\begin{aligned} x_{k+1,h+1} \in\ & A_{k+1,h}^{(1)} \mathscr{X}\left(\hat{x}_{k+1,h}, \tau_{k+1,h} P_{k+1,h}\right) \\ & \oplus A_{k,h+1}^{(2)} \mathscr{X}\left(\hat{x}_{k,h+1}, \tau_{k,h+1} P_{k,h+1}\right) \\ & \oplus \mathscr{X}\left(0, Q_{k+1,h}\right) \oplus \mathscr{X}\left(0, Q_{k,h+1}\right). \end{aligned} \tag{24}$$

By utilizing Lemma 1 and affine transformation technique [18], one computes an ellipsoidal set that contains the convex set in (24):

$$\begin{aligned} & A_{k+1,h}^{(1)} \mathscr{X}\left(\hat{x}_{k+1,h}, \tau_{k+1,h} P_{k+1,h}\right) \oplus \mathscr{X}\left(0, Q_{k+1,h}\right) \\ & \oplus A_{k,h+1}^{(2)} \mathscr{X}\left(\hat{x}_{k,h+1}, \tau_{k,h+1} P_{k,h+1}\right) \oplus \mathscr{X}\left(0, Q_{k,h+1}\right) \\ & \subseteq \mathscr{X}(\vec{x}_{k+1,h+1}, \Phi_{k+1,h+1}). \end{aligned} \tag{25}$$

Therefore, we have

$$x_{k+1,h+1} \in \mathscr{X}(\vec{x}_{k+1,h+1}, \Phi_{k+1,h+1}). \tag{26}$$

2) *Correction Step.* Utilizing (2) and Lemma 2, one has

$$\begin{aligned} & \mu_{k+1,h+1} - C_{k+1,h+1} x_{k+1,h+1} \\ & \in \mathscr{X}\left(0, R_{k+1,h+1}\right) \oplus \mathscr{X}\left(0, E_{k+1,h+1}\right). \end{aligned} \tag{27}$$

Similarly, by resorting to Lemma 1, we obtain an ellipsoidal set that contains the convex set in (27):

$$\mathscr{X}\left(0, R_{k+1,h+1}\right) \oplus \mathscr{X}\left(0, E_{k+1,h+1}\right) \subseteq \mathscr{X}\left(0, \bar{R}_{k+1,h+1}\right). \tag{28}$$

Clearly, it follows from (27) and (28) that

$$C_{k+1,h+1} x_{k+1,h+1} \in \mathscr{X}\left(\mu_{k+1,h+1}, \bar{R}_{k+1,h+1}\right). \tag{29}$$

Next, from (26) and (29), we know that the system state $x_{k+1,h+1}$ resides in the intersection of $\mathscr{X}(\vec{x}_{k+1,h+1}, \Phi_{k+1,h+1})$ and $\mathscr{X}(\mu_{k+1,h+1}, \bar{R}_{k+1,h+1})$:

$$x_{k+1,h+1} \in \mathscr{C}_{k+1,h+1} \tag{30}$$

where $\mathscr{C}_{k,h} \triangleq \mathscr{X}(\vec{x}_{k,h}, \Phi_{k,h}) \cap \mathscr{X}(\mu_{k,h}, \bar{R}_{k,h})$. Then, it is obvious that the intersection of ellipsoidal set $\mathscr{C}_{k+1,h+1}$ is bounded by the following ellipsoidal set:

$$\mathscr{B}_{k+1,h+1} \triangleq \{x_{k+1,h+1} \mid g(x_{k+1,h+1}) \leq 0\} \tag{31}$$

where $\vec{\vartheta}_{k,h} \triangleq x_{k,h} - \vec{x}_{k,h}$, $\theta_{k,h} \triangleq \mu_{k,h} - C_{k,h} x_{k,h}$, and

$$g(x_{k,h}) \triangleq \bar{\rho}_{k,h} \vec{\vartheta}_{k,h}^{\mathrm{T}} \Phi_{k,h}^{-1} \vec{\vartheta}_{k,h} + \rho_{k,h} \theta_{k,h}^{\mathrm{T}} \bar{R}_{k,h}^{-1} \theta_{k,h} - 1. \tag{32}$$

After performing simple calculations and substituting $\vec{\vartheta}_{k,h} = \hat{\vartheta}_{k,h} + K_{k,h} \zeta_{k,h}$, $\theta_{k,h} = \zeta_{k,h} - C_{k,h} \vec{\vartheta}_{k,h}$ into (32), one obtains

$$\hat{\vartheta}_{k+1,h+1}^{\mathrm{T}} \big(\bar{\rho}_{k+1,h+1} \Phi_{k+1,h+1}^{-1} + \rho_{k+1,h+1} C_{k+1,h+1}^{\mathrm{T}}$$

$$
\begin{aligned}
&\times \bar{R}^{-1}_{k+1,h+1}C_{k+1,h+1}\big)\hat{\vartheta}_{k+1,h+1} + \zeta^{\mathrm{T}}_{k+1,h+1}K^{\mathrm{T}}_{k+1,h+1} \\
&\times \big(\bar{\rho}_{k+1,h+1}\Phi^{-1}_{k+1,h+1} + \rho_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1} \\
&\times C_{k+1,h+1}\big)K_{k+1,h+1}\zeta_{k+1,h+1} + 2\hat{\vartheta}^{\mathrm{T}}_{k+1,h+1}\big(\bar{\rho}_{k+1,h+1} \\
&\times \Phi^{-1}_{k+1,h+1} + \rho_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1}C_{k+1,h+1}\big) \\
&\times K_{k+1,h+1}\zeta_{k+1,h+1} - 2\rho_{k+1,h+1}\hat{\vartheta}^{\mathrm{T}}_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1} \\
&\times \bar{R}^{-1}_{k+1,h+1}\zeta_{k+1,h+1} - 2\rho_{k+1,h+1}\zeta^{\mathrm{T}}_{k+1,h+1}K^{\mathrm{T}}_{k+1,h+1} \\
&\times C^{\mathrm{T}}_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1}\zeta_{k+1,h+1} + \rho_{k+1,h+1}\zeta^{\mathrm{T}}_{k+1,h+1} \\
&\times \bar{R}^{-1}_{k+1,h+1}\zeta_{k+1,h+1} \le 1. \qquad (33)
\end{aligned}
$$

In the subsequent proof, simplification calculations will be performed on (33). By leveraging matrix inverse lemma and (22), it is easy to have

$$
\begin{aligned}
&\big(\bar{\rho}_{k+1,h+1}\Phi^{-1}_{k+1,h+1} + \rho_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1} \\
&\times \bar{R}^{-1}_{k+1,h+1}C_{k+1,h+1}\big)^{-1} \\
&= \bar{\rho}^{-1}_{k+1,h+1}(I - K_{k+1,h+1}C_{k+1,h+1})\Phi_{k+1,h+1} \qquad (34)
\end{aligned}
$$

which together with (20) results in

$$
\begin{aligned}
&\bar{\rho}_{k+1,h+1}\Phi^{-1}_{k+1,h+1} + \rho_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1} \\
&\times \bar{R}^{-1}_{k+1,h+1}C_{k+1,h+1} = P^{-1}_{k+1,h+1}. \qquad (35)
\end{aligned}
$$

Next, multiplying $P_{k+1,h+1}P^{-1}_{k+1,h+1}$ on the right-hand side of (22) and combining it with (35) yields

$$
K_{k+1,h+1} = \rho_{k+1,h+1}P_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1}. \qquad (36)
$$

Utilizing matrix inverse lemma to $S_{k+1,h+1}$ yields

$$
\begin{aligned}
&S^{-1}_{k+1,h+1} \\
&= \rho_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1} - \rho_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1}C_{k+1,h+1} \\
&\quad \times (\bar{\rho}_{k+1,h+1}\Phi^{-1}_{k+1,h+1} + \rho_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1} \\
&\quad \times C_{k+1,h+1})^{-1}\rho_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1}
\end{aligned}
$$

which together with (35) easily yields that

$$
\begin{aligned}
S^{-1}_{k+1,h+1} &= \rho_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1} - \rho^2_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1} \\
&\quad \times C_{k+1,h+1}P_{k+1,h+1}C^{\mathrm{T}}_{k+1,h+1}\bar{R}^{-1}_{k+1,h+1}. \qquad (37)
\end{aligned}
$$

Substituting (23), (35)–(37) into (33), one has

$$
\hat{\vartheta}^{\mathrm{T}}_{k+1,h+1}P^{-1}_{k+1,h+1}\hat{\vartheta}_{k+1,h+1} \le \tau_{k+1,h+1}. \qquad (38)
$$

Thus, (21) is guaranteed, which completes the proof. ■

3.2 Optimization of the EDSMF Parameter

Theorem 1 outlines the recursive steps of calculating the filter gain $K_{k+1,h+1}$. It should be noted that the variables $\varrho^{(\iota)}_{k+1,h+1}$ ($\iota = 1, 2, \ldots, 6$) and $\rho_{k+1,h+1}$ would influence the shape of the ellipsoidal set, and it is therefore possible to tune these variables in order to minimize the size of the obtained ellipsoidal set, thereby optimizing the filtering performance. The following theorems present feasible methods for calculating the parameters of the optimal filter in terms of matrix trace.

Theorem 2 *The trace* $\mathrm{Tr}(\Phi_{k+1,h+1})$ *is minimized when the the positive variables* $\varrho^{(\iota)}_{k+1,h+1}$ *are tuned as*

$$
\varrho^{(\iota)}_{k+1,h+1} = \frac{\sqrt{\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1})}}{\sum_{\iota=1}^{4}\sqrt{\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1})}}, \quad \iota = 1, 2, 3, 4. \qquad (39)
$$

Furthermore, the corresponding $\Phi_{k+1,h+1}$ *is given by*

$$
\Phi_{k+1,h+1} = \sum_{\iota=1}^{4}\sqrt{\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1})}\sum_{\iota=1}^{4}\frac{\check{P}^{(\iota)}_{k+1,h+1}}{\sqrt{\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1})}} \qquad (40)
$$

where $\check{P}^{(1)}_{k+1,h+1} \triangleq \tau_{k+1,h}A^{(1)}_{k+1,h}P_{k+1,h}(A^{(1)}_{k+1,h})^{\mathrm{T}}$, $\check{P}^{(2)}_{k+1,h+1} \triangleq \tau_{k,h+1}A^{(2)}_{k,h+1}P_{k,h+1}(A^{(2)}_{k,h+1})^{\mathrm{T}}$, $\check{P}^{(3)}_{k+1,h+1} \triangleq Q_{k+1,h}$*, and* $\check{P}^{(4)}_{k+1,h+1} \triangleq Q_{k,h+1}$.

Proof: It follows from (19) that

$$
\mathrm{Tr}(\Phi_{k+1,h+1}) = \sum_{\iota=1}^{4}(\varrho^{(\iota)}_{k+1,h+1})^{-1}\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1}). \qquad (41)
$$

By considering the constraint

$$
\sum_{\iota=1}^{4}\varrho^{(\iota)}_{k+1,h+1} = 1, \qquad (42)
$$

we define the Lagrangian function as follows:

$$
\begin{aligned}
\mathcal{L}_{k+1,h+1} &= \sum_{\iota=1}^{4}(\varrho^{(\iota)}_{k+1,h+1})^{-1}\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1}) \\
&\quad + \lambda_{k+1,h+1}\left(\sum_{\iota=1}^{4}\varrho^{(\iota)}_{k+1,h+1} - 1\right) \qquad (43)
\end{aligned}
$$

where $\lambda_{k,h}$ is a variable called the Lagrange multiplier.

According to the method of Lagrange multipliers, one obtains the minimal value of $\mathrm{Tr}(\Phi_{k+1,h+1})$ by solving the following equations:

$$
\frac{\partial \mathcal{L}_{k+1,h+1}}{\partial \varrho^{(\iota)}_{k+1,h+1}} = 0, \qquad \frac{\partial \mathcal{L}_{k+1,h+1}}{\partial \lambda_{k+1,h+1}} = 0. \qquad (44)
$$

Solving equations in (44), it is easy to obtain that

$$
\begin{cases}
\varrho^{(\iota)}_{k+1,h+1} = \dfrac{\sqrt{\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1})}}{\sum_{\iota=1}^{4}\sqrt{\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1})}} \\
\lambda_{k+1,h+1} = \left(\sum_{\iota=1}^{4}\sqrt{\mathrm{Tr}(\check{P}^{(\iota)}_{k+1,h+1})}\right)^2.
\end{cases} \qquad (45)
$$

By some straightforward algebraic manipulations, (39) and (40) can be obtained from (45), and the proof is therefore complete. ■

Theorem 3 *The trace* $\mathrm{Tr}(\bar{R}_{k+1,h+1})$ *is minimized when the positive variables* $\varrho^{(5)}_{k+1,h+1}$ *and* $\varrho^{(6)}_{k+1,h+1}$ *are tuned as*

$$
\varrho^{(5)}_{k+1,h+1} = \frac{\sqrt{\mathrm{Tr}(R_{k+1,h+1})}}{\sqrt{\mathrm{Tr}(R_{k+1,h+1})} + \sqrt{\mathrm{Tr}(E_{k+1,h+1})}} \qquad (46)
$$

$$
\varrho^{(6)}_{k+1,h+1} = \frac{\sqrt{\mathrm{Tr}(E_{k+1,h+1})}}{\sqrt{\mathrm{Tr}(R_{k+1,h+1})} + \sqrt{\mathrm{Tr}(E_{k+1,h+1})}}. \qquad (47)
$$

Furthermore, the $\bar{R}_{k+1,h+1}$ *is*

$$\bar{R}_{k+1,h+1} = \Big(\sqrt{\mathrm{Tr}(R_{k+1,h+1})} + \sqrt{\mathrm{Tr}(E_{k+1,h+1})}\Big) \times\Big(\frac{R_{k+1,h+1}}{\sqrt{\mathrm{Tr}(R_{k+1,h+1})}} + \frac{E_{k+1,h+1}}{\sqrt{\mathrm{Tr}(E_{k+1,h+1})}}\Big). \quad (48)$$

Proof: The proof of this theorem is similar to that of Theorem 2 and is thus omitted here due to the limitation of pages. For the detailed proof process, we refer the readers to [57]. ■

Before giving the next theorem, let us first determine the upper bound of $\tau_{k+1,h+1}$. In light of (23) and Theorems 2 and 3, $\tau_{k+1,h+1}$ can be rewritten as follows:

$$\begin{aligned}&\tau_{k+1,h+1}\\ &= 1 - \rho_{k+1,h+1}\bar{\rho}_{k+1,h+1}\zeta^{\mathrm{T}}_{k+1,h+1}\check{R}^{\mathrm{T}}_{k+1,h+1}(\bar{\rho}_{k+1,h+1}\\ &\quad \times I + \rho_{k+1,h+1}\Phi_{k+1,h+1})^{-1}\check{R}_{k+1,h+1}\zeta_{k+1,h+1}\\ &\le \bar{\tau}_{k+1,h+1}\end{aligned} \quad (49)$$

where $\vec{\zeta}_{k,h} \triangleq \check{R}_{k,h}\zeta_{k,h}$ and

$$\Phi_{k,h} \triangleq \check{R}_{k,h}C_{k,h}\Phi_{k,h}C^{\mathrm{T}}_{k,h}\check{R}^{\mathrm{T}}_{k,h}$$

$$\bar{\tau}_{k,h} \triangleq 1 - \frac{\rho_{k,h}\bar{\rho}_{k,h}\vec{\zeta}^{\mathrm{T}}_{k,h}\vec{\zeta}_{k,h}}{\bar{\rho}_{k,h} + \rho_{k,h}\bar{\phi}_{k,h}}.$$

Here, $\check{R}_{k,h}$ is a factorization of $\bar{R}^{-1}_{k,h}$, i.e., $\bar{R}^{-1}_{k,h} = \check{R}^{\mathrm{T}}_{k,h}\check{R}_{k,h}$ and $\bar{\phi}_{k,h}$ is the maximum eigenvalue of $\Phi_{k,h}$.

Theorem 4 *The positive variable* $\bar{\tau}_{k+1,h+1}$ *is minimized when the positive variable* $\rho_{k+1,h+1}$ *is tuned as*

$$\rho_{k+1,h+1} = \begin{cases} 0, & \vec{\zeta}^{\mathrm{T}}_{k+1,h+1}\vec{\zeta}_{k+1,h+1} = 0\\ \dfrac{1}{1+\sqrt{\bar{\phi}_{k+1,h+1}}}, & \textit{otherwise}. \end{cases} \quad (50)$$

Proof: First, taking the first and second derivatives of $\bar{\tau}_{k+1,h+1}$ with respect to $\rho_{k+1,h+1}$, we have

$$\begin{aligned}\frac{\mathrm{d}\bar{\tau}_{k+1,h+1}}{\mathrm{d}\rho_{k+1,h+1}} &= \frac{\rho^2_{k+1,h+1}\bar{\phi}_{k+1,h+1} - \bar{\rho}^2_{k+1,h+1}}{(\bar{\rho}_{k+1,h+1} + \rho_{k,h}\bar{\phi}_{k+1,h+1})^2}\\ &\quad \times \vec{\zeta}^{\mathrm{T}}_{k+1,h+1}\vec{\zeta}_{k+1,h+1}\end{aligned} \quad (51)$$

$$\frac{\mathrm{d}^2\bar{\tau}_{k+1,h+1}}{\mathrm{d}\rho^2_{k+1,h+1}} = \frac{2\bar{\phi}_{k+1,h+1}\vec{\zeta}^{\mathrm{T}}_{k+1,h+1}\vec{\zeta}_{k+1,h+1}}{(\bar{\rho}_{k+1,h+1} + \rho_{k+1,h+1}\bar{\phi}_{k+1,h+1})^3}. \quad (52)$$

It is easy to see that $\frac{\mathrm{d}^2\bar{\tau}_{k+1,h+1}}{\mathrm{d}\rho^2_{k+1,h+1}} \ge 0$ for any $0 \le \rho_{k+1,h+1} < 1$, which implies that $\frac{\mathrm{d}\bar{\tau}_{k+1,h+1}}{\mathrm{d}\rho_{k+1,h+1}}$ is non-decreasing as $\rho_{k+1,h+1}$ increases. Then, let us prove Theorem 4 based on the following cases.

Case I: $\vec{\zeta}^{\mathrm{T}}_{k+1,h+1}\vec{\zeta}_{k+1,h+1} = 0$. In this case, it follows from (51) that

$$\frac{\mathrm{d}\bar{\tau}_{k+1,h+1}}{\mathrm{d}\rho_{k+1,h+1}} = 0, \quad \forall\rho_{k+1,h+1} \in [0,1). \quad (53)$$

Thus, $\bar{\tau}_{k+1,h+1}$ is a constant function, and $\rho_{k+1,h+1} = 0$ is satisfied.

Case II: $\vec{\zeta}^{\mathrm{T}}_{k+1,h+1}\vec{\zeta}_{k+1,h+1} \ne 0$. From (51), one has

$$\frac{\mathrm{d}\bar{\tau}_{k+1,h+1}}{\mathrm{d}\rho_{k+1,h+1}}\Big|_{\rho_{k+1,h+1}=0} = -\vec{\zeta}^{\mathrm{T}}_{k+1,h+1}\vec{\zeta}_{k+1,h+1} < 0 \quad (54)$$

$$\frac{\mathrm{d}\bar{\tau}_{k+1,h+1}}{\mathrm{d}\rho_{k+1,h+1}}\Big|_{\rho_{k+1,h+1}=1} = \vec{\zeta}^{\mathrm{T}}_{k+1,h+1}\vec{\zeta}_{k+1,h+1} > 0 \quad (55)$$

which means that there exists a minimum value in $[0,1)$. It is obvious that $\bar{\tau}_{k+1,h+1}$ is minimized when $\frac{\mathrm{d}\bar{\tau}_{k+1,h+1}}{\mathrm{d}\rho_{k+1,h+1}} = 0$, that is, $\rho_{k+1,h+1} = \frac{1}{1+\sqrt{\bar{\phi}_{k+1,h+1}}}$. Therefore, the proof is complete. ■

In Theorems 2–4, the parameters $\Phi_{k,h}$, $\bar{R}_{k,h}$ and $\varrho_{k,h}$ are optimized in the sense of matrix trace. Thus, based on these optimized parameters, the desired filter gain matrix $K_{k,h}$ is obtained, and the second objective of this paper is achieved.

3.3 Boundedness Analysis

In the following, we will analyze the uniform boundedness of the ellipsoidal shape-defining matrix $P_{k,h}$.

Assumption 2 *There exist positive scalars* $\underline{a}_1$, $\bar{a}_1$, $\underline{a}_2$, $\bar{a}_2$, $\underline{c}$, $\bar{c}$, $\underline{q}$, $\bar{q}$, $\underline{r}$, *and* $\bar{r}$ *such that, for* $\forall\, k,h \in \mathbb{N}$, *the following conditions are satisfied:*

$$\underline{a}_1^2 I \le (A^{(1)}_{k,h})^{\mathrm{T}}A^{(1)}_{k,h} \le \bar{a}_1^2 I \quad (56)$$

$$\underline{a}_2^2 I \le (A^{(2)}_{k,h})^{\mathrm{T}}A^{(2)}_{k,h} \le \bar{a}_2^2 I \quad (57)$$

$$\underline{c}^2 I \le C^{\mathrm{T}}_{k,h}C_{k,h} \le \bar{c}^2 I \quad (58)$$

$$\underline{q} I \le Q_{k,h} \le \bar{q} I \quad (59)$$

$$\underline{r} I \le \bar{R}_{k,h} \le \bar{r} I. \quad (60)$$

Lemma 3 *For two positive definite matrices* $\mathcal{X}$ *and* $\mathcal{Y}$, *there exists a scalar* $0 < \gamma < 1$ *such that*

$$(\mathcal{X} + \mathcal{Y})^{-1} > \gamma\mathcal{X}^{-1}. \quad (61)$$

Proof: The proof is straightforward and is thus omitted. ■

Theorem 5 *Under Assumption 2, there exists a positive scalar* $\underline{p}$ *such that*

$$P_{k,h} \ge \underline{p} I, \quad \forall\, k,h \in \mathbb{N} \quad (62)$$

where $\underline{p} \triangleq (\underline{q}^{-1} + \bar{c}^2\underline{r}^{-1})^{-1}$.

Proof: From (19), it follows that

$$\Phi^{-1}_{k,h} \le Q^{-1}_{k,h-1}. \quad (63)$$

Then, considering (35) and (63), one has

$$P^{-1}_{k,h} \le \underline{q}^{-1} + \bar{c}^2\underline{r}^{-1} \quad (64)$$

which implies $P_{k,h} \ge \underline{p}I$ and ends the proof. ■

Theorem 6 *Under Assumption 2, suppose that there exist a positive scalar* $\underline{\omega}$ *and an integer* N *such that, for any* $k > N$ *and* $h > N$, *the following condition holds:*

$$\sum_{i=k-N}^{k}\sum_{j=h-N}^{h}\Big(\rho_{k,h}\Psi^{\mathrm{T}}_{k,h}(k-i,h-j)$$

$$\times C_{k,h}^{\mathrm{T}} C_{k,h} \Psi_{k,h}(k-i,h-j)\Big) \geq \underline{\omega} I. \tag{65}$$

Then, there exists a positive scalar $\bar{p}$ such that

$$P_{k,h} \leq \bar{p} I \tag{66}$$

where $\bar{p} \triangleq \bar{r}\underline{\omega}^{-1}$, $\Psi_{0,0}(i,j) = I$, $i \geq 0$, $j \geq 0$, and

$$\begin{aligned}
\Psi_{k,h}(i,j) &= \sqrt{\hat{\gamma}_{k,h-1}^{(1)}}(A_{k,h-1}^{(1)})^{-1}\Psi_{k,h-1}(i,j-1) \\
&\quad + \sqrt{\hat{\gamma}_{k-1,h}^{(2)}}(A_{k-1,h}^{(2)})^{-1}\Psi_{k-1,h}(i-1,j) \\
\hat{\gamma}_{k,h-1}^{(1)} &\triangleq \bar{\rho}_{k,h}\bar{\gamma}_{k,h}\vec{\gamma}_{k,h}(\gamma_{k,h-1}^{(1)})^{-1}, \;\; 0 < \bar{\gamma}_{k,h} < 1 \\
\hat{\gamma}_{k-1,h}^{(2)} &\triangleq \bar{\rho}_{k,h}\bar{\gamma}_{k,h}\vec{\gamma}_{k,h}(\gamma_{k-1,h}^{(2)})^{-1}, \;\; 0 < \vec{\gamma}_{k,h} < 1 \\
\gamma_{k,h-1}^{(1)} &\triangleq (\varrho_{k,h}^{(1)})^{-1}, \;\; \gamma_{k-1,h}^{(2)} \triangleq (\varrho_{k,h}^{(2)})^{-1}.
\end{aligned}$$

Proof: From (19) and (35), one has

$$\begin{aligned}
P_{k,h}^{-1} = \bar{\rho}_{k,h}\Big(&(\varrho_{k,h}^{(1)})^{-1}\tau_{k,h-1}A_{k,h-1}^{(1)}P_{k,h-1}(A_{k,h-1}^{(1)})^{\mathrm{T}} \\
&+ (\varrho_{k,h}^{(2)})^{-1}\tau_{k-1,h}A_{k-1,h}^{(2)}P_{k-1,h}(A_{k-1,h}^{(2)})^{\mathrm{T}} \\
&+ (\varrho_{k,h}^{(3)})^{-1}Q_{k,h-1} + (\varrho_{k,h}^{(4)})^{-1}Q_{k-1,h}\Big)^{-1} \\
&+ \rho_{k,h}C_{k,h}^{\mathrm{T}}\bar{R}_{k,h}^{-1}C_{k,h}.
\end{aligned} \tag{67}$$

Then, it follows from (67) that

$$\begin{aligned}
P_{k,h}^{-1} \geq \bar{\rho}_{k,h}\Big(&\gamma_{k,h-1}^{(1)}A_{k,h-1}^{(1)}P_{k,h-1}(A_{k,h-1}^{(1)})^{\mathrm{T}} \\
&+ \gamma_{k-1,h}^{(2)}A_{k-1,h}^{(2)}P_{k-1,h}(A_{k-1,h}^{(2)})^{\mathrm{T}} \\
&+ (\varrho_{k,h}^{(3)})^{-1}Q_{k,h-1} + (\varrho_{k,h}^{(4)})^{-1}Q_{k-1,h}\Big)^{-1} \\
&+ \rho_{k,h}C_{k,h}^{\mathrm{T}}\bar{R}_{k,h}^{-1}C_{k,h}.
\end{aligned} \tag{68}$$

Applying Lemma 3 to (68) results in

$$\begin{aligned}
P_{k,h}^{-1} \geq\; &\hat{\gamma}_{k,h-1}^{(1)}\Big(A_{k,h-1}^{(1)}P_{k,h-1}(A_{k,h-1}^{(1)})^{\mathrm{T}}\Big)^{-1} \\
&+ \hat{\gamma}_{k-1,h}^{(2)}\Big(A_{k-1,h}^{(2)}P_{k-1,h}(A_{k-1,h}^{(2)})^{\mathrm{T}}\Big)^{-1} \\
&+ \rho_{k,h}C_{k,h}^{\mathrm{T}}\bar{R}_{k,h}^{-1}C_{k,h}.
\end{aligned} \tag{69}$$

Next, according to (69), one has

$$\begin{aligned}
&P_{k,h}^{-1} \\
&\geq \hat{\gamma}_{k,h-1}^{(1)}((A_{k,h-1}^{(1)})^{-1})^{\mathrm{T}}P_{k,h-1}^{-1}(A_{k,h-1}^{(1)})^{-1} \\
&\quad + \hat{\gamma}_{k-1,h}^{(2)}((A_{k-1,h}^{(2)})^{-1})^{\mathrm{T}}P_{k-1,h}^{-1}(A_{k-1,h}^{(2)})^{-1} \\
&\quad + \rho_{k,h}C_{k,h}^{\mathrm{T}}\bar{R}_{k,h}^{-1}C_{k,h} \\
&\geq \cdots \\
&\geq \sum_{i=k-N}^{k}\Psi_{k,h}^{\mathrm{T}}(k-i,h-1)P_{k-i,h-1}^{-1}\Psi_{k,h}(k-i,h-1) \\
&\quad + \sum_{j=h-N}^{h}\Psi_{k,h}^{\mathrm{T}}(k-1,h-j)P_{k-1,h-j}^{-1}\Psi_{k,h}(k-1,h-j) \\
&\quad + \sum_{i=k-N}^{k}\sum_{j=h-N}^{h}\Big(\rho_{k,h}\Psi_{k,h}^{\mathrm{T}}(k-i,h-j)C_{k,h}^{\mathrm{T}}\bar{R}_{k,h}^{-1}C_{k,h} \\
&\quad \times \Psi_{k,h}(k-i,h-j)\Big) \\
&\geq \bar{r}^{-1}\sum_{i=k-N}^{k}\sum_{j=h-N}^{h}\Big(\rho_{k,h}\Psi_{k,h}^{\mathrm{T}}(k-i,h-j)C_{k,h}^{\mathrm{T}}C_{k,h} \\
&\quad \times \Psi_{k,h}(k-i,h-j)\Big).
\end{aligned} \tag{70}$$

Then, it can be derived from (65) and (70) that

$$P_{k,h} \leq \bar{r}\underline{\omega}^{-1} I \tag{71}$$

which is the same as (66) and the proof is complete. ■

Remark 5 *In Theorems 1–6, the primary focus has been on the EDM-based set-membership filtering problem for 2-D systems with UBB noises, which has significant theoretical significance. The theoretical contributions are summarized are follows. 1) Novel filtering algorithm: the new EDSMF is designed in Theorem 1 with the assistance of set theory. 2) Analytic Optimization: the corresponding filter parameters are optimized in Theorems 2–4 by applying the Lagrange multiplier and monotonicity analysis methods. 3) Performance Analysis: the uniform boundedness of the proposed filtering algorithm is fully analyzed in Theorems 5 and 6.*

Remark 6 *So far, the EDSMF design issue has been addressed for the considered 2-D system for the first time. Compared with the rich literature on 2-D systems and filtering problems, e.g., [5,34–38], the distinguishing features/novelties of our results are summarized as follows: 1) the addressed EDSMF design problem is new and has both theoretical significance and engineering background; 2) the XOR-based EDM is initially developed, which ensures the security of data transmission and the efficiency of network communication; 3) the secure set-membership filtering scheme is dedicatedly designed, in which the EDM and UBB noises are considered; and 4) the boundedness analysis problem is fully analyzed, which gives the quantitative results on the uniform boundedness of the ellipsoidal shape-defining matrix $P_{k,h}$.*

4 Illustrative Example

In this section, the effectiveness of the proposed set-membership filtering algorithm is verified by an example with practical background on networked heat exchanger. The differential equation of the heat exchanger is given as $\frac{\partial E_{\hat{s},\hat{t}}}{\partial \hat{s}} + \frac{\partial E_{\hat{s},\hat{t}}}{\partial \hat{t}} = a(1+\varsigma_a)E_{\hat{s},\hat{t}} + \varpi_{\hat{s},\hat{t}}$, where $E_{\hat{s},\hat{t}}$ is the temperature at the space $\hat{s}$ and time $\hat{t}$, $\varpi_{\hat{s},\hat{t}}$ is a force function, a is the heat transfer coefficient, and ς_a is the uncertain parameter.

By denoting $x_{k,h} \triangleq \begin{bmatrix} E_{k,h}^{\mathrm{T}} & E_{k-1,h}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ with $E_{k,h} \triangleq E_{k\Delta\hat{s},h\Delta\hat{t}}$, the differential equation can be converted into a general F-M second model with the following parameters:

$$A^{(1)} = \begin{bmatrix} 0 & 0 \\ \frac{\Delta \hat{t}}{\Delta \hat{s}} & 1 - \frac{\Delta \hat{t}}{\Delta \hat{s}} + a\Delta \hat{t} \end{bmatrix}, \quad A^{(2)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Let the parameters be $\Delta \hat{t} = 0.1$, $\Delta \hat{s} = 0.4$, and $a = \sin(0.1(k+h)) - 4$. Then, the system parameters are given as follows:

$$\begin{aligned} A_{k,h}^{(1)} &= \begin{bmatrix} 0.1\mathrm{e}^{-k} & 0 \\ 0.25 & 0.35 + 0.1\sin(0.1(k+h)) \end{bmatrix} \\ A_{k,h}^{(2)} &= \begin{bmatrix} 0 & 1 \\ 0.15\cos(0.5(k+h)) & 0 \end{bmatrix} \\ C_{k,h} &= \begin{bmatrix} 0.9 + 0.1\mathrm{e}^{-k-h} & 1.2 \end{bmatrix}. \end{aligned}$$

The process noise $w_{k,h}$ and the measurement noise $v_{k,h}$ are selected as $w_{k,h} = 0.35\sin(0.1k + 0.15h)$ and $v_{k,h} = 0.2\sin(0.15k) + 0.18\cos(0.15h)$.

The shape-defining matrices are $Q_{k,h} = 0.15I$ and $R_{k,h} = 0.2I$. In the EDM, the parameters are selected as $\flat_{k,h} = 1.2$, $\mathcal{R} = 2^7$, and $g_{k,h}^{[1]} = g_{k,h}^{[2]} = 0.3\chi_{k,h}$. Let the boundary conditions be given as follows:

$$I: \; x_{k,0} = x_{0,h} = \begin{bmatrix} 2.3 & 3 \end{bmatrix}^{\mathrm{T}}, \; \hat{x}_{k,0} = \hat{x}_{0,h} = \begin{bmatrix} 1.8 & 2.7 \end{bmatrix}^{\mathrm{T}}.$$

$$II: \begin{cases} x_{k,0} = \begin{bmatrix} 0.5\sin(k) & -0.6\,\mathrm{e}^{0.1(1-k)} \end{bmatrix}^{\mathrm{T}} \\ x_{0,h} = \begin{bmatrix} \cos(h) & 0.9\,\mathrm{e}^{0.2(1-h)} \end{bmatrix}^{\mathrm{T}} \\ \hat{x}_{k,0} = \begin{bmatrix} 0.8\sin(k) & -0.3\,\mathrm{e}^{0.1(1-k)} \end{bmatrix}^{\mathrm{T}} \\ \hat{x}_{0,h} = \begin{bmatrix} 0.85\cos(h) & \mathrm{e}^{0.2(1-h)} \end{bmatrix}^{\mathrm{T}}. \end{cases}$$

Fig. 1 depicts the dynamic trajectories of system states and their estimates under the boundary condition I, which shows that the proposed SMF achieves a satisfactory level of performance under the effects of UBB noises and EDM. Then, the estimation errors are given in Fig. 2, which indicates that the estimation error of the proposed filtering algorithm and the ellipsoidal shape-defining matrix are bounded. Furthermore, to show the general effectiveness of the proposed filtering algorithm, we also provide simulation results under the boundary condition II. From the simulations in Fig. 3, it is observed that the SMF is also capable of achieving the desired performance under the boundary condition II.

On the other hand, to verify the effectiveness of the designed EDM, corresponding simulation results are given in Figs. 4–6. Specifically, Fig. 4 shows the amplitude changes of the measurement output $y_{k,h}$, the encoder output $\xi_{h,k}$, and the decoder output $\mu_{h,k}$. It is clear that the amplitude of the encoder output $\xi_{h,k}$ is smaller than the measurement output $y_{k,h}$, which means that the proposed EDM can reduce the communication burden. Fig. 5 shows that the error between the decoding data

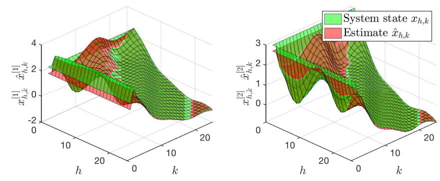

Fig. 1. State variable $x_{k,h}$ and its estimate $\hat{x}_{k,h}$ (Case I).

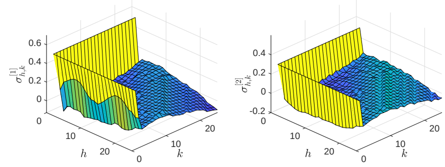

Fig. 2. Estimation error $\sigma_{k,h} \triangleq x_{k,h} - \hat{x}_{k,h}$ (Case I).

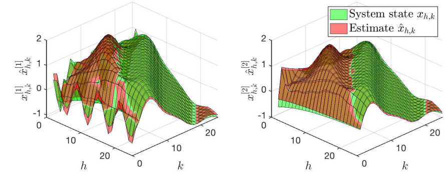

Fig. 3. State variable $x_{k,h}$ and its estimate $\hat{x}_{k,h}$ (Case II).

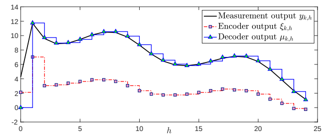

Fig. 4. Measurement output, encoder output and decoder output on $k = 8$.

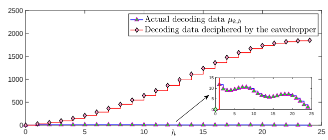

Fig. 5. Actual decoding data $\mu_{k,h}$ and the decoding data deciphered by the eavesdropper on $k = 8$.

deciphered by the eavesdropper and the actual decoding data is unstable, which means that the security/privacy performance of the designed EDM is guaranteed. In addition, the estimation errors of the eavesdropper are depicted in Fig. 6, which shows that the eavesdropper can not obtain the satisfactory/accurate system information (i.e., the estimation). Thus, Figs. 1–6 show the effectiveness of the proposed EDSMF.

5 Conclusion

This paper has addressed the encryption-decryption-based set-membership filtering issue for systems subject to UBB noises. An XOR-based EDM has been proposed for 2-D systems to preserve data privacy. An encryption-decryption-based set-membership filtering scheme has

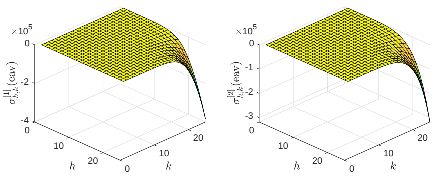

Fig. 6. Estimation error of the eavesdropper.

been developed for considered 2-D systems such that the true system state is confined to an ellipsoidal set, and the corresponding filter parameters have been obtained by computing a set of recursive equations. It has been proved that the ellipsoidal shape-defining matrix in the proposed filtering scheme is uniformly bounded. Finally, the usefulness of the designed filtering scheme has been validated by a heat exchanger example.

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