Ultimately Bounded Output Feedback Control for Networked Nonlinear Systems with Unreliable Communication Channel: A Buffer-Aided Strategy

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***Abstract*—This paper concerns ultimately bounded output-feedback control problems for networked systems with unknown nonlinear dynamics. Sensor-to-observer signal transmission is facilitated over networks that has communication constraints. These transmissions are carried out over an unreliable communication channel. In order to enhance the utilization rate of measurement data, a buffer-aided strategy is novelly employed to store historical measurements when communication networks are inaccessible. Using the neural network technique, a novel observer-based controller is introduced to address effects of signal transmission behaviors and unknown nonlinear dynamics. Through the application of stochastic analysis and Lyapunov stability, a joint framework is constructed for analyzing resultant system performance under the introduced controller. Subsequently, existence conditions for the desired output-feedback controller are delineated. The required parameters for the observer-based controller are then determined by resolving some specific matrix inequalities. Finally, a simulation example is showcased to confirm method efficacy.**

***Index Terms*—Output-feedback control, nonlinear control, neural networks, unreliable communication channel, buffer-aided strategy.**

Abbreviations and Notations

HJB	Hamilton-Jacobi-Bellman
ADP	adaptive dynamic programming
NN	Neural network
NCSs	Networked control systems
NNW	Neural network weight
LMI	Linear matrix inequality
$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{R}^{n\times m}$	The set of all $n \times m$ real matrices
$\mathbb{N}$	The set of nonnegative integers
$U \geq F$	$U - F$ is positive semi-definite
$U > F$	$U - F$ is positive definite
S^T	The transpose of the matrix S
$\mathrm{tr}\{S\}$	The trace of the matrix S
$\|S\|$	The Frobenius norm of the matrix S
$\lambda_{\min}(S)$	The minimum eigenvalue of S
$b^{-1}(\cdot)$	The inverse function of $b(\cdot)$
$\mathrm{Prob}\{\cdot\}$	The occurrence probability of the random event "·"
$\mathbb{E}\{x\}$	The expectation of the stochastic variable x
$\mathbb{E}\{x\|y\}$	The expectation of x conditional on y
0	Zero matrix of compatible dimension
I	Identity matrix of compatible dimension
$\mathrm{diag}\{\cdots\}$	The block-diagonal matrix
"$*$"	The symmetric parts in the symmetric block matrix

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I. Introduction

Over the past few decades, a wide interest has been shown in optimal control problems due to their significance in fields of finance, ecology, power systems, and aerospace [16], [17], [34]. The optimal control is to minimize (or maximize) certain performance index function for a given system while adhering to certain physical constraints. It is widely recognized that gains of optimal controllers are typically derived from solving Hamilton-Jacobi-Bellman (HJBs) equations. In linear cases, HJB equation simplify to Riccati equations, allowing the controller's gain matrix to be parameterized upon solving this equation. However, for nonlinear systems, solving HJB equations becomes notably challenging because of the complexities introduced by inherent nonlinearities [43].

In recent years, adaptive dynamic programming (ADP) has gradually gained much research attention. Leveraging actor/critic neural networks (NNs) known for their superior approximation capabilities, ADP has been extensively employed to tackle optimal control problems with both known and unknown nonlinear dynamics [19], [35], [38], [39]. The ADP-based algorithms have garnered significant research attention, and numerous notable results can be found in the literature [12], [18], [23]. Although much of the research on ADP-based control has centered on state feedback, practical engineering often limits access to full state information of systems. This

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limitation, caused either by budget constraints or complex external environments, has steered engineers towards favoring ADP-based output-feedback control strategies [8].

Networked control systems (NCSs) denote dynamical systems in which distinct system components communicate through a network characterized by limited bandwidth [5], [21], [32], [47]. Over the past two decades, rapid advancements in network-based communication technology have significantly expanded the potential of NCSs [3], [6], [45]. Enhanced data transmission rates, improved error correction methods, and the rise of machine learning techniques for network optimization have all combined to elevate the capabilities of these systems. As a result, NCSs have permeated a myriad of practical engineering fields including spacecrafts, smart grids, mobile robots, and unmanned underwater vehicles [11], [22], [42], [45], [46]. Each of these applications underscores the versatility and transformative potential of NCSs in modern engineering landscapes.

In the deployment of NCSs, the reliability of signal transmissions is significantly impacted by pervasive communication constraints. Such constraints are often manifested as limited bandwidth or finite bit rates [10], [14], [24], [28], [37]. Issues such as congestion or packet dropping can be caused by constraints like limited communication capacity. As a result, the reliability of signal transmissions can be substantially compromised, leading to diminished or even devastated estimation/control performance [25]. Due to these challenges, attention has now been drawn to control problems associated with NCSs operating over unreliable communication channels from both control and signal processing communities. Consequently, numerous research outcomes have been documented [1], [36].

In response to the challenges posed by unreliable communication channels, the *buffer-aided strategy*, which has gained widespread acceptance in practical applications. This strategy aims to enhance the transmission of measurement signals during specific transmission instants. Initially, newly generated signals are stored in the buffer and, following this, all the signals stored (i.e., both current and historical instant signals) are transmitted to the receiver (e.g. observer) simultaneously at the designated transmission instant (often, the present moment). Once the transmission is completed, the buffer is cleared to create space for measurement signals generated in the ensuing instants [40]. Leveraging this method, a greater number of measurement signals can be harnessed by the observer for the estimation procedure. The buffer-aided strategy not only ensures a more judicious use of resources but also facilitates the attainment of the desired estimation outcomes [39].Unfortunately, even with its profound engineering ramifications and broad application prospects, the control problems of NCSs using a buffer-aided strategy over unreliable communication channels have yet to receive the research attention they deserve.

Motivated by the aforementioned considerations, our objective is to delve into ultimately bounded output-feedback control problems, which holds both theoretical and practical significance, for nonlinear NCSs that employ a buffer-aided strategy over unreliable communication channels. The output-feedback control problem under investigation presents three anticipated yet foundational challenges: 1) how to quantify transmission unreliability and buffer-aided strategy effects? 2) how to design the tuning laws for the neural-network-weights (NNWs) for networked nonlinear systems that use a buffer-aided strategy over unreliable communication channels? and 3) how to analyze bounded stability of considered networked nonlinear systems with a buffer-aided strategy to counteract the limited communication capacity? The primary drive of this research is, therefore, to address these challenges through a comprehensive examination.

The primary contributions are enumerated as follows.

1) The ultimately bounded output-feedback control problem is first concerned for networked nonlinear systems under a buffer-aided strategy over unreliable communication channels.
2) An intricately devised ADP-based output-feedback control scheme is introduced to address system dynamics constrained by limited communication capacity and the buffer-aided strategy.
3) An adaptive tuning law is designed for the controller.
4) The ultimate boundedness affected by unreliable communication channels and the buffer-aided strategy are rigorously analyzed.

II. Problem Formulation and Preliminaries

In this paper, a nonlinear NCS is examined in which sensor-to-controller transmission is facilitated through an unreliable communication network. A buffer-aided strategy is integrated with aim to optimize efficiency of measurement data utilization by archiving historical measurements during instances when the communication channel becomes inaccessible. This section is dedicated to providing an in-depth delineation of the nonlinear NCS, the peculiarities of transmission behaviors and the control methodology employed.

A. System Model and Signal Transmissions

Consider the following nonlinear system:

$$\begin{cases} x_{k+1} = Ax_k + f(x_k) + Bu_k + E\omega_k \\ \quad y_k = Cx_k + D\omega_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $y_k \in \mathbb{R}^{n_y}$ and $u_k \in \mathbb{R}^{n_u}$ represent, respectively, the system state, the measurement signal and the control input. $f(\cdot)$ is an unknown but bounded smooth nonlinear function on a compact set $\Omega \in \mathbb{R}^n$. $\omega(k) \in \mathbb{R}^{n_\omega}$ denotes the bounded stochastic noise with zero-mean and known variance $\bar{Q} = \tilde{Q}\tilde{Q}^T$. Matrices A, B, C, D and E are known.

The communication network is now introduced. Communication between sensors and controllers transpires via an unreliable network channel, which is prone to intermittent packet dropouts during signal transmissions. Traditionally, if the communication channel is inaccessible, the measurement signals, which the sensors produce, would be lost. This sporadic packet dropout, in contrast to continuous transmission, inevitably impairs the estimation/control performance, attributed mainly to the "low utilization efficiency" of measurement

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data. The estimation/control challenges arising from unreliable or lossy networks have been the subject of extensive research. For instance, in [7], the non-fragile estimation challenge was explored for a complex networks subset with a dynamic event-based transmission mechanism. Similarly, [33] tackled the NN-based control problem for a nonlinear system faced with intermittent packet dropouts caused by denial-of-service attacks.

To mitigate unreliable transmission, a buffer-aided mechanism is proposed to boost utilization efficiency of measurement signals. Specifically, this mechanism operates in two distinct modes: the storage mode and the delivery mode. In the storage mode, when the communication network is inaccessible, measurement signals are retained in the buffer, which has a designated *maximum buffer capacity* denoted as Q. If the buffer is filled to its capacity, the "oldest" measurement signal stored therein will be displaced by the most recently generated signal. Conversely, in the delivery mode, when the communication network becomes accessible, all the measurement signals retained in the buffer are concurrently dispatched over the communication channel. Subsequent to this transmission, the buffer undergoes a clearing process to remove all the signals it previously held. This approach ensures that a larger volume of measurement signals are employed for control as compared to traditional methods where generated measurements are instantly discarded if the communication channel is out of service.

In this paper, the characteristics of unreliable signal transmissions are described in the following assumptions.

Assumption 1: (Transmission interval) [40] Let $h(i)$ be the transmission interval between $t(i)$ and $t(i-1)$, i.e., $h(i) \triangleq t(i)-t(i-1)$ $(h(i) \in \mathbb{N}^{+})$. For $i \in \mathbb{N}^{+}$, $h(i)$ satisfies

$$h(i) \in \mathcal{H} \triangleq \{1, 2, \cdots, H\}$$

where constant H is known and positive representing a maximum transmission interval.

Assumption 2: The transmission intervals $\{h(i)\}_{i\geq 0}$ is a sequence of random variables which are independently and identically distributed. The disturbance noise ω_k and transmission intervals $h(i)$ are mutually uncorrelated stochastic vectors. The occurrence probability of $h(i)=\chi$ $(\forall \chi \in \{1,...,H\})$ is partially unknown, i.e.,

$$\begin{cases} \text{Prob}\{h(i)=\iota\} = p_{\iota}, & \text{if } \iota \in \mathcal{H}_a \\ \text{Prob}\{h(i)=\tau\} = ?, & \text{if } \tau \in \mathcal{H}_b \end{cases} \tag{2}$$

where $0 \leq p_{\iota} \leq 1$ and "?", respectively, are the known and unknown probabilities with $\sum_{h(i)=1}^{H} p_{h(i)} = 1$. $\mathcal{H}_a \triangleq \{\iota \mid p_{\iota} \text{ is known}\}$ and $\mathcal{H}_b \triangleq \{\tau \mid p_{\tau} \text{ is unknown}\}$. Obviously, it is easy to observe that $\mathcal{H}_a \cup \mathcal{H}_b = \mathcal{H}$ and $\mathcal{H}_a \cap \mathcal{H}_b = \varnothing$.

Remark 1: Assumptions 1 and 2 are quite reasonable in real-world applications. Assumptions 1 is proposed based on the intermittent characteristic of the signal transmissions under the impact of the unreliable communication channels. In engineering practice, it obvious that the transmission intervals of networked systems are upper bounded. Assumption 2 shows the typical characteristics of the buffer (i.e., limited capacity), which is preferred in practical applications in order to save economic costs. In these cases, it is of practical significance to assume that the number of signals transmitted is bounded.

Let us now consider the measurement data received by the controller. It is clear that data can only be received by the controller at transmission instants. Specifically, at each transmission instant $t(i)$, the number of measurement signals received by the controller is dictated by the amount of data retained in the buffer. By designating $q(i)$ as the count of signals preserved in the buffer at $t(i)$, it can be deduced that:

$$q(i) = \min\{Q, h(i)\}, \ i \in \mathbb{N}^{+}.$$

Accordingly, the received measurement data for the controller at time k (defined as $\mathcal{Y}_k$) is

$$\mathcal{Y}_k = \begin{cases} \{y_{k-j}\}_{j=0,1,\cdots,q(i)-1}, & \text{if } \{i|k=t(i), i\geq 0\} \neq \varnothing \\ \varnothing, & \text{if } \{i|k=t(i), i\geq 0\} = \varnothing \end{cases}.$$

B. Observer-Based Controller

In this study, an observer-based control strategy is employed to control the plant as defined in (1), considering the influences of both the buffer-aided strategy and unreliable signal transmissions. To address the unknown nonlinearity $f(\cdot)$, an NN-based observer is initially introduced to produce estimates, followed by presentation of the observer-based controller policy.

According to [12], an NN is utilized to approximate $f(\cdot)$ via $W_f\varphi_f(x_k)+\zeta_{f,k}$, where $\varphi_f(\cdot)$, $W_f \in \mathbb{R}^{n_x\times n_x}$ and $\zeta_{f,k} \in \mathbb{R}^{n_x}$ denote the activation function, the ideal weight matrix and the approximation error of the NN, respectively. Thus, we have

$$\begin{cases} x_{k+1} = Ax_k + W_f\varphi_f(x_k) + Bu_k + E\omega_k + \zeta_{f,k} \\ \quad y_k = Cx_k + D\omega_k \end{cases} \tag{3}$$

Here, it is reasonable to assume that

$$\|W_*\| \leq \bar{W}_*, \ \|\varphi_*(\cdot)\| \leq \bar{\varphi}_*, \ \|\zeta_{*,k}\| \leq \bar{\zeta}_*$$

where $\bar{W}_*$, $\bar{\varphi}_*$, and $\bar{\zeta}_*$ are known positive constants, and $*$ represents f or other symbols.

According to the received measurement data $\mathcal{Y}_k$, the following observer is utilized to acquire desired estimates:

$$\begin{cases} \textbf{Case 1:} \text{ if } \{i|k=t(i), i\geq 0\} = \varnothing \\ \hat{x}_{k+1} = A\hat{x}_k + \hat{W}_{f,k}\varphi_f(\hat{x}_k) + Bu_k \\ \textbf{Case 2:} \text{ if } \{i|k=t(i), i\geq 0\} \neq \varnothing \\ \vec{x}_{j+1} = A\vec{x}_j + \vec{W}_{f,j}\varphi_f(\vec{x}_j) + Bu_j + L_{h(i)}(y_j \\ \qquad - C\vec{x}_j), \quad t(i)-q(i)+1 \leq j \leq t(i), \\ \hat{x}_{k+1} = \vec{x}_{k+1} \end{cases} \tag{4}$$

where $\{\vec{x}_{j+1}\}_{t(i)-q(i)+1\leq j\leq t(i)}$ are the so-called "reorganized" state estimates with $\vec{x}_{t(i)-q(i)+1} = \hat{x}_{t(i)-q(i)+1}$, $\hat{x}_k$ and $\hat{W}_{f,k}$ are the estimates of x_k and W_f, respectively. $\vec{W}_{f,j}$ is the reorganized estimate value of W_f. Here, $L_{h(i)}$ is the observer gain.

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The adaptive tuning law is

$$\begin{cases} \textbf{Case 1:} \text{ if } \{i|k=t(i), i\geq 0\}=\varnothing \\ \hat{W}_{f,k+1} = \hat{W}_{f,k} - \alpha_1\alpha_2\hat{W}_{f,k} \\ \textbf{Case 2:} \text{ if } \{i|k=t(i), i\geq 0\}\neq\varnothing \\ \vec{W}_{f,j+1} = \vec{W}_{f,j} + \alpha_1\big(C^T(y_{j+1}-C\vec{x}_{j+1})\tilde{\varphi}_f^T(\vec{x}_j) \\ \qquad\qquad -\alpha_2\vec{W}_{f,j}\big), \quad t(i)-q(i)+1\leq j\leq t(i), \\ \hat{W}_{k+1} = \vec{W}_{k+1} \end{cases} \tag{5}$$

where $\vec{W}_{t(i)-q(i)+1} = \hat{W}_{t(i)-q(i)+1}$, α_1 and α_2 are two positive tuning scalars, and $\tilde{\varphi}_f(\vec{x}_k) \triangleq \varphi_f(\vec{x}_k)/(\|1+\varphi_f^T(\vec{x}_k)\varphi_f(\vec{x}_k)\|\|C^TC\|)$.

Now, we are ready to consider the observer-based control strategy based on $\hat{x}_k$. The desired control input is calculated by minimizing $J(x_k)$ (i.e. $u_k = \arg\min\{J(x_k)\}$), where

$$J(x_k) \triangleq \sum_{j=k}^{\infty} l(x_j,u_j), \tag{6}$$

with the utility function $l(x_k,u_k) \triangleq x_k^TMx_k + u_k^TRu_k$, $l(0,0)=0$, and $l(x_k,u_k)\geq 0$ for any x_k and u_k. This paper aims to design a suboptimal control strategy to optimize (6). Unfortunately, such a minimization problem is quite difficult to solve since the value of x_k is unknown. An alternative method is to generate the desired control input by minimizing an approximated cost function $\hat{J}(\hat{x}_k)$.

According to the universal approximation property of the NN, it is easy to see that $J(x_k)$ can be approximated by an NN (namely, the critic NN):

$$J(x_k) = W_J^T\varphi_J(x_k) + \zeta_{J,k} \tag{7}$$

where W_J is the ideal weight, $\varphi_J(x_k)$ is the corresponding active function, and $\zeta_{J,k}$ is the bounded approximation error. Similarly, the ideal control input (i.e. $u_k = \arg\min\{J(x_k)\}$) can also be approximated by an NN (namely, the actor NN):

$$u(x_k) = W_u^T\varphi_u(x_k) + \zeta_{u,k} \tag{8}$$

where W_u is the ideal weight matrix for the actor NN, $\varphi_u(x_k)$ is the corresponding active function, and $\zeta_{u,k}$ is the bounded approximation error.

Since the plant state is inaccessible, the developed control strategy is based on state estimates $\hat{x}_k$. Accordingly, the approximated cost function $\hat{J}(\hat{x}_k)$ and control input are

$$\hat{J}(\hat{x}_k) = \hat{W}_{J,k}^T\varphi_J(\hat{x}_k) \tag{9}$$

and

$$\hat{u}(\hat{x}_k) = \hat{W}_{u,k}^T\varphi_u(\hat{x}_k) \tag{10}$$

where $\hat{W}_{J,k}$ and $\hat{W}_{u,k}$ denote the estimate of W_J and W_u, respectively. The detailed design procedure about the parameters $\hat{W}_{J,k}$ and $\hat{W}_{u,k}$ will be introduced in Section III-B.

Remark 2: In linear cases, HJB equations can be reduced to Riccati equations, which is straightforward to solve. However, for nonlinear systems, finding a solution to the HJB equation often proves challenging due to the presence of intricate nonlinearities within the system. In response, the ADP algorithm, leveraging the actor/critic NNs, has been introduced as an optimal control solution to for these nonlinear systems. A detailed description regarding control strategy design will be provided subsequently.

Before proceeding further, we shall introduce some performance requirements about exponential ultimate boundedness in mean square.

Definition 1: [48] The discrete nonlinear system (1) is said to be exponentially ultimately bounded (EUB) in mean square if there exist positive constants $\vartheta>0$, $0\leq\varrho<1$ and $\varsigma>0$ such that, for any solution x_k with the initial condition x_0, the following is true:

$$\mathbb{E}[\|x_k\|^2] \leq \vartheta\|x_0\|^2\varrho^k + \varsigma,\ k\geq 0$$

where ς is an asymptotic upper bound in mean square of (1).

The objectives are twofold.

1) Design the observer parameter $L_{h(i)}$ such that the estimation error (i.e. $x_k - \hat{x}_k$) is EUB in mean square.
2) Design the weight update laws and analyze ultimate boundedness.

III. Main Results

A. Observer Design

Utilizing a buffer-aided strategy, an NN-based observer will be constructed to address unreliable signal transmission scenarios. Since the suboptimal control strategy u_k is derived based on $\hat{x}_k$, the error dynamics proves crucial for achieving precise control. Subsequently, a joint analysis on the EUB of estimation errors for both state and NNW will be undertaken.

Defining $\tilde{W}_{f,k} \triangleq W_f - \hat{W}_{f,k}$ and $\check{W}_{f,k} \triangleq W_f - \vec{W}_{f,k}$ as the estimation error and the reorganized estimated error of nonlinear NNW, respectively, the error dynamics is

$$\begin{cases} \textbf{Case 1:} \text{ if } \{i|k=t(i), i\geq 0\}=\varnothing \\ \tilde{W}_{f,k+1} = (1-\alpha_1\alpha_2)\tilde{W}_{f,k} + \alpha_1\alpha_2W_f \\ \textbf{Case 2:} \text{ if } \{i|k=t(i), i\geq 0\}\neq\varnothing \\ \check{W}_{f,j+1} = (1-\alpha_1\alpha_2)\check{W}_{f,j} - \alpha_1C^TC\check{W}_{f,j}\varphi_f(\vec{x}_j)\tilde{\varphi}_f^T(\vec{x}_j) \\ \qquad -\alpha_1C^TD\omega_{j+1}\tilde{\varphi}_f^T(\vec{x}_j) - \alpha_1C^TC\check{\zeta}_j\tilde{\varphi}_f^T(\vec{x}_j) \\ \qquad +\alpha_1\alpha_2W_f - \alpha_1C^TC\bar{E}_{h(i)}\omega_j\tilde{\varphi}_f^T(\vec{x}_j) \\ \qquad -\alpha_1C^TC\bar{A}_{h(i)}\tilde{x}_j\tilde{\varphi}_f^T(\vec{x}_j), \\ \qquad\qquad t(i)-q(i)+1\leq j\leq t(i), \\ \tilde{W}_{f,k+1} = \check{W}_{f,k+1} \end{cases} \tag{11}$$

with $\tilde{W}_{t(i)-q(i)+1} = \check{W}_{t(i)-q(i)+1}$, $\bar{A}_{h(i)} \triangleq A - L_{h(i)}C$, $\bar{E}_{h(i)} \triangleq E - L_{h(i)}D$ and $\check{\zeta}_{f,k} \triangleq W\big(\varphi_f(x_k)-\varphi_f(\vec{x}_k)\big)+\zeta_{f,k}$.

Let the estimation error and reorganized estimated error be $\tilde{x}_k \triangleq x_k - \hat{x}_k$ and $\check{x}_k \triangleq x_k - \vec{x}_k$. The error dynamics is governed by

$$\begin{cases} \textbf{Case 1:} \text{ if } \{i|k=t(i), i\geq 0\}=\varnothing \\ \tilde{x}_{k+1} = A\tilde{x}_k + \tilde{W}_{f,k}\varphi_f(\hat{x}_k) + E\omega_k + \check{\zeta}_{f,k} \\ \textbf{Case 2:} \text{ if } \{i|k=t(i), i\geq 0\}\neq\varnothing \\ \check{x}_{j+1} = \bar{A}_{h(i)}\check{x}_j + \check{W}_{f,j}\varphi_f(\vec{x}_j) + \bar{E}_{h(i)}\omega_j + \check{\zeta}_{f,j}, \\ \qquad\qquad t(i)-q(i)+1\leq j\leq t(i), \\ \tilde{x}_{k+1} = \check{x}_{k+1} \end{cases} \tag{12}$$

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with $\breve{x}_{t(i)-q(i)+1} = \tilde{x}_{t(i)-q(i)+1}$.

Theorem 1: Let gain $L_{h(i)}$ be given. Assume that there exist scalars $\delta > 0$, $\mu_1 > 0$, $0 < \mu_2 < 1$, $0 < \alpha_i < 1$ $(i = 1, 2)$, $\sigma_s > 0$ $(s = 1, 2, 3, 4, 5)$, and positive definite matrices P, Φ_l $(l = 1, 2, \cdots, 7)$ such that the following conditions hold:

$$\begin{cases} \Pi_1 < 0 & (13) \\ \Pi_2 < 0 & (14) \\ \Xi_1 < 0 & (15) \\ \Xi_2 < 0 & (16) \end{cases}$$

$$C^T C P C^T C - \sigma_3 \|C^T C\|^2 P \le 0 \tag{17}$$

$$\sum_{s=M+1}^{H} \bar{p}_s (1+\mu_1)^{s-M} (1-\mu_2)^M + \sum_{s=1}^{M} \bar{p}_s (1-\mu_2)^s < 1 \tag{18}$$

$$\bar{p}_s \triangleq \begin{cases} p_s, & \text{if } s \in \mathcal{H}_a \\ 1 - \sum_{\iota \in \mathcal{H}_a} \bar{p}_\iota, & \text{if } s \in \mathcal{H}_b \end{cases} \tag{19}$$

where

$$\Pi_1 \triangleq \begin{bmatrix} \Pi_1^{11} & \Pi_1^{12} \\ * & \Pi_1^{22} \end{bmatrix},$$

$$\Pi_2 \triangleq \begin{bmatrix} \Pi_2^{11} & 0 & 0 \\ * & \Pi_2^{22} & 0 \\ * & * & \Pi_2^{33} \end{bmatrix},$$

$$\Xi_1 \triangleq \begin{bmatrix} \Xi_1^{11} & \Xi_1^{12} & 0 & \Xi_1^{14} \\ * & \Xi_1^{22} & 0 & \Xi_1^{24} \\ * & * & \Xi_1^{33} & 0 \\ * & * & * & \Xi_1^{44} \end{bmatrix},$$

$$\Xi_2 \triangleq \begin{bmatrix} \Xi_2^{11} & \Xi_2^{12} & 0 & \Xi_2^{14} \\ * & \Xi_2^{22} & 0 & \Xi_2^{24} \\ * & * & \Xi_2^{33} & 0 \\ * & * & * & \Xi_2^{44} \end{bmatrix},$$

$$\begin{aligned}
\varepsilon_1 &\triangleq \delta\tilde{\alpha}\bar{\varepsilon}, \bar{\varepsilon} \triangleq 1 - \alpha_1\alpha_2 + 4\alpha_1 + \alpha_1\sigma_4 + \alpha_1\alpha_2\sigma_5, \\
\varepsilon_4 &\triangleq 1/\|C^T C\|^2, \bar{\alpha} \triangleq 1 - \alpha_1\alpha_2 + 4\alpha_1, \varepsilon_5 = 1 + \alpha_1^2 \\
\varepsilon_6 &\triangleq \delta\alpha_1\sigma_3\sigma_4^{-1}\bar{\alpha} + 2\sigma_3\alpha_1^2, \varepsilon_2 \triangleq \varepsilon_3 \triangleq \delta\alpha_1\sigma_3\bar{\varepsilon}, \\
\varepsilon_7 &\triangleq \delta\alpha_1\alpha_2\sigma_5^{-1}\bar{\alpha} + 2\alpha_1^2\alpha_2^2, \\
\Pi_1^{11} &\triangleq \delta\big(\tilde{\alpha}^2 - (1+\mu_1)\big)P + \sigma_1\bar{\varphi}_f^2 I, \\
\Pi_1^{12} &\triangleq \delta\tilde{\alpha}\alpha_1\alpha_2 P, \Xi_1^{24} \triangleq P, \Xi_1^{12} \triangleq A^T P, \\
\Pi_1^{22} &\triangleq \delta\alpha_1^2\alpha_2^2 P - \Phi_3, \Xi_1^{11} \triangleq A^T P A - (1+\mu_1)P, \\
\Xi_1^{22} &\triangleq P - \sigma_1 I, \Xi_1^{33} \triangleq E^T P E - \Phi_1, \Xi_1^{44} \triangleq P - \Phi_2, \\
\Xi_1^{14} &\triangleq A^T P, \ \Pi_2^{22} \triangleq \varepsilon_4 D^T C P C^T D - \Phi_6, \\
\Pi_2^{11} &\triangleq \varepsilon_1 + \varepsilon_2\bar{\varphi}_f^2 - \delta(1-\mu_1)\big)P + \sigma_2\bar{\varphi}_f^2 I, \\
\Xi_2^{11} &\triangleq (1+\varepsilon_3)\bar{A}_{h(i)}^T P \bar{A}_{h(i)} - (1-\mu_2)P, \\
\Xi_2^{24} &\triangleq P, \Xi_2^{12} \triangleq \Xi_2^{14} \triangleq \bar{A}_{h(i)}^T P, \Pi_2^{33} \triangleq \varepsilon_7 P - \Phi_7, \\
\Xi_2^{44} &\triangleq (1+\varepsilon_6)P - \Phi_5, \Xi_2^{22} \triangleq P - \sigma_2 I, \\
\Xi_2^{33} &\triangleq \varepsilon_5 \bar{E}_{h(i)}^T P \bar{E}_{h(i)} - \Phi_4.
\end{aligned}$$

Then, both the error dynamics (11) and (12) are EUB in mean square subject to ω_k.

Proof: To begin with, we construct the following Lyapunov-like function:

$$V_k \triangleq V_{1,k} + V_{2,k} \tag{20}$$

where

$$V_{1,k} \triangleq \tilde{x}_k^T P \tilde{x}_k, \quad V_{2,k} \triangleq \delta\text{tr}\{\tilde{W}_{f,k}^T P \tilde{W}_{f,k}\}.$$

Since the observer has no measurement signal to utilize when $k \neq t(i)$, the error dynamics (11) and (12) would undergo an increment. Fortunately, at $t(i)$, the buffer signal packet would be transmitted to the observer. With the aid of the signal packet, the estimation value of system state and nonlinear NNW from $t(i) - q(i) + 2$ to $t(i)$ would be regenerated, and then those regenerated estimates would be utilized to generate the state estimate of $t(i) + 1$ (as seen in (4) and (5)). In this way, the increment would be compensated by the decrement, and the overall error dynamics (for both state and NNW estimation) would be EUB in mean square. Therefore, the following analysis of the error dynamics of state and NNW estimation is implemented based on (11) and (12). Consider *two cases*.

Case 1: $\{i|k = t(i), i \ge 0\} = \varnothing$

In this case, there exists a positive scalar i satisfying $t(i) < k \le t(i+1) - q(i)$. Denote ΔV_k as the difference between V_{k+1} and V_k, i.e.,

$$\Delta V_k = \sum_{r=1}^{2} \Delta V_{r,k} = \sum_{r=1}^{2} (V_{r,k+1} - V_{r,k}). \tag{21}$$

According to the estimation error dynamics (12), by calculating the mathematical expectation of $\mathbb{E}\{\Delta V_k - \mu_1 V_k\}$, we can easily obtain that

$$\begin{aligned}
&\mathbb{E}\{\Delta V_k - \mu_1 V_k\} \\
=&\mathbb{E}\{V_{1,k+1} - (1+\mu_1)V_{1,k} + V_{2,k+1} - (1+\mu_1)V_{2,k}\}
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
&\mathbb{E}\{V_{1,k+1} - (1+\mu_1)V_{1,k}\} \\
=&\mathbb{E}\Big\{2\tilde{x}_k^T A^T P \tilde{W}_{f,k}\varphi_f(\hat{x}_k) + 2\tilde{x}_k^T A^T P \check{\zeta}_{f,k} + \tilde{x}_k^T A^T P A \tilde{x}_k \\
&+ 2\varphi_f^T(\hat{x}_k)\tilde{W}_{f,k}^T P \check{\zeta}_{f,k} + \varphi_f^T(\hat{x}_k)\tilde{W}_{f,k}^T P \tilde{W}_{f,k}\varphi_f(\hat{x}_k) \\
&+ \omega_k^T \Phi_1 \omega_k + \omega_k^T(E^T P E - \Phi_1)\omega_k + \check{\zeta}_{f,k}^T \Phi_2 \check{\zeta}_{f,k} \\
&+ \check{\zeta}_{f,k}^T (P - \Phi_2)\check{\zeta}_{f,k} - (1+\mu_1)\tilde{x}_k^T P \tilde{x}_k\Big\}
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
&\mathbb{E}\{V_{2,k+1} - (1+\mu_1)V_{2,k}\} \\
=&\delta\text{tr}\Big\{\mathbb{E}\Big\{(1-\alpha_1\alpha_2)^2\tilde{W}_{f,k}^T P \tilde{W}_{f,k} + 2(1-\alpha_1\alpha_2) \\
&\times \alpha_1\alpha_2\tilde{W}_{f,k}^T P W_f - (1+\mu_1)\tilde{W}_k^T P \tilde{W}_{f,k} \\
&+ W_f^T(\alpha_1^2\alpha_2^2 P - \Phi_3)W_f + W_f^T \Phi_3 W_f\Big\}\Big\}.
\end{aligned} \tag{24}$$

Subsequently, by means of

$$\sigma_1\varphi_f^T(\hat{x}_k)\tilde{W}_{f,k}^T\tilde{W}_{f,k}\varphi_f(\hat{x}_k) - \sigma_1\bar{\varphi}_f^2\text{tr}\{\tilde{W}_{f,k}^T\tilde{W}_{f,k}\} \le 0 \tag{25}$$

and considering (22) to (25), we have

$$\mathbb{E}\{\Delta V_k - \mu_1 V_k\}$$

$$\begin{aligned}\le&\mathbb{E}\Big\{2\tilde{x}_k^T A^T P\tilde{W}_{f,k}\varphi_f(\hat{x}_k)+2\tilde{x}_k^T A^T P\check{\zeta}_{f,k}\\&+2\varphi_f^T(\hat{x}_k)\tilde{W}_{f,k}^T P\check{\zeta}_{f,k}+\tilde{x}_k^T A^T PA\tilde{x}_k\\&+\varphi_f^T(\hat{x}_k)\tilde{W}_{f,k}^T P\tilde{W}_{f,k}\varphi_f(\hat{x}_k)+\check{\zeta}_{f,k}^T\Phi_2\check{\zeta}_{f,k}\\&+\omega_k^T\Phi_1\omega_k+\omega_k^T(E^TPE-\Phi_1)\omega_k\\&+\check{\zeta}_{f,k}^T(P-\Phi_2)\check{\zeta}_{f,k}-(1+\mu_1)\tilde{x}_k^TP\tilde{x}_k\\&+\sigma_1\bar{\varphi}_f^2\text{tr}\{\tilde{W}_{f,k}^T\tilde{W}_{f,k}\}+\delta\text{tr}\{\tilde{\alpha}^2\tilde{W}_{f,k}^TP\tilde{W}_{f,k}\\&+2\tilde{\alpha}\alpha_1\alpha_2\tilde{W}_{f,k}^TPW_f-(1+\mu_1)\tilde{W}_{f,k}^TP\tilde{W}_{f,k}\\&+W_f^T(\delta\alpha_1^2\alpha_2^2P-\Phi_3)W_f+W_f^T\Phi_3W_f\}\Big\}\\\le&\mathbb{E}\{\gamma_k^T\Pi_1\gamma_k+\eta_k^T\Xi_1\eta_k\}+d_1\end{aligned}\tag{26}$$

where

$$\begin{aligned}\gamma_k&\triangleq\begin{bmatrix}\tilde{W}_{f,k}^T & W_f^T\end{bmatrix}^T,\\\eta_k&\triangleq\begin{bmatrix}\tilde{x}_k^T & \varphi^T(\hat{x}_k)\tilde{W}_{f,k}^T & \omega_k{}^T & \check{\zeta}_k^T\end{bmatrix}^T,\\d_1&\triangleq\text{tr}\{\tilde{Q}^T\Phi_1\tilde{Q}+\Phi_2\tilde{\zeta}^2+\Phi_3\bar{W}_f^2\},\\\tilde{\zeta}&\triangleq2\bar{W}_f\bar{\varphi}_f+\bar{\zeta}_f,\ \tilde{\alpha}\triangleq1-\alpha_1\alpha_2.\end{aligned}$$

Taking (13), (15) and (26) into account, we arrive at

$$\mathbb{E}\{\Delta V_k-\mu_1V_k\}\le\gamma_k^T\Pi_1\gamma_k+\eta_k^T\Xi_1\eta_k+d_1\le d_1.\tag{27}$$

Therefore, for any $t(i)+1\le k<t(i+1)-q(i+1)+1$ and positive scalar π_1, we have

$$\begin{aligned}&\pi_1^{k+1}V_{k+1}-\pi_1^kV_k=\pi_1^{k+1}(V_{k+1}-V_k)+\pi_1^k(\pi_1-1)V_k\\\le&\pi_1^k(\pi_1+\mu_1\pi_1-1)V_k+\pi_1^{k+1}d_1.\end{aligned}\tag{28}$$

Defining $\bar{\pi}_1\triangleq1/(1+\mu_1)$ and calculating the sum of both sides of (28) from $t(i)+1$ to $t(i+1)-q(i+1)+1$ with respect to k, we have

$$\begin{aligned}&\bar{\pi}_1^{t(i+1)-q(i+1)+1}V_{t(i+1)-q(i+1)+1}-\bar{\pi}_1^{t(i)+1}V_{t(i)+1}\\\le&d_1\sum_{\phi=t(i)+2}^{t(i+1)-q(i+1)+1}\bar{\pi}_1^{\phi}=d_1\frac{\bar{\pi}_1^{t(i)+2}-\bar{\pi}_1^{t(i+1)-q(i+1)+2}}{1-\bar{\pi}_1},\end{aligned}$$

which implies

$$V_{t(i+1)-q(i+1)+1}\le\bar{\pi}_1^{q(i+1)-h(i+1)}V_{t(i)+1}+\bar{d}_1\tag{29}$$

where $\bar{d}_1\triangleq d_1\frac{\bar{\pi}_1^{q(i+1)-h(i+1)+1}-\bar{\pi}_1}{1-\bar{\pi}_1}$.

Case 2: $\{i|k=t(i),i\ge0\}\ne\varnothing$

In this case, there exists a positive scalar i such that $k=t(i+1)$. Furthermore, under the effects of buffer-aided strategy, the available measurement signals (i.e., $\mathcal{Y}_{t(i+1)}=\{y_{t(i+1)},y_{t(i+1)-1},\cdots,y_{t(i+1)-q(i+1)+1}\}$) are utilized to facilitate the state estimation process, where the reorganized estimated states and NNWs are acquired (as shown in (4) and (5)). Then, the desired state estimate $\hat{x}_{t(i+1)+1}$ is generated based on the reorganized estimated states.

For $t(i+1)-q(i+1)+1\le j<t(i+1)+1$, letting $\check{V}_j\triangleq\check{V}_{1,j}+\check{V}_{2,j}\triangleq\check{x}_j^TP\check{x}_j+\delta\text{tr}\{\check{W}_{f,j}^TP\check{W}_{f,j}\}$ and calculating the mathematical expectation of $\mathbb{E}\{\check{V}_{j+1}-\check{V}_j\}$, we have

$$\mathbb{E}\{\Delta\check{V}_j\}=\mathbb{E}\{\check{V}_{1,j+1}+\check{V}_{2,j+1}-\check{V}_{1,j}-\check{V}_{2,j}\}\tag{30}$$

where

$$\begin{aligned}&\mathbb{E}\{\check{V}_{1,j+1}-\check{V}_{1,j}\}\\=&\mathbb{E}\Big\{2\check{x}_j^T\bar{A}_{h(i)}^TP\check{W}_{f,j}\varphi_f(\vec{x}_j)+2\check{x}_j^T\bar{A}_{h(i)}^TP\check{\zeta}_{f,j}+\check{x}_j^T\bar{A}_{h(i)}^T\\&\times PA_{h(i)}\check{x}_j+2\varphi_f^T(\vec{x}_j)\check{W}_{f,j}^TP\check{\zeta}_{f,j}+\varphi_f^T(\vec{x}_j)\check{W}_{f,j}^TP\check{W}_{f,j}\\&\times\varphi_f(\vec{x}_j)+2\omega_j^T\bar{E}_{h(i)}^TP\check{\zeta}_{f,j}+\omega_j^T(\bar{E}_{h(i)}^TP\bar{E}_{h(i)}-\Phi_4)\omega_j\\&+\check{\zeta}_{f,j}^T(P-\Phi_5)\check{\zeta}_{f,j}+\omega_j^T\Phi_4\omega_j+\check{\zeta}_{f,j}^T\Phi_5\check{\zeta}_{f,j}\\&-(1-\mu_2)\check{x}_j^TP\check{x}_j-\mu_2\check{x}_j^TP\check{x}_j\Big\}\end{aligned}\tag{31}$$

and

$$\begin{aligned}&\mathbb{E}\{\check{V}_{2,j+1}-\check{V}_{2,j}\}\\=&\delta\text{tr}\Big\{\mathbb{E}\Big\{\big(\tilde{\alpha}\check{W}_{f,j}-\alpha_1C^TC\check{W}_{f,j}\varphi_f(\vec{x}_j)\tilde{\varphi}_f^T(\vec{x}_j)\\&-\alpha_1C^TC\bar{A}_{h(i)}\check{x}_j\tilde{\varphi}_f^T(\vec{x}_j)-\alpha_1C^TD\omega_{j+1}\tilde{\varphi}_f^T(\vec{x}_j)\\&-\alpha_1C^TC\bar{E}_{h(i)}\omega_j\tilde{\varphi}_f^T(\vec{x}_j)-\alpha_1C^TC\check{\zeta}_j\tilde{\varphi}_f^T(\vec{x}_j)\\&+\alpha_1\alpha_2W_f\big)P\big(\tilde{\alpha}\check{W}_{f,j}-\alpha_1C^TC\check{W}_{f,j}\varphi_f(\vec{x}_j)\tilde{\varphi}_f^T(\vec{x}_j)\\&-\alpha_1C^TC\bar{A}_{h(i)}\check{x}_j\tilde{\varphi}_f^T(\vec{x}_j)-\alpha_1C^TD\omega_{j+1}\tilde{\varphi}_f^T(\vec{x}_j)\\&-\alpha_1C^TC\bar{E}_{h(i)}\omega_j\tilde{\varphi}_f^T(\vec{x}_j)-\alpha_1C^TC\check{\zeta}_j\tilde{\varphi}_f^T(\vec{x}_j)\\&+\alpha_1\alpha_2W_f\big)-(1-\mu_2+\mu_2)\check{W}_j^TP\check{W}_j\Big\}\Big\}.\end{aligned}\tag{32}$$

Furthermore, by means of $C^TCPC^TC\le\sigma_3\|C^TC\|^2P$, (32) can be calculated as

$$\begin{aligned}&\mathbb{E}\{\check{V}_{2,j+1}-\check{V}_{2,j}\}\\\le&\delta\text{tr}\Big\{\mathbb{E}\Big\{\varepsilon_1\check{W}_{f,j}^TP\check{W}_{f,j}+\varepsilon_2\bar{\varphi}_f^2\check{W}_{f,j}^TP\check{W}_{f,j}+\varepsilon_3\check{x}_j^T\\&\times\bar{A}_{h(i)}^TP\bar{A}_{h(i)}\check{x}_j+\varepsilon_4\omega_{j+1}^TD^TCPC^TD\omega_{j+1}\\&+\varepsilon_5\omega_j^T\bar{E}_{h(i)}^TP\bar{E}_{h(i)}\omega_j+\varepsilon_6\check{\zeta}_j{}^TP\check{\zeta}_j+\varepsilon_7W_f^TP\\&\times W_f-(1-\mu_2)\check{W}_{f,j}^TP\check{W}_{f,j}-\mu_2\check{W}_{f,j}^TP\check{W}_{f,j}\Big\}\Big\}.\end{aligned}\tag{33}$$

Afterwards, with the help of the inequity

$$\sigma_2\varphi_f^T(\vec{x}_j)\check{W}_{f,j}^T\check{W}_{f,j}\varphi_f(\vec{x}_j)-\sigma_2\bar{\varphi}_f^2\text{tr}\{\check{W}_{f,j}^T\check{W}_{f,j}\}\le0,\tag{34}$$

we substitute (33) and (31) into (30) to obtain

$$\begin{aligned}&\mathbb{E}\{\Delta\check{V}_j\}\\\le&\mathbb{E}\Big\{2\check{x}_j^T\bar{A}_{h(i)}^TP\check{W}_{f,j}\varphi_f(\vec{x}_j)+2\check{x}_j^T\bar{A}_{h(i)}^TP\check{\zeta}_{f,j}\\&+(1+\varepsilon_3)\check{x}_j^T\bar{A}_{h(i)}^TP\bar{A}_{h(i)}\check{x}_j+2\varphi_f^T(\vec{x}_j)\\&\times\check{W}_{f,j}^TP\check{\zeta}_{f,j}+\varphi_f^T(\vec{x}_j)\check{W}_{f,j}^TP\check{W}_{f,j}\varphi_f(\vec{x}_j)\\&-(1-\mu_2)\check{x}_j^TP\check{x}_j-\mu_2\check{x}_j^TP\check{x}_j-\sigma_2\varphi_f^T(\vec{x}_j)\\&\times\check{W}_{f,j}^T\check{W}_{f,j}\varphi_f(\vec{x}_j)+\omega_j^T\big((1+\varepsilon_5)\bar{E}_{h(i)}^T\\&\times P\bar{E}_{h(i)}-\Phi_4\big)\omega_j+\omega_j^T\Phi_4\omega_j+\check{\zeta}_{f,j}^T\Phi_5\check{\zeta}_{f,j}\\&+\check{\zeta}_{f,j}^T\big((1+\varepsilon_6)P-\Phi_5\big)\check{\zeta}_{f,j}+\delta\text{tr}\{\varepsilon_1\check{W}_{f,j}^TP\\&\times\check{W}_{f,j}+\delta^{-1}\sigma_2\bar{\varphi}_f^2\check{W}_{f,j}^T\check{W}_{f,j}+\varepsilon_2\bar{\varphi}_f^2\check{W}_{f,j}^TP\\&\times\check{W}_{f,j}+\varepsilon_4\omega_{j+1}^TD^TCPC^TD\omega_{j+1}+\varepsilon_7W_f^TP\\&\times W_f-(1-\mu_2)\check{W}_{f,j}^TP\check{W}_{f,j}-\mu_2\check{W}_{f,j}^TP\check{W}_{f,j}\}\Big\}\\\le&\mathbb{E}\{\bar{\gamma}_j^T\Pi_2\bar{\gamma}_j+\eta_j^T\Xi_2\eta_j-\mu_2\check{V}_j\}+d_2\end{aligned}\tag{35}$$

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where $\bar{\gamma}_j \triangleq \begin{bmatrix}\check{W}_{f,j}^T & \omega_{j+1}^T & W_f^T\end{bmatrix}^T$, $d_2 \triangleq \mathrm{tr}\{\tilde{Q}^T(\Phi_4+\Phi_6)\tilde{Q}+\Phi_5\tilde{\zeta}^2+\Phi_7\bar{W}_f^2\}$.

It can be observed from (14), (16) and (35) that

$$\begin{aligned}\mathbb{E}\{\Delta\check{V}_j\} \leq & \bar{\gamma}_j^T\Pi_2\bar{\gamma}_j + \eta_j^T\Xi_2\eta_j - \mu_2\mathbb{E}\{\check{V}_j\} + d_2 \\ \leq & -\mu_2\mathbb{E}\{\check{V}_j\} + d_2.\end{aligned}$$

Obviously, for any $t(i+1)-q(i+1)+1 \leq k < t(i+1)$ and positive scalar μ_2, one has

$$\begin{aligned}&\bar{\mu}_2^{j+1}\check{V}_{j+1} - \bar{\mu}_2^j\check{V}_j \\ =&\bar{\mu}_2^{j+1}(\check{V}_{j+1}-\check{V}_j) + \bar{\mu}_2^j(\bar{\mu}_2-1)\check{V}_j \\ \leq&\bar{\mu}_2^j(\bar{\mu}_2-\bar{\mu}_2\mu_2-1)\check{V}_j + \bar{\mu}_2^{j+1}d_2.\end{aligned} \tag{36}$$

Denoting $\bar{\mu}_2 \triangleq 1/(1-\mu_2)$ and calculating the summation in (36) from $t(i+1)-q(i+1)+1$ to $t(i+1)+1$ in respect to k, one has

$$\begin{aligned}&\bar{\mu}_2^{t(i+1)+1}\check{V}_{t(i+1)+1} - \bar{\mu}_2^{t(i+1)-q(i+1)+1}\check{V}_{t(i+1)-q(i+1)+1} \\ \leq& d_2\sum_{\phi=t(i+1)-q(i+1)+2}^{t(i+1)+1}\bar{\mu}_2^{\phi} = d_2\frac{\bar{\mu}_2^{t(i+1)-q(i+1)+2}-\bar{\mu}_2^{t(i+1)+2}}{1-\bar{\mu}_2}.\end{aligned}$$

Furthermore, it is obvious that $V_{t(i+1)+1} = \check{V}_{t(i+1)+1}$ and $V_{t(i+1)-q(i+1)+1} = \check{V}_{t(i+1)-q(i+1)+1}$. Then, we have

$$V_{t(i+1)+1} \leq \bar{\mu}_2^{-q(i+1)}V_{t(i+1)-q(i+1)+1} + \bar{d}_2 \tag{37}$$

where $\bar{d}_2 \triangleq d_2\frac{\bar{\mu}_2^{-q(i+1)+1}-\bar{\mu}_2}{1-\bar{\mu}_2}$.

Aggregation of Case 1 and Case 2

We now aggregate the results obtained in the analysis of *Case 1* and *Case 2*. In the following part of this subsection, we will show that the EUB of the error dynamics (11) and (12) can be simultaneously guaranteed. To this end, it is easily obtained from (29) and (37) that

$$V_{t(i+1)+1} \leq \tilde{\mu}V_{t(i)+1} + \bar{\mu}_2^{-q(i+1)}\bar{d}_1 + \bar{d}_2 \tag{38}$$

where $\tilde{\mu} \triangleq \bar{\mu}_1^{q(i+1)-h(i+1)}\bar{\mu}_2^{-q(i+1)}$.

Considering (19), we have from $0 \leq p_n \leq 1-\sum_{\iota\in\mathcal{H}_a}p_\iota$ that

$$\begin{aligned}&\mathbb{E}\{\bar{\mu}_1^{q(i+1)-h(i+1)}\bar{\mu}_2^{-q(i+1)}\} \\ =&\sum_{s=1}^{M}p_s\bar{\mu}_1^{s-s}\bar{\mu}_2^{-s} + \sum_{s=M+1}^{H}p_s\bar{\mu}_1^{M-s}\bar{\mu}_2^{-M} \\ \leq&\sum_{s=M+1}^{H}\bar{p}_s(1+\mu_1)^{s-M}(1-\mu_2)^M + \sum_{s=1}^{M}\bar{p}_s(1-\mu_2)^s \triangleq \hat{\mu},\end{aligned} \tag{39}$$

and

$$\begin{aligned}&\mathbb{E}\{\bar{\mu}_2^{-q(i+1)}\bar{d}_1 + \bar{d}_2\} \\ =&\mathbb{E}\left\{\bar{\mu}_2^{-q(i+1)}d_1\frac{\bar{\mu}_1^{q(i+1)-h(i+1)+1}-\bar{\mu}_1}{1-\bar{\mu}_1} + d_2\frac{\bar{\mu}_2^{-q(i+1)+1}-\bar{\mu}_2}{1-\bar{\mu}_2}\right\} \\ \leq&\sum_{s=M+1}^{H}\bar{p}_s\left(\bar{\mu}_2^{-M}d_1\frac{\bar{\mu}_1^{M-s+1}-\bar{\mu}_1}{1-\bar{\mu}_1} + d_2\frac{\bar{\mu}_2^{-M+1}-\bar{\mu}_2}{1-\bar{\mu}_2}\right) \\ &+\sum_{s=1}^{M}\bar{p}_s\left(\bar{\mu}_2^{-s}d_1d_2\frac{\bar{\mu}_2^{-s+1}-\bar{\mu}_2}{1-\bar{\mu}_2}\right) \triangleq \hat{d}.\end{aligned} \tag{40}$$

Then, calculate the conditional expectation of (38), and from (39) and (40), we have

$$\mathbb{E}\{V_{t(i+1)+1}|t(i),\tilde{x}_{t(i)}\} \leq \hat{\mu}\mathbb{E}\{V_{t(i)+1}|t(i),\tilde{x}_{t(i)}\} + \hat{d}. \tag{41}$$

Take the mathematical expectation of (41).

$$\mathbb{E}\{V_{t(i+1)+1}\} \leq \hat{\mu}\mathbb{E}\{V_{t(i)+1}\} + \hat{d}. \tag{42}$$

Next, for any positive scalar $\bar{\mu}$, one has

$$\begin{aligned}&\bar{\mu}^{m+1}\mathbb{E}\{V_{t(i+1)+1}\} - \bar{\mu}^m\mathbb{E}\{V_{t(i)+1}\} \\ \leq&\bar{\mu}^m(\bar{\mu}-\bar{\mu}(1-\hat{\mu})-1)\mathbb{E}\{V_{t(i)+1}\} + \bar{\mu}^{m+1}\hat{d}.\end{aligned} \tag{43}$$

Subsequently, denoting $\bar{\mu} = 1/\hat{\mu}$ and summing up (43) from $t(0)+1$ to $t(z)+1$ in respect to z, one has

$$\bar{\mu}^z\mathbb{E}\{V_{t(0)}\} - \mathbb{E}\{V_{t(0)+1}\} \leq \hat{d}\frac{\bar{\mu}-\bar{\mu}^{z+1}}{1-\bar{\mu}}$$

which results in

$$\begin{aligned}\mathbb{E}\{V_{t(z)+1}\} \leq& \hat{\mu}^z\mathbb{E}\{V_{t(0)+1}\} + \hat{d}\frac{1-\hat{\mu}^z}{1-\hat{\mu}} \\ \leq&\hat{\mu}^z(1-\mu_2)\mathbb{E}\{V_{t(0)}\} + \hat{\mu}^z d_2 - \frac{\hat{\mu}^z\hat{d}}{1-\hat{\mu}} + \frac{\hat{d}}{1-\hat{\mu}}.\end{aligned}$$

Therefore, $\mathbb{E}\{V_{t(z)+1}\}$ is ultimately bounded, i.e.,

$$\lim_{z\to+\infty}\mathbb{E}\{V_{t(z)+1}\} = \frac{\hat{d}}{1-\hat{\mu}} < +\infty.$$

Then, for any $t(z)+1 \leq k < t(z+1)+1$, one has $\mathbb{E}\{V_k\} \leq \mathbb{E}\{V_{t(z)+H}\}$, and

$$\begin{aligned}\mathbb{E}\{V_k\} \leq&\mathbb{E}\{V_{t(z)+H}\} \\ \leq&\hat{\mu}^z\bar{\mu}_1^{1-H}(1-\mu_2)V_0 + \tilde{d} + \hat{\mu}^z\bar{\mu}_1^{1-H}\big(d_2 - \frac{\hat{d}}{1-\hat{\mu}}\big)\end{aligned} \tag{44}$$

where $\tilde{d} \triangleq \frac{\hat{d}\bar{\mu}_1^{1-H}}{1-\hat{\mu}} + \frac{d_1(\bar{\mu}_1-\bar{\mu}_1^{2-H})}{\bar{\mu}_1-1}$. Finally,

$$\lim_{k\to+\infty}\mathbb{E}\{\|\tilde{x}_k\|^2\} \leq \frac{\tilde{d}}{\lambda_{\min}(P)}$$

which ends the proof. ■

Theorem 2: For the error dynamics (11) and (12), assume that there exist scalars $\delta > 0$, $\mu_1 > 0$, $0 < \mu_2 < 1$, $0 < \alpha_i < 1$ $(i = 1, 2)$, $\sigma_s > 0$ $(s = 1, 2, 3, 4, 5)$, positive definite matrices P, Φ_l $(l = 1, 2, \cdots, 7)$, and observer gain matrix $L_{h(i)}$ satisfying (13), (14), (15), (17), (18), (19) and the following matrix inequality

$$\tilde{\Xi}_2 < 0 \tag{45}$$

where

$$\tilde{\Xi}_2 \triangleq \begin{bmatrix}\tilde{\Xi}_2^{11} & 0 & 0 & 0 & \tilde{\Xi}_2^{15} & \tilde{\Xi}_2^{16} \\ * & \tilde{\Xi}_2^{22} & 0 & 0 & \tilde{\Xi}_2^{25} & 0 \\ * & * & \tilde{\Xi}_2^{33} & 0 & 0 & \tilde{\Xi}_2^{36} \\ * & * & * & \tilde{\Xi}_2^{44} & 0 & 0 \\ * & * & * & * & \tilde{\Xi}_2^{55} & 0 \\ * & * & * & * & * & \tilde{\Xi}_2^{66}\end{bmatrix},$$

$\tilde{\Xi}_2^{11} \triangleq -(1-\mu_2)P$, $\tilde{\Xi}_2^{15} \triangleq A^T - C^TL_{h(i)}^T$,
$\tilde{\Xi}_2^{16} \triangleq \left[\sqrt{1+\varepsilon_3}(A^T - C^TL_{h(i)}^T) \quad 0\right]$, $\tilde{\Xi}_2^{33} \triangleq -\Phi_4$,

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$$\tilde{\Xi}_2^{25} \triangleq I,\ \tilde{\Xi}_2^{36} \triangleq \left[0 \quad \sqrt{1+\varepsilon_5}(E^T - D^T L_{h(i)}^T)\right],$$
$$\tilde{\Xi}_2^{44} \triangleq (1+\varepsilon_6)P - \Phi_5, \tilde{\Xi}_2^{55} \triangleq P - 2I,$$
$$\tilde{\Xi}_2^{22} \triangleq -\sigma_2 I, \tilde{\Xi}_2^{66} \triangleq \text{diag}\{P-2I, P-2I\},$$

where $\varepsilon_s (s = 3,5,6)$ are defined in Theorem 1. Then, (11) and (12) are EUB in mean square subject to ω_k.

Proof: The proof follows from Theorem 1 and Schur Complement Lemma. ■

Remark 3: Because $\mu_1 > 0$ and $0 < \mu_2 < 1$, error dynamics undergoes an increment since the observer has no measurement signal to utilize when implementing the observation task. Fortunately, during $t(i) - q(i) + 1 \leq k < t(i+1) + 1$, a decrement would be utilized to compensate the increment. In this way, the EUB of the error dynamics (11) and (12) can be jointly guaranteed.

B. Controller design

In this subsection, we design controller parameters. Furthermore, the error dynamics EUB about actor/critic-NNWs will be simultaneously analyzed.

With the help of the Bellman's principle of optimality, $J(x_k)$ can be rewritten as

$$\begin{aligned} J(x_k) =& l(x_k, u_k) + \sum_{j=k+1}^{\infty} l(x_j, u_j) \\ =& l(x_k, u_k) + J(x_{k+1}). \end{aligned} \tag{46}$$

Considering the approximation of $J(x_k)$ shown in (7), let $Z_J(W_J) \triangleq J(\hat{x}_k) - J(x_k)$ be the residual error produced during the approximation process of critic NN. Then, we have

$$\begin{aligned} Z_J(W_J) &= l\big(\hat{x}_k, \hat{u}(\hat{x}_k)\big) + J(\hat{x}_{k+1}) - J(x_k) \\ &\approx l\big(\hat{x}_k, \hat{u}(\hat{x}_k)\big) + W_J^T \Delta\varphi_J(\hat{x}_k) \end{aligned}$$

where $\Delta\varphi_J(x_k) \triangleq \varphi_J(\hat{x}_{k+1}) - \varphi_J(x_k)$. By minimizing $\frac{1}{2} Z_J^T(W_J) Z_J(W_J)$, we obtain the update law of $\hat{W}_{J,k}$ for critic NNs based on gradient descent.

$$\hat{W}_{J,k+1} = \hat{W}_{J,k} - \beta_1 \Delta\varphi_J(\hat{x}_k) r_J(\hat{x}_k) Z_J^T(\hat{W}_{J,k}) \tag{47}$$

where β_1 is the tuning scalar of the update law and $r_J(\hat{x}_k)$ is the step length used to adjust the updated amplitude, $r_J(\hat{x}_k) = 1/\big(1 + \|\Delta\varphi_J(\hat{x}_k)^T \Delta\varphi_J(\hat{x}_k)\|\big)$.

Next, we are in a position to design the weight update law. Based on (10) and the Bellman's principle of optimality, one desired "optimal" control policy is governed by

$$\frac{\partial l\big(\hat{x}_k, \hat{u}(\hat{x}_k)\big)}{\partial \hat{u}(\hat{x}_k)} = -\frac{\partial J(\hat{x}_{k+1})}{\partial \hat{u}(\hat{x}_k)}.$$

Define $g\big(\hat{u}(\hat{x}_k)\big)$ as the derivative function of $l\big(\hat{x}_k, \hat{u}(\hat{x}_k)\big)$ which is invertible, i.e., $g\big(\hat{u}(\hat{x}_k)\big) \triangleq \partial l\big(\hat{x}_k, \hat{u}(\hat{x}_k)\big)/\partial \hat{u}(\hat{x}_k)$. As shown in [8], the approximated value of $u(\hat{x}_k)$ (i.e. $\mathcal{U}(\hat{x}_k)$) is calculated based on an inverse function of $g\big(\hat{u}(\hat{x}_k)\big)$:

$$\begin{aligned} \mathcal{U}(\hat{x}_k) =& g^{-1}\Big(\frac{\partial l\big(\hat{x}_k, \hat{u}(\hat{x}_k)\big)}{\partial \hat{u}(\hat{x}_k)}\Big) \\ =& -\frac{1}{2} R^{-1} B^T \nabla\varphi_J^T(\hat{x}_{k+1}) \hat{W}_{J,k} \end{aligned}$$

where $\nabla\varphi_J^T(\hat{x}_{k+1})$ represents the gradient operation of $\varphi_J^T(\hat{x}_{k+1})$. Let $Z_u(\hat{x}_k)$ be the control input error represented by

$$\begin{aligned} Z_u(\hat{x}_k) =& \hat{u}(\hat{x}_k) - \mathcal{U}(\hat{x}_k) \\ =& \hat{W}_{u,k}^T \varphi_u(\hat{x}_k) + \frac{1}{2} R^{-1} B^T \nabla\varphi_J^T(\hat{x}_{k+1}) \hat{W}_{J,k}. \end{aligned}$$

Similarly, using gradient descent, we obtain a weight update law by minimizing $\frac{1}{2} Z_u^T(\hat{x}_k) Z_u(\hat{x}_k)$, i.e.,

$$\hat{W}_{u,k+1} = \hat{W}_{u,k} - \beta_2 \varphi_u(\hat{x}_k) Z_u^T(\hat{x}_k). \tag{48}$$

Define $\tilde{W}_{J,k} = \hat{W}_{J,k} - W_J$ as the estimation error of critic-NNW.

$$\begin{aligned} \tilde{W}_{J,k+1} =& \hat{W}_{J,k+1} - W_J \\ =& \tilde{W}_{J,k} - \beta_1 \Delta\varphi_J(\hat{x}_k) r_J(\hat{x}_k) Z_J^T(\hat{x}_k) \\ =& \tilde{W}_{J,k} - \beta_1 \Delta\varphi_J(\hat{x}_k) r_J(\hat{x}_k) \\ & \times \big(\Delta\varphi_J^T(\hat{x}_k) \tilde{W}_{J,k} + \Upsilon_J\big) \end{aligned} \tag{49}$$

where $\Upsilon_J \triangleq l^T\big(\hat{x}_k, \hat{u}(\hat{x}_k)\big) + \Delta\varphi_J^T(\hat{x}_k) W_J$.

Letting $\tilde{W}_{u,k} = \hat{W}_{u,k} - W_u$ be the estimation error of the actor-NNW, (48) indicates

$$\begin{aligned} \tilde{W}_{u,k+1} =& \hat{W}_{u,k+1} - W_u \\ =& \tilde{W}_{u,k} - \beta_2 \varphi_u(\hat{x}_k) Z_u^T(\hat{x}_k) \\ =& \tilde{W}_{u,k} - \beta_2 \varphi_u(\hat{x}_k) \varphi_u^T(\hat{x}_k) \tilde{W}_{u,k} - \beta_2 \Upsilon_u \\ & - \frac{1}{2} \beta_2 \varphi_u(\hat{x}_k) \tilde{W}_{J,k}^T \nabla\varphi_J(\hat{x}_{k+1}) B (R^{-1})^T \end{aligned} \tag{50}$$

where $\Upsilon_u \triangleq \frac{1}{2} \varphi_u(\hat{x}_k) \big(W_J^T \nabla\varphi_J(\hat{x}_{k+1}) B (R^{-1})^T + \varphi_u^T(\hat{x}_k) \times W_u\big)$.

The following theorem presents the selection scheme on the tuning scalars β_1 and β_2, which ensures that the error dynamics (49) and (50) are EUB in mean square.

Theorem 3: Let the initial control input (i.e. $\hat{u}_0(\hat{x}_k) \triangleq \hat{W}_{u,0}^T \varphi_u(\hat{x}_k)$) be admissible and the initial actor- and critic-NNW (i.e. $\hat{W}_{J,0}$ and $\hat{W}_{u,0}$) be selected from a compact set which includes the ideal weights. Assume that there exist scalars $\beta_1 > 0$, $\beta_2 > 0$, $0 < \mu_j < 1$ $(j = 3,4)$, $\sigma_s > 0$ $(s = 6,7,8,9)$ and positive matrices Γ_l $(l = 1,2,3)$ such that

$$\begin{cases} \Xi_3 < 0 & (51) \\ \Xi_4 < 0 & (52) \end{cases}$$

where

$$\Xi_3 \triangleq \begin{bmatrix} \Xi_3^{11} & 0 \\ * & \Xi_3^{22} \end{bmatrix}, \Xi_4 \triangleq \begin{bmatrix} \Xi_4^{11} & \Xi_4^{12} \\ * & \Xi_4^{22} \end{bmatrix},$$
$$\begin{aligned} \Xi_3^{11} \triangleq& -\big(\beta_1(2 - \sigma_6 - 4\beta_1\sigma_6\bar{\varphi}_J^2 - 4\beta_1\bar{\varphi}_J^2) - \mu_3\big) \\ & + \beta_2\bar{\varphi}_u^2\bar{\varphi}_J^2(\beta_2 + \sigma_7^{-1} + \sigma_8^{-1} + \sigma_9^{-1})\|R^{-1}B^T\|^2, \\ \Xi_3^{22} \triangleq& \beta_1(\sigma_6^{-1} + 4\beta_1\sigma_6^{-1}\bar{\varphi}_J^2 + 4\beta_1\bar{\varphi}_J^2) - \Gamma_1, \\ \Xi_4^{11} \triangleq& (-2\beta_2\bar{\varphi}_u^2 + \beta_2^2\bar{\varphi}_u^4 + \beta_2\sigma_7 + \beta_2\bar{\varphi}_u^2\sigma_8 + \mu_4), \\ \Xi_4^{12} \triangleq& -\beta_2 + \beta_2^2\bar{\varphi}_u^2, \Xi_4^{22} \triangleq \beta_2^2 + \beta_2^2\sigma_9 - \Gamma_2. \end{aligned}$$

Then, both estimation errors for critic/actor-NNWs are EUB in mean square.

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Proof: For the critic NN with update law (47) and the actor NN with update law (48), we construct Lyapunov functions

$$V_{3,k} \triangleq \text{tr}\{\tilde{W}_{J,k}^T \tilde{W}_{J,k}\}, \quad V_{4,k} \triangleq \text{tr}\{\tilde{W}_{u,k}^T \tilde{W}_{u,k}\}.$$

Taking the mathematical expectation along the trajectory of (49) and (50) leads to

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V_{3,k}|\hat{x}_k, \hat{W}_{u,k}, \hat{W}_{J,k}\Big\} \\
=&\mathbb{E}\Big\{V_{3,k+1}|\hat{x}_k, \hat{W}_{u,k}, \hat{W}_{J,k}\Big\} - V_{3,k} \\
\leq&\text{tr}\Big\{\mathbb{E}\Big\{-\big(\beta_1(2-\sigma_6-4\beta_1\sigma_6\bar{\varphi}_J^2-4\beta_1\bar{\varphi}_J^2)-\mu_3\big) \\
&\times \tilde{W}_{J,k}^T\tilde{W}_{J,k} - \mu_3\tilde{W}_{J,k}^T\tilde{W}_{J,k} + \big(\tilde{W}_{J,k}^T\Delta\varphi_J(\hat{x}_k)+\Upsilon_J^T\big) \\
&\times \Big(\beta_1(\sigma_6^{-1}+4\beta_1\sigma_6^{-1}\bar{\varphi}_J^2+4\beta_1\bar{\varphi}_J^2)-\Gamma_1\Big) \\
&\times \big(\Delta\varphi_J^T(\hat{x}_k)\tilde{W}_{J,k}+\Upsilon_J\big) + \big(\tilde{W}_{J,k}^T\Delta\varphi_J(\hat{x}_k)+\Upsilon_J^T\big) \\
&\times \Gamma_1\big(\Delta\varphi_J^T(\hat{x}_k)\tilde{W}_{J,k}+\Upsilon_J\big)\Big\}\Big\}
\end{aligned} \tag{53}$$

and

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V_{4,k}|\hat{x}_k, \hat{W}_{u,k}, \hat{W}_{J,k}\Big\} \\
=&\mathbb{E}\Big\{V_{4,k+1}|\hat{x}_k, \hat{W}_{u,k}, \hat{W}_{J,k}\Big\} - V_{4,k} \\
\leq&\text{tr}\Big\{\mathbb{E}\Big\{(-2\beta_2\bar{\varphi}_u^2+\beta_2^2\bar{\varphi}_u^4+\beta_2\sigma_7+\beta_2\bar{\varphi}_u^2\sigma_8+\mu_4)\tilde{W}_{u,k}^T \\
&\times \tilde{W}_{u,k} + 2(-\beta_2+\beta_2^2\bar{\varphi}_u^2)\tilde{W}_{u,k}^T\Upsilon_u + \Upsilon_u^T(\beta_2^2+\beta_2^2\sigma_9 \\
&-\Gamma_2)\Upsilon_u + \Upsilon_u^T\Gamma_2\Upsilon_u + \frac{1}{4}\beta_2\bar{\varphi}_u^2(\beta_2+\sigma_7^{-1}+\sigma_8^{-1}+\sigma_9^{-1}) \\
&\times R^{-1}B^T\nabla\varphi_J^T(\hat{x}_{k+1})\tilde{W}_{J,k}\tilde{W}_{J,k}^T\nabla\varphi_J(\hat{x}_{k+1})B(R^{-1})^T \\
&-\mu_4\tilde{W}_{u,k}^T\tilde{W}_{u,k}\Big\}\Big\}.
\end{aligned} \tag{54}$$

(53) and (54) indicate

$$\begin{aligned}
&\text{tr}\Big\{\mathbb{E}\Big\{\Delta V_{3,k}+\Delta V_{4,k}|\hat{x}_k, \tilde{W}_{u,k}, \tilde{W}_{J,k}\Big\}\Big\} \\
&\leq \text{tr}\Big\{\mathbb{E}\Big\{\xi_k^T\Xi_3\xi_k - \mu_3\tilde{W}_{J,k}^T\tilde{W}_{J,k} + d_3 + \bar{\xi}_k^T\Xi_4\bar{\xi}_k \\
&-\mu_4\tilde{W}_{u,k}^T\tilde{W}_{u,k} + d_4\Big\}\Big\}
\end{aligned}$$

where

$$\begin{aligned}
\xi_k \triangleq& \begin{bmatrix}\tilde{W}_{J,k}^T & \varrho_k^T\end{bmatrix}^T, \bar{\xi}_k \triangleq \begin{bmatrix}\tilde{W}_{u,k}^T & \Upsilon_u^T\end{bmatrix}^T, \\
\varrho_k \triangleq& l^T\big(\hat{x}_k, \hat{u}(\hat{x}_k)\big) + \Delta\varphi_J^T(\hat{x}_k)W_J, d_3 \triangleq \text{tr}\{3\bar{W}_J\bar{\varphi}_J\Gamma_1\}, \\
d_4 \triangleq& \text{tr}\{(\|R^{-1}B^T\|^2\bar{\varphi}_u^2\bar{\varphi}_J^2\bar{W}_J^2+\bar{\varphi}_u^4\bar{W}_u^2)\Gamma_2\}.
\end{aligned}$$

(51) and (52) indicate

$$\begin{aligned}
&\text{tr}\Big\{\mathbb{E}\Big\{\Delta V_{3,k}+\Delta V_{4,k}|\hat{x}_k, \tilde{W}_{u,k}, \tilde{W}_{J,k}\Big\}\Big\} \\
&\leq \text{tr}\big\{-\mu_3 V_{3,k} + d_3 - \mu_4\tilde{V}_{4,k} + d_4\big\},
\end{aligned} \tag{55}$$

and the proof is complete. ■

Remark 4: Utilizing the universal approximation property, (9) and (10) are used to suitably approximate (7) and (8), respectively. By this approach, the NN-based control algorithm can be realized. Furthermore, based on Lyapunov stability, the boundedness of both critic-NNW and actor-NNW is assured.

C. Boundedness Analysis for the Nonlinear NCSs

In this subsection, stability analysis will be conducted.

Theorem 4: Let the initial control input (i.e. $\hat{u}_0(\hat{x}_k) \triangleq \hat{W}_{u,0}^T\varphi_u(\hat{x}_k)$) be admissible and the initial actor- and critic-NN weights (i.e. $\hat{W}_{J,0}$ and $\hat{W}_{u,0}$) be selected from a compact set which includes the ideal weights. Suppose that there exist scalars $0 < \aleph < 1$, $\sigma_{10} > 0$, $0 < \mu_5 < 1$ and positive matrices Γ_l $(l = 4, 5)$ such that

$$\Pi_5 < 0 \tag{56}$$

where

$$\Pi_5 \triangleq \begin{bmatrix}\Pi_5^{11} & 0 & 0 \\ * & \Pi_5^{22} & 0 \\ * & * & \Pi_5^{33}\end{bmatrix},$$

$$\begin{aligned}
\Pi_5^{11} \triangleq& \aleph(1+2\sigma_{10})-1+\mu_5, \Pi_5^{22} \triangleq (2+\sigma_{10}^{-1})B^TB-\Gamma_3, \\
\Pi_5^{33} \triangleq& (2+\sigma_{10}^{-1})B^TB-\Gamma_4.
\end{aligned}$$

Then, system (1) with control policy (10) is EUB in mean square.

Proof: In light of the optimal control theory, (8) will stabilize (in the sense of input-to-state stability) the following system on a compact set [8]:

$$x_{k+1} = Ax_k + f(x_k) + Bu_k + E\omega_k = \Lambda(x_k) + E\omega_k$$

In other words, there exists a positive constant $\aleph < 1$ such that

$$\mathbb{E}\{\|\Lambda(x_k)\|^2\} \leq \aleph\mathbb{E}\{\|x_k\|^2\} + \|E\omega_k\|. \tag{57}$$

Considering the observer-based control framework, in view of (10), we have the following *actual closed-loop system*:

$$\begin{aligned}
x_{k+1} =& \Lambda(x_k) - BW_u^T\varphi_u(x_k) - B\zeta_{u,k} + B\hat{W}_{u,k}^T\varphi_u(\hat{x}_k) \\
=& \Lambda(x_k) - B\zeta_{u,k} - BW_u^T\grave{\varphi}_u(x_k) + B\tilde{W}_{u,k}^T\varphi_u(\hat{x}_k)
\end{aligned} \tag{58}$$

where $\grave{\varphi}_u(x_k) \triangleq \varphi_u(x_k) - \varphi_u(\hat{x}_k)$. Let us construct

$$V_{5,k} \triangleq \text{tr}\{x_k^Tx_k\}.$$

Seeking for the mathematical expectation implies

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V_{5,k}\Big\} \\
=&\mathbb{E}\Big\{V_{5,k+1}|\hat{x}_k, \hat{W}_{u,k}, \hat{W}_{J,k}\Big\} - V_{5,k} \\
\leq&\text{tr}\Big\{\mathbb{E}\Big\{\xi_k^T\Pi_5\xi_k + \big(\zeta_{u,k}+W_u^T\grave{\varphi}_u(x_k)\big)^T\Gamma_3 \\
&\times \big(\zeta_{u,k}+W_u^T\grave{\varphi}_u(x_k)\big)^T + \varphi_u^T(\hat{x}_k)\tilde{W}_{u,k} \\
&\times \Gamma_4\tilde{W}_{u,k}^T\varphi_u(\hat{x}_k) - \mu_5x_k^Tx_k + \tilde{Q}^T\tilde{Q}\Big\}\Big\} \\
\leq&\text{tr}\Big\{\mathbb{E}\Big\{\tilde{\xi}_k^T\Pi_5\tilde{\xi}_k - \mu_5x_k^Tx_k + d_5\Big\}\Big\}
\end{aligned} \tag{59}$$

where $\tilde{\xi}_k \triangleq \begin{bmatrix}x_k^T & \zeta_{u,k}^T+\grave{\varphi}_u^T(x_k)W_u & \tilde{W}_{u,k}^T\end{bmatrix}^T$, $d_5 \triangleq \text{tr}\{(\bar{\zeta}_u + 2\bar{W}_u\bar{\varphi}_u)^T\Gamma_3(\bar{\zeta}_u+2\bar{W}_u\bar{\varphi}_u) + \bar{\varphi}_u^2\Gamma_4(d_3+d_4) + \tilde{Q}^T\tilde{Q}\}$.

Taking (56) into consideration, it follows from (59) that

$$\mathbb{E}\{\Delta V_{5,k}\} \leq -\mu_5\mathbb{E}\{V_{5,k}\} + d_5. \tag{60}$$

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Now, let us consider (55) and (60). It is obvious that

$$\sum_{r=3}^{5}\mathbb{E}\{V_{r,k}\} \leq \sum_{r=3}^{5}(\bar{\mu}_r V_{r,k-1} + d_r) \leq \sum_{r=3}^{5}\left(\bar{\mu}_r^k V_{r,0} + d_r\frac{1-\bar{\mu}_r^k}{1-\bar{\mu}_r}\right) \quad (61)$$

where $\bar{\mu}_r \triangleq 1 - \mu_r$.

By constructing the following Lyapunov-like function

$$\mathcal{V}_k \triangleq \sum_{r=1}^{5} V_{r,k}$$

and considering (44) and (61), we have

$$\lim_{k\to+\infty}\mathbb{E}\{\mathcal{V}_k\} <\tilde{d} + d_3\frac{1}{1-\bar{\mu}_3} + d_4\frac{1}{1-\bar{\mu}_4} + d_5\frac{1}{1-\bar{\mu}_5} < +\infty.$$

■

Remark 5: In Theorems 1-4, we have explored the ultimately bounded output-feedback control for nonlinear NCSs by a buffer-aided strategy amidst inconsistent communication channels. Specifically, we have quantitatively modeled the unreliable signal transmissions and evaluated the impact of the buffer-aided approach, designed the tuning laws for the NNWs, and also ensured the bounded stability.

Remark 6: Compared with existing results, the salient features of can be summarized as follows. 1) This work pioneers the exploration into the NN-based output-feedback control for networked nonlinear systems utilizing a buffer-aided strategy amidst unreliable signal transmissions. 2) Given the nature of unreliable signal transmissions and the incorporation of the buffer-aided strategy, this paper introduces innovative adaptive tuning laws of the nonlinear/critic/actor-NNWs. Furthermore, NN tuning scalars have been tailored to ensure a commendable approximation of unknown nonlinearities and the critic/actor NNs. 3) In the face of unreliable signal transmissions, the EUB of the system states, along with error dynamics of system states, nonlinear/critic/actor-NNWs, have been collectively assured.

Remark 7: It should be mentioned that the signal transmissions of a typical network system are implemented via a digital communication channel, where an encoding-decoding mechanism is utilized to encode signals. By now, various encoding-decoding schemes have been reported in the literature (e.g. the quantization-based encoding-decoding schemes and symbolic-based encoding-decoding schemes) [37]. Different encoding-decoding mechanisms would lead to different "decoding errors", which will affect the resultant accuracy of the control system. One of our future research topics is to study the design of optimal buffer-aided control strategy for networked systems with unreliable communication channels and encoding-decoding mechanisms.

IV. Illustrative Example

Consider a networked nonlinear system (1) where

$$A = \begin{bmatrix} 0.5 & 0 & -0.6 \\ 0 & 1.01 & 0 \\ 0 & 0.5 & 0.2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, E = \begin{bmatrix} 0.1 \\ -0.1 \\ 0.05 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.8 & -0.8 & 0 \\ -0.7 & 0 & -0.7 \end{bmatrix}, D = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}.$$

The variance of ω_k is set as 0.2, and $f_k = 8\begin{bmatrix}\sin(x_{1,k}) & \sin(x_{2,k}) & \sin(x_{2,k})\cos(x_{3,k})\end{bmatrix}^T$.

Let the maximum capacity of the buffer be $Q = 2$. The transmission interval $h(i)$ is selected from the set $\mathbb{H} = \{1, 2, 3, 4\}$, whose known occurrence probabilities is taken as $p_1 = 0.2$ and $p_2 = 0.4$.

Set $\delta = 0.1$, $\mu_1 = 1.2$, $\mu_2 = 0.75$, $\alpha_1 = 5$, $\alpha_2 = 1.2$, $\sigma_1 = 9$, $\sigma_2 = 1.2$, $\sigma_3 = 1.3$, $\sigma_4 = 0.2$ and $\sigma_5 = 0.5$. Using MATLAB LMI Toolbox, the desired solution to the matrix inequalities (13)-(15), (17)-(19) and (45) is

$$P = \begin{bmatrix} 24.1891 & -1.3302 & -1.4385 \\ -1.3302 & 24.1972 & -0.0034 \\ -1.6385 & -0.0034 & 24.1876 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0.3017 & 0.1487 \\ -0.2577 & -0.0482 \\ -0.1327 & -0.0617 \end{bmatrix}, L_2 = \begin{bmatrix} 0.5680 & 0.3322 \\ -1.2492 & -0.4145 \\ -0.5034 & -0.4423 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 0.19070 & 0.2279 \\ -0.2764 & -0.4392 \\ -0.2997 & -0.1672 \end{bmatrix}, L_4 = \begin{bmatrix} 0.3740 & 0.3322 \\ -0.6433 & -0.4392 \\ -0.3007 & -0.1742 \end{bmatrix}.$$

Set $\xi = 0.6$, $\mu_3 = 0.01$, $\mu_4 = 0.01$, $\mu_5 = 0.01$, $\beta_1 = 0.99$, $\beta_2 = 0.99$, $\sigma_6 = 0.8$, $\sigma_7 = 0.2$, $\sigma_8 = 0.2$, $\sigma_9 = 0.2$ and $\sigma_{10} = 0.2$. Therefore, the matrix inequalities (51), (52), and (56) hold. In what follows, let us validate this ADP-based control strategy. The utility function is selected as $l(x_k, u_k) = x_k^T M x_k + u_k^T R u_k$ where $M = 1.6I$ and $R = 1.2I$. The activation functions are selected as

$$\varphi_f(\hat{x}_k) = 0.01\begin{bmatrix}\tanh(\hat{x}_{1,k}) & \tanh(\hat{x}_{2,k}) & \tanh(\hat{x}_{3,k})\end{bmatrix}^T,$$
$$\varphi_v(\hat{x}_k) = 0.4\begin{bmatrix}\tanh(\hat{x}_{1,k}^2) & \tanh(\hat{x}_{2,k}\hat{x}_{3,k}) & \tanh(\hat{x}_{3,k})\end{bmatrix}^T,$$
$$\varphi_u(\hat{x}_k) = 0.4\begin{bmatrix}\tanh(\hat{x}_{1,k}) & \tanh(0.2\hat{x}_{2,k}) & \tanh(0.2\hat{x}_{3,k})\end{bmatrix}^T.$$

The initial values are

$$x_0 = \begin{bmatrix}0.9 & -0.6 & 0.6\end{bmatrix}^T, \hat{x}_0 = \begin{bmatrix}-0.24 & 0.12 & -0.36\end{bmatrix}^T,$$
$$\hat{W}_{f,0} = \begin{bmatrix}0.1 & 0.1 & 0.1\end{bmatrix}^T, \hat{W}_{J,0} = \begin{bmatrix}-1 & -1 & 1.8\end{bmatrix}^T,$$
$$\hat{W}_{u,0} = \begin{bmatrix}-1.52 & -4.24 & 7.6\end{bmatrix}.$$

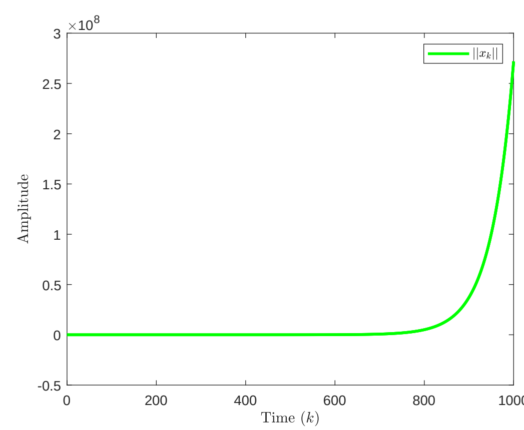

Fig. 1: Norm of the state vector of the open-loop system.

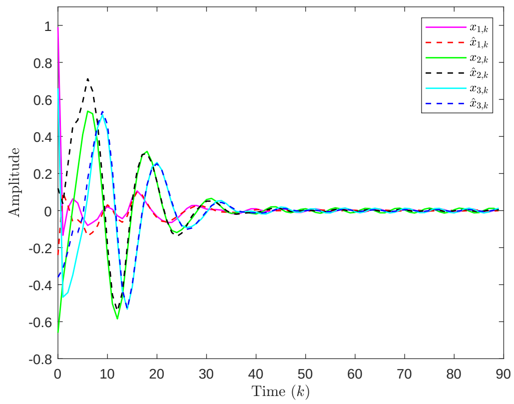

Fig. 2: States and their estimates of the closed-loop system.

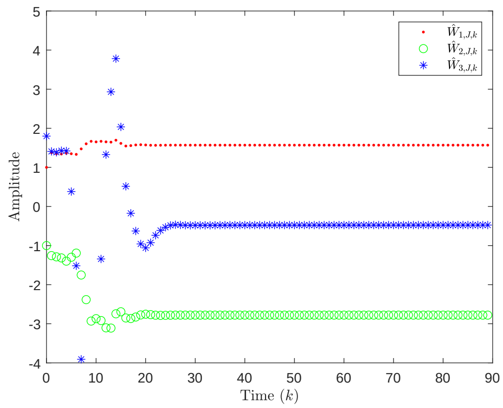

Fig. 3: The weight estimate of critic NN.

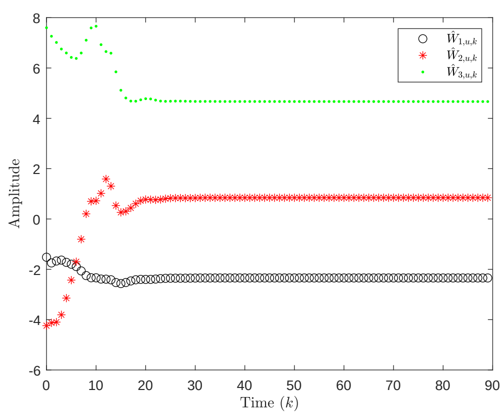

Fig. 4: The weight estimate of actor NN.

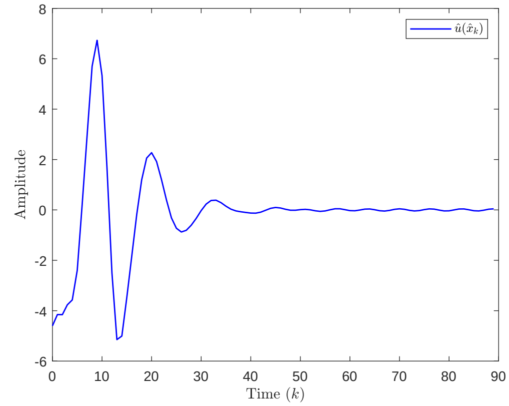

Fig. 5: The control input.

The validity and efficacy of our proposed approach are visually substantiated through results explained as follows.

1) To begin, Fig. 1 showcases the norm of state trajectories for the open-loop system. It becomes evident that the open-loop system is inherently unstable, which motivates the need for an effective control strategy even more apparent.
2) Transitioning to the closed-loop system, we have displayed both the state trajectories and estimates in Fig. 2, which provides a clear testament to the feasibility of the NN-based output-feedback control strategy developed in our study. The trajectories closely align with their estimates, underscoring the controller's ability to maintain system stability and accurately track the desired states.
3) Delving into the neural network details, Fig. 3 and Fig. 4 depict the estimates of the actor/critic NNWs, respectively, and this provides insight into the dynamic adaptation and learning process that the networks undergo as they interact with the system. The control input, crucial for achieving the desired system behavior, is represented in Fig. 5, from which one can verify the controller's responsiveness and precision in action.
4) Collectively, these simulation outcomes show that the proposed NN-based control strategy achieves satisfactory performance, and our developed approach not only addresses the inherent instability of the system but also provides commendable precision and adaptability.

V. Conclusions

In this study, we have examined the ultimately bounded output-feedback control for networked nonlinear systems employing a buffer-aided strategy over unreliable communication channels was explored. Given the unreliable nature of signal transmission, we have used a buffer-aided strategy to relay a greater number of measurements. To obtain the coveted control strategy, an NN-based observer has been devised for state estimation. In addition, an observer-based ADP algorithm has been introduced to approximate the ideal solution for the suboptimal control issue. Utilizing the Lyapunov stability, sufficient conditions have been identified that jointly ensure that the close-loop system, state estimates and critic/actor-NNW estimates are all the EUB in mean square. Numerical examples have been presented to reinforce the efficacy of the outlined control strategy. Potential avenues for future investigations include the extension of the proposed control strategy to systems with buffer-aided strategy and other phenomena such as complex networks [2], [9], [31], wireless sensor networks [13], multiagent systems [20], and others [4], [15], [27], [29], [30], [41], [44].

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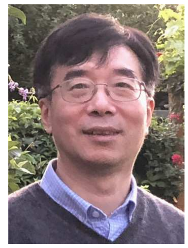

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