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# Broadband shock-associated noise modelling for high-area-ratio under-expanded ${\rm jets}^{\rm a)}$

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#### **ABSTRACT:**

Broadband shock-associated noise (BBSAN) is an important component of supersonic jet noise for jets at off-design conditions when the pressure at the nozzle exit is different from the ambient. Two high-area-ratio under-expanded supersonic jets at nozzle pressure ratios (NPRs) 3.4 and 4.2 are considered. The jets correspond to conditions of the experiment in the Laboratory for Turbulence Research in Aerospace and Combustion (LTRAC) in the Supersonic Jet Facility of Monash University. Flow solutions are obtained by the large eddy simulation (LES) and Reynolds averaged Navier–Stokes (RANS) methods. The solutions are validated against the particle image velocimetry (PIV) data. For noise spectra predictions, the LES solution is combined with the time-domain Ffowcs Williams–Hawkings method. To probe the accuracy of the reduced-order method based on acoustic analogy, the RANS solutions are substituted in the Morris and Miller BBSAN method, where different options for modelling of the acoustic correlation scales are investigated. The noise spectra predictions are compared with the experimental data from the non-anechoic LTRAC facility and the NASA empirical sJet model. Apart from the low frequencies influenced by the jet mixing noise, the RANS-based acoustic predictions align with those from LES for most frequencies in the range of Strouhal numbers (St) 0.4 < St < 2 within  $1-2 \, \text{dB}$ . © 2021 Acoustical Society of America. https://doi.org/10.1121/10.0005976

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#### NOMENCLATURE

- $a_{\infty}$  Speed of sound in the far-field
- D<sub>ef</sub> Effective jet diameter
- D<sub>i</sub> Nozzle exit diameter
- M<sub>i</sub> Acoustic Mach number
- M<sub>c</sub> Convective Mach number
- U<sub>i</sub> Fully expanded jet velocity
- U<sub>e</sub> Sonic velocity at the nozzle exit
- k1 Axial wavenumber component
- 1 Turbulence length scale in stream-wise direction
- $l_{\perp}$  Turbulence length scale in cross-stream direction
- p<sub>s</sub> Shock pressure
- $p_{\infty}$  Ambient pressure
- $\hat{p}_s$  Fourier transform of shock pressure
- R Distance to the observer
- r Radial coordinate
- St Strouhal number
- T<sub>0</sub> Ambient temperature
- ū<sub>c</sub> Mean convective velocity
- **x** Position vector of the observer
- y Position vector of the source
- y1 Axial coordinate
- ε Turbulent dissipation rate

- $\theta$  Polar angle (observer angle)
- к Turbulent kinetic energy
- $\tau$  Time delay
- $\tau_s$  Turbulence time scale
- $\rho_{\infty}$  Free-stream density
- $\phi$  Azimuthal angle
- ω Angular frequency (2πf)
- cov Covariance
- NPR Nozzle plenum pressure ratio
- NTR Nozzle plenum temperature ratio

#### I. INTRODUCTION

The pressure mismatch at the nozzle exit develops a series of standing shock waves, or shock-cells, in supersonic jets. The interaction between turbulent flow and the shockcell structure in the jet shear layer is known to generate broadband shock-associated noise (BBSAN). BBSAN is distinctly different from jet mixing noise that occurs due to the turbulence interaction that primarily radiates in the jet downstream direction. Since jet mixing noise masks the shock-associated noise at small angles to the jet flow, it is the upstream direction where the contribution of the shockassociated noise to the overall aircraft noise is most significant. Occasionally, when a feedback occurs between the upstream propagating acoustic waves and the boundary condition at the nozzle lip, there is also a tonal component of the shock-associated noise present, also known as "jet screech."

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Various aspects of the shock-associated noise have been a subject of investigation since the 1970s experimentally, theoretically, and computationally (Harper-Bourne and Fisher, 1973; Tam, 1975; Zorumski, 1982a, 1982b). Despite this rich history, due to complexity of the underlying fluid mechanics, phenomena that involve high-Reynolds number turbulence, and quasi-stationary shock waves whose structure is highly sensitive to the flow conditions, shockassociated noise has remained a fascinating subject for investigation. In addition to the aspects of fundamental fluid mechanics, the study of shock-associated jet noise has an important practical dimension. This type of noise can be a psycho-physiologically important part of the aircraft cabin noise in high altitude conditions when the temperature drops, and the engine fan operates at supersonic off-design conditions (Bodony et al., 2006). Reduction of supersonic jet noise also plays an important role for military aircraft or during rocket launches, which typically operate at highthrust conditions, since high levels of acoustic fluctuations are harmful to the personnel exposed to such noise as well as leading to structural fatigue of the aircraft structures (Powell, 1956). Shock-associated noise is also an important factor to avoid in the design of future civil supersonic aircraft (Cowart, 2013).

For BBSAN modelling, Harper-Bourne and Fisher (1973) considered a series of noise sources that are located along the jet lip-line located at the shock fronts and radiate noise with a phase delay in accordance with turbulence convection speed and the distance between the shock fronts. For calibration of their semi-empirical model, they used pressure measurements obtained with a phased array. The model can correctly capture the main directivity features of the shock-associated noise as well as its fundamental frequencies as a function of the nozzle pressure ratio (NPR) that defines the standing wave pattern and the jet shear layer growth rate in agreement with earlier studies (Pack, 1950; Davies *et al.*, 1963).

Following the idea of coherent "large-scale" disturbances interacting with the quasi-periodic standing waves (Pack, 1950), Tam and Tanna (1982) proposed a BBSAN model based on instability waves interacting with the shockcells, which gives similar predictions of peak frequency compared to the Harper-Bourne and Fisher (1973) model but is much more physics grounded. Based on the stochastic shear layer model (Tam and Chen, 1979), Tam (1987) improved the original shock-associated noise model to take turbulence in the shear layer into account. They considered the solution of the Euler equations that was based on decomposing the flow variables into the background mean-flow, the shock-cell fluctuation, the perturbation due to turbulence, and the perturbation due to the interaction between turbulence and shock-cells. The magnitudes of the standing waves in the model were matched with those of Pack (1950). The governing equations were rearranged to linearized Euler equations (LEE), which were Fourier transformed and solved with the method of matched asymptotic expansions (van Dyke, 1975). An expression for the far-field noise spectral density was obtained based on the auto-correlation of near-field pressure, where an empirical analytical Gaussian function, the so-called similarity spectrum of the near-field pressure, was used. The underlying assumption was that the noise source spectra have no intrinsic length or time scale. In the follow-on works (Tam and Jackson, 1985; Tam, 1987), the same model was extended to include the effect of a slowly diverging jet flow, the dissipative effects of the turbulence on the shock-cells, and the effects in slightly heated jets. The last two modifications were based on semi-empirical approaches such as adding viscosity terms to mimic the smoothing out of sharp velocity and density gradients of the shock-cells by turbulence and applying various scaling corrections for the strength of the shock-cell and the far-field noise spectral density.

Building on the semi-empirical models of Tam (1975) and Harper-Bourne and Fisher (1973), Morris and Miller (2010) developed a BBSAN prediction scheme that is based on the flow decomposition into the constant background flow, the steady shock profile, the unsteady solution component due to the turbulence, and the acoustic component. Using the Lighthill acoustic analogy (Lighthill, 1952, 1954), this formulation rearranges the governing Navier-Stokes equations and consistently takes into account the nozzle geometry and the realistic jet turbulence by solving a twoequation Reynolds averaged Navier-Stokes (RANS) model numerically. The Morris and Miller (2010) model separates the effects of spatially variable mean-flow velocity gradient from the effective source function that scales with the nearfield pressure and reduces the effective source function to the velocity auto-correlation function. The latter is approximated by an analytical Gaussian exponential shape function following Tam and Auriault (1999). Following this approach, the relevant correlation scales and the amplitude of the acoustic source are approximated by turbulence scales obtained from the RANS solution with some calibration parameters based on matching the model predictions to the far-field noise spectra. More recent modifications of the same model make use of frequency-dependent correlation scales for which semi-empirical relations adopted from the jet mixing noise literature were implemented (Miller, 2013). As with the original Tam shock-associated noise model (Tam, 1975), the Morris and Miller (2010) model captures the general shape and the peak of the shock-associated noise well but tends to under-predict the roll-off of the acoustic spectra at high frequencies. To remedy this, Kalyan and Karabasov (2017) proposed an improved version of the Morris and Miller (2010) model by including both the turbulence/turbulence interaction and the scatter by the shock wave in the definition of the corresponding acoustic source correlation scales. The Kalyan and Karabasov (2017) model was shown to improve the predictions of the Morris and Miller (2010) model in a number of over-expanded and under-expanded supersonic jet cases.

In addition to the theoretical aeroacoustics methods, the availability of high-resolution methods such as large eddy simulations (LESs) for a range of supersonic jet flows



opened up another direction in the research of shockassociated jet noise (Paliath and Morris, 2005; Shur et al., 2006; Pineau and Bogey, 2019; Pérez Arroyo et al., 2019; Pérez Arroyo and Moreau, 2019). In scale-resolving solutions of the governing Navier-Stokes equations, the shocks, turbulence, and their interaction leading to shock-associated noise are automatically included in the unsteady flow solution. The resulting solutions provide a wealth of space-time resolved data, which can be very useful for validation of theoretical models. Despite making considerable progress in developing accurate algorithms supported by the increased availability of computer power, validation of unsteady flow solutions of supersonic turbulent jets remains highly challenging. This is because supersonic flows are highly sensitive to the jet inflow conditions, which are usually not fully reported in experiments. Hence, the motivation of the current work is to partially fill this gap by developing a range of computational models for supersonic jet flow cases corresponding to the experiments in the Laboratory for Turbulence Research in Aerospace and Combustion (LTRAC) conducted in the Supersonic Jet Facility of Monash University, Australia (Mitchell et al., 2013; Tan et al., 2019a,b). The LTRAC jets correspond to both the BBSAN and screeching noise behavior. The jets are well documented for aerodynamics using the high-resolution particle image velocimetry (PIV) measurements and partly for aeroacoustics. In comparison with the existing supersonic jet databases, which have been used for validation of high-resolution flow and noise solutions in the literature (Pérez Arroyo et al., 2019; Pérez Arroyo and Moreau, 2019), the convergent nozzle used in the LTRAC experiment has a much higher area ratio. The resulting strong flow compression at the nozzle exit leads to prominent shockcells not only along the jet centerline but also at the jet lipline locations. For the highest NPR = 4.2 case, the compression and the following expansion waves are so strong that a Mach disk appears downstream of the nozzle exit, which adds an additional challenge for computational modelling.

To simulate the broadband dynamics of the LTRAC jets at NPR = 3.4 and 4.2, the high-resolution CABARET LES method accelerated on graphics processing units (GPUs) is used. Following the preliminary work reported in Markesteijn et al. (2017), the inflow boundary conditions are specified at the nozzle exit (the throat of the convergent nozzle) using the PIV data from the experiment, where sonic condition is assumed. For the far-field spectra predictions, the LES solution is combined with the Ffowcs Williams-Hawkings (FW-H) method following the standard jet noise modelling practice (Ffowcs Williams and Hawkings, 1969; Shur et al., 2005). To investigate the effect of the LES grid sensitivity on flow and noise solution for the most challenging LTRAC NPR = 4.2 case, the LES solutions on three grid resolutions are obtained and cross-validated in comparison with the available experimental data.

In addition to the LES solutions, the same jet flows are computed using the RANS method based on the  $\kappa - \omega$  SST turbulence model. The RANS solutions are used in the

acoustic analogy model of Morris and Miller (2010) for farfield noise spectra predictions. In the LES- and RANS-based noise spectra predictions, the acoustic results are compared with the available acoustic measurements from the LTRAC facility. Since the experimental facility is known to be nonanechoic, the noise predictions are also validated in comparison with the "consensus" BBSAN results for the same temperature and NPR conditions in accordance with the NASA sJet model (Khavaran and Bridges, 2009a,b). The latter is an empirical scaling-law model based on interpolations over a large database of NASA jets of various Mach numbers and temperature ratios. The sJet model includes both the BBSAN and mixing noise components.

#### II. HIGH-SPEED JET NOISE LTRAC CASE: CONDITIONS, FLOW, AND FAR-FIELD MODELLING

#### A. Summary of the experimental set-up

In the LTRAC facility, compressed air is supplied to the plenum chamber at approximately  $T_0 = 288 \text{ K}$ . The compressed air is connected directly to the mixing chamber at normal atmospheric conditions where PIV measurements were taken. The air issues from an axisymmetric nozzle with a blunt lip of 5 mm thickness (Fig. 1). The diameter of the nozzle at the exit,  $D_i$ , is equal to 15 mm. The nozzle has an inlet to exit area ratio of 93.44 with a short purely converging section so that the flow is sonic at the exit with a velocity of  $U_e = 310$  m/s. The converging section consists of a contoured wall with a radius of curvature of 67.15 mm and a short parallel section at the exit. A complete description of the facility and the PIV system can be found in Edgington-Mitchell et al. (2014) and Tan et al. (2019b). The operating conditions of the jets considered in this study are summarised in Table I.

In addition to the PIV measurements, acoustic microphone measurements were obtained at several distances from the nozzle exit in the LTRAC experiment. Because of the non-anechoic acoustic facility, the low-frequency part of the noise spectra measured at observer distances from 30 to 100 nozzle diameters (Markesteijn *et al.*, 2017) were shown to not satisfy the standard scaling on  $\mathbb{R}^2$ , where R is the observer distance from the nozzle exit, for observer angles to the jet flow of 90° and higher (Tan *et al.*, 2016). Hence, the low frequencies of the LTRAC noise spectra are believed to be contaminated by noise not related to BBSAN



FIG. 1. (Color online) Rendered views of the computer-assisted design (CAD) model of the LTRAC nozzle (Tan *et al.*, 2019a,b).

TABLE I. Operating conditions of the LTRAC jets.

NPR	3.4	4.2
Mj	1.45	1.59
$U_j(m/s)$	412	440
NTR	1	1
$D_{ef}\left(mm\right)$	16.02	16.73

or jet mixing. At the same time, the mid- to high-frequency noise measurements in the range of relevance to BBSAN remain useful for validation of the jet noise modelling as will be discussed in Secs. II B–II E.

#### **B. LES modelling**

For unsteady simulation of the LTRAC jets, LESs are performed and the far-field noise spectra are computed based on the FW-H method with multiple closing disks (Shur et al., 2005). In all simulations, the high-resolution CABARET method (Goloviznin and Samarskii, 1998; Karabasov and Goloviznin, 2009, Chintagunta et al., 2017) is used for solving the Navier-Stokes equations on unstructured meshes with good preservation of linear wave propagation properties (Tucker and Karabasov, 2009; Goloviznin et al., 2013, Chintagunta et al., 2018). Particular features of the CABARET method include compactness of the computational stencil and asynchronous time stepping for efficient calculations on non-uniform space-time grids (Semiletov and Karabasov, 2013). The CABARET solver was validated for a range of jet flows (Semiletov et al., 2016; Faranosov et al., 2013). The LES implementation on GPUs further enabled a considerable reduction of the solution time in comparison with conventional LES approaches (Markesteijn and Karabasov, 2018a,b, 2019; Markesteijn et al., 2020). The method utilises the so-called split-hexa meshes, which correspond to hanging node-type cells, which are available in the semi-automatic mesh generation process using the

TABLE II. LES grid details.

OpenFOAM utility "snappyHexMesh" (sHM). This mesh utility enables hexa-dominant mesh generation from triangulated surface geometries (e.g., CAD geometry), which is an essential need for the LES treatment of complex geometries.

In the most challenging jet case of NPR = 4.2, three LES grids are considered, the details of which are summarised in Table II. Three regions of specific local grid refinement include the following zones: the jet plume, the region outside of the jet core, and the FW-H surface region. The three non-uniform grids considered correspond to 24, 40, and  $70 \times 10^6$  cells. For all of them, zones of uniform Cartesian grid cells imbedded in the unstructured snappyHex mesh are generated in the jet shear layer at locations similar to our previous simulations of complex round jet flows (Markesteijn et al., 2020). The mesh containing  $40 \times 10^6$  cells has a finer resolution in the potential core region than the one having  $24 \times 10^6$  cells but has a more aggressive grid de-refinement outside of the jet core. The mesh containing  $70 \times 10^6$  cells is the finest one both in the jet plume and in the FW-H surface region. For the NPR = 3.4 case, the intermediate  $40 \times 10^6$  LES grid is considered.

The simulations are performed on a single computer workstation equipped with two GPU cards [NVidia (Santa Clara, CA) Titan RTX 24GB]. For example, in the case of the  $70 \times 10^6$  cell mesh, the solution spin-out time is 300 convective time units (TUs), and a further 1000 TUs are simulated for statistical averaging. Here, 1 TU of the simulation corresponds to the time taken by a turbulent eddy travelling at a speed equal to the jet velocity to cover the distance equal to one diameter of the nozzle exit. The total time to solution in this case is 39 h. This utilises the current solver capability to perform flow and noise predictions using single-precision floating point accuracy without a notable loss in accuracy, due to a modified time-update procedure (Markesteijn and Karabasov, 2018b), as well as the speedup

Grid	$24 \times 10^6$ cells	$40 \times 10^6$ cells	$70  imes 10^6$ cells
Mesh resolution at the nozzle lip			
$\Delta x/D_{i} \sim \Delta y/D_{i} \sim \Delta z/D_{i} =$	0.04	0.0267	0.0267
Mesh resolution at the nozzle exit region			
of the FW-H surface and the			
maximum frequency resolved based on 8 ppw			
$\Delta x/D_j \sim \Delta y/D_j \sim \Delta z/D_j =$	0.08	0.107	0.053
	St = 1.7	St = 1.3	St = 2.6
Mesh resolution at the end closing disk region			
of the FW–H surface and the			
maximum frequency resolved based on 8 ppw			
$\Delta x/D_j \sim \Delta y/D_j \sim \Delta z/D_j =$	0.107	0.16	0.08
	St = 1.3	St = 0.86	St = 1.7
Mesh resolution at the nozzle lip			
$\Delta x/D_i \sim \Delta y/D_i \sim \Delta z/D_i =$	0.04	0.0267	0.0267
Simulation run times, number of TUs generated,	300 TUs (spin-out)	400 TUs (spin-out)	300 TUs (spin-out) + 1000 TUs
and the total time to solution	+ 1200 TUs (averaging), 20 h	+ 800 TUs (averaging), 26 h	(averaging), 39 h



FIG. 2. (Color online) Comparison of the LES solution (bottom half) with the PIV data (top half) for the NPR = 3.4 jet: axial mean-flow velocity normalised by jet exit velocity.

obtained, due to the use of an asynchronous time stepping algorithm. In accordance with the latter algorithm, the cells of the LES grid are distributed into several groups, which are updated at different rates depending on the cell size in accordance with an approximately constant local Courant–Friedrichs–Lewy (CFL) number, which is kept as close to optimal as possible. It can be noted that the optimal CFL number for CABARET is 0.5; this is where the scheme becomes exact for one-dimensional linear wave propagation. For the time stepping, seven dyadic update groups are used, where the local time step changes from  $\Delta t$  to  $64\Delta t$  and where  $\Delta t$  is the minimum time step.

Details of the three LES grids and the corresponding times to solution of the GPU LES solver accelerated on two NVidia Titan RTX cards are summarised in Table II. The maximum frequency resolution on the grid corresponding to 8 points per wavelength (ppw) is defined in terms of the dimensionless Strouhal number,  $St = fD_j/U_j$ , which is based on the fully expanded jet velocity  $U_j$  and the nozzle exit diameter  $D_j$ .

#### C. RANS modelling

Following Tan *et al.* (2019a), the RANS solutions of the same LTRAC jets were obtained using the  $\kappa - \omega$  SST turbulence model, which includes the compressibility corrections and accounts for the adverse pressure gradients typical of supersonic flows. The three-dimensional (3D) axisymmetric RANS equations are solved, imposing an inlet boundary condition at the inflow region of the nozzle, free-stream boundary conditions at the far-field domains, and a downstream pressure condition for the jet outflow. The computational domain



is designed to be sufficiently large to ensure that numerical open boundary condition effect is negligible. RANS calculations were carried out on several initial coarse grids corresponding to a range of computational domain sizes, 60-100 nozzle diameters axially and 20-40 nozzle diameters radially, to ensure that the pressure and Mach number profiles in the jet core region remain virtually insensitive to the domain size. Away from the jet flow region, the grid is kept coarse to optimize the total grid count. Grid refinement was carried out by iteratively running several RANS calculations to verify that the target  $y^+$  value in the wall-normal units is close to 1 as recommended in the literature. The final computational mesh includes  $165 \times 10^3$  elements and extends  $60D_i$  and  $20D_i$ , in axial and transverse dimensions, respectively. The governing RANS equations are solved using the density-based solver in ANSYS Fluent. For spatial discretization, a second-order Roe scheme is used and an implicit integration scheme in time. The iterations are stopped when the solution residuals reduce to  $10^{-5}$  of the initial values.

#### D. Flow-field comparison with experiment

The flow modelling results of the NPR =  $3.4 (40 \times 10^6 \text{ cell LES grid})$  and NPR =  $4.2 \text{ jets} (24 \text{ and } 70 \times 10^6 \text{ cell LES grid})$  are presented in this section. For comparing different datasets, the velocities and distances are normalised on nozzle diameter D<sub>i</sub> and jet exit velocity U<sub>e</sub>.

#### 1. NPR = 3.4 jet

Figure 2 compares the distributions of the mean axial velocity component from the LES and the PIV data in the jet symmetry plane. A detailed comparison of LES and RANS solutions with the PIV data is presented in Figs. 3 and 4, which show the distributions of the time-averaged axial velocity and turbulent kinetic energy along jet center-line and lip-line  $(r/D_j = 0.5)$  locations. Notably, the LES and RANS time-averaged velocity solutions are in good agreement with the experiment: the location and the amplitude of the first 4–5 shock-cells are well predicted [Fig. 3(a)]. On the one hand, the LES solution more accurately captures the jet velocity profile in comparison with the



FIG. 3. (Color online) Comparison of the LES and RANS solutions along the jet centerline versus the PIV data for the NPR = 3.4 jet: axial mean-flow velocity distributions (a) and turbulent kinetic energy (b).





FIG. 4. (Color online) Comparison of the LES and RANS solution along the jet lip-line versus the PIV data for the NPR = 3.4 jet: axial mean-flow velocity distributions (a) and turbulent kinetic energy (b).

RANS solution especially for the centerline distribution. On the other hand, the LES solution tends to over-amplify the turbulent kinetic energy distribution (especially the first prominent peak) in comparison with the RANS solution for the jet lip-line locations [Fig. 4(b)], which may be related to the initially laminar inflow condition effect in the LES simulation. At the same time, the RANS solutions fail to resolve any turbulence along the jet centerline [Fig. 3(b)] as typical for the RANS models, where the turbulence generation is related to the mean-flow vorticity, which can be regarded as a general deficiency of the RANS models.

#### 2. NPR = 4.2 jet

The fastest LTRAC jet of NPR = 4.2 corresponds to a strong expansion, which generates shock-cells in the entire jet extending to the jet lip-line locations as well as a Mach

disk downstream of the nozzle exit. The latter features typical of strongly under-expanded jets are often missing in the laboratory-scale experiments, such as the jet case at a relatively low NPR = 2.27 recently considered by Pérez Arroyo *et al.* (2019).

Figures 5 and 6 compare the two-dimensional (2D) distributions of the time-averaged axial velocity component and turbulent kinetic energy of the LES solutions on the two grids in the jet symmetry plane with the PIV data. It can be noted that both the coarse- and fine-grid LES solution correctly capture the location of the first four shock-cells as well as the Mach disk. The LES solution on the  $70 \times 10^6$  cell grid leads to a notable improvement of the turbulent kinetic energy distribution in comparison with the  $24 \times 10^6$  LES grid solution [Fig. 6(b)]. The latter shows a strong amplification of the turbulent kinetic energy levels on the coarse grid due to the lack of the mesh resolution in the shear layers, thereby leading to insufficient turbulent mixing.



FIG. 5. (Color online) Comparison of the LES solution (bottom half) with the PIV data (top half) for the NPR = 4.2 jet: axial mean-flow velocity normalised by jet exit velocity for  $70 \times 10^6$  mesh (a) and  $24 \times 10^6$  mesh (b).



FIG. 6. (Color online) Comparison of the LES solution (bottom half) with the PIV data (top half) for the NPR = 4.2 jet: turbulent kinetic energy normalised by  $U_e^2$  for  $70 \times 10^6$  mesh (a) and  $24 \times 10^6$  mesh (b).

Figures 7 and 8 compare the jet centerline timeaveraged axial velocity and turbulent kinetic energy distributions of the LES solutions on the 24 and  $70 \times 10^6$  meshes with the RANS solutions and with the experiment.

Both the LES solutions capture the mean-flow velocity distributions in amplitude and phase in comparison with the PIV data up to the fourth shock-cell [Fig. 7(a)]. For further shock-cells, numbers 5 and 6, the fine-grid LES solutions still preserve the correct amplitude of the shock-cells in comparison with the experiment while the phase error of up to 50% is accumulated.

The RANS solution captures the Mach disk correctly but under-predicts the potential core of the jet, which is especially notable for the centerline jet location where the shock-cell structure predicted by RANS is very different from the experiment [Fig. 7(a)].

The LES solution of turbulent kinetic energy on the  $70 \times 10^6$  grid is in encouraging agreement with the experiment. Recalling that the jet core resolution of the  $70 \times 10^6$ 

mesh is the same as the  $40 \times 10^6$  mesh, it can be suggested that initially laminar flow condition is less of a problem for the fastest NPR = 4.2 case in comparison with the NPR = 3.4 case [compare Figs. 7(b) and 8(b) with Figs. 3(b) and 4(b)]. This can be attributed to a thinner boundary layer and a faster transition to turbulence, which can be expected in the faster NPR = 4.2 jet in comparison with the NPR = 3.4 jet.

Still, the RANS solution of the turbulent kinetic energy distribution along the lip-line is closer to the experiment in comparison with the LES solution on the fine  $70 \times 10^6$  grid [Fig. 8(b)], which suggests that the latter LES grid needs to be further refined to correctly resolve the turbulent mixing.

#### E. Far-field acoustic modelling

#### 1. FW-H method based on LES solution

For far-field noise spectra computation, the LES solution is combined with the time-domain FW-H method based



FIG. 7. (Color online) Comparison of the LES and RANS solutions along the jet centerline versus the PIV data for the NPR = 4.2 jet: axial mean-flow velocity distributions (a) and turbulent kinetic energy (b).





FIG. 8. (Color online) Comparison of the LES and RANS solutions along the jet lip-line versus the PIV data for the NPR = 4.2 jet: axial mean-flow velocity distributions (a) and turbulent kinetic energy (b).

on a permeable acoustic integration surface of a funnel shape. The control surface includes 16 closing disks over which the signal is averaged to reduce the pseudo-sound effects (Shur *et al.*, 2005; Semiletov *et al.*, 2016). The acoustic signal is computed at the distance of the acoustic microphone used in the experiment ( $R/D_j = 100$ ). The time-domain pressure signal is Fourier transformed using the single-sided spectra definition, and the resulting sound pressure power spectral density (PSD) is computed using the Welch method, the details of which can be found in Semiletov and Karabasov (2018). The common definition of PSD is used with 1 Hz reference frequency and 20  $\mu$ Pa reference pressure.

# 2. Acoustic analogy method based on the RANS solution

Following Morris and Miller (2010) and Kalyan and Karabasov (2017), the RANS-based acoustic analogy model of BBSAN is implemented. First, by ignoring the mean-flow sound propagation effect for angles of interest with respect to BBSAN that are close to 90°, the governing Navier–Stokes equations are rearranged to the homogeneous linear wave propagation form,

$$\begin{split} &\frac{\partial \pi'}{\partial t} + \frac{\partial v'_i}{\partial y_i} = 0, \\ &\frac{\partial v'_i}{\partial t} + \bar{a}^2 \frac{\partial \pi'}{\partial y_i} = f^v_i, \quad i.j = 1, 2, 3, \end{split} \tag{1}$$

where

$$f_{i}^{v} = -v_{sj}\frac{\partial v_{ti}}{\partial y_{j}} - v_{tj}\frac{\partial v_{si}}{\partial y_{j}}$$
(2)

is the unsteady force per unit volume associated with interactions between the shock and turbulent velocity fluctuations. The latter scales with the shock pressure and the unsteady velocity fluctuations,  $f_i^v \sim p_s v_t / \rho_\infty a_\infty l$ . The shock pressure is defined as the difference between the local static pressure and the ambient,

$$\mathbf{p}_{s}(\mathbf{y}) = \mathbf{p}(\mathbf{y}) - \mathbf{p}_{\infty}(\mathbf{y}). \tag{3}$$

The acoustic propagation equations are solved analytically using the free-space Green's function method. A cylindrical-polar coordinate system,  $\mathbf{y} = (\mathbf{x}, \mathbf{r} \cos \phi, \mathbf{r} \sin \phi)$ , is considered and is schematically illustrated in Fig. 9.

By making use of the Proudman form (Proudman and Taylor, 1952) for the cross correlation, the expression for the far-field sound PSD is derived,

$$\begin{split} \mathbf{S}(\mathbf{x},\omega) &= \frac{\omega^2}{16\pi^2 a_{\infty}^4 |\mathbf{x}|^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{l^2} \mathbf{p}_{s}(\mathbf{y}) \\ &\times \mathbf{p}_{s}(\mathbf{y}+\mathbf{\eta}) \mathbf{R}^{\mathrm{v}}(\mathbf{y},\mathbf{\eta},\tau) \\ &\times \exp\left[\mathrm{i}\omega \left(\tau - \frac{\mathbf{x}\mathbf{\eta}}{a_{\infty}|\mathbf{x}|}\right)\right] \mathrm{d}\tau \mathrm{d}\mathbf{\eta} \mathrm{d}\mathbf{y}, \end{split} \tag{4}$$

where

$$\mathbf{R}^{\mathrm{v}}(\mathbf{y}, \Delta, \tau) = \overline{\mathbf{v}_{t2}(\mathbf{y}, t)\mathbf{v}_{t2}(\mathbf{y} + \mathbf{\eta}, t + \tau)}$$
(5)

is the near-field velocity auto-covariance function, which is modelled in accordance with the analytical Gaussian/exponential shape function (Tam and Auriault, 1999),

$$\mathbf{R}^{\mathbf{v}}(\mathbf{y}, \mathbf{\eta}, \tau) \approx \mathbf{R}^{\mathbf{v}}(\mathbf{y}, 0, 0) \exp\left[-\frac{|\tau|}{\tau_{s}}\right]$$
$$\times \exp\left[-\frac{(\mathbf{\eta}_{1} - \bar{\mathbf{u}}_{c}\tau)^{2}}{l^{2}}\right] \exp\left[\frac{-(\mathbf{\eta}_{2}^{2} + \mathbf{\eta}_{3}^{2})^{2}}{l_{\perp}}\right].$$
(6)



FIG. 9. Definition of the cylindrical-polar coordinate system.

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In the above equation,  $\eta_1$  is the Cartesian coordinate in the stream-wise (jet flow) direction,  $(\eta_2, \eta_3)$  are in the transverse plane,  $(\eta_1, \eta_2)$  are in-plane with the far-field observer, and  $\bar{u}_c$  is a characteristic turbulence convection speed, which can be scaled with the jet velocity at the nozzle exit,  $U_e$ , via a proportionality coefficient,  $\alpha_c = 0.7-0.8$ . It can be noted that both of the considered LTRAC jets are underexpanded; hence, the equivalent convection speed scaling based on the fully expanded jet velocity, which is 30–40%

greater than the jet velocity at the nozzle exit, would be in the range of 0.5–0.6, which is in reasonable agreement with the jet mixing noise literature (Harper-Bourne, 2003; Semiletov and Karabasov, 2018). To close the acoustic model, the convective Mach number is defined as  $M_c = \bar{u}_c/a_\infty$ , where the acoustic Mach number  $M_j = U_e/a_\infty$  is based on the sonic velocity at the nozzle exit.

By evaluating the inner integral over  $\tau$  analytically, the acoustic integral reduces to

$$S(\mathbf{x},\omega) = \frac{1}{16\pi\sqrt{\pi}a_{\infty}^{4}R^{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{c_{A}\kappa l_{\perp}^{2}}{l\tau_{s}} p_{s}(\mathbf{y})\hat{p}_{s}(\mathbf{k}_{1},\mathbf{y}_{2},\mathbf{y}_{3}) \exp(i\mathbf{k}_{1}\mathbf{y}_{1}) \\ \times \frac{\tau_{s}^{2}\omega^{2}\exp\left[-l^{2}(\mathbf{k}_{1}-\omega\cos\theta/a_{\infty})^{2}/4-\omega^{2}l_{\perp}^{2}\sin^{2}\theta/4a_{\infty}^{2}\right]}{1+(1-M_{c}\cos\theta+\bar{u}_{c}\mathbf{k}_{1}/\omega)^{2}\omega^{2}\tau_{s}^{2}} d\mathbf{k}_{1}d\mathbf{y},$$
(7)

where  $k_1$  is the axial wavenumber component and  $\hat{p}_s$  is the wavenumber transform of  $p_s$  in the stream-wise direction.

The temporal correlation scale of  $\tau_s$  and the spatial correlation scales of  $l_{\perp}$  and l, which partly capture the anisotropy effect, are evaluated as the corresponding integral scales of the velocity auto-correlation function  $R^v(\mathbf{y}, \Delta, \tau) = \overline{v_{t2}(\mathbf{y}, t)v_{t2}(\mathbf{y} + \Delta, t + \tau)}$ . In the framework of RANS modelling, the correlation scales are modelled by assuming a linear relationship between the integral scales and some characteristic flow scale in the turbulent jet flow. For example, in the Morris and Miller (2010) model, following the jet mixing noise model (Tam and Auriault, 1999), the turbulent length and time scales are modelled dimensionally and are related to the large scales from equating the rate of energy production at the large scales to the rate of viscous dissipation at the smallest scales.

That is,

$$\tau_{\rm s} = {\rm c}_{\tau} \frac{\kappa}{\varepsilon}, \quad {\rm l} = {\rm c}_{\rm l} \frac{\kappa^{3/2}}{\varepsilon}, \quad {\rm l}_{\perp} = {\rm c}_{\perp} {\rm l}$$
 (8)

and

$$\mathbf{R}^{\mathsf{v}}(\mathbf{y},0,0) = \overline{\mathbf{v}_{t2}(\mathbf{y},t)^2} = \mathbf{c}_{\mathsf{A}}\kappa,\tag{9}$$

where  $\kappa$  and  $\varepsilon$  are the turbulence kinetic energy and the dissipation rate obtained from the RANS solution, and  $c_1$ ,  $c_{\perp}$ ,  $c_{\tau}$ , and  $c_A$  are some calibration parameters. The calibration parameters are obtained by fitting the model predictions to the reference far-field noise spectrum measurements for one jet condition and one observer angle to the jet flow (and then keeping the same parameters for all other observer angles and jet conditions). In the current LTRAC cases, we have used the NASA sJet model (Khavaran and Bridges, 2009a,b) for the reference noise spectrum of the NPR = 3.4 jet at a 90° angle. The resulting values of the model constants are as follows:

$$c_l = 2.9, \quad c_\perp = 0.3, \quad c_\tau = 0.5, \quad c_A = 0.5.$$
 (10)

Additionally, following the two-scale model of Kalyan and Karabasov (2017), two mechanisms of shock-associated noise are considered. One corresponds to a jet-mixing-noise-like process and has the time correlation scale  $\tau_{s1}$ , which scales with turbulence energy dissipation rate [same as in the Morris and Miller (2010) model], and the other corresponds to the acoustic scattering of turbulent eddies by the shock-cell, which has the time correlation scale  $\tau_{s2}$  that depends on the stream-wise mean-flow velocity gradient. Then the resulting correlation time scale is given by Ghahramani (2005),

$$\tau_{\mathrm{s}} = \tau_{\mathrm{s}1} \cdot \mathrm{f}(\tau_{\mathrm{s}1}, \tau_{\mathrm{s}2}),\tag{11}$$

where the function

$$f(\tau_{s1}, \tau_{s2}) = \sqrt{1 + \left(\frac{\tau_{s1}}{\tau_{s2}}\right)^2 + \frac{\text{cov}(X(\tau_{s1}), Y(\tau_{s2}))}{\tau_{s1}^2}} \quad (12)$$

is approximated by a model based on the dimensional analysis,

$$f(\tau_{s1}, \tau_{s2}) = \left(\frac{\tau_{s1}}{\tau_{s2}}\right)^{\alpha} \text{ if } \frac{\tau_{s1}}{\tau_{s2}} > 1 \quad \text{and}$$
  
$$f(\tau_{s1}, \tau_{s2}) = 1, \text{ otherwise}, \tag{13}$$

where  $\alpha$  is a model parameter that needs to be specified along with the rest of the space and time scale functions similar to the Morris and Miller (2010) model,

$$\begin{aligned} \tau_{s1} &= \frac{\mathbf{c}_{\tau} \kappa(\mathbf{y})}{\varepsilon(\mathbf{y})}, \quad \tau_{s2} = \mathbf{c}_{\tau} \left| \frac{\partial \overline{\mathbf{v}}_{1}(\mathbf{y})}{\partial \mathbf{y}_{1}} \right|^{-1}, \\ \mathbf{l} &= \mathbf{c}_{1} \sqrt{\kappa(\mathbf{y})} \tau_{s}(\mathbf{y}), \quad \mathbf{l}_{\perp} = \mathbf{c}_{\perp} \mathbf{l}(\mathbf{y}). \end{aligned}$$
(14)

. . . . . . . . . . . . . . . . . . https://doi.org/10.1121/10.0005976 0.5 0 -3000 6 8 10 -2000 -1000 0 1000 2000 3000 x/D k(1/m) 2500 3500 4000 0.4 0.6 0.8 1.2 1.4 500 1000 1500 2000 3000 16 (a) (b)

FIG. 10. (Color online) The acoustic integration part of the RANS simulation domain of the NPR = 4.2 jet case: the physical domain showing contours of the mean-flow axial jet velocity normalised by U<sub>j</sub> (a) and the Fourier domain showing the shock pressure magnitude,  $|\hat{p}_s|$ , from 0 to 0.45p<sub> $\infty$ </sub> (b).

So, for example, for  $\tau_{s1}/\tau_{s2} > 1$ , the effect associated with the acoustic scattering of turbulent eddies by the shockcell is neglected in comparison with the turbulent energy dissipation, and the Kalyan and Karabasov (2017) model automatically switches to the Morris and Miller (2010) model.

To obtain the Fourier transform of the shock pressure,  $\hat{p}_s$ , the RANS flow domain is symmetrically extended to the negative  $y_1$ -coordinate region (x-coordinate in the Cartesian-polar coordinate system) with respect to x = 0. The numerical Fourier transform in the stream-wise direction is computed with a Hanning window applied to minimize the finite computational domain effect. Before integration, the RANS solution is interpolated to a uniform Cartesian grid in the (x - r) plane. Figure 10(a) shows one-half of the acoustic integration domain (the physical solution domain corresponding to  $y_1 > 0$ ) and the Fourier domain [Fig. 10(b)] used in the spectral part of the acoustic model.

To investigate the importance of the two BBSAN mechanisms for the LTRAC jets, the jet lip-line distribution of the characteristic scale ratio,  $\tau_{s1}/\tau_{s2} = [\kappa(\mathbf{y})/\varepsilon(\mathbf{y})]|\partial \bar{v}_1(\mathbf{y})/\partial y_1|$ , is computed for both LTRAC jet cases (Fig. 11). The RANS solutions are compared with the PIV data on the same plots. Notably, in both cases,  $\tau_{s1}/\tau_{s2} < 1$ , which means that the turbulent energy dissipation is the dominant mechanism defining the correlation time and length scales for these jets, hence making the original Morris and Miller (2010) model applicable for the considered jet cases.

It can be noted that the complete dominance of the turbulence dissipation mechanism in acoustic scales makes the LTRAC jets different in comparison with the convergent and convergent-divergent Pennsylvania State University (PSU) jet cases (Morris and Miller, 2010). For the PSU jets, Kalyan and Karabasov (2017) reported a large region of  $\tau_{s1}/\tau_{s2} > 1$  along the jet lip-line, which corresponds to the turbulent eddy/shock-cell scattering process. It can be hypothesised that the difference between the LTRAC and PSU jets is due to the strong screeching component of the LTRAC jets, which effectively (in the time-average sense) diffuses the jet shear layers, thereby reducing the mean-flow velocity gradients.

#### 3. Comparison with far-field noise measurements

Figures 12 and 13 show the far-field noise PSD predictions based on the LES solution coupled with the FW–H method for the NPR = 3.4 jet on the  $40 \times 10^6$  grid and the NPR = 4.2 jet on the  $70 \times 10^6$  grid. All LES-FW–H model predictions are compared with the RANSbased Morris and Miller (2010) BBSAN model, the results of the NASA empirical sJet model (Khavaran and Bridges, 2009a,b), and the LTRAC acoustic measurements. The noise spectra correspond to 90° and 120° observer angles, which are most typical of the BBSAN component of supersonic jet noise. All acoustic data are presented at the observer distance of 100 nozzle diameters from the nozzle



FIG. 11. (Color online) Jet lip-line distributions of the dimensionless function  $\tau_{s1}/\tau_{s2}$ , which detects the zones of importance of the shock-scattering mechanism in BBSAN ( $\tau_{s1}/\tau_{s2}$ ) > 1 for the LTRAC jets corresponding to NPR = 3.4 (a) and NPR = 4.2 (b). Both the PIV data and the RANS solutions predict the dominance of the turbulence dissipation mechanism for the LTRAC jets.

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FIG. 12. (Color online) Comparison of the PSD predictions of the LES-FW–H and RANS-based acoustic models with the NASA sJet model and the LTRAC experiment for NPR = 3.4 at  $90^{\circ}$  (a) and  $120^{\circ}$  (b) observer angles.

exit. For the most challenging NPR = 4.2 jet case, the farfield spectra predictions are computed using three LES grids (24, 40, and  $70 \times 10^6$  cells) and compared with the experiment (Fig. 14).

First, it can be noted that the noise spectra predictions of the LES-FW–H models are within 1–2 dB from the RANS-based predictions for NPR = 3.4 and 4.2 over the high-frequency range corresponding to 0.6 < St < 2 (Figs. 12 and 13). For the  $70 \times 10^6$  grid for NPR = 4.2, the LES-FW–H solution is also within 2–3 dB from the LTRAC experiment for the same range of high frequencies. For frequencies higher than St = 2, the LES-FW–H solutions are under-resolved, while the LTRAC acoustic measurements show a faster drop off in comparison with the NASA sJet model predictions. This is because, in contrast to the LTRAC measurements, the sJet model includes a correction for atmospheric attenuation.

For low frequencies (0.04 < St < 0.5), where the LTRAC data are contaminated by noise, the LES-FW–H solution is within 2–3 dB from the reference NASA sJet solution.

In addition, the LES correctly captures the screech tone frequencies of the experiment of St = 0.6 and St = 0.9 for the 90° spectrum of the NPR = 3.4 LTRAC jet and also those of St = 0.2 and St = 0.9 for the 120° spectrum of the same jet. Furthermore, the LES-FW–H solution of the NPR = 4.2 LTRAC jet resolves the screech tone frequency of St = 0.4 for both the 90° and 120° observer angles.

It can also be noticed, as the grid resolution increases from  $24 \times 10^6$  to  $70 \times 10^6$  cells, the predictions of the LES-FW–H model of the NPR = 4.2 jet for high frequencies gradually approach the LTRAC experiment spectra (Fig. 14).

The RANS-based BBSAN model predictions at frequencies lower than 0.7 for the 90° observer angle fall off more rapidly than the LES and the sJet spectra. This can be explained by the jet mixing noise contribution, which is not accounted for in the pure BBSAN acoustic analogy model.

#### **III. CONCLUSION**

The flow and noise predictions of the two strongly under-expanded jets issuing from a high-area-ratio



FIG. 13. (Color online) Comparison of the PSD predictions of the LES-FW–H and RANS-based acoustic models with the NASA sJet model and the LTRAC experiment for NPR =  $4.2 \text{ at } 90^{\circ}$  (a) and  $120^{\circ}$  (b) observer angles.



FIG. 14. (Color online) Comparison of the PSD predictions of the LES-FW–H model on three different grid resolutions with the LTRAC experiment for NPR =  $4.2 \text{ at } 90^{\circ}$  (a) and  $120^{\circ}$  (b) observer angles.

convergent nozzle corresponding to the LTRAC experiment at Monash University are obtained using the LES and RANS methods. The LES is based on the high-resolution CABARET solver accelerated on GPUs with asynchronous time stepping. The LES solution starts from the nozzle exit, where the inlet boundary condition is imposed using the available PIV data from the LTRAC experiment. For the far-field noise predictions, the LES solution is coupled with the FW-H method based on a permeable control surface with multiple closing disks. The RANS model is based on solving the 3D-axisymmetric equations using the  $\kappa - \omega$ SST turbulence model, which includes the compressibility corrections and accounts for the adverse pressure gradients typical of supersonic flows. For the far-field noise modelling with the RANS solution, two versions of the Morris and Miller (2010) model are implemented: accounting for and not accounting for the mechanism of sound scattering by the shock-cells in the definition of the acoustic correlation scale. As it turns out, both models converge to the same result for the LTRAC jets considered, which can be attributed to the screeching mechanism of these jets.

For sufficient grid resolution, the LES solution shows good agreement with the LTRAC PIV data for the timeaveraged velocity fields, accurately capturing the shock-cell structure over the first four shock-cells for both the NPR = 3.4 and NPR = 4.2 jet cases. In the latter case, the Mach disk region is also accurately captured. In comparison with the LES, the RANS solution is less accurate for resolving the shock-cell structure, especially at the centerline of the high-speed NPR = 4.2 jet. However, the RANS model better predicts the turbulent kinetic energy distribution in the jet shear layers in comparison with the LES solution, which over-predicts the turbulent kinetic energy levels. The reason for the turbulence intensity amplification could be due to the insufficient resolution of the LES solution and also the initially laminar turbulent inflow condition effect. Hence, future work will involve further LES grid refinement studies and imposing a synthetic turbulence inflow condition

upstream of the nozzle exhaust. The current predictions of the LES-FW–H models are within 1–2 dB from the RANSbased shock-associated noise model predictions and within 2–3 dB from the LTRAC experiment over the high frequency range 0.6 < St < 2, which is sufficiently above the frequencies influenced by jet noise. For low frequencies (0.04 < St < 0.5), where the LTRAC data are contaminated by noise, the LES-FW–H solution is within 2–3 dB from the sJet solution, which is based on empirical laws stemming from the NASA jet noise database.

In addition, the LES solution correctly captures the screech tone frequencies of the experiment at St = 0.6 and St = 0.9 for the 90° noise spectrum of the NPR = 3.4 LTRAC jet and also those at St = 0.2 and St = 0.9 for the 120° spectrum of the same jet. Furthermore, the LES-FW–H solution of the NPR = 4.2 LTRAC jet resolves the screech tone frequency of St = 0.4 for both the 90° and 120° observer angles. Notably, as the LES grid resolution increase from  $24 \times 10^6$  to  $70 \times 10^6$  cells, the noise spectra predictions of the LES-FW–H model of the NPR = 4.2 jet consistently approach the LTRAC experimental spectra at high frequencies.

The favorable comparison of the LES-FW–H results for the LTRAC jets with those obtained by the RANS-based acoustic analogy model within the range of frequencies relevant for BBSAN suggests that the acoustic analogy approach remains an attractive tool for quick-turn-aroundtime evaluations of shock-associated jet noise.

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