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Asynchronous PID Control for T-S Fuzzy Systems Over Gilbert-Elliott Channels Utilizing Detected Channel Modes

Yezheng Wang, Zidong Wang, Lei Zou, Quanbo Ge, and Hongli Dong

***Abstract*—This paper is concerned with the H_∞ proportional-integral-derivative (PID) control problem for Takagi-Sugeno fuzzy systems over lossy networks that are characterized by the Gilbert-Eillott model. The communication quality is reflected by the presence of two channel modes (i.e., "bad" mode and "good" mode), which switch randomly according to a Markov process. In the "bad" mode, packet dropouts are governed by a stochastic variable sequence. Considering the inaccessibility of channel modes, a mode detector is utilized to estimate the communication situation. The relationship between the actual channel mode and the estimated mode is depicted in terms of certain conditional probabilities. Moreover, a comprehensive model is constructed to represent the probability uncertainties arising from statistical errors in channel mode switching, packet dropouts, and mode detection processes. Subsequently, a robust asynchronous PID controller, based on the detected channel mode, is proposed. Sufficient conditions are then derived to ensure the mean-square stability of the closed-loop system while maintaining the desired H_∞ performance. Finally, the efficacy of the proposed design approach is demonstrated through a simulation example.**

***Index Terms*—Fuzzy system, proportional-integral-derivative control, Takagi-Sugeno fuzzy model, Gilbert-Elliott channel, asynchronous control.**

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I. Introduction

The Takagi-Sugeno (T-S) fuzzy technique emerges as a potent tool for addressing the challenges of identification, control, and filtering in nonlinear plants. This technique relies on simple linear submodels to establish the local input-output relationship of the nonlinear dynamics through T-S fuzzy modeling. Consequently, the global system is approximated via fuzzy blending. T-S fuzzy models/systems not only achieve the requisite approximation accuracy for a wide array of complex nonlinear functions, but they also feature a clear and comprehensible structure, thereby garnering increasing research interest. For instance, nonlinear systems have been represented by a class of affine T-S fuzzy models in [1], where the H_∞ control problem has been addressed through an effective non-synchronous fuzzy controller. Additional notable contributions can be found in [1]–[9].

Fuzzy control, a fundamental aspect of control theory, focuses on developing appropriate fuzzy controllers to fulfill specific performance criteria in controlled nonlinear systems. To date, various mature fuzzy control strategies have been established which predominantly encompass state-feedback-based and output-feedback-based control methods. The output-feedback controller, in contrast to its state-feedback counterpart, is deemed more practical due to its reliance on system measurement outputs, which are readily obtained via sensors. The proportional-integral-derivative (PID) controller, a prevalent form of output-feedback controller, boasts significant advantages such as robustness, fault-tolerance, and a well-established systematic parameter tuning theory, which has led to its widespread adoption in diverse industrial processes [10]–[13]. In recent years, the fuzzy PID control problem has attracted increased attention, with research efforts concentrated on enhancing system performance and addressing complexities in systems, see e.g. [14]–[18].

Networked control systems (NCSs) have recently found widespread applications in various domains including smart homes, automated traffic control, and geological exploration. Utilizing wireless communication techniques, these systems interconnect components like sensors and controllers through shared networks. This approach offers benefits such as extended transmission distances, high feasibility, and reduced costs [19]–[22]. From a system design standpoint, the primary challenges in analyzing NCSs arise from network-induced phenomena (NIP) which include, but are not limited to, channel fadings, packet dropouts, and transmission delays.

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NIP significantly complicates the transmission process, being a principal factor in degrading NCS performance. Consequently, these challenges have gained special attention within the control community [23]–[34]. Particularly, the fuzzy control is not only capable of modeling complex nonlinearities but also provides a proper framework of dealing with kinds of NIP, and thus has proven to be effective for nonlinear NCSs [1], [2], [7].

Packet dropout is a prevalent NIP in NCSs arising from various factors. Firstly, in communication networks with limited capacity, bandwidth constraints can result in data congestion when there are simultaneous and numerous network access requests. This congestion often leads to the failure of data packets to reach their intended destinations. Secondly, the open nature of NCSs exposes them to cyber threats such as denial-of-service attacks. In these scenarios, a deluge of interference signals is intentionally directed into the communication channels, deliberately causing data congestion. In addressing these issues, the Bernoulli-type model has been extensively adopted in the literature to characterize packet dropouts. This model serves as a foundation for tackling numerous control and filtering challenges in NCSs, see e.g. [35]–[37].

In real-world scenarios, packet dropouts in NCSs are intricately linked to the fluctuating channel conditions that are often impacted by a dynamic and sometimes abruptly changing communication environment. Consequently, the probability of packet loss varies across different time periods. The traditional Bernoulli-type model, which assumes a constant packet loss probability, does not fully capture this variability. In contrast, the Gilbert and Elliott model, introduced in [38], [39], offers a more comprehensive representation. This model effectively characterizes burst errors in wireless communication by describing the channel condition through two distinct modes: “good” and “bad.” The “good” mode represents a channel state free of data losses, whereas the “bad” mode is characterized by burst errors leading to data losses with certain probabilities. By accounting for the temporary nature of channel conditions, the Gilbert-Elliott model provides a more realistic depiction of communication in NCSs.

The exploration of optimization, control, and filtering issues in NCSs operating over Gilbert-Elliott channels has seen a surge in research interest in recent years, resulting in a substantial body of literature. A key focus in these studies is the channel mode or state, which is crucial in determining the quality of communication. This aspect has been given special attention in various research endeavors. For example, a mode-dependent filter has been developed in [40] for multi-rate systems in sensor networks. In [41], a mode-independent filter has been proposed to address the challenges posed by packet dropouts and measurement anomalies. In [42], a distributed channel access scheme has been designed that is applicable to both known and unknown Gilbert-Elliott channels. Furthermore, an optimal linear filter has been put forward in [43] for Gilbert-Elliott channels, specifically tailored to handle unobservable packet losses.

Upon reviewing the literature related to Gilbert-Elliott channels, several key observations have been made. 1) A number of papers have focused on channel-mode-dependent filtering, operating under the assumption that channel modes are accurately and timely obtained [40]. However, this assumption often does not hold in complex environments due to difficulties in precisely identifying the channel situation. 2) Mode-independent methods have been employed to obtain some results [41], [43], which tend to introduce significant conservatism due to the lack of channel information consideration. 3) In current Gilbert-Elliott models, the probability value of packet dropouts and mode switching are presumed to be *exact*, but such an assumption fails to account for statistical errors that may arise from experimental data [40], [41], [43], [44]. 4) The majority of controllers discussed in the literature are of the proportional type [44], [45], and there appears to be a lack of comprehensive research on PID control problems, especially for general nonlinear systems over uncertain Gilbert-Elliott channels due primarily to complexities in design.

Summarizing the previous discussions, our research focuses on exploring the H_∞ PID control problem for T-S fuzzy systems operating over Gilbert-Elliott channels. The primary challenges in this area include: 1) developing a viable channel model that supports the effective implementation of the PID controller; 2) analyzing the performance of the closed-loop system particularly in the context of uncertain probability information; and 3) determining optimal controller gains while comprehensively considering all complex factors encountered. In addressing these challenges, the following key contributions are made in this paper.

1) This study represents the first endeavor to tackle the fuzzy PID control issue for T-S fuzzy systems over Gilbert-Elliott channels with multiple uncertain information, where the encountered complexities have not been dealt with systematically in the existing works.
2) A thorough transmission model is proposed, which not only encapsulates the core characteristics of Gilbert-Elliott packet dropouts but also accurately represents the uncertainties associated with data loss probabilities, mode transitions, and mode detection. The proposed method relaxes the strict assumption of accurately known channel information used in the literature, and the built model is more general than the existing models [40]–[44].
3) An asynchronous fuzzy PID controller is developed, ensuring the H_∞ performance of the fuzzy system. Compared to the existing proportional-type (P-type) controller [4], [6], [23], the proposed controller enjoys more design freedom which could improve the system performance under the unknown transmission environment. In addition, the devised fuzzy controller processes a flexible structure due to the independent construction of membership functions for the system and controller.

The structure of the remainder of this paper is outlined as follows. Section II offers a detailed description of various components, including the system model, transmission model, controller structure, and the performance index. Section III introduces two key theorems: the first relates to system analysis, and the second pertains to controller design. Section IV provides simulation results to validate the effectiveness of

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the proposed method. Finally, Section V concludes the paper by summarizing the main findings and implications of this research.

II. Problem Statement and Preliminaries

A. Discrete-time Fuzzy System

Consider the following class of T-S fuzzy systems:

$$\begin{cases} x(n+1) = \sum_{o=1}^{\bar{o}} m_o(\theta(n))\big(A_o x(n) + B_o u(n) + E_o v(n)\big) \\ z(n) = \sum_{o=1}^{\bar{o}} m_o(\theta(n)) N_o x(n) \\ y(n) = Cx(n) + Dv(n) \end{cases} \tag{1}$$

where n is the sampling time instant; the positive integral $\bar{o}$ refers to the total number of fuzzy rules; $x(n) \in \mathbb{R}^{l_x}$ is the system state; $y(n) \in \mathbb{R}^{l_y}$ is the measurement output that is collected by sensors; $z(n) \in \mathbb{R}^{l_z}$ is the variable controlled to satisfy certain performance requirements; $v(n) \in \mathbb{R}^{l_v}$ is the energy-bounded noise; $u(n) \in \mathbb{R}^{l_u}$ is the control input; A_o, B_o, E_o, C, D and N_o are known matrices with proper dimensions; $\theta(n)$ is the premise variable vector; and $m_o(\theta(n))$ is the membership function related to the o-th rule with the following properties:

$$\sum_{o=1}^{\bar{o}} m_o(\theta(n)) = 1, \quad m_o(\theta(n)) \geq 0, \quad o = 1, 2, \cdots, \bar{o}.$$

B. Network-Based Signal Transmissions

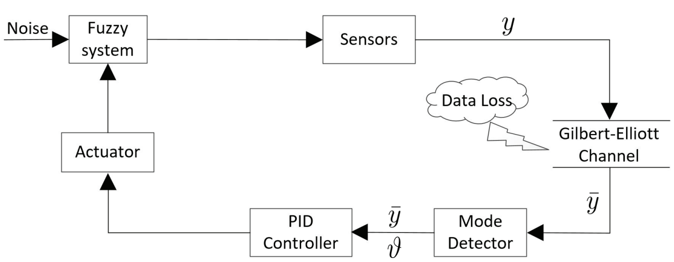

Fig. 1: Remote PID control over Gilbert-Elliott channel

The overall system chart is depicted in Fig. 1, illustrating the process where the measurement outputs are relayed to the remote controller via a lossy channel, characterized by the Gilbert-Elliott model. This setup includes the consideration of packet dropouts. As a result, the transmitted measurement, which accounts for the effects of these packet dropouts, is described as follows:

$$\bar{y}(n) = \delta(\alpha(n), 1)y(n) + \delta(\alpha(n), 2)\beta(n)y(n) \tag{2}$$

where $\bar{y}(n)$ denotes the information received by the controller, $\delta(\cdot)$ is the Kronecker delta function, $\alpha(n) \in \{1, 2\}$ is a Markov chain standing for the channel mode, and $\beta(n) \in \{0, 1\}$ is the independent and identically distributed stochastic variable satisfying:

$$\mathbf{Prob}\{\beta(n) = 1\} \triangleq \bar{\beta} + \Delta_\beta,$$

$$\mathbf{Prob}\{\beta(n) = 0\} \triangleq 1 - \bar{\beta} - \Delta_\beta.$$

Here, $\bar{\beta} \in (0, 1)$ is a precisely known scalar and Δ_β is an unknown scalar representing the uncertainty. It is assumed that $\underline{\Delta}_\beta \leq \Delta_\beta \leq \bar{\Delta}_\beta$ with $\underline{\Delta}_\beta$ and $\bar{\Delta}_\beta$ being two known scalars, and $0 < \bar{\beta} + \Delta_\beta < 1$, $0 < 1 - \bar{\beta} - \Delta_\beta < 1$. The transition probability matrix of $\alpha(n)$ is given as follows:

$$\Pi \triangleq \begin{bmatrix} \bar{\pi}_{1,1} & \bar{\pi}_{1,2} \\ \bar{\pi}_{2,1} & \bar{\pi}_{2,2} \end{bmatrix}$$

where $\mathbf{Prob}\{\alpha(n+1) = e|\alpha(n) = p\} \triangleq \bar{\pi}_{p,e}$, $\bar{\pi}_{p,1} \triangleq \pi_{p,1} + \Delta_p^{(\pi)}$, $\bar{\pi}_{p,2} = 1 - \bar{\pi}_{p,1}$ ($p = 1, 2,\ e = 1, 2$); $\pi_{p,1}$ is known and $\Delta_p^{(\pi)}$ is unknown but satisfies $|\Delta_p^{(\pi)}| \leq 2\epsilon_p$ with ϵ_p being known scalars. Clearly, $0 < \pi_{p,1} + \Delta_p^{(\pi)} < 1$ and $0 < 1 - \pi_{p,1} - \Delta_p^{(\pi)} < 1$ should hold, and it is easy to see that $\bar{\pi}_{p,1} + \bar{\pi}_{p,2} = 1$.

Remark 1: The Gilbert-Elliott model (2) serves as a comprehensive framework for characterizing lossy communication channels. This model captures the dynamic nature of communication quality through a Markov process denoted as $\alpha(n)$. Within this framework, $\alpha(n) = 1$ signifies a "good" channel condition indicating a stable and reliable transmission environment where data is less likely to be lost, and $\alpha(n) = 2$ corresponds to a "bad" channel condition implying a scenario where the transmitted data is prone to experiencing losses. Notably, under specific conditions, this model can be simplified into other well-known forms: 1) if $\alpha(n) \equiv 2$, the model simplifies to the Bernoulli-type packet dropout model [35]–[37], which is a common representation for channels with consistent data loss; and 2) if $\bar{\beta} + \Delta_\beta = 0$, the model converges to the simplified version of the Gilbert-Elliott model [41], [43], [46], and this variant is particularly relevant for scenarios with less variability in the channel conditions.

By defining $\tilde{\beta}(n) \triangleq \beta(n) - \bar{\beta} - \Delta_\beta$, the output model (2) can be rewritten by

$$\begin{aligned} \bar{y}(n) = & \Big(\delta(\alpha(n), 1) + \delta(\alpha(n), 2)\big(\tilde{\beta}(n) + \bar{\beta} + \Delta_\beta\big)\Big) Cx(n) \\ & + \Big(\delta(\alpha(n), 1) + \delta(\alpha(n), 2)\big(\tilde{\beta}(n) + \bar{\beta} + \Delta_\beta\big)\Big) \\ & \times Dv(n). \end{aligned} \tag{3}$$

C. Fuzzy PID Controller

In practice, it is difficult to measure the channel mode accurately at each time instant. Thus, we employ a mode detector to observe the channel mode. Let $\vartheta(n) \in \{1, 2\}$ be the detected channel mode at the controller side. The relation between $\vartheta(n)$ and the real channel mode $\alpha(n)$ is described by

$$\mathbf{Prob}\{\vartheta(n) = q|\alpha(n) = p\} \triangleq \bar{\kappa}_{p,q},\ p = 1, 2,\ q = 1, 2$$

where $\bar{\kappa}_{p,q} \triangleq \kappa_{p,q} + \Delta_\kappa^{(p)}$; $\kappa_{p,q}$ is a known scalar; $\Delta_\kappa^{(p)}$ denotes the uncertainty and satisfies $|\Delta_\kappa^{(p)}| \leq 2\upsilon_p$ with $\upsilon_p > 0$ being a known scalar. It is assumed that $0 < \kappa_{p,q} + \Delta_\kappa^{(p)} < 1$, $0 < 1 - \kappa_{p,q} - \Delta_\kappa^{(p)} < 1$, $\bar{\kappa}_{p,1} + \bar{\kappa}_{p,2} = 1$, and $\beta(n)$ is mutually independent with $\alpha(n)$ and $\vartheta(n)$.

Remark 2: In this paper, we recognize that the probability information of stochastic variables is typically obtained through extensive statistical experiments. However, due to limitations in measurement equipment and calculation methods,

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the derived probability information can often be inaccurate. To address this issue, our approach incorporates bounded uncertainties into the modeling of three key processes.

1) *The Packet Dropout Process*: Here, we acknowledge and account for potential inaccuracies in estimating the probability of packet loss, a critical factor in lossy communication channels.
2) *The Mode Switching Process of Channels*: This aspect of the model includes uncertainties related to predicting the transitions between different channel states (e.g., from "good" to "bad" modes), which are pivotal for understanding and managing data transmission reliability.
3) *The Mode Detection Process*: In this part, we introduce uncertainties to reflect the challenges in accurately detecting the current operational mode of the channel, and this is particularly important given the variable nature of network conditions and the limitations of detection technologies.

By utilizing the detected mode $\vartheta(n)$ and the output $\bar{y}(n)$, we adopt the following fuzzy PID controller:

$$u(n)=\sum_{s=1}^{\hat{o}}\bar{m}_s(\bar{\theta}(n))\bigg(K^P_{s,\vartheta(n)}\bar{y}(n)+K^I_{s,\vartheta(n)}\sum_{\tau=0}^{n-1}\bar{y}(\tau) + K^D_{s,\vartheta(n)}\big(\bar{y}(n)-\bar{y}(n-1)\big)\bigg) \tag{4}$$

where $K^P_{s,\vartheta(n)}$, $K^I_{s,\vartheta(n)}$ and $K^D_{s,\vartheta(n)}$ are proportional, integral and derivative gains, respectively; $\hat{o}>0$ is an integer denoting the number of fuzzy rules; and $\bar{m}_s(\bar{\theta}(n))$ is the membership function of the controller.

For the purpose of simplifying the system description, an auxiliary variable is defined as follows:

$$x_a(n)\triangleq\begin{cases}0, & n=0\\ \sum_{\tau=0}^{n-1}\bar{y}(\tau), & n>0.\end{cases}$$

Then, it can be deduced that

$$x_a(n+1)=\sum_{\tau=0}^{n}\bar{y}(\tau)=x_a(n)+\bar{y}(n).$$

By substituting (4) into the fuzzy system (1) and using the special augmentation method, we derive the closed-loop system as follows:

$$\xi(n+1)=\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\bigg(\Big(\mathcal{A}^{o,s}_{\alpha(n),\vartheta(n)} + \big(\tilde{\beta}(n)+\Delta_\beta\big)\mathcal{C}^{o,s}_{\alpha(n),\vartheta(n)}\Big)\xi(n)+\Big(\mathcal{E}^{o,s}_{\alpha(n),\vartheta(n)} + \big(\tilde{\beta}(n)+\Delta_\beta\big)\mathcal{F}^{o,s}_{\alpha(n),\vartheta(n)}\Big)v(n)\bigg) \tag{5}$$

and

$$z(k)=\sum_{o=1}^{\bar{o}}m_o(\theta(n))\mathcal{N}_o\xi(n) \tag{6}$$

where

$$\xi(n)\triangleq\begin{bmatrix}x^T(n) & x_a^T(n) & \bar{y}^T(n-1)\end{bmatrix}^T,$$

$$\mathcal{A}^{o,s}_{\alpha(n),\vartheta(n)}\triangleq\begin{bmatrix}\bar{\mathcal{A}}^{o,s}_{\alpha(n),\vartheta(n)} & B_oK^I_{s,\vartheta(n)} & -B_oK^D_{s,\vartheta(n)}\\ \bar{\mathcal{A}}^{(2,1)}_{\alpha(n)} & I & 0\\ \bar{\mathcal{A}}^{(3,1)}_{\alpha(n)} & 0 & 0\end{bmatrix},$$

$$\bar{\mathcal{A}}^{o,s}_{\alpha(n),\vartheta(n)}\triangleq A_o+\delta(\alpha(n),1)\Big(B_oK^P_{s,\vartheta(n)}C+B_oK^D_{s,\vartheta(n)}C\Big) + \delta(\alpha(n),2)\bar{\beta}\Big(B_oK^P_{s,\vartheta(n)}C+B_oK^D_{s,\vartheta(n)}C\Big),$$

$$\bar{\mathcal{A}}^{(2,1)}_{\alpha(n)}\triangleq\delta(\alpha(n),1)C+\delta(\alpha(n),2)\bar{\beta}C,\ \bar{\mathcal{A}}^{(3,1)}_{\alpha(n)}=\bar{\mathcal{A}}^{(2,1)}_{\alpha(n)},$$

$$\mathcal{C}^{o,s}_{\alpha(n),\vartheta(n)}\triangleq\begin{bmatrix}\bar{\mathcal{C}}^{o,s}_{\alpha(n),\vartheta(n)} & 0 & 0\\ \delta(\alpha(n),2)C & 0 & 0\\ \delta(\alpha(n),2)C & 0 & 0\end{bmatrix},\ \mathcal{N}_o\triangleq\begin{bmatrix}N_o & 0 & 0\end{bmatrix},$$

$$\bar{\mathcal{C}}^{o,s}_{\alpha(n),\vartheta(n)}\triangleq\delta(\alpha(n),2)\Big(B_oK^P_{s,\vartheta(n)}C+B_oK^D_{s,\vartheta(n)}C\Big),$$

$$\mathcal{E}^{o,s}_{\alpha(n),\vartheta(n)}\triangleq\begin{bmatrix}\bar{\mathcal{E}}^{o,s}_{\alpha(n),\vartheta(n)}\\ \delta(\alpha(n),1)D+\delta(\alpha(n),2)\bar{\beta}D\\ \delta(\alpha(n),1)D+\delta(\alpha(n),2)\bar{\beta}D\end{bmatrix},$$

$$\bar{\mathcal{E}}^{o,s}_{\alpha(n),\vartheta(n)}\triangleq E_o+\delta(\alpha(n),1)\Big(B_oK^P_{s,\vartheta(n)}D+B_oK^D_{s,\vartheta(n)}D\Big) + \delta(\alpha(n),2)\bar{\beta}\Big(B_oK^P_{s,\vartheta(n)}D+B_oK^D_{s,\vartheta(n)}D\Big),$$

$$\mathcal{F}^{o,s}_{\alpha(n),\vartheta(n)}\triangleq\begin{bmatrix}\delta(\alpha(n),2)\Big(B_oK^P_{s,\vartheta(n)}D+B_oK^D_{s,\vartheta(n)}D\Big)\\ \delta(\alpha(n),2)D\\ \delta(\alpha(n),2)D\end{bmatrix}.$$

For the considered nonlinear systems subject to energy-bounded noises over Gilbert-Elliott channels with multiple unknown information. Our purpose is to design a fuzzy PID controller such that the desired stability and robustness can be achieved, i.e.:

1) the closed-loop system (5) is mean-square asymptotically stable in the absence of external noises; and
2) given a scalar $\gamma>0$, for any energy-bounded noises $v(n)\neq 0$, the following H_∞ performance requirement is satisfied under the zero initial condition:

$$\mathbb{E}\left\{\frac{\sum_{n=0}^{\infty}z^T(n)z(n)}{\sum_{n=0}^{\infty}v^T(n)v(n)}\right\}<\gamma^2. \tag{7}$$

III. Main Results

The following lemma is introduced before presenting the main results.

Lemma 1: [47] Given any real number ϵ and any square matrix P, the following inequality holds for any matrix T with proper dimension:

$$\epsilon(P+P^T)\leq\epsilon^2T+PT^{-1}P^T.$$

In Theorem 1 given below, we present sufficient conditions that ensure the mean-square stability as well as the prescribed H_∞ performance for the controlled system with given specific controller parameters.

Theorem 1: Consider the fuzzy system (1) and the fuzzy PID controller (4). Let the parameters $K^P_{s,q}$, $K^I_{s,q}$ and $K^D_{s,q}$ be given. Then, the closed-loop system (5) is mean-square asymptotically stable and the H_∞ performance is achieved if, for $o,s=1,2,\cdots,\bar{o}$, $p,q=1,2$, there exist symmetric positive definite matrices $P_p\in\mathbb{R}^{l_x+2l_y}$, $T_p\in\mathbb{R}^{5(l_x+2l_y)+l_z}$, $M_p\in$

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$\mathbb{R}^{5(l_x+2l_y)+l_z}$, $R_{p,q} \in \mathbb{R}^{l_x+2l_y+n_v}$ and $S_p \in \mathbb{R}^{l_x+2l_y+n_v}$ such that

$$\pi_{p,1}(\check{P}_1 - \check{P}_2) + \check{P}_2 + \bar{T}_p < M_p \tag{8}$$

$$\left(\tilde{\mathcal{A}}^{o,s}_{p,q}\right)^T M_p \tilde{\mathcal{A}}^{o,s}_{p,q} - R_{p,q} < 0 \tag{9}$$

$$\kappa_{p,1}(R_{p,1} - R_{p,2}) + R_{p,2} + \bar{S}_p < 0 \tag{10}$$

where

$$\begin{aligned}
\check{P}_p &\triangleq \mathrm{diag}\{\underbrace{P_p, \cdots, P_p}_{5}, I\},\\
\bar{T}_p &\triangleq \epsilon_p^2 T_p + (\check{P}_1 - \check{P}_2)T_p^{-1}(\check{P}_1 - \check{P}_2),\\
\tilde{\mathcal{A}}^{o,s}_{p,q} &\triangleq \begin{bmatrix} \mathcal{A}^{o,s}_{p,q} & \mathcal{E}^{o,s}_{p,q} \\ \sqrt{2\hat{\beta} + \frac{13}{4}\bar{\Delta}_\beta^2}\mathcal{C}^{o,s}_{p,q} & 0 \\ 0 & \sqrt{2\hat{\beta} + 4\bar{\Delta}_\beta^2}\mathcal{F}^{o,s}_{p,q} \\ \frac{\sqrt{5}}{2}\mathcal{A}^{o,s}_{p,q} & 0 \\ 0 & \frac{\sqrt{5}}{2}\mathcal{E}^{o,s}_{p,q} \\ \mathcal{N}_o & 0 \end{bmatrix},\\
\bar{S}_p &\triangleq \upsilon_p^2 S_p + (R_{p,1} - R_{p,2})S_p^{-1}(R_{p,1} - R_{p,2}) + \Gamma_p,\\
\Gamma_p &\triangleq \mathrm{diag}\{-P_p, -\gamma^2 I\},\ \hat{\beta} \triangleq \bar{\beta} + \bar{\Delta}_\beta - (\bar{\beta} + \underline{\Delta}_\beta)^2.
\end{aligned}$$

Proof: Choose the following mode-dependent Lyapunov function candidate:

$$V(n, \alpha(n)) \triangleq \xi^T(n) P_{\alpha(n)} \xi(n).$$

The conditional mathematical expectation of the difference of $V(k, \alpha(n))$ is calculated by

$$\begin{aligned}
&\mathbb{E}\left\{V(n+1, \alpha(n+1)) - V(n, \alpha(n)) | \xi(n), \alpha(n)\right\}\\
=&\mathbb{E}\left\{V(n+1, \alpha(n+1)) | \xi(n), \alpha(n)\right\} - V(n, \alpha(n))\\
=&\mathbb{E}\left\{\xi^T(n+1) P_{\alpha(n+1)} \xi(n+1) | \xi(n), \alpha(n)\right\}\\
&- \zeta^T(n) P_{\alpha(n)} \zeta(n)\\
=&\mathbb{E}\Bigg\{\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}\sum_{i=1}^{\bar{o}}\sum_{j=1}^{\hat{o}} m_o(\theta(n))\bar{m}_s(\bar{\theta}(n)) m_i(\theta(n))\bar{m}_j(\bar{\theta}(n))\\
&\times \Bigg(\left(\mathcal{A}^{o,s}_{\alpha(n),\vartheta(n)} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{C}^{o,s}_{\alpha(n),\vartheta(n)}\right)\xi(n)\\
&+ \left(\mathcal{E}^{o,s}_{\alpha(n),\vartheta(n)} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{F}^{o,s}_{\alpha(n),\vartheta(n)}\right)v(n)\Bigg)^T P_{\alpha(n+1)}\\
&\times \Bigg(\left(\mathcal{A}^{i,j}_{\alpha(n),\vartheta(n)} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{C}^{i,j}_{\alpha(n),\vartheta(n)}\right)\xi(n)\\
&+ \left(\mathcal{E}^{i,j}_{\alpha(n),\vartheta(n)} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{F}^{i,j}_{\alpha(n),\vartheta(n)}\right)v(n)\Bigg)\\
&- \zeta^T(n) P_{\alpha(n)} \zeta(n) | \xi(n), \alpha(n)\Bigg\}.
\end{aligned}$$

Using the elementary inequality $2X^TOY \le X^TOX + Y^TOY$ (for any real matrices X, Y and $O > 0$ with proper dimensions) and letting $\alpha(n) = p$ ($p \in \{1, 2\}$), one has

$$\begin{aligned}
&\mathbb{E}\left\{V(n+1, \alpha(n+1)) - V(n, \alpha(n)) | \xi(n), \alpha(n) = p\right\}\\
\le&\mathbb{E}\Bigg\{\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}} m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\\
&\times \Bigg(\left(\mathcal{A}^{o,s}_{\alpha(n),\vartheta(n)} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{C}^{o,s}_{\alpha(n),\vartheta(n)}\right)\xi(n)\\
&+ \left(\mathcal{E}^{o,s}_{\alpha(n),\vartheta(n)} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{F}^{o,s}_{\alpha(n),\vartheta(n)}\right)v(n)\Bigg)^T \tilde{P}_p\\
&\times \Bigg(\left(\mathcal{A}^{o,s}_{\alpha(n),\vartheta(n)} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{C}^{o,s}_{\alpha(n),\vartheta(n)}\right)\xi(n)\\
&+ \left(\mathcal{E}^{o,s}_{\alpha(n),\vartheta(n)} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{F}^{o,s}_{\alpha(n),\vartheta(n)}\right)v(n)\Bigg)\\
&- \xi^T(n) P_{\alpha(n)} \xi(n) | \xi(n), \alpha(n) = p\Bigg\}
\end{aligned} \tag{11}$$

where

$$\tilde{P}_p \triangleq \sum_{e=1}^{2} \bar{\pi}_{p,e} P_e.$$

From (11), one can further obtain that

$$\begin{aligned}
&\mathbb{E}\left\{V(n+1, \alpha(n+1)) - V(n, \alpha(n)) | \xi(n), \alpha(n) = p\right\}\\
\le&\mathbb{E}\Bigg\{\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}} m_o(\theta(n))\bar{m}_s(\bar{\theta}(n)) \sum_{q=1}^{2}\bar{\kappa}_{p,q}\Bigg(\Big(\mathcal{A}^{o,s}_{p,q} + (\tilde{\beta}(n)\\
&+ \Delta_\beta)\mathcal{C}^{o,s}_{p,q}\Big)\xi(n) + \left(\mathcal{E}^{o,s}_{p,q} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{F}^{o,s}_{p,q}\right)v(n)\Bigg)^T \tilde{P}_p\\
&\times \Bigg(\left(\mathcal{A}^{o,s}_{p,q} + (\tilde{\beta}(n) + \Delta_\beta)\mathcal{C}^{o,s}_{p,q}\right)\xi(n) + \Big(\mathcal{E}^{o,s}_{p,q} + (\tilde{\beta}(n)\\
&+ \Delta_\beta)\mathcal{F}^{o,s}_{p,q}\Big)v(n)\Bigg) - \xi^T(n) P_p \xi(n) | \xi(n), \alpha(n) = p\Bigg\}\\
=&\mathbb{E}\Bigg\{\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}} m_o(\theta(n))\bar{m}_s(\bar{\theta}(n)) \sum_{q=1}^{2}\bar{\kappa}_{p,q}\Bigg(\xi^T(n)\Big((\mathcal{A}^{o,s}_{p,q})^T\\
&\times \tilde{P}_p\mathcal{A}^{o,s}_{p,q} + \tilde{\beta}^2(n)(\mathcal{C}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{C}^{o,s}_{p,q} + \Delta_\beta^2(\mathcal{C}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{C}^{o,s}_{p,q}\\
&+ 2\tilde{\beta}(n)(\mathcal{A}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{C}^{o,s}_{p,q} + 2\Delta_\beta(\mathcal{A}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{C}^{o,s}_{p,q} + 2\tilde{\beta}(n)\Delta_\beta\\
&\times (\mathcal{C}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{C}^{o,s}_{p,q}\Big)\xi(n) + v^T(n)\Big((\mathcal{E}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{E}^{o,s}_{p,q} + \tilde{\beta}^2(n)\\
&\times (\mathcal{F}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q} + \Delta_\beta^2(\mathcal{F}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q} + 2\tilde{\beta}(n)(\mathcal{E}^{o,s}_{p,q})^T\tilde{P}_p\\
&\times \mathcal{F}^{o,s}_{p,q} + 2\Delta_\beta(\mathcal{E}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q} + 2\tilde{\beta}(n)\Delta_\beta(\mathcal{F}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q}\Big)\\
&\times v(n) + 2\xi^T(n)\Big((\mathcal{A}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{E}^{o,s}_{p,q} + \tilde{\beta}(n)(\mathcal{A}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q}\\
&+ \Delta_\beta(A^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q} + \tilde{\beta}(n)(\mathcal{C}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{E}^{o,s}_{p,q} + \tilde{\beta}^2(n)\\
&\times (\mathcal{C}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q} + \tilde{\beta}(n)\Delta_\beta(\mathcal{C}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q} + \Delta_\beta(\mathcal{C}^{o,s}_{p,q})^T\\
&\times \tilde{P}_p\mathcal{E}^{o,s}_{p,q} + \tilde{\beta}(n)\Delta_\beta(\mathcal{C}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q} + \Delta_\beta^2(\mathcal{C}^{o,s}_{p,q})^T\tilde{P}_p\mathcal{F}^{o,s}_{p,q}\Big)\\
&\times v(n) - \xi^T(n) P_p \xi(n) | \xi(n), \alpha(n) = p\Bigg\}.
\end{aligned} \tag{12}$$

It follows from the definition of $\tilde{\beta}(n)$ that

$$\begin{aligned}
\mathbb{E}\{\tilde{\beta}(n)\} =& \mathbb{E}\{\beta(n) - \bar{\beta} - \Delta_\beta\} = 0,\\
\mathbb{E}\{\tilde{\beta}^2(n)\} =& (\bar{\beta} + \Delta_\beta)(1 - \bar{\beta} - \Delta_\beta)\\
=& \bar{\beta} + \Delta_\beta - \bar{\beta}^2 - 2\bar{\beta}\Delta_\beta - \Delta_\beta^2\\
=& \bar{\beta} + \Delta_\beta - (\bar{\beta} + \Delta_\beta)^2\\
\le& \bar{\beta} + \bar{\Delta}_\beta - (\bar{\beta} + \underline{\Delta}_\beta)^2 = \hat{\beta}.
\end{aligned}$$

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The following inequalities can be established:

$$\begin{aligned}
&\xi^T(n)\Delta_\beta^2(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{C}_{p,q}^{o,s}\xi(n)\\
&\quad\le\xi^T(n)\bar{\Delta}_\beta^2(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{C}_{p,q}^{o,s}\xi(n),\\
&v^T(n)\Delta_\beta^2(\mathcal{F}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}v(n)\\
&\quad\le v^T(n)\bar{\Delta}_\beta^2(\mathcal{F}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}v(n),\\
&\xi^T(n)(\mathcal{A}_{p,q}^{o,s})^T\tilde{P}_p\Delta_\beta\mathcal{F}_{p,q}^{o,s}v(n)\\
&\quad=2\xi^T(n)\left(\frac{1}{2}\mathcal{A}_{p,q}^{o,s}\right)^T\tilde{P}_p\Delta_\beta\mathcal{F}_{p,q}^{o,s}v(n)\\
&\quad\le\frac{1}{4}\xi^T(n)(\mathcal{A}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{A}_{p,q}^{o,s}\xi(n)\\
&\qquad+v^T(n)\bar{\Delta}_\beta^2(\mathcal{F}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}v(n),\\
&\xi^T(n)\Delta_\beta(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{E}_{p,q}^{o,s}v(n)\\
&\quad\le\xi^T(n)\bar{\Delta}_\beta^2(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{C}_{p,q}^{o,s}\xi(n)\\
&\qquad+\frac{1}{4}v^T(n)(\mathcal{E}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{E}_{p,q}^{o,s}v(n),\\
&\xi^T(n)\Delta_\beta^2(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}v(n)\\
&\quad\le\frac{1}{4}\xi^T(n)\bar{\Delta}_\beta^2(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{C}_{p,q}^{o,s}\xi(n)\\
&\qquad+v^T(n)\bar{\Delta}_\beta^2(\mathcal{F}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}v(n),\\
&2\xi^T(n)(\mathcal{A}_{p,q}^{o,s})^T\tilde{P}_p\Delta_\beta\mathcal{C}_{p,q}^{o,s}\xi(n)\\
&\quad\le\xi^T(n)\Big((\mathcal{A}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{A}_{p,q}^{o,s}\xi(n)+\bar{\Delta}_\beta^2(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{C}_{p,q}^{o,s}\Big)\xi(n),\\
&2v^T(n)\Delta_\beta(\mathcal{E}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}v(n)\\
&\quad\le v^T(n)\Big((\mathcal{E}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{E}_{p,q}^{o,s}+\bar{\Delta}_\beta^2(\mathcal{F}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}\Big)v(n).
\end{aligned}$$

Furthermore, we have from (12) that

$$\begin{aligned}
&\mathbb{E}\left\{V(n+1,\alpha(n+1))-V(n,\alpha(n))|\xi(n),\alpha(n)=p\right\}\\
&\le\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\sum_{q=1}^{2}\bar{\kappa}_{p,q}\Bigg(\Big(\mathcal{A}_{p,q}^{o,s}\xi(n)\\
&\quad+\mathcal{E}_{p,q}^{o,s}v(n)\Big)^T\tilde{P}_p\Big(\mathcal{A}_{p,q}^{o,s}\xi(n)+\mathcal{E}_{p,q}^{o,s}v(n)\Big)+\frac{5}{4}\xi^T(n)\\
&\quad\times(\mathcal{A}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{A}_{p,q}^{o,s}\xi(n)+\frac{5}{4}v^T(n)(\mathcal{E}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{E}_{p,q}^{o,s}v(n)\\
&\quad+\left(2\hat{\beta}+\frac{13}{4}\bar{\Delta}_\beta^2\right)\xi^T(n)(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{C}_{p,q}^{o,s}\xi(n)+(2\hat{\beta}+4\bar{\Delta}_\beta^2)\\
&\quad\times v^T(n)(\mathcal{F}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}v(n)\Bigg)-\xi^T(n)P_p\xi(n).
\end{aligned}\tag{13}$$

For the purpose of stability analysis, we first let $v(0)\equiv 0$ and then obtain from (13) that

$$\begin{aligned}
&\mathbb{E}\left\{V(n+1,\alpha(n+1))-V(n,\alpha(n))|\xi(n),\alpha(n)=p\right\}\\
&\le\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\sum_{q=1}^{2}\bar{\kappa}_{p,q}\Big(\xi^T(n)(\mathcal{A}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{A}_{p,q}^{o,s}\\
&\quad\times\xi(n)+\frac{5}{4}\xi^T(n)(\mathcal{A}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{A}_{p,q}^{o,s}\xi(n)+\left(2\hat{\beta}+\frac{13}{4}\bar{\Delta}_\beta^2\right)\\
&\quad\times\xi^T(n)(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{C}_{p,q}^{o,s}\xi(n)\Big)-\xi^T(n)P_p\xi(n)\\
&=\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\sum_{q=1}^{2}\bar{\kappa}_{p,q}\xi^T(n)\Big((\vec{\mathcal{A}}_{p,q}^{o,s})^T\\
&\quad\times\left(\bar{\pi}_{p,1}\vec{P}_1+\bar{\pi}_{p,2}\vec{P}_2\right)\vec{\mathcal{A}}_{p,q}^{o,s}-P_p\Big)\xi(n).
\end{aligned}\tag{14}$$

where

$$\vec{\mathcal{A}}_{p,q}^{o,s}\triangleq\begin{bmatrix}\mathcal{A}_{p,q}^{o,s}\\ \sqrt{2\hat{\beta}+\frac{13}{4}\bar{\Delta}_\beta^2}\,\mathcal{C}_{p,q}^{o,s}\\ \frac{\sqrt{5}}{2}\mathcal{A}_{p,q}^{o,s}\end{bmatrix},\quad\vec{P}_p\triangleq\operatorname{diag}\left\{P_p,P_p,P_p\right\}.$$

By using Lemma 1, one obtains that

$$\begin{aligned}
&\bar{\pi}_{p,1}\vec{P}_1+\bar{\pi}_{p,2}\vec{P}_2\\
&\quad=\left(\pi_{p,1}+\Delta_p^{(\pi)}\right)\vec{P}_1+\left(1-\pi_{p,1}-\Delta_p^{(\pi)}\right)\vec{P}_2\\
&\quad=\pi_{p,1}\left(\vec{P}_1-\vec{P}_2\right)+\vec{P}_2+\Delta_p^{(\pi)}\left(\vec{P}_1-\vec{P}_2\right)\\
&\quad=\pi_{p,1}\left(\vec{P}_1-\vec{P}_2\right)+\vec{P}_2+\frac{1}{2}\Delta_p^{(\pi)}\left(\vec{P}_1-\vec{P}_2\right)\\
&\qquad+\frac{1}{2}\Delta_p^{(\pi)}\left(\vec{P}_1-\vec{P}_2\right)\\
&\quad\le\pi_{p,1}\left(\vec{P}_1-\vec{P}_2\right)+\vec{P}_2+\left(\frac{1}{2}\Delta_p^{(\pi)}\right)^2T_p^{(1)}\\
&\qquad+\left(\vec{P}_1-\vec{P}_2\right)\left(T_p^{(1)}\right)^{-1}\left(\vec{P}_1-\vec{P}_2\right)\\
&\quad\le\pi_{p,1}\left(\vec{P}_1-\vec{P}_2\right)+\vec{P}_2+\epsilon_p^2T_p^{(1)}\\
&\qquad+\left(\vec{P}_1-\vec{P}_2\right)\left(T_p^{(1)}\right)^{-1}\left(\vec{P}_1-\vec{P}_2\right)
\end{aligned}$$

where $T_p^{(1)}\in\mathbb{R}^{3l_x+6l_y}$ is the 1×1-th block of T_p defined by

$$T_p\triangleq\begin{bmatrix}T_p^{(1)}&*\\ T_p^{(2)}&T_p^{(3)}\end{bmatrix}.$$

Choose M_p with the following structure:

$$M_p\triangleq\begin{bmatrix}M_p^{(1)}&*\\ M_p^{(2)}&M_p^{(3)}\end{bmatrix},\quad M_p^{(1)}\in\mathbb{R}^{3l_x+6l_y}.$$

Then, by conducting simple matrix manipulation according to (8), one obtains

$$\bar{\pi}_{p,1}\vec{P}_1+\bar{\pi}_{p,2}\vec{P}_2<M_p^{(1)},$$

which leads to

$$\begin{aligned}
&\mathbb{E}\left\{V(n+1,\alpha(n+1))-V(n,\alpha(n))|\xi(n),\alpha(n)=p\right\}\\
&\le\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\sum_{q=1}^{2}\bar{\kappa}_{p,q}\xi^T(n)\Big((\vec{\mathcal{A}}_{p,q}^{o,s})^T\\
&\quad\times M_p^{(1)}\vec{\mathcal{A}}_{p,q}^{o,s}-P_p\Big)\xi(n).
\end{aligned}\tag{15}$$

Similarly, choose $R_{p,q}$ with the following structure:

$$R_{p,q}\triangleq\begin{bmatrix}R_{p,q}^{(1)}&*\\ R_{p,q}^{(2)}&R_{p,q}^{(3)}\end{bmatrix},\quad R_{p,q}^{(1)}\in\mathbb{R}^{l_x+2l_y}.$$

Then, it can be deduced from (9) that

$$\begin{aligned}
&\mathbb{E}\left\{V(n+1,\alpha(n+1))-V(n,\alpha(n))|\xi(n),\alpha(n)=p\right\}\\
&\le\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\xi^T(n)\Big(\bar{\kappa}_{p,1}R_{p,1}^{(1)}+\bar{\kappa}_{p,2}R_{p,2}^{(1)}\\
&\quad-P_p\Big)\xi(n).
\end{aligned}$$

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Note that

$$\begin{aligned}&\bar{\kappa}_{p,1}R_{p,1}^{(1)}+\bar{\kappa}_{p,2}R_{p,2}^{(1)}\\&=\left(\kappa_{p,1}+\Delta_p^{(\kappa)}\right)R_{p,1}^{(1)}+\left(1-\kappa_{p,1}-\Delta_p^{(\kappa)}\right)R_{p,2}^{(2)}\\&\leq\kappa_{p,1}\left(R_{p,1}^{(1)}-R_{p,2}^{(1)}\right)+R_{p,2}^{(1)}+\upsilon_p^2S_p^{(1)}\\&\quad+\left(R_{p,1}^{(1)}-R_{p,2}^{(1)}\right)\left(S_p^{(1)}\right)^{-1}\left(R_{p,1}^{(1)}-R_{p,2}^{(1)}\right).\end{aligned}$$

Considering the property of fuzzy membership functions and using condition (10), one obtains

$$\mathbb{E}\left\{V(n+1,\alpha(n+1))-V(n,\alpha(n))|\xi(n),\alpha(n)=p\right\}<0.$$

By taking the mathematical expectation of both sides of the above inequality, one further has

$$\mathbb{E}\left\{V(n+1,\alpha(n+1))-V(n,\alpha(n))\right\}<0$$

which implies that the closed-loop system (5) is mean-square asymptotically stable.

For analyzing the H_∞ performance, we assume that the system's initial value is zero and the noise is energy-bounded. To proceed, we define an auxiliary variable:

$$f(r)\triangleq\sum_{n=0}^{r}\left(z^T(n)z(n)-\gamma^2v^T(n)v(n)\right). \quad (16)$$

Under the facts of $V(r+1,\alpha(r+1))\geq 0$ and $V(0,\alpha(0))=0$, we have from (16) that

$$\begin{aligned}f(r)\leq&\sum_{n=0}^{r}\Big(z^T(n)z(n)-\gamma^2v^T(n)v(n)+V(n+1,\alpha(n+1))\\&-V(n,\alpha(n))\Big).\end{aligned}$$

Then, the following conditional mathematical expectation can be calculated according to (13):

$$\begin{aligned}&\mathbb{E}\{f(r)|\xi(n),\alpha(n)=p\}\\&\leq\sum_{n=0}^{r}\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\sum_{q=1}^{2}\bar{\kappa}_{p,q}\Big(\Big(\mathcal{A}_{p,q}^{o,s}\xi(n)\\&\quad+\mathcal{E}_{p,q}^{o,s}v(n)\Big)^T\tilde{P}_p\Big(\mathcal{A}_{p,q}^{o,s}\xi(n)+\mathcal{E}_{p,q}^{o,s}v(n)\Big)+\frac{5}{4}\xi^T(n)\\&\quad\times(\mathcal{A}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{A}_{p,q}^{o,s}\xi(n)+\frac{5}{4}v^T(n)(\mathcal{E}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{E}_{p,q}^{o,s}v(n)\\&\quad+\left(2\hat{\beta}+\frac{13}{4}\bar{\Delta}_\beta^2\right)\xi^T(n)(\mathcal{C}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{C}_{p,q}^{o,s}\xi(n)+(2\hat{\beta}+4\bar{\Delta}_\beta^2)\\&\quad\times v^T(n)(\mathcal{F}_{p,q}^{o,s})^T\tilde{P}_p\mathcal{F}_{p,q}^{o,s}v(n)-\xi^T(n)P_p\xi(n)\\&\quad+\xi^T(n)\mathcal{N}_o^T\mathcal{N}_o\xi(n)-\gamma^2v^T(n)v(n)\Big)\\&=\sum_{n=0}^{r}\sum_{o=1}^{\bar{o}}\sum_{s=1}^{\hat{o}}m_o(\theta(n))\bar{m}_s(\bar{\theta}(n))\sum_{q=1}^{2}\bar{\kappa}_{p,q}\Big(\bar{\xi}^T(n)\left(\tilde{\mathcal{A}}_{p,q}^{o,s}\right)^T\\&\quad\times\hat{P}_p\tilde{\mathcal{A}}_{p,q}^{o,s}\bar{\xi}(n)+\Gamma_p\Big).\end{aligned}$$

where

$$\bar{\xi}(n)\triangleq\begin{bmatrix}\xi^T(n) & v^T(n)\end{bmatrix}^T,\quad \hat{P}_p\triangleq\sum_{e=1}^{2}\bar{\pi}_{p,e}\check{P}_e.$$

Along a similar line of the stability analysis, the conditions (8)-(10) ensure

$$\mathbb{E}\{f(r)|\xi(n),\alpha(n)=p\}<0.$$

By taking the mathematical expectation on both sides of the above inequality, and letting $r\to\infty$, one finally arrives at

$$\mathbb{E}\left\{\sum_{n=0}^{\infty}\left(z^T(n)z(n)-\gamma^2v^T(n)v(n)\right)\right\}<0,$$

which ends the proof. ■

In Theorem 1, we have provided a sufficient condition for evaluating system performance with predetermined controller gains. This evaluation specifically addresses the effects of packet dropouts and fluctuating channel modes, particularly in scenarios with uncertain probabilities. Theorem 1 is essential for ensuring that the system maintains effective performance despite the network-induced challenges. Furthermore, Theorem 2 given below will present a sufficient condition related to the feasibility of the desired controller. This condition, derived from the inequality criteria established in Theorem 1, is crucial for determining the practicality of implementing a controller that meets the specified performance targets.

Theorem 2: Consider the fuzzy system (1) and the fuzzy PID controller (4). The closed-loop system (5) is mean-square asymptotically stable and the H_∞ performance is achieved if, for $o,s=1,2,\cdots,\bar{o}$, $p,q=1,2$, there exist symmetric positive definite matrices $P_p\in\mathbb{R}^{l_x+2l_y}$, $T_p\in\mathbb{R}^{5(l_x+2l_y)+l_z}$, $M_p\in\mathbb{R}^{5(l_x+2l_y)+l_z}$, $Q_p\in\mathbb{R}^{5(l_x+2l_y)+l_z}$, $R_{p,q}\in\mathbb{R}^{l_x+2l_y+n_v}$, $S_p\in\mathbb{R}^{l_x+2l_y+n_v}$, and matrices $K_{s,q}^P$, $K_{s,q}^I$, $K_{s,q}^D$ such that

$$\begin{bmatrix}\pi_{p,1}(\check{P}_1-\check{P}_2)+\check{P}_2+\epsilon_p^2T_p-M_p & *\\ \check{P}_1-\check{P}_2 & -T_p\end{bmatrix}<0 \quad (17)$$

$$\begin{bmatrix}-R_{p,q} & *\\ \acute{\mathcal{A}}_{p,q}^{o,s} & -Q_p\end{bmatrix}<0 \quad (18)$$

$$\begin{bmatrix}\kappa_{p,1}(R_{p,1}-R_{p,2})+R_{p,2}+\upsilon_p^2S_p+\Gamma_p & *\\ R_{p,1}-R_{p,2} & -S_p\end{bmatrix}<0 \quad (19)$$

$$M_pQ_p=I \quad (20)$$

where

$$\acute{\mathcal{A}}_{p,q}^{o,s}\triangleq\tilde{I}_1\mathcal{A}_{p,q}^{o,s}\bar{I}_1+\tilde{I}_2\mathcal{C}_{p,q}^{o,s}\bar{I}_1+\tilde{I}_3\mathcal{E}_{p,q}^{o,s}\bar{I}_2+\tilde{I}_4\mathcal{F}_{p,q}^{o,s}\bar{I}_2+\vec{\mathcal{N}}_o,$$

$$\tilde{I}_1\triangleq\begin{bmatrix}I\\0\\0\\\frac{\sqrt{5}}{2}I\\0\\0\end{bmatrix},\quad \tilde{I}_2\triangleq\begin{bmatrix}0\\\sqrt{2\hat{\beta}+\frac{13}{4}\bar{\Delta}_\beta^2}I\\0\\0\\0\\0\end{bmatrix},\quad \bar{I}_1=\begin{bmatrix}I & 0\end{bmatrix},$$

$$\tilde{I}_3\triangleq\begin{bmatrix}I\\0\\0\\0\\\frac{\sqrt{5}}{2}I\\0\end{bmatrix},\quad \tilde{I}_4\triangleq\begin{bmatrix}0\\0\\\sqrt{2\hat{\beta}+4\bar{\Delta}_\beta^2}I\\0\\0\\0\end{bmatrix},\quad \bar{I}_2=\begin{bmatrix}0 & I\end{bmatrix},$$

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$$\vec{\mathcal{N}}_o \triangleq \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \mathcal{N}_o & 0 \end{bmatrix}, \quad \hat{I}_1 \triangleq \begin{bmatrix} I & 0 & 0 \end{bmatrix}, \quad \hat{I}_2 \triangleq \begin{bmatrix} 0 & I & 0 \end{bmatrix},$$

$$\begin{aligned}
\mathcal{A}^{o,s}_{p,q} \triangleq & \begin{bmatrix} A_o & 0 & 0 \\ \delta(p,1)C + \delta(p,2)\bar{\beta}C & I & 0 \\ \delta(p,1)C + \delta(p,2)\bar{\beta}C & 0 & 0 \end{bmatrix} + \hat{I}_1^T \delta(p,1)\big(B_o \\
& \times K^P_{s,q}C + B_o K^D_{s,q}C\big)\hat{I}_1 + \hat{I}_1^T \delta(p,2)\bar{\beta}\big(B_o K^P_{s,q}C \\
& + B_o K^D_{s,q}C\big)\hat{I}_1 + \hat{I}_1^T B_o K^I_{s,q}\hat{I}_2 - \hat{I}_1^T B_o K^D_{s,q}\hat{I}_3, \\
\mathcal{C}^{o,s}_{p,q} \triangleq & \begin{bmatrix} 0 & 0 & 0 \\ \delta(p,2)C & 0 & 0 \\ \delta(p,2)C & 0 & 0 \end{bmatrix} + \hat{I}_1^T \delta(p,2)\big(B_o K^P_{s,q}C \\
& + B_o K^D_{s,q}C\big)\hat{I}_1, \quad \hat{I}_3 \triangleq \begin{bmatrix} 0 & 0 & I \end{bmatrix}, \\
\mathcal{E}^{o,s}_{p,q} \triangleq & \begin{bmatrix} E_o \\ \delta(p,1)D + \delta(p,2)\bar{\beta}D \\ \delta(p,1)D + \delta(p,2)\bar{\beta}D \end{bmatrix} + \hat{I}_1^T \delta(p,1)\big(B_o K^P_{s,q}D \\
& + B_o K^D_{s,q}D\big) + \hat{I}_1^T \delta(p,2)\bar{\beta}\big(B_o K^P_{s,q}D + B_o K^D_{s,q}D\big), \\
\mathcal{F}^{o,s}_{p,q} \triangleq & \begin{bmatrix} 0 \\ \delta(p,2)D \\ \delta(p,2)D \end{bmatrix} + \hat{I}_1^T \delta(p,2)\big(B_o K^P_{s,q}D + B_o K^D_{s,q}D\big).
\end{aligned}$$

Proof: It is easy to see that $\acute{\mathcal{A}}^{o,s}_{p,q} = \tilde{\mathcal{A}}^{o,s}_{p,q}$. Then, letting $Q_p = M_p^{-1}$ and utilizing the Schur Complement Lemma to inequalities presented in Theorem 1, (8)-(10) are guaranteed by (17)-(19), respectively, and the proof is complete. ■

In order to deal with the equality constraint (20), we employ the well-known cone-complementarity-linearization technique [48] by defining

$$\bar{M}_p \triangleq \begin{bmatrix} M_p & * \\ I & Q_p \end{bmatrix}, \quad \vec{M}_p \triangleq \begin{bmatrix} -R_{p,q} & * \\ \acute{\mathcal{A}}^{o,s}_{p,q} & -M_p^{-1} \end{bmatrix},$$

where the corresponding algorithm (Algorithm 1) is proposed to provide a feasible method for calculating the controller gains within the linear matrix inequality framework.

Algorithm 1: Mode-dependent Fuzzy PID Control

Step 1. Set $k = 0$. Derive initial values $P_p^{(0)}, T_p^{(0)}, M_p^{(0)}, Q_p^{(0)}, R_{p,q}^{(0)}$, $S_p^{(0)}, K^P_{s,q,0}, K^I_{s,q,0}, K^D_{s,q,0}$, by solving (17)-(19) and $\bar{M}_p > 0$.

Step 2. Solve min tr $\sum_{p=1}^{2}(M_p Q_p^{(k)} + M_p^{(k)} Q_p)$ subject to (17)-(19) and $\bar{M}_p > 0$ to derive $P_p, T_p, M_p, Q_p, R_{p,q}, S_p, K^P_{s,q}, K^I_{s,q}$, $K^D_{s,q}$. Set $\sigma = |0.5\text{trace}\left(\sum_{p=1}^{2}(M_p Q_p)\right) - 5(l_x + 2l_y) - l_z|$.

Step 3. Substitute obtained matrices into $\vec{M}_p < 0$. If it holds and σ is less than a small scalar, then output these variables and exit.

Step 4. If $k > k_{\max}$, where $k_{\max}$ is the allowed maximum number of iterations, exit. Else, set $k = k + 1$, and $P_p^{(k)} = P_p$, $T_p^{(k)} = T_p$, $M_p^{(k)} = M_p$, $Q_p^{(k)} = Q_p$, $R_{p,q}^{(k)} = R_{p,q}$, $S_p^{(k)} = S_p$, $K^P_{s,q,k} = K^P_{s,q}$, $K^I_{s,q,k} = K^I_{s,q}$, $K^D_{s,q,k} = K^D_{s,q}$. Go to *Step 2*.

Remark 3: In addressing the H_∞ fuzzy PID control problem for nonlinear systems operating over Gilbert-Elliott channels, this study has made significant strides as follows. 1) A mode detector has been employed to observe and identify the current state of the channel mode. This tool is crucial for accurately reflecting the real channel situation, albeit with certain probabilities. 2) We have incorporated bounded uncertainties in the model, specifically in the probability information of data losses, channel mode switching, and mode detection. This addition enhances the realism and applicability of the model in dynamic, real-world communication environments. 3) Leveraging the detected channel mode, a robust fuzzy PID controller has been formulated. This controller is designed to achieve the desired level of disturbance attenuation, adapting to the varying conditions of the communication channel. 4) The main results presented in Theorems 1 and 2 encompass all elements that influence system dynamics, which include parameter matrices, probability information, and the bounds of uncertainties. Such a comprehensive approach ensures that the controller design and system analysis are robust and reliable, accounting for a wide range of variables and conditions.

Remark 4: In this work, a robust method has been proposed for dealing with the inaccurate probability information, where only the bounds of uncertainties have been used in the performance analysis and controller design. Our method is applicable for all uncertainties within the considered bounds. For the case of totally accurate probability information, it can be regarded that all uncertainties are zero which is a special case of what we have discussed. Thus, the proposed method can deal with the case of totally accurate probability information.

Remark 5: This work makes distinctive contributions to the field of Gilbert-Elliott-channel-related research, setting it apart from existing studies. The key technical contributions are as follows.

1) *Innovative Exploration of H_∞ Fuzzy PID Control Problem*: This research represents the first attempt to investigate the H_∞ fuzzy PID control problem for nonlinear systems over Gilbert-Elliott channels by utilizing detected channel modes. A novel aspect of this study is the comprehensive consideration and handling of uncertain probability information, which arises due to statistical errors.
2) *Development of a Special Augmentation Technique*: The study introduces a unique augmentation technique designed to simplify the system's description. This method is not only instrumental in facilitating the application of the fuzzy PID controller under mode-dependent data loss conditions but also significantly reduces the computational load during the performance analysis process.

IV. Simulation Example

In this section, we present one application-motivated example along with comparative results to validate the effectiveness of the proposed control approach.

Consider a truck-trailer control system modeled as follows [16]:

$$\begin{cases} x_1(n+1) = \left(1 - \dfrac{\omega\epsilon}{W}\right)x_1(n) + \dfrac{\omega\epsilon}{w}u(n) + 0.1v(n) \\ x_2(n+1) = \dfrac{\omega\epsilon}{W}x_1(n) + x_2(n) + 0.1v(n) \\ x_3(n+1) = \omega\epsilon\sin\left(\dfrac{\omega\epsilon}{2W}x_1(n) + x_2(n)\right) + x_3(n) \end{cases}$$

where $x_1(n)$ is the angle difference between the truck and the trailer; $x_2(n)$ is the angle of the trailer; $x_3(n)$ is the vertical

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position of the rear end of the trailer; $u(n)$ is the steering angle; $v(n)$ is the external disturbance; $w = 2.8m$ is the length of the truck; $W = 5.5m$ is the length of the trailer; $\epsilon = 1s$ is the sampling time; and $\omega = -0.5m/s$ is the constant speed of backing up.

In terms of the key points $0°$, $\pm 30°$, $\pm 180°$ and by using the standard fuzzy modeling technique, we can obtain the following discrete-time T-S fuzzy model:

$$x(n+1) = \sum_{o=1}^{3} m_o(\theta(n))\Big(A_o x(n) + Bu(n) + Ev(n)\Big) \tag{21}$$

where

$$A_1 \triangleq \begin{bmatrix} 1-\frac{\omega\epsilon}{W} & 0 & 0 \\ \frac{\omega\epsilon}{W} & 1 & 0 \\ \frac{\omega^2\epsilon^2}{2W} & \omega\epsilon & 1 \end{bmatrix}, A_2 \triangleq \begin{bmatrix} 1-\frac{\omega\epsilon}{W} & 0 & 0 \\ \frac{\omega\epsilon}{W} & 1 & 0 \\ \frac{3\omega^2\epsilon^2}{2\pi N} & \frac{3\omega\epsilon}{\pi} & 1 \end{bmatrix},$$

$$A_3 \triangleq \begin{bmatrix} 1-\frac{\omega\epsilon}{W} & 0 & 0 \\ \frac{\omega\epsilon}{W} & 1 & 0 \\ \frac{\omega^2 T}{200W^2} & \frac{\omega}{100W} & 1 \end{bmatrix}, \quad x(n) \triangleq \begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \end{bmatrix},$$

$$B \triangleq \begin{bmatrix} \frac{\omega\epsilon}{w} \\ 0 \\ 0 \end{bmatrix}, \quad \theta(n) \triangleq \frac{\omega\epsilon}{2W}x_1(n) + x_2(n), \;\; E \triangleq \begin{bmatrix} 0.1 \\ 0.1 \\ 0 \end{bmatrix}.$$

It is assumed that there is one sensor with $C = \begin{bmatrix} 5 & -2 & 1 \end{bmatrix}$ and $D = 0.3$. The measurement outputs are sent to the controller through the Gilbert-Elliott channel with two modes $\alpha(n) \in \{1, 2\}$, where mode "1" represents the "good" channel condition while mode "2" denotes the "bad" channel condition. The state transition probabilities for $p = 1, 2$ are assumed to be

$$\begin{aligned} &\mathbf{Prob}\{\alpha(n+1) = 1|\alpha(n) = p\} = \pi_{p,1} + \Delta_p^{(\pi)}, \\ &\mathbf{Prob}\{\alpha(n+1) = 2|\alpha(n) = p\} = 1 - \pi_{p,1} - \Delta_p^{(\pi)}, \end{aligned}$$

where $\pi_{1,1} = 0.5, \;\pi_{2,1} = 0.5, \;\Delta_1^{(\pi)} = 0.17, \;\Delta_2^{(\pi)} = 0$ with known bounds -0.2 and 0.2.

When the channel is in the "bad" condition, the transmitted signals would suffer from packet dropouts governed by a stochastic variable $\beta(n)$ with the following properties:

$$\begin{aligned} \mathbf{Prob}\{\beta(n) = 1\} &= \bar{\beta} + \Delta_\beta, \\ \mathbf{Prob}\{\beta(n) = 0\} &= 1 - \bar{\beta} - \Delta_\beta \end{aligned}$$

where $\bar{\beta} = 0.6$, $\Delta_\beta = 0.09$ with known bounds -0.1 and 0.1.

On the controller side, the detected channel mode is denoted by $\vartheta(n)$ that would be asynchronous with $\alpha(n)$. The relation between $\alpha(n)$ and $\vartheta(n)$ is given as follows:

$$\mathbf{Prob}\{\vartheta(n) = 1|\alpha(n) = p\} = \kappa_{p,1} + \Delta_p^{(\kappa)}, \;\; p = 1, 2$$

where $\kappa_{1,1} = 0.8$, $\kappa_{2,1} = 0.2$ and $\Delta_p^{(\kappa)} = 0$ with known bounds -0.19 and 0.19.

Note that, only the information of $\pi_{p,e}$, $\kappa_{p,q}$ $(p, q, e = 1, 2)$, $\bar{\beta}$ and the bounds of uncertainty are available for the controller designer.

In this example, we aim to develop a fuzzy PID controller in the form of (4) by utilizing the network-affected measurements and detected channel modes, such that the considered system can be stabilized in the mean-square sense with a prescribed performance index parameter $\gamma = 1.2$.

Set the simulation length to be $n_{\max} = 1000$. Assume that the external noise is $v(n) = 0.5\sin(n)/n$. The desired controller gains are derived by Algorithm 1, under which simulation results are presented in Figs. 2–3. The state evolution of the open-loop system (without any control law) is showcased in Fig. 2, from which we can see that the original system is rather unstable and the state goes far away from the origin fast. By using the proposed control approach, the state evolution is shown in Fig. 3. It can be seen that all state components converge to the equilibrium point with a desired speed.

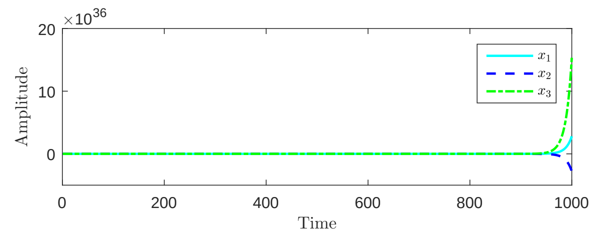

Fig. 2: System state evolution without control

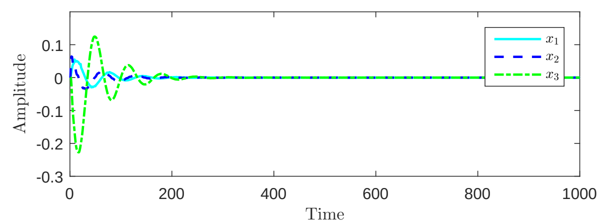

Fig. 3: System state evolution with fuzzy PID control

To check the H_∞ performance of the closed-loop system, we define the following auxiliary parameter:

$$\gamma_{\text{real}} \triangleq \sqrt{\sum_{n=0}^{n_{\max}} z^T(n)z(n) \Big/ \sum_{n=0}^{n_{\max}} v^T(n)v(n)}.$$

Then, it can be calculated that $\gamma_{\text{real}} = 0.4259 < 1.2$, implying the achievement of the H_∞ performance.

Compared to the traditional P-type controller, i.e., the static output-feedback (SOF) one, the devised fuzzy PID controller enjoys more design degrees of freedom and utilizes more system information, which is reliable in dealing with NIP. To validate the superiority of the proposed fuzzy PID controller, some comparative results are given in Table I where the fuzzy nominal controller (NC) means the design method without considering unknown information. In this table, the obtained γ_{real} is listed under different noises, from which one can observe that the value of γ_{real} under the fuzzy PID control is smaller than that under the SOF one, reflecting better control performance. In addition, one can conclude that the fuzzy NC fails to achieve the prescribed performance requirement as $\gamma_{\text{real}} > 1.2$. These simulation results verify the effectiveness of the proposed method in dealing with uncertainties.

It is known that adding integral and derivative terms to the controller could reduce steady state error and improve response time. In order to show the improvement of the

TABLE I: Disturbance Attenuation Capability of Different Control Methods

Noise $v(n)$	$\frac{0.5\sin n}{n}$	$\frac{0.8\cos n}{n}$	$\frac{0.4\cos n}{n}+\frac{0.6\sin n}{n}$
γ_{real} (fuzzy PID)	0.4259	0.0568	0.3354
γ_{real} (fuzzy SOF)	0.4812	0.0644	0.3644
γ_{real} (fuzzy NC)	57.87	70.87	35.99

convergence of the system state by using the PID-inspired multi-variable controller (4), we set the external noise to be zero and the initial state to be $x(0) = \begin{bmatrix}1.2 & 2 & 1.5\end{bmatrix}^T$. Then, the comparison figures are given in Figs. 4–6, from which we can see that the proposed control method has a faster rate of convergence than the existing method.

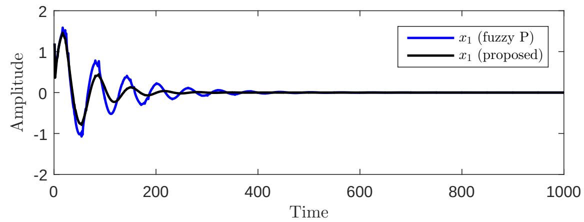

Fig. 4: Comparison of the state $x_1(n)$

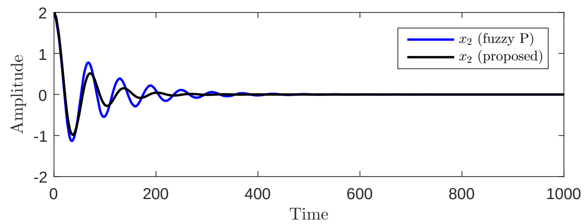

Fig. 5: Comparison of the state $x_2(n)$

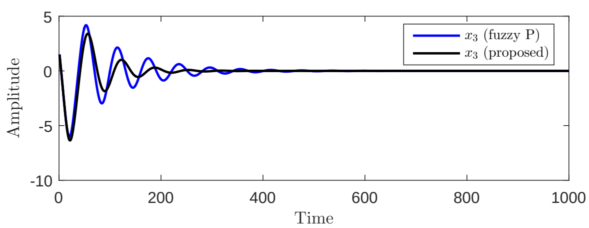

Fig. 6: Comparison of the state $x_3(n)$

V. Conclusion

In this paper, an H_∞ PID controller has been designed for T-S fuzzy systems with data transmission over Gilbert-Elliott channels. The quality of communication has been modeled by a two-state Markov process by incorporating uncertain transition probabilities. In the channel's "bad" mode, packet dropouts have been handled using stochastic variable sequences by considering the probability inaccuracies from statistical errors. The detection of channel modes has been described as a hidden Markov process, enhancing the understanding of channel dynamics. The relationship between real and detected channel modes has been defined through conditional probabilities. An asynchronous fuzzy PID controller, based on detected channel modes, has been proposed. Sufficient conditions, rooted in stochastic analysis theory, have been set for assessing system performance. The effectiveness of our developed approach has been validated through a simulation to demonstrate its practical applicability. One of the future research directions would be to extend the main results of this paper to more general systems with more specific application insights [49]–[52].

References

[1] W. Ji, J. Qiu, C. Song and Y. Fu, New results on nonsynchronous-observer-based output-feedback control of fuzzy-affine-model-based discrete-time nonlinear systems, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 8, pp. 2836–2847, Aug. 2023.
[2] R. V. Aravind and P. Balasubramaniam, Membership-function-dependent design of quantized fuzzy sampled-data controller for semi-Markovian jump systems with actuator faults, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 1, pp. 40–52, Jan. 2023.
[3] L. Ding and W. Sun, Predefined time fuzzy adaptive control of switched fractional-order nonlinear systems with input saturation, *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 4, art. no. 100019, Dec. 2023.
[4] P. Y. Reddy and L. C. Saikia, Hybrid AC/DC control techniques with improved harmonic conditions using DBN based fuzzy controller and compensator modules, *Systems Science and Control Engineering*, vol. 11, no. 1, art. no. 2188406, Mar. 2023.
[5] S. Ma and Y. Li, Adaptive fuzzy fault-tolerant control for active seat suspension systems with full-state constraints, *Systems Science and Control Engineering*, vol. 11, no. 1, art. no. 2153391, Jan. 2023.
[6] J. Liu, E. Gong, L. Zha, E. Tian and X. Xie, Observer-based security fuzzy control for nonlinear networked systems under weighted try-once-discard protocol, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 11, pp. 3853–3865, Nov. 2023.
[7] Y. Wu, J. Cheng and Z.-G. Wu, Fuzzy-affine-model-based filtering design with memory-based dynamic event-triggered protocol, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 4, pp. 1340–1351, Apr. 2023.
[8] Z. Xiang, P. Li, M. Chadli and W. Zou, Fuzzy optimal control for a class of discrete-time switched nonlinear systems, *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 4, pp. 2297–2306, Apr. 2024.
[9] D. Cui and Z. Xiang, Nonsingular fixed-time fault-tolerant fuzzy control for switched uncertain nonlinear systems, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 1, pp. 174–183, Jan. 2023.
[10] K. H. Ang, G. Chong and Y. Li, PID control system analysis, design, and technology, *IEEE Transactions on Control Systems Technology*, vol. 13, no. 4, pp. 559–579, Jul. 2005.
[11] E. N. Goncalves, R. M. Palhares and R. H. C. Takahashi, A novel approach for H_2/H_∞ robust PID synthesis for uncertain systems, *Journal of Process Control*, vol. 18, no. 1, pp. 19–26, Jan. 2008.
[12] G.-P. Liu, Tracking control of multi-agent systems using a networked predictive PID tracking scheme, *IEEE/CAA Journal of Automatica Sinica*, vol. 10, no. 1, pp. 216–225, Jan. 2023.
[13] F. d. O. Souza, L. A. Mozelli, M. C. d. Oliveira and R. M. Palhares, LMI design method for networked-based PID control, *International Journal of Control*, vol. 89, no. 10, pp. 1962–1971, Mar. 2016.
[14] C.-S. Chiu and T.-S. Chiang, H_∞ output-feedback fuzzy proportional-integral control of fully delayed input/output systems, *ISA Transactions*, vol. 66, pp. 22–31, Jan. 2017.
[15] L. Pan and X. Wang, Variable pitch control on direct-driven PMSG for offshore wind turbine using Repetitive-TS fuzzy PID control, *Renewable Energy*, vol. 159, pp. 221–237, Oct. 2020.
[16] Y. Wang, Z. Wang, L. Zou and H. Dong, H_∞ PID control for discrete-time fuzzy systems with infinite-distributed delays under round-robin communication protocol, *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 6, pp. 1875–1888, Jun. 2022.
[17] Y. Wang, Z. Wang, L. Zou, L. Ma and H. Dong, Ultimately bounded PID control for T-S fuzzy systems under FlexRay communication protocol, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 12, pp. 4308–4320, Dec. 2023.
[18] H. Zhang and J. Liu, Event-triggered fuzzy flight control of a two-degree-of-freedom helicopter system, *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 10, pp. 2949–2962, Oct. 2021.

[19] R. Caballero-Águila, A. Hermoso-Carazo and J. Linares-Pérez, Covariance-based fusion filtering for networked systems with random transmission delays and non-consecutive losses, *International Journal of General Systems*, vol. 46, no. 7, pp. 752–771, Jun. 2017.

[20] R. Caballero-Águila, A. Hermoso-Carazo, and J. Linares-Pérez, Networked fusion estimation with multiple uncertainties and time-correlated channel noise, *Information Fusion*, vol. 54, pp. 161–171, Feb. 2020.

[21] Y. Wang, H.-J. Liu and H.-L. Tan, An overview of filtering for sampled-data systems under communication constraints, *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 3, art. no. 100011, Sept. 2023.

[22] Y.-A. Wang, B. Shen, L. Zou and Q.-L. Han, A survey on recent advances in distributed filtering over sensor networks subject to communication constraints, *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 2, art. no. 100007, Jun. 2023.

[23] F. Han, J. Liu, J. Li, J. Song, M. Wang and Y. Zhang, Consensus control for multi-rate multi-agent systems with fading measurements: the dynamic event-triggered case, *Systems Science and Control Engineering*, vol. 11, no. 1, art. no. 2158959, Jan. 2023.

[24] Event-based adaptive fuzzy tracking control for nonlinear systems with input magnitude and rate saturations, *International Journal of Systems Science*, vol. 54, no. 16, pp. 3045–3058, Dec. 2023.

[25] X. Li and D. Ye, Dynamic event-triggered distributed filtering design for interval type-2 fuzzy systems over sensor networks under deception attacks, *International Journal of Systems Science*, vol. 54, no. 15, pp. 2875–2890, Nov. 2023.

[26] M. Thummalapeta and Y.-C. Liu, Survey of containment control in multi-agent systems: concepts, communication, dynamics, and controller design, *International Journal of Systems Science*, vol. 54, no. 14, pp. 2809–2835, Aug. 2023.

[27] Y. Guo, Z. Wang, J.-Y. Li and Y. Xu, State estimation for Markovian jump neural networks under probabilistic bit flips: allocating constrained bit bates, *IEEE Transactions on Neural Networks and Learning Systems*, in press, DOI: 10.1109/TNNLS.2024.3411484.

[28] Z. Huang, W. Lv, C. Liu, Y. Xu, L. Rutkowski and T. Huang, Event-triggered distributed moving horizon estimation over wireless sensor networks, *IEEE Transactions on Industrial Informatics*, vol. 20, no. 3, pp. 4218–4226, Mar. 2024.

[29] H. Peng, B. Zeng, L. Yang, Y. Xu and R. Lu, Distributed extended state estimation for complex networks with nonlinear uncertainty, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 34, no. 9, pp. 5952–5960, Sept. 2023.

[30] A. Rajagopal and S. Chitraganti, State estimation and control for networked control systems in the presence of correlated packet drops, *International Journal of Systems Science*, vol. 54, no. 11, pp. 2352–2365, Aug. 2023.

[31] C. Yang, J. Liang and W. Xu, PI observer-based synchronization control for singularly perturbed complex dynamical networks: an improved coding-decoding protocol scheme, *IEEE Transactions on Network Science and Engineering*, vol. 11, no. 1, pp. 858–871, Jan./Feb. 2024.

[32] X. Yi, H. Yu, Z. Fang and L. Ma, Probability-guaranteed state estimation for nonlinear delayed systems under mixed attacks, *International Journal of Systems Science*, vol. 54, no. 9, pp. 2059–2071, Jul. 2023.

[33] M. Yao and G. Wei, Dynamic event-triggered control of continuous-time systems with random impulses, *IEEE/CAA Journal of Automatica Sinica*, vol. 10, no. 12, pp. 2292–2299, Dec. 2023.

[34] Y. Xu, W. Lv, W. Lin, R. Lu and D. E. Quevedo, On extended state estimation for nonlinear uncertain systems with round-robin protocol, *Automatica*, vol. 138, art. no. 110154, Apr. 2022.

[35] R. Datta and Y. H. Joo, Fuzzy memory sampled-data controller design for PMSG-based WECS with stochastic packet dropouts, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 12, pp. 4421–4434, Dec. 2023.

[36] A. Elahi, A. Alf and H. Modares, H_∞ Consensus of homogeneous vehicular platooning systems with packet dropout and communication delay, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 6, pp. 3680–3691, Jun. 2022.

[37] Y. Yan, H. Du, Q.-L. Han and W. Li, Discrete multi-objective switching topology sliding mode control of connected autonomous vehicles with packet loss, *IEEE Transactions on Intelligent Vehicles*, vol. 8, no. 4, pp. 2926–2938, Apr. 2023.

[38] E. O. Elliott, Estimates of error rates for codes on burst-noise channels, *The Bell System Technical Journal*, vol. 42, no. 5, pp. 1977–1997, Sept. 1963.

[39] E. N. Gilbert, Capacity of a burst-noise channel, *The Bell System Technical Journal*, vol. 39, no. 5, pp. 1253–1265, Sept. 1960.

[40] Y. Shen, Z. Wang, B. Shen and F. E. Alsaadi, Nonfragile H_∞ filtering for discrete multirate time-delayed systems over sensor networks characterized by Gilbert-Elliott models, *International Journal of Robust and Nonlinear Control*, vol. 30, no. 8, pp. 3194–3214, May 2020.

[41] Q. Zhang, Y. Wang, J. Cheng, H. Yan and K. Shi, Improved filtering of interval type-2 fuzzy systems over Gilbert-Elliott channels, *Information Sciences*, vol. 627, pp. 132–146, May 2023.

[42] T. Farjam, H. Wymeersch and T. Charalambous, Distributed channel access for control over known and unknown Gilbert-Elliott channels, *IEEE Transactions on Automatic Control*, vol. 68, no. 12, pp. 7405–7419, Dec. 2023.

[43] Y. Li, J. Lam and H. Li, On stability and performance of the optimal linear filter over Gilbert-Elliott channels with unobservable packet losses, *IEEE Transactions on Control of Network Systems*, vol. 9, no. 2, pp. 1029–1039, Jun. 2022.

[44] H. Song, S.-C. Chen and Y. Yam, Sliding mode control for discrete-time systems with Markovian packet dropouts, *IEEE Transactions on Cybernetics*, vol. 47, no. 11, pp. 3669–3679, Nov. 2017.

[45] A. Yogarathinam and N. R. Chaudhuri, Wide-area damping control using reduced copy under intermittent observation: a novel performance measure, *IEEE Transactions on Control Systems Technology*, vol. 27, no. 1, pp. 434–442, Jan. 2019.

[46] D. Spinello, Asymptotic agreement in a class of networked Kalman filters with intermittent stochastic communications, *IEEE Transactions on Automatic Control*, vol. 61, no. 4, pp. 1093–1098, Apr. 2016.

[47] J. Xiong and J. Lam, Robust H_2 control of Markovian jump systems with uncertain switching probabilities, *International Journal of Systems Science*, vol. 40, no. 3, pp. 255–265, Mar. 2009.

[48] D. Mao, Y. Niu, J. Li and Y. Yang, Security sliding mode control for networked control system under multi-channel Markovian hybrid cyber-attacks, *International Journal of Robust and Nonlinear Control*, vol. 34, pp. 773–792, Jan. 2024.

[49] X. Yi and T. Xu, Distributed event-triggered estimation for dynamic average consensus: A perturbation-injected privacy-preservation scheme, *Information Fusion*, vol. 108, art. no. 102396, Aug. 2024.

[50] X. Yi, Z. Wang, S. Liu and Q. Tang, Acceleration model considering multi-stress coupling effect and reliability modeling method based on nonlinear Wiener process, *Quality and Reliability Engineering International*, vol. 40, no. 6, pp. 3055–3078, May 2024.

[51] X. Yi, H. Yu and T. Xu, Solving multi-objective weapon-target assignment considering reliability by improved MOEA/D-AM2M, *Neurocomputing*, vol. 563, art. no. 126906, Jan. 2024.

[52] X. Yi, C. Xu, X. Fang and S. Liu, A new reliability analysis method for software-intensive systems with degradation accumulation effect based on goal oriented methodology, *Quality and Reliability Engineering International*, vol. 40, no. 1, pp. 236–260, May 2023.

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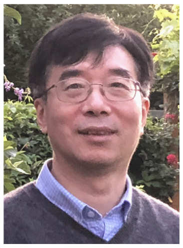

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