ACCOUNTING BASED VALUATIONS WITH MULTIPLE "ANCHORS"

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Abstract

Each accounting based valuation uses an anchor such equity book value or capitalised earnings, whose merits are shown by past literature. This paper extends accounting based valuations, so that each valuation can have different anchors at different points in time, for example an initial anchor and a "terminal" anchor. Each anchor is an accounting variable multiplied by a valuation multiple typically derived from comparable companies. These extended valuations combine residual income valuation and AEG valuation, and reinterpret the generalisation of AEG valuations by Ohlson and Johannesson (2016).

Cash flow based valuations with multiple anchors are also presented.

Key words: accounting based valuations, valuation multiples, valuation anchors, residual income valuation, AEG valuation.

JEL classification: G12; G13.

1 Introduction and literature

Accounting based valuations are typically based on an accounting "anchor". A typical example is the residual income valuation (RIV) of Ohlson (1995), which anchors on the initial book value of equity, the only anchor in that valuation. That anchor has well known merits, such as the need to only forecast the "residual" part of future income, such as reducing the forecasting horizon, and such as naturally explaining the price-to-book multiple.

A theoretical breakthrough took place when AEG valuation was introduced by Ohlson and Juettner-Nauroth (2005) and Ohlson (2005). The breakthrough was to use multiple anchors in the same valuation, indeed a different anchor in each period, each anchor being capitalised forward earnings. Another way to view this breakthrough, whose merits are explained for example in Ohlson (2005), is the forfeiting of the clean surplus accounting relation that RIV assumed. This paper revisits this theoretical breakthrough and extends it to other valuations that use different anchors at different points in time. In fact this paper presents the general algebra of these multi-anchor valuations, as well as interesting special cases.

Some of the multi-anchor valuations reinterpret the generalised AEG valuations of Ohlson and Johannesson (2016) and Ohlson (2022), and the related extensions of Lai (2020). Generalised AEG valuations anchor on a multiple of forward earnings that need not be capitalised forward earnings as in the AEG valuation of Ohlson and Juettner-Nauroth (2005) and Ohlson (2005). These generalised AEG valuations significantly extend AEG valuation, but rely on a definition of generalised AEG that has proved not very intuitive. The multi-anchor valuations in this paper provide a new interpretation of generalised AEG that seems quite intuitive. In a given period generalised AEG is the sum of a generalised version of residual income plus the gain or loss due to the change in the valuation at the start of the period.

Multi-anchor valuations extend the generalised AEG valuations of Ohlson and Johannesson (2016), Ohlson (2022) and Lai (2020) in two ways.

First the valuation multiple on which to anchor may change over time. For example the initial anchor may multiply the company's forward earnings by the corrent normal price-to-earnings multiple of peers, while the anchor to compute terminal value may multiply the company's terminal forward earnings by the expected terminal normal price-to-earnings multiple of peers. The initial and the terminal price-to-earnings multiples of peers may well differ.

Second in multi-anchor valuations the anchor may or may not change in each period. Instead valuations in the literature typically either assume no change of anchor or assume that the anchor changes each period, such as in AEG valuation.

Some multi-anchor valuations may anchor on average past and/or future earnings, since average earnings are often perceived as more meaningful than single period earnings. For example the initial anchor may multiply average earnings in the last three years by a multiple, while the terminal value anchor may multiply average earnings in the previous five years by the same multiple. The multiple could be the average price to average earnings multiple of peers.

Other multi-anchor valuations may anchor on a multiple of equity book value. As equity book value changes each period, the anchor changes each period. The resulting valuations extend the abnormal book value growth valuation of Ohlson (2009).

Yet other multi-anchor valuations may intially anchor on earnings and later on equity book value. For example the valuation of a firm currently making losses, but with positive book value of equity, may have an equity book value as initial anchor, and then a terminal value anchor linked to expected positive long term earnings.

Still other multi-anchor valuations estimate firm value rather than equity value, and some can anchor on sales revenues. For example the valuation of a firm making operating losses and with negative net operating assets at present, may have an initial anchor on sales and a then a terminal anchor on expected positive long term operating income.

Some multi-anchor valuations can forecast cash flows rather than earnings. These valuations can anchor on free cash flow, on EBITDA or on other accounting variables. These anchors entail that only "residual" cash flows need forecasting, and thus can shorten the long forecast horizon that is typical of cash flow based valuations.

Many of the multi-anchor valuations are hybrid valuations close in spirit to those successfully tested empirically by Gao and others (2019).

In general multi-anchor valuations seem to provide helpful flexibility to analysts and researchers. In particular the multiple anchors can be chosen so as to shorten the valuation forecast horizon.

Following most of the accounting based valuation literature, in this paper all valuations assume a constant discount rate, which is a "quasi-yield to maturity for equity" as per Penman and others (2023). Although the assumption of a constant discount rate has been criticised, it is still deemed as a practical and useful approximation. According to Penman and others (2023) "a series of changing discount rates can be summarized by an “average” rate over the term" .. "With a time-varying cost of capital for equity so elusive, the ICC can serve the same function." ICC is the implied cost of capital, which is a constant discount rate.

This paper is organised as follows. An introductory example presents a valuation with two anchors, an initial one and a terminal one, linked to forward earnings. Then the general algebra of valuation models based on multiple anchors is presented. Then various special cases of this general valuation algebra are presented. Then multi-anchor valuations under constant growth assumptions are presented. The conclusions follow.

2 Introductory example: anchoring on forward P/E multiples

This section presents the example of a valuation based on two anchors. Time 0 is the time of the valuation. The first anchor is at time 0 and is $m_0 \cdot Earn_1$, where $Earn_1$ are the earnings forecasted in year 1, and m_0 is the "normal" forward P/E ratio at time 0 derived from peer companies. The second anchor helps us estimate terminal value at time $T > 0$ and is $m_T \cdot Earn_{T+1}$, where m_T is the expected "normal" forward P/E ratio for peer companies at time T. Let V_0^e be the fundamental value of equity at time 0, r_e be the cost of equity capital and d_t be dividends at time t.

2.1 Valuation with one anchor

If we anchor our estimate of V_0^e only on $m_0 \cdot Earn_1$ at time 0, as in the generalised AEG valuation of Ohlson and Johannesson (2016), hereafter OJ (2016), it can be shown that

$$V_0^e = \sum_{t=1}^{\infty} \frac{d_t}{(1+r_e)^t} = m_0 \cdot Earn_1 + \sum_{t=1}^{\infty} \frac{RI_t^0}{(1+r_e)^t}$$
$$RI_{t+1}^0 = Earn_{t+1} - r_e \cdot \Theta_t$$
$$\Theta_0 = m_0 \cdot Earn_1$$
$$\Theta_{t+1} = \Theta_t + Earn_{t+1} - d_{t+1}.$$

Θ_0 is the only anchor in this valuation. Θ_{t+1} for $t \geq 0$ is a "balance" variable, whose definition emulates the clean surplus relation in RIV. RI_{t+1}^0 can be viewed as a generalisation of residual income during $[t, t+1]$. The superscript 0 next to RI highlights that the anchor is set at time 0. RI is forecasted and discounted similarly to how residual income is forecasted and discounted in RIV. However this valuation anchors on forward earnings $Earn_1$, not on the current book value of equity B_0 as RIV does. When $\Theta_0 = B_0$ this valuation becomes RIV.

When $RI_{t+1}^0 = (1+g) \cdot RI_t^0$ for $t \geq 1$, then $V_0^e = m_0 \cdot Earn_1 + \frac{RI_1^0}{r_e - g}$. In this case $\lim_{t \to \infty} (V_t^e - \Theta_t) \to 0$ if $g < 0$.

Note that $RI_{t+1}^0 - RI_t^0 = AEG_t$, AEG_t is abnormal earnings growth in period $[t-1, 1]$ as per Ohlson (2005). When $AEG_{t+1} = (1+g) \cdot AEG_t$ for $t \geq 2$ and $m_0 = \frac{1}{r_e}$, then under regularity conditions we obtain $V_0^e = \frac{Earn_1}{r_e} + \frac{1}{r_e} \frac{AEG_2}{r_e - g}$. The latter is the well known version of AEG valuation with constant growth.

2.2 Valuation with two anchors

Now we extend the previous valuation to two anchors, one at time 0 and one at time $T \geq 1$. It can be shown that

$$V_0^E = \sum_{t=1}^{\infty} \frac{d_t}{(1+r_e)^t} = \Theta_0^0 + \sum_{t=1}^{T} \frac{RI_t^0}{(1+r_e)^t} + \frac{\Theta_T^T - \Theta_T^0}{(1+r_e)^T} + \sum_{t=T+1}^{\infty} \frac{RI_t^T}{(1+r_e)^t}$$
$$\Theta_0^0 = m_0 \cdot Earn_1$$
$$\Theta_T^T = m_T \cdot Earn_{T+1}$$
$$\Theta_t^0 = m_0 \cdot Earn_1 + \sum_{i=1}^{t} (Earn_i - d_i) \text{ for } 1 \leq t \leq T$$
$$\Theta_t^T = m_T \cdot Earn_{T+1} + \sum_{i=T+1}^{t} (Earn_i - d_i) \text{ for } t \geq T+1$$
$$RI_{t+1}^0 = Earn_{t+1} - r_e \cdot \Theta_t^0 \text{ for } 0 \leq t \leq T-1$$
$$RI_{t+1}^T = Earn_{t+1} - r_e \cdot \Theta_t^T \text{ for } t \geq T.$$

The two anchors are Θ_0^0 and Θ_T^T. The anchor at T helps us estimate the terminal value V_T^E. An appeal of this valuation is that, if $\Theta_T^T = V_T^e$, then the forecast horizon stops at T. Then if $m_0 = m_T = m$ we obtain

$$\begin{aligned} \Theta_T^T - \Theta_T^0 &= m \cdot \sum_{i=1}^{T} \left(Earn_{i+1} - Earn_i - \frac{1}{m} (Earn_i - d_i) \right) \\ &= m \cdot \sum_{i=2}^{T+1} AEG_i^{MAV} \end{aligned}$$

$$AEG^{MAV}_{t+2} = Earn_{t+2} - Earn_{t+1} - \frac{1}{m}\left(Earn_{t+1} - d_{t+1}\right).$$

AEG^{MAV} stands for AEG according to the multi-anchor valuation (MAV). Later we show the link between AEG^{MAV} and the generalised AEG of Ohlson and Johannesson (2016). If no re-anchoring takes place at time T, then $\Theta^T_T - \Theta^0_T = 0$ and this valuation becomes the one-anchor valuation seen above.

This example assumes just one re-anchoring at time T and links the two anchors to the popular forward P/E multiple. The next section presents the general algebra of valuations with multiple anchors.

3 The general multi-anchor valuation (MAV)

This section shows the algebra of the multi-anchor valuation. In this section the variables V_0, r, z have different meanings in equity valuations and in firm valuations as follows

Table 1	Equity valuation	Firm valuation
V_0	$= V^e_0$	$= V^f_0$
r	$= r_e$	$= r_f$
z	$= d$	$= C - I.$

V^f_0 is the value of the whole firm, i.e. enterprise value, at time 0. r_f is the cost of capital of the whole firm. z is a cash flow, either dividend d or free cash flow $C - I$. C denotes operating cash flows. I denotes net cash invested/divested in/from operating fixed assets. C_{t+1}, I_{t+1} refer to the period $[t, t+1]$. The algebra of multi-anchor valuation can be summarised as

$$V_t = \frac{V_{t+1} + z_{t+1}}{1+r} = \Theta^{\tau(t)}_t + \frac{\theta_{t+1} - r \cdot \Theta^{\tau(t)}_t + V_{t+1} - \left(\theta_{t+1} - z_{t+1} + \Theta^{\tau(t)}_t\right)}{1+r}$$

$$\Theta^{\tau(t)}_{t+1} = \theta_{t+1} - z_{t+1} + \Theta^{\tau(t)}_t$$
$$RI^{\tau(t)}_{t+1} = \theta_{t+1} - r \cdot \Theta^{\tau(t)}_t$$

where:

- θ is a measure of value added, a flow variable, such as earnings or cash flows; θ_{t+1} refers to the period $[t, t+1]$;

- z_{t+1} is a cash flow during the the period $[t, t+1]$;

- $\tau(t)$ is the time of the most recent anchor "change", where most recent is defined from the point of view of time t; note that $\tau(t) \leq t$; an anchor "change" has a specific meaning defined below;

- $\Theta^{\tau(t)}_{t+1}$ and $\Theta^{\tau(t)}_t$ assume that the anchor last "changed" at time $\tau(t) \leq t$ and satisfy a kind of clean surplus relation.

$\Theta_{t+1}^{\tau(t+1)} \neq \Theta_{t+1}^{\tau(t)}$ if and only if the anchor changes at time $t+1$, so that $\tau(t+1) = t+1$ and $\tau(t+1) \neq \tau(t)$ and so that

$$\Theta_{t+1}^{\tau(t+1)} \neq \Theta_{t+1}^{\tau(t)} = \theta_{t+1} - z_{t+1} + \Theta_t^{\tau(t)}. \tag{1}$$

Each anchor can be a multiple of earnings, cash flows, sales or book values. An anchor is often of the type

$$\Theta_t^{\tau(t)} = m_t \cdot \theta_{t+1}.$$

m_t is the time t value of a multiple derived from peers. For example in an equity valuation m_t can be derived from the P/E of peers at time t and θ_{t+1} a forecast of earnings. Anchors can change over time because m_t changes with t and/or because θ_{t+1} changes.

The general MAV of this section can also be written as

$$V_0 = \sum_{t=1}^{\infty} \frac{z_t}{(1+r)^t} = \Theta_0^0 + \sum_{t=0}^{\infty} \frac{RI_{t+1}^{\tau(t)} + \Theta_{t+1}^{\tau(t+1)} - \Theta_{t+1}^{\tau(t)}}{(1+r)^{1+t}}.$$

This equation states that present value V_0 is computed by discounting a stream of generalised residual income RI and a stream of gains/losses $\Theta_{t+1}^{\tau(t+1)} - \Theta_{t+1}^{\tau(t)}$ due to anchor changes. The latter gains/losses are zero when the anchor does not change. By analogy with RIV, the term $V_{t+1} - \Theta_{t+1}^{\tau(t)}$ can be interpreted as a "kind of goodwill" at time $t+1$.

The concept of an anchor based on a valuation multiple, and the related concept of generalised residual income RI are similar to those in Realdon (2023), but he only assumes an initial anchor at time 0 with no later re-anchoring, so that for $t \geq 0$

$$\tau(t+1) = \tau(t) = 0$$
$$\Theta_{t+1}^{\tau(t+1)} = \Theta_{t+1}^{\tau(t)}.$$

MAV with only two anchors, a initial anchor Θ_0^0 and a terminal value anchor Θ_T^T at $T \geq 1$, can be written as

$$V_0 = \sum_{t=1}^{\infty} \frac{z_t}{(1+r)^t} = \Theta_t^0 + \sum_{t=1}^{T} \frac{RI_t^0}{(1+r)^t} + \frac{\Theta_T^T - \Theta_T^0}{(1+r)^T} + \sum_{t=T+1}^{\infty} \frac{RI_t^T}{(1+r)^t}.$$

$\Theta_T^T - \Theta_T^0$ is the gain or loss due to re-anchoring at time T. If we further assume

$$\Theta_0^0 = m \cdot \theta_1$$
$$\Theta_T^T = m \cdot \theta_{T+1}$$

then the gain or loss due to re-anchoring can be written as

$$\Theta_T^T - \Theta_T^0 = m \cdot \sum_{i=2}^{T+1} AEG_i^{MAV}$$
$$AEG_i^{MAV} = \theta_i - \theta_{i-1} - \frac{1}{m} \cdot (\theta_{i-1} - z_{i-1}).$$

This section has presented the MAV algebra. The next section discusses the reasons to re-anchor, i.e. to use multiple anchors in the same valuation.

3.1 Some reasons for multiple anchors

As mentioned above, re-anchoring helps us estimate terminal values at future dates. For example if $V_T^e = \Theta_T^T$ we need not forecast beyond time T. Thus we can choose the anchors so as to reduce the forecast horizon to time T. Different anchors at different future dates may correspond to different growth stages of the company.

Θ_0^0 and Θ_T^T may be linked to different multiples. For example Θ_0^0 may be linked to the normal P/B multiple of peer companies, while Θ_T^T may be linked to the normal P/E of peer companies. This makes sense if, for the company to be valued, book value is gauged to be a more reliable initial anchor, while earnings are gauged to be a more reliable terminal anchor.

Re-anchoring can also take place each period, so that $\Theta_t^t \neq \Theta_t^{t-1}$ for all t. AEG valuation is one such example, as mentioned earlier. Another example is when $\Theta_t^t = m_t \cdot \theta_{t+1}$ so that the anchor changes because both m_t and θ_{t+1} change each period. The valuation multiple m_t may change over time for a number of reasons, for example macro-economic forecasts of time-varying inflation rates or time-varying discount rates because the Government bond yield curve is not flat and changes over time.

Next we consider a set of valuations that are special cases of the general MAV presented above.

4 MAV's in which the value added measure is income

This section concerns MAV's of particular interest in which the value added measure is income, such that $\theta_{t+1} = Earn_{t+1}$ for all t in equity valuations, and such that $\theta_{t+1} = OI_{t+1}$ for all t in firm valuations. OI_{t+1} denotes operating income during $[t, t+1]$. The first such valuation, AEG-MAV, extends and reinterprets generalised AEG valuations. Other MAV's that anchor on earnings follows. Then a MAV that extends the abnormal book value growth (ABG) valuation of Ohlson (2009) is presented. Then various valuations with only two anchors are discussed, as well as valuations with no anchor at all.

4.1 AEG-MAV

OJ (2016), Lai (2020) and Ohlson (2022) present what we refer to as generalised AEG valuations, which extend the AEG valuations of Ohlson and Juettner-Nauroth (2005) and Ohlson (2005). This section presents a MAV that is akin to generalised AEG valuations and in which the anchor is $\Theta_t^t = m_t \cdot Earn_{t+1}$ for each t, such that

$$
\begin{aligned}
V_0^e &= \sum_{t=1}^{\infty} \frac{d_t}{(1+r_e)^t} \\
&= m_0 \cdot Earn_1 + \sum_{t=0}^{\infty} \frac{Earn_{t+1} - r_e \cdot m_t \cdot Earn_{t+1}}{(1+r_e)^{1+t}} + \\
&\quad + \sum_{t=0}^{\infty} \frac{m_{t+1} \cdot Earn_{t+2} - m_t \cdot Earn_{t+1} - (Earn_{t+1} - d_{t+1})}{(1+r_e)^{1+t}}.
\end{aligned}
$$

In this valuation the anchor is a multiple of forward earnings and changes in each period. The second line discounts a stream of generalised residual income. The third line discounts a stream of gains/losses due to re-anchoring each period. m_t may change with t to reflect the forecasted dynamics of the "normal" P/E multiple of peer companies, possibly due to a non-flat Government bond yield curve. If $m_t = m$, then the valuation becomes the generalised AEG valuation of OJ (2016) since

$$
\begin{aligned}
V_0^e &= m \cdot Earn_1 + \sum_{t=0}^{\infty} \frac{Earn_{t+1} - r_e \cdot m \cdot Earn_{t+1} + m \cdot (Earn_{t+2} - Earn_{t+1}) - (Earn_{t+1} - d_{t+1})}{(1+r_e)^{1+t}} \\
&= m \cdot Earn_1 + \sum_{t=0}^{\infty} \frac{RI_{t+1}^t + m \cdot AEG_{t+2}^{MAV}}{(1+r_e)^{1+t}} \\
&= m \cdot Earn_1 + m \cdot \sum_{t=0}^{\infty} \frac{AEG_{t+2}^{OJ}}{(1+r_e)^{1+t}} \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
RI_{t+1}^t &= Earn_{t+1} - r_e \cdot m \cdot Earn_{t+1} \\
AEG_{t+2}^{MAV} &= Earn_{t+2} - Earn_{t+1} - \frac{1}{m} \cdot (Earn_{t+1} - d_{t+1}) \\
AEG_{t+2}^{OJ} &= Earn_{t+2} + d_{t+1}/m - (1+r_e) \cdot Earn_{t+1}
\end{aligned}
$$

$$
RI_{t+1}^t + m \cdot AEG_{t+2}^{MAV} = m \cdot AEG_{t+2}^{OJ}. \qquad (3)
$$

AEG_{t+2}^{OJ} is AEG as per OJ (2016) and line 2 is their generalised AEG valuation.

$m \cdot AEG_{t+2}^{MAV}$ is the gain or loss due to the change of anchor at time $t+1$, itself driven by changed expected earnings and by the earnings re-investment $Earn_{t+1} - d_{t+1}$.

RI_{t+1}^t is akin to residual income because it is the difference between earnings and required earnings in period $t+1$, where the latter are the cost of capital r_e multiplied by the valuation anchor at time t, which is $m \cdot Earn_{t+1}$ and which is a measure of equity holders' investment at time t.

Equation 3 provides an interpretation of $m \cdot AEG_2^{OJ}$ as the sum of RI_{t+1}^t and $m \cdot AEG_{t+2}^{MAV}$. This interpretation seems helpful, because AEG_2^{OJ}, which appeared in OJ (2016), has so far not had a simple economic interpretation.

This lack of interpretation may explain the limited use of the generalised AEG valuation of OJ (2016) so far, despite it being a significant extension of the more well known AEG valuation that makes it easier to compute the implied cost of capital form market prices.

A point to note is also that, while AEG equals the change in the residual earnings,

$$RI_{t+2}^{t+1} - RI_{t+1}^{t} = (Earn_{t+2} - Earn_{t+1})(1 - r_e \cdot m) \neq AEG_{t+2}^{OJ}, AEG_{t+2}^{MAV}.$$

If $m_t = m = \frac{1}{r_e}$ for all t, the AEG-MAV of this section becomes "classic" AEG valuation, namely

$$V_0^e = \frac{Earn_1}{r_e} + \sum_{t=0}^{\infty} \frac{Earn_{t+2} - Earn_{t+1} - r_e (Earn_{t+1} - d_{t+1})}{r_e (1 + r_e)^{1+t}}$$

and in this case $AEG_{t+2}^{OJ}, AEG_{t+2}^{MAV} = RI_{t+2}^{t} - RI_{t+1}^{t}$.

Equation 3 provides new flexibility in modelling expected growth. $RI_{t+1}^{t}, AEG_{t+2}^{MAV}$ can each have different growth rates, while OJ (2016) model growth in AEG_t^{OJ}.

For example OJ (2016) assume constant growth such that:

a1) $AEG_{t+1}^{OJ} = h \cdot AEG_t^{OJ}$ for $t \geq 2$; h is a constant such that $1 + r_e > h$ and $0 < h < 1$;

a2) $V_0^e = m \cdot Earn_1 + m \cdot \frac{AEG_2^{OJ}}{1+r_e-h}$ with $m > 0$ and $\frac{1}{m} < r_e$.

On the other hand AEG-MAV can assume two constant growth rates such that:

b1) $AEG_{t+1}^{MAV} = h \cdot AEG_t^{MAV}$ for $t \geq 2$; again h is a constant such $1 + r_e > h$ and $0 < h < 1$;

b2) $RI_{t+2}^{t+1} = h_{ri} \cdot RI_{t+1}^{t}$ for $t \geq 0$; h_{ri} is another constant such $1 + r_e > h_{ri}$ and $0 < h_{ri} < 1$;

b3) $V_0^e = m \cdot Earn_1 + m \cdot \frac{AEG_{t+2}^{MAV}}{1+r_e-h} + \frac{RI_1^0}{1+r_e-h_{ri}}$ with $m > 0$.

Note that in equation b3) the restriction $\frac{1}{m} < r_e$ is not needed, while in equation a2) it is. If $h = h_{ri}$ equation b3) becomes equation a2). h and h_{ri} determine how quickly AEG^{MAV} and generalised RI fade toward zero. If $RI_1^0 = 0$ in condition b2), generalised residual income is expected to be zero at all times, and all value in equation b3) is due to the initial anchor and to re-anchoring each period.

Lai (2020) extended the generalised AEG valuation of OJ (2016) to firm valuation by effectively replacing $Earn$ with operating income, EBITDA or sales. The MAV of this section can be similarly extended to firm valuation by replacing $Earn$ with operating income, EBITDA or sales, while equation 3 and the interpretation it provides still hold.

The generalised AEG valuation of OJ (2016) as well as the AEG-MAV of this section assume a re-anchoring each period, such that $\Theta_t^t = m \cdot Earn_{t+1}$ for all t, but in the general MAV re-anchoring need not take place each period. This is the novelty. To stress this point, note that the valuation of section 2.2 anchors on forward earnings, but the anchor does not change each period, it only changes at T. Unlike in the generalised AEG valuation of OJ (2016), in MAV the anchor need not change each period.

The MAV of this section can be extended in that each anchor need not be a multiple of forward earnings, but can be a multiple of some average of earnings, as described in an Appendix.

4.2 Anchoring on trailing P/E adjusted for dividends

The trailing P/E multiple is often adjusted for dividends and computed as $\frac{P_0+d_0}{Earn_0}$. If m is derived from this multiple of peer companies, then $m \cdot Earn_0$ can serve as anchor for $V_0^e + d_0$. When re-anchoring on the trailing P/E multiple in this way each period, the MAV becomes

$$V_0^e = \sum_{t=1}^{\infty} \frac{d_t}{(1+r_e)^t} = \Theta_0^0 + \sum_{t=0}^{\infty} \frac{Earn_{t+1} - r_e \cdot \Theta_t^t + \Theta_{t+1}^{t+1} - \Theta_{t+1}^t}{(1+r_e)^{1+t}}$$
$$\Theta_0^0 = m \cdot Earn_0 - d_0$$
$$\Theta_t^t = m \cdot Earn_t - d_t$$
$$\Theta_{t+1}^{t+1} = m \cdot Earn_{t+1} - d_{t+1}$$
$$\Theta_{t+1}^t = \Theta_t^t + Earn_{t+1} - d_{t+1}.$$

An Appendix details how we can similarly anchor firm valuation on the enterprise version of the trailing P/E multiple.

4.3 No initial anchor

Another special case of MAV generalises the abnormal earnings valuation of Realdon (2019). This is the case when $\theta = Earn$ and

$$\Theta_0^0 = 0$$

so that the valuation has no initial anchor. Realdon (2019) assumes no re-anchoring, so that his valuation forecasts and discounts residual income as if $B_0 = 0$.

Instead a MAV can accommodate re-anchoring. For example the terminal anchor Θ_T^T at time $T \geq 1$ can be freely chosen so as to shorten the forecast horizon. Instead the initial anchor can be set to zero if an initial comparison with peers makes little sense, for example because the company to be valued is a young start-up or close to bankrupt.

4.4 Extending abnormal book value growth valuation

Another special case of MAV generalises the abnormal book value growth (ABG) valuation of Ohlson (2009) by setting the anchor $\Theta_t^t = m \cdot B_{t+1}$ at each time t, so that

$$
\begin{aligned}
V_0^e &= \sum_{t=1}^{\infty} \frac{d_t}{(1+r_e)^t} = m \cdot B_0 + \sum_{t=0}^{\infty} \frac{RI_{t+1}^t + m \cdot GABG_{t+2}}{(1+r_e)^{1+t}} \\
&= m \cdot B_0 + \sum_{t=0}^{\infty} \frac{m \cdot B_{t+1} + d_{t+1} - (1+r_e) \cdot m \cdot B_t}{(1+r_e)^{1+t}}
\end{aligned}
$$

$$
\begin{aligned}
&RI_{t+1}^t = Earn_{t+1} - r_e \cdot m \cdot B_t \\
&GABG_{t+2} = B_{t+1} - B_t - (Earn_{t+1} - d_{t+1})/m.
\end{aligned}
$$

$GABG$ stands for generalised ABG. Each period value gained or lost due to re-anchoring is $m \cdot GABG_{t+2}$ and other value gained or lost is RI_{t+1}^t. When $m = 1$ we obtain the ABG valuation of Ohlson (2009)

$$
V_0^e = B_0 + \sum_{t=0}^{\infty} \frac{B_{t+1} + d_{t+1} - (1+r_e) \cdot B_t}{(1+r_e)^{1+t}}.
$$

$B_{t+1} - B_t + d_{t+1}$ is "cum-dividend" abnormal book value growth. $r_e \cdot B_t$ is required book value growth. As Ohlson (2009) noted, ABG valuation does not need any clean surplus assumption. The fact that in GABG valuation $m > 1$ can reflect the effect of conservative accounting in peers balance sheets, and may also shorten the forecast horizon by reducing the time for RI_{t+1}^t to fade to approximately zero.

Valuations that anchor on equity book value, such as the MAV of this section, are of particular interest when anchoring on earnings is not meaningful, for example when earnings in the near future are expected to be negative for the company to valued.

The MAV of this section can also be of interest to value banks, as for banks the P/B multiple is a popular metric. Fair value accounting often entails that P/B is less than 1 for banks, so that $m \cdot B_0$ with $m < 1$ can be a more fitting valuation anchor than B_0.

Again a novelty that MAV can provide to ABG valuation is that the re-anchoring on equity book value need not take place each period.

Finally the MAV of this section can also be used to estimate the value of the whole firm V_0^f, if only we assume that the firm has no financial asset and no financial liability at any time.

4.5 Anchoring equity valuations on P/B and P/E

To estimate V_0^e we can choose an initial anchor Θ_0^0 and a different terminal anchor Θ_T^T that approximates V_T^e so as to shorten the forecast horizon, as stated above. In particular one anchor may be based on the P/B and the other on the P/E multiple of peer companies, or both anchors may be based on P/B. Then we can still assume $\theta = Earn$ and

$$
\begin{aligned}
V_0^e &= \sum_{t=1}^{\infty} \frac{d_t}{(1+r_e)^t} \\
&= \Theta_0^0 + \sum_{t=0}^{T-1} \frac{Earn_{t+1} - r_e \cdot \Theta_t^0}{(1+r_e)^{1+t}} + \frac{\Theta_T^T - \Theta_T^0}{(1+r_e)^T} + \sum_{t=T}^{\infty} \frac{Earn_{t+1} - r_e \cdot \Theta_t^T}{(1+r_e)^{1+t}}
\end{aligned}
$$

$$
\begin{aligned}
\Theta_{t+1}^0 &= \Theta_t^0 + Earn_{t+1} - d_{t+1},\ 0 \le t \le T-1 \\
\Theta_{t+1}^T &= \Theta_t^T + Earn_{t+1} - d_{t+1},\ t \ge T.
\end{aligned}
$$

Then we can set:

- $\Theta_0^0 = m_0 \cdot B_0$ and $\Theta_T^T = m_T \cdot B_T$, where m_0, m_T are respectively the current and the long term normal P/B of peer companies; this valuation anchors on equity book value and re-anchoring only takes place once at time T;

- $\Theta_0^0 = m_0 \cdot Earn_1$ and $\Theta_T^T = m_T \cdot B_T$; in this case m_0 is the current normal P/E and m_T is the expected normal P/B of peer companies; this valuation can be of interest when earnings after time T are expected to be negative or close to zero;

- $\Theta_0^0 = m_0 \cdot B_0$ and $\Theta_T^T = m_T \cdot Earn_{T+1}$; in this case m_0 is the current normal P/B multiple of peer companies and m_T is the expected normal P/E multiple of peer companies at $T > 0$; this valuation is of interest when forward earnings $Earn_1$ do not provide a reliable initial anchor, for example because of some temporary macroeconomic shock currently depresses market valuations and the company's earnings, while long term expected earnings $Earn_T$ provide a more reliable terminal anchor; else the company may be being restructured, so that near future earnings may not provide a reliable anchor; in general this valuation seems of interest when the company's forward earnings are currently expected to be negative or close to zero.

Thus a MAV can initially anchor on the P/E multiple and later re-anchor on the P/B multiple, or vice versa.

4.6 MAV's with two anchors for firm valuation

In this section we briefly review special cases of the general MAV that estimate the value of the firm/enterprise V_0^f and have only two anchors, namely Θ_0^0 and Θ_T^T. Again Θ_T^T may be chosen to approximate V_T^f. In the enterprise valuations we consider in this section we set $\theta = OI$ and then set the two anchors.

We can set $\Theta_0^0 = m_0 \cdot OI_1$ and $\Theta_T^T = m_T \cdot OI_{T+1}$. In this case m_0 is the current normal EV/OI of peer companies, and m_T is the long term normal EV/OI of peers. EV/OI stands for the enterprise price to operating income multiple, effectively the enterprise version of the P/E multiple.

We can set $\Theta_0^0 = m_0 \cdot S_1$ and $\Theta_T^T = m_T \cdot S_{T+1}$, where S_t denotes forecasted sales for year t. In this case m_0 is the current normal EV/S and m_T is the long term normal EV/S of peers. EV/S stands for the enterprise value to sales revenues multiple.This valuation anchors on sales, which seems of interest when the firm's operating income and net operating assets are both negative or close to zero, both in the short term and in the long term, and thus may be misleading valuation anchors.

We can set $\Theta_0^0 = m_0 \cdot OI_1$ and $\Theta_T^T = m_T \cdot S_{T+1}$. In this case m_T is be the long term normal EV/S and m_0 the current normal EV/OI of peers. This valuation seems of interest when, even as short term operating income provides a reliable anchor, terminal sales at time $T+1$ can be forecasted more reliably than terminal operating income, either for the company to be valued or for the peers from which m_T is estimated. This may be due to substantial uncertainty about long term operating margins as opposed to short term margins.

We can set $\Theta_0^0 = m_0 \cdot S_1$ and $\Theta_T^T = m_T \cdot OI_{T+1}$ where m_T and m_0 are to be re-interpreted accordingly. This valuation seems of interest when the firm's operating income is expected to be negative in near future, but in the long term when terminal value is estimated.

We can link one or both anchors to net operating assets book value. For example we can set $\Theta_0^0 = m_0 \cdot NOA_0$ and $\Theta_T^T = m_T \cdot NOA_T$ where NOA denotes net operating assets, while m_T and m_0 are to be reinterpreted accordingly. Provided net operating assets are positive, this valuation is of interest for firms making operating losses, both in the short term and in the long term.

We can link one of both anchors to EBITDA. For example we can set $\Theta_0^0 = m_0 \cdot EBITDA_1$ and $\Theta_T^T = m_T \cdot EBITDA_{T+1}$ where $EBITDA_t$ denotes earnings before interest, tax, depreciation and amortisation in year t, while m_T and m_0 are to be re-interpreted accordingly. This valuation seems of interest when EBITDA proxies the more persistent component of operating income, especially when forward operating income is expected to be affected by transitory items in depreciation and amortisation expenses. This valuation is also of interest when anchoring on the company's operating cash flow, which EBITDA somehow proxies, can be misleading if it is affected by temporary growth or contraction of working capital.

Finally we can set $\Theta_0^0 = 0$ while Θ_T^T can be freely chosen, or we can set $\Theta_0^0 = \Theta_T^T = 0$, giving a valuation with no anchor that discounts residual operating income as if $NOA_0 = 0$. This firm valuation is of interest when no reliable anchor seems available, either in the short or on the long term.

5 MAV forecasting income and re-anchoring each period under constant growth

As the previous section, also this section presents MAV's that forecast income, be it net income or operating income, but the MAV's in this section all assume re-anchoring each period as well as constant growth. Constant growth is a common assumption in the literature and in valuation practice, and often deemed a reasonable approximation for valuation purposes. This section shows that MAV's that decompose generalised AEG as in equation 3 can give some new insight into value-relevant growth. Such MAV's can explain why the valuation multiple of an equity or firm differs from the normal level of the same multiple for its peers.

In this section some variables have different meanings in equity valuations and in firm valuations as follows

Table 2	Equity valuation	Firm valuation
V_0	$= V_0^e$	$= V_0^f$
r	$= r_e$	$= r_f$
z	$= d$	$= C - I$
θ	$= Earn$	$= OI$
$Book$	$= B$	$= NOA.$

Moreover A denotes the accounting variable on which the valuation is anchored.

5.1 The general MAV under constant growth and re-anchoring each period

In this sub-section:

- A_{t+1} can be $Earn_{t+1}$ or B_t in equity valuations;
- A_{t+1} can be OI_{t+1} or NOA_t in firm valuations.

The general MAV under constant growth and re-anchoring each period assumes $\Theta_t^t = m \cdot A_{t+1}$ for all t such that

$$\begin{aligned}
&V_0 = m \cdot A_1 + \textstyle\sum_{t=0}^{\infty} \dfrac{RI_{t+1}^t + m \cdot AEG_{t+2}^{MAV}}{(1+r)^{1+t}} \\
&RI_{t+1}^t = \theta_{t+1} - r \cdot m \cdot A_{t+1} \\
&AEG_{t+2}^{MAV} = A_{t+2} - A_{t+1} - \frac{1}{m} \cdot (\theta_{t+1} - z_{t+1}) .
\end{aligned}$$

Under the clean surplus relation $z_{t+1} = \theta_{t+1} - (Book_{t+1} - Book_t)$

$$AEG_{t+2}^{MAV} = A_{t+2} - A_{t+1} - \frac{1}{m} \cdot (Book_{t+1} - Book_t) .$$

Recall that in equity valuations $Book_0$ is B_0, while in firm valuations $Book_0$ is NOA_0.

Under the additional assumptions that $Q_1 = \frac{\theta_{t+1}}{A_{t+1}}$ and $Q_2 = \frac{A_{t+1}}{Book_t}$ are constant for all $t \geq 0$, it follows that

$$RI_{t+1}^t + m \cdot AEG_{t+2}^{MAV} = (Q_1 - r \cdot m) \cdot A_{t+1} + \left(m - \frac{1}{Q_2}\right) \cdot (A_{t+2} - A_{t+1}) .$$

Further assuming constant growth $A_{t+2} = (1+g) \cdot A_{t+1}$ for $t \geq 0$ implies that

$$\textstyle\sum_{t=0}^{\infty} \frac{RI_{t+1}^{t} + m \cdot AEG_{t+2}^{MAV}}{(1+r)^{1+t}} = \textstyle\sum_{t=0}^{\infty} \frac{A_1 \cdot Q \cdot (1+g)^t}{(1+r)^{1+t}} = \frac{Q}{r-g} \cdot A_1$$

$$Q = Q_1 - r \cdot m + \left(m - \frac{1}{Q_2}\right) \cdot g$$

$$V_0 = \left(m + \frac{Q}{r-g}\right) \cdot A_1.$$

The plausibility condition $r > g$ requires that expected growth in A be less than the cost of capital r. This MAV can explain the V_0/A_1 multiple of an equity or firm as a function of the normal level m of the same multiple for its peers.

In an equity valuation, when $A_1 = Earn_1$, then $Q_1 = 1$ and Q_2 is return on equity.

In a firm valuation, when $A_1 = OI_1$, then $Q_1 = 1$ and Q_2 is return on net operating assets.

In an equity valuation, when $A_1 = B_0$, then $Q_2 = 1$ and Q_1 is return on equity.

In a firm valuation, when $A_1 = NOA_0$, then $Q_2 = 1$ and Q_1 is return on net operating assets.

In this section, return on equity in equity valuation is assumed constant over time. Similarly return on net operating assets in firm valuation is assumed constant over time.

The term $\frac{Q}{r-g}$ in the formula for V_0 can explain the difference between the P/A multiple of the company to be valued from the normal P/A multiple of peers, which is m:

- only when $Q > 0$ should the P/A multiple of the company be higher than the normal P/A multiple of its peers;

- unless A_1 is negative, growth in A only adds value when $m > \frac{1}{Q_2}$;

- generalised residual income is only positive when $Q_1 > r \cdot m$.

For example, if in an equity valuation $A_1 = Earn_1$ generalised residual income is positive only if $\frac{1}{r_e} > m$ and earnings growth g only increases V_0^e when return on equity r_{oe} is high enough, i.e. when $r_{oe} > \frac{1}{m}$. Only when g and r_{oe} are high enough so that $1 - r_e \cdot m + \left(m - \frac{1}{r_{oe}}\right) \cdot g > 0$ should the forward P/E multiple of the company be higher than the normal forward P/E multiple of its peers m.

Alternatively, if in an equity valuation $A_1 = B_0$, generalised residual income is positive only if $r_{oe} > r_e \cdot m$ and equity book value growth g only increases V_0^e when $m > 1$. Only when g and r_{oe} are high enough that $r_{oe} - r_e \cdot m + (m-1) \cdot g > 0$ should the P/B multiple of the company be higher than the normal P/B multiple of its peers m.

This section assumes that all the parameters m, r, g, Q_1, Q_2 be constant over time. When this assumption is not accurate enough, the valuation can assume that these parameters be functions of time. Then V_0 would still be proportional to A_1 provided the time-varying parameters be such that V_0 is comprised between zero and a finite positive number.

Next we consider another case of this general MAV under constant growth.

5.2 MAV re-anchoring firm valuation on sales each period

This section presents a MAV for firm valuation that each period re-anchors on forward sales. Other things as in the previous section, now

$$A_{t+1} = S_{t+1}$$
$$Q_2 = r_{os} = \frac{OI_{t+1}}{A_{t+1}}$$
$$Q_2 = a_{to} = \frac{A_{t+1}}{NOA_t}.$$

r_{os} is return on sales and a_{to} is assets turnover. Both r_{os} and a_{to} are assumed to be constant over time for all $t \geq 0$. Then this MAV assumes that $\Theta_t^t = m \cdot S_{t+1}$ for all t and $\theta = OI$, such that

$$V_0^f = m \cdot S_1 + \sum_{t=0}^{\infty} \frac{RI_{t+1}^t + m \cdot AEG_{t+2}^{MAV}}{(1+r_f)^{1+t}}$$
$$RI_{t+1}^t = OI_{t+1} - r_f \cdot m \cdot S_{t+1}$$
$$AEG_{t+2}^{MAV} = S_{t+2} - S_{t+1} - \frac{1}{m} \cdot (OI_{t+1} - (C_{t+1} - I_{t+1})).$$

Under the clean surplus relation $C_{t+1} - I_{t+1} = OI_{t+1} - (NOA_{t+1} - NOA_t)$

$$AEG_{t+2}^{MAV} = S_{t+2} - S_{t+1} - \frac{1}{m} \cdot (NOA_{t+1} - NOA_t).$$

Since r_{os} and a_{to} are constant for all $t \geq 0$, it follows that

$$RI_{t+1}^t + m \cdot AEG_{t+2}^{MAV} = (r_{os} - r_f \cdot m) \cdot S_{t+1} + \left(m - \frac{1}{a_{to}}\right) \cdot (S_{t+2} - S_{t+1}).$$

Further assuming constant sales growth $S_{t+2} = (1+g) \cdot S_{t+1}$ for $t \geq 0$ implies that

$$Q = r_{os} - r_f \cdot m + \left(m - \frac{1}{a_{to}}\right) \cdot g$$
$$V_0^f = \left(m + \frac{Q}{r_f - g}\right) \cdot S_1.$$

The plausibility condition $r_f > g$ requires that expected sales growth be less than the firm's cost of capital. This MAV can explain the P/S multiple of a firm as a function of the normal P/S multiple m of its peers.

When $m = \frac{1}{a_{to}}$ re-anchoring on forward sales each period adds no value, and sales growth only adds value when $m > \frac{1}{a_{to}}$, unless expected sales are negative. If $r_{os} = r_f \cdot m$ generalised residual income is zero, and is only positive when $r_{os} > r_f \cdot m$, unless expected sales are negative. When $m = \frac{1}{a_{to}}$ and $r_{os} = r_f \cdot m$, then $V_0^f = m \cdot S_1$ so that the P/S multiple of the firm should equal the normal P/S multiple of its peers. Finally, only when $Q > 0$ should the P/S multiple of the firm be higher than the normal P/S multiple of its peers, unless expected sales are negative.

While this section assumes that all the parameters $m, r_f, g, r_{os}, a_{to}$ be constant over time, these parameters may be functions of time and yet V_0^f could still be proportional to S_1.

5.3 The independence of the growth rate and the discount rate r

A merit of the just seen MAV's under constant growth is that the growth rate g refers to A, which is an easily observable accounting variable. Another merit is that AEG^{MAV} grows at the same rate as A and does not depend on the cost of capital r. Instead AEG or AEG^{OJ} or residual income all depend on r. Penman and others (2023) have highlighted that computing the implied cost of capital r through a valuation model that assumes constant growth in AEG or AEG^{OJ} or residual income, all of which depend on r, poses an undesirable circularity. g and r would not be independent inputs to the valuation model. Instead the above MAV's under constant growth bypass this circularity, because AEG^{MAV} does not depend on r. Thus g and r would be independent inputs. This is a convenient simplification when computing the implied cost of capital r.

6 MAV's in which the value added measure is cash flow

The MAV's seen so far have assumed that the value added variable θ is income, be it net income or operating income. Instead this section concerns MAV's in which the value added variable is a cash flow such that $\theta_{t+1} = z_{t+1}$ for all t. As in Table 1, also in this section the variables V_0, r, z have different meanings in equity valuations and in firm valuations. These MAV's can be summarised as follows

$$
\begin{aligned}
V_0 &= \textstyle\sum_{t=1}^{\infty} \frac{z_t}{(1+r)^t} \\
&= \Theta_0^0 + \textstyle\sum_{t=0}^{\infty} \frac{RI_{t+1}^t + m \cdot AEG_{t+2}^{MAV}}{(1+r)^{1+t}} \\
&= \Theta_0^0 + m \cdot \textstyle\sum_{t=0}^{\infty} \frac{AEG_{t+2}^{OJ}}{(1+r)^{1+t}}.
\end{aligned}
$$

In equity valuation $z = d$, while in firm valuation $z = C - I$. Moreover if $\Theta_t^t = m \cdot z_{t+1}$ for all t, i.e. if re-anchoring takes place each period and each anchor is a multiple of forward cash flow, then

$$
\begin{aligned}
RI_{t+1}^t &= z_{t+1} - r \cdot m \cdot z_{t+1} \\
AEG_{t+2}^{MAV} &= z_{t+2} - z_{t+1} - \frac{1}{m} \cdot (z_{t+1} - z_{t+1}) = z_{t+2} - z_{t+1} \\
AEG_{t+2}^{OJ} &= z_{t+2} + z_{t+1}/m - (1+r) \cdot z_{t+1}.
\end{aligned}
$$

These valuations forecast and discount only cash flows and can be viewed as a weighted average of forecasted cash flows of different future periods.

Again of special interest are also valuations in which re-anchoring only takes place at time T. Note that in these valuations:

- $\Theta_t^0 = \Theta_0^0 + \sum_{i=1}^t z_i - \sum_{i=1}^t z_i = \Theta_0^0$ for $1 \leq t \leq T$;
- $\Theta_t^T = \Theta_T^T + \sum_{i=T+1}^t z_i - \sum_{i=T+1}^t z_i = \Theta_T^T$ for $t \geq T+1$.

An example of one such firm valuation that re-anchors only at time T would assume

$$
\begin{aligned}
\Theta_0^0 &= m_0 \cdot (C_1 - I_1) \\
\Theta_T^T &= m_T \cdot (C_{T+1} - I_{T+1}).
\end{aligned}
$$

Here m_0 and m_T is the "normal" EV-to-free-cash-flow multiples of peers time 0 and time T.

This section has focused on valuations whereby $\theta_{t+1} = z_{t+1}$ for all t. While these valuations anchor on multiples of forward cash flows, they may also anchor on average past and/or future cash flows. When these valuations have no anchor at all, all anchors are set to zero and the valuations become standard discounted cash flow valuations. Moreover these valuations can also accommodate anchors linked to accounting variables such as EBITDA, so that

$$
\begin{aligned}
\Theta_0^0 &= m_0 \cdot EBITDA_1 \\
\Theta_T^T &= m_T \cdot EBITDA_{T+1}
\end{aligned}
$$

where m_0 and m_T are the current and expected future level of the normal EV/EBITDA multiple derived from peer firms. A merit of anchoring on $EBITDA$ is that the EV/EBITDA multiple is a popular valuation metric, even as Penman (2012) highlights its shortcomings. Then $EBITDA$ is more likely to be positive than is free cash flow $C - I$, does not depend on depreciation and amortisation policies, and is less sensitive than is C to fluctuations in working capital. These reasons too suggest that a valuation that discounts free cash flows may anchor on $EBITDA$, even as $EBITDA$ itself is not exactly a cash flow.

We finally note that the MAV algebra allows a valuation that discounts free cash flow to anchor also on net operating assets, or on operating income, or on sales. For example such a valuation may use the two anchors $\Theta_0^0 = m_0 \cdot OI_1$ and $\Theta_T^T = m_T \cdot OI_{T+1}$. Linking the initial anchor at time 0 to net operating assets, or to operating income, or to sales can help when near future free cash flows are negative, so that anchoring on such free cash flows would not be meaningful. Then, linking the terminal anchor at time T to net operating assets, or to operating income, or to sales can shorten the forecast horizon, which tends to be notoriously long in valuations that discount free cash flows.

7 Conclusion

This paper has presented the algebra of accounting based valuations with multiple valuation "anchors", i.e. with different anchors in different time periods. Each anchor is an estimate of value typically based on a multiple such as price-earnings, price-to-book, price-to-sales of peer companies. Anchors may or may not change in each period, which generates a variety of new valuations that analysts and researchers can choose from, for example for estimating the implied cost of capital. Anchors can be chosen so as to shorten the valuation forecast horizon. The main focus has been on valuations in which the anchor changes each period, and on valuations that have two anchors, i.e. an initial anchor and a different anchor for terminal value.

The multi-anchor valuation algebra encompasses and generalises the RIV of Ohlson (1995) and the AEG valuation of Ohlson and Juettner-Nauroth (2005) and Ohlson (2005). The said algebra also encompasses and reinterprets the generalised AEG valuations of Ohlson and Johannesson (2016), Lai (2020) and Ohlson (2022). The multi-anchor valuation algebra is applicable also to cash flow based valuations.

A Appendix

A.1 Anchoring on average earnings

In MAV each anchor need not be a multiple of forward earnings, but can be a multiple of some average of earnings, be they past earning and/or forecasted future earnings. If so, assuming re-anchoring in each period, each anchor can be

$$\Theta_t^t = m_t \cdot \overline{Earn}_t$$

where $\overline{Earn}_t$ is a time t weighted average of earnings. Average earnings are often perceived as more reliable indication of profitability than the earnings of a single period. For example $\overline{Earn}$ may follow some EWMA process such that $\overline{Earn}_t = \alpha \cdot Earn_{t-1} + (1-\alpha) \cdot \overline{Earn}_{t-1}$ or such that $\overline{Earn}_t = \alpha \cdot Earn_t + (1-\alpha) \cdot \overline{Earn}_{t-1}$. If $m_t = m$ re-anchoring in each period is only due to the updating of $\overline{Earn}_t$ and implies

$$V_0^e = \sum_{t=1}^{\infty} \frac{d_t}{(1+r_e)^t} = \Theta_0^0 + \sum_{t=0}^{\infty} \frac{Earn_{t+1} - r_e \cdot \Theta_t^t + \Theta_{t+1}^{t+1} - \Theta_{t+1}^t}{(1+r_e)^{1+t}}$$
$$\Theta_0^0 = m \cdot \overline{Earn}_0$$
$$\Theta_t^t = m \cdot \overline{Earn}_t$$
$$\Theta_{t+1}^t = m \cdot \overline{Earn}_t + Earn_{t+1} - d_{t+1}$$
$$\Theta_{t+1}^{t+1} = m \cdot \overline{Earn}_{t+1}.$$

This MAV again separately forecasts generalised residual income $RI_{t+1}^t = Earn_{t+1} - r_e \cdot \Theta_t^t$ and gains/losses due to re-anchoring, namely

$$\Theta_{t+1}^{t+1} - \Theta_{t+1}^t = m \cdot AEG_{t+2}^{MAV} = m \cdot \left(\overline{Earn}_{t+1} - \overline{Earn}_t\right) - \left(Earn_{t+1} - d_{t+1}\right).$$

Note also that forecasted forward earnings may often be set as $Earn_{t+1} = \overline{Earn}_t$.

A.2 Anchoring firm valuation on the enterprise version of the trailing P/E multiple

We can anchor firm valuation on the enterprise version of the trailing P/E multiple, so that $m \cdot OI_0$ can serve as anchor for $V_0^f + C_0 - I_0$. Here we follow Penman's (2012) textbook notation. V_0^f is enterprise value at time 0. OI_0, C_0, I_0 refer to the period $[-1, 0]$. OI_0 denotes operating income. C_0 denotes operating cash flows. I_0 denotes net cash invested/divested in/from operations. m can be derived from the enterprise version of the trailing P/E multiple of peer companies. Then the MAV becomes

$$V_0^f = \sum_{t=1}^{\infty} \frac{C_t - I_t}{(1+r_f)^t} = \Theta_0^0 + \sum_{t=0}^{\infty} \frac{OI_{t+1} - r_f \cdot \Theta_t^t + \Theta_{t+1}^{t+1} - \Theta_{t+1}^t}{(1+r_f)^{1+t}}$$
$$\Theta_0^0 = m \cdot OI_0 - (C_0 - I_0)$$
$$\Theta_t^t = m \cdot OI_t - (C_t - I_t)$$
$$\Theta_{t+1}^{t+1} = m \cdot OI_{t+1} - (C_{t+1} - I_{t+1})$$
$$\Theta_{t+1}^t = \Theta_t^t + OI_{t+1} - (C_{t+1} - I_{t+1}).$$

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