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A Topology Detector Based Power Flow Approach for Radial and Weakly Meshed Distribution Networks

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***Abstract*—Power distribution networks may need to be switched from one radial configuration to another radial structure, providing better technical and economic benefits. Or, they may also need to switch from a radial configuration to a meshed one and vice-versa due to operational purposes. Thus the detection of the structure of the grid is important as this detection will improve the operational efficiency, provide technical benefits, and optimize economic performance. Accurate detection of the grid structure is needed for effective load flow analysis, which becomes increasingly computationally expensive as the network size increases. To perform a proper load flow analysis, one has to build the distribution load flow (DLF) matrix from scratch cost of which is unavoidable with the growing size of the network. This will considerably increase the computation time when the system size increases, compromising applicability in online implementations. In this study we introduce a novel graph-based model designed to rapidly detect transitions between radial and weakly meshed systems. By leveraging the characteristic properties of Sparse Matrix-Vector product (SpMV) operations, we accelerate power flow calculations without necessitating the complete reconstruction of the DLF matrix. With this approach we aim to reduce the computational costs and to improve the feasibility of near-online implementations.**

***Index Terms*—Graph Laplacian matrix, power distribution networks, topology detection, weakly meshed distribution systems.**

I. Introduction

The operation philosophy of electrical power distribution networks is changing with intelligent control devices. Apart from the apparent advantages of penetration of renewable energy sources (RES), additional operational problems are occurring due to the fluctuations in voltage magnitudes. Traditionally, distribution networks were designed to operate in radial mode. Due to reliability issues, this type of operation is not the best choice [1]. Thus, with switching operations, when needed, the systems may operate in a weakly meshed structure as well. Due to the known fact that especially with the use of DGs and the new loads such as EVs, the load and generation structure changes rapidly. Thus, to improve the efficiency one may need to change the topology of the system at hand and may need to perform a fast power flow.

The power flow calculations are the basis of the analysis of distribution grids. According to [2], power flow studies on distribution networks are grouped as the forward-backward sweep algorithm, compensation methods, implicit Z_{BUS} Gauss method, modified Newton-like methods, and miscellaneous power flow methods. Unlike transmission systems, Newton-Raphson-based power flow calculation is inappropriate due to high R/X ratios [3]; Kirchoff voltage (KVL) and current (KCL) law-based ladder iterative method is preferred [4]. When the network is reconfigured with a few switching operations, the ladder iterative network method is not applicable; direct methods that convert power flow injections to equivalent current injections are used [5].

Several studies focus on developing methodologies for solving power flow problems in radial distribution networks. Most of these methods are based on KVL and KCL. From those [6] incorporates linear proportional principle after the backward step to represent the voltage at the substation node in terms of the specified voltage. The method is validated on three-phase distribution networks, but weakly-meshed conditions were not considered. Another one improves the computational time of the backward-forward sweep algorithm-based load flow by

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using the breadth-first search method to create a modified incidence matrix [7]; this paper has not considered the weakly meshed cases as well.

Also, approximate linear models have been proposed for distribution networks. In [8], the authors propose a linear approximate solution to the power flow equations for a balanced distribution network using fixed-point interpretation. A similar approach is used in [9] by extending the idea to unbalanced distribution networks. In [10], the authors propose an approximate power flow considering the ZIP model of the loads and PV nodes. The experiments performed on the IEEE 37-bus distribution network showed low numerical errors.

The studies on developing power flow methodologies for weakly meshed distribution systems vary. One of the initial works proposes a method that can solve both radial and weakly meshed distribution networks [11]. To be able to solve weakly meshed networks, the method uses breakpoints to create loops and converts the system to a radial one, then applies Kirchoff voltage and current laws. In [12], the authors have developed an iterative load flow methodology suitable for both radial and weakly meshed systems. The method models the loads as impedances and solves the system by using the the fact that the voltages and currents are related to unknowns linearly. The literature on solving power flow problem using graph theory based applications is not very rich, a recent study [13] applies graph theory to solve radial and weakly meshed systems, another one models the power flow problem as a network-flow problem and solves by using a maximum-flow algorithm [14].

This paper aims to develop a model for the cases when the power distribution system changes from a radial one to a weakly meshed one and vice-versa from an operational point of view. In the distribution systems, the equations will change if there is at least one cycle (or loop). When these changes happen, to perform load flow, one has to build the distribution load flow (DLF) matrix from scratch. When the system size increases, this will add additional unavoidable computational costs.

The developed model integrates graph theory with power system analysis, which enables efficient detection and handling of grid structure changes. The model addresses the computational challenges and provides high accuracy with a scalable structure to larger networks. With eliminating the need for repetitive DLF matrix reconstructions, the proposed model represents a robust solution for managing dynamic grid structures. Compared to the previous papers mentioned above, the main contributions of this study are summarized as follows.

- A fast novel graph-based approach to detect the switches from radial to weakly meshed systems (or vice-versa).
- A novel power flow method that reuses the previous calculation results after transformation to a weakly meshed network from the radial network (or vice-versa) utilizing the characteristic properties of Sparse Matrix-Vector product (SpMV) operations.

The rest of the paper is organized as follows. In the methodology section, we give the topology detector model, bus current injections to branch currents (BIBC) and the branch current to bus voltages (BCBV) matrix-based models, the construction of mesh-related block matrices, and the proposed algorithm. Section III details the experimental results. Finally, we conclude the paper in the Conclusion section.

II. Methodology

A. Topology Detector Model

We propose a graph-based novel approach based on the numerical properties of the Graph Laplacian matrix to detect the topology of the distribution system. One can define a graph as an ordered pair $G = (H, E)$ where H and E corresponds to the set of vertices (nodes, points) and edges (links, lines), respectively. The element-wise definition of Graph Laplacian is as follows:

$$L_{ij} = \begin{cases} deg(h_i) & \text{if} \quad i = j \\ -1 & \text{if} \quad i \neq j \text{ and } h_i \text{ adjacent to } h_j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $h_i, h_j \in H$, n is the number of the nodes in set H and $deg(h_i)$ corresponds to the connections to or from the related vertex. Note that i and j should be smaller or equal to n [15]. The Laplacian matrix is a semi-positive definite matrix with at least one zero eigenvalue. In other words, the rank of the Laplacian matrix is at most $n-1$. However, if there is more than one component in the graph, zero eigenvalues increase. In graph theory, a component of an undirected graph is a connected subgraph that is not part of a larger connected subgraph. Moreover, the diagonal elements of a Laplacian matrix correspond to the degree of each vertex, as can be seen from its definition. Therefore, the trace of the Laplacian matrix is equal to the number of connections (twice the number of edges if it is an undirected graph) in a graph, which can be written as $trace(L) = 2|E|$. The number of components in the graph G equals the $n - rank(L)$ [15]. If the number of edges in a graph is at least

$$|E| = n - (n - rank(L)) + 1 = rank(L) + 1 \tag{2}$$

we can conclude that G contains at least one cycle if $|E|$ is strictly larger than the rank of the graph Laplacian matrix. Combining this fact with $trace(L) = 2|E|$, we can claim that an indirect graph G is acyclic if and only if the following equality holds,

$$\frac{1}{2} trace(L) = rank(L) \tag{3}$$

The algorithmic structure of the topology detector is given in Alg. 1. Note that the computational cost of rank determination can be high for very large systems. However, randomized linear algebra routines can be helpful in avoiding the memory and time complexity of the proposed approach. For example, a low-cost method based on orthogonal polynomials can be used for numerical rank estimation of Laplacian [16]. Moreover, the trace computation can be done without explicit Laplacian construction via the randomized Hutchinson trace estimator [17].

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Algorithm 1 Topology Detector

```
Build adjacency matrix for the distribution network
Construct the Laplacian matrix,
Calculate the rank and the trace of the Laplacian matrix
if ½trace(L) = rank(L) then
    The system is radial.
else
    The system is weakly meshed.
end if
```

B. Exploiting the Block Structure of BIBC and BCBV Matrices

The structure of the BIBC matrix relies on the connections of the associated graph. Applying the Kirchoff laws to the connections between the buses will create a well-defined sparse block structure on BIBC matrices. The goal of the first part of the study is to exploit this structure to reduce the computational cost of the overall algorithm. In general, the direct method for solving the distribution systems starts with the computation of the distribution load flow (DLF) matrix as $[DLF] = [BCBV][BIBC]$. The general algorithm of the direct approach for radial systems proposed by [5] is listed in Alg. 2. Note that in all all algorithms below, i, represents the node in the system, and k shows the iteration number.

Algorithm 2 Direct Approach

```
1: Build the BIBC and BCBV matrices
2: Form the [DLF] = [BCBV][BIBC].
3: k = 0
4: while |V^{k+1} - V^k| > τ do
5:     I_i^k = ((P_i + jQ_i)/V_i)^*
6:     ΔV^{k+1} = DLF × I^k
7:     V^{k+1} = V^k + ΔV^{k+1}
8:     k=k+1
9: end while
```

The most important advantage of the Alg. 2 arises from its direct approach to obtaining the voltages without solving a linear equation system at each iteration. On the other hand, there are still some possibilities to improve the computational efficiency of this algorithm. For instance, the block sparsity structure of the $BIBC$ and $BCBV$ matrices will get lost during the computation, and preserving that sparsity structure will enhance the speed of the calculation. The first step of the proposed algorithm will focus on this problem.

The radial structure of the distribution system will form an acyclic graph by its definition. It can also be considered that whole branch endpoints should be connected to the first bus of the physical system once we assume that the system is fully connected. This fact can also be observed from the number of zero eigenvalues in the Laplacian graph, which equals one. By using this observation, one can easily apply one of the well-known shortest-path algorithms (such as Dijkstra Algorithm [18]) to determine the branch connection scheme of the given distribution system. In Fig. 1, the determined branch tree structures are shown via the Dijkstra algorithm for the 33-bus distribution system.

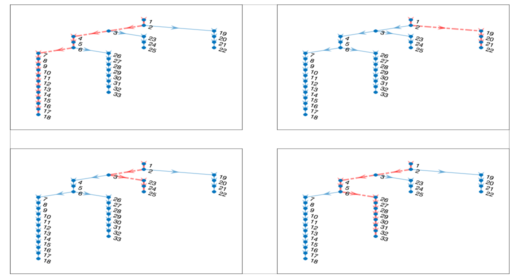

Fig. 1. Application of Dijkstra algorithm to the 33-bus radial distribution system topology. Note that the red color in the graph corresponds to the algorithm's selection path for each branch.

Once the whole trees in the radial network are determined, one can easily decompose the BIBC (and BCBV) matrices into diagonal blocks. Note that in Fig. 2, there is also an upper diagonal block which shows the mutual effect of the common edges of each tree. Step 6 of the Alg. 2 can be rewritten as $\Delta V^{k+1} = BCBV \times y$ where $y = BIBC \times I^k$. The vector y will have r blocks where r corresponds to the number of trees in the radial system and can be computed as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} BIBC_1 & & BIBC_0 & \\ & BIBC_2 & & \\ & & \ddots & \\ & & & BIBC_r \end{bmatrix} \begin{bmatrix} I^1_{node} \\ I^2_{node} \\ \vdots \\ I^r_{node} \end{bmatrix} \tag{4}$$

Here, $BIBC_0$ represents the mutual injection of the common edges and $BIBC_p$ where $p = 1, \ldots, r$ corresponds to the independent injection of the edges for r trees that appear in the radial system. By using this block decomposition, the whole computations can be realized independently as,

$$\begin{aligned} y_1 &= BIBC_1 I^1_{node} + BIBC_0 \hat{I}_{node} \\ y_p &= BIBC_p I^p_{node} \quad i = 2, \ldots, r \end{aligned} \tag{5}$$

where $\hat{I}_{node} = [I^2_{node} \quad I^3_{node} \quad \ldots \quad I^r_{node}]^T$. The graphical illustration of equation 5 for the 33-bus system is depicted in Fig. 2. For this radial system, $r = 4$ and the trees found by Dijkstra's algorithm are shown in Fig. 1.

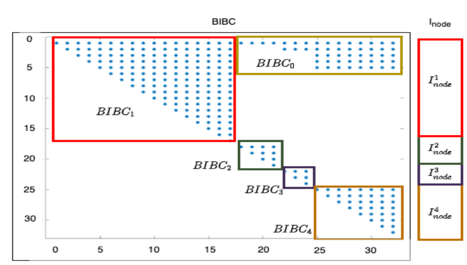

Fig. 2. Illustration of the sparse decomposition of the BIBC matrix of radial 33-bus distribution network

Then, for obtaining the ΔV^{k+1}, one should perform the second sparse matrix-vector product operation as $\Delta V^{k+1} =$

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$BCBV \times y$. The algorithm of the modified direct approach is listed in Alg. 3.

Algorithm 3 Direct Approach with Block SpMVs

```
Build the BIBC and BCBV matrices
Run Dijkstra Algorithm to obtain the block structure
k = 0
while |V^{k+1} - V^k| > τ do
    I_i^k = ((P_i + jQ_i)/V_k)^*
    Apply Eq. 5 to obtain y
    ΔV^{k+1} = BCBV × y
    V^{k+1} = V^0 + ΔV^{k+1}
end while
```

C. Construction of Mesh-Related Block Matrices

The existence of a loop in meshed (or weakly meshed) systems creates extra blocks in the matrix structure given in Eq. 4. However, the inner block structure will remain the same. Due to this fact, we focused on creating the loop-related blocks only and involving those blocks to reduce the computational burden of the overall meshed distribution system simulation. We will reuse the radial system results in the proposed approach and update the solution with only a few new sparse matrix-vector products. Note that for the power flow calculations in the radial system, the numerical updates of the voltages using the direct approach are defined as follows:

$$\Delta v = [DLF_{\text{radial}}] I_{node} \tag{6}$$

where $[DLF_{\text{radial}}] = [BCBV][BIBC]$. On the other hand, for the weakly meshed systems, the update will be transformed into a block matrix format where B, C, and D can be formed by the loops in the meshed system.

$$\begin{bmatrix} \Delta v \\ 0 \end{bmatrix} = \begin{bmatrix} DLF_{\text{radial}} & B \\ C & D \end{bmatrix} \begin{bmatrix} I_{node} \\ I_{loop} \end{bmatrix} \tag{7}$$

By applying the basic substitution in equation (7), it can be rewritten as $\Delta v = (DLF_{\text{radial}} - BD^{-1}C) I_{node}$. Moreover, we can derive that $\Delta v = DLF_{\text{radial}} I_{node} - BD^{-1}CI_{node}$. Note that the second part refers only to the loops in the meshed system. Our proposed mechanism is based on detecting the existence and location of a cycle in a distribution system. In this way, the recalculation of the DLF matrix can be easily avoided by forming only loop-related block matrices in Eq. 7. Although the Alg. 1 gives the current information related to the system connection state either in radial or meshed, the main issue for constructing the loop-related matrices is to figure out the locations of the new loop automatically. The Laplacians of the current and next connection graphs can be used for this purpose. The diagonal elements of the Laplacian matrix show the degree of each vertex. Assume that the L_1 and L_2 represent the Laplacians of the radial and meshed systems, respectively. $|diag(L_1) - diag(L_2)| > 0$ gives the node numbers of the newly connected meshes. Once the connection nodes are determined, the rest of the procedure will calculate the current directions of each edge in the loop to determine the negative coefficients for the required edges. In Fig.3, the loop is colored according to its directions for the 33-bus distribution system. Note that the green lines represent the negative directions in the current after establishing a new connection between nodes 12 and 22. To obtain the mesh-related blocks in $BIBC$ and $BCBV$ matrices, one can rewrite the equation 7 as follows,

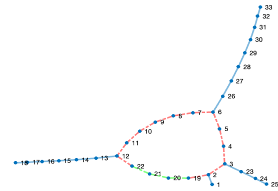

Fig. 3. A new topology for the meshed structure between nodes 12 and 22 at the 33-bus system. Here, the green and red colors correspond to the new connections in the graph after the applied operation.

$$\begin{bmatrix} \Delta v \\ 0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} I_{node} \\ I_{loop} \end{bmatrix} \tag{8}$$

where A_1 and A_2 are the $BCBV$ and $BIBC$ matrices of the radial system. Moreover, $B_1, B_2 \in \mathbb{C}^{n \times l}$, $C_1, C_2 \in \mathbb{C}^{l \times n}$, $D_1, D_2 \in \mathbb{C}^{l \times l}$ where l and n represents the number of the loops and vertices respectively. To compute the voltage updates in a meshed system effectively, Eq. 8 can be rewritten as,

$$\begin{bmatrix} \Delta V^{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & D_1D_2 \end{bmatrix} \begin{bmatrix} I_{node}^k \\ I_{loop}^k \end{bmatrix} \tag{9}$$

If the Eq. 7 is reconsidered, and one writes the blocks w.r.t. those notation,

$$\begin{aligned} DLF_{radial} &= A_1A_2 + B_1C_2 \\ B &= A_1B_2 + B_1D_2 \\ C &= C_1A_2 + D_1C_2 \\ D &= D_1D_2 \end{aligned} \tag{10}$$

can be obtained. Note that, B_1 and C_2 should be zero vectors to keep the upper and lower triangular forms of the meshed system BIBC and BCBV matrices. Hence, the overall voltage updates can be rewritten as,

$$\Delta V^{k+1} = (A_1A_2 - [A_1B_2(D_1D_2)^{-1}C_1A_2]) I_{node}^k \tag{11}$$

Since the $(A_1A_2)I_{node}$ is already calculated in radial system computations, after the switch from the radial to the meshed system, the voltage updates can be calculated very efficiently by applying only a few sparse-matrix vector products. Note that B_2 and C_1 will be formed according to the newly constructed loop by implementing the Kirchhoff laws, D_1 and D_2 will be a $l \times l$ matrix of related impedance and ones, respectively.

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D. Proposed Algorithm

The proposed algorithm has three main building blocks. During the execution of the simulation, firstly, the Alg. 1 will be executed to detect any instant switches from the radial to the meshed (or vice-versa) system. If the system is in radial mode, the Alg. 2 will be used to obtain the simulation results. However, if the Alg. 1 detects any change in the system state, the loop-related block matrices will be created, and the voltage updates will be computed via the Eq. 11. The numerical experiments show that the proposed mechanism has lesser computational time in most cases w.r.t. regular direct method and Newton-Raphson-based approaches. The overall approach is illustrated in Fig. 4.

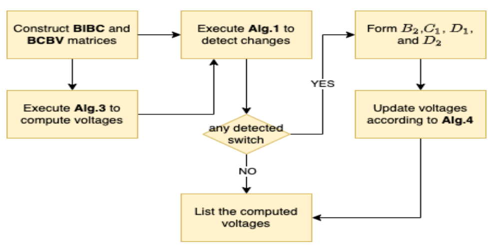

Fig. 4. Flowchart of the overall approach

On the other hand, the change in the computational order of the matrix-vector products in the proposed procedure produces slightly different results with an acceptable error rate mostly below 1%. The overall algorithm is listed in Alg. 4. Note that, the convergence criterion is selected equal to the default $\tau = 10^{-8}$ value of MatPower.

Algorithm 4 Proposed Algorithm

```
Build the BIBC and BCBV matrices
Execute Alg. 3 to obtain voltages
if (any switch detected by Alg. 1) then
    Form B_2, C_1, D_1, and D_2
    while |V^{k+1} − V^k| > τ do
        I_i^k = ((P_i + jQ_i)/V_k)^*
        Apply Eq. 5 to obtain y
        ΔV̂^{k+1} = BCBV × y
        ΔV^{k+1} = ΔV̂^{k+1} − [A_1 B_2 (D_1 D_2)^{−1} C_1 A_2]
        V^{k+1} = V^k + ΔV^{k+1}
    end while
end if
```

III. Experimental Results

A. Detection of Mesh Structure

We performed several simulations to test the Alg. 1. Table I shows the ranks and traces of the different test systems with different topologies. As shown in the last column, detection based on the Graph Laplacian's numerical properties yields the correct system topology in all test cases.

TABLE I
Experimental results for the Alg. 1

System	Connection Type	rank(L)	trace(L)	Result
7-bus [19]	meshed	6	14	meshed
7-bus [19]	radial	6	12	radial
33-bus [20]	radial	32	64	radial
33-bus [20]	meshed	32	66	meshed
33-bus [20]	meshed	32	68	meshed
33-bus [20]	meshed	32	72	meshed

B. Weakly Meshed System Simulation Results

To demonstrate the time efficiency of the Alg.4, several simulations are realized with 33-bus and 69-bus distribution systems. In the numerical experiments, we used five meshed system and execute each experiment 100 times to have a more reliable measurement for the CPU times. All simulations are realized in an 8-core M1 chip with 16 GB of memory. Matlab is used for the coding, and for the Newton-Raphson simulation, MatPower [21] is used.

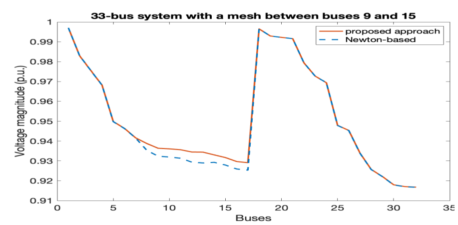

Fig. 5. Simulation results for the 33-bus system.

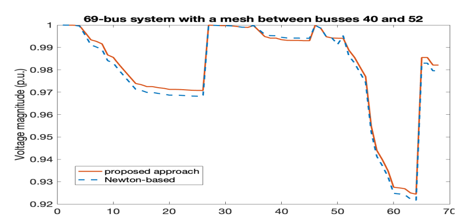

Fig. 6. Simulation results for the 69-bus system.

In Figs. 5 and 6, we compared the simulation results for the meshed 33-bus [20] and 69-bus [22] system with new connections between the buses 26-69 and 8-21, respectively. Besides the graphical illustration of the results, in Table II, the error and the average CPU times are listed for whole scenarios. The error term is simply the 2-norm of the difference between the amplitudes of the computed voltages and is defined as $err = ||abs(V_P) - abs(V_{NR})||_2$ where V_P and V_{NR} correspond to the amplitude of the voltages computed by the proposed approach and Newton-Raphson method respectively. Similarly, $time_P$ and $time_{NR}$ correspond to the mean value of 100 executions of the proposed approach and Newton-Raphson method. As seen from the table, the proposed approach has a clear advantage in computational time with an acceptable error for the Newton-Raphson approach. A more detailed analysis of the computational time of the algorithm for different scenarios

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is depicted in Figs. 7 and 8 where the statistical execution results of the CPU times including the mean CPU times and the outliers are represented for several cases.

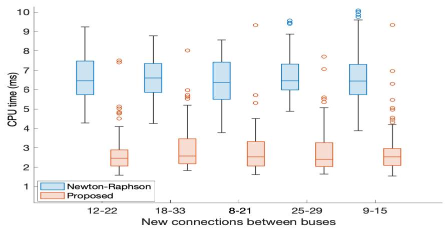

Fig. 7. Timing comparison of Newton-Raphson and proposed method for the 33-bus system. Note that these results show 100 executions for each approach.

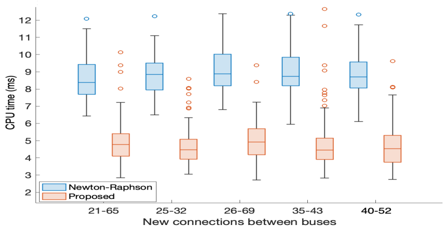

Fig. 8. Timing comparison of Newton-Raphson and proposed method for the 69-bus system. Note that these results show 100 executions for each approach.

TABLE II
EXPERIMENTAL RESULTS FOR THE ALG. 4

System	Meshed Buses	$err(\%)$	$time_P(ms)$	$time_{NR}(ms)$
33-bus	8-21	0.99	**3.8**	7.8
	9-15	1.33	**3.7**	6.8
	12-22	0.96	**3.3**	7.0
	18-33	0.03	**3.2**	6.7
	25-29	1.07	**3.0**	6.8
69-bus	21-65	0.89	**6.3**	10.0
	25-32	1.29	**5.3**	9.1
	26-69	0.10	**5.8**	9.9
	35-43	0.01	**5.0**	9.3
	40-52	1.66	**4.9**	9.5

IV. CONCLUSION

This paper proposed a novel graph-based approach to detect radial or weakly meshed systems and a power flow method for radial and weakly meshed distribution networks based on reusing the results after transformation to a weakly meshed network from the radial network (or vice-versa) utilizing the characteristic properties of Sparse Matrix-Vector product (SpMV) operations. Simulation results were performed for several distribution networks with radial and weakly meshed cases. From the numerical results, we observe that the proposed method not only gives good numerical accuracy but also provides better computational time. We plan to extend this study and apply the proposed method to larger test systems incluuding multiple the ones with multiple-ring topologies in the future.

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