Stochastic Reliable Control of a Class of Uncertain Time-Delay Systems with Unknown Nonlinearities

Zidong Wang, Biao Huang, and K. J. Burnham

***Abstract*—This paper investigates the robust reliable control problem for a class of nonlinear time-delay stochastic systems. The system under study involves stochastics, state time-delay, parameter uncertainties, possible actuator failures and unknown nonlinear disturbances, which are often encountered in practice and the sources of instability. Our attention is focused on the design of linear state feedback memoryless controllers such that, for all admissible uncertainties as well as actuator failures occurring among a prespecified subset of actuators, the plant remains stochastically exponentially stable in mean square, independent of the time delay. Sufficient conditions are proposed to guarantee the desired robust reliable exponential stability despite possible actuator failures, which are in terms of the solutions to algebraic Riccati inequalities. An illustrative example is exploited to demonstrate the applicability of the proposed design approach.**

***Index Terms*—Exponential stability, nonlinear systems, reliable control, robust control, stochastic control, time-delay systems.**

I. Introduction

The dynamic behavior of many industrial processes contains inherent time delays. Control of time-delay systems has been a subject of great practical importance which has attracted a great deal of interest for several decades [1]. Moreover, due to the unavoidable parameter uncertainties in modeling dynamical systems, in the past few years, considerable attention has been given to both the problems of robust stabilization and robust control for linear systems with certain types of time-delays, see [3] for a survey. On the other hand, since the traditional feedback control designs for a multiple input multiple output (MIMO) plant may result in unsatisfactory system performance, the study on the reliable control problem has received much attention in the past decade, see [6], [8] and the references therein.

It is now well known that stochastic modeling has come to play an important role in many branches of engineering applications. An area of particular interest has been the control of stochastic systems, with consequent emphasis being placed on the stabilization of the stochastic model in terms of various definitions of stochastic stability [2], [5]. So far, there are very few papers dealing with the reliable stabilization for *general stochastic* systems, not to mention the consideration of the case where time-delay, parameter uncertainty and nonlinear disturbance simultaneously exist in the system model, due to the complexity of such a challenging problem. This motivates us to investigate the multiobjective realization problem of robustness and reliability for stochastic uncertain time-delay systems with nonlinear disturbances, that is, to generalize the results of [6] to the stochastic case.

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In this paper, we consider the problem of robust reliable control design for a class of stochastic nonlinear uncertain state delayed systems. The class of the stochastic time-delay systems is described by a state-space model with real time-varying norm-bounded parameter uncertainties and nonlinear disturbances meeting the boundedness condition. Here, attention is focused on the design of state feedback controllers which guarantee, for all admissible uncertainties as well as actuator failures occurring among a prespecified subset of actuators, the stochastical exponential stability of the nonlinear plant, independent of the time delay. We show that the problem addressed can be solved in terms of some algebraic Riccati matrix inequalities, and the resulting nonlinear time-delay control systems provide guaranteed robust reliable exponential stability despite possible actuator failures.

Notation: The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. We let $h > 0$ and $C([-h, 0]; \mathbb{R}^n)$ denote the family of continuous functions φ from $[-h, 0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-h\leq\theta\leq 0} |\varphi(\theta)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a matrix, denote by $\|A\|$ its operator norm, i.e., $\|A\| = \sup\{|Ax| : |x| = 1\} = \sqrt{\lambda_{\max}(A^TA)}$ where $\lambda_{\max}(\cdot)$ [respectively, $\lambda_{\min}(\cdot)$] means the largest (respectively, smallest) eigenvalues of A. $l_2[0, \infty]$ is the space of square integrable vector. Moreover, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Denote by $L^p_{\mathcal{F}_0}([-h, 0]; \mathbb{R}^n)$ the family of all $\mathcal{F}_0$-measurable $C([-h, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -h \leq \theta \leq 0\}$ such that $\sup_{-h\leq\theta\leq 0} \mathbb{E}|\xi(\theta)|^p < \infty$ where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P.

II. Problem Formulation and Assumptions

We consider a nonlinear uncertain continuous-time state delayed stochastic system described by

$$dx(t) = [(A + \Delta A(t))\,x(t) + (A_d + \Delta A_d(t))\,x(t-h) + Bu(t) + f\,(x(t))]\,dt + E\,dw(t) \tag{1}$$

$$x(t) = \varphi(t), \qquad t \in [-h, 0] \tag{2}$$

where

$x(t) \in \mathbb{R}^n$	state;
$u(t) \in \mathbb{R}^m$	control input;
$f(\cdot)\colon \mathbb{R}^n \to \mathbb{R}^{n_f}$	unknown nonlinear function;
h	*unknown* deterministic state delay;
$\varphi(t)$	known continuous vector valued initial function.

Here, $w(t)$ is a scalar Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. A, A_d, B, E are known constant matrices with appropriate dimensions, ΔA, ΔA_d are real-valued matrix functions representing norm-bounded parameter uncertainties.

Remark 1: Note, that the system (1) and (2) can be used to represent many important physical systems subject to inherent state delays, parameter uncertainties, deterministic nonlinear disturbances (which may result from linearization process of an originally nonlinear plant), and stochastic exogenous noises with known statistics [3].

Assumption 1: The admissible parameter uncertainties are of the norm-bounded form

$$[\,\Delta A(t) \quad \Delta A_d(t)\,] = M\Xi(t)\,[\,N_1 \quad N_2\,] \tag{3}$$

where M, N_1 and N_2 are known real constant matrices with proper dimensions and $\Xi(t) \in \mathbb{R}^{n \times j}$ is an unknown time-varying matrix which contains the uncertain parameters in the linear part of the system and is bounded by $\Xi^T(t)\Xi(t) \leq I$. The parameter uncertainty structure as in (3) has been widely used in the problems of robust control and robust filtering of uncertain systems.

Assumption 2: There exists a known real constant matrix G such that the unknown nonlinear vector function $f(\cdot)$ satisfies the boundedness condition $|f(x(t))| \leq |Gx(t)|$ for any $x(t) \in \mathbb{R}^n$.

Next, let $x(t;\, \xi)$ denote the state trajectory from the initial data $x(\theta) = \xi(\theta)$ on $-h \leq \theta \leq 0$ in $L^2_{\mathcal{F}_0}([-h,\, 0];\, \mathbb{R}^n)$. Clearly, the system (1) and (2) with $u(t) \equiv 0$ admits a trivial solution $x(t;\, 0) \equiv 0$ corresponding to the initial data $\xi = 0$. We introduce the following stability and stabilizability concepts.

Definition 1: For system (1) and (2) with $u(t) \equiv 0$ and every $\xi \in L^2_{\mathcal{F}_0}([-h,\, 0];\, \mathbb{R}^n)$, the trivial solution is asymptotically stable in mean square if $\lim_{t\to\infty} \mathbb{E}|x(t;\, \xi)|^2 = 0$; and the trivial solution is exponentially stable in mean square if there exist constants $\alpha > 0$ and $\beta > 0$ such that $\mathbb{E}|x(t;\, \xi)|^2 \leq \alpha e^{-\beta t} \sup_{-h\leq\theta\leq 0} \mathbb{E}|\xi(\theta)|^2$.

Definition 2: We say that the system (1) and (2) is asymptotically stabilizable in mean square (respectively, exponentially stabilizable in mean square) if, for every $\xi \in L^2_{\mathcal{F}_0}([-h,\, 0];\, \mathbb{R}^n)$, there exists a linear feedback control law $u(t) = Kx(t)$ such that the resulting closed-loop system is asymptotically stable in mean square (respectively, exponentially stable in mean square).

We are now in a position to discuss the reliability with respect to actuator outages which are restricted to occur within a preselected subset of the control channel. The set of actuators which are susceptible to failures is denoted as $\Sigma \subseteq \{1,\, 2,\, \ldots,\, m\}$. The set of actuators which are not subject to failure (i.e., robust to failures and essential to stabilize the plant) is denoted as $\overline{\Sigma} \subseteq \{1,\, 2,\, \ldots,\, m\} - \Sigma$. Introduce the decomposition $B = B_\Sigma + B_{\overline{\Sigma}}$ where B_Σ denotes the control matrix associated with the set Σ and $B_{\overline{\Sigma}}$ denotes the control matrix associated with the complementary subset of control inputs. Furthermore, let $\sigma \subseteq \Sigma$ correspond to a particular subset of susceptible actuators that actually experience failures, and assume that the actuators failures are modeled as control input failures, that is $u_i = 0$, $i \in \sigma$. To this end, we give the notation $B = B_\sigma + B_{\overline{\sigma}}$ where B_σ and $B_{\overline{\sigma}}$ have meanings analogous to those of B_Σ and $B_{\overline{\Sigma}}$.

The problem addressed in this paper aims at designing a linear state feedback memoryless controller of the form $u(t) = Kx(t)$ such that, for all admissible uncertainties as well as actuator failures occurring among the prespecified subset B_Σ, the controlled system is robustly exponentially stable in mean square, independent of the unknown time delay.

III. Main Results and Proofs

It is shown in the following theorem that the robust stochastic exponential stability of the nonlinear time-delay system (1) and (2) can be guaranteed if a positive definite solution to a modified algebraic Riccati-like matrix inequality (quadratic matrix inequality) exists. This theorem plays a key role in the design of the expected controllers.

Theorem 1: Let the controller gain F be given. If there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 > 0$ and a positive definite matrix $P > 0$ such that the following matrix inequality

$$\begin{aligned}(A+BF)^T P &+ P(A+BF)\\ &+ P\left[(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4^{-1})I\right]P + \varepsilon_1^{-1}A_d^T A_d\\ &+ \lambda_{\max}(M^T M)(\varepsilon_2^{-1}N_2^T N_2 + \varepsilon_3^{-1}N_1^T N_1)\\ &+ \varepsilon_4 G^T G < 0 \end{aligned} \tag{4}$$

holds, then the nonlinear uncertain stochastic state delayed system (1) and (2) is exponentially stabilized (in mean square) by the state feedback control law $u(t) = Fx(t)$.

Proof: For simplicity, we make the definitions

$$\begin{aligned} A_1(t) &= A + BF + \Delta A(t) = A + BF + M\Xi(t)N_1\\ A_{1d}(t) &= A_d + \Delta A_d(t) = A_d + M\Xi(t)N_2. \end{aligned} \tag{5}$$

and then the closed-loop system is governed by

$$dx(t) = [A_1(t)x(t) + A_{1d}(t)x(t-h) + f(x(t))]\, dt + E\, dw(t). \tag{6}$$

Fix $\xi \in L^2_{\mathcal{F}_0}([-h,\, 0];\, \mathbb{R}^n)$ arbitrarily and write $x(t;\, \xi) = x(t)$. For $(x(t),\, t) \in \mathbb{R}^n \times \mathbb{R}_+$, we define the Lyapunov function candidate

$$Y(x(t),\, t) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(s)Qx(s)\, ds \tag{7}$$

where P is the positive definite solution to the matrix inequality (4) and $Q > 0$ is defined by

$$Q := \varepsilon_1^{-1}A_d^T A_d + \lambda_{\max}(M^T M)\varepsilon_2^{-1}N_2^T N_2. \tag{8}$$

By Itô's formula, the stochastic differential of Y along a given trajectory is obtained as

$$\begin{aligned} dY(x(t),\, t) = \Big\{ & x^T(t)\Big[(A+BF)^T P + P(A+BF)\\ &\qquad + Q\Big]x(t)\\ &+ x^T(t)\Big[(\Delta A(t))^T P + P(\Delta A(t))\Big]x(t)\\ &+ x^T(t-h)A_d^T Px(t) + x^T(t)PA_d x(t-h)\\ &+ x^T(t-h)(\Delta A_d(t))^T Px(t)\\ &+ x^T(t)P(\Delta A_d(t))x(t-h)\\ &- x^T(t-h)Qx(t-h)\\ &+ f^T(x(t))Px(t) + x^T(t)Pf(x(t))\Big\}\, dt\\ &+ 2x^T(t)PE\, dw(t). \end{aligned} \tag{9}$$

Let $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ be positive scalars. Then the matrix inequality

$$\left[\varepsilon_1^{1/2}x^T(t)P - \varepsilon_1^{-1/2}x^T(t-h)A_d^T\right] \cdot \left[\varepsilon_1^{1/2}x^T(t)P - \varepsilon_1^{-1/2}x^T(t-h)A_d^T\right]^T \geq 0$$

yields

$$\begin{aligned} &x^T(t-h)A_d^T Px(t) + x^T(t)PA_d x(t-h)\\ &\quad\leq \varepsilon_1 x^T(t)P^2 x(t) + \varepsilon_1^{-1}x^T(t-h)A_d^T A_d x(t-h). \end{aligned} \tag{10}$$

Moreover, noticing that $\Delta A_d(t) = M\Xi N_2$ and $\Xi^T\Xi \leq I$, it follows from

$$\begin{aligned} (\Delta A_d(t))^T(\Delta A_d(t)) &\leq \lambda_{\max}(M^T M)N_2^T(\Xi^T\Xi)N_2\\ &\leq \lambda_{\max}(M^T M)N_2^T N_2 \end{aligned}$$

$$\Psi_1 := \varepsilon_2^{1/2}x^T(t)P - \varepsilon_2^{-1/2}x^T(t-h)(\Delta A_d(t))^T, \qquad \Psi_1\Psi_1^T \geq 0$$

that

$$\begin{aligned} x^T(t-h)&(\Delta A_d(t))^T Px(t)\\ &+ x^T(t)P(\Delta A_d(t))x(t-h)\\ &\leq \varepsilon_2 x^T(t)P^2 x(t) + \varepsilon_2^{-1}\lambda_{\max}(M^T M)\\ &\quad\cdot x^T(t-h)N_2^T N_2 x(t-h). \end{aligned} \tag{11}$$

Similarly, it results from

$$\Psi_2 := \varepsilon_3^{1/2} x^T(t)P - \varepsilon_3^{-1/2} x^T(t)\,(\Delta A(t))^T, \qquad \Psi_2\Psi_2^T \geq 0$$

that

$$x^T(t)\,(\Delta A(t))^T\,Px(t) + x^T(t)P\,(\Delta A(t))\,x(t) \leq \varepsilon_3 x^T(t)P^2x(t) + \varepsilon_3^{-1}\lambda_{\max}(M^TM)x^T(t)N_1^TN_1x(t). \tag{12}$$

Furthermore, from

$$\left[\varepsilon_4^{1/2} f^T(x(t)) - \varepsilon_4^{-1/2}x^T(t)P\right] \cdot \left[\varepsilon_4^{1/2} f^T(x(t)) - \varepsilon_4^{-1/2}x^T(t)P\right]^T \geq 0$$

and the Assumption 2, we have

$$\begin{aligned} f^T(x(t))\,Px(t) &+ x^T(t)Pf(x(t)) \\ &\leq \varepsilon_4 f^T(x(t))\,f(x(t)) + \varepsilon_4^{-1}x^T(t)P^2x(t) \\ &\leq \varepsilon_4\,|Gx(t)|^2 + \varepsilon_4^{-1}x^T(t)P^2x(t) \\ &= \varepsilon_4 x^T(t)G^TGx(t) + \varepsilon_4^{-1}x^T(t)P^2x(t). \end{aligned} \tag{13}$$

Noticing the condition (4) and definition (8), we denote

$$\begin{aligned} \Upsilon := &(A+BF)^TP + P(A+BF) \\ &+ P\left[(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4^{-1})I\right]P \\ &+ \varepsilon_1^{-1}A_d^TA_d + \lambda_{\max}(M^TM) \\ &\cdot(\varepsilon_2^{-1}N_2^TN_2 + \varepsilon_3^{-1}N_1^TN_1) + \varepsilon_4G^TG < 0. \end{aligned} \tag{14}$$

Then, substituting (10)–(13) into (9) results in

$$\begin{aligned} dY(x(t),t) &\leq \left(x^T(t)\Upsilon x(t)\right)dt + 2x^T(t)PE\,dw(t) \\ &\leq -\lambda_{\min}(-\Upsilon)x^T(t)x(t)\,dt + 2x^T(t)PE\,dw(t) \end{aligned} \tag{15}$$

which means that the nonlinear uncertain stochastic state delayed system (1) and (2) is asymptotically stabilized (in mean square) by the state feedback control law $u(t) = Fx(t)$.

The required exponential stability (in mean square) of the closed-loop system can be proved by making some standard manipulations on the relation (15), see [2]. ∎

Remark 2: The theoretical basis is provided in Theorem 1 for the controller design of the nonlinear uncertain time-delay stochastic systems. The result may be conservative due to the use of the inequalities (10)–(13). However, such conservativeness can be significantly reduced by properly selecting the parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ in a matrix norm sense. The relevant discussion and corresponding numerical algorithm can be found in [7] and references therein.

Remark 3: Compared to the existing results, this paper includes the consideration of stochastic disturbances, and moreover, different techniques are used to tackle the uncertainty ΔA_d in the delayed state term. Accordingly, in Theorem 1 only one quadratic matrix inequality is involved while in the similar result of [6] we need to solve two matrix inequalities. We point out that the result of Theorem 1 can be easily extended to the multiple state delayed case.

Remark 4: Note that the result of Theorem 1 is also applicable to the more general case

$$\begin{aligned} dx(t) = &[(A+\Delta A(t))\,x(t) + (A_d+\Delta A_d(t))\,x(t-h) \\ &+ Bu(t) + f(x(t))]\,dt + \sum_{i=1}^{r} E_i\,dw_i(t) \end{aligned} \tag{16}$$

$$x(t) = \varphi(t), \quad t \in [-h, 0] \tag{17}$$

where $(w_1, w_2, \ldots, w_m)$ is an m-dimensional Brownian motion, instead of a scalar one in system (1) and (2). The reason why we discuss on the system (1) and (2) is just for simpler notations.

We are now ready to enforce the reliability requirement of the closed-loop system subjected to possible actuator failures. In this study, the outputs of faulty actuators are assumed to be zero, and therefore the controlled system (1) and (2) can be expressed as

$$\begin{aligned} dx(t) = &[(A+\Delta A(t))\,x(t) + (A_d+\Delta A_d(t))\,x(t-h) \\ &+ B_{\overline{\sigma}}u(t) + f(x(t))]\,dt + E\,dw(t) \end{aligned} \tag{18}$$

$$x(t) = \varphi(t), \quad t \in [-h, 0]. \tag{19}$$

The following theorem implies that the mixed robustness and reliability constraints for the addressed nonlinear stochastic time-delay systems can be guaranteed when the positive definite solution to an algebraic Riccati matrix inequality is known to exist.

Theorem 2: If there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5 > 0$ such that the following matrix inequality

$$\begin{aligned} A^TP + PA &+ P\left[(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4^{-1})I - \varepsilon_5^{-1}B_{\overline{\Sigma}}B_{\overline{\Sigma}}^T\right]P \\ &+ \varepsilon_1^{-1}A_d^TA_d + \lambda_{\max}(M^TM)(\varepsilon_2^{-1}N_2^TN_2 + \varepsilon_3^{-1}N_1^TN_1) \\ &+ \varepsilon_4G^TG < 0 \end{aligned} \tag{20}$$

has a positive definite solution $P > 0$, then the state feedback control law

$$u(t) = Fx(t), \qquad F = -0.5\varepsilon_5^{-1}B^TP \tag{21}$$

exponentially stabilizes (in mean square) the uncertain time-delay system (1) and (2), independent of the unknown delay h, for all admissible uncertainties as well as all actuator failures corresponding to $\sigma \in \Sigma$.

Proof: It follows from $B_{\overline{\Sigma}}B_{\overline{\Sigma}}^T = B_{\overline{\sigma}}B_{\overline{\sigma}}^T - B_{\Sigma-\sigma}B_{\Sigma-\sigma}^T$ that $B_{\overline{\Sigma}}B_{\overline{\Sigma}}^T \leq B_{\overline{\sigma}}B_{\overline{\sigma}}^T$. Recall that the control input $u(t)$ acts only through the normal actuators and the outputs of faulty actuators are known as zero. Therefore, by applying the control law (21), we obtain that

$$\begin{aligned} (A+BF)^TP + P(A+BF) &= A^TP + PA - \varepsilon_5^{-1}B_{\overline{\sigma}}B_{\overline{\sigma}}^T \\ &\leq A^TP + PA - \varepsilon_5^{-1}B_{\overline{\Sigma}}B_{\overline{\Sigma}}^T \end{aligned}$$

and

$$\begin{aligned} (A+BF)^TP &+ P(A+BF) \\ &+ P\left[(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4^{-1})I\right]P + \varepsilon_1^{-1}A_d^TA_d \\ &+ \lambda_{\max}(M^TM)(\varepsilon_2^{-1}N_2^TN_2 + \varepsilon_3^{-1}N_1^TN_1) + \varepsilon_4G^TG \\ \leq A^TP &+ PA \\ &+ P\left[(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4^{-1})I - \varepsilon_5^{-1}B_{\overline{\Sigma}}B_{\overline{\Sigma}}^T\right]P \\ &+ \varepsilon_1^{-1}A_d^TA_d + \lambda_{\max}(M^TM)(\varepsilon_2^{-1}N_2^TN_2 + \varepsilon_3^{-1}N_1^TN_1) \\ &+ \varepsilon_4G^TG < 0. \end{aligned}$$

Finally, the proof of this theorem follows from that of Theorem 1 immediately. ∎

Remark 5: Note that (20) is a quadratic matrix inequality (Riccati-like inequality) and the relevant detailed discussion on the existence of a positive definite solution to such a matrix inequality can be found in [6].

Remark 6: The positive scalar ε_5 can be chosen to meet the low-energy control input requirement since a high-gain controller may be unstable for the state delayed systems. We may assume that the output of a failed actuator to be any arbitrary energy-bounded signal different from the normal controller output [4], and suppress the signals on the system outputs caused by faulty actuators as well as other possible disturbance inputs below a given level. This gives one of the future research subjects.

The following corollary, which results easily from [2], reveals that for the linear delay stochastic control system (1) and (2), the exponential stabilizability in mean square implies the almost surely exponential stabilizability.

Corollary 1: Under the conditions of Theorem 2, the state feedback control law (21) almost surely exponentially stabilizes the uncertain time-delay system (1) and (2), independent of the unknown delay h, for all admissible uncertainties as well as all actuator failures corresponding to $\sigma \in \Sigma$, that is, the trivial solution $x(t;\ \xi)$ of the closed-loop system (18) is almost surely exponentially stable.

IV. A Numerical Example

Consider the nonlinear uncertain stochastic state delayed system (1) and (2) with parameters as follows

$$A = \begin{bmatrix} -2.5 & 0.2 & -0.2 \\ -0.3 & -3 & -0.4 \\ 1.5 & -0.4 & -5 \end{bmatrix}$$

$$A_d = \begin{bmatrix} 0.03 & 0.01 & 0.01 \\ 0.01 & -0.04 & 0 \\ -0.01 & 0.01 & -0.02 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.6 & 0.1 & 0.02 \\ 1.6 & -2.1 & 0.03 \\ 1.1 & 1.4 & 1.2 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$f(x) = \begin{bmatrix} 0.01 \sin x_2 \\ 0.01 \sin x_1 \\ 0.02 \sin x_1 + 0.02 \sin x_2 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.45 & 0 & 0.05 \\ 0 & 0.45 & 0 \\ 0.15 & 0 & 0.15 \end{bmatrix}$$

$$N_1 = \begin{bmatrix} 0.02 & 0.02 & 0 \\ 0 & 0 & 0.02 \\ 0 & 0.02 & 0 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} 0 & 0.06 & 0 \\ 0 & 0 & 0.06 \\ 0.02 & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.3 & 0 & 0.01 \\ 0 & 0.2 & 0 \\ 0.01 & 0 & 0.4 \end{bmatrix}$$

$$\Xi(t) = \sin t I_3 \quad h = 0.1 \quad \varphi(t) = 0.1 \quad t \in [-0.1\ \ 0].$$

It is desired to design a linear state feedback memoryless controller such that, for all admissible uncertainties as well as actuator failures occurring among the prespecified subset B_Σ, the controlled system is robustly exponentially stable in mean square, independent of the unknown time delay.

Case 1: The third actuator is susceptible to failure. We have $\Sigma = \{3\}$. Set $\varepsilon_1, \ldots, \varepsilon_5$ as follows

$$\varepsilon_1 = 8.2375, \quad \varepsilon_2 = 1.9836, \quad \varepsilon_3 = 2.0065,$$
$$\varepsilon_4 = 0.2523, \quad \varepsilon_5 = 0.6045.$$

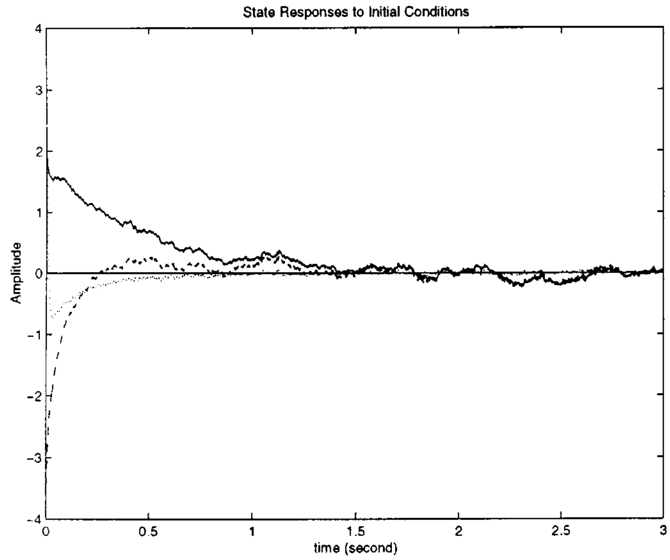

Fig. 1. x_1 (solid), x_2 (point), x_3 (dashed).

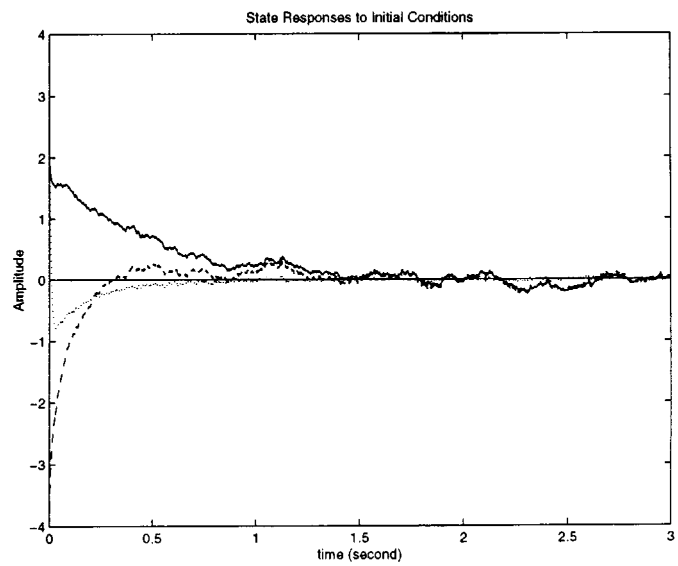

Fig. 2. x_1 (solid), x_2 (point), x_3 (dashed).

Subsequently, the symmetric positive definite solution P to the Riccati matrix inequality (20) and the feedback gain matrix F in (21) can be obtained as

$$P = \begin{bmatrix} 1.2960 & 4.3672 & -1.1492 \\ 4.3672 & 22.8177 & -5.1367 \\ -1.1492 & -5.1367 & 2.4548 \end{bmatrix}$$

$$F = \begin{bmatrix} -5.3772 & -27.6908 & 5.1348 \\ 8.8093 & 45.2207 & -11.6699 \\ 1.0109 & 4.4600 & -2.2900 \end{bmatrix}.$$

With the feedback gain matrix obtained above, simulation results show that the resulting closed-loop system is guaranteed to have desired robust faculty staff member [exponential stability (in mean square). When there are no actuator failures (i.e., all actuators are normal), the state responses are shown in Fig. 1, and when there is a failure of third actuator (i.e., an actuator failure corresponding to $\sigma \in \Sigma = \{3\}$ occurs), the state responses are displayed in Fig. 2.

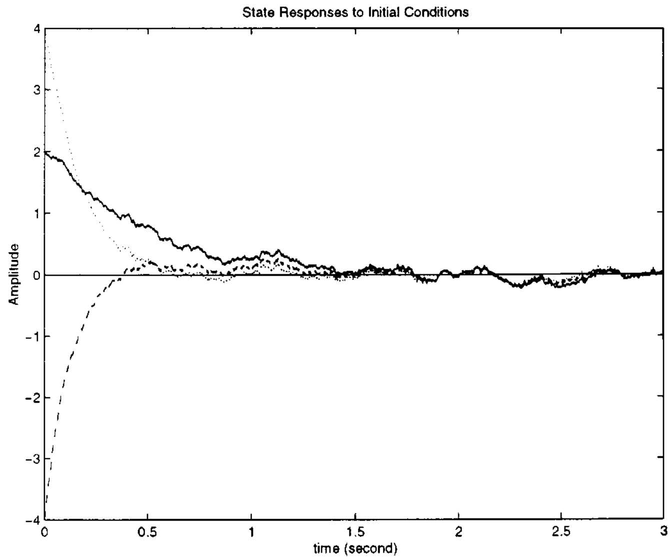

Fig. 3. x_1 (solid), x_2 (point), x_3 (dashed).

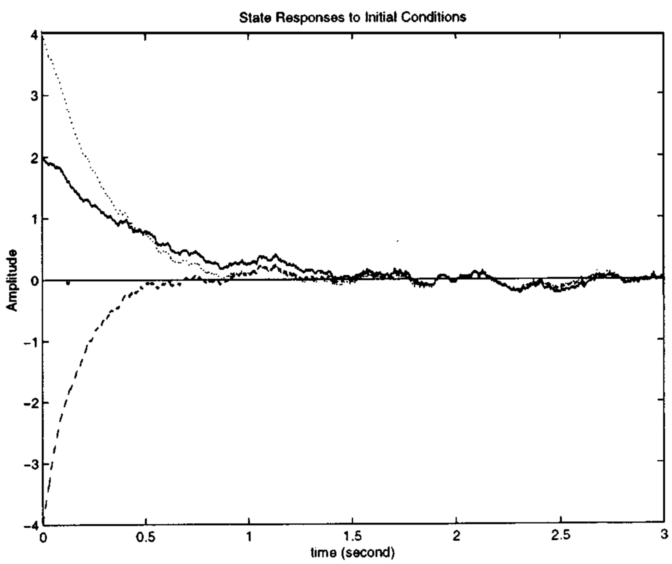

Fig. 4. x_1 (solid), x_2 (point), x_3 (dashed).

Case 2: The second actuator is susceptible to failure. In this case, it is clear that $\Sigma = \{2\}$. We choose

$$\begin{aligned} \varepsilon_1 &= 6.2365 \quad \varepsilon_2 = 1.8076 \quad \varepsilon_3 = 2.0156 \\ \varepsilon_4 &= 0.3368 \quad \varepsilon_5 = 1.0045. \end{aligned}$$

and the it follows from (20) and (21) that

$$P = \begin{bmatrix} 0.5536 & 0.0549 & -0.1507 \\ 0.0549 & 0.8630 & 0.3568 \\ -0.1507 & 0.3568 & 1.3390 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.1265 & -0.8991 & -0.9723 \\ 0.1348 & 0.6507 & -0.5526 \\ 0.0837 & -0.2266 & -0.8036 \end{bmatrix}.$$

The simulation results imply that the desired goal is achieved. Fig. 3 shows the state responses with respect to the case when there are no actuator failures, and Fig. 4 illustrates the state responses with respect to the case when there is a failure of second actuator.

Case 3: The first actuator is susceptible to failure. Similar to the previous cases, we have $\Sigma = \{1\}$ and then obtain

$$P = \begin{bmatrix} 0.9137 & 1.5977 & -1.7559 \\ 1.5977 & 7.8571 & -6.8633 \\ -1.7559 & -6.8633 & 7.9023 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.8375 & -4.2699 & 2.3864 \\ 4.0855 & 18.5270 & -18.0641 \\ 1.4571 & 5.6892 & -6.5984 \end{bmatrix}.$$

V. Conclusions

We have investigated the reliable stabilization problem for a class of uncertain nonlinear state delayed stochastic systems. A robust reliable static control design methodology has been presented to achieve the exponential stability (in mean square or almost surely) for all admissible parameter uncertainties, independent of the time-delay, not only when the system is operating properly, but also in the presence of certain actuator failures. The nonlinearities are assumed to satisfy the boundedness condition, and the parameter uncertainties are allowed to be time-varying unstructured. We have constructed the desired state feedback gains in terms of a positive definite solution to a parameter-dependent algebraic Riccati-like inequality. The existing results on robust and/or reliable control for linear systems have been extended to the nonlinear time-delay stochastic systems.

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References

[1] M. Malek-Zavarei and M. Jamshidi, *Time-Delay Systems: Analysis, Optimation and Application*. Amsterdam, The Netherlands: North-Holland, 1987.

[2] X Mao, *Stochastic Differential Equations and Applications*. Chichester, U.K.: Horwood, 1997.

[3] S.-I. Niculescu, E. I. Verriest, L. Dugard, and J. M Dion *et al.*, "Stability and robust stability of time-delay systems: A guided tour," in *Stability and Control of Time-Delay Systems*. ser. Lect. Notes Control Inf. Sci., L. Dugard *et al.*, Eds. Berlin, Germany: Springer-Verlag, 1998, vol. 228, pp. 1–71.

[4] C.-J. Seo and B. K. Kim, "Robust and reliable H_∞ control for linear systems with parameter uncertainty and actuator failure," *Automatica*, vol. 32, no. 3, pp. 465–467, 1996.

[5] P. Shi, E.-K. Boukas, and R. K. Agarwal, "Control of Markovian jump discrete-time systems with norm-bounded uncertainty and unknown delay," *IEEE Trans. Automat. Contr.*, vol. 44, pp. 2139–2144, Nov. 1999.

[6] Z. Wang, B. Huang, and H. Unbehauen, "Robust reliable control for a class of uncertain nonlinear state delayed systems," *Automatica*, vol. 35, no. 5, pp. 955–963, 1999.

[7] L. Xie and Y. C. Soh, "Robust Kalman filtering for uncertain systems," *Systems Control Lett.*, vol. 22, no. 2, pp. 123–129, 1994.

[8] G.-H. Yang, J. Lam, and J. Wang, "Reliable H_∞ control for affine nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 43, pp. 1112–1116, Aug. 1998.