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Reputation-based Distributed Filtering Over Sensor Networks Subject to Stochastic Nonlinearity and Network-Induced Quantization

Chaoqing Jia, Zidong Wang, Jun Hu, and Hongli Dong

***Abstract*—In sensor networks, due to inevitable sensor faults, malfunctions, or deliberate attacks, sensors may transmit erroneous, inaccurate, or misleading data, thereby degrading overall system performance. To address this issue, an effective approach is to assign reputation scores to sensors based on their trustworthiness, historical performance, or reliability. In this paper, the reputation-based distributed filtering (RBDF) problem is considered for a class of stochastic nonlinear systems over sensor networks with network-induced quantization. A reputation mechanism is employed to mitigate the adverse effects caused by noisy, faulty, or malicious sensors. Specifically, reputations are allocated by each sensor to the data received from its neighbors, ensuring that abnormal data are assigned smaller reputation values and may even be discarded. For the first time, a recursive RBDF algorithm is proposed, wherein an upper bound of the filtering error covariance (UBFEC) is derived by solving two matrix equations. Subsequently, the filter gain is determined by minimizing the trace of UBFEC at each step. Furthermore, a sufficient condition is presented to ensure the uniform boundedness of the filtering error dynamics. Finally, a simulation example is provided to verify the feasibility and validity of the developed RBDF algorithm.**

***Index Terms*—Sensor networks, distributed filtering, reputation mechanism, network-induced quantization, boundedness analysis.**

I. Introduction

Sensor networks (SNs) are composed of numerous low-cost sensor nodes that are dispersedly arranged in a region and equipped with communication and computing capabilities. In recent years, SNs have received significant attention, as they enable a wide range of critical applications, including target tracking, surveillance, transportation, and weather forecasting [48]. The analysis and synthesis of dynamical systems encompass, but are not limited to, control [20], [21], [44], synchronization [9], and filtering [2], [4], [5], [52]. In particular, the filtering problem has been extensively studied within the framework of networked systems due to its broad applicability across various domains, such as industrial monitoring, integrated navigation, and power generation. Specifically, distributed fusion filtering problem for nonlinear time-varying systems over SNs has been addressed in [30], where the dynamic-event-triggering mechanism has been employed to regulate data transmission and alleviate communication burdens. Furthermore, finite-time H_∞ filtering problems for discrete-time nonlinear stochastic systems over SNs, subject to varying topologies and two-channel malicious attacks, have been investigated in [13] and [53].

Among various filtering algorithms, distributed filtering emerges as a particularly suitable technique for dynamical systems over SNs, which are characterized by large-scale structures, low-energy components, and limited computational resources. This approach leverages the collaborative capabilities of sensor nodes to process data efficiently while reducing communication overhead. It is generally applied to vehicle tracking, indoor localization, environmental monitoring, orbit determination, navigation, and other critical fields that require accurate state estimation in real-time. The objective of distributed filtering is to estimate the unknown state based on local observations and neighboring interactions under given topology structures, ensuring robustness and scalability; see [10], [17] and the references therein.

The commonly used distributed filtering methods can be broadly categorized into distributed extended Kalman filtering [28], [40], distributed particle filtering [41], distributed cubature filtering [54], distributed H_∞ filtering [16], and other variants. These methods are designed to address different system complexities and practical constraints. For example, the event-based distributed filtering problem has been addressed in [37] for a class of discrete-time nonlinear systems with actuator saturation, where two recursive equations have been derived and the filter gain has been designed in the sense of minimum mean-square error. It is worth mentioning that the distributed Kalman-type filtering algorithm remains a significant area of research among the various distributed filtering schemes, requiring ongoing enrichment, theoretical advancements, and

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practical implementations to further enhance performance.

Within the framework of the distributed filtering algorithm, each node can receive the local information of its neighbors through a shared network channel. Unfortunately, sudden environmental changes, hardware malfunctions, or communication interference can make this information unreliable. Furthermore, the information from neighbors may become vulnerable to malicious cyber attacks [34], such as denial-of-service attacks, data injection, or replay attacks, which can severely degrade filtering performance and system stability. As a result, the reliable/secure distributed filtering problem has attracted significant research attention. For instance, the consensus-based distributed filtering issue has been investigated in [19], where performance evaluation under replay attacks has been comprehensively analyzed, highlighting the vulnerabilities of systems under persistent threats.

To address the aforementioned data trustworthiness challenges, reputation mechanisms have emerged as an effective strategy for mitigating the impact of unreliable or malicious data. In this approach, whether a node accepts data from its neighbors depends on the assigned reputation scores, which are determined based on factors such as trustworthiness, data consistency, and historical performance. Specifically, unreliable data are assigned lower reputation scores and may be discarded by the receiver to ensure system reliability; see [35], [36] for further details. Despite the promising potential of reputation-based approaches, it is worth noting that there are currently very few results focusing on the reputation-based distributed filtering (RBDF) problem, particularly in the context of stochastic nonlinear systems with quantization effects. This noticeable gap in the literature strongly motivates further research to develop effective RBDF strategies that simultaneously ensure reliability and enhance filtering accuracy.

As is well-known, nonlinearity is a universal feature in many practical systems, often leading to undesirable behaviors such as oscillations, instability, and other complications. Extensive research on nonlinearity has been reported and discussed; see [7], [8], [11], [22], [38], [51] for more details. Notably, nonlinear disturbances typically arise from sudden environmental changes, random failures of physical components, or communication constraints. In such scenarios, the so-called stochastic nonlinearity becomes inevitable in engineering, often involving state-multiplicative noise and second-order moments. In recent years, significant research has been conducted on the analysis and synthesis of networked control systems subject to stochastic nonlinearity. For instance, the maximum correntropy Kalman filtering problem for time-varying system with stochastic nonlinearity has been investigated in [39], where detailed analyses of uniqueness have been presented.

Another significant research area involves quantization of transmitted data in digital channels, particularly under bandwidth or energy limitations. Quantization introduces distortions due to finite word lengths, which can degrade the performance of filtering algorithms. As a result, considerable attention has been given to the design of quantization-based filtering algorithms; see [14], [23], [32], [46], [49] for more details. Quantization techniques are typically classified into uniform quantization [26], [47] and logarithmic quantization [18], [24], [45], where the key challenge lies in effectively handling quantization errors [46]. Notably, quantization values are often determined probabilistically. For instance, the feedback quadratic distributed filtering problem in [31] has incorporated probabilistic-uniform quantization between adjacent targets. However, research on distributed filtering over SNs with probabilistic quantization effects remains limited, which serves as a key motivation for this paper.

Encouraged by the above discussions, this paper aims to investigate the RBDF problem for a class of stochastic nonlinear time-varying systems (SNTVS) in the simultaneous presence of stochastic nonlinearity, probabilistic quantization, and a reputation mechanism. Compared with the current results, the key challenges of this study can be listed as 1) a novel reputation principle is established to distinguish and discard the abnormal data caused by equipment failure or malicious attacks, thereby improving the reliability and robustness of the designed distributed filtering algorithm; 2) the reputation-dependent upper bound of the filtering error covariance (UBFEC) is determined by solving two recursive matrix equations, whose trace is minimized by parameterizing the filter gain in the sense of minimum mean-square error; and 3) a performance evaluation is given to verify the uniform boundedness of filtering error.

In response to the difficulties and challenges, the main contributions of this paper are listed as follows.

1) A novel reputation mechanism suitable for distributed filtering problem is proposed, effectively identifying and discarding the neighboring data interfered by sensor faults, malicious attacks, or sudden network environment.
2) The UBFEC is determined recursively by solving the matrix equations and the filter gain is further selected by minimizing the trace of UBFEC, enhancing the filtering algorithm performance.
3) A sufficient condition is given to verify the uniform boundedness of filtering error dynamics based on some mild assumptions.

This paper is organized as follows. Section III introduces the problem formulation, focusing on the RBDF problem for SNTVS with stochastic nonlinearity, probabilistic quantization, and a reputation mechanism. In Section IV, the RBDF algorithm is presented, where the recursion for UBFEC is derived, and the filter gain is designed by minimizing the trace of UBFEC. Section V provides a sufficient criterion to ensure that the UBFEC is uniformly bounded. An illustrative example is given in Section VI to demonstrate the validity of the proposed RBDF algorithm. Finally, Section VII concludes the paper by summarizing the main results.

Notations. The notations used in this paper are standard. I and 0 represent the identity matrix and the zero matrix, respectively, with appropriate dimensions. $\mathbb{E}\{*\}$ denotes the mathematical expectation. The superscripts $(*)^T$ and $(*)^{-1}$ indicate the transpose and inverse operations, respectively. The notation $\mathrm{tr}(*)$ refers to the trace, which is the sum of the diagonal elements of a square matrix.

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II. Related Work

Recently, a large amount of literature has focused on the attack-resistant distributed nonlinear filtering problems over SNs, ensuring and improving the filtering performance in the uncertain and unreliable network environment. Consequently, the related literature can be divided into two aspects: distributed nonlinear filtering [3], [27], [53] and anti-attack strategy [6], [33], [43].

On one hand, a novel secure distributed set-membership filtering algorithm has been proposed in [4] for a class of dynamical systems, where a homogeneous Markov chain has been adopted to describe the switching topologies and an event-triggered scheduling strategy has been employed to regulate communication frequency and alleviate channel burden. In addition, the filter has been designed by solving convex optimization problem and boundedness conditions with respect to the filtering error have been derived completely. Another representative example [3] has investigated the distributed H_∞ filtering problem for a class of discrete-time nonlinear systems with unknown parameters and energy-bounded disturbances, where the asymptotic stability of filtering error has been analyzed and the H_∞ performance index has been guaranteed by means of Lyapunov theory and stochastic analysis. It should be emphasized that most of the literature mainly discusses and solves the distributed set-membership filtering and distributed H_∞ filtering problems subject to unknown-but-bounded noises [55] or energy-bounded noises [1], [12], but lacks the research on recursive filtering dealing with Gaussian white noises especially in the situations of network-induced quantization and anti-attack scheme.

On the other hand, the current node usually can receive the data from its neighbors by shared networks, typically bringing challenges to data security owing to the uncertain and unreliable network environment. For example, the distributed H_∞-consensus filtering method has been proposed in [15] for a class of time-varying systems subject to sector-like-bounded attacks, where robust performance under attack scenarios has been ensured. Similar to [15], the anti-attack distributed filtering methods generally construct attack-information-dependent filters to ensure the desired estimation performance, see [29], [42], [50] for more details. Furthermore, some literature has been concerned with the design of distributed filtering algorithm, thereby detecting, identifying and removing the abnormal data contaminated by malicious attacks [33]. Different with the current results, we aim to develop a remarkable reputation mechanism and propose a novel anti-attack RBDF algorithm in our work, and then give the boundedness analysis of filtering error dynamic. The comparison with representative works is presented in Table I to emphasize the distinctive novelties and advantages of the proposed RBDF method.

III. Problem Formulation

In this paper, the SNs consisting of N sensor nodes are utilized to measure the target information. The topology of the SNs is represented by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$, where $\mathscr{V} = \{1, 2, \ldots, N\}$ denotes the set of nodes, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ represents the set of edges and $\mathscr{A} = [a_{iz}]_{N\times N}$ with $a_{iz} \geq 0$ stands for the adjacency matrix. The condition $a_{iz} > 0 \Leftrightarrow (i, z) \in \mathscr{E}$ indicates that information transmission exists from the z-th node to the i-th node. Furthermore, $\mathscr{N}_i = \{z \in \mathscr{V} | (i, z) \in \mathscr{E}\}$ denotes the set of neighbor nodes of the i-th node, while $|\mathscr{N}_i|$ represents the number of neighbors of the i-th node.

Consider the following SNTVS:

$$x_{s+1} = A_s x_s + f(s, x_s, \theta_s) + B_s \omega_s, \tag{1}$$
$$y_{i,s} = C_{i,s} x_s + \upsilon_{i,s}, \tag{2}$$

where $x_s \in \mathbb{R}^{n_x}$ represents the model state to be estimated and $y_{i,s} \in \mathbb{R}^{n_y}$ denotes the measurement output. The system matrices $A_s \in \mathbb{R}^{n_x\times n_x}$, $B_s \in \mathbb{R}^{n_x\times n_\omega}$ and $C_{i,s} \in \mathbb{R}^{n_y\times n_x}$ are given with appropriate dimensions. The process noise $\omega_s \in \mathbb{R}^{n_\omega}$ is a sequence of Gaussian white noise with zero mean and covariance Θ_s, while $\upsilon_{i,s}$ represents the measurement noise, which is also a Gaussian white noise with zero mean and covariance $R_{i,s} > 0$.

The term $f(s, x_s, \theta_s) \in \mathbb{R}^{n_x}$ with $f(s, 0, \theta_s) = 0$ describes the *stochastic nonlinearity* satisfying

$$\begin{aligned} &\mathbb{E}\left\{f(s, x_s, \theta_s)|x_s\right\} = 0, \\ &\mathbb{E}\left\{f(s, x_s, \theta_s)f^T(\ell, x_\ell, \theta_\ell)|x_s\right\} = 0, \quad s \neq \ell \\ &\mathbb{E}\left\{f(s, x_s, \theta_s)f^T(s, x_s, \theta_s)|x_s\right\} = \sum_{h=1}^{m} \Pi_{h,s} x_s^T \Lambda_{h,s} x_s, \end{aligned} \tag{3}$$

where $m > 0$ is a given integer, $\Pi_{h,s}$ and $\Lambda_{h,s}$ are known matrices with proper dimensions. Throughout this paper, it is assumed that ω_s, $\upsilon_{i,s}$ and θ_s are mutually independent.

For the sake of convenience, let

$$y_{i,s} = \begin{bmatrix} y_{i1,s} & y_{i2,s} & \ldots & y_{in_y,s} \end{bmatrix}^T.$$

In practice, the raw measurements may be quantized using a probabilistic uniform-type quantizer. Such a quantizer is defined as follows:

$$Q(y_{i,s}) = [Q_1(y_{i1,s}) \;\; Q_2(y_{i2,s}) \cdots Q_{n_y}(y_{in_y,s})]^T.$$

For each $Q_\mu(y_{i\mu,s})$ ($\mu = 1, 2, \ldots, n_y$), the quantization levels are described by the set $\mathscr{U}_{i\mu} = \{\phi_{i\mu,p} | \phi_{i\mu,p} \triangleq p\kappa_{i\mu},\ \kappa_{i\mu} > 0,\ p = 0, \pm1, \pm2, \ldots\}$. If $\phi_{i\mu,p} \leq y_{i\mu,s} \leq \phi_{i\mu,p+1}$ holds, the current output $y_{i\mu,s}$ is quantized probabilistically as follows:

$$\begin{aligned} &\mathrm{Prob}\{Q_\mu(y_{i\mu,s}) = \phi_{i\mu,p} | r_{i\mu}\} = 1 - r_{i\mu}, \\ &\mathrm{Prob}\{Q_\mu(y_{i\mu,s}) = \phi_{i\mu,p+1} | r_{i\mu}\} = r_{i\mu}, \end{aligned} \tag{4}$$

where

$$r_{i\mu} = \frac{y_{i\mu,s} - \phi_{i\mu,p}}{\kappa_{i\mu}}.$$

Denoting the quantization error as $q_{i\mu,s} = Q_\mu(y_{i\mu,s}) - y_{i\mu,s}$, it follows from (4) that

$$\begin{aligned} &\mathrm{Prob}\{q_{i\mu,s} = -r_{i\mu}\kappa_{i\mu} | r_{i\mu}\} = 1 - r_{i\mu}, \\ &\mathrm{Prob}\{q_{i\mu,s} = (1 - r_{i\mu})\kappa_{i\mu} | r_{i\mu}\} = r_{i\mu}. \end{aligned} \tag{5}$$

As noted in [31], the quantization error satisfies the following properties:

$$\begin{aligned} &\mathbb{E}\{q_{i\mu,s}\} = 0, \\ &\mathbb{E}\left\{(q_{i\mu,s})^2\right\} \leq \frac{\kappa_{i\mu}^2}{4}, \end{aligned}$$

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TABLE I
COMPARISON WITH REPRESENTATIVE WORKS IN ANTI-ATTACK DISTRIBUTED FILTERING.

Ref.	Nonlinear System	Performance Index	Anti-attack strategy	Anti-attack type	Performance Analysis
[3]	Nonlinearity	H_∞ performance	✗	✗	Boundedness
[42]	Nonlinearity	H_∞ performance	Attack confrontation	Passive	✗
[15]	✗	H_∞ performance	Attack confrontation	Passive	✗
[29]	✗	Ellipsoid constraint	Attack confrontation	Passive	✗
[33]	✗	Variance constraint	Attack detection	Active	Boundedness
[50]	✗	Variance constraint	Attack detection	Active	Stability
[27]	Nonlinearity	Variance constraint	✗	✗	✗
Our Work	Nonlinearity	Variance constraint	Reputation mechanism	Active	Boundedness

$$\mathbb{E}\left\{q_{i\mu_1,s}q_{i\mu_2,s}\right\}=0$$

for $\mu_1 \neq \mu_2$ and $\mu_1, \mu_2 \in \{1, 2, \ldots, n_y\}$.

Remark 1: Due to the shared networks with limited capacities of transmission, storage and computation, the data to be transmitted are frequently quantized. It is essential to deal with the quantization error appropriately, otherwise it will seriously deteriorate the filtering performance. Different from the deterministic quantization (e.g. uniform quantization), the network-induced probabilistic-type quantization is common in engineering, which transforms the quantized data into stochastic variables. The probabilistic-type quantization could embody the statistical characteristics of signals adequately, reflecting the randomness and uncertainty inherent in practical scenarios and enhancing the robustness of the designed distributed filtering algorithm.

The following distributed filter is constructed:

$$\begin{aligned}\hat{x}_{i,s+1|s} =& A_s\hat{x}_{i,s|s}, \quad (6)\\ \hat{x}_{i,s+1|s+1} =& \hat{x}_{i,s+1|s} + \Delta_{i,s+1}[Q(y_{i,s+1}) - C_{i,s+1}\hat{x}_{i,s+1|s}]\\ &+ \sum_{z\in\mathscr{N}_i} a_{iz}\varphi_{iz,s+1}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}), \quad (7)\end{aligned}$$

where $\hat{x}_{i,s|s}$ and $\hat{x}_{i,s+1|s}$ represent the state estimation and prediction of x_s at the i-th sensor, respectively. $\Delta_{i,s+1}$ is the filter gain to be determined, and $\varphi_{iz,s+1}$ is the reputation-based parameter, which will be defined later.

In the distributed filtering problem, each node can obtain prediction information from its neighbors via a shared wireless channel. However, the data received from neighboring nodes may be contaminated due to sensor faults, hardware malfunctions, communication errors, or deliberate malicious attacks. Such contamination can significantly degrade the performance of the distributed filtering algorithm. To address this issue, a reputation mechanism is introduced to identify unreliable data and improve filtering accuracy. Specifically, the *reputation-based parameter* is firstly calculated by

$$\vec{\psi}_{iz,s+1} = -\sum_{j\in\mathscr{N}_i\cup\{i\}} \frac{||\hat{x}_{z,s+1|s} - \hat{x}_{j,s+1|s}||}{|\mathscr{N}_i|} \quad (8)$$

where $\vec{\psi}_{iz,s+1} = 0$ if $z \notin \mathscr{N}_i$. Here, $\vec{\psi}_{iz,s+1}$ represents the reputation assigned by node i to node z. Next, the calculated reputation is normalized as follows:

$$\psi_{iz,s+1} = \frac{\vec{\psi}_{iz,s+1} - \min\limits_{j\in\mathscr{N}_i}\vec{\psi}_{ij,s+1}}{\zeta_{ij,s+1}}, \quad (9)$$

where

$$\zeta_{ij,s+1} = \max_{j\in\mathscr{N}_i}\vec{\psi}_{ij,s+1} - \min_{j\in\mathscr{N}_i}\vec{\psi}_{ij,s+1}.$$

Finally, the reputation-based parameter is determined by

$$\varphi_{iz,s+1} = \begin{cases}\psi_{iz,s+1}, & \text{if} \quad \zeta_{ij,s+1} \geq \eta_{ij,s+1}\\ \dfrac{1}{|\mathscr{N}_i|}, & \text{otherwise}\end{cases} \quad (10)$$

where $\eta_{ij,s+1}$ is a positive scalar threshold.

Remark 2: A two-step recursive filter is designed in (6)–(7) including prediction, innovation and neighboring information. Note that some research results have focused on the distributed filtering problems with equal weights, where $\varphi_{iz,s+1}$ has been set to a specific value within the range $(0,\ 1/\max_i|\mathscr{N}_i|)$. In contrast, a reputation-based anti-unreliability scheme is proposed in this paper to identify and remove the abnormal data from neighbors. In this approach, each sensor functions as a local fusion center, capable of assigning individual reputations to the local predictions received from its neighbors, following the definitions in (8)–(10). Specifically, a mean-based evaluation rule is defined in (8) to derive the reputation parameter. Notably, the allocated reputation will decrease as the deviation calculated by (8) increases, which is further normalized by means of (9). It can be seen from (8)–(9) that the predictions with low reputations are identified as abnormal data subject to cyber attacks or sensor faults. The proposed reputation strategy effectively reduces the adverse impact of unreliable sensory data, thereby enhancing the robustness, resilience, and accuracy of the distributed filtering algorithm.

The objective of this paper is to propose an RBDF algorithm such that

(1) The reputation-dependent recursion regarding UBFEC is determined by solving some matrix equations;
(2) The filter gain is parameterized by minimizing the trace of UBFEC to ensure locally optimal filtering performance.
(3) A sufficient condition is established to guarantee the uniform boundedness of the filtering error dynamics.

Before proceeding, the following lemmas are presented for the subsequent derivation.

Lemma 1: [45] For two given vectors $a \in \mathbb{R}^n$ and $b \in \mathbb{R}^n$, the following inequality holds:

$$ab^T + ba^T \leq oaa^T + o^{-1}bb^T,$$

where $o > 0$ is a known scalar.

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Lemma 2: [31] Let $\mathcal{G}$, $\mathcal{H}$, $\mathcal{F}$ and $\mathcal{D}$ be matrices with appropriate dimensions. Then, we have

$$\frac{\partial \text{tr}(\mathcal{GHF})}{\partial \mathcal{H}} = \mathcal{G}^T\mathcal{F}^T, \quad \frac{\partial \text{tr}(\mathcal{GHF})}{\partial \mathcal{H}^T} = \mathcal{FG},$$
$$\frac{\partial \text{tr}((\mathcal{GHF})\mathcal{D}(\mathcal{GHF})^T)}{\partial \mathcal{H}} = 2\mathcal{G}^T\mathcal{GHFDF}^T.$$

IV. Main Results

This section is devoted to obtaining the UBFEC and determining the filter gain in the sense of minimum mean-square error. First of all, the prediction error is given as follows:

$$\begin{aligned} e_{i,s+1|s} &= x_{s+1} - \hat{x}_{i,s+1|s} \\ &= A_s e_{i,s|s} + f(s, x_s, \theta_s) + B_s\omega_s. \end{aligned} \tag{11}$$

Furthermore, the filtering error dynamics is presented by

$$\begin{aligned} e_{i,s+1|s+1} =& x_{s+1} - \hat{x}_{i,s+1|s+1} \\ =& (I - \Delta_{i,s+1}C_{i,s+1})e_{i,s+1|s} - \Delta_{i,s+1}q_{i,s+1} \\ &- \Delta_{i,s+1}\upsilon_{i,s+1} - \sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,s+1}\psi_{iz,s+1} \\ &\times (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) - \sum_{z\in\mathcal{N}_i} a_{iz}(1-\hbar_{i,s+1}) \\ &\times \frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}), \end{aligned} \tag{12}$$

where $q_{i,s+1} = [q_{i1,s+1}\ q_{i2,s+1} \cdots q_{in_y,s+1}]^T$ and $\hbar_{i,s+1}$ is a binary variable satisfying

$$\hbar_{i,s+1} = \begin{cases} 1, & \text{if } \zeta_{ij,s+1} \geq \eta_{ij,s+1} \\ 0. & \text{otherwise} \end{cases}$$

The following lemma provides the covariances of the prediction error and filtering error.

Lemma 3: The prediction error covariance and filtering error covariance evolve, respectively, as follows:

$$\begin{aligned} P_{i,s+1|s} =& A_s P_{i,s|s}A_s^T + \mathbb{E}\left\{\sum_{h=1}^{m}\Pi_{h,s}x_s^T\Lambda_{h,s}x_s\right\} \\ &+ B_s\Theta_s B_s^T \end{aligned} \tag{13}$$

and

$$\begin{aligned} P_{i,s+1|s+1} =& (I - \Delta_{i,s+1}C_{i,s+1})P_{i,s+1|s}(I - \Delta_{i,s+1}C_{i,s+1})^T \\ &+ \Delta_{i,s+1}\mathbb{E}\{q_{i,s+1}q_{i,s+1}^T\}\Delta_{i,s+1}^T \\ &+ \sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,s+1}\psi_{iz,s+1}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \\ &\times \left[\sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,s+1}\psi_{iz,s+1}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})\right]^T \\ &+ \Delta_{i,s+1}R_{i,s+1}\Delta_{i,s+1}^T + \sum_{z\in\mathcal{N}_i} a_{iz}(1-\hbar_{i,s+1}) \\ &\times \frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})\left[\sum_{z\in\mathcal{N}_i} a_{iz}\right. \\ &\left.\times (1-\hbar_{i,s+1})\frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})\right]^T \\ &- M_{1,s+1} - M_{1,s+1}^T - M_{2,s+1} - M_{2,s+1}^T \\ &+ M_{3,s+1} + M_{3,s+1}^T, \end{aligned} \tag{14}$$

where

$$\begin{aligned} M_{1,s+1} =& \mathbb{E}\Bigg\{(I - \Delta_{i,s+1}C_{i,s+1})e_{i,s+1|s}\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,s+1} \\ &\times \psi_{iz,s+1}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})\Bigg]^T\Bigg\}, \\ M_{2,s+1} =& \mathbb{E}\Bigg\{(I - \Delta_{i,s+1}C_{i,s+1})e_{i,s+1|s}\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\frac{1}{|\mathcal{N}_i|} \\ &\times (1-\hbar_{i,s+1})(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})\Bigg]^T\Bigg\}, \\ M_{3,s+1} =& \Delta_{i,s+1}\mathbb{E}\{q_{i,s+1}\upsilon_{i,s+1}^T\}\Delta_{i,s+1}^T. \end{aligned}$$

Proof: According to the error dynamics in (11) and (12), it is easy to have

$$\begin{aligned} P_{i,s+1|s} =& \mathbb{E}\{e_{i,s+1|s}e_{i,s+1|s}^T\} \\ =& A_s P_{i,s|s}A_s^T + \mathbb{E}\{f(s,x_s,\theta_s)f^T(s,x_s,\theta_s)\} \\ &+ \mathbb{E}\{B_s\omega_s\omega_s^T B_s^T\} + N_{1,s} + N_{1,s}^T \\ &+ N_{2,s} + N_{2,s}^T + N_{3,s} + N_{3,s}^T, \end{aligned}$$

and

$$\begin{aligned} &P_{i,s+1|s+1} \\ =& \mathbb{E}\{e_{i,s+1|s+1}e_{i,s+1|s+1}^T\} \\ =& (I - \Delta_{i,s+1}C_{i,s+1})P_{i,s+1|s}(I - \Delta_{i,s+1}C_{i,s+1})^T \\ &+ \Delta_{i,s+1}\mathbb{E}\{q_{i,s+1}q_{i,s+1}^T\}\Delta_{i,s+1}^T \\ &+ \sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,s+1}\psi_{iz,s+1}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \\ &\times \left[\sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,s+1}\psi_{iz,s+1}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})\right]^T \\ &+ \Delta_{i,s+1}R_{i,s+1}\Delta_{i,s+1}^T + \sum_{z\in\mathcal{N}_i} a_{iz}(1-\hbar_{i,s+1}) \\ &\times \frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})\left[\sum_{z\in\mathcal{N}_i} a_{iz}\right. \\ &\left.\times (1-\hbar_{i,s+1})\frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})\right]^T \\ &- M_{1,s+1} - M_{1,s+1}^T - M_{2,s+1} - M_{2,s+1}^T \\ &+ M_{3,s+1} + M_{3,s+1}^T - M_{4,s+1} - M_{4,s+1}^T \\ &- M_{5,s+1} - M_{5,s+1}^T + M_{6,s+1} + M_{6,s+1}^T \\ &+ M_{7,s+1} + M_{7,s+1}^T + M_{8,s+1} + M_{8,s+1}^T \\ &+ M_{9,s+1} + M_{9,s+1}^T + M_{10,s+1} + M_{10,s+1}^T, \end{aligned}$$

where

$$\begin{aligned} N_{1,s} =& \mathbb{E}\{A_s e_{i,s|s}f^T(s,x_s,\theta_s)\}, \\ N_{2,s} =& \mathbb{E}\{A_s e_{i,s|s}\omega_s^T B_s^T\}, \\ N_{3,s} =& \mathbb{E}\{f(s,x_s,\theta_s)\omega_s^T B_s^T\}, \\ M_{4,s+1} =& \mathbb{E}\{(I - \Delta_{i,s+1}C_{i,s+1})e_{i,s+1|s}q_{i,s+1}^T\Delta_{i,s+1}^T\}, \\ M_{5,s+1} =& \mathbb{E}\{(I - \Delta_{i,s+1}C_{i,s+1})e_{i,s+1|s}\upsilon_{i,s+1}^T\Delta_{i,s+1}^T\}, \end{aligned}$$

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$$
\begin{aligned}
M_{6,s+1} =&\mathbb{E}\Bigg\{ \Delta_{i,s+1}q_{i,s+1}\Bigg[\sum_{z\in\mathscr{N}_i} a_{iz}(1-\hbar_{i,s+1}) \\
&\times \frac{1}{|\mathscr{N}_i|}(\hat{x}_{z,s+1|s}-\hat{x}_{i,s+1|s})\Bigg]^T\Bigg\}, \\
M_{7,s+1} =&\mathbb{E}\Bigg\{ \Delta_{i,s+1}q_{i,s+1}\Bigg[\sum_{z\in\mathscr{N}_i} a_{iz}\hbar_{i,s+1}\psi_{iz,s+1} \\
&\times (\hat{x}_{z,s+1|s}-\hat{x}_{i,s+1|s})\Bigg]^T\Bigg\}, \\
M_{8,s+1} =&\mathbb{E}\Bigg\{ \Delta_{i,s+1}\upsilon_{i,s+1}\Bigg[\sum_{z\in\mathscr{N}_i} a_{iz}\hbar_{i,s+1}\psi_{iz,s+1} \\
&\times (\hat{x}_{z,s+1|s}-\hat{x}_{i,s+1|s})\Bigg]^T\Bigg\}, \\
M_{9,s+1} =&\mathbb{E}\Bigg\{ \Delta_{i,s+1}\upsilon_{i,s+1}\Bigg[\sum_{z\in\mathscr{N}_i} a_{iz}(1-\hbar_{i,s+1}) \\
&\times \frac{1}{|\mathscr{N}_i|}(\hat{x}_{z,s+1|s}-\hat{x}_{i,s+1|s})\Bigg]^T\Bigg\}, \\
M_{10,s+1} =& \sum_{z\in\mathscr{N}_i} a_{iz}\hbar_{i,s+1}\psi_{iz,s+1}(\hat{x}_{z,s+1|s}-\hat{x}_{i,s+1|s}) \\
&\times \Bigg[\sum_{z\in\mathscr{N}_i} a_{iz}(1-\hbar_{i,s+1})\frac{1}{|\mathscr{N}_i|} \\
&\times (\hat{x}_{z,s+1|s}-\hat{x}_{i,s+1|s})\Bigg]^T.
\end{aligned}
$$

Notice that $\mathbb{E}\{f(s,x_s,\theta_s)\}=0$, $\mathbb{E}\{\omega_s\}=0$, $\mathbb{E}\{q_{i,s+1}\}=0$, $\mathbb{E}\{\upsilon_{i,s+1}\}=0$ and $\hbar_{i,s+1}(1-\hbar_{i,s+1})=0$. It is obvious that $N_{1,s}$, $N_{2,s}$, $N_{3,s}$, $M_{4,s+1}$, $M_{5,s+1}$, $M_{6,s+1}$, $M_{7,s+1}$, $M_{8,s+1}$, $M_{9,s+1}$ and $M_{10,s+1}$ are all zero-valued matrices by means of the independent characteristics of stochastic variables, and the proof is then complete. ■

Remark 3: Taking network-induced probabilistic-type quantization, stochastic nonlinearity and reputation mechanism into account simultaneously, a comprehensive filter is designed recursively, resulting in the extremely complexity in determining the exact value of the filtering error covariance. Consequently, it is imperative to obtain an expression for the UBFEC by handling quantization errors, cross-product terms, and other nonlinear effects, facilitating the further calculation and analysis.

The covariance upper bounds of the prediction error and filtering error are concretized in the following theorem. In addition, the filter gain is parameterized to minimize the trace of the upper bound of the UBFEC.

Theorem 1: Consider the SNTVS (1)–(2) and reputation-based distributed filter (6)–(7). For known scalars $o_1>0$, $o_2>0$, $o_3>0$ and $o_4>0$, under initial condition $\mathcal{P}_{i,0|0}\geq P_{i,0|0}>0$, assume that the following recursive matrix equations:

$$
\begin{aligned}
&\mathcal{P}_{i,s+1|s} \\
&=A_s\mathcal{P}_{i,s|s}A_s^T+\sum_{h=1}^{m}\Pi_{h,s}\mathrm{tr}(\mathcal{X}_{i,s|s}\Lambda_{h,s}) \\
&\quad +B_s\Theta_sB_s^T
\end{aligned}
\tag{15}
$$

and

$$
\begin{aligned}
&\mathcal{P}_{i,s+1|s+1} \\
&=(1+o_2+o_3)(I-\Delta_{i,s+1}C_{i,s+1})\mathcal{P}_{i,s+1|s} \\
&\quad\times (I-\Delta_{i,s+1}C_{i,s+1})^T+2(1+o_2^{-1}) \\
&\quad\times \hbar_{i,s+1}\sum_{j\in\mathscr{N}_i}a_{ij}\psi_{ij,s+1}\sum_{z\in\mathscr{N}_i}a_{iz}\psi_{iz,s+1} \\
&\quad\times (\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s})+2(1+o_3^{-1}) \\
&\quad\times (1-\hbar_{i,s+1})\frac{1}{|\mathscr{N}_i|^2}\sum_{j\in\mathscr{N}_i}a_{ij}\sum_{z\in\mathscr{N}_i}a_{iz} \\
&\quad\times (\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s})+\Delta_{i,s+1} \\
&\quad\times [(1+o_4)\aleph+(1+o_4^{-1})R_{i,s+1}]\Delta_{i,s+1}^T
\end{aligned}
\tag{16}
$$

have solutions $\mathcal{P}_{i,s+1|s+1}>0$ and $\mathcal{P}_{i,s+1|s}>0$, where

$$
\begin{aligned}
\mathcal{X}_{i,s|s}&=(1+o_1)\mathcal{P}_{i,s|s}+(1+o_1^{-1})\hat{x}_{i,s|s}\hat{x}_{i,s|s}^T, \\
\aleph&=\mathrm{diag}\left\{\frac{\kappa_{i1}^2}{4},\frac{\kappa_{i2}^2}{4},\cdots,\frac{\kappa_{in_y}^2}{4}\right\}.
\end{aligned}
\tag{17}
$$

Then, $\mathcal{P}_{i,s+1|s+1}$ is an upper bound of $P_{i,s+1|s+1}$, i.e., $P_{i,s+1|s+1}\leq\mathcal{P}_{i,s+1|s+1}$ at each sampling step. Furthermore, if the selected filter gain satisfies

$$
\begin{aligned}
\Delta_{i,s+1}=&(1+o_2+o_3)\mathcal{P}_{i,s+1|s}C_{i,s+1}^T\Big[(1+o_2+o_3) \\
&\times C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T+(1+o_4)\aleph \\
&+(1+o_4^{-1})R_{i,s+1}\Big]^{-1},
\end{aligned}
\tag{18}
$$

then the index $\mathrm{tr}(\mathcal{P}_{i,s+1|s+1})$ can be minimized at each sampling step and the minimized $\mathcal{P}_{i,s+1|s+1}$ can be recursively calculated by

$$
\begin{aligned}
&\mathcal{P}_{i,s+1|s+1} \\
&=(1+o_2+o_3)\mathcal{P}_{i,s+1|s}-(1+o_2+o_3)^2\mathcal{P}_{i,s+1|s}C_{i,s+1}^T \\
&\quad\times\Big[(1+o_2+o_3)C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T+(1+o_4)\aleph \\
&\quad+(1+o_4^{-1})R_{i,s+1}\Big]^{-1}C_{i,s+1}\mathcal{P}_{i,s+1|s}+2(1+o_2^{-1}) \\
&\quad\times\hbar_{i,s+1}\sum_{j\in\mathscr{N}_i}a_{ij}\psi_{ij,s+1}\sum_{z\in\mathscr{N}_i}a_{iz}\psi_{iz,s+1} \\
&\quad\times(\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s})+2(1+o_3^{-1})(1-\hbar_{i,s+1}) \\
&\quad\times\frac{1}{|\mathscr{N}_i|^2}\sum_{j\in\mathscr{N}_i}a_{ij}\sum_{z\in\mathscr{N}_i}a_{iz}(\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s}).
\end{aligned}
\tag{19}
$$

Proof: By means of Lemma 1, one has

$$
\begin{aligned}
\mathbb{E}\{x_sx_s^T\}&=\mathbb{E}\{(\hat{x}_{i,s|s}+e_{i,s|s})(\hat{x}_{i,s|s}+e_{i,s|s})^T\} \\
&\leq(1+o_1)P_{i,s|s}+(1+o_1^{-1})\hat{x}_{i,s|s}\hat{x}_{i,s|s}^T \\
&\triangleq X_{i,s|s},
\end{aligned}
\tag{20}
$$

where $o_1>0$ is a known scalar. It is obvious that

$$
\begin{aligned}
\mathbb{E}\left\{\sum_{h=1}^{m}\Pi_{h,s}x_s^T\Lambda_{h,s}x_s\right\}&=\sum_{h=1}^{m}\Pi_{h,s}\mathrm{tr}(\mathbb{E}\{x_sx_s^T\}\Lambda_{h,s}) \\
&\leq\sum_{h=1}^{m}\Pi_{h,s}\mathrm{tr}(X_{i,s|s}\Lambda_{h,s}).
\end{aligned}
\tag{21}
$$

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Substituting (21) into (13) yields

$$P_{i,s+1|s} \leq A_s P_{i,s|s} A_s^T + \sum_{h=1}^{m} \Pi_{h,s} \mathrm{tr}(X_{i,s|s}\Lambda_{h,s}) + B_s \Theta_s B_s^T. \tag{22}$$

Recalling the fundamental inequality in Lemma 1, we have

$$\begin{aligned} &-M_{1,s+1} - M_{1,s+1}^T \\ \leq & o_2 (I - \Delta_{i,s+1} C_{i,s+1}) P_{i,s+1|s} (I - \Delta_{i,s+1} C_{i,s+1})^T \\ &+ o_2^{-1} \sum_{z \in \mathcal{N}_i} a_{iz} \hbar_{i,s+1} \psi_{iz,s+1} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \\ &\times \Big[\sum_{z \in \mathcal{N}_i} a_{iz} \hbar_{i,s+1} \psi_{iz,s+1} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big]^T, \end{aligned} \tag{23}$$

$$\begin{aligned} &-M_{2,s+1} - M_{2,s+1}^T \\ \leq & o_3 (I - \Delta_{i,s+1} C_{i,s+1}) P_{i,s+1|s} (I - \Delta_{i,s+1} C_{i,s+1})^T \\ &+ o_3^{-1} \sum_{z \in \mathcal{N}_i} a_{iz} \frac{1}{|\mathcal{N}_i|} (1 - \hbar_{i,s+1}) \\ &\times (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big[\sum_{z \in \mathcal{N}_i} a_{iz} \frac{1}{|\mathcal{N}_i|} \\ &\times (1 - \hbar_{i,s+1}) (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big]^T \end{aligned} \tag{24}$$

and

$$M_{3,s+1} + M_{3,s+1}^T \leq o_4 \Delta_{i,s+1} \mathbb{E}\{q_{i,s+1} q_{i,s+1}^T\} \Delta_{i,s+1}^T + o_4^{-1} \Delta_{i,s+1} R_{i,s+1} \Delta_{i,s+1}^T, \tag{25}$$

where $o_2 > 0$, $o_3 > 0$ and $o_4 > 0$ are known scalars. Substituting (23)–(25) into (14) leads to

$$\begin{aligned} &P_{i,s+1|s+1} \\ \leq & (1 + o_2 + o_3)(I - \Delta_{i,s+1} C_{i,s+1}) P_{i,s+1|s} \\ &\times (I - \Delta_{i,s+1} C_{i,s+1})^T + (1 + o_4) \Delta_{i,s+1} \\ &\times \mathbb{E}\{q_{i,s+1} q_{i,s+1}^T\} \Delta_{i,s+1}^T + (1 + o_2^{-1}) \\ &\times \sum_{z \in \mathcal{N}_i} a_{iz} \hbar_{i,s+1} \psi_{iz,s+1} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \\ &\times \Big[\sum_{z \in \mathcal{N}_i} a_{iz} \hbar_{i,s+1} \psi_{iz,s+1} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big]^T \\ &+ (1 + o_4^{-1}) \Delta_{i,s+1} R_{i,s+1} \Delta_{i,s+1}^T + (1 + o_3^{-1}) \\ &\times \sum_{z \in \mathcal{N}_i} a_{iz} (1 - \hbar_{i,s+1}) \frac{1}{|\mathcal{N}_i|} (\hat{x}_{z,s+1|s} \\ &- \hat{x}_{i,s+1|s}) \Big[\sum_{z \in \mathcal{N}_i} a_{iz} (1 - \hbar_{i,s+1}) \frac{1}{|\mathcal{N}_i|} \\ &\times (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big]^T. \end{aligned} \tag{26}$$

Obviously, the quantization error satisfies

$$\mathbb{E}\{q_{i,s+1} q_{i,s+1}^T\} \leq \aleph, \tag{27}$$

where $\aleph$ is defined in (17). It is not difficult to verify that

$$\begin{aligned} &P_{i,s+1|s+1} \\ \leq & (1 + o_2 + o_3)(I - \Delta_{i,s+1} C_{i,s+1}) P_{i,s+1|s} \\ &\times (I - \Delta_{i,s+1} C_{i,s+1})^T + (1 + o_2^{-1}) \\ &\times \sum_{z \in \mathcal{N}_i} a_{iz} \hbar_{i,s+1} \psi_{iz,s+1} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \\ &\times \Big[\sum_{z \in \mathcal{N}_i} a_{iz} \hbar_{i,s+1} \psi_{iz,s+1} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big]^T \\ &+ (1 + o_3^{-1}) \sum_{z \in \mathcal{N}_i} a_{iz} (1 - \hbar_{i,s+1}) \frac{1}{|\mathcal{N}_i|} \\ &\times (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big[\sum_{z \in \mathcal{N}_i} a_{iz} (1 - \hbar_{i,s+1}) \\ &\times \frac{1}{|\mathcal{N}_i|} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big]^T + \Delta_{i,s+1} \\ &\times [(1 + o_4)\aleph + (1 + o_4^{-1}) R_{i,s+1}] \Delta_{i,s+1}^T. \end{aligned} \tag{28}$$

Subsequently, the following inequality can be established

$$\begin{aligned} &\sum_{z \in \mathcal{N}_i} a_{iz} \hbar_{i,s+1} \psi_{iz,s+1} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \\ &\times \Big[\sum_{z \in \mathcal{N}_i} a_{iz} \hbar_{i,s+1} \psi_{iz,s+1} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big]^T \\ \leq & \sum_{z \in \mathcal{N}_i} \sum_{j \in \mathcal{N}_i} a_{iz} a_{ij} \hbar_{i,s+1}^2 \psi_{iz,s+1} \psi_{ij,s+1} \\ &\times (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})(\hat{x}_{j,s+1|s} - \hat{x}_{i,s+1|s})^T \\ \leq & \frac{1}{2} \sum_{z \in \mathcal{N}_i} \sum_{j \in \mathcal{N}_i} a_{iz} a_{ij} \hbar_{i,s+1}^2 \psi_{iz,s+1} \psi_{ij,s+1} \\ &\times \Big[(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})(\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s})^T \\ &+ (\hat{x}_{j,s+1|s} - \hat{x}_{i,s+1|s})(\hat{x}_{j,s+1|s} - \hat{x}_{i,s+1|s})^T \Big] \\ \leq & 2\hbar_{i,s+1} \sum_{j \in \mathcal{N}_i} a_{ij} \psi_{ij,s+1} \sum_{z \in \mathcal{N}_i} a_{iz} \psi_{iz,s+1} \\ &\times (P_{z,s+1|s} + P_{i,s+1|s}). \end{aligned} \tag{29}$$

Similarly, we have

$$\begin{aligned} &\sum_{z \in \mathcal{N}_i} a_{iz} (1 - \hbar_{i,s+1}) \frac{1}{|\mathcal{N}_i|} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \\ &\times \Big[\sum_{z \in \mathcal{N}_i} a_{iz} (1 - \hbar_{i,s+1}) \frac{1}{|\mathcal{N}_i|} (\hat{x}_{z,s+1|s} - \hat{x}_{i,s+1|s}) \Big]^T \\ \leq & 2(1 - \hbar_{i,s+1}) \sum_{j \in \mathcal{N}_i} a_{ij} \frac{1}{|\mathcal{N}_i|} \sum_{z \in \mathcal{N}_i} a_{iz} \frac{1}{|\mathcal{N}_i|} \\ &\times (P_{z,s+1|s} + P_{i,s+1|s}). \end{aligned} \tag{30}$$

Hence, we arrive at

$$\begin{aligned} P_{i,s+1|s+1} \leq & (1 + o_2 + o_3)(I - \Delta_{i,s+1} C_{i,s+1}) P_{i,s+1|s} \\ &\times (I - \Delta_{i,s+1} C_{i,s+1})^T + 2(1 + o_2^{-1}) \\ &\times \hbar_{i,s+1} \sum_{j \in \mathcal{N}_i} a_{ij} \psi_{ij,s+1} \sum_{z \in \mathcal{N}_i} a_{iz} \psi_{iz,s+1} \\ &\times (P_{z,s+1|s} + P_{i,s+1|s}) + 2(1 + o_3^{-1}) \\ &\times (1 - \hbar_{i,s+1}) \sum_{j \in \mathcal{N}_i} a_{ij} \frac{1}{|\mathcal{N}_i|} \sum_{z \in \mathcal{N}_i} a_{iz} \end{aligned}$$

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$$\times \frac{1}{|\mathcal{N}_i|}(P_{z,s+1|s} + P_{i,s+1|s}) + \Delta_{i,s+1} \times [(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}]\Delta_{i,s+1}^T. \quad (31)$$

Based on (15), (16), (22), (31) and by means of mathematical induction, one has

$$\begin{aligned} &P_{i,s+1|s+1} \\ \leq&(1+o_2+o_3)(I-\Delta_{i,s+1}C_{i,s+1})\mathcal{P}_{i,s+1|s} \\ &\times (I-\Delta_{i,s+1}C_{i,s+1})^T + 2(1+o_2^{-1}) \\ &\times \hbar_{i,s+1}\sum_{j\in\mathcal{N}_i} a_{ij}\psi_{ij,s+1}\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,s+1} \\ &\times (\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s}) + 2(1+o_3^{-1}) \\ &\times (1-\hbar_{i,s+1})\sum_{j\in\mathcal{N}_i} a_{ij}\frac{1}{|\mathcal{N}_i|}\sum_{z\in\mathcal{N}_i} a_{iz} \\ &\times \frac{1}{|\mathcal{N}_i|}(\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s}) + \Delta_{i,s+1} \\ &\times [(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}]\Delta_{i,s+1}^T \\ =&\mathcal{P}_{i,s+1|s+1}. \end{aligned} \quad (32)$$

Now, it remains to design a filter gain to minimize the filtering index $\mathrm{tr}(\mathcal{P}_{i,s+1|s+1})$. It follows from Lemma 2 that

$$\begin{aligned} &\frac{\partial \mathrm{tr}(\mathcal{P}_{i,s+1|s+1})}{\partial \Delta_{i,s+1}} \\ =&\frac{\partial}{\partial \Delta_{i,s+1}}\mathrm{tr}\Bigg((1+o_2+o_3)(I-\Delta_{i,s+1}C_{i,s+1}) \\ &\times \mathcal{P}_{i,s+1|s}(I-\Delta_{i,s+1}C_{i,s+1})^T + 2(1+o_2^{-1}) \\ &\times \hbar_{i,s+1}\sum_{j\in\mathcal{N}_i} a_{ij}\psi_{ij,s+1}\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,s+1} \\ &\times (\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s}) + 2(1+o_3^{-1}) \\ &\times (1-\hbar_{i,s+1})\sum_{j\in\mathcal{N}_i} a_{ij}\frac{1}{|\mathcal{N}_i|}\sum_{z\in\mathcal{N}_i} a_{iz} \\ &\times \frac{1}{|\mathcal{N}_i|}(\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s}) + \Delta_{i,s+1} \\ &\times [(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}]\Delta_{i,s+1}^T\Bigg) \\ =&\frac{\partial}{\partial \Delta_{i,s+1}}\mathrm{tr}\Big((1+o_2+o_3)(I-\Delta_{i,s+1}C_{i,s+1}) \\ &\times \mathcal{P}_{i,s+1|s}(I-\Delta_{i,s+1}C_{i,s+1})^T + \Delta_{i,s+1} \\ &\times [(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}]\Delta_{i,s+1}^T\Big) \\ =&-2(1+o_2+o_3)(I-\Delta_{i,s+1}C_{i,s+1})\mathcal{P}_{i,s+1|s} \\ &\times C_{i,s+1}^T + 2\Delta_{i,s+1}[(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}]. \end{aligned} \quad (33)$$

Letting $\frac{\partial \mathrm{tr}(\mathcal{P}_{i,s+1|s+1})}{\partial \Delta_{i,s+1}} = 0$, we have

$$\begin{aligned} \Delta_{i,s+1} =&(1+o_2+o_3)\mathcal{P}_{i,s+1|s}C_{i,s+1}^T\Big[(1+o_2+o_3) \\ &\times C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T + (1+o_4)\aleph \\ &+ (1+o_4^{-1})R_{i,s+1}\Big]^{-1}. \end{aligned} \quad (34)$$

It follows from (16) and (34) that the minimized $\mathcal{P}_{i,s+1|s+1}$ satisfies

$$\begin{aligned} &\mathcal{P}_{i,s+1|s+1} \\ =&(1+o_2+o_3)\mathcal{P}_{i,s+1|s} - (1+o_2+o_3)^2\mathcal{P}_{i,s+1|s}C_{i,s+1}^T \\ &\times \Big[(1+o_2+o_3)C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T + (1+o_4)\aleph \\ &+ (1+o_4^{-1})R_{i,s+1}\Big]^{-1}C_{i,s+1}\mathcal{P}_{i,s+1|s} + 2(1+o_2^{-1}) \\ &\times \hbar_{i,s+1}\sum_{j\in\mathcal{N}_i} a_{ij}\psi_{ij,s+1}\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,s+1} \\ &\times (\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s}) + 2(1+o_3^{-1})(1-\hbar_{i,s+1}) \\ &\times \sum_{j\in\mathcal{N}_i} a_{ij}\frac{1}{|\mathcal{N}_i|}\sum_{z\in\mathcal{N}_i} a_{iz}\frac{1}{|\mathcal{N}_i|}(\mathcal{P}_{z,s+1|s}+\mathcal{P}_{i,s+1|s}), \end{aligned} \quad (35)$$

and the proof is now complete. ■

Remark 4: Up to now, the recursion of UBFEC is derived by solving two matrix equations. Furthermore, the trace of UBFEC serves as a performance index within the framework of distributed Kalman-type filtering, which is optimized by selecting the filter gain properly. The parameterization of the filter gain is implemented by calculating (33), ensuring the minimization of filtering error dynamics at each sampling instant.

V. Boundedness Analysis

This section is devoted to providing the uniform boundedness analysis for the filtering error dynamics. To begin, it is essential to introduce the following assumptions.

Assumption 1: There exist positive scalars $\bar{a}$, $\underline{b}$, $\bar{b}$, $\bar{x}$, $\underline{\theta}$, $\bar{\theta}$, $\bar{\pi}$, $\bar{\lambda}$, $\underline{c}$, $\bar{c}$ and $\underline{r}$ such that the following inequalities

$$\begin{aligned} &A_sA_s^T \leq \bar{a}I,\ \ \underline{b}I \leq B_sB_s^T \leq \bar{b}I,\ \ \underline{\theta}I \leq \Theta_s \leq \bar{\theta}I, \\ &\mathcal{X}_{i,s|s} \leq \bar{x}I,\ \ \Pi_{h,s} \leq \bar{\pi}I,\ \ \Lambda_{h,s} \leq \bar{\lambda}I, \\ &\underline{c}I \leq C_{i,s}C_{i,s}^T \leq \bar{c}I,\ \ R_{i,s} \geq \underline{r}I, \end{aligned}$$

hold for every s, h and i.

Assumption 2: $C_{i,s+1}$ is a row-full rank matrix and $C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T$ is invertible.

The uniform boundedness of the filtering error dynamics is established in the following theorem with the help of Assumptions 1 and 2.

Theorem 2: For the considered SNTVS (1)–(2) as well as the constructed reputation-based distributed filter (6)–(7), if there exists scalar $\bar{p} > 0$ satisfying

$$2(1+o_2+o_3)\bar{p}_1 + \eta + \phi \leq \bar{p}, \quad (36)$$

where

$$\begin{aligned} &\bar{p}_1 = \bar{a}\bar{p} + \bar{x}\bar{\lambda}\bar{\pi}mn_x + \bar{b}\bar{\theta},\ \ \underline{p}_1 = \underline{b}\underline{\theta}, \\ &\eta = 4\left[(1+o_2^{-1})|\mathcal{N}_i|^2 + (1+o_3^{-1})\right]\bar{p}_1 I, \\ &\phi = (1+o_2+o_3)^2\bar{p}_1^2\bar{c}\left[\frac{1}{\underline{c}\underline{p}_1} + \frac{1}{(1+o_4^{-1})\underline{r}}\right], \end{aligned}$$

then $\mathcal{P}_{i,s+1|s+1}$ is uniformly bounded under the initial condition $\mathcal{P}_{i,0|0} \leq \bar{p}I$, i.e., $\mathcal{P}_{i,s+1|s+1} \leq \bar{p}I$ for each instant s.

Proof: By means of mathematical induction, starting from the initialization step $\mathcal{P}_{i,0|0} \leq \bar{p}I$, we assume that $\mathcal{P}_{i,s|s} \leq \bar{p}I$

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and then proceed to verify $\mathcal{P}_{i,s+1|s+1} \leq \bar{p}I$. First of all, by resorting to Assumption 1, we have

$$\begin{aligned}\mathcal{P}_{i,s+1|s} =&A_s\mathcal{P}_{i,s|s}A_s^T + \sum_{h=1}^{m}\Pi_{h,s}\text{tr}(\mathcal{X}_{i,s|s}\Lambda_{h,s})\\ &+ B_s\Theta_sB_s^T\\ \leq&\bar{a}\bar{p}I + \bar{x}\bar{\lambda}\bar{\pi}mn_xI + \bar{b}\bar{\theta}I\\ \triangleq&\bar{p}_1I\end{aligned} \tag{37}$$

and

$$\mathcal{P}_{i,s+1|s} \geq B_s\Theta_sB_s^T \geq \underline{b}\underline{\theta}I \triangleq \underline{p}_1I \tag{38}$$

for each i and s. It follows from Lemma 1 that

$$\begin{aligned}&(I-\Delta_{i,s+1}C_{i,s+1})P_{i,s+1|s}(I-\Delta_{i,s+1}C_{i,s+1})^T\\ \leq&2P_{i,s+1|s} + 2\Delta_{i,s+1}C_{i,s+1}P_{i,s+1|s}C_{i,s+1}^T\Delta_{i,s+1}^T.\end{aligned} \tag{39}$$

Consequently, we can obtain

$$\begin{aligned}&\mathcal{P}_{i,s+1|s+1}\\ \leq&2(1+o_2+o_3)(\mathcal{P}_{i,s+1|s} + \Delta_{i,s+1}C_{i,s+1}\mathcal{P}_{i,s+1|s}\\ &\times C_{i,s+1}^T\Delta_{i,s+1}^T) + 2(1+o_2^{-1})\hbar_{i,s+1}\\ &\times \sum_{j\in\mathcal{N}_i}a_{ij}\psi_{ij,s+1}\sum_{z\in\mathcal{N}_i}a_{iz}\psi_{iz,s+1}\\ &\times(\mathcal{P}_{z,s+1|s} + \mathcal{P}_{i,s+1|s}) + 2(1+o_3^{-1})\\ &\times(1-\hbar_{i,s+1})\sum_{j\in\mathcal{N}_i}a_{ij}\frac{1}{|\mathcal{N}_i|}\sum_{z\in\mathcal{N}_i}a_{iz}\\ &\times\frac{1}{|\mathcal{N}_i|}(\mathcal{P}_{z,s+1|s} + \mathcal{P}_{i,s+1|s}) + \Delta_{i,s+1}\\ &\times[(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}]\Delta_{i,s+1}^T\\ \leq&2(1+o_2+o_3)\bar{p}_1I + 2(1+o_2+o_3)\Delta_{i,s+1}\\ &\times C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T\Delta_{i,s+1}^T + 4(1+o_2^{-1})\\ &\times|\mathcal{N}_i|^2\bar{p}_1I + 4(1+o_3^{-1})\bar{p}_1I + \Delta_{i,s+1}\\ &\times[(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}]\Delta_{i,s+1}^T.\end{aligned} \tag{40}$$

For convenience, we denote

$$\begin{aligned}\bar{\Sigma}_{i,s+1} =&(1+o_2+o_3)C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T\\ &+ (1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1},\\ \underline{\Sigma}_{i,s+1}^{(1)} =&C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T,\\ \underline{\Sigma}_{i,s+1}^{(2)} =&(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}.\end{aligned}$$

On the basic of Assumptions 1 and 2, one has

$$\begin{aligned}&\Delta_{i,s+1}C_{i,s+1}\mathcal{P}_{i,s+1|s}C_{i,s+1}^T\Delta_{i,s+1}^T\\ =&(1+o_2+o_3)^2\mathcal{P}_{i,s+1|s}C_{i,s+1}^T\bar{\Sigma}_{i,s+1}^{-1}\underline{\Sigma}_{i,s+1}^{(1)}\\ &\times\bar{\Sigma}_{i,s+1}^{-T}C_{i,s+1}\mathcal{P}_{i,s+1|s}\\ \leq&(1+o_2+o_3)^2\mathcal{P}_{i,s+1|s}C_{i,s+1}^T(\underline{\Sigma}_{i,s+1}^{(1)})^{-1}\\ &\times C_{i,s+1}\mathcal{P}_{i,s+1|s}\\ \leq&\frac{(1+o_2+o_3)^2\bar{p}_1^2\bar{c}}{\underline{c}\underline{p}_1}I\end{aligned} \tag{41}$$

and

$$\begin{aligned}&\Delta_{i,s+1}[(1+o_4)\aleph + (1+o_4^{-1})R_{i,s+1}]\Delta_{i,s+1}^T\\ =&(1+o_2+o_3)^2\mathcal{P}_{i,s+1|s}C_{i,s+1}^T\bar{\Sigma}_{i,s+1}^{-1}\underline{\Sigma}_{i,s+1}^{(2)}\\ &\times\bar{\Sigma}_{i,s+1}^{-T}C_{i,s+1}\mathcal{P}_{i,s+1|s}\\ \leq&(1+o_2+o_3)^2\mathcal{P}_{i,s+1|s}C_{i,s+1}^T(\underline{\Sigma}_{i,s+1}^{(2)})^{-1}\\ &\times C_{i,s+1}\mathcal{P}_{i,s+1|s}\\ \leq&\frac{(1+o_2+o_3)^2\bar{p}_1^2\bar{c}}{(1+o_4^{-1})\underline{r}}I.\end{aligned} \tag{42}$$

Finally, we conclude that

$$\begin{aligned}&\mathcal{P}_{i,s+1|s+1}\\ \leq&2(1+o_2+o_3)\bar{p}_1I + 4(1+o_2^{-1})|\mathcal{N}_i|^2\bar{p}_1I\\ &+ 4(1+o_3^{-1})\bar{p}_1I + (1+o_2+o_3)^2\bar{p}_1^2\bar{c}\\ &\times\left[\frac{1}{\underline{c}\underline{p}_1} + \frac{1}{(1+o_4^{-1})\underline{r}}\right]I\\ \leq&\bar{p}I.\end{aligned} \tag{43}$$

Up to now, the proof of this theorem is complete. ■

Remark 5: Until now, the reputation-dependent UBFEC has been determined by solving two recursive matrix equations, and the filter gain has been parameterized by minimizing the trace of the UBFEC at each step. With the given initial values, the proposed RBDF algorithm can be implemented recursively. Furthermore, the performance of the developed RBDF algorithm is evaluated through a rigorous boundedness analysis. Specifically, a sufficient condition is presented to ensure that the filtering error dynamics are maintained within the known desirable parameters. From the perspective of practical application, this analysis provides a significant foundation for ensuring system stability, reliability and consistent filtering performance under varying conditions, including stochastic disturbances, probabilistic quantization, and reputation mechanism.

Remark 6: The RBDF algorithm is developed recursively for SNTVS with stochastic interferences and signal quantization. Notice that it is extremely difficult to guarantee that the filtering error converges to a fixed value due to the system complexity or the external disturbances such as the stochastic noises/nonliearity, signal quantization and reputation mechanism. Hence, we are devoted to presenting the boundedness analysis to ensure that the filtering error fluctuates within a certain range rather than unlimited growth under the complicated situations considered in this paper.

Remark 7: Compared to the existing results, the distinctive novelties and advantages in this paper can be summarized as: 1) a reputation-based evaluation scheme is introduced to discern and eliminate the neighboring data with low reputation scores by assigning different reputation weights to the data from neighbors, which differs from the distributed filtering problem with uniform weights; 2) the UBFEC is obtained by dealing with stochastic nonlinearity, network-induced probabilistic-type quantization, and reputation mechanism; 3) the filter gain is selected by optimizing the trace of UBFEC at each step for the purpose of improving filtering accuracy; and 4) a sufficient condition is given to ensure that the filtering error dynamics are uniformly bounded.

VI. SIMULATION RESULTS

In this section, a simulation example is adopted to demonstrate the effectiveness, superiority and extensibility of the developed RBDF algorithm over heterogeneous SNs with six nodes. The network topologies are described by

$$\begin{aligned}\mathscr{V} =&\{1,2,3,4,5,6\},\\ \mathscr{E} =&\{(1,2),(1,3),(1,4),(2,1),(2,3),(2,4)\\ &(3,2),(4,1),(5,2),(5,6),(6,2),(6,5)\}.\end{aligned}$$

The system matrices are parameterized by

$$\begin{aligned}A_s &= \begin{bmatrix}-0.351-0.2\sin(s) & 0.429\\ 0.546-\sin(s)\cos(s) & -0.39+0.4\cos(s)\end{bmatrix},\\ B_s &= \begin{bmatrix}-0.24\\ 0.6-0.2\sin(s)\end{bmatrix},\\ C_{1,s} &= \begin{bmatrix}-0.1 & -0.07\end{bmatrix},\ C_{2,s} = \begin{bmatrix}-0.25 & 0.5\end{bmatrix},\\ C_{3,s} &= \begin{bmatrix}0.6 & -0.6\end{bmatrix},\ C_{4,s} = \begin{bmatrix}-0.5 & 0.8\end{bmatrix}.\end{aligned}$$

The stochastic nonlinearity satisfies

$$\begin{aligned}&f(s,x_s,\theta_s)\\ =&\begin{bmatrix}0.1\\ 0.2\end{bmatrix}[0.1\text{sign}(x_{1,s})x_{1,s}\theta_{1,s}+0.2\text{sign}(x_{2,s})x_{2,s}\theta_{2,s}],\end{aligned}$$

where $x_{1,s}$ and $x_{2,s}$ are both the elements of x_s, $\theta_{1,s}$ and $\theta_{2,s}$ denote uncorrelated zero-mean Gaussian white noises with unity variances.

The vectors $\hat{x}_{i,s|s} = [\hat{x}_{i1,s|s}\ \ \hat{x}_{i2,s|s}]^T$ $(i = 1,2,3,4)$ are the estimations of states at instant s. The corresponding initial values are set as $\hat{x}_{1,s|s} = \hat{x}_{2,s|s} = \hat{x}_{3,s|s} = \hat{x}_{4,s|s} = \hat{x}_{5,s|s} = \hat{x}_{6,s|s} = [0.8\ \ 0.8]^T$, $\mathcal{P}_{1,0|0} = 5I_2$, $\mathcal{P}_{2,0|0} = I_2$, $\mathcal{P}_{3,0|0} = 2I_2$, $\mathcal{P}_{4,0|0} = 4I_2$, $\mathcal{P}_{5,0|0} = \mathcal{P}_{6,0|0} = 3I_2$. Other parameters are set as $\Theta_s = 0.2$, $R_{1,s} = R_{2,s} = R_{3,s} = R_{4,s} = 0.5$, $\kappa_{11} = \kappa_{21} = 5\times10^{-5}$, $\kappa_{31} = \kappa_{41} = 1\times10^{-5}$, $o_1 = 0.5$, $o_2 = 0.8$, $o_3 = 0.7$, $o_4 = 0.2$, $\eta_{ij,s} = 0.01$.

The considered heterogeneous SNs are composed of two types of sensors. The type I sensors are capable of measurement as well as calculation and type II sensors without measurement capability function as extending network operation time and maintaining overall network stability. The nodes 1–4 belong to the type I sensors and the filter conforms to the structure of (6)–(7), i.e.,

$$\begin{aligned}\hat{x}_{u_1,s+1|s} =& A_s\hat{x}_{u_1,s|s},\\ \hat{x}_{u_1,s+1|s+1} =& \hat{x}_{u_1,s+1|s} + \Delta_{u_1,s+1}[Q(y_{u_1,s+1}) - C_{u_1,s+1}\\ &\times\hat{x}_{u_1,s+1|s}] + \sum_{z\in\mathscr{N}_{u_1}} a_{u_1z}\varphi_{u_1z,s+1}\\ &\times(\hat{x}_{z,s+1|s} - \hat{x}_{u_1,s+1|s}).\end{aligned}$$

For the type II sensors including nodes 5 and 6, the filter is designed by

$$\begin{aligned}\hat{x}_{u_2,s+1|s} =& A_s\hat{x}_{u_2,s|s},\\ \hat{x}_{u_2,s+1|s+1} =& \hat{x}_{u_2,s+1|s} + \sum_{z\in\mathscr{N}_{u_2}} a_{u_2z}\varphi_{u_2z,s+1}(\hat{x}_{z,s+1|s}\\ &-\hat{x}_{u_2,s+1|s}).\end{aligned}$$

The validity and superiority of the proposed RBDF algorithm are demonstrated by a simulation example over heterogeneous SNs, and the simulation results are depicted in Figs. 1–6. Among them, Figs. 1 and 2 depict the trajectories of the real states and their corresponding estimation trajectories from six filters, demonstrating the close alignment between the two. Figs. 3 and 4 reveal that the mean-square errors (MSE1 and MSE2) consistently remain below their respective upper bounds. Here, MSE1 and MSE2 denote the sum of the mean-square errors of $x_{1,s}$ and $x_{2,s}$, respectively.

It is worth mentioning that the RBDF algorithm developed in this paper is an improvement of the traditional distributed recursive filtering schemes without reputation mechanism (see, e.g. [27]). In order to clarify the advantages relative to the traditional distributed recursive filtering methods, a simulation comparison is presented to show the superiority of reputation mechanism in response to network attacks. As mentioned in [25], the stochastic-type false data injection attack is typically discussed and considered in the design of filtering algorithm, which is modeled here by

$$\tilde{\xi}_{i,s} = (1-\beta_{i,s})\vec{\xi}_{i,s} + \beta_{i,s}\epsilon_{i,s},$$

where $\vec{\xi}_{i,s}$ is the original data sent by neighbors, and $\epsilon_{i,s}$ is a zero-mean attack signal with variance $\tau_s = 50$. $\beta_{i,s}$ is a Bernoulli distributed stochastic variable to characterize the attack frequency. Specifically, $\beta_{i,s} = 1$ means that the attack occurs and otherwise no attack occurs. Here, the expectation of $\beta_{i,s}$ is set as 0.95. Now, under a scenario where $\hat{x}_{4,s+1|s}$ is subject to network attacks, the comparative results are provided to validate the effectiveness of the proposed reputation mechanism. For *Case I*, the reputation-based filter described in (7) is employed, while for *Case II*, adopt the filter in traditional distributed recursive filtering method, where $\varphi_{iz,s}$ is fixed at 0.2. Figs. 5 and 6 demonstrate that the reputation mechanism significantly improves estimation accuracy.

Furthermore, it can be seen from (8)–(10) that the local prediction $\hat{x}_{z,s+1|s}$ contaminated by malicious attacks can be identified and removed effectively when $\varphi_{iz,s} = 0$. Otherwise, different reputation parameter will be allocated based on the calculation principle in (8). The successful identification and removal situations under different variances of attack signal are shown in Table II, where the first (second) element of vector represents the number of $\varphi_{14,s} = 0$ ($\varphi_{24,s} = 0$) and the corresponding probability.

TABLE II
THE SUCCESSFUL IDENTIFICATION SITUATIONS UNDER DIFFERENT VARIANCES OF ATTACKS.

τ_s	The number of $\varphi_{14,s} = 0$ and $\varphi_{24,s} = 0$	Probability
0.5	[57 44]	[0.63 0.49]
1	[62 47]	[0.69 0.52]
4	[64 48]	[0.71 0.53]
16	[66 49]	[0.73 0.54]

VII. CONCLUSION

In this paper, we have proposed an RBDF algorithm for stochastic nonlinear time-varying systems over sensor networks subject to probabilistic quantization and a reputation

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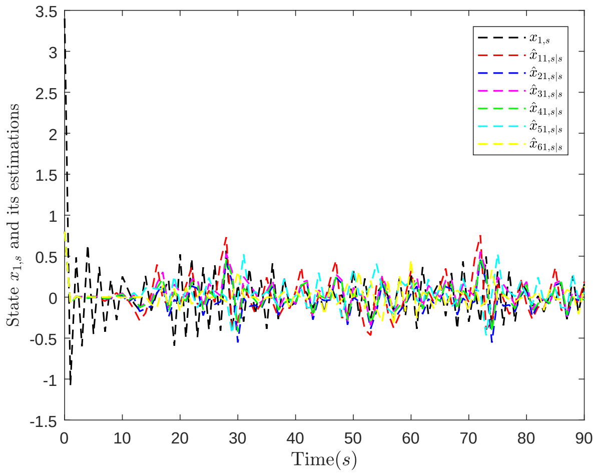

Fig. 1. State $x_{1,s}$ and its estimations.

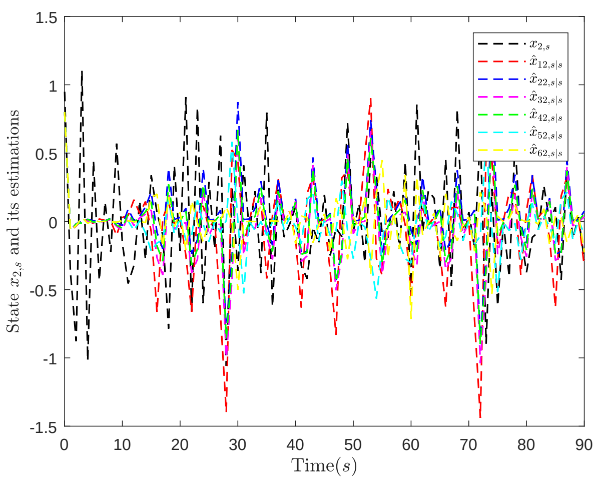

Fig. 2. State $x_{2,s}$ and its estimations.

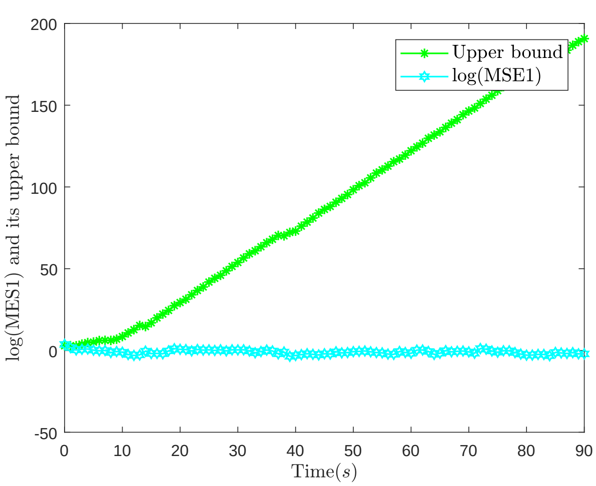

Fig. 3. Log(MSE1) and its upper bound.

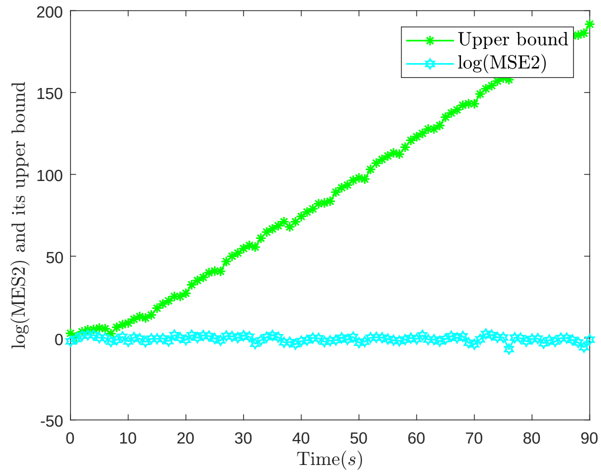

Fig. 4. Log(MSE2) and its upper bound.

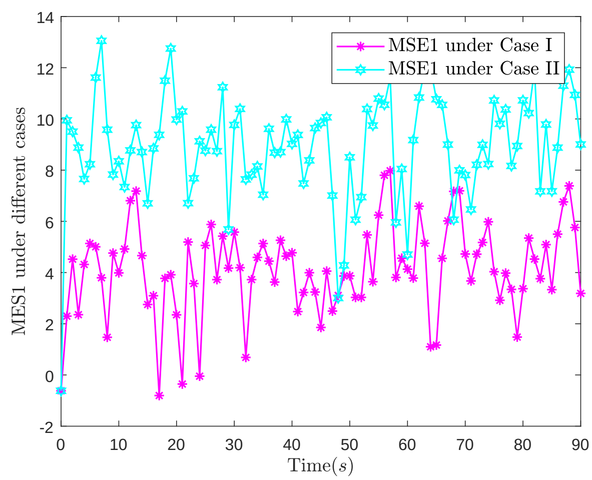

Fig. 5. MSE1 under different cases.

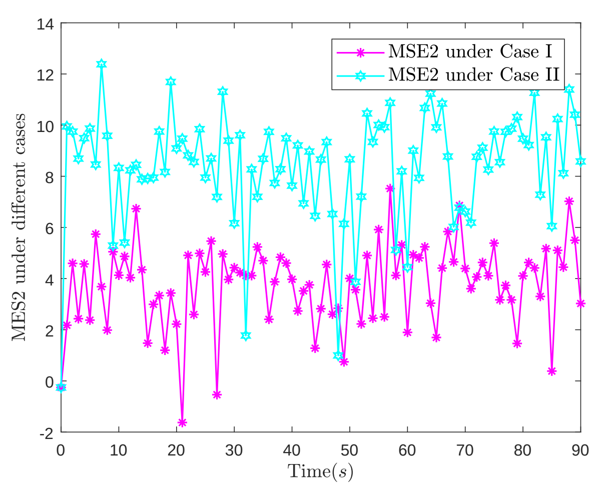

Fig. 6. MSE2 under different cases.

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mechanism. First, a uniform-type probabilistic quantization strategy has been employed to preprocess the data before transmission, thereby effectively addressing communication constraints. Subsequently, a reputation mechanism has been introduced to identify and mitigate abnormal data, which may arise due to sensor faults or cyber attacks. The RBDF problem has been tackled under the simultaneous presence of stochastic nonlinearity, probabilistic quantization, and the reputation mechanism. A reputation-dependent UBFEC has been derived recursively by solving two matrix equations. The filter gain has then been optimally parameterized to minimize the trace of the UBFEC, ensuring improved estimation performance in the mean-square error sense. Furthermore, sufficient conditions guaranteeing the uniform boundedness of the UBFEC have been established under mild assumptions. Finally, an illustrative example has demonstrated the effectiveness and superiority of the proposed RBDF algorithm, particularly in handling unreliable and attack-contaminated data. The future extended directions could include the designs of ensemble or unscented Kalman-type filtering algorithm subject to reputation mechanism and communication protocol.

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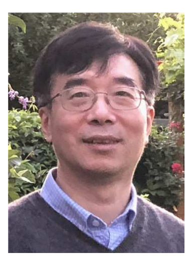

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