

Neural-Network-Based Recursive State Estimation for Nonlinear Networked Systems With Binary-Encoding Mechanisms

Yuhan Zhang, Zidong Wang, Lei Zou, Wei Qian, and Shuxin Du

Abstract—This work addresses the problem of recursive state estimation for the networked control systems with unknown nonlinearities and binary-encoding mechanisms (BEMs). To enhance transmission reliability and reduce network resource consumption, BEMs are used to convert measurement signals into binary bit strings (BBSs) of limited length, which are then transmitted to the estimator through noisy communication channels. During transmission, random bit errors may occur in the BBSs due to channel noise. For the considered nonlinear networked control systems affected by random bit errors, a neural-network-based recursive estimation strategy is proposed, where a neural network with a time-varying tuning scalar is employed to approximate the unknown nonlinearity of the networked control systems. By using the proposed strategy, the upper bounds of the estimation error of the system state and the trace of the estimation error of the neural network weight (NNW) are first derived. These bounds are then minimized by recursively designing both the estimator gain matrix and the tuning scalar of the NNW. Finally, the effectiveness of the proposed estimation strategy is demonstrated through a numerical example.

Index Terms—Networked nonlinear systems, neural network-
s, unknown nonlinearities, recursive state estimation, binary-
encoding mechanism.

Abbreviations and Notations

NCSs	Networked control systems
BBS	Binary bit string
BEM	Binary-encoding mechanism
MBSCs	Memoryless binary symmetric channels
NN	Neural network

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Y. Zhang is with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China. (Email: zhangyh_sdust@163.com).

Z. Wang is with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China, and is also with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom (Email: Zidong.Wang@brunel.ac.uk).

L. Zou is with the College of Information Science and Technology, Donghua University, Shanghai 201620, China, and is also with the Engineering Research Center of Digitalized Textile and Fashion Technology, Ministry of Education, Shanghai 201620, China. (Email: zouleicup@gmail.com).

W. Qian is with the School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, China. (Email: qwei@hpu.edu.cn).

S. Du is with the Huzhou Key Laboratory of Intelligent Sensing and Optimal Control for Industrial Systems, School of Engineering, Huzhou University, Huzhou 313000, China. (Email: shxdu@zjhu.edu.cn).

NNW	Neural network weight
$\mathbb{R}^z$	The Euclidean space of dimension z
$\mathbb{R}^{s \times z}$	The set of real matrices of dimension $s \times z$
$G \geq H$	Matrix $G - H$ is positive semi-definite
$G > H$	Matrix $G - H$ is positive definite
M^T	The transposition of a matrix M
$\ M\ $	The Frobenius norm of the matrix M
$\text{tr}\{M\}$	The trace of the a matrix M
$\mathbb{P}\{\cdot\}$	The occurrence probability of the random event “.”
$\mathbb{V}\{c\}$	The variance of random variable c
$\mathbb{E}\{d\}$	The mathematical expectation of random variable d
$\mathbb{E}\{c d\}$	The mathematical expectation of c conditional on d
I	Identity matrix of appropriate dimension

I. INTRODUCTION

Networked control systems (NCSs) have been the subject of extensive research over the past decades [28]. Different from traditional point-to-point communication mechanisms, this network-based communication technology enables data exchange among system components via a shared communication network [3], [23], [33], [47]. The use of network-based communication technology can reduce hardwiring, simplify installation, and lower implementation costs. These advantages have led to its widespread application in various fields such as smart homes, unmanned vehicles, and regional exploration. In engineering practice, when performing tasks such as remote control, fault diagnosis, or consensus, obtaining accurate state information is a critical first step [43]. Unfortunately, the state information of an NCS is typically not directly available, and only the system’s measurements can be accessed, which motivates research into state estimation issues for NCSs. To date, a wide range of state estimation approaches has been developed to generate state information for NCSs based on the available measurements, with popular methods including the H_∞ estimation technique, the set-membership estimation method, and the minimum-variance state estimation algorithm [22], [39], [44].

Unlike traditional point-to-point systems, the use of network-based communication technology in NCSs inevitably introduces network-induced phenomena (e.g., packet losses, time delays, and bit-rate constraints), which can result in

performance degradation or even system instability, thereby posing significant challenges to state estimation tasks for NCSs [5], [12], [14], [29], [45], [49]. Therefore, the corresponding state estimation approaches must be capable of mitigating the negative impacts caused by these network-induced phenomena [20], [31], [37]. Many researchers have thus focused on addressing the estimation issues for NCSs affected by such phenomena, leading to the publication of numerous significant research results. Some of the notable works in this field can be found in [2], [19], [36].

Among the various network-induced phenomena, binary encoding mechanisms (BEMs) have gained prominence in engineering practice, primarily due to their advantages in reducing network resource consumption and the efficiency of binary bit strings (BBSs) in supporting encryption [10], [56]. The implementation of BEMs primarily relies on a set of encoder-decoder pairs [18]. With the help of encoders, the original signals are first quantized and then transformed into BBSs of finite length before being transmitted. These BBSs are sent through memoryless binary symmetric channels (MBSCs), after which decoders are used to reconstruct the original signals based on the received BBSs. Finally, estimators generate state estimates using the reconstructed signals. It should be noted that channel noise within MBSCs inevitably leads to random bit errors (i.e., each binary bit may flip from 0 to 1 or 1 to 0 with a small probability), meaning the reconstructed signals will differ from the original ones. These discrepancies can degrade the performance of the estimator. To ensure the performance of the estimator, the estimation approaches must be able to mitigate the negative effects of these bit errors [24], [25].

The nonlinear estimation issues for NCSs have attracted significant attention in the system science and control communities due to their widespread presence in practical applications [11], [35], [50]. A variety of effective approaches have been developed to address the challenges posed by inherent nonlinearities. These methods can generally be classified into three categories: sector-bounded-condition (SBC)-based approaches [26], Taylor-expansion (TE)-based techniques [30], [48], and neural-network (NN)-based methods [32]. In SBC-based methods, sector-bounded-like conditions are used to analyze the effects of nonlinearities, while TE-based techniques employ Taylor polynomials to handle them. However, the implementation of these two types of approaches depends on prior knowledge of the nonlinearities. In other words, these approaches implicitly assume that the nonlinear dynamics are already known. In practice, obtaining full information about the nonlinearities is often difficult due to harsh application environments, low engineering budgets, or the complexity of the system itself. NNs can approximate the unknown nonlinear dynamics with arbitrary precision when the unknown nonlinear functions are continuous [4]. Consequently, the NN-based approximation method has become the most popular nonlinear estimation approach, primarily because of its excellent capability to approximate unknown nonlinearities and adaptively update neural network weights (NNWs) [1], [41]. As a result, attention has now been drawn to utilize the NN-based approximation technique to solve the estimation/control

problems of NCSs with unknown nonlinearities and numerous research outcomes have been documented [7], [46].

Despite the theoretical and practical significance of NN-based state estimation, there has been limited investigation into the use of NNs for addressing recursive state estimation issues in NCSs, particularly with the simultaneous consideration of BEMs and random bit errors. Therefore, the main objective of this paper is to bridge this gap. In light of the considerations mentioned earlier, this paper aims to solve the recursive state estimation problem for NCSs with unknown nonlinearities and BEMs. The following potential challenges are identified: 1) how to quantify the effects of bit errors on estimation performance? 2) how to develop an appropriate law to update the NN weights (NNWs) for unknown nonlinearities in NCSs? and 3) how to propose an easy-to-implement NN-based recursive estimation algorithm for NCSs with unknown nonlinearities and BEM, ensuring the minimization of the upper bound of the estimation error covariance for both system states and NNWs?

The primary contributions of this paper are as follows.

- 1) The recursive state estimation problem is, for the first time, addressed in the context of NCSs with unknown nonlinearities and BEM.
- 2) An NN approximation approach and a covariance-based NNW updating strategy are employed to address the unknown nonlinearities present in nonlinear NCSs.
- 3) The effects of bit errors in BEM on estimation performance are quantitatively analyzed.
- 4) An NN-based recursive estimation algorithm is developed for recursively calculating the estimator gains and NNW tuning scalars within a unified framework.

The remainder of this paper is structured as follows. Section II formulates the estimation problem, introducing the nonlinear systems model, the characterization of the communication network, and the developed NN-based recursive state estimator. Section III presents four theorems that ensure the ultimate boundedness of the estimation errors for both the system state and the NNW. In Section IV, a numerical example is provided to illustrate the effectiveness of the proposed estimation strategy. Finally, conclusions are drawn in Section V.

II. PROBLEM FORMULATION

A. Nonlinear NCS Model

Consider a type of nonlinear plant modeled by the following NCS:

$$\begin{cases} x_{k+1} = Ax_k + g(x_k) + B\omega_k \\ y_k = Cx_k + Dv_k \end{cases} \quad (1)$$

where $x_k \in \mathbb{R}^n$ represents the system state and $y_k \in \mathbb{R}^z$ is the measurement signal before transmission. $g(\cdot)$ is an unknown but bounded smooth nonlinear function on a compact set $\Omega \in \mathbb{R}^n$, which satisfies $\|g(\cdot)\| \leq \bar{g}$ where $\bar{g}$ is a known constant. The matrices A , B , C , and D are known system parameters with appropriate dimensions. The disturbance noises $\omega_k \in \mathbb{R}^b$ and $v_k \in \mathbb{R}^q$ have zero mean and covariances Q_k and S_k , respectively. In addition, it is assumed that ω_k and v_k are mutually uncorrelated, with $\|\omega_k\| \leq \bar{\omega}$ and $\|v_k\| \leq \bar{v}$, where

$\bar{\omega}$ and $\bar{v}$ are constants that are uncorrelated with the initial state. The mean $\bar{x}_0$, $\bar{M}_0$, and covariance Υ_0 of the initial state, as well as Ξ_0 of the initial NNW, are given.

Remark 1: As an extensively existed non-Gaussian noise, norm-bounded disturbance has attracted a great deal of research attention. Compared with the widely studied Gaussian noises, most disturbances in practical applications are non-Gaussian. To capture the boundedness nature of disturbance signals, in this paper, we assume that both the process noise and measurement noise are norm-bounded.

B. Communication Network

As shown in Fig. 1, communication between the sensors and the recursive estimator takes place via a network under the BEM, which is widely used in practical applications due to the significant advantages that BBSs offer, including enhanced transmission robustness and ease of implementation. Under the BEM, a set of encoders is first employed to convert the original measurements into BBSs, which are then transmitted through memoryless Binary Symmetric Channels (MBSCs). Based on the received BBSs, the original measurements are reconstructed via a decoding scheme. Subsequently, a recovery scheme is implemented to compensate for the effects of channel noise present in the memoryless MBSCs. Finally, the recursive estimator receives the recovered measurements for further processing. The detailed workflow of BEM is introduced as follows.

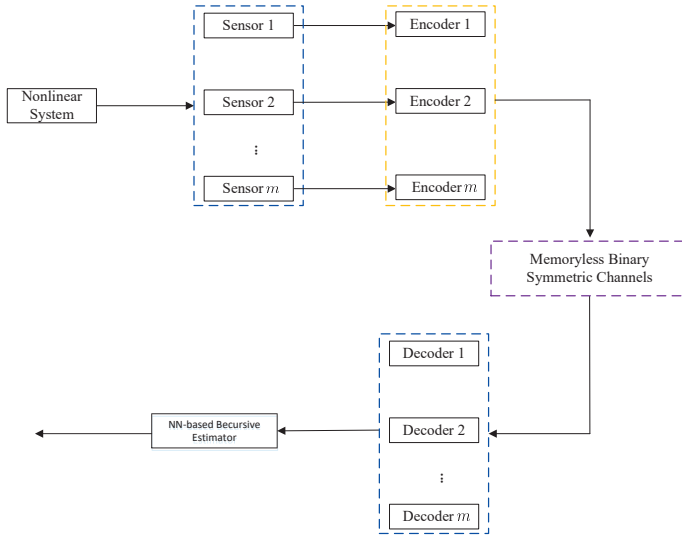


Fig. 1: Nonlinear NCS with BEM.

Step 1. Encoding

During the encoding process, a group of probabilistic quantizer are first adopted to pre-treat the original measurements, and then encoding functions are employed to transform the quantized measurements into certain BBSs [25].

The original measurement y_k can be represented by

$$y_k \triangleq [y_{1,k} \ y_{2,k} \ \cdots \ y_{z,k}]^T$$

where $y_{m,k}$ ($m \in \{1, 2, \dots, z\}$) is the m th scalar element of y_k . In this paper, we assume that $-\mu \leq y_{m,k} \leq \mu$, where

$\mu > 0$ is an application-dependent scalar. In the encoding process, the measurement signal $y_{m,k}$ would be quantized first, and subsequently, a group of encoders are utilized to transform the quantized signal into a BBS with length H . Given the interval $[-\mu, \mu]$, we define the set of 2^H encoding levels as follows:

$$\mathcal{R} \triangleq \{\varepsilon^{(1)}, \varepsilon^{(2)}, \dots, \varepsilon^{(2^H)}\}$$

where $\varepsilon^{(i)} \triangleq -\mu + (i-1)\epsilon$ represents the i th encoding level in $\mathcal{R}$, and $\epsilon \triangleq 2\mu/(2^H - 1)$ is the encoding interval. Obviously, we can see $\epsilon = \varepsilon^{(i+1)} - \varepsilon^{(i)}$.

During the encoding process, a quantization function is firstly utilized to pretreat $y_{m,k}$. Define $\mathcal{Q}_m(\cdot)$ as the m th quantization function with the following form

$$\begin{cases} \mathbb{P}\{\mathcal{Q}_m(y_{m,k}) = \varepsilon^{(i)}\} = 1 - \imath_{m,k} \\ \mathbb{P}\{\mathcal{Q}_m(y_{m,k}) = \varepsilon^{(i+1)}\} = \imath_{m,k} \end{cases} \quad (2)$$

where $\imath_{m,k}$ is defined by $\imath_{m,k} \triangleq (\bar{y}_{m,k} - \varepsilon^{(i)})/\epsilon$ with $0 \leq \imath_{m,k} < 1$. Then, the quantized signal for $y_{m,k}$ is described by

$$\hat{y}_{m,k} = \mathcal{Q}_m(y_{m,k}).$$

Given the quantized measurement $\hat{y}_{m,k}$, an encoding function is subsequently utilized to generate the BBS. Letting the m th encoding function be $\mathcal{E}_m(\cdot)$ and $\mathcal{B}_{m,k}$ be the generated codeword set of $\hat{y}_{m,k}$, we have

$$\mathcal{B}_{m,k} \triangleq \mathcal{E}_m(\hat{y}_{m,k}) \triangleq \{\Psi_{1,m,k}, \Psi_{2,m,k}, \dots, \Psi_{H,m,k}\} \quad (3)$$

where $\Psi_{s,m,k} \in \{0, 1\}$ ($s \in \{1, 2, \dots, H\}$) is the s th element of the codeword set $\mathcal{B}_{m,k}$. According to the encoding mechanism, the following condition holds for any quantized signal $\hat{y}_{m,k}$:

$$\hat{y}_{m,k} = -\mu + \sum_{s=1}^H \Psi_{s,m,k} 2^{s-1} \epsilon.$$

Step 2. Transmission executed via the MBSC

After the encoding process, the BBSs $\mathcal{B}_{m,k}$ is transmitted to the decoder via the MBSC [25]. Owing to the existence of the channel noise, every bit might flip with a small crossover probability. Denote the received BBS as $\hat{\mathcal{B}}_{m,k}$ and $\hat{\Psi}_{s,m,k}$ as the s th element of $\hat{\mathcal{B}}_{m,k}$, i.e.,

$$\hat{\mathcal{B}}_{m,k} \triangleq \{\hat{\Psi}_{1,m,k}, \hat{\Psi}_{2,m,k}, \dots, \hat{\Psi}_{H,m,k}\} \quad (4)$$

where

$$\begin{aligned} \hat{\Psi}_{s,m,k} &\triangleq \gamma_{s,m,k}(1 - \Psi_{s,m,k}) \\ &\quad + (1 - \gamma_{s,m,k})\Psi_{s,m,k}. \end{aligned} \quad (5)$$

Here, $\gamma_{s,m,k}$ is a binary variable defined as

$$\gamma_{s,m,k} \triangleq \begin{cases} 1, & \text{if the } s\text{th bit is flipped} \\ 0, & \text{if the } s\text{th bit is not flipped} \end{cases} \quad (6)$$

In this paper, we assume that $\gamma_{s,m,k}$ ($s \in \{1, 2, \dots, H\}$) are mutually independent and identically distributed, where $\mathbb{P}\{\gamma_{s,m,k} = 1\} = p$ and p is a known scalar [17].

Step 3. Decoding and recovery

Based on the received BBSs $\{\hat{\mathcal{B}}_{m,k}\}_{m=1,2,\dots,z}$, a group of decoding functions (which are defined as $\{\mathcal{D}_m(\cdot)\}_{m=1,2,\dots,z}$) are employed to reconstruct the measurement signals (which are defined as $\{\hat{y}_{m,k}\}_{m=1,2,\dots,z}$):

$$\hat{y}_{m,k} = \mathcal{D}_m(\hat{\mathcal{B}}_{m,k}) \triangleq -\mu + \sum_{s=1}^H \hat{\Psi}_{s,m,k} 2^{s-1} \epsilon. \quad (7)$$

Due to the existence of bit flips, it is easy to observe that $\hat{y}_{m,k} \neq y_{m,k}$. The statistical properties of $\hat{y}_{m,k}$ can be described in the following Lemma.

Lemma 1: [25] The mean and variance of the received signal $\hat{y}_{m,k}$ are given by

$$\mathbb{E}\{\hat{y}_{m,k}\} = (1 - 2p)y_{m,k} \quad (8)$$

and

$$\mathbb{V}\{\hat{y}_{m,k}\} = \frac{4\mu^2(p - p^2)(4^H - 1)}{3(2^H - 1)^2} \quad (9)$$

Proof: Taking the mathematical expectation of $\hat{y}_{m,k}$ over the random variables $\gamma_{s,m,k}$, we have

$$\begin{aligned} \mathbb{E}\{\hat{y}_{m,k}\} &= \mathbb{E}\{-\mu + \sum_{s=1}^H \hat{\Psi}_{s,m,k} 2^{s-1} \epsilon\} \\ &= -\mu + \sum_{s=1}^H (p(1 - \Psi_{s,m,k}) \\ &\quad + (1 - p)\Psi_{s,m,k}) 2^{s-1} \epsilon \\ &= y_{m,k} + p \sum_{s=1}^H (1 - 2\Psi_{s,m,k}) 2^{s-1} \epsilon \end{aligned}$$

Considering $\hat{y}_{m,k} = -\mu + \sum_{s=1}^H \Psi_{s,m,k} 2^{s-1} \epsilon$, we obtain

$$\begin{aligned} &p \sum_{s=1}^H (1 - 2\Psi_{s,m,k}) 2^{s-1} \epsilon \\ &= p \sum_{s=1}^H 2^{s-1} \epsilon - 2p \sum_{s=1}^H \Psi_{s,m,k} 2^{s-1} \epsilon \\ &= 2p(\mu - \sum_{s=1}^H \Psi_{s,m,k} 2^{s-1} \epsilon) \\ &= -2p\hat{y}_{m,k} \end{aligned}$$

Moreover, the variance of $\hat{y}_{m,k}$ is calculated by:

$$\begin{aligned} \mathbb{V}\{\hat{y}_{m,k}\} &= \mathbb{E}\{(-\mu + \sum_{s=1}^H \hat{\Psi}_{s,m,k} 2^{s-1} \epsilon)^2\} - \mathbb{E}\{\hat{y}_{m,k}\}^2 \\ &= \mathbb{E}\{(\mathbb{E}\{\hat{y}_{m,k}\} + \sum_{s=1}^H (\hat{\Psi}_{s,m,k} \\ &\quad - \mathbb{E}\{\hat{\Psi}_{s,m,k}\}) 2^{s-1} \epsilon)^2\} - \mathbb{E}\{\hat{y}_{m,k}\}^2 \\ &= \mathbb{E}\{(\sum_{s=1}^H (\hat{\Psi}_{s,m,k} - \mathbb{E}\{\hat{\Psi}_{s,m,k}\}) 2^{s-1} \epsilon)^2\} \end{aligned}$$

According to (6), one has

$$\begin{aligned} &\mathbb{E}\{(\sum_{s=1}^H (\hat{\Psi}_{s,m,k} - \mathbb{E}\{\hat{\Psi}_{s,m,k}\}) 2^{s-1} \epsilon)^2\} \\ &= \sum_{s=1}^H (\mathbb{E}\{\hat{\Psi}_{s,m,k}^2\} - \mathbb{E}\{\hat{\Psi}_{s,m,k}\}^2) 2^{2s-2} \epsilon^2 \\ &= \sum_{s=1}^H (p(1 - \Psi_{s,m,k})^2 + (1 - p)\Psi_{s,m,k}^2 \\ &\quad - p^2(1 - \Psi_{s,m,k})^2 - (1 - p)^2\Psi_{s,m,k}^2 \\ &\quad - 2p(1 - p)(1 - \Psi_{s,m,k})\Psi_{s,m,k}) 2^{2s-2} \epsilon^2 \\ &= \frac{4\mu^2(p - p^2)(4^H - 1)}{3(2^H - 1)^2}, \end{aligned}$$

and the proof is now complete. $\blacksquare$

According to the results in Lemma 1, we adopt the following function to generate the recovered measurements (denoted as $\{\tilde{y}_{m,k}\}_{m=1,2,\dots,z}$):

$$\tilde{y}_{m,k} = \mathcal{Z}(\hat{y}_{m,k}) = \Gamma^{-1} \hat{y}_{m,k}$$

where $\Gamma \triangleq 1 - 2p$. In this way, the mean of the recovered measurement is equal to the original measurement, i.e., $\mathbb{E}\{\tilde{y}_{m,k}\} = y_{m,k}$.

Define $\aleph_{m,k}$ as the quantization error of the measurement signal $\hat{y}_{m,k}$ (i.e., $\aleph_{m,k} \triangleq \hat{y}_{m,k} - y_{m,k}$) and denote $\iota_{m,k}$ as the difference between the recovered measurement $\tilde{y}_{m,k}$ and the quantized measurement $\hat{y}_{m,k}$ (i.e., $\iota_{m,k} \triangleq \tilde{y}_{m,k} - \hat{y}_{m,k}$). Then, we have

$$\tilde{y}_{m,k} = \aleph_{m,k} + \iota_{m,k} + y_{m,k}. \quad (10)$$

Moreover, by denoting

$$\begin{aligned} \aleph_k &\triangleq [\aleph_{1,k}^T \ \aleph_{2,k}^T \ \cdots \ \aleph_{z,k}^T]^T, \\ \iota_k &\triangleq [\iota_{1,k}^T \ \iota_{2,k}^T \ \cdots \ \iota_{z,k}^T]^T, \\ \hat{y}_k &\triangleq [\hat{y}_{1,k}^T \ \hat{y}_{2,k}^T \ \cdots \ \hat{y}_{z,k}^T]^T, \\ \tilde{y}_k &\triangleq [\tilde{y}_{1,k}^T \ \tilde{y}_{2,k}^T \ \cdots \ \tilde{y}_{z,k}^T]^T, \end{aligned}$$

one has

$$\tilde{y}_k = \aleph_k + \iota_k + y_k. \quad (11)$$

Based on the above discussion, it is obvious that under the BEM, the recovered measurement $\tilde{y}_k$ inevitably experiences a certain degree of distortion compared to the original measurement y_k . In the following Lemmas, the characteristics of these distortions will be quantitatively analyzed in detail.

Lemma 2: [25] The quantization error $\aleph_k$ is of zero mean and bounded variance, i.e.,

$$\mathbb{E}\{\aleph_k\} = 0$$

and

$$\mathbb{E}\{\aleph_k^T \aleph_k\} \leq \bar{\aleph}$$

where $\bar{\aleph} \triangleq z\epsilon^2/4$.

Proof: Considering the quantization function (2), the encoding function (3) and the encoding function (7), we can readily infer that

$$\begin{aligned}\mathbb{P}\{\aleph_{m,k} = \varepsilon_i \epsilon - y_{m,k}\} \\ = \mathbb{P}\{-\iota_{m,k} \epsilon\} = 1 - \iota_{m,k}\end{aligned}$$

and

$$\begin{aligned}\mathbb{P}\{\aleph_{m,k} = \varepsilon_{i+1} \epsilon - y_{m,k}\} \\ = \mathbb{P}\{(1 - \iota_{m,k}) \epsilon\} = \iota_{m,k}.\end{aligned}$$

Therefore, we have

$$\begin{aligned}\mathbb{E}\{\aleph_{m,k}\} = -\iota_{m,k} \epsilon (1 - \iota_{m,k}) \\ + (1 - \iota_{m,k}) \epsilon \iota_{m,k}\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}\{\aleph_{m,k}^2\} = (-\iota_{m,k} \epsilon)^2 (1 - \iota_{m,k}) \\ + (1 - \iota_{m,k})^2 \epsilon^2 \iota_{m,k} \\ = \epsilon^2 \iota_{m,k} (1 - \iota_{m,k}),\end{aligned}$$

which indicate that the quantization error $\aleph_{m,k}$ has zero mean and bounded variance, i.e.,

$$\mathbb{E}\{\aleph_{m,k}\} = 0, \quad \mathbb{E}\{\aleph_{m,k}^2\} \leq \epsilon^2/4.$$

Therefore, we conclude that

$$\mathbb{E}\{\aleph_k\} = 0, \quad \mathbb{E}\{\aleph_k^T \aleph_k\} \leq \bar{\aleph}.$$

Lemma 3: [25] The equivalent noise coming from the bit errors in the MBSC satisfies

$$\mathbb{E}\{\iota_k\} = 0 \quad (12)$$

and

$$\mathbb{V}\{\iota_k\} \leq \bar{\iota}^2 \quad (13)$$

where

$$\bar{\iota}^2 \triangleq z \bar{\iota}_m^2, \quad \bar{\iota}_m^2 \triangleq \frac{4\mu^2 p(1-p)(4^H - 1)}{3(2^H - 1)^2(1 - 2p)^2}.$$

Proof: Noting

$$\mathbb{E}\{\check{y}_{m,k}\} = \dot{y}_{m,k},$$

we have

$$\mathbb{E}\{\iota_{m,k}\} = \mathbb{E}\{\check{y}_{m,k} - \dot{y}_{m,k}\} = 0.$$

According to (8), it is readily obtained that

$$\mathbb{V}\{\iota_{m,k}\} = \frac{\mathbb{V}\{\check{y}_{m,k}\}}{(1 - 2p)^2} = \bar{\iota}_m^2$$

Denoting

$$\begin{aligned}\iota_k &\triangleq \check{y}_k - \dot{y}_k, \\ \iota_k &\triangleq [\iota_{1,k}^T \quad \iota_{2,k}^T \quad \cdots \quad \iota_{z,k}^T]^T,\end{aligned}$$

we obtain

$$\mathbb{E}\{\iota_k\} = 0, \quad \mathbb{E}\{\iota_k^T \iota_k\} \leq \bar{\iota}^2.$$

Remark 2: In this paper, the communication from the sensors to the recursive estimator is affected by the influence of the BEM. Specifically, the stochastic quantization errors, resulting from probabilistic quantization, have zero expectation and bounded variance. Stochastic bit errors, which occur due to the presence of channel noise, also exhibit zero expectation and bounded variance. Both the quantization errors $\aleph_k$ and the bit errors ι_k contribute to discrepancies between the reconstructed measurements and the original measurements, leading to reduced transmission reliability and, consequently, degrading the overall estimation performance.

C. NN-Based Recursive State Estimator

Before carrying out the design process of the NN-based recursive state estimator, the following lemma is introduced to address the unknown nonlinearities in NCSs.

Lemma 4: [16] NNs are employed to approximate the unknown nonlinearities using the form

$$M\vartheta(x_k) + \eta_k$$

where $\vartheta(\cdot) \in \mathbb{R}^n$, $M \in \mathbb{R}^{n \times n}$ and $\eta_k \in \mathbb{R}^n$ denote the activation function, the ideal weight matrix and the approximation error of the NN, respectively.

Consequently, system (1) can be rewritten as:

$$\begin{cases} x_{k+1} = Ax_k + M\vartheta(x_k) + B\omega_k + \eta_k \\ y_k = Cx_k + Dv_k \end{cases} \quad (14)$$

Assumption 1: The connecting NNW matrix M , the NN activation function $\vartheta(\cdot)$, and the NN approximate error η_k satisfy the following condition

$$\|M\| \leq \bar{M}, \quad \|\vartheta(\cdot)\| \leq \bar{\vartheta}, \quad \|\eta(\cdot)\| \leq \bar{\eta}$$

where $\bar{M}$, $\bar{\vartheta}$, and $\bar{\eta}$ are known positive constants.

Remark 3: The unknown nonlinearities considered in this paper are bounded. As proved in [4], NNs can approximate unknown nonlinearities with arbitrary precision when the unknown nonlinear functions are continuous. Therefore, it is reasonable to assume that the activation function, the connecting matrix, and the approximation error of the NN are bounded.

Let $\hat{x}_k$ be the estimate of x_k and $\hat{M}_k$ be the estimates of M at time instant k . Define L_k as the estimator gain to be designed. We adopt the following NN-based recursive estimator to generate the state estimates for the NCSs with unknown nonlinearities: (1):

$$\begin{cases} \hat{x}_{k+1} = A\hat{x}_k + \hat{M}_k\vartheta(\hat{x}_k) + L_k(\check{y}_k - C\hat{x}_k) \\ \hat{M}_k = \hat{M}_{k-1} + \theta_k C^T (\check{y}_k - C\hat{x}_k) \tilde{\vartheta}^T(\hat{x}_{k-1}) \end{cases} \quad (15)$$

where θ_k is tuning parameter to be designed and $\tilde{\vartheta}(\hat{x}_k) \triangleq \vartheta(\hat{x}_k) / (1 + \vartheta^T(\hat{x}_k) \vartheta(\hat{x}_k) \|C^T C\|)$.

It can be found from (15) that M needs to be estimated during the training process, and the tuning law of $\hat{M}_k$ is obtained with the help of the gradient descent method.

Remark 4: Unlike existing state estimation methods, the developed NN-based recursive estimation strategy offers significant advantages in improving adaptability from the perspective of engineering practice. This is mainly due to two

reasons: 1) the proposed approach combines the NN method with the recursive state estimation approach, which allows for both the estimator gain and the NNW tuning parameter to be adjusted recursively within a unified framework; and 2) the NN-based recursive state estimation strategy provides greater design flexibility. Specifically, the NN tuning parameter θ_k which recursively updated directly affect the updating of NNWs, which have significant influence on the approximation performance of the NNs. Notably, this strategy is not only applicable to systems with unknown nonlinearities, but also to systems with known nonlinearities and linear systems.

Next, defining $\tilde{M}_k \triangleq M - \hat{M}_k$ as the estimation error of the NNW, we have

$$\begin{aligned} \tilde{M}_{k+1} = & \tilde{M}_k + \theta_{k+1} C^T C (L_k \iota_k - (A - L_k C) \tilde{x}_k - \tilde{M}_k \vartheta(\hat{x}_k) \\ & - \mathfrak{S}_k - B\omega_k + L_k \mathfrak{N}_k + L_k Dv_k) \tilde{\vartheta}^T(\hat{x}_k) \\ & - \theta_{k+1} C^T (Dv_{k+1} + \iota_{k+1} + \mathfrak{N}_{k+1}) \tilde{\vartheta}^T(\hat{x}_k) \end{aligned} \quad (16)$$

where $\mathfrak{S}_k \triangleq M(\vartheta(x_k) - \vartheta(\hat{x}_k)) + \eta_k$.

Denote $\tilde{x}_k \triangleq x_k - \hat{x}_k$ as the state estimation error whose dynamics is governed by

$$\begin{aligned} \tilde{x}_{k+1} = & (A - L_k C) \tilde{x}_k + \tilde{M}_k \vartheta(\hat{x}_k) + B\omega_k \\ & - L_k (Dv_k + \mathfrak{N}_k + \iota_k) + \mathfrak{S}_k \end{aligned} \quad (17)$$

Now, we are in a position to highlight the aims of this work:

- 1) investigate the influences of the quantization errors and bit errors in a quantitatively manner; and
- 2) develop an effective algorithm to design the estimator gain L_k and the NNW tuning parameter θ_k within a unified framework such that the upper bounds for the covariance of the state estimation error (i.e., $\Upsilon_k \triangleq \mathbb{E}\{\tilde{x}_k \tilde{x}_k^T\}$) and the trace of the NNW estimation error (i.e., $\Xi_k \triangleq \text{tr}\{\mathbb{E}\{\tilde{M}_k \tilde{M}_k^T\}\}$) are locally minimized at each time instant.

III. MAIN RESULTS

To generate the estimates of the system state and NNW, an NN-based recursive estimator is proposed. In this section, we will first introduce the design process for the tuning parameter of the NNW. Then, the calculation method for the NN-based recursive estimator gain will be presented.

A. Design of the NNW Tuning Scalar

Before further proceeding, the following lemma are introduced and will be utilized in the following design process.

Lemma 5: [21] For any real-valued matrices $\mathcal{H}_1$ and $\mathcal{H}_2$ with any scalar $d > 0$, the following inequality holds:

$$\mathcal{H}_1 \mathcal{H}_2^T + \mathcal{H}_2 \mathcal{H}_1^T \leq d \mathcal{H}_1 \mathcal{H}_1^T + d^{-1} \mathcal{H}_2 \mathcal{H}_2^T. \quad (18)$$

Theorem 1: Let the positive scalars $\alpha_1, \alpha_2, \alpha_3$ and α_4 be given. Assume that there exists a set of real-valued matrices $\bar{\Xi}_k$ (with the initial condition $\bar{\Xi}_0 = \Xi_0$) satisfying the following recursive equation:

$$\begin{aligned} \bar{\Xi}_{k+1} = & (1 + \theta_{k+1} \alpha_1 + \theta_{k+1} \alpha_2) \bar{\Xi}_k \\ & + \theta_{k+1} \{ \text{tr}\{ \alpha_1^{-1} (A - L_k C) \Upsilon_k (A^T - C^T L_k^T) \} \\ & + \alpha_2^{-1} \text{tr}\{ (1 + \alpha_4) \bar{g}^2 + (1 + \alpha_4^{-1}) \\ & \times (\bar{\vartheta}^2 \hat{M}_k \hat{M}_k^T) \} \} + \theta_{k+1}^2 \text{tr}\{ (1 + \alpha_3) (A - L_k C) \Upsilon_k \\ & \times (A^T - C^T L_k^T) + (1 + \alpha_3^{-1}) (1 + \alpha_4) \bar{g}^2 \\ & + (1 + \alpha_3^{-1}) (1 + \alpha_4^{-1}) \bar{\vartheta}^2 \hat{M}_k \hat{M}_k^T + 2\bar{v}^2 L_k L_k^T \\ & + 2\mathfrak{N}^2 L_k \bar{L}_k^T + BQ_k B^T + L_k D S_k D^T L_k^T \\ & + \frac{2\bar{v}^2 + 2\mathfrak{N}^2 + D S_{k+1} D^T}{\|C^T C\|} + 2\mathfrak{N}^2 L_k \bar{L}_k^T + BQ_k B^T \\ & + L_k D S_k D^T L_k^T + 2\bar{v}^2 L_k L_k^T \} \end{aligned} \quad (19)$$

Then, $\bar{\Xi}_k$ is an upper bound on the NNW estimation error covariance Ξ_k .

Proof: Defining the trace of the estimation error covariance of NNW as

$$\Xi_k = \text{tr}\{\mathbb{E}\{\tilde{M}_k \tilde{M}_k^T\}\},$$

Ξ_{k+1} can be calculated as

$$\begin{aligned} \Xi_{k+1} = & \text{tr}\{\mathbb{E}\{\tilde{M}_{k+1} \tilde{M}_{k+1}^T\}\} \\ = & \text{tr}\{\mathbb{E}\{\tilde{M}_k \tilde{M}_k^T + \theta_{k+1}^2 C^T C B \omega_k \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \omega_k^T B^T C^T C \\ & + 2\theta_{k+1}^2 C^T C (\tilde{M}_k \vartheta(\hat{x}_k) + \mathfrak{S}_k) \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \tilde{x}_k^T (A \\ & - L_k C)^T C^T C - 2\theta_{k+1} C^T C (A - L_k C) \tilde{x}_k \tilde{\vartheta}^T(\hat{x}_k) \\ & \times \tilde{M}_k^T \tilde{M}_k^T - 2\theta_{k+1} C^T C (\tilde{M}_k \vartheta(\hat{x}_k) + \mathfrak{S}_k) \tilde{\vartheta}^T(\hat{x}_k) \tilde{M}_k^T \\ & + \theta_{k+1}^2 C^T C (A - L_k C) \tilde{x}_k \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \tilde{x}_k^T (A - L_k C)^T \\ & \times C^T C + \theta_{k+1}^2 C^T C (\tilde{M}_k \vartheta(\hat{x}_k) + \mathfrak{S}_k) \\ & \times \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) (\mathfrak{S}_k^T + \vartheta^T(\hat{x}_k) \tilde{M}_k^T) C^T C \\ & + \theta_{k+1}^2 C^T C L_k \iota_k \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \iota_k^T L_k^T C^T C \\ & - 2\theta_{k+1}^2 C^T C L_k \iota_k \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \iota_{k+1}^T C \\ & + \theta_{k+1}^2 C^T \iota_{k+1} \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \iota_{k+1}^T C \\ & + \theta_{k+1}^2 C^T C L_k \mathfrak{N}_k \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \mathfrak{N}_k^T L_k^T C^T C \\ & + \theta_{k+1}^2 C^T \mathfrak{N}_{k+1} \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \mathfrak{N}_{k+1}^T C \\ & - 2\theta_{k+1}^2 C^T \mathfrak{N}_{k+1} \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) \mathfrak{N}_k^T L_k^T C^T C \\ & + \theta_{k+1}^2 C^T D v_{k+1} \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) v_{k+1}^T D^T C \\ & + \theta_{k+1}^2 C^T C L_k D v_k \tilde{\vartheta}^T(\hat{x}_k) \tilde{\vartheta}(\hat{x}_k) v_k^T D^T L_k^T C^T C\} \end{aligned} \quad (20)$$

With the help of Lemma 5, we can obtain from (20) that

$$\begin{aligned} \Xi_{k+1} \leq & (1 + \theta_{k+1} \alpha_1 + \theta_{k+1} \alpha_2) \Xi_k + \theta_{k+1} \{ \text{tr}\{ \alpha_1^{-1} (A - L_k C) \Upsilon_k \\ & \times (A^T - C^T L_k^T) \} + \alpha_2^{-1} \text{tr}\{ (1 + \alpha_4) \bar{g}^2 + (1 + \alpha_4^{-1}) \\ & \times (\bar{\vartheta}^2 \hat{M}_k \hat{M}_k^T) \} \} + \theta_{k+1}^2 \text{tr}\{ (1 + \alpha_3) (A - L_k C) \Upsilon_k \\ & \times (A^T - C^T L_k^T) + (1 + \alpha_3^{-1}) (1 + \alpha_4) \bar{g}^2 \\ & + (1 + \alpha_3^{-1}) (1 + \alpha_4^{-1}) \bar{\vartheta}^2 \hat{M}_k \hat{M}_k^T + 2\bar{v}^2 L_k L_k^T \\ & + 2\mathfrak{N}^2 L_k \bar{L}_k^T + BQ_k B^T + L_k D S_k D^T L_k^T \\ & + \frac{2\bar{v}^2 + 2\mathfrak{N}^2 + D S_{k+1} D^T}{\|C^T C\|} \} \end{aligned} \quad (21)$$

Finally, it follows from (19) that

$$\Xi_{k+1} \leq \bar{\Xi}_{k+1},$$

which ends the proof. ■

Now, we proceed with designing the NNW tuning parameter in a recursive manner, ensuring that the upper bound derived in Theorem 1 is minimized.

Theorem 2: The upper bound of the NNW estimation error covariance can be minimized with the following NNW tuning parameter:

$$\begin{aligned} \theta_{k+1} = & - \left((\alpha_1 + \alpha_2) \Xi_k + \text{tr} \{ \alpha_1^{-1} (A - L_k C) \Upsilon_k (A^T - C^T L_k^T) \right. \\ & + \alpha_2^{-1} (1 + \alpha_4) \bar{g}^2 + \alpha_2^{-1} (1 + \alpha_4^{-1}) (\bar{\vartheta}^2 \hat{M}_k \hat{M}_k^T) \} \\ & \times \text{tr} \{ (1 + \alpha_3) (A - L_k C) \Upsilon_k (A^T - C^T L_k^T) + (1 + \alpha_3^{-1}) \\ & \times (1 + \alpha_4) \bar{g}^2 + (1 + \alpha_3^{-1}) (1 + \alpha_4) \bar{g}^2 (1 + \alpha_4^{-1}) \bar{\vartheta}^2 \hat{M}_k \hat{M}_k^T \\ & + 2L_k \bar{\iota}^2 L_k^T + 2L_k \bar{\aleph}^2 L_k^T + L_k D S_k D^T L_k^T + B Q_k B^T \\ & \left. + \frac{2\bar{\iota}^2 + 2\bar{\aleph}^2 + D S_{k+1} D^T}{\|C^T C\|} \right\}^{-1} \end{aligned} \quad (22)$$

Proof: Taking the partial derivative of $\bar{\Xi}_{k+1}$ with respect to θ_{k+1} , we have

$$\begin{aligned} \frac{\partial \bar{\Xi}_{k+1}}{\partial \theta_{k+1}} = & (\alpha_1 + \alpha_2) \bar{\Xi}_k + \text{tr} \{ \alpha_1^{-1} (A - L_k C) \bar{\Upsilon}_k (A^T - C^T L_k^T) \\ & + \alpha_2^{-1} (1 + \alpha_4) \bar{g}^2 + \alpha_2^{-1} (1 + \alpha_4^{-1}) (\bar{\vartheta}^2 \hat{M}_k \hat{M}_k^T) \} \\ & + \theta_{k+1} \text{tr} \{ (1 + \alpha_3) (A - L_k C) \bar{\Upsilon}_k (A^T - C^T L_k^T) \\ & + (1 + \alpha_3^{-1}) (1 + \alpha_4) \bar{g}^2 + (1 + \alpha_3^{-1}) (1 + \alpha_4^{-1}) \bar{\vartheta}^2 \hat{M}_k \hat{M}_k^T \\ & + 2L_k \bar{\iota}^2 L_k^T + 2L_k \bar{\aleph}^2 L_k^T + L_k D S_k D^T L_k^T + B Q_k B^T \\ & \left. + \frac{2\bar{\iota}^2 + 2\bar{\aleph}^2 + D S_{k+1} D^T}{\|C^T C\|} \right\}. \end{aligned}$$

Then, letting

$$\frac{\partial \bar{\Xi}_{k+1}}{\partial \theta_{k+1}} = 0,$$

it is easy to see that $\bar{\Xi}_{k+1}$ is minimized if the NNW tuning scalar θ_{k+1} is selected as (22). ■

Remark 5: We have now completed the design of the NNW tuning parameter for the unknown nonlinearities. The NNW tuning parameter is computed recursively by minimizing the trace of the estimation error covariance of the NNW (i.e., $\Xi_k = \text{tr} \{ \mathbb{E} \{ \tilde{M}_k \tilde{M}_k^T \} \}$). Unlike a time-invariant NNW tuning parameter, the recursively calculated NNW tuning parameter can achieve better approximation performance, as it adapts to the transient properties of the unknown nonlinearities. Notably, the accurate estimation error covariance is difficult to obtain in this paper because the biases caused by quantization errors and bit errors only have upper bounds on variance, without accurate variance values. Therefore, minimizing the estimation error covariance of the NNW is both theoretically and practically justified.

B. Design of the Filter Gain Matrix

In this subsection, an effective algorithm will be developed to parameterize the recursive estimator gains to minimize the upper bound of $\Upsilon_k = \mathbb{E} \{ \tilde{x}_k \tilde{x}_k^T \}$.

Theorem 3: Let the positive scalars ς be given. Assume that there exists a set of real-valued matrices $\bar{\Upsilon}_k$ (with the initial condition $\bar{P}_0 = P_0$) satisfying the following recursive equation

$$\begin{aligned} \bar{\Upsilon}_{k+1} = & (1 + \varsigma) (A - L_k C) \bar{\Upsilon}_k (A^T - C^T L_k^T) + (2 + \varsigma^{-1} + \varsigma) \bar{g}^2 \\ & + (1 + \varsigma^{-1})^2 \bar{\vartheta}^2 \text{tr} \{ \hat{M}_k \hat{M}_k^T \} + 2L_k \bar{\iota}^2 L_k^T + 2L_k \bar{\aleph}^2 L_k^T \\ & + B Q_k B^T + L_k D S_k D^T L_k^T. \end{aligned} \quad (23)$$

Then, $\bar{\Upsilon}_k$ is an upper bound of the estimation error covariance Υ_k .

Proof: The estimation error covariance Υ_{k+1} is calculated as

$$\begin{aligned} \Upsilon_{k+1} = & \mathbb{E} \{ \tilde{x}_{k+1} \tilde{x}_{k+1}^T \} \\ = & \mathbb{E} \{ ((A - L_k C) \tilde{x}_k + (\tilde{M}_k \vartheta(\hat{x}_k) + \mathfrak{S}_k) - L_k (D v_k + \aleph_k \\ & + \iota_k) + B \omega_k) (\tilde{x}_k^T (A^T - C^T L_k^T) + (\vartheta^T(\hat{x}_k) \tilde{M}_k^T + \mathfrak{S}_k^T) \\ & - (v_k^T D^T + \aleph_k^T + \iota_k^T) L_k^T + \omega_k^T B^T) \} \end{aligned} \quad (24)$$

With the help of Lemma 5, we obtain from (24) that

$$\begin{aligned} \Upsilon_{k+1} \leq & (1 + \varsigma) (A - L_k C) \bar{\Upsilon}_k (A^T - C^T L_k^T) + (2 + \varsigma^{-1} \\ & + \varsigma) \bar{g}^2 I + (1 + \varsigma^{-1})^2 \bar{\vartheta}^2 \text{tr} \{ \hat{M}_k \hat{M}_k^T \} I + 2L_k \bar{\iota}^2 L_k^T \\ & + 2L_k \bar{\aleph}^2 L_k^T + B Q_k B^T + L_k D S_k D^T L_k^T, \end{aligned} \quad (25)$$

where ς is a given positive scalar. Finally, it follows from (25) that

$$\Upsilon_{k+1} \leq \bar{\Upsilon}_{k+1}$$

which ends the proof. ■

Now, we are ready to design the gain matrix of the NN-based recursive estimator which ensure that the upper bound obtained in Theorem 3 is minimized.

Theorem 4: The upper bound of the estimation error covariance can be minimized with the following estimator gain parameter:

$$\begin{aligned} L_k = & ((1 + \varsigma) A \bar{\Upsilon}_k C^T) (C \bar{\Upsilon}_k C^T \\ & + D S_k D^T + 2\bar{\iota}^2 + 2\bar{\aleph}^2)^{-1} \end{aligned} \quad (26)$$

Proof: Taking the partial derivative of $\text{tr} \{ \bar{\Upsilon}_{k+1} \}$ with respect to L_k , one has

$$\begin{aligned} \frac{\partial \bar{\Upsilon}_{k+1}}{\partial L_k} = & - 2(1 + \varsigma) (A - L_k C) \bar{\Upsilon}_k C^T \\ & + 2L_k D S_k D^T + 4L_k \bar{\iota}^2 + 4L_k \bar{\aleph}^2 \end{aligned}$$

Then, by letting the derivative be zero, it is easy to see that $\bar{\Upsilon}_{k+1}$ is minimized if the estimator gain L_k is selected as (26). ■

Based on the above discussion, the parameters (i.e., L_k and θ_k) of the recursive state estimator for the nonlinear NCSs with the BEM have been computed by solving a minimization issue. The following algorithm provides the steps to compute L_k and θ_k .

NN-based Recursive Estimation Algorithm:

- Step 1. Initialization: Given the initial value y_0 , $\hat{M}_0 = \bar{M}_0$, $\hat{x}_0 = \bar{x}_0$, θ_0 , Ξ_0 , Υ_0 .
- Step 2. Calculate the NNW tuning parameter θ_k .
- Step 3. Calculate the estimate of NNW $\hat{M}_k$.
- Step 4. Calculate the upper bound of the estimation error covariance of NNW $\bar{\Xi}_k$.
- Step 5. Calculate the recursive estimator gain L_k .
- Step 6. Calculate the upper bound of the estimation error covariance of system state $\bar{\Upsilon}_{k+1}$.
- Step 7. Calculate the state estimate $\hat{x}_{k+1}$.
- Step 8. $k = k + 1$.
-

In the NN-based recursive estimation algorithm, the first step is to set the initial value. The second step is to calculate the NNW tuning parameter θ_k based on (22). Then, the estimates of the NNW $\hat{M}_k$ can be obtained. Up to now, the estimates of the unknown nonlinearities (i.e., $\hat{M}_k \vartheta(\hat{x}_k)$) can be calculated, and the upper bound of the trace of the estimation error covariance of NNW $\bar{\Xi}_k$ can be obtained. Next, according to (26), we can derive the gain matrix L_k of the NN-based recursive estimator, based on which the upper bound of the estimation error covariance of system state $\bar{\Upsilon}_{k+1}$ can be obtained. Finally, based on $\hat{M}_k \vartheta(\hat{x}_k)$ and L_k , we can generate the state estimate $\hat{x}_{k+1}$. It is worth mentioning that $\bar{\Xi}_{k+1} = \Xi_{k+1}$ and $\bar{\Upsilon}_{k+1} = \Upsilon_{k+1}$ in specific case.

Remark 6: In this paper, we have developed an NN-based recursive estimation strategy to address the state estimation problem for nonlinear NCSs with unknown nonlinearities and BEMs. We have first analyzed the effects of quantization errors $\aleph_k$ and bit errors ι_k introduced by BEM, and analyzed their impact on the system's state estimation performance. Then, a novel recursive estimator has been proposed that integrates neural networks to approximate unknown nonlinearities. This approach adapts to the transient behavior of nonlinear systems by recursively updating the NNWs and estimator gains. Subsequently, we have designed the NNW tuning parameter θ_k and the estimator gain L_k to minimize the upper bounds of the estimation error covariance for both the system state and the NNWs. These parameters have been computed recursively to ensure optimal estimation performance of both the system states and the unknown nonlinearities over time. Overall, the proposed estimation strategy has enhanced adaptability and performance, offering a flexible solution for systems with unknown nonlinearities.

Remark 7: The distinctive contributions of this paper, as compared to existing results, are highlighted as follows:

- 1) Unlike most existing works, which focus on state estimation without considering the impact of communication mechanisms, this paper addresses the recursive state estimation problem for nonlinear NCSs under BEM and random bit errors. This integration leads to a more realistic model for modern networked systems, where such phenomena frequently occur.
- 2) This work leverages NNs to approximate unknown nonlinearities in NCSs. Unlike traditional methods that rely on known nonlinear dynamics or linearization techniques, the proposed method uses NNs to handle unknown nonlinearities without requiring prior knowledge, enhancing

flexibility and adaptability.

- 3) A key novelty of this paper lies in the recursive adjustment of both the NNW and the estimator gain. Such a dynamic updating mechanism, embedded in a unified framework, significantly improves approximation accuracy and estimation performance as compared to time-invariant or static approaches used in other works.
- 4) The paper provides a detailed quantitative analysis of the distortions caused by both quantization and bit errors in BEM, which could degrade the estimation performance. Unlike prior research that often assumes ideal communication, this work thoroughly examines how these errors affect the estimator and proposes a strategy to mitigate their negative impact.

IV. ILLUSTRATIVE EXAMPLE

In this section, a simulation example is presented to verify the effectiveness and correctness of the developed NN-based state estimation algorithm.

Consider a nonlinear NCSs (1) with the following parameters:

$$A = \begin{bmatrix} 0.7 & 0.4 & 0 \\ 0.3 & 1.025 & 0.1 \\ 0 & 0.3 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 \\ -0.1 \\ -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.45 & -0.2 & 0 \\ -0.98 & 0 & -0.2 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

The nonlinear function is selected as

$$g(x_k) = 4 \begin{bmatrix} \sin(x_{1,k}) \\ \cos(x_{2,k}) \\ \sin(x_{3,k}) \cos(x_{3,k}) \end{bmatrix}.$$

Denote $x_{r,k}$ as the r th row of x_k . The covariances of the process noise ω_k and the measurement noise v_k are chosen as $S_k = 0.09$ and $Q_k = 0.04$, respectively. Under the BEM, we set $H = 8$ and $p = 0.1$ and the interval length $\epsilon = 0.6$. Then, we let $\alpha_1 = 0.9$, $\alpha_2 = 0.8$, $\alpha_3 = 0.8$, $\alpha_4 = 0.7$, and select the activation function of the NN as

$$\vartheta(\hat{x}_k) = [0.1 \tanh(\hat{x}_{1,k}) \quad 0.1 \tanh(\hat{x}_{2,k}) \quad 0.1 \tanh(\hat{x}_{3,k})]^T.$$

The initial values are chosen as

$$x_0 = [3.5 \quad 3.85 \quad 3.15]^T,$$

$$\hat{x}_0 = [0.7 \quad 2.1 \quad 0.7]^T,$$

$$\hat{M}_0 = \begin{bmatrix} 1 & 1 & 0.95 \\ 0.22 & -0.41 & 0.1 \\ 0.1 & 0.9 & 0.1 \end{bmatrix}.$$

The goal of this paper is to develop an NN-based recursive state estimator to minimize the upper bound of $\mathbb{E}\{\tilde{x}_k \tilde{x}_k^T\}$ and $\text{tr}\{\mathbb{E}\{\tilde{M}_k \tilde{M}_k^T\}\}$. By applying Algorithm 1, the parameters for the NNW tuning scalars and the estimator gain matrices are obtained, with their values listed in Tables I and II. The corresponding simulation results are presented in Figs. 2-6, demonstrating the effectiveness of the proposed estimator.

Figs. 2-4 display the trajectories of the system states and their corresponding estimates. The dynamics of the state estimation error is shown in Fig. 5, clearly demonstrating

TABLE I: The time-varying NNW tuning scalars

k	1	2	3	4	5	6	7	8	9	10
θ_k	-0.7369	-0.7564	-0.8963	-0.8849	-0.8963	-0.8849	-0.8193	-0.8963	-0.8849	-0.8849
k	11	12	13	14	15	16	17	18	19	20
θ_k	-0.8963	-0.8849	-0.7564	-0.8963	-0.8849	-0.8963	-0.8849	-0.8193	-0.8963	-0.8167
k	21	22	23	24	25	26	27	28	29	30
θ_k	-0.6753	-0.6489	-0.8963	-0.8849	-0.7564	-0.8963	-0.8849	-0.8963	-0.8849	-0.7164
k	31	32	33	34	35	36	37	38	39	40
θ_k	-0.6073	-0.7648	-0.6158	-0.6482	-0.7347	-0.8321	-0.7995	-0.8734	-0.7156	-0.6539
k	41	42	43	44	45	46	47	48	49	50
θ_k	-0.8492	-0.6592	-0.6975	-0.7539	-0.8528	-0.8904	-0.8532	-0.7863	-0.6056	-0.8167

TABLE II: The time-varying recursive estimator gain matrices

k	1	2	3	4	5	...
L_k	-3.0668 -3.0656 -5.7618 -3.8880 -0.9776 -1.2741	0.4630 -2.0193 0.4561 -2.0693 0.2235 -0.9109	5.1628 -0.9048 3.8194 -1.2699 -1.4578 -1.3082	5.3038 -0.8711 3.1489 -1.4280 -2.1259 -1.4661	5.0492 -0.9090 2.1370 -1.6597 -2.7030 -1.6173	...
k	11	12	13	14	15	...
L_k	2.2277 -1.6528 -2.6865 -2.9621 -3.9504 -1.9711	0.4854 -2.1914 -4.8375 -3.6266 -4.0498 -2.0015	-0.6531 -2.5440 -6.1397 -4.0294 -4.0306 -1.9951	0.9289 -2.0483 -4.1994 -3.4211 -3.9507 -1.9699	1.4144 -1.8949 -3.6842 -3.2579 -3.9916 -1.9824	...
k	21	22	23	24	25	...
L_k	0.0566 -2.3165 -5.4124 -3.7943 -4.1113 -2.0194	0.2087 -2.2688 -5.2201 -3.7340 -4.0989 -2.0156	0.2120 -2.2677 -5.2119 -3.7313 -4.0953 -2.0144	-0.1216 -2.3721 -5.5948 -3.8511 -4.0908 -2.0130	-0.1023 -2.3660 -5.5269 -3.8298 -4.0539 -2.0014	...
k	31	32	33	34	35	...
L_k	0.7109 -2.1114 -4.4930 -3.5061 -3.9830 -1.9792	0.6893 -2.1182 -4.5279 -3.5170 -3.9909 -1.9817	0.6222 -2.1392 -4.6139 -3.5440 -3.9973 -1.9837	0.7253 -2.1069 -4.5019 -3.5089 -4.0040 -1.9858	0.6581 -2.1280 -4.5979 -3.5390 -4.0184 -1.9903	...
k	41	42	43	44	45	...
L_k	0.4329 -2.1984 -4.9007 -3.6337 -4.0515 -2.0007	0.6581 -2.1280 -4.5979 -3.5390 -4.0184 -1.9903	0.5502 -2.1617 -4.7417 -3.5840 -4.0332 -1.9949	0.5123 -2.1736 -4.7937 -3.6002 -4.0396 -1.9969	0.4856 -2.1819 -4.8306 -3.6118 -4.0444 -1.9985	...

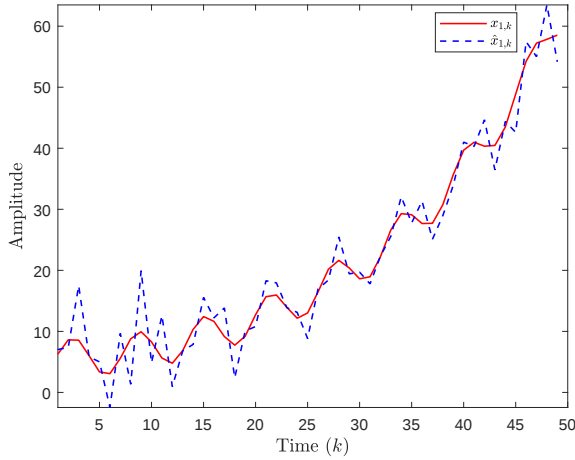


Fig. 2: State $x_{1,k}$ and its estimate.

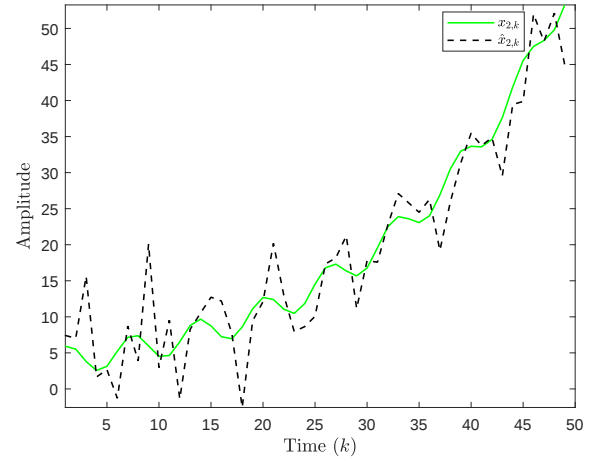


Fig. 3: State $x_{2,k}$ and its estimate.

that the upper bound of the state estimation error covariance is minimized. Fig. 6 illustrates the estimation error for the unknown nonlinearities (i.e., $\tilde{g}(x_k) \triangleq g(x_k) - \hat{M}_k \vartheta(\hat{x}_k)$), confirming that the developed NN is capable of effectively approximating $g(\cdot)$. Overall, the simulation results indicate that the proposed NN-based recursive state estimator delivers satisfactory performance. TABLE I and II give the recursive updating process of the parameters (i.e., L_k and θ_k) of the recursive state estimator.

V. CONCLUSIONS

In this study, we have addressed the recursive state estimation problem for a class of NCSs with unknown nonlinearities and BEM, where the BEM is utilized to convert measurement signals into BBSs of limited length, which are then transmitted to the recursive state estimator through noisy communication channels. NNs have been employed to approximate the unknown nonlinearities of the NCSs. A recursive state estimator with a suitable structure has been proposed, where both the

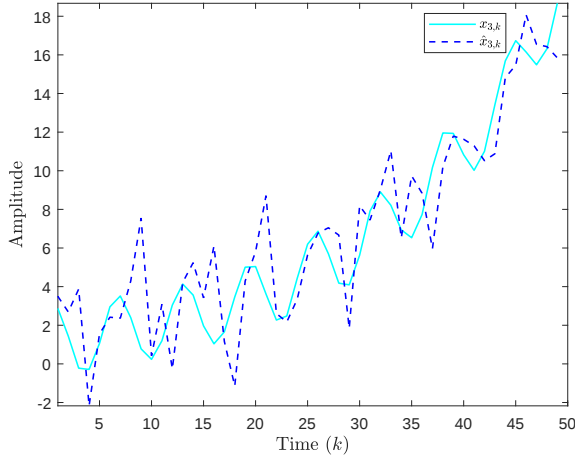


Fig. 4: State $x_{3,k}$ and its estimate.

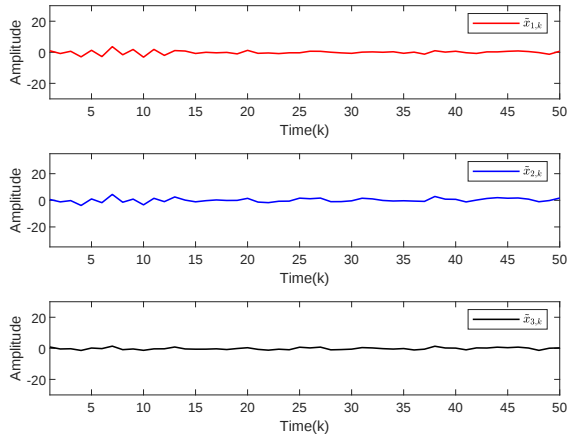


Fig. 5: State estimation error

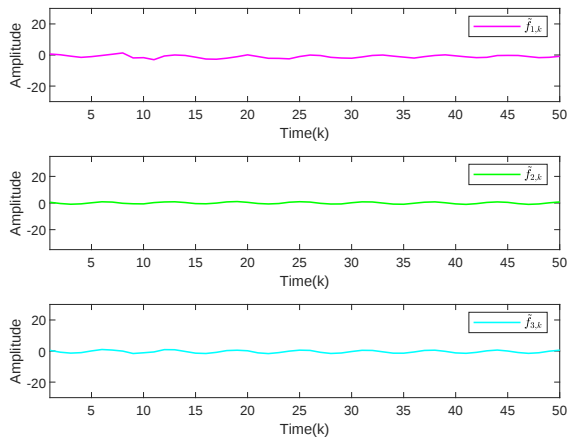


Fig. 6: Nonlinear function estimation error

estimator gain parameters and the NNW tuning scalars have been computed recursively within a unified framework. Sufficient conditions have been derived to ensure the mean-square boundedness of the estimation errors for both the system states

and the NNWs. Numerical examples have been provided to demonstrate the effectiveness of the proposed state estimation strategy. Future research directions could include extending the proposed estimation algorithm to systems with BEM and other phenomena, such as wireless sensor networks [38], [40], multi-agent systems [15], and additional applications [13], [27], [42], [51]–[55].

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Yuhan Zhang received the B.Eng. degree in electronic information science and technology from the Shandong University of Science and Technology, Qingdao, China, in 2016, the M.Sc. degree in marketing from the University of Nottingham, Nottingham, UK, in 2017, and the Ph.D. degree in control science and engineering in 2024 from Shandong University of Science and Technology, Qingdao, China.

Her current research interests include the control and filtering of networked systems, reinforcement learning, and neural networks.

She is a very active reviewer for many international journals.



Wei Qian (Senior Member, IEEE) received the M.S. degree in control theory and control engineering from Southeast University, Nanjing, China, and the Ph.D. degree in control science and engineering from the State Key Laboratory of Industrial Control Technology, Zhejiang University, Hangzhou, China, in 2005 and 2009, respectively.

He is currently a Professor with the School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo, China. His current research interests include time-delay systems, nonlinear systems, networked control systems, and multiagent system.



Zidong Wang (SM'03-F'14) received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently a Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, Uxbridge, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the U.K. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published a number of papers in international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, the Editor-in-Chief for *Systems Science & Control Engineering*, and an Associate Editor for 12 international journals including *IEEE Transactions on Automatic Control*, *IEEE Transactions on Control Systems Technology*, *IEEE Transactions on Neural Networks*, *IEEE Transactions on Signal Processing*, and *IEEE Transactions on Systems, Man, and Cybernetics-Part C*. He is a Member of the Academia Europaea, a Member of the European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a Fellow of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.



Shuxin Du received his B.S. degree in industrial electrical automation from Jiangsu University, Zhenjiang, China, in 1989, and M.S. degree in theory and application of automatic control from Xi'an Jiaotong University, Xi'an, China, in 1992, and Ph.D. degree in aircraft control, guidance and simulation from Northwestern Polytechnical University, Xi'an, China, in 1995.

From September 1995 to September 1997, he worked as a postdoctoral fellow in the Institute of Industrial Process Control, Zhejiang University, Hangzhou, China. From September 1997 to September 2015, he was an associate professor in the Department of Control Science and Engineering, Zhejiang University, Hangzhou, China. He is currently a professor in the Engineering College, Huzhou University, Huzhou, China. His research interests include control theory and application, pattern recognition and intelligent systems, on-line measurement of quality parameters based on spectrum.



Lei Zou (Senior Member, IEEE) received the Ph.D. degree in control science and engineering in 2016 from Harbin Institute of Technology, Harbin, China. From October 2013 to October 2015, he was a visiting Ph.D. student with the Department of Computer Science, Brunel University London, Uxbridge, U.K.

He is currently a Professor with the College of Information Science and Technology, Donghua University, Shanghai, China. His research interests include control and filtering of networked systems, moving-horizon estimation, state estimation subject

to outliers, and secure state estimation.

Prof. Zou serves (or has served) as an Associate Editor for *Neurocomputing*, *International Journal of Systems Science*, *IEEE/CAA Journal of Automatica Sinica*, *IET Control Theory & Applications*, and *International Journal of Control, Automation and Systems*. He is a Senior Member of *IEEE* and *Chinese Association of Automation*, a Regular Reviewer of *Mathematical Reviews*, and a very active reviewer for many international journals.