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Proportional-Integral-Observer-Based Fusion Estimation for Artificial Neural Networks: Implementing A One-Bit Encoding Scheme

Kaiqun Zhu, Zidong Wang, Derui Ding, Jun Hu, and Hongli Dong

***Abstract*—The paper is concerned with the proportional-integral-observer-based (PIO-based) fusion estimation problem for a class of artificial neural networks equipped with multiple sensors, which are constrained by bandwidth and subjected to unknown-but-bounded noises. For the purpose of efficient information communication, an approach known as the one-bit encoding mechanism (OBEM) is proposed that enables the encoding of scalar data using merely a single bit. Then, a local PIO-based set-membership estimator is devised for each sensor node, with the aim of achieving the desired estimation task while considering the possible data distortion due to OBEM and the existence of unknown-but-bounded noises. Subsequently, sufficient conditions are established to ensure the existence and effectiveness of the PIO-based set-membership estimator. Moreover, to enhance the global estimation performance, an ellipsoid-based fusion rule is introduced for all local PIO-based set-membership estimators. The performance of fusion estimation is then analyzed using set theory and the optimization method, leading to the determination of relevant parameters. Finally, the effectiveness and advantages of the proposed estimation algorithm are demonstrated through a simulation example.**

***Index Terms*—Artificial neural networks, set-membership state estimation, fusion estimation, proportional-integral-observer, one-bit encoding mechanism.**

Abbreviations and Notations

ANN	Artificial neural network
OBEM	One-bit encoding mechanism
PIO	Proportional-integral observer
UBBN	Unknown-but-bounded noise
$P > 0$	P is a positive-definite matrix
$\mathrm{Tr}[P]$	The trace of a matrix P
$\mathrm{col}\{\cdot\}$	A column vector operator
$\mathrm{diag}\{\cdot\}$	A block-diagonal matrix operator
$\mathbf{0}_n$	$\big[\underbrace{0, 0, \ldots, 0}_{n}\big]$ with appropriate dimensions
$\mathbb{O}_n$	$\mathrm{diag}\{\underbrace{0, 0, \ldots, 0}_{n}\}$ with appropriate dimensions

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I. INTRODUCTION

Emulating the biological neural systems, the Artificial Neural Network (ANN) has recently gained widespread attention in the fields of control theory and state estimation [5], [6], [8], [11], [44], [49]. Known for its proficiency in extracting valuable information from vast amounts of sample data (through adaptive learning and training techniques), the ANN is instrumental in modeling and analyzing industrial systems [20], [27]. Typically comprising an input layer, several hidden layers and an output layer, the ANN is adept at learning the nonlinear mappings between system inputs and outputs. This capability distinguishes it from traditional linear models, allowing for a more precise description of the dynamic attributes and nonlinear behaviors of complex systems, which is essential for system analysis and synthesis [9], [31], [38], [39].

State estimation techniques are fundamental in control theory and its applications as they facilitate the analysis of system behavior through precise state estimation, which is crucial for achieving precise control [1], [4], [10], [16], [28]. Common methods for addressing the state estimation problems include H_∞ estimation [17], [29], Kalman-type filtering [30], [33], and particle filtering algorithms [23]. Accordingly, the state estimation problem for ANNs has attracted considerable research attention in the past few decides [26], [40], [42], [46]. For instance, a particle filter has been specifically designed in [32] for ANNs with saturation constraints and redundant channels. Despite these advancements, existing state estimation algorithms often face challenges in achieving optimal estimation performance particularly when the system is affected by unknown-but-bounded noises (UBBNs).

Set-membership state estimation emerges as a preferred solution for systems influenced by UBBNs due primarily to two significant benefits: i) it circumvents the necessity for detailed knowledge of statistical characteristics of the noises, and ii) it ensures the identification of an ellipsoidal set that confidently encompasses the true state of the system with

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100% certainty [21], [22], [24], [43], [47], [51], [52]. In another aspect, to address issues related to imperfect information, the proportional-integral observer (PIO) approach has been applied in various contexts [7], [14], [48]. Motivated by these observations, the integration of PIO techniques, set-membership estimation methods, and multi-sensor systems shows promising potential for enhancing both robustness and accuracy of state estimation. However, such an integrated approach remains largely unexplored in the existing literature, thereby providing a key motivation for the current study.

In practical systems, the wide application of network communication technology is pivotal for achieving remote state estimation. A notable concern in this context is the transmission of system data over communication networks with limited capacity, which can lead to congestion issues [12], [34], [35], [45]. Such problems have the potential to impair data integrity, thereby negatively impacting the system's estimation performance [15], [19], [41]. To mitigate these challenges, encoding mechanisms are implemented in networked systems to enhance data transmission efficiency and integrity [13], [18], [36], [37], where most existing mechanisms limit the encoding of system data to a finite bit count. This paper introduces a one-bit encoding mechanism (OBEM) to further refine the data encoding process. Nevertheless, it is important to recognize that data distortion could occur during encoding and decoding which, if not properly addressed, might compromise the accuracy of state estimation. Therefore, examining the effects of OBEM on state estimation performance becomes a critical element of our research.

Summarizing the preceding discussions, this paper concentrates on examining the PIO-based fusion estimation problem by taking into account the influence of the OBEM. This investigation is challenging due to several key factors: 1) how to design an encoding mechanism that can effectively compress scalar data into a single bit; 2) how to construct an appropriate state estimator by considering the data distortion induced by the encoding mechanism as well as the presence of UBBNs; and 3) how to develop a fusion estimation rule to ensure that the performance of the fused state estimation surpasses that of individual local state estimation.

In response to the outlined challenges, this paper makes the following notable contributions.

1) A new OBEM, specifically designed for systems constrained by limited bandwidth, is introduced to efficiently encode scalar data using just a single bit. Compared with the existing encoding technique, e.g., [13], [18], [37], [51], this method significantly improves the efficiency of information transmission.
2) A novel PIO-based state estimator is formulated within the set-membership estimation framework, which marks the first exploration of the PIO-based set-membership estimation problem in the context of systems affected by both UBBNs and OBEM. Unlike the method presented in [9], [21], [48], our approach offers significant advantages in addressing bandwidth limitations and UBBNs.
3) An original fusion estimation rule is established for local PIO-based set-membership estimators drawing upon set theory, and the performance of this fusion estimation is thoroughly analyzed in order to ensure that the fused estimation results are superior to those of any individual local sensor.

These contributions collectively enhance the efficiency and accuracy of fusion estimation in the presence of UBBNs and OBEM. Utilizing PIO, set-membership estimation and set theory, this work addresses the challenges posed by bandwidth constraints and achieves reliable estimation results.

II. System Description and Preliminaries

For the convenience of the later discussion, we first give the following definition.

Definition 1: [2] An ellipsoidal set is defined as

$$\mathscr{E}(c,P) \triangleq \{x|(x-c)^{\mathrm{T}}P^{-1}(x-c)\le 1\} \tag{1}$$

where c is the center of the ellipsoidal set and $P>0$ is called the shape-matrix of the ellipsoidal set.

A. The Description of Artificial Neural Networks

Consider a kind of ANNs of the following form:

$$x_{t+1} = \Psi_t x_t + W_t\phi(V_t x_t) + w_t \tag{2}$$

where $x_t\in\mathbb{R}^n$ is the state vector of the ANN; $\phi(\cdot)$ is the neuron activation function which is twice continuously differentiable; Ψ_t, W_t and V_t are known matrices with appropriate dimensions; and $w_t\in\mathbb{R}^n$ is the process noise satisfying

$$w_t\in\mathscr{E}(0,Q_t)\triangleq\{w_t|w_t^{\mathrm{T}}Q_t^{-1}w_t\le 1\}. \tag{3}$$

Here, $Q_t>0$ is a shape-matrix and $\mathscr{E}(0,Q_t)$ characterizes the ellipsoidal set constraining the process noise.

To overcome the inherent limitations of single-sensor systems in measurement and state estimation capabilities, this paper introduces a multi-sensor system architecture aimed at enhancing the overall performance of state estimation. The measurement information of the mth sensor node is

$$y_{m,t} = C_{m,t}x_t + v_{m,t},\quad m=1,2,\ldots,M \tag{4}$$

where $y_{m,t}\in\mathbb{R}^l$ is the measurement output of the mth sensor node, $C_{m,t}$ is the known matrix with appropriate dimension, and $v_{m,t}\in\mathbb{R}^l$ is the measurement noise satisfying

$$v_{m,t}\in\mathscr{E}(0,R_{m,t})\triangleq\{v_{m,t}|v_{m,t}^{\mathrm{T}}R_{m,t}^{-1}v_{m,t}\le 1\}. \tag{5}$$

Here, $R_{m,t}>0$ is a shape-matrix and $\mathscr{E}(0,R_{m,t})$ characterizes the ellipsoidal set constraining the measurement noise.

B. The One-Bit Encoding Mechanism

Considering the bandwidth limitations of communication networks and the power restrictions prevalent in multi-sensor systems, it becomes essential to encode the measurement data from each sensor node into a limited number of bits prior to its transmission to a remote estimator. To tackle this challenge, this paper introduces a novel *1-bit* data encoding mechanism. This mechanism is aimed at significantly enhancing the efficiency of the data encoding process, thereby optimizing data transmission within the constraints of network bandwidth and power availability in multi-sensor systems. The detailed

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implementation of the encoding and decoding processes is presented as follows.

The encoding mechanism for the ith element of measurement output $y_{m,t}$ is described as

$$\xi_{m,t}^{[i]} \triangleq \mathrm{Enc}(y_{m,t}^{[i]}) = \begin{cases} 1, & y_{m,t}^{[i]} \geq d_m^{[i]} \\ -1, & y_{m,t}^{[i]} < d_m^{[i]} \end{cases} \quad (6)$$

for $i = 1, 2, \ldots, l$ and $m = 1, 2, \ldots, M$, where $\xi_{m,t}^{[i]}$ is the encoded data to be transmitted through the communication network, $y_{m,t}^{[i]}$ is the ith element of the vector $y_{m,t}$, and $d_m^{[i]}$ is the ith element of the threshold vector $d_{m,t}$ with

$$d_m \triangleq \begin{bmatrix} d_m^{[1]} & d_m^{[2]} & \cdots & d_m^{[l]} \end{bmatrix}^{\mathrm{T}}.$$

It is easy to see from (6) that, under the encoding mechanism, the continuous-amplitude signal $y_{m,t}^{[i]}$ is encoded into a 1-bit discrete-amplitude data $\xi_{m,t}^{[i]}$.

According to (6), the encoded data for the measurement output vector $y_{m,t}$ is represented as

$$\xi_{m,t} \triangleq \mathrm{Enc}(y_{m,t}) = \begin{bmatrix} \xi_{m,t}^{[1]} & \xi_{m,t}^{[2]} & \cdots & \xi_{m,t}^{[l]} \end{bmatrix}^{\mathrm{T}}. \quad (7)$$

Then, the encoded data $\mathrm{Enc}(y_{m,t})$ of the mth sensor node is transmitted to the remote decoder via the bandwidth-limited communication network.

In the decoder side, the decoding mechanism is given as

$$\mathrm{Dec}(\xi_{m,t}) = \xi_{m,t} \quad (8)$$

where $\xi_{m,t}$ is the output of the decoder.

Remark 1: Over recent years, the one-bit encoding method has been increasingly adopted in the field of compressive sensing, particularly for the encoding of static signals [3], [25]. This method, which encodes signals using a single bit, significantly reduces the amount of data required for storage and transmission while preserving the essential characteristics of the original signal. However, a notable limitation of existing one-bit encoding techniques is their focus predominantly on static signals. This highlights the need for a new mechanism that is well-suited for *dynamic* systems. Developing such a mechanism would enable a comprehensive analysis of how encoding methods interact with system dynamics and affect state estimation performance.

Remark 2: This paper presents a novel encoding mechanism, termed OBEM, specifically designed for systems operating under limited bandwidth constraints and applicable to dynamic signals. The OBEM stands out for its capability to significantly boost the efficiency of the data encoding process, thereby effectively reducing the communication burden. Furthermore, this initiative marks the first effort to integrate encoding mechanisms into the realm of state estimation for dynamic systems, representing a substantial step forward along this direction.

C. OBEM-Based Proportional-Integral Observer

In this paper, a novel OBEM-based PIO technique is applied to estimate the system state where, for $m = 1, 2, \ldots, M$, the state estimator structure for the mth sensor node is designed as follows:

$$\begin{aligned} \hat{x}_{m,t+1} &= \Psi_t \hat{x}_{m,t} + W_t \phi(V_t \hat{x}_{m,t}) \\ &\quad + K_{m,t}^P (d_m - C_m \hat{x}_{m,t}) + K_{m,t}^I \mu_{m,t} \quad (9) \\ \mu_{m,t+1} &= \mu_{m,t} + F_{m,t}(d_m - C_{m,t}\hat{x}_{m,t}). \quad (10) \end{aligned}$$

Here, $\hat{x}_{m,t} \in \mathbb{R}^n$ is the estimate of x_t on sensor m; $\mu_{m,t} \in \mathbb{R}^l$ means the integral of the output estimation error; and $K_{m,t}^P$, $K_{m,t}^I$, $F_{m,t}$ are estimator gain matrices to be determined.

In the following, in order to take full advantage of the system information, the variable d_m in estimator (9)–(10) will be transformed into a form represented by the measurement output $y_{m,t}$ and the decoder output $\xi_{m,t}$. The detailed principle is described according to the following two cases.

Case I: $\xi_{m,t}^{[i]} \xi_{m,t-1}^{[i]} < 0$.

In accordance with the encoding rule (6), it is clear that the variable $d_m^{[i]}$ falls between $y_{m,t}^{[i]}$ and $y_{m,t-1}^{[i]}$ in this case. Accordingly, there exists $\delta_{m,t}^{[i]} \in (-0.5, 0.5)$ such that

$$d_m^{[i]} = (0.5 + \delta_{m,t}^{[i]}) y_{m,t}^{[i]} + (0.5 - \delta_{m,t}^{[i]}) y_{m,t-1}^{[i]}. \quad (11)$$

Case II: $\xi_{m,t}^{[i]} \xi_{m,t-1}^{[i]} \geq 0$.

Following the same method, we konw that the variable $d_m^{[i]}$ falls between $y_{m,t}^{[i]}$ and $-y_{m,t-1}^{[i]}$. In this case, $d_m^{[i]}$ can be described as

$$d_m^{[i]} = (0.5 + \delta_{m,t}^{[i]}) y_{m,t}^{[i]} - (0.5 - \delta_{m,t}^{[i]}) y_{m,t-1}^{[i]} \quad (12)$$

where $\delta_{m,t}^{[i]}$ is a certain variable satisfying $\delta_{m,t}^{[i]} \in (-0.5, 0.5)$.

By consolidating the aforementioned two cases, we can establish a unified formulation to characterize variable $d_m^{[i]}$:

$$\begin{aligned} d_m^{[i]} &= (0.5 + \delta_{m,t}^{[i]}) y_{m,t}^{[i]} \\ &\quad + \mathrm{sign}(-\xi_{m,t}^{[i]} \xi_{m,t-1}^{[i]})(0.5 - \delta_{m,t}^{[i]}) y_{m,t-1}^{[i]} \quad (13) \end{aligned}$$

where $\mathrm{sign}(\cdot)$ means a signum function and

$$\mathrm{sign}(-\xi_{m,t}^{[i]} \xi_{m,t-1}^{[i]}) = \begin{cases} 1, & -\xi_{m,t}^{[i]} \xi_{m,t-1}^{[i]} > 0 \\ -1, & -\xi_{m,t}^{[i]} \xi_{m,t-1}^{[i]} \leq 0. \end{cases}$$

Subsequently, by substituting (13) into (9)–(10), we derive a state estimator structure that incorporates both system measurement information and decoder output information, which can be formulated as:

$$\begin{aligned} \hat{x}_{m,t+1} &= \Psi_t \hat{x}_{m,t} + W_t \phi(V_t \hat{x}_{m,t}) + K_{m,t}^P \big((0.5I + \Delta_{m,t}) \\ &\quad \times y_{m,t} + A_{m,t}(0.5I - \Delta_{m,t}) y_{m,t-1} - C_m \hat{x}_{m,t}\big) \\ &\quad + K_{m,t}^I \mu_{m,t} \quad (14) \\ \mu_{m,t+1} &= \mu_{m,t} + F_{m,t}\big((0.5I + \Delta_{m,t}) y_{m,t} \\ &\quad + A_{m,t}(0.5I - \Delta_{m,t}) y_{m,t-1} - C_m \hat{x}_{m,t}\big) \quad (15) \end{aligned}$$

where

$$\begin{aligned} \alpha_{m,t}^{[i]} &\triangleq \mathrm{sign}(-\xi_{m,t}^{[i]} \xi_{m,t-1}^{[i]}) \\ A_{m,t} &\triangleq \mathrm{diag}\{\alpha_{m,t}^{[1]}, \alpha_{m,t}^{[2]}, \ldots, \alpha_{m,t}^{[l]}\} \\ \Delta_{m,t} &\triangleq \mathrm{diag}\{\delta_{m,t}^{[1]}, \delta_{m,t}^{[2]}, \ldots, \delta_{m,t}^{[l]}\}. \end{aligned}$$

In ANNs, the presence of nonlinear activation functions poses significant theoretical challenges to the state estimation problem. To overcome this technical difficulty, we employ the Taylor series expansion method. Specifically, the nonlinear activation function of neurons $\phi(V_t x_t)$ is linearized through Taylor expansion into the following form:

$$\phi(V_t x_t) = \phi(V_t \hat{x}_{m,t}) + \Xi_{m,t} V_t (x_t - \hat{x}_{m,t}) + \rho_{m,t} \tag{16}$$

where $\Xi_{m,t}$ is the Jacobian matrix and $\rho_{m,t}$ is the high-order Lagrange remainder. Here,

$$\begin{aligned}
\Xi_{m,t} &\triangleq \left.\frac{\partial \phi(V_t x_t)}{\partial (V_t x)}\right|_{x=\hat{x}_{m,t}} \\
\rho_{m,t} &\triangleq \frac{1}{2}\mathrm{diag}_n\big\{(x_t - \hat{x}_{m,t})^{\mathrm{T}}\big\} \left.\frac{\partial^2 \phi(V_t x_t)}{\partial (V_t x)^2}\right|_{x=\varepsilon_{m,t}} \\
&\quad \times (x_t - \hat{x}_{m,t}) \\
\varepsilon_{m,t} &\triangleq \Lambda_{m,t} x_t + (I - \Lambda_{m,t})\hat{x}_{m,t} \\
\Lambda_{m,t} &\triangleq \mathrm{diag}\{\lambda_{m,t}^{[1]}, \lambda_{m,t}^{[2]}, \ldots, \lambda_{m,t}^{[n]}\} \\
\lambda_{m,t}^{[o]} &\in [0,1], \quad o = 1, 2, \ldots, n.
\end{aligned}$$

Then, letting the estimation error be $e_{m,t} \triangleq x_t - \hat{x}_{m,t}$, one obtains

$$\begin{aligned}
e_{m,t+1} = {} & (\Psi_t + \Xi_{m,t} V_t) e_{m,t} + w_t + \rho_{m,t} - K_{m,t}^P \\
& \times \big((0.5I + \Delta_{m,t}) y_{m,t} + A_{m,t}(0.5I - \Delta_{m,t}) \\
& \times y_{m,t-1} - C_m \hat{x}_{m,t}\big) - K_{m,t}^I \mu_{m,t}.
\end{aligned} \tag{17}$$

Remark 3: Acknowledging the OBEM-induced data distortion problem, achieving the desired state estimation task becomes challenging when only utilizing the variable d_m. In this context, an innovative approach is employed that utilizes both the system measurement information $y_{m,t}$ and the encoding information $\xi_{m,t}$ to reconstruct the variable d_m within the OBEM framework (as shown in (13)). This method aims to enhance the estimation performance of the system as much as possible. It is also important to note that including the OBEM poses significant challenges in parameter solving and performance analysis for the designed PIO-based fusion estimation scheme.

The main objectives of this paper are highlighted as follows.

1) Determine estimator parameters $K_{m,t}^P$, $K_{m,t}^I$ and $F_{m,t}$ such that, under the influence of the OBEM, for each sensor node m, the estimation error is confined to the local ellipsoidal set:

$$\zeta_{m,t} \in \mathscr{E}(0, P_{m,t}), \quad m = 1, 2, \ldots, M \tag{18}$$

where $P_{m,t} > 0$ is the shape-matrix of the ellipsoidal set constraining the variable $\zeta_{m,t}$, and $\hat{P}_{m,t} > 0$ and $\check{P}_{m,t} > 0$ represent, respectively, the shape-matrices of the ellipsoidal sets that bound the estimation error $e_{m,t}$ and the integral of the output estimation error $\mu_{m,t}$ with

$$\zeta_{m,t} \triangleq \begin{bmatrix} e_{m,t}^{\mathrm{T}} & \mu_{m,t}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \ P_{m,t} \triangleq \mathrm{diag}\{\hat{P}_{m,t}, \check{P}_{m,t}\}.$$

2) Develop a fusion rule for all sensor nodes, such that the fused shape-matrix $P_{f,t}$ is smaller than all local shape-matrices $\hat{P}_{m,t}$ in the sense of matrix trace:

$$\mathrm{Tr}[P_{f,t}] < \mathrm{Tr}[\hat{P}_{m,t}], \quad m = 1, 2, \ldots, M \tag{19}$$

where $P_{f,t} > 0$ represents the shape-matrix of the ellipsoidal set that bounds the fused estimation error defined in (42).

III. PIO-based Set-Membership Fusion Estimator under OBEM

In this section, we first design a PIO-based set-membership state estimation scheme under the OBEM framework, and then determine a desired fusion rule for sensor nodes by solving an optimization problem. This approach aims to effectively integrate the OBEM with the PIO-based fusion estimation methodology, thereby enhancing the overall state estimation accuracy and efficiency.

In order to facilitate the derivation of our main results, it is essential to first present a set of lemmas.

Lemma 1: [50] For any matrices $\mathcal{M}$, $\mathcal{N}$, and Δ_t with $\|\Delta_t\|^2 \le 1$, there exist a positive scalar κ such that

$$\mathcal{M}\Delta_t\mathcal{N} + (\mathcal{M}\Delta_t\mathcal{N})^{\mathrm{T}} \le \kappa\mathcal{M}\mathcal{M}^{\mathrm{T}} + \kappa^{-1}\mathcal{N}^{\mathrm{T}}\mathcal{N}.$$

We introduce the following notation simplifications for improved readability. All variables involved herein have been previously defined in the preceding sections.

$$\begin{aligned}
&\mathcal{N}_{m,t}^{[1]} \triangleq \begin{bmatrix} C_m\hat{x}_{m,t} & 0 & \frac{1}{2}C_m\hat{\Omega}_{m,t} & \mathbf{0}_5 \end{bmatrix}, \ \mathcal{N}_{m,t}^{[4]} \triangleq \begin{bmatrix} \mathbf{0}_7 & \frac{1}{2}I \end{bmatrix} \\
&\mathcal{N}_{m,t}^{[2]} \triangleq \begin{bmatrix} \frac{1}{2}C_{m,t}\hat{x}_{m,t-1} & \mathbf{0}_2 & \frac{1}{2}C_{m,t}\hat{\Omega}_{m,t-1} & \mathbf{0}_4 \end{bmatrix} \\
&\mathcal{N}_{m,t}^{[3]} \triangleq \begin{bmatrix} \mathbf{0}_6 & \frac{1}{2}I & 0 \end{bmatrix}, \ \mathcal{N}_{m,t}^{[5]} = \mathcal{N}_{m,t}^{[1]}, \ \mathcal{N}_{m,t}^{[6]} = \mathcal{N}_{m,t}^{[2]} \\
&\mathcal{N}_{m,t}^{[7]} = \mathcal{N}_{m,t}^{[3]}, \ \mathcal{N}_{m,t}^{[8]} = \mathcal{N}_{m,t}^{[4]}, \ \mathcal{M}_{m,t}^{[3]} = \mathcal{M}_{m,t}^{[1]} \\
&\mathcal{M}_{m,t}^{[1]} \triangleq \mathrm{col}\{-K_{m,t}^P, \mathbf{0}_7^{\mathrm{T}}\}, \ \mathcal{M}_{m,t}^{[2]} \triangleq \mathrm{col}\{K_{m,t}^P A_{m,t}, \mathbf{0}_7^{\mathrm{T}}\} \\
&\mathcal{M}_{m,t}^{[5]} \triangleq \mathrm{col}\{F_{m,t}, \mathbf{0}_7^{\mathrm{T}}\}, \ \mathcal{M}_{m,t}^{[6]} \triangleq \mathrm{col}\{-F_{m,t}A_{m,t}, \mathbf{0}_7^{\mathrm{T}}\} \\
&\mathcal{M}_{m,t}^{[4]} = \mathcal{M}_{m,t}^{[2]}, \ \mathcal{M}_{m,t}^{[7]} = \mathcal{M}_{m,t}^{[5]}, \ \mathcal{M}_{m,t}^{[8]} = \mathcal{M}_{m,t}^{[6]}.
\end{aligned}$$

A. Design of the PIO-based Set-Membership Estimator

In this subsection, we aim to design a local set-membership state estimator based on PIO structure for each sensor node, and derive the sufficient condition that guarantees estimation error satisfies the performance index (18).

Theorem 1: Let the positive scalar d_m be given and $\zeta_{m,0} \in \mathscr{E}(0, P_{m,0})$. Consider the ANN (2), the OBEM (7), and the PIO-based state estimator (9)–(10). If

$$\zeta_{m,t} \in \mathscr{E}(0, P_{m,t}), \tag{20}$$

then the variable $\zeta_{m,t+1}$ belongs to the local ellipsoidal set $\mathscr{E}(0, P_{m,t+1})$ and the estimator gains are obtained by solving

$$\begin{bmatrix} -\vec{\Upsilon}_{m,t} & * & * \\ \vec{\Theta}_{m,t} & -P_{m,t+1} & * \\ \mathcal{M}_{m,t}^{\mathrm{T}} & 0 & -\kappa_{m,t}I \end{bmatrix} \le 0 \tag{21}$$

where

$$\begin{aligned}
\vec{\Upsilon}_{m,t} \triangleq \mathrm{diag}\Big\{ & 1 - \sum_{u=1}^{7}\pi_{m,t}^{[u]}, \pi_{m,t}^{[1]}I, \pi_{m,t}^{[2]}I, \pi_{m,t}^{[3]}I, \\
& \pi_{m,t}^{[4]}S_{m,t}^{-1}, \pi_{m,t}^{[5]}Q_t^{-1}, \pi_{m,t}^{[6]}R_{m,t}^{-1}, \pi_{m,t}^{[7]}R_{m,t-1}^{-1}\Big\}
\end{aligned}$$

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$$-\sum_{s=1}^{8}\kappa_{m,t}^{[s]}(\mathcal{N}_{m,t}^{[s]})^{\mathrm{T}}\mathcal{N}_{m,t}^{[s]},\quad \Upsilon^{[0]}\triangleq\mathrm{diag}\{1,\mathbb{O}_7\}$$

$$\pi_{m,t}^{[u]}>0,\ u=1,2,\ldots,7,\ \kappa_{m,t}^{[s]}>0,\ s=1,2,\ldots,8$$

$$\vec{\Theta}_{m,t}\triangleq\begin{bmatrix}\vec{\Theta}_{m,t}^{[1,1]} & \vec{\Theta}_{m,t}^{[1,2]} & \vec{\Theta}_{m,t}^{[1,3]} & \vec{\Theta}_{m,t}^{[1,4]} & I & I & \vec{\Theta}_{m,t}^{[1,7]}\\ \vec{\Theta}_{m,t}^{[2,1]} & \vec{\Theta}_{m,t}^{[2,2]} & \vec{\Theta}_{m,t}^{[2,3]} & \vec{\Theta}_{m,t}^{[2,4]} & 0 & 0 & \vec{\Theta}_{m,t}^{[2,7]}\end{bmatrix}$$

$$\vec{\Theta}_{m,t}^{[1,1]}\triangleq -K_{m,t}^{P}C_{m,t}\hat{x}_{m,t}-0.5K_{m,t}^{P}A_{m,t}C_{m,t}\hat{x}_{m,t-1}$$

$$\vec{\Theta}_{m,t}^{[1,2]}\triangleq -K_{m,t}^{I}\check{\Omega}_{m,t},\ \vec{\Theta}_{m,t}^{[1,4]}\triangleq -0.5K_{m,t}^{P}A_{m,t}C_{m,t}\hat{\Omega}_{m,t-1}$$

$$\vec{\Theta}_{m,t}^{[1,3]}\triangleq(\Psi_t+\Xi_{m,t}V_t)\hat{\Omega}_{m,t}-0.5K_{m,t}^{P}C_{m,t}\hat{\Omega}_{m,t}$$

$$\vec{\Theta}_{m,t}^{[1,7]}\triangleq\begin{bmatrix}-0.5K_{m,t}^{P} & -0.5K_{m,t}^{P}A_{m,t}\end{bmatrix}$$

$$\vec{\Theta}_{m,t}^{[2,1]}\triangleq -0.5F_{m,t}C_{m,t}\hat{x}_{m,t}+0.5F_{m,t}A_{m,t}C_{m,t}\hat{x}_{m,t-1}$$

$$\vec{\Theta}_{m,t}^{[2,2]}\triangleq\check{\Omega}_{m,t},\quad \vec{\Theta}_{m,t}^{[2,3]}\triangleq 0.5F_{m,t}C_{m,t}\hat{\Omega}_{m,t}$$

$$\vec{\Theta}_{m,t}^{[2,4]}\triangleq 0.5F_{m,t}A_{m,t}C_{m,t}\hat{\Omega}_{m,t-1}$$

$$\vec{\Theta}_{m,t}^{[2,7]}\triangleq\begin{bmatrix}0.5F_{m,t} & 0.5F_{m,t}A_{m,t}\end{bmatrix}$$

$$\mathcal{M}_{m,t}\triangleq\begin{bmatrix}\mathcal{M}_{m,t}^{[1]} & \mathcal{M}_{m,t}^{[2]} & \cdots & \mathcal{M}_{m,t}^{[8]}\end{bmatrix}$$

$$\kappa_{m,t}\triangleq\mathrm{diag}\{\kappa_{m,t}^{[1]},\kappa_{m,t}^{[2]},\ldots,\kappa_{m,t}^{[8]}\}.$$

Proof: First, we shall construct the ellipsoidal constraint set that bounds the Lagrange remainder term. Combining Definition 1, it is readily deduced from (20) that

$$(x_t-\hat{x}_{m,t})^{\mathrm{T}}\hat{P}_{m,t}^{-1}(x_t-\hat{x}_{m,t})\le 1 \tag{22}$$

where $\hat{P}_{m,t}>0$ denotes the shape-matrix that characterizes the ellipsoidal set of estimation errors. Then, it is obvious that, for $\varepsilon_{m,t}=\Lambda_{m,t}x_t+(I-\Lambda_{m,t})\hat{x}_{m,t}$, we have

$$(\varepsilon_{m,t}-\hat{x}_{m,t})^{\mathrm{T}}\hat{P}_{m,t}^{-1}(\varepsilon_{m,t}-\hat{x}_{m,t})\le 1 \tag{23}$$

which further indicates that

$$\varepsilon_{m,t}^{[o]}\in\chi_{m,t}^{[o]}\triangleq\left[\hat{x}_{m,t}^{[o]}-\sqrt{\hat{P}_{m,t}^{[o,o]}},\hat{x}_{m,t}^{[o]}+\sqrt{\hat{P}_{m,t}^{[o,o]}}\right] \tag{24}$$

for $o=1,2,\ldots,n$, with $\varepsilon_{m,t}^{[o]}$ and $\hat{x}_{m,t}^{[o]}$ being, respectively, the oth element of vectors $\varepsilon_{m,t}$ and $\hat{x}_{m,t}$, and $\hat{P}_{m,t}^{[o,o]}$ being the (o,o)-element of the matrix $\hat{P}_{m,t}$. Obviously, for the multi-variable case, we obtain the following interval vector:

$$\mathcal{X}_{m,t}=\frac{1}{2}\mathrm{diag}_n\{(\chi_{m,t}-\hat{x}_{m,t})^{\mathrm{T}}\}\begin{bmatrix}\mathcal{H}_1(\chi_{m,t})\\ \mathcal{H}_2(\chi_{m,t})\\ \vdots\\ \mathcal{H}_n(\chi_{m,t})\end{bmatrix}\times(\chi_{m,t}-\hat{x}_{m,t}) \tag{25}$$

where, for $o=1,2,\ldots,n$, $\mathcal{H}_o(\cdot)$ is the Hessian matrix of the activation function $\phi_o(\cdot)$ with $\phi_o(\cdot)$ being the oth element of the function $\phi(\cdot)$.

With the assistance of the interval analysis technique and combining with (25), it is known that the Lagrange remainder $\rho_{m,t}$ in (17) can be bounded by an ellipsoidal set. The ellipsoidal set with the minimized volume is given as follows:

$$\rho_{m,t}\in\mathscr{E}(0,S_{m,t}) \tag{26}$$

where

$$S_{m,t}^{[o,p]}=\begin{cases}2(\mathcal{X}_{m,t}^{[o,+]}-\mathcal{X}_{m,t}^{[o,-]})^2, & o=p\\ 0, & o\neq p\end{cases}$$

with $\mathcal{X}_{m,t}^{[o,-]}$ and $\mathcal{X}_{m,t}^{[o,+]}$ being the minimum and maximum values of the oth interval vector $\mathcal{X}_{m,t}^{[o]}$, and $S_{m,t}^{[o,p]}$ being the (o,p)-element of $S_{m,t}$.

Next, we analyze the set-membership estimation performance index associated with the variable $\zeta_{m,t}$. By utilizing (20) again, it is obvious that there exist vectors $\hat{z}_{m,t}$ and $\check{z}_{m,t}$ (with $\|\hat{z}_{m,t}\|\le 1$ and $\|\check{z}_{m,t}\|\le 1$) such that the following equations

$$x_t-\hat{x}_{m,t}=\hat{\Omega}_{m,t}\hat{z}_{m,t} \tag{27}$$

and

$$\mu_{m,t}=\check{\Omega}_{m,t}\check{z}_{m,t} \tag{28}$$

are satisfied, where $\hat{\Omega}_{m,t}$ and $\check{\Omega}_{m,t}$ are, respectively, the factorizations of the matrices $\hat{P}_{m,t}$ and $\check{P}_{m,t}$ with $\hat{P}_{m,t}=\hat{\Omega}_{m,t}\hat{\Omega}_{m,t}^{\mathrm{T}}$ and $\check{P}_{m,t}=\check{\Omega}_{m,t}\check{\Omega}_{m,t}^{\mathrm{T}}$. Then, by substituting (28) into (15), we obtain

$$\begin{aligned}\mu_{m,t+1}=\ &\check{\Omega}_{m,t}\check{z}_{m,t}+F_{m,t}\big((0.5I+\Delta_{m,t})(C_{m,t}(\hat{x}_{m,t}\\ &+\hat{\Omega}_{m,t}\hat{z}_{m,t})+v_{m,t})+A_{m,t}(0.5I-\Delta_{m,t})\\ &\times((C_{m,t}(\hat{x}_{m,t-1}+\hat{\Omega}_{m,t-1}\hat{z}_{m,t-1})\\ &+v_{m,t-1}))-C_{m,t}\hat{x}_{m,t}\big).\end{aligned} \tag{29}$$

Similarly, by substituting (27)–(28) into (17), we have

$$\begin{aligned}e_{m,t+1}=\ &(\Psi_t+\Xi_{m,t}V_t)\hat{\Omega}_{m,t}\hat{z}_{m,t}+w_t+\rho_{m,t}-K_{m,t}^{P}\\ &\times\big((0.5I+\Delta_{m,t})(C_{m,t}(\hat{x}_{m,t}+\hat{\Omega}_{m,t}\hat{z}_{m,t})\\ &+v_{m,t})+A_{m,t}(0.5I-\Delta_{m,t})(C_{m,t}(\hat{x}_{m,t-1}\\ &+\hat{\Omega}_{m,t-1}\hat{z}_{m,t-1})+v_{m,t-1})-C_{m,t}\hat{x}_{m,t}\big)\\ &-K_{m,t}^{I}\check{\Omega}_{m,t}\check{z}_{m,t}.\end{aligned} \tag{30}$$

Combining (29) and (30) together, one has the following compact form:

$$\zeta_{m,t+1}=\Theta_{m,t}\eta_{m,t} \tag{31}$$

where

$$\eta_{m,t}\triangleq\mathrm{col}\{1,\check{z}_{m,t},\hat{z}_{m,t},\hat{z}_{m,t-1},\rho_{m,t},w_t,v_{m,t},v_{m,t-1}\}$$

$$\Theta_{m,t}\triangleq\begin{bmatrix}\Theta_{m,t}^{[1,1]} & \Theta_{m,t}^{[1,2]} & \Theta_{m,t}^{[1,3]} & \Theta_{m,t}^{[1,4]} & I & I & \Theta_{m,t}^{[1,7]}\\ \Theta_{m,t}^{[2,1]} & \Theta_{m,t}^{[2,2]} & \Theta_{m,t}^{[2,3]} & \Theta_{m,t}^{[2,4]} & 0 & 0 & \Theta_{m,t}^{[2,7]}\end{bmatrix}$$

$$\begin{aligned}\Theta_{m,t}^{[1,1]}\triangleq\ &-K_{m,t}^{P}((B_{m,t}C_{m,t}+C_{m,t})\hat{x}_{m,t}\\ &+A_{m,t}D_{m,t}C_{m,t}\hat{x}_{m,t-1}),\quad \Theta_{m,t}^{[1,2]}\triangleq -K_{m,t}^{I}\check{\Omega}_{m,t}\end{aligned}$$

$$\Theta_{m,t}^{[1,3]}\triangleq(\Psi_t+\Xi_{m,t}V_t)\hat{\Omega}_{m,t}-K_{m,t}^{P}B_{m,t}C_{m,t}\hat{\Omega}_{m,t}$$

$$\Theta_{m,t}^{[1,4]}\triangleq -K_{m,t}^{P}A_{m,t}D_{m,t}C_{m,t}\hat{\Omega}_{m,t-1}$$

$$\Theta_{m,t}^{[1,7]}\triangleq\begin{bmatrix}-K_{m,t}^{P}B_{m,t} & -K_{m,t}^{P}A_{m,t}D_{m,t}\end{bmatrix}$$

$$\begin{aligned}\Theta_{m,t}^{[2,1]}\triangleq\ &F_{m,t}((B_{m,t}C_{m,t}-C_{m,t})\hat{x}_{m,t}\\ &+A_{m,t}D_{m,t}C_{m,t}\hat{x}_{m,t-1})\end{aligned}$$

$$\Theta_{m,t}^{[2,2]}\triangleq\check{\Omega}_{m,t},\quad \Theta_{m,t}^{[2,3]}\triangleq F_{m,t}B_{m,t}C_{m,t}\hat{\Omega}_{m,t}$$

$$\Theta_{m,t}^{[2,4]}\triangleq F_{m,t}A_{m,t}D_{m,t}C_{m,t}\hat{\Omega}_{m,t-1}$$

$$\Theta_{m,t}^{[2,7]}\triangleq\begin{bmatrix}F_{m,t}B_{m,t} & F_{m,t}A_{m,t}D_{m,t}\end{bmatrix}$$

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$$B_{m,t} \triangleq 0.5I + \Delta_{m,t}, \quad D_{m,t} \triangleq 0.5I - \Delta_{m,t}.$$

Then, according to (3), (5) and (26)–(27), it is apparent that the following conditions are satisfied:

$$\eta_{m,t}^{\mathrm{T}} \Upsilon_{m,t}^{[u]} \eta_{m,t} \le 0, \quad u = 1, 2, \ldots, 7 \tag{32}$$

where

$$\begin{aligned}
\Upsilon_{m,t}^{[1]} &\triangleq \mathrm{diag}\{-1, I, \mathbb{O}_6\} \\
\Upsilon_{m,t}^{[2]} &\triangleq \mathrm{diag}\{-1, 0, I, \mathbb{O}_5\} \\
\Upsilon_{m,t}^{[3]} &\triangleq \mathrm{diag}\{-1, \mathbb{O}_2, I, \mathbb{O}_4\} \\
\Upsilon_{m,t}^{[4]} &\triangleq \mathrm{diag}\{-1, \mathbb{O}_3, S_{m,t}^{-1}, \mathbb{O}_3\} \\
\Upsilon_{m,t}^{[5]} &\triangleq \mathrm{diag}\{-1, \mathbb{O}_4, Q_t^{-1}, \mathbb{O}_2\} \\
\Upsilon_{m,t}^{[6]} &\triangleq \mathrm{diag}\{-1, \mathbb{O}_5, R_{m,t}^{-1}, 0\} \\
\Upsilon_{m,t}^{[7]} &\triangleq \mathrm{diag}\{-1, \mathbb{O}_6, R_{m,t-1}^{-1}\}.
\end{aligned}$$

By utilizing the Schur complement lemma, it is derived from (21) that the following equation holds:

$$-\vec{\Upsilon}_{m,t} + \vec{\Theta}_{m,t}^{\mathrm{T}} P_{m,t+1}^{-1} \vec{\Theta}_{m,t} + \kappa_{m,t}^{-1} \mathcal{M}_{m,t} \mathcal{M}_{m,t}^{\mathrm{T}} \le 0, \tag{33}$$

which further indicates that

$$\begin{aligned}
&\vec{\Theta}_{m,t}^{\mathrm{T}} P_{m,t+1}^{-1} \vec{\Theta}_{m,t} + \kappa_{m,t}^{-1} \mathcal{M}_{m,t} \mathcal{M}_{m,t}^{\mathrm{T}} \\
&\le \bar{\Upsilon}_{m,t} - \sum_{s=1}^{8} \kappa_{m,t}^{[s]} (\mathcal{N}_{m,t}^{[s]})^{\mathrm{T}} \mathcal{N}_{m,t}^{[s]}.
\end{aligned} \tag{34}$$

Here,

$$\begin{aligned}
\bar{\Upsilon}_{m,t} \triangleq \mathrm{diag}\Big\{ &1 - \sum_{u=1}^{7} \pi_{m,t}^{[u]}, \pi_{m,t}^{[1]} I, \pi_{m,t}^{[2]} I, \pi_{m,t}^{[3]} I, \\
&\pi_{m,t}^{[4]} S_{m,t}^{-1}, \pi_{m,t}^{[5]} Q_t^{-1}, \pi_{m,t}^{[6]} R_{m,t}^{-1}, \pi_{m,t}^{[7]} R_{m,t-1}^{-1} \Big\}.
\end{aligned}$$

With the aid of Lemma 1, it is inferred from (34) that

$$\begin{aligned}
&\vec{\Theta}_{m,t}^{\mathrm{T}} P_{m,t+1}^{-1} \vec{\Theta}_{m,t} + \mathcal{M}_{m,t} \Delta_{m,t} \mathcal{N}_{m,t} \\
&\quad + (\mathcal{M}_{m,t} \Delta_{m,t} \mathcal{N}_{m,t})^{\mathrm{T}} \\
&\le \vec{\Theta}_{m,t}^{\mathrm{T}} P_{m,t+1}^{-1} \vec{\Theta}_{m,t} + \kappa_{m,t}^{-1} \mathcal{M}_{m,t} \mathcal{M}_{m,t}^{\mathrm{T}} \\
&\quad + \sum_{s=1}^{8} \kappa_{m,t}^{[s]} (\mathcal{N}_{m,t}^{[s]})^{\mathrm{T}} \mathcal{N}_{m,t}^{[s]} \le \bar{\Upsilon}_{m,t}.
\end{aligned} \tag{35}$$

Based on the definitions of $\vec{\Theta}_{m,t}$ and $\Theta_{m,t}$ in (21) and (31), we can directly derive the following result:

$$\begin{aligned}
&\vec{\Theta}_{m,t}^{\mathrm{T}} P_{m,t+1}^{-1} \vec{\Theta}_{m,t} + \mathcal{M}_{m,t} \Delta_{m,t} \mathcal{N}_{m,t} \\
&\quad + (\mathcal{M}_{m,t} \Delta_{m,t} \mathcal{N}_{m,t})^{\mathrm{T}} \\
&= \Theta_{m,t}^{\mathrm{T}} P_{m,t+1}^{-1} \Theta_{m,t} \le \bar{\Upsilon}_{m,t}
\end{aligned} \tag{36}$$

with $\mathcal{N}_{m,t} \triangleq \mathrm{col}\{\mathcal{N}_{m,t}^{[1]}, \mathcal{N}_{m,t}^{[2]}, \ldots, \mathcal{N}_{m,t}^{[8]}\}$. Then, it follows from (36) that

$$\eta_{m,t}^{\mathrm{T}} \Theta_{m,t}^{\mathrm{T}} P_{m,t+1}^{-1} \Theta_{m,t} \eta_{m,t} \le \eta_{m,t}^{\mathrm{T}} \bar{\Upsilon}_{m,t} \eta_{m,t}, \tag{37}$$

which, together with (31) and (32), implies

$$\zeta_{m,t+1}^{\mathrm{T}} P_{m,t+1}^{-1} \zeta_{m,t+1} \le 1. \tag{38}$$

According to (38), we know that the variable $\zeta_{m,t+1}$ is confined in the local ellipsoidal set $\mathscr{E}(0, P_{m,t+1})$. By leveraging the method of mathematical induction, it becomes evident that the recursive feasibility of the estimation algorithm is ensured. Therefore, the proof is now complete. ■

B. Optimization of the PIO-based Set-Membership Estimator

From Theorem 1, it is known that

$$\zeta_{m,t}^{\mathrm{T}} \zeta_{m,t} \le \mathrm{Tr}\left[P_{m,t}\right] \tag{39}$$

which means that the estimation performance of the mth sensor node is related to $\mathrm{Tr}[P_{m,t}]$. Specifically, a smaller value of $\mathrm{Tr}[P_{m,t}]$ implies a reduction in the Euclidean norm of the estimation error, thereby achieving superior state estimation performance. Accordingly, to obtain the desired estimator gain matrices, the following theorem is presented, which solves the corresponding parameters by optimizing $\mathrm{Tr}[P_{m,t}]$.

Theorem 2: Let the positive scalar d_m be given and $\zeta_{m,0} \in \mathscr{E}(0, P_{m,0})$. Consider the ANN (2), the OBEM (7), and the PIO-based set-membership state estimator (9) and (10). The local ellipsoidal set $\mathscr{E}(0, P_{m,t})$ is minimized in the sense of matrix trace if the estimator gains are solved from the following optimization problem:

$$\begin{aligned}
\mathrm{OP\ 1}: \min_{K_{m,t}^P, K_{m,t}^I, F_{m,t}} \ &\mathrm{Tr}\left[P_{m,t}\right] \\
\text{s.t.} \ &(9),\ (10),\ \text{and}\ (21).
\end{aligned} \tag{40}$$

Proof: The proof is straightforward and is, therefore, omitted for the conciseness. ■

C. Design of the Fusion Rule

In order to fully leverage the local estimates $\hat{x}_{m,t}$ $(m = 1, 2, \ldots, M)$ for enhancing the global estimation performance, this subsection proposes a state estimation fusion rule for all local PIO-based set-membership state estimators.

First, the fusion rule is designed of the following form:

$$\hat{x}_{f,t} = \sum_{m=1}^{M} H_{m,t} \hat{x}_{m,t} \tag{41}$$

where $\hat{x}_{f,t} \in \mathbb{R}^n$ is the fused estimate and $H_{m,t}$ represents the fusion weight to be determined.

By defining $e_{f,t} \triangleq x_t - \hat{x}_{f,t}$ as the global fused estimation error, it is easy to obtain that

$$e_{f,t} = x_t - \sum_{m=1}^{M} H_{m,t} \hat{x}_{m,t}. \tag{42}$$

In addition, combining Definition 1 and Theorem 1, we known that the global estimation error $e_{f,t}$ resides in the *intersection* of all local ellipsoidal sets:

$$e_{f,t} \in \bigcap_{m=1}^{M} \mathscr{E}(0, \hat{P}_{m,t}). \tag{43}$$

It is obvious that the intersection set has a smaller volume than all local ellipsoidal sets, thereby guaranteeing a better estimation performance. The following theorem presents a sufficient condition to ensure the existence of parameters $H_{m,t}$ and $P_{f,t}$ such that the intersection of all local ellipsoidal sets is contained within the fused ellipsoidal set $\mathscr{E}(0, P_{f,t})$.

Theorem 3: For $m = 1, 2, \ldots, M$, let local shape-matrices $P_{m,t} > 0$, local state estimates $\hat{x}_{m,t}$, and scalars $\varpi_{m,t} \geq 0$ with $\sum_{m=1}^{M} \varpi_{m,t} = 1$ be given. Then, the intersection of all local ellipsoidal sets is confined to the fused ellipsoidal set if the fusion weights are selected as

$$H_{m,t} = \left(\sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \right)^{-1} \varpi_{m,t} \hat{P}_{m,t}^{-1}. \tag{44}$$

Furthermore, the fused shape-matrix is

$$P_{f,t} = (1 - \wp_t) \left(\sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \right)^{-1} \tag{45}$$

where

$$\wp_t \triangleq \sum_{m=1}^{M} \varpi_{m,t} \hat{x}_{m,t}^{\mathrm{T}} \hat{P}_{m,t}^{-1} \hat{x}_{m,t} - \hat{x}_{f,t}^{\mathrm{T}} \sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \hat{x}_{f,t}.$$

Proof: From Definition 1 and Theorem 1, we know that

$$x_t \in \mathscr{E}(\hat{x}_{m,t}, \hat{P}_{m,t}), \quad m = 1, 2, \ldots, M \tag{46}$$

are satisfied for all sensor nodes, which implies that the system state resides within the intersection of all ellipsoidal sets:

$$x_t \in \bigcap_{m=1}^{M} \mathscr{E}(\hat{x}_{m,t}, \hat{P}_{m,t}). \tag{47}$$

Then, according to set theory principles, the intersection of ellipsoidal sets in (47) is contained within the following convex set:

$$\bigcap_{m=1}^{M} \mathscr{E}(\hat{x}_{m,t}, \hat{P}_{m,t}) \subset \left\{ x_t \middle| \sum_{m=1}^{M} \varpi_{m,t} e_{m,t}^{\mathrm{T}} \hat{P}_{m,t}^{-1} e_{m,t} \leq 1 \right\}. \tag{48}$$

Next, by rather straightforward calculations and transformations, one has

$$\begin{aligned} &\sum_{m=1}^{M} \varpi_m (x_t - \hat{x}_{m,t})^{\mathrm{T}} \hat{P}_{m,t}^{-1} (x_t - \hat{x}_{m,t}) \\ &= x_t^{\mathrm{T}} \hat{\mathbf{P}}_t^{-1} x_t + \hat{\mathbf{x}}_t^{\mathrm{T}} \hat{\mathbf{P}}_t \hat{\mathbf{x}}_t - \hat{\mathbf{x}}_t^{\mathrm{T}} \hat{\mathbf{P}}_t \hat{\mathbf{x}}_t - 2 x_t^{\mathrm{T}} \hat{\mathbf{x}}_t \\ &\quad + \sum_{m=1}^{M} \varpi_{m,t} \hat{x}_{m,t}^{\mathrm{T}} \hat{P}_{m,t}^{-1} \hat{x}_{m,t} \\ &= \big(x_t - \hat{\mathbf{P}}_t \hat{\mathbf{x}}_t\big)^{\mathrm{T}} \hat{\mathbf{P}}_t^{-1} \big(x_t - \hat{\mathbf{P}}_t \hat{\mathbf{x}}_t\big) - \big(\hat{\mathbf{P}}_t \hat{\mathbf{x}}_t\big)^{\mathrm{T}} \hat{\mathbf{P}}_t^{-1} \\ &\quad \times \big(\hat{\mathbf{P}}_t \hat{\mathbf{x}}_t\big) + \sum_{m=1}^{M} \varpi_{m,t} \hat{x}_{m,t}^{\mathrm{T}} \hat{P}_{m,t}^{-1} \hat{x}_{m,t}. \end{aligned} \tag{49}$$

where

$$\hat{\mathbf{x}}_t \triangleq \sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \hat{x}_{m,t}, \quad \hat{\mathbf{P}}_t^{-1} \triangleq \sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1}.$$

Then, by substituting (44) and (45) into (49), we have

$$\left(x_t - \sum_{m=1}^{M} H_{m,t} \hat{x}_{m,t} \right)^{\mathrm{T}} P_{f,t}^{-1} \left(x_t - \sum_{m=1}^{M} H_{m,t} \hat{x}_{m,t} \right) \leq 1, \tag{50}$$

which, together with (42) and (48), yields

$$\begin{aligned} &\{x_t | e_{f,t}^T P_{f,t}^{-1} e_{f,t} \leq 1\} \\ &= \left\{ x_t \middle| \sum_{m=1}^{M} \varpi_{m,t} e_{m,t}^{\mathrm{T}} \hat{P}_{m,t}^{-1} e_{m,t} \leq 1 \right\}. \end{aligned} \tag{51}$$

Thus, it follows from (48) and (51) that

$$\left(\bigcap_{m=1}^{M} \mathscr{E}(0, \hat{P}_{m,t}) \right) \subset \{x_t | e_{f,t}^T P_{f,t}^{-1} e_{f,t} \leq 1\}, \tag{52}$$

which completes the proof. ■

Theorems 1–3 establish the theoretical framework of the PIO-based set-membership fusion estimation algorithm, and its detailed implementation procedure is systematically presented in Algorithm 1.

Algorithm 1 PIO-based fusion estimation algorithm

1: **Input**. $y_{m,t}$, d_m, Q_t, $R_{m,t}$
2: **Output**. $\hat{x}_{m,t}$, $\hat{x}_{f,t}$, $\varpi_{m,t}$, $K_{m,t}^P$, $K_{m,t}^I$, $F_{m,t}$, $H_{m,t}$
3: *Step 1*. Utilize OBEM (6)–(8) to process measurement outputs before they are transmitted to the remote estimator.
4: *Step 2*. Calculate the estimator gains by solving the optimization problem (40) in Theorem 2.
5: *Step 3*. Calculate the fusion weights according to Theorems 3–4.
6: *Step 4*. Compute the fused estimate according to (41).
7: **Return** the fused estimate $\hat{x}_{f,t}$.

IV. Fusion Estimation Performance Analysis

The following theorem is given to show that, based on the fusion estimation scheme proposed in Theorem 3, the fused shape-matrix $P_{f,t}$ is smaller than all the local shape-matrices $P_{m,t}$ in the sense of matrix trace (i.e., (19)), indicating that the fusion estimation performance outperforms that of any local estimation node. Before proceeding further, the following lemma is needed.

Lemma 2: For $m = 1, 2, \ldots, M$, let local shape-matrices $\hat{P}_{m,t} > 0$, local state estimates $\hat{x}_{m,t}$, and scalars $\varpi_{m,t} \geq 0$ with $\sum_{m=1}^{M} \varpi_{m,t} = 1$ be given. Then, we have

$$P_{f,t} < \left(\sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \right)^{-1}. \tag{53}$$

Proof: For $m = 1, 2, \ldots, M$, by applying $\hat{P}_{m,t}^{-1} > 0$, one obtains

$$\sum_{m=1}^{M} \left(\varpi_{m,t} \begin{bmatrix} \hat{x}_{m,t}^{\mathrm{T}} \\ 1 \end{bmatrix} \hat{P}_{m,t}^{-1} \begin{bmatrix} \hat{x}_{m,t} & 1 \end{bmatrix} \right) > 0, \tag{54}$$

that is,

$$\begin{bmatrix} \sum_{m=1}^{M} \varpi_{m,t} \hat{x}_{m,t}^{\mathrm{T}} \hat{P}_{m,t}^{-1} \hat{x}_{m,t} & * \\ \sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \hat{x}_{m,t} & \sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \end{bmatrix} > 0. \tag{55}$$

By utilizing Schur complement lemma, we derive from (45) and (55) that

$$\wp_t > 0 \tag{56}$$

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which, together with (45) again, yields

$$P_{f,t} < \left(\sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \right)^{-1}. \tag{57}$$

Therefore, the proof is now complete. ■

Theorem 4: For $m = 1, 2, \ldots, M$, let local shape-matrices $\hat{P}_{m,t} > 0$ and local state estimates $\hat{x}_{m,t}$ be given. The fused shape-matrix $P_{f,t}$ is smaller than all local shape-matrices $\hat{P}_{m,t}$ in the presence of matrix trace, i.e.,

$$\mathrm{Tr}[P_{f,t}] < \mathrm{Tr}[\hat{P}_{m,t}]. \tag{58}$$

In addition, the scalars $\varpi_{m,t}$ can be optimized by solving the following optimization problem:

$$\begin{aligned} \text{OP 2}: \quad & \min_{\varpi_{m,t}} \ \mathrm{Tr}\left[(1-\wp_t) \left(\sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \right)^{-1} \right] \\ & \text{s.t.} \quad \sum_{m=1}^{M} \varpi_{m,t} = 1. \end{aligned} \tag{59}$$

Proof: It is obvious that there exists a set of feasible solutions $\varpi_{m,t} \geq 0$ with $\sum_{m=1}^{M} \varpi_{m,t} = 1$ to optimization problem (59) such that the inequality

$$\sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \geq \hat{P}_{m,t}^{-1} \tag{60}$$

holds for all $m \in \{1, 2, \ldots, M\}$. Then, based on Lemma 2, we can derive that

$$P_{f,t} < \left(\sum_{m=1}^{M} \varpi_{m,t} \hat{P}_{m,t}^{-1} \right)^{-1} \leq \hat{P}_{m,t}, \tag{61}$$

which indicates

$$\mathrm{Tr}[P_{f,t}] < \mathrm{Tr}[\hat{P}_{m,t}], \quad m = 1, 2, \ldots, M. \tag{62}$$

Thus, the proof is now complete. ■

Remark 4: In Theorems 1–4 of this paper, the primary focus has been on the PIO-based set-membership fusion estimation for ANNs in the context of the OBEM. Compared with the literature on state estimation and encoding-encoding problems, e.g., [9], [13], [21], [26], [48], [51], this paper involves several key developments outlined as follows.

1) *Novel Encoding Method*: A new encoding technique has been proposed, which efficiently utilizes just a single bit to encode the signal. This method is particularly significant in the context of bandwidth-limited scenarios and is introduced for the first time in this paper.
2) *Innovative OBEM-based PIO*: An advanced PIO has been developed within the set-membership estimation framework, specifically tailored to address the challenges arising from limited bandwidth, data distortion and UBBN, and this PIO is designed to be robust in the face of these constraints.
3) *Original Fusion Estimation Scheme*: A fusion estimation strategy for ANNs has been established by employing principles from set theory and optimization methods, and such scheme is strategically designed to ensure that the overall fusion estimation performance is superior to that of any individual local sensor node.

Theorems 1–4 collectively contribute to the advancement of set-membership fusion estimation techniques, particularly in environments challenged by encoding constraints and system uncertainties.

Remark 5: Thus far, the PIO-based fusion estimation issue has been addressed for ANNs with OBEM. Notably, our research exhibits several distinctive features: 1) the proposed OBEM is novel, effectively addressing problems stemming from bandwidth constraints; 2) the explored PIO-based fusion estimation challenge is pioneering through taking into account the OBEM, the PIO, and the ellipsoid-based fusion estimation problem; and 3) the performance analysis of the fusion estimation is rigorous in ensuring the efficacy of the proposed fusion algorithm.

V. Illustrative Example

In this section, an example is presented to demonstrate the usefulness of the proposed PIO-based fusion estimation algorithm. Let the parameters of the ANN (2) be given by

$$\Psi_t = \begin{bmatrix} 0.45 + 0.1\sin(0.15t) & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.4 + 0.1\mathrm{e}^{-t} \end{bmatrix}$$
$$W_t = \begin{bmatrix} 0.4 & 0.3 & 0.4 \\ 0.5 & 0.3 & 0.3 \\ 0.4 & 0.3 & 0.25 + 0.15\cos(0.2t) \end{bmatrix}$$
$$V_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.75 + 0.2\cos(0.1t) & 0 \\ 0 & 0 & 1.25 \end{bmatrix}.$$

For the sensor nodes, the parameters are

$$C_{1,t} = \begin{bmatrix} 1 & 0.15 & 1 \end{bmatrix}, \ C_{2,t} = \begin{bmatrix} 0.9 & 0.1\sin(0.12t) & 1 \end{bmatrix}$$
$$C_{3,t} = \begin{bmatrix} 1.2 & 0.08\sin(0.15t) & 1 \end{bmatrix}, \ C_{4,t} = \begin{bmatrix} 1.1 & 0.05 & 1 \end{bmatrix}.$$

The process noise w_t and the measurement noises $v_{m,t}$ $(m = 1, 2, 3, 4)$ are set as follows:

$$w_t = \begin{bmatrix} 0.8\mathrm{u}(0.5)\sin(0.1t) \\ -\mathrm{u}(0.3)\sin(0.15t) \\ \mathrm{u}(0.15)\cos(0.15t) \end{bmatrix}$$
$$v_{1,t} = \mathrm{u}(0.15)\sin(0.1t), \quad v_{2,t} = -\mathrm{u}(0.1)\sin(0.15t)$$
$$v_{3,t} = -\mathrm{u}(0.15)\cos(0.15t), \quad v_{4,t} = \mathrm{u}(0.2)\sin(0.1t)$$

where $\mathrm{u}(\cdot)$ obeys the uniform distribution. The other parameters are selected as $Q_t = \mathrm{diag}\{0.35, 0.35, 0.35\}$ and $R_{m,t} = \mathrm{diag}\{0.2, 0.2, 0.2\}$, $(m = 1, 2, 3, 4)$.

The simulation results are given in Figs. 1–9. Figs. 1–4 plot the dynamics of system state x_t and its estimates $\hat{x}_{m,t}$ $(m = 1, 2, 3, 4)$ from each sensor node under the influence of UBBNs and OBEM, which shows that the proposed PIO-based set-membership estimation algorithm attains a desirable level of performance. The estimation errors of each sensor node are given in Figs. 5–8. It is shown that the estimation errors are bounded, which further means that, for each sensor node, the estimation error is confined to a bounded ellipsoidal set. To verify the effectiveness of the designed fusion estimation

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method, corresponding result is given in Fig. 9. It is obvious that the trace of the fused shape matrix $P_{f,t}$ is smaller than that of all other local shape matrices $P_{m,t}$. In conclusion, Figs. 1–9 show the usefulness of the proposed PIO-based fusion estimation algorithm.

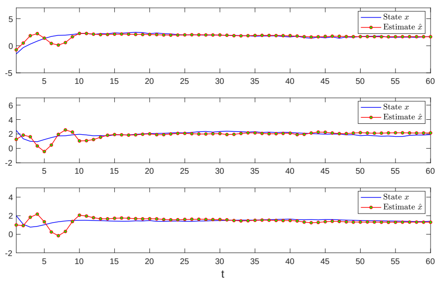

Fig. 1: System state x and its estimate $\hat{x}$ on sensor node 1.

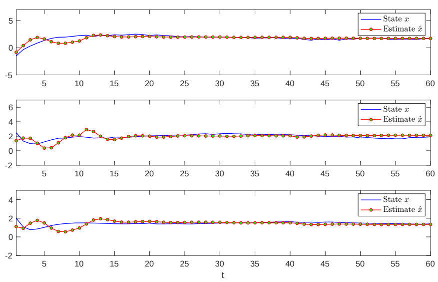

Fig. 2: System state x and its estimate $\hat{x}$ on sensor node 2.

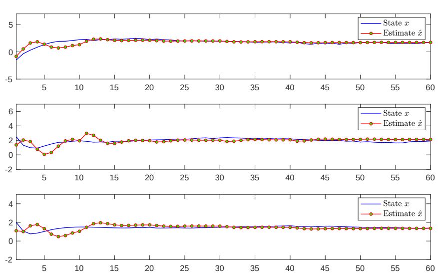

Fig. 3: System state x and its estimate $\hat{x}$ on sensor node 3.

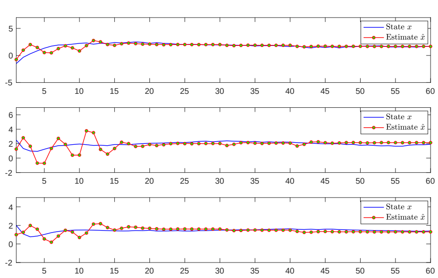

Fig. 4: System state x and its estimate $\hat{x}$ on sensor node 4.

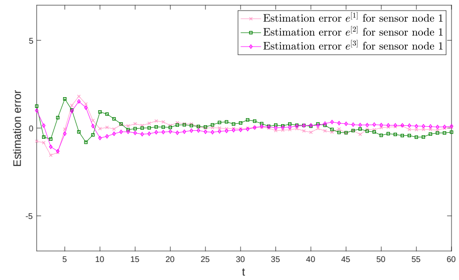

Fig. 5: Estimation error on sensor node 1.

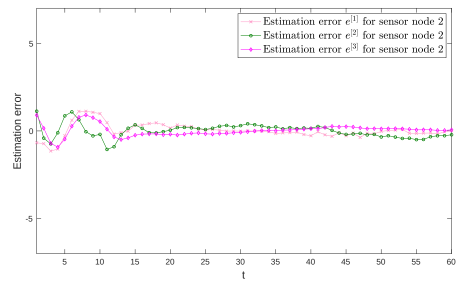

Fig. 6: Estimation error on sensor node 2.

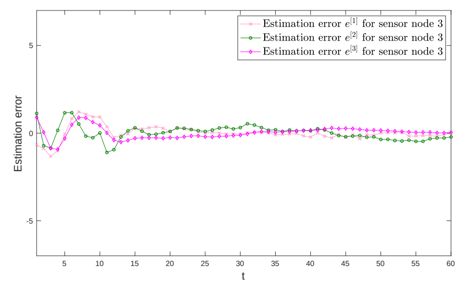

Fig. 7: Estimation error on sensor node 3.

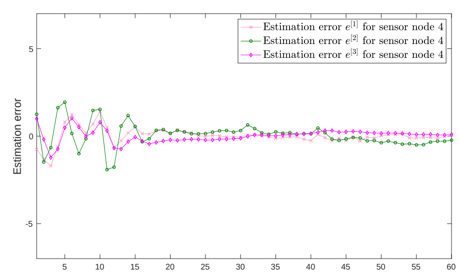

Fig. 8: Estimation error on sensor node 4.

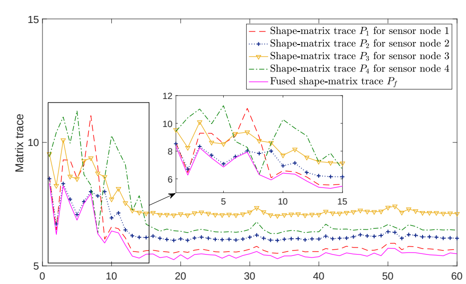

Fig. 9: Fusion estimation performance.

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VI. Conclusion

This paper has presented an investigation into the PIO-based set-membership fusion estimation for multi-sensor ANNs in the presence of OBEM. To address the effect caused by the capacity-limited communication network, a new OBEM has been proposed, which can encode the scalar data with only one bit, thereby improving the coding efficiency. Then, based on the OBEM, a PIO-based estimator has been designed for the state estimation task, and sufficient conditions have been derived to ensure the existence of desired estimators. For improving the global estimation performance, an ellipsoid-based fusion rule has been developed leveraging set theory and optimization technique, and the fusion estimation performance has been analyzed. Lastly, through a simulation example, the usefulness of the proposed estimation scheme has been demonstrated.

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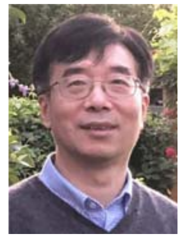

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