Robust H_∞ Filter Design With Variance Constraints and Parabolic Pole Assignment

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***Abstract*—In this letter, we consider a multiobjective filtering problem for uncertain linear continuous time-invariant systems subject to error variance constraints. A linear filter is used to estimate a linear combination of the system states. The problem addressed is the design of a filter such that, for all admissible parameter uncertainties, the following three objectives are simultaneously achieved: 1) the filtering process is $\mathcal{P}$-stable, i.e., the poles of the filtering matrix are located inside a parabolic region; 2) the steady-state variance of the estimation error of each state is not more than the individual prespecified value; and 3) the transfer function from exogenous noise inputs to error state outputs meets the prespecified H_∞ norm upper-bound constraint. An effective algebraic matrix inequality approach is developed to derive both the existence conditions and the explicit expression of the desired filters. An illustrative example is used to demonstrate the usefulness of the proposed design approach.**

***Index Terms*—Algebraic matrix inequality, error variance constraints, H_∞ filtering, Kalman filtering, pole assignment.**

I. Introduction

THE FILTERING problem has been playing an important role in signal processing and control engineering. Among various filtering schemes, the celebrated Kalman filtering approach minimizes the H_2 norm of the estimation error, under the assumptions that an exact model is available and the noise processes have exactly known statistical properties. To improve the robustness of Kalman filters, in the past decade, the robust H_∞ performance of the designed filters has become an important issue (see, e.g., [2], [3], [10], and [11]). On the other hand, it is quite common in filtering problems, such as the tracking of a maneuvering target and recognition of flight paths from multiple sources, to have performance objectives that are directly expressed as *upper bounds* on the variances of the estimation error [6]. These prescribed variance restrictions may not be minimal but should meet engineering requirements, and therefore, the obtained filters are often nonunique [7]–[9], [12].

It is well known that, by constraining the poles of the filtering matrix to lie inside a prescribed region in the open left-half plane, the filter designed would have expected transient performance. Besides, regional pole assignment can also provide indirect tolerance against plant uncertainties. In the past few years, the filter design problem with regional pole placement has received initial research attention (see, e.g., [11] and references therein). As indicated in [4], for linear time-invariant continuous systems, a parabolic region is directly related to the maximum percent overshoot and the rise time. Therefore, assigning the poles of the filtering matrix inside a prespecified parabolic region would guarantee satisfactory transient behavior of the filtering dynamics. It should be pointed out that a parabolic region cannot be simply represented by the intersections of linear matrix inequality (LMI) regions. Hence, the methods developed in [11] are not applicable for parabolic pole assignment.

Motivated by the above discussion, in this letter, it is our intention to deal with the robust H_∞ filtering problem for uncertain linear continuous time-invariant systems with both error variance and parabolic pole constraints, so that the resulting filtering process will be provided with expected transient property, steady-state error variance constraint, and disturbance rejection behavior, in the presence of parameter uncertainties.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The superscript "T" denotes the transpose. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. $Re(\lambda)$ and $Im(\lambda)$ represent the real and imaginary parts of the complex number λ, respectively.

II. Problem Formulation and Assumptions

Consider the following class of linear uncertain continuous-time systems

$$\dot{x}(t) = (A + \Delta A)x(t) + D_1 w(t) \quad (1)$$

$$y(t) = (C + \Delta C)x(t) + D_2 w(t) \quad (2)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, and A, C, D_1, and D_2 are known constant matrices. $w(t)$ is a zero-mean Gaussian white noise process with covariance $I > 0$. The initial state $x(0)$ has the mean $\bar{x}(0)$ and covariance $P(0)$ and is uncorrelated with $w(t)$. ΔA and ΔC are real-valued perturbation matrices satisfying

$$\begin{bmatrix}\Delta A\\ \Delta C\end{bmatrix} = \begin{bmatrix}M_1\\ M_2\end{bmatrix} FN \quad (3)$$

Manuscript received June 6, 2005; revised September 13, 2005. This work was supported in part by the Engineering and Physical Sciences Research Council (EPSRC) of the U.K. under Grant GR/S27658/01, in part by the Nuffield Foundation of the U.K. under Grant NAL/00630/G, and in part by the Alexander von Humboldt Foundation of Germany. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Amy E. Bell.

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Digital Object Identifier 10.1109/LSP.2005.862618

where $F \in \mathbb{R}^{i\times j}$ is a real uncertain matrix satisfying $F^TF \le I$, and M_1, M_2, and N are known constant matrices of appropriate dimensions.

Assumption 1: The system matrix A is Hurwitz stable, and the matrix D_2 or M_2 is of full-row rank.

The linear full-order filter is given by

$$\dot{\hat{x}}(t) = G\hat{x}(t) + Ky(t) \tag{4}$$

where $\hat{x}(t)$ denotes the state estimation, and G and K are filter parameters to be determined.

The estimation error covariance in the steady state is denoted by $P := \lim_{t\to\infty} P(t) := \lim_{t\to\infty} E[e(t)e^T(t)]$, where $e(t) = x(t) - \hat{x}(t)$, if the limit exists. By defining

$$\begin{aligned} x_f(t) &:= \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, & A_f &:= \begin{bmatrix} A & 0 \\ A-G-KC & G \end{bmatrix} \\ D_f &:= \begin{bmatrix} D_1 \\ D_1 - KD_2 \end{bmatrix}, & M_f &:= \begin{bmatrix} M_1 \\ M_1 - KM_2 \end{bmatrix} \\ N_f &:= [N \quad 0], & \Delta A_f &= M_f F(t) N_f \end{aligned}$$

and considering (1), (2), and (4), we obtain the following augmented system:

$$\dot{x}_f(t) = (A_f + \Delta A_f)x_f(t) + D_f w(t). \tag{5}$$

When the system (5) is robustly asymptotically stable, the steady-state covariance defined by

$$X := \lim_{t\to\infty} X(t) := \lim_{t\to\infty} E\left[x_f(t)x_f^T(t)\right] := \begin{bmatrix} X_{xx} & X_{xe} \\ X_{xe}^T & P \end{bmatrix} \tag{6}$$

exists and satisfies the following Lyapunov matrix equation:

$$(A_f + \Delta A_f)X + X(A_f + \Delta A_f)^T + D_f D_f^T = 0. \tag{7}$$

Fact 1: Let $a > 0$ and $b > 0$ be two positive scalars. If a complex number λ satisfies

$$Im^2(\lambda) \le -\frac{2}{b}\left(Re(\lambda) + a\right) \tag{8}$$

then λ is located within a parabolic region $\mathcal{P}(a,b)$ in the left-hand side of the complex plane where the vertex is at $-a$, as shown in Fig. 1.

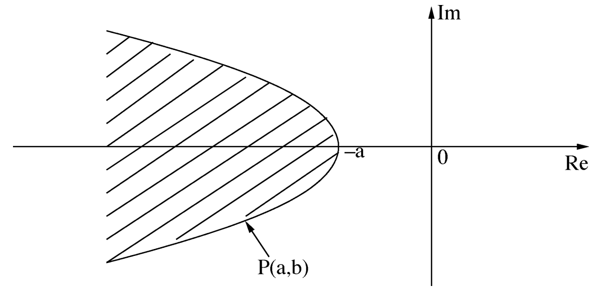

Fig. 1. Parabolic region $\mathcal{P}(a,b)$.

Assume that the error state outputs are represented by $Le(k)$, where L is a known constant matrix of appropriate dimension. Given a desired parabolic pole region $\mathcal{P}(a,b)$, we are now ready to state the variance-constrained H_∞ filtering problem considered for uncertain continuous systems. Our objective is to seek the filter parameters G and K such that for all admissible parameter uncertainties, the following three requirements are simultaneously satisfied:

1) The system (5) is $\mathcal{P}$-stable, i.e., all poles of the filtering matrix $A - KC$ remain within the parabolic pole region $\mathcal{P}(a,b)$.
2) The steady-state error covariance P meets

$$[P]_{ii} \le \sigma_i^2, \quad i = 1,2,\cdots,n \tag{9}$$

where $[P]_{ii}$ means the ith diagonal element of P, i.e., the steady-state variance of the ith state. $\sigma_i^2 (i = 1,2,\cdots,n)$ denotes the prespecified steady-state error estimation variance constraint on the ith state and can be determined by the practical performance requirements.
3) The H_∞ norm of the transfer function $H(s) = C_f[sI - (A_f + \Delta A_f)]^{-1}D_f$ from disturbances $w(t)$ to error state outputs $Le(t)$ (or $C_f x_f(t)$) satisfies the constraint $\|H(s)\|_\infty \le \lambda$, where L is the known error state output matrix, $C_f := [0 \quad L]$, where $\|H(s)\|_\infty = \sup_{\omega\in R} \sigma_{\max}[H(j\omega)]$ and $\sigma_{\max}[\cdot]$ denotes the largest singular value of $[\cdot]$; and γ is a given positive constant.

III. Main Results and Proofs

Lemma 1: [7] Let A, D, E, F, and P be real matrices of appropriate dimensions with $P > 0$ and F satisfying $F^TF \le I$. Then, for any scalars $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ where $DPD^T \le \varepsilon_2 I$, we have: 1) $DFE + (DFE)^T \le \varepsilon_1 DD^T + \varepsilon_1^{-1}E^TE$; and 2) $(A + DFE)^T P(A + DFE) \le A^T(P^{-1} - \varepsilon_2^{-1}DD^T)^{-1}A + \varepsilon_2 E^TE$.

The main results of this letter are given as follows, which show that the $\mathcal{P}$-stability, H_∞ performance, and the steady-state constraints on the filtering process are closely related to the positive definite solutions to a pair of Riccati-like matrix equations.

For notational simplicity, we first make the following definitions:

$$\begin{aligned}
\Gamma &:= \left(Q_1^{-1} - \varepsilon_1^{-1}N^TN\right)^{-1} \\
\Phi &:= b\Gamma^T A^T + Q_1 && (10)\\
\widehat{A} &:= A + \left[(b\varepsilon_1 + \varepsilon_2)M_1M_1^T + D_1D_1^T\right]\Phi^{-1} \\
\widehat{C} &:= C + \left[(b\varepsilon_1 + \varepsilon_2)M_2M_1^T + D_2D_1^T\right]\Phi^{-1} && (11)\\
R &:= b(C - \widehat{C})\Gamma(C - \widehat{C})^T + b\widehat{C}Q_2\widehat{C}^T \\
&\quad + (b\varepsilon_1 + \varepsilon_2)M_2M_2^T + D_2D_2^T && (12)\\
\Theta &:= b(A - \widehat{A})\Gamma(C - \widehat{C})^T + b\widehat{A}Q_2\widehat{C}^T \\
&\quad + Q_2\widehat{C}^T + (b\varepsilon_1 + \varepsilon_2)M_1M_2^T + D_1D_2^T && (13)\\
\Upsilon &:= b(A - \widehat{A})\Gamma(A - \widehat{A})^T + b\widehat{A}Q_2\widehat{A}^T + \widehat{A}Q_2 \\
&\quad + Q_2\widehat{A}^T + 2aQ_2 + \gamma^{-2}Q_2L^TLQ_2 \\
&\quad + (b\varepsilon_1 + \varepsilon_2)M_2M_2^T + D_1D_1^T && (14)\\
\Pi &:= b(A_f + \Delta A_f)Q(A_f + \Delta A_f)^T \\
&\quad + (A_f + \Delta A_f + aI)Q + Q(A_f + \Delta A_f + aI)^T \\
&\quad + \gamma^{-2}QC_f^TC_fQ + D_fD_f^T. && (15)
\end{aligned}$$

Theorem 1: Assume that the H_∞ norm upper-bound $\gamma > 0$ and the parabolic region $\mathcal{P}(a,b)$ are given. Let $\delta_1 > 0$ and $\delta_2 > 0$ be sufficiently small constants and $U \in \mathbb{R}^{p\times p}$ be an arbitrary orthogonal matrix. If there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and a matrix $H \in \mathbb{R}^{n\times p}$ such that $NQ_1N^T \le \varepsilon_1 I$ and the Riccati-like matrix equations

$$\begin{aligned}(A+aI)Q_1 + Q_1(A+aI)^T + bA\Gamma A^T + (b\varepsilon_1+\varepsilon_2)M_1M_1^T\\ + \varepsilon_2^{-1}Q_1N^TNQ_1 + D_1D_1^T + \delta_1 I = 0 \quad (16)\end{aligned}$$

$$\begin{aligned}(\widehat{A}+aI)Q_2 + Q_2(\widehat{A}+aI)^T + b\widehat{A}Q_2\widehat{A}^T\\ + \gamma^{-2}Q_2L^TLQ_2 + b(A-\widehat{A})\Gamma(A-\widehat{A})^T\\ + (b\varepsilon_1+\varepsilon_2)M_2M_2^T + D_1D_1^T\\ - \Theta R^{-1}\Theta^T + HH^T + \delta_2 I = 0 \quad (17)\end{aligned}$$

have positive definite solutions $Q_1 > 0$ and $Q_2 > 0$, respectively, where Γ, Φ, $\widehat{A}$, $\widehat{C}$, R, and Θ are defined in (10)–(13), then with the parameters determined by

$$K = \Theta R^{-1} + HUR^{-\frac{1}{2}}, \quad G = \widehat{A} - K\widehat{C} \quad (18)$$

the filter (4) will be such that, for all admissible perturbations ΔA and ΔC: 1) the filtering matrix $A_f + \Delta A_f$ satisfies the parabolic pole constraint $\sigma(A_f+\Delta A_f) \subset \mathcal{P}(a,b)$; 2) $\|H(s)\|_\infty \le \gamma$; and 3) the steady-state error covariance P exists and meets $P < Q_2$.

Proof: We first show that $\Pi < 0$, where Π is defined in (15). It follows from Lemma 1 that

$$\begin{aligned}(Af+\Delta A_f)Q(A_f+\Delta A_f)^T\\ \le A_f\left(Q^{-1}-\varepsilon_1^{-1}N_f^TN_f\right)^{-1}A_f^T + \varepsilon_1 M_fM_f^T \quad (19)\end{aligned}$$

$$\begin{aligned}(\Delta A_f)Q + Q(\Delta A_f)^T\\ \le \varepsilon_2 M_fM_f^T + \varepsilon_2^{-1}QN_f^TN_fQ. \quad (20)\end{aligned}$$

Set $Q :=$ Block-diag(Q_1, Q_2). After tedious calculation, we have from (19) and (20) that

$$\Pi \le \Psi := \begin{bmatrix}\Psi_{11} & \Psi_{12}\\ \Psi_{12}^T & \Psi_{22}\end{bmatrix} \quad (21)$$

where

$$\begin{aligned}\Psi_{11} =& (A+aI)Q_1 + Q_1(A+aI)^T + bA\Gamma A^T + (b\varepsilon_1+\varepsilon_2)\\ &\cdot M_1M_1^T + \varepsilon_2^{-1}Q_1N^TNQ_1 + D_1D_1^T \quad (22)\\ \Psi_{12} =& (bA\Gamma + Q_1)(A-G-KC)^T + (b\varepsilon_1+\varepsilon_2)\\ &\cdot M_1(M_1-KM_2)^T + D_1(D_1-KD_2)^T \quad (23)\\ \Psi_{22} =& b(A-G-KC)\Gamma(A-G-KC)^T + bGQ_2G^T\\ &+ (b\varepsilon_1+\varepsilon_2)(M_1-KM_2)(M_1-KM_2)^T\\ &+ GQ_2 + Q_2G^T + 2aQ_2 + \gamma^{-2}Q_2L^TLQ_2\\ &+ (D_1-KD_2)(D_1-KD_2)^T. \quad (24)\end{aligned}$$

Equation (16) means that $\Psi_{11} = -\delta_1 I < 0$, and the expression of G in (18) implies $\Psi_{12} = 0$. From Assumption 1, we know that R is invertible. Now, substituting the expression $G = \widehat{A} - K\widehat{C}$ into (24), we can verify that

$$\Psi_{22} = \Upsilon - K\Theta^T - \Theta K^T + KRK^T \quad (25)$$

where Θ and Υ are defined in (13) and (14), respectively.

Note that (17) is actually the same as $\Upsilon - \Theta R^{-1}\Theta^T + HH^T + \delta_2 I = 0$. Using the expression of K in (18) and fact $UU^T = I$, we obtain from (25) that

$$\begin{aligned}\Psi_{22} &= \Upsilon - \Theta R^{-1}\Theta^T + \left(KR^{\frac{1}{2}} - \Theta R^{-\frac{1}{2}}\right)\left(KR^{\frac{1}{2}} - \Theta R^{-\frac{1}{2}}\right)^T\\ &= \Upsilon - \Theta R^{-1}\Theta^T + (HU)(HU)^T\\ &= \Upsilon - \Theta R^{-1}\Theta^T + HH^T = -\delta_2 I < 0.\end{aligned}$$

Now, we have the conclusion that $\Pi \le \Psi < 0$. Let us first prove that $\sigma(A_f + \Delta A_f) \subset \mathcal{P}(a,b)$.

1) Denote $\Xi = -\Pi > 0$. Let $Z := \gamma^{-2}QC_f^TC_fQ + D_fD_f^T$, and it follows from (15) that

$$\begin{aligned}b(A_f+\Delta A_f)Q(A_f+\Delta A_f)^T + (A_f+\Delta A_f+aI)Q+\\ Q(A_f+\Delta A_f+aI)^T + Z + \Xi = 0. \quad (26)\end{aligned}$$

Let λ be an eigenvalue of $(A_f + \Delta A_f)^T$ and η be the associated eigenvector. Then, we have

$$(A_f+\Delta A_f)^T\eta = \lambda\eta, \quad \eta^*(A_f+\Delta A_f) = \bar{\lambda}\eta^* \quad (27)$$

where η^* denotes the complex conjugate transpose. Premultiplying and post-multiplying (26) by η^* and η, respectively, yield

$$(b\lambda\bar{\lambda} + \lambda + \bar{\lambda} + 2a)\eta^*Q\eta = -\eta^*(Z+\Xi)\eta \quad (28)$$

or $Im^2(\lambda) \le (-2/b)(Re(\lambda)+a)$, which implies from Fact 1 that the eigenvalues of $A_f+\Delta A_f$ are situated inside the parabolic region $\mathcal{P}(a,b)$.

2) Rearrange (26) as

$$\begin{aligned}(A_f+\Delta A_f)Q + Q(A_f+\Delta A_f)^T + D_fD_f^T+\\ \gamma^{-2}QC_f^TC_fQ + Y = 0 \quad (29)\end{aligned}$$

where $Y = b(A_f+\Delta A_f)Q(A_f+\Delta A_f)^T + 2aQ + \Xi > 0$. Then, the proof of $\|H(s)\|_\infty \le \gamma$ can be completed by a standard manipulation of (29); see [7] for more details.

3) Equation (26) can be further transformed into

$$(A_f+\Delta A_f)Q + Q(A_f+\Delta A_f)^T + D_fD_f^T + \Omega = 0 \quad (30)$$

where $\Omega := \gamma^{-2}QC_f^TC_fQ + b(A_f+\Delta A_f)Q(A_f+\Delta A_f)^T + 2aQ + \Xi > 0$. Notice that A_f is asymptotically stable. Subtract (7) from (30) to give $Q - X = (A_f+\Delta A_f)(Q-X) + (Q-X)(A_f+\Delta A_f)^T + \Omega$, or equivalently

$$Q - X = \int_0^\infty e^{(A_f+\Delta A_f)t}\Xi e^{(A_f+\Delta A_f)^Tt}dt \ge 0$$

which means that $X \le Q$ and $P \le Q_2$. The proof of Theorem 1 is then completed. ■

In view of Theorem 1, if the positive definite solutions Q_1 and Q_2 to (16) and (17) exist, and $Q_2 > 0$ meets $[Q_2]_{ii} \le \sigma_i^2$, $i = 1, 2, \cdots, n$, we will have the following conclusions: 1) the augmented system (5) is $\mathcal{P}$-stable; 2) $\|H(s)\|_\infty \le \gamma$; and 3) $[P]_{ii} < [Q_2]_{ii} \le \sigma_i^2$, $i = 1, 2, \cdots, n$. Hence, with the filter (4) whose parameters K and G are determined by (18), the variance-constrained robust H_∞ filtering gain design task will be accomplished.

Finally, a solution to the addressed filter design problem is given as a corollary from Theorem 1.

Corollary 1: Let the desired parabolic pole region $\mathcal{P}(a,b)$, the H_∞ disturbance attenuation constraint $\gamma > 0$, and the steady-state error variance constraints σ_i^2 $(i = 1,2,\cdots,n)$ be given. If there exist positive definite matrices $Q_1 > 0$ and $Q_2 > 0$ such that the conditions of Theorem 1 are all satisfied, then a desired filter gain K for the addressed filter design problem can be obtained by (18).

Remark 1: In practical applications, it is very desirable to directly solve Riccati matrix equations (16) and (17), subject to the constraint $[Q_2]_{ii} \le \sigma_i^2$ $(i = 1,2,\cdots,n)$, and then obtain the expected filter parameters readily from (18). We can see that the key step in designing the expected filters is to consider the solvability of the Riccati-like equations (16) and (17). When we deal with (16) and (17), the local numerical searching algorithms suggested in [1] are very effective for a relatively low-order model. A related discussion of the solving algorithms for Riccati-like equations can also be found in [5].

IV. Numerical Example

In this section, a simple design example is presented to illustrate the usefulness and exibility of the theory developed in this letter.

Consider a linear continuous-time uncertain stochastic system (1) and (2), with parameters as follows:

$$A = \begin{bmatrix} -2.5 & 0.3 \\ 0.4 & -1.5 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}$$
$$M_1 = \begin{bmatrix} 0.8 \\ 0.8 \end{bmatrix}, \quad M_2 = 0.8$$
$$N = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad L = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}.$$

It is desired to design robust filter (4) such that: 1) the poles of the filtering matrix $A_f + \Delta A_f$ are all constrained to lie inside the parabolic region $\mathcal{P}(0.5, 0.5)$; 2) the transfer function $H(s)$ from disturbances $w(t)$ to error state outputs $Le(t)$ satisfies the constraint $\|H(s)\|_\infty \le \gamma = 0.1$; and 3) the steady-state covariance P exists and satisfies $[P]_{11} \le \sigma_1^2 = 0.3$, $[P]_{22} \le \sigma_1^2 = 0.7$.

By setting $\varepsilon_1 = \varepsilon_2 = 0.58$, $\delta_1 = \delta_2 = 0.01$, and $E = [0.1 \quad 0.1]^T$, we solve the Riccati-like matrix equations (16) and (17) and obtain

$$Q_1 = \begin{bmatrix} 1.2682 & -0.0031 \\ -0.0031 & 1.6053 \end{bmatrix}$$
$$Q_2 = \begin{bmatrix} 0.2476 & -0.0255 \\ -0.0255 & 0.5789 \end{bmatrix}.$$

Clearly, $[Q_2]_{ii} \le \sigma_i^2$ $(i = 1,2)$. Then, for orthogonal "matrices" $U_1 = 1$ and $U_2 = -1$, the corresponding expected filter parameters in these two cases can be obtained from (18), respectively, as the following:

$$K_1 = \begin{bmatrix} 0.3868 \\ 0.3025 \end{bmatrix}, \quad G_1 = \begin{bmatrix} -2.2816 & 0.1753 \\ 0.7458 & -1.4981 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 0.4869 \\ 0.4106 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -2.3886 & 0.1702 \\ 0.6293 & -1.5101 \end{bmatrix}.$$

It is easy to verify that all specified performance requirements are achieved.

V. Conclusion

In this letter, we have considered a parabolic pole and variance-constrained robust H_∞ filtering problem for linear continuous-time systems. It has been shown that this filtering problem can be converted into an auxiliary problem that is related to the solutions of two Riccati-like matrix equations. The existence conditions and the analytical expression of desired estimators have been characterized, and a numerical example has been exploited to show the effectiveness of the proposed design method. Note that only sufficient conditions have been obtained in our main results, and one of our future research topics would be how to reduce the conservatism of the design.

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