Synchronization and State Estimation for Discrete-Time Complex Networks With Distributed Delays

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***Abstract*—In this paper, a synchronization problem is investigated for an array of coupled complex discrete-time networks with the simultaneous presence of both the discrete and distributed time delays. The complex networks addressed which include neural and social networks as special cases are quite general. Rather than the commonly used Lipschitz-type function, a more general sector-like nonlinear function is employed to describe the nonlinearities existing in the network. The distributed infinite time delays in the discrete-time domain are first defined. By utilizing a novel Lyapunov–Krasovskii functional and the Kronecker product, it is shown that the addressed discrete-time complex network with distributed delays is synchronized if certain linear matrix inequalities (LMIs) are feasible. The state estimation problem is then studied for the same complex network, where the purpose is to design a state estimator to estimate the network states through available output measurements such that, for all admissible discrete and distributed delays, the dynamics of the estimation error is guaranteed to be globally asymptotically stable. Again, an LMI approach is developed for the state estimation problem. Two simulation examples are provided to show the usefulness of the proposed global synchronization and state estimation conditions. It is worth pointing out that our main results are valid even if the nominal subsystems within the network are unstable.**

***Index Terms*—Complex networks, discrete time delays, distributed time delays, linear matrix inequality (LMI), Lyapunov–Krasovskii functional, neural networks, state estimation, synchronization.**

I. Introduction

COMPLEX networks are composed of a large number of highly interconnected dynamical units and therefore exhibit very complicate dynamics. Examples of such complex networks include the Internet, which is a network of routers or domains, the World Wide Web, which is a network of web sites, the brain, which is a network of neurons, and an organization, which is a network of people. Since the introduction of the small-world network principle by Watts and Strogatz [35], a great deal of research has been focused on the dependence of the asymptotic behavior of interconnected oscillatory agents on the structural properties of complex networks. It has been found out that the general structure of the interaction network may play a crucial role in the emergence of synchronization phenomena in various fields such as physics, technology, and the life sciences. Synchronization is attracting more and more research attention due to its ubiquity in many system models of the natural world, for example, the large-scale and complex networks of chaotic oscillators [15], [19], [26], [32], the coupled systems exhibiting spatiotemporal chaos and autowaves [28], [29], and the array of coupled neural networks [1], [14], [18], [27], [36], [37].

Time delays occur commonly in complex networks because of the network traffic congestions as well as the finite speed of signal transmission over the links [4], [8], [9], [12], [13], [29], [30], [38]. Also, the network couplings often give rise to delays in biological neural networks, gene regulatory networks, communication networks, and electrical power grids [2], [9], [17], [19]. It has been recognized that time delays can cause complex dynamics such as periodic or quasi-periodic motions, Hopf bifurcation, and higher dimensional chaos. In recent years, the synchronization problem for various types of networks with delayed coupling has been extensively studied [2], [9], [17], [19], [23], [37]. For example, the synchronization criteria have been established in [17] for complex dynamical network models with coupling delays for both continuous and discrete-time cases, which have further been improved in [9] by using less conservative delay-dependent techniques. A variational method has been used in [23] to deal with the synchronization problem for an array of linearly coupled identical connected neural networks with delays, whereas the similar problem has been addressed in [37] for an array of coupled nonlinear systems with delay and nonreciprocal time-varying coupling. It is worth mentioning that most of the reported results have addressed the synchronization problem for networks with discrete time delays. Another kind of time delay, namely, continuously distributed delays, has started to gain research attention in the context of synchronization because a complex network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of node sizes and lengths. Very recently, in [22], the synchronization problem has been investigated for coupled networks with both discrete and distributed time delays.

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One of the important yet challenging issues for understanding the interaction topology of complex networks has to do with the discrete nature of network topology [31]. The reason is mainly threefold: 1) the discretization process of a continuous-time network cannot preserve the dynamics of the continuous-time part even for small sampling periods; 2) a discrete-time network is in a better position to model digitally transmitted signals in a dynamical way than its continuous-time analog, and 3) the discrete-time networks have already been applied in a wide range of areas, such as image processing, time series analysis, quadratic optimization problems, and system identification. Recently, the synchronization problem for discrete-time networks has received some initial research interests. For example, the master–slave synchronization has been discussed in [19] where the activation function was assumed to be of the traditional Lipschitz type. In [24] and [25], the synchronization problem has been studied for an array of discrete-time coupled complex networks in a systematic way and a series of results was obtained by using innovative manifold/graph approaches.

Although the synchronization problem for discrete-time complex networks is now drawing increasing research attention, there are still several open problems deserving further investigation. First, despite their importance in modeling the distribution of propagation delays over a period of sampling time, the distributed time delays have not yet been addressed in the synchronization problems for discrete-time complex networks. The main reason is that it is nontrivial to represent the distributed time delays in the discrete-time domain and establish a unified framework to handle both the discrete and distributed time delays. Second, for large-scale complex networks, it is quite common that only partial information about the network nodes (states) is accessible from the network outputs. Therefore, in order to make use of key network nodes in practice, it becomes necessary to estimate the network nodes through available measurements. Note that the state estimation problem for neural networks (a special class of complex networks) was first addressed in [34] and has then drawn particular research interests, see, e.g., [11] and [12], where the networks are deterministic and continuous time. Unfortunately, the state estimation problem for discrete-time complex networks with or without distributed delays has not been researched yet. Therefore, the aim of this paper is to deal with the synchronization and state estimation problems for discrete-time complex networks with distributed delays.

In this paper, we investigate the synchronization problem for an array of coupled complex discrete-time networks with the simultaneous presence of both the discrete and distributed time delays. Rather than the commonly used Lipschitz-type function, a more general sectorlike nonlinear function is employed to describe the nonlinearities existing in the network. We first define the distributed time delays for the complex networks in the discrete-time domain. By utilizing a novel Lyapunov–Krasovskii functional and the Kronecker product, we show that the addressed synchronization problem can be converted into the feasibility problem of a set of linear matrix inequalities (LMIs). We then turn to the state estimation problem for the same complex networks. Through available output measurements, we aim to design a state estimator to estimate the network states such that, for all admissible discrete and distributed delays, the dynamics of the estimation error is guaranteed to be globally asymptotically stable. Again, an LMI approach is used with the main proof omitted for the state estimation case. Two simulation examples are provided to show the usefulness of the proposed the- ory. It is worth pointing out that our main results are valid even if the nominal subsystems within the network are unstable.

Notations: The notations are quite standard. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The superscript "T" denotes matrix transposition, and the notation $X \geq Y$ ($X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (positive definite). For vector or matrix z, $z \succeq 0$ means that each entry of z is nonnegative. I_n is the $n\times n$ identity matrix. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. The Kronecker product of an $n\times m$ matrix X and a $p\times q$ matrix Y is defined by an $np\times mq$ matrix $X\otimes Y$. If A is a matrix, denote by $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) the largest (smallest) eigenvalue of A. Matrices, if not explicitly specified, are assumed to have compatible dimensions. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

II. Problem Formulation

Consider the following discrete-time delayed complex network consisting of N coupled nodes of the form:

$$\begin{aligned} x_i(k+1) = {} & f\left(x_i(k)\right) + g\left(x_i\left(k-\tau(k)\right)\right) \\ & + \sum_{m=1}^{+\infty} \mu_m h\left(x_i(k-m)\right) + \sum_{j=1}^{N} w_{ij}\Gamma x_j(k), \\ & i = 1,2,\ldots,N \end{aligned} \tag{1}$$

where $x_i(k) = (x_{i1}(k), x_{i2}(k), \ldots, x_{in}(k))^{\mathrm{T}}$ is the state vector of the ith node. $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are nonlinear vector-valued functions satisfying certain conditions given later. The positive integer $\tau(k)$ denotes the discrete time-varying delay satisfying

$$\tau_m \leq \tau(k) \leq \tau_M, \qquad k \in \mathbb{N} \tag{2}$$

where τ_m and τ_M are known positive integers. The constants $\mu_m \geq 0\,(m=1,2,\ldots)$ satisfy the following convergent conditions:

$$\sum_{m=1}^{+\infty} \mu_m < +\infty \qquad \sum_{m=1}^{+\infty} m\mu_m < +\infty. \tag{3}$$

$\Gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\} \geq 0$ is a matrix linking the jth state variable if $\gamma_j \neq 0$, and $W = (w_{ij}) \in \mathbb{R}^{N\times N}$ is the coupled configuration matrix of the network with $w_{ij} \geq 0$ $(i \neq j)$ but not all zero. As usual, the coupling configuration matrix $W = (w_{ij})$ is symmetric (i.e., $W = W^{\mathrm{T}}$) and satisfies

$$\sum_{l=1}^{N} w_{kl} = \sum_{l=1}^{N} w_{lk} = 0, \qquad k = 1,2,\ldots,N. \tag{4}$$

Remark 1: The model (1) includes the term of the distributed time delays $\sum_{m=1}^{+\infty} \mu_m h(x_i(k-m))$ in the discrete-time setting. Such a term is proposed, to the best of the authors'

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knowledge, for the first time for complex networks and the model and the model (1) can be interpreted as the discrete analog of the following continuous-time complex network with mixed time delay:

$$\frac{dx_i(t)}{dt} = f(x_i(t)) + g(x_i(t-\tau(t))) + \int_{-\infty}^{t} k(t-s)h(x_i(s))\,ds + \sum_{j=1}^{N} w_{ij}\Gamma x_j(t).$$

As can be seen in the sequel, the inclusion of such a distributed delay term will bring additional difficulty in the analysis and a special inequality will need to be developed.

Remark 2: The convergent condition (3) is used to make sure that the term of $\sum_{m=1}^{+\infty}\mu_m h(x_i(k-m))$ in (1) as well as the Lyapunov functional (to be constructed later) are convergent.

Remark 3: The addressed discrete-time delayed complex networks (1) that include many different kinds of networks (e.g., neural and social networks) as special cases are quite general. For example, we consider the following n-neuron discrete-time neural network with discrete and distributed delays of the form:

$$u_i(k+1)=a_iu(k)+\sum_{j=1}^{n} b_{ij}\hat{f}_j(u_j(k))+\sum_{j=1}^{n} c_{ij}\hat{g}_j(u_j(k-\tau(k))) + \sum_{j=1}^{n} d_{ij}\sum_{m=1}^{+\infty}\mu_m\hat{h}_j(u_j(k-m)) \tag{5}$$

or, in an equivalent vector form

$$u(k+1) = Au(k) + B\hat{F}(u(k)) + C\hat{G}(u(k-\tau(k))) + D\sum_{m=1}^{+\infty}\mu_m\hat{H}(u(k-m)) \tag{6}$$

where $u(k) = (u_1(k), u_2(k), \ldots, u_n(k))^{\mathrm{T}}$ is the neural state vector, $A = \mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ with $|a_i| < 1$ is the state feedback coefficient matrix, the $n \times n$ matrices $B = [b_{ij}]_{n\times n}$, $C = [c_{ij}]_{n\times n}$, and $D = [d_{ij}]_{n\times n}$ are the connection weight matrix, the discretely delayed connection weight matrix, and distributively delayed connection weight matrix, respectively. The positive integer $\tau(k)$ is the same as in (2). In (6), $\hat{F}(u(k))$, $\hat{G}(u(k))$, and $\hat{H}(u(k))$ denote the neuron activation functions. μ_m $(m = 1, 2, \ldots)$ are scalar constants. It is obvious to see that the neural network model (5) is just a subnetwork of an array of coupled neural networks described by (1).

In the literature concerning the synchronization problem for complex networks, the nonlinearities are usually assumed to satisfy the Lipschitz-type conditions. In the following, we adopt a more general sectorlike nonlinear function to describe the nonlinearities existing in the networks.

Assumption 1: The nonlinear vector-valued functions f, g, and h are continuous and satisfy [20], [33]

$$[f(x) - f(y) - B_1(x-y)]^{\mathrm{T}}[f(x) - f(y) - B_2(x-y)] \le 0 \quad \forall\, x,\, y \in \mathbb{R}^n \tag{7}$$

$$[g(x) - g(y) - D_1(x-y)]^{\mathrm{T}}[g(x) - g(y) - D_2(x-y)] \le 0 \quad \forall\, x,\, y \in \mathbb{R}^n \tag{8}$$

$$[h(x) - h(y) - V_1(x-y)]^{\mathrm{T}}[h(x) - h(y) - V_2(x-y)] \le 0 \quad \forall\, x,\, y \in \mathbb{R}^n \tag{9}$$

where B_1, B_2, D_1, D_2, V_1, and V_2 are constant matrices.

Remark 4: The conditions (7)–(9) are known as sector-like descriptions of the nonlinearities, which are in a more general form than the usual Lipschitz functions. By adopting such a presentation, it would be possible to reduce the conservatism of the main results caused by quantifying the nonlinear functions via an LMI technique.

For notation simplicity, we let

$$\begin{aligned} x(k) &= \left(x_1^{\mathrm{T}}(k), x_2^{\mathrm{T}}(k), \ldots, x_N^{\mathrm{T}}(k)\right)^{\mathrm{T}} \\ F(x(k)) &= \left(f^{\mathrm{T}}(x_1(k)), f^{\mathrm{T}}(x_2(k)), \ldots, f^{\mathrm{T}}(x_N(k))\right)^{\mathrm{T}} \\ G(x(k)) &= \left(g^{\mathrm{T}}(x_1(k)), g^{\mathrm{T}}(x_2(k)), \ldots, g^{\mathrm{T}}(x_N(k))\right)^{\mathrm{T}} \\ H(x(k)) &= \left(h^{\mathrm{T}}(x_1(k)), h^{\mathrm{T}}(x_2(k)), \ldots, h^{\mathrm{T}}(x_N(k))\right)^{\mathrm{T}}. \end{aligned}$$

With the matrix Kronecker product, we can rewrite the network (1) in the following compact form:

$$x(k+1) = F(x(k)) + G(x(k-\tau(k))) + \sum_{m=1}^{+\infty}\mu_m H(x(k-m)) + (W\otimes\Gamma)x(k). \tag{10}$$

Definition 1: The discrete-time complex network (1) or (10) is said to be globally synchronized if, for all addressed discrete and distributed delays, the following holds:

$$\lim_{k\to+\infty}|x_i(k) - x_j(k)| = 0, \qquad 1 \le i < j \le N.$$

In the rest of this paper, we shall focus on the synchronization problem and the state estimation problem for the discrete-time complex network (10) with both discrete and distributed time delays. By utilizing new Lyapunov–Krasovskii functionals, we develop an LMI approach to derive sufficient conditions under which the discrete-time complex network (10) is globally synchronized, and then, we further extend the results obtained to design the desired state estimator for the same complex network through available network output.

III. Synchronization of Discrete-Time Complex Network

In this section, let us deal with the synchronization problem for the complex network (10). First, we introduce several lemmas to be used in the sequel.

Lemma 1: Let X and Y be any n-dimensional real vectors, and let P be an $n \times n$ positive semidefinite matrix. Then, the following matrix inequality holds:

$$2X^{\mathrm{T}}PY \le X^{\mathrm{T}}PX + Y^{\mathrm{T}}PY.$$

Lemma 2: Let $M \in \mathbb{R}^{n\times n}$ be a positive semidefinite matrix, $\mathbf{x}_i \in \mathbb{R}^n$, and scalar constant $a_i \ge 0$ $(i = 1, 2, \ldots)$. If the series concerned is convergent, then the following inequality holds:

$$\left(\sum_{i=1}^{+\infty} a_i\mathbf{x}_i\right)^{\mathrm{T}} M\left(\sum_{i=1}^{+\infty} a_i\mathbf{x}_i\right) \le \left(\sum_{i=1}^{+\infty} a_i\right)\sum_{i=1}^{+\infty} a_i\mathbf{x}_i^{\mathrm{T}}M\mathbf{x}_i. \tag{11}$$

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Proof: Letting m be a positive integer, we have

$$\begin{aligned}\left(\sum_{i=1}^{m} a_i\mathbf{x}_i\right)^{\mathrm{T}} M\left(\sum_{i=1}^{m} a_i\mathbf{x}_i\right) &= \left(\sum_{i=1}^{m} a_i\mathbf{x}_i\right)^{\mathrm{T}} M\left(\sum_{j=1}^{m} a_j\mathbf{x}_j\right)\\ &= \sum_{i=1}^{m}\sum_{j=1}^{m} a_i a_j \mathbf{x}_i^{\mathrm{T}} M\mathbf{x}_j\\ &\le \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{1}{2}a_i a_j\left(\mathbf{x}_i^{\mathrm{T}}M\mathbf{x}_i+\mathbf{x}_j^{\mathrm{T}}M\mathbf{x}_j\right)\\ &\quad \text{(By Lemma 1)}\\ &= \left(\sum_{i=1}^{m} a_i\right)\sum_{i=1}^{m} a_i\mathbf{x}_i^{\mathrm{T}}M\mathbf{x}_i\end{aligned}$$

and then (11) follows directly by letting $m \to +\infty$, which completes the proof. ∎

Lemma 3: (Schur Complement) Given constant matrices Ω_1, Ω_2, and Ω_3, where $\Omega_1 = \Omega_1^{\mathrm{T}}$ and $\Omega_2 > 0$, then

$$\Omega_1 + \Omega_3^{\mathrm{T}}\Omega_2^{-1}\Omega_3 < 0$$

if only if

$$\begin{bmatrix}\Omega_1 & \Omega_3^{\mathrm{T}}\\ \Omega_3 & -\Omega_2\end{bmatrix} < 0.$$

Lemma 4: Let $\mathcal{U} = (\alpha_{ij})_{N\times N}$, $P \in \mathbb{R}^{n\times n}$, $x = (x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_N^{\mathrm{T}})^{\mathrm{T}}$, and $y = (y_1^{\mathrm{T}}, y_2^{\mathrm{T}}, \ldots, y_N^{\mathrm{T}})^{\mathrm{T}}$ with x_k, $y_k \in \mathbb{R}^n (k = 1, 2, \ldots, N)$. If $\mathcal{U} = \mathcal{U}^{\mathrm{T}}$ and each row sum of $\mathcal{U}$ is zero, then

$$x^{\mathrm{T}}(\mathcal{U}\otimes P)y = -\sum_{1\le i<j\le N}\alpha_{ij}(x_i - x_j)^{\mathrm{T}}P(y_i - y_j).$$

Lemma 5: Let $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{n\times q}$, and $C = (c_{ij})_{m\times q} = AB$. If the sum of all elements in each column of A (row of B) is zero, then the sum of all elements in each column (row) of C is zero. Moreover, if A is a symmetric matrix and the sum of all elements in each row of A is zero, then, for any positive integer n, the sum of all elements in each row of A^n is zero.

For notation simplicity, we denote by $w_{ij}^{(2)}$ the (i, j) entry of the matrix W^2. Now, we give our main result in this paper as follows.

Theorem 1: Under Assumption 1, the discrete-time complex network (10) is globally asymptotically synchronized if there exist three scalar constants $\delta_1 > 0$, $\delta_2 > 0$, and $\delta_3 > 0$ and three positive definite matrices P, Q, and R such that the LMIs shown at the bottom of the page hold, where

$$\breve{B}_1 = \left(B_1^{\mathrm{T}}B_2 + B_2^{\mathrm{T}}B_1\right)/2 \qquad \breve{B}_2 = \left(B_1^{\mathrm{T}} + B_2^{\mathrm{T}}\right)/2 \tag{13}$$

$$\breve{D}_1 = \left(D_1^{\mathrm{T}}D_2 + D_2^{\mathrm{T}}D_1\right)/2 \qquad \breve{D}_2 = \left(D_1^{\mathrm{T}} + D_2^{\mathrm{T}}\right)/2 \tag{14}$$

$$\breve{V}_1 = \left(V_1^{\mathrm{T}}V_2 + V_2^{\mathrm{T}}V_1\right)/2 \qquad \breve{V}_2 = \left(V_1^{\mathrm{T}} + V_2^{\mathrm{T}}\right)/2 \tag{15}$$

$$\Xi = (\tau_M - \tau_m + 1)Q - \delta_2 I \qquad \bar{\mu} = \sum_{k=1}^{+\infty}\mu_k \tag{16}$$

$$\Theta_{ij} = -P - Nw_{ij}^{(2)}\Gamma P\Gamma - \delta_1\breve{B}_1 - \delta_2\breve{D}_1 - \delta_3\breve{V}_1. \tag{17}$$

Proof: Let $\mathbf{x}_{ij}(k) = x_i(k) - x_j(k)$, $\mathbf{f}_{ij}(k) = f(x_i(k)) - f(x_j(k))$, $\mathbf{g}_{ij}(k) = g(x_i(k)) - g(x_j(k))$, $\mathbf{h}_{ij}(k) = h(x_i(k)) - h(x_j(k))$, and $\widehat{\mathbf{h}}_{ij}(k) = \sum_{m=1}^{+\infty}\mu_m h(x_i(k - m)) - \sum_{m=1}^{+\infty}\mu_m h(x_j(k - m))$. From (7), it follows readily that

$$\begin{aligned}&\begin{bmatrix} x_i(k) - x_j(k)\\ f(x_i(k)) - f(x_j(k))\end{bmatrix}^{\mathrm{T}}\\ &\times\begin{bmatrix}\left(B_1^{\mathrm{T}}B_2 + B_2^{\mathrm{T}}B_1\right)/2 & -\left(B_1^{\mathrm{T}} + B_2^{\mathrm{T}}\right)/2\\ -(B_1 + B_2)/2 & I\end{bmatrix}\\ &\times\begin{bmatrix} x_i(k) - x_j(k)\\ f(x_i(k)) - f(x_j(k))\end{bmatrix} \le 0\end{aligned}$$

namely

$$\begin{bmatrix}\mathbf{x}_{ij}(k)\\ \mathbf{f}_{ij}(k)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\breve{B}_1 & -\breve{B}_2\\ -\breve{B}_2^{\mathrm{T}} & I\end{bmatrix}\begin{bmatrix}\mathbf{x}_{ij}(k)\\ \mathbf{f}_{ij}(k)\end{bmatrix} \le 0. \tag{18}$$

Similarly, from (8) and (9), we have

$$\begin{bmatrix}\mathbf{x}_{ij}(k)\\ \mathbf{g}_{ij}(k)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\breve{D}_1 & -\breve{D}_2\\ -\breve{D}_2^{\mathrm{T}} & I\end{bmatrix}\begin{bmatrix}\mathbf{x}_{ij}(k)\\ \mathbf{g}_{ij}(k)\end{bmatrix} \le 0 \tag{19}$$

$$\begin{bmatrix}\mathbf{x}_{ij}(k)\\ \mathbf{h}_{ij}(k)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\breve{V}_1 & -\breve{V}_2\\ -\breve{V}_2^{\mathrm{T}} & I\end{bmatrix}\begin{bmatrix}\mathbf{x}_{ij}(k)\\ \mathbf{h}_{ij}(k)\end{bmatrix} \le 0. \tag{20}$$

To deal with the synchronization of the network (10), we introduce the following Lyapunov–Krasovskii functional:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) \tag{21}$$

$$\Phi_{ij} = \begin{bmatrix}\Theta_{ij} & -Nw_{ij}\Gamma P + \delta_1\breve{B}_2 & \delta_2\breve{D}_2 & -Nw_{ij}\Gamma P & \delta_3\breve{V}_2 & -Nw_{ij}\Gamma P\\ * & P - \delta_1 I & 0 & P & 0 & P\\ * & * & \Xi & 0 & 0 & 0\\ * & * & * & P - Q & 0 & P\\ * & * & * & * & \bar{\mu}R - \delta_3 I & 0\\ * & * & * & * & * & P - \frac{1}{\bar{\mu}}R\end{bmatrix} < 0, \qquad 1 \le i < j \le N \tag{12}$$

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where

$$V_1(k) = x^{\mathrm{T}}(k)(U \otimes P)x(k) \tag{22}$$

$$V_2(k) = \sum_{i=k-\tau(k)}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \tag{23}$$

$$V_3(k) = \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \tag{24}$$

$$V_4(k) = \sum_{i=1}^{+\infty} \mu_i \sum_{j=k-i}^{k-1} H^{\mathrm{T}}(x(j))(U \otimes R)H(x(j)) \tag{25}$$

with

$$U = \begin{bmatrix} N-1 & -1 & \cdots & -1 \\ -1 & N-1 & \cdots & -1 \\ \cdots & \cdots & \cdots & \cdots \\ -1 & -1 & \cdots & N-1 \end{bmatrix}_{N\times N}.$$

Notice that from the condition (3), $V_4(k)$ is convergent. Calculating the difference of $V(k)$ along the system (10), we have

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) \tag{26}$$

where

$$\begin{aligned}
\Delta V_1(k) &= V_1(k+1) - V_1(k) \\
&= \Bigg(F(x(k)) + G(x(k-\tau(k))) \\
&\qquad + \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) + (W \otimes \Gamma)x(k)\Bigg)^{\mathrm{T}} \\
&\quad \times (U \otimes P)\Bigg(F(x(k)) + G(x(k-\tau(k))) \\
&\qquad + \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) + (W \otimes \Gamma)x(k)\Bigg) \\
&\quad - x^{\mathrm{T}}(k)(U \otimes P)x(k) \\
&= F^{\mathrm{T}}(x(k))(U \otimes P)F(x(k)) + G^{\mathrm{T}}(x(k-\tau(k))) \\
&\quad \times (U \otimes P)G(x(k-\tau(k))) \\
&\quad + \left(\sum_{m=1}^{+\infty} \mu_m H(x(k-m))\right)^{\mathrm{T}} \\
&\quad \times (U \otimes P)\sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \\
&\quad + x^{\mathrm{T}}(k)(W \otimes \Gamma)^{\mathrm{T}}(U \otimes P) \\
&\quad \times (W \otimes \Gamma)x(k) + 2F^{\mathrm{T}}(x(k))(U \otimes P) \\
&\quad \times G(x(k-\tau(k))) + 2F^{\mathrm{T}}(x(k))(U \otimes P) \\
&\quad \times \sum_{m=1}^{+\infty} \mu_m H(x(k-m)) + 2F^{\mathrm{T}}(x(k))(U \otimes P) \\
&\quad \times (W \otimes \Gamma)x(k) + 2(G(x(k-\tau(k))))^{\mathrm{T}} \\
&\quad \times (U \otimes P)\sum_{m=1}^{+\infty} \mu_m H(x(k-m)) \\
&\quad + 2(G(x(k-\tau(k))))^{\mathrm{T}}(U \otimes P)(W \otimes \Gamma)x(k) \\
&\quad + 2\left(\sum_{m=1}^{+\infty} \mu_m H(x(k-m))\right)^{\mathrm{T}} \\
&\quad \times (U \otimes P)(W \otimes \Gamma)x(k) - x^{\mathrm{T}}(k)(U \otimes P)x(k)
\end{aligned} \tag{27}$$

$$\begin{aligned}
\Delta V_2(k) &= V_2(k+1) - V_2(k) \\
&= \sum_{i=k+1-\tau(k+1)}^{k} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&\quad - \sum_{i=k-\tau(k)}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&= G^{\mathrm{T}}(x(k))(U \otimes Q)G(x(k)) \\
&\quad - G^{\mathrm{T}}(x(k-\tau(k)))(U \otimes Q)G(x(k-\tau(k))) \\
&\quad + \sum_{i=k-\tau(k+1)+1}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&\quad - \sum_{i=k-\tau(k)+1}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&= G^{\mathrm{T}}(x(k))(U \otimes Q)G(x(k)) \\
&\quad - G^{\mathrm{T}}(x(k-\tau(k)))(U \otimes Q)G(x(k-\tau(k))) \\
&\quad + \sum_{i=k-\tau_m+1}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&\quad + \sum_{i=k-\tau(k+1)+1}^{k-\tau_m} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&\quad - \sum_{i=k-\tau(k)+1}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&\le G^{\mathrm{T}}(x(k))(U \otimes Q)G(x(k)) \\
&\quad - G^{\mathrm{T}}(x(k-\tau(k)))(U \otimes Q)G(x(k-\tau(k))) \\
&\quad + \sum_{i=k-\tau_M+1}^{k-\tau_m} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i))
\end{aligned} \tag{28}$$

$$\begin{aligned}
\Delta V_3(k) &= V_3(k+1) - V_3(k) \\
&= \sum_{j=k-\tau_M+2}^{k-\tau_m+1} \sum_{i=j}^{k} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&\quad - \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&= \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j+1}^{k} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&\quad - \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} G^{\mathrm{T}}(x(i))(U \otimes Q)G(x(i)) \\
&= \sum_{j=k-\tau_M+1}^{k-\tau_m} \big(G^{\mathrm{T}}(x(k))(U \otimes Q)G(x(k)) \\
&\qquad - G^{\mathrm{T}}(x(j))(U \otimes Q)G(x(j))\big)
\end{aligned}$$

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$$\begin{aligned}&=(\tau_M-\tau_m)G^{\mathrm{T}}\left(x(k)\right)(U\otimes Q)G\left(x(k)\right)\\&\quad-\sum_{i=k-\tau_M+1}^{k-\tau_m}G^{\mathrm{T}}\left(x(i)\right)(U\otimes Q)G\left(x(i)\right)\end{aligned}\tag{29}$$

$$\begin{aligned}\Delta V_4(k)&=V_4(k+1)-V_4(k)\\&=\sum_{i=1}^{+\infty}\mu_i\sum_{j=k+1-i}^{k}H^{\mathrm{T}}\left(x(j)\right)(U\otimes R)H\left(x(j)\right)\\&\quad-\sum_{i=1}^{+\infty}\mu_i\sum_{j=k-i}^{k-1}H^{\mathrm{T}}\left(x(j)\right)(U\otimes R)H\left(x(j)\right)\\&=\sum_{i=1}^{+\infty}\mu_i\Big(H^{\mathrm{T}}\left(x(k)\right)(U\otimes R)H\left(x(k)\right)\\&\qquad\qquad-H^{\mathrm{T}}(x(k-i))(U\otimes R)H\left(x(k-i)\right)\Big)\\&=\sum_{i=1}^{+\infty}\mu_iH^{\mathrm{T}}\left(x(k)\right)(U\otimes R)H\left(x(k)\right)\\&\quad-\sum_{i=1}^{+\infty}\mu_iH^{\mathrm{T}}\left(x(k-i)\right)(U\otimes R)H\left(x(k-i)\right)\\&\qquad\text{(By Lemma 2)}\\&\le\bar{\mu}H^{\mathrm{T}}\left(x(k)\right)(U\otimes R)H\left(x(k)\right)\\&\quad-\frac{1}{\bar{\mu}}\left(\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right)\right)^{\mathrm{T}}(U\otimes R)\\&\quad\times\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right).\end{aligned}\tag{30}$$

In view of

$$\begin{aligned}(W\otimes\Gamma)^{\mathrm{T}}(U\otimes P)(W\otimes\Gamma)&=(W^{\mathrm{T}}\otimes\Gamma^{\mathrm{T}})(U\otimes P)(W\otimes\Gamma)\\&=(W^{\mathrm{T}}UW)\otimes(\Gamma^{\mathrm{T}}P\Gamma)\\&=NW^2\otimes(\Gamma P\Gamma)\\(U\otimes P)(W\otimes\Gamma)&=(UW)\otimes(P\Gamma)\\&=NW\otimes(P\Gamma)\end{aligned}$$

we substitute (27)–(30) into (26) and obtain

$$\begin{aligned}\Delta V(k)\le\;&F^{\mathrm{T}}\left(x(k)\right)(U\otimes P)F\left(x(k)\right)\\&+G^{\mathrm{T}}\left(x\left(k-\tau(k)\right)\right)(U\otimes P)G\left(x\left(k-\tau(k)\right)\right)\\&+\left(\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right)\right)^{\mathrm{T}}(U\otimes P)\\&\times\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right)\\&+x^{\mathrm{T}}(k)\left(NW^2\otimes(\Gamma P\Gamma)\right)x(k)\\&+2F^{\mathrm{T}}\left(x(k)\right)(U\otimes P)G\left(x\left(k-\tau(k)\right)\right)\\&+2F^{\mathrm{T}}\left(x(k)\right)(U\otimes P)\\&\times\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right)\\&+2F^{\mathrm{T}}\left(x(k)\right)(NW\otimes P\Gamma)x(k)\\&+2G^{\mathrm{T}}\left(x(k-\tau(k))\right)(U\otimes P)\\&\times\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right)\\&+2G^{\mathrm{T}}\left(x(k-\tau(k))\right)(NW\otimes P\Gamma)x(k)\\&+\left(\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right)\right)^{\mathrm{T}}(NW\otimes P\Gamma)x(k)\\&-x^{\mathrm{T}}(k)(U\otimes P)x(k)\\&+(1+\tau_M-\tau_m)G^{\mathrm{T}}\left(x(k)\right)(U\otimes Q)G\left(x(k)\right)\\&-G^{\mathrm{T}}\left(x\left(k-\tau(k)\right)\right)(U\otimes Q)G\left(x\left(k-\tau(k)\right)\right)\\&+\bar{\mu}H^{\mathrm{T}}\left(x(k)\right)(U\otimes R)H\left(x(k)\right)\\&-\frac{1}{\bar{\mu}}\left(\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right)\right)^{\mathrm{T}}(U\otimes R)\\&\times\left(\sum_{m=1}^{+\infty}\mu_mH\left(x(k-m)\right)\right).\end{aligned}\tag{31}$$

It follows from Lemma 4, Lemma 5, and (31) that

$$\begin{aligned}\Delta V(k)\le&\sum_{1\le i<j\le N}\Bigg\{-\mathbf{x}_{ij}^{\mathrm{T}}P\mathbf{x}_{ij}-\mathbf{x}_{ij}^{\mathrm{T}}\left(Nw_{ij}^{(2)}\Gamma P\Gamma\right)\mathbf{x}_{ij}\\&\quad+\mathbf{f}_{ij}^{\mathrm{T}}(k)P\mathbf{f}_{ij}(k)+\mathbf{g}_{ij}^{\mathrm{T}}\left(k-\tau(k)\right)\\&\quad\times P\mathbf{g}_{ij}\left(k-\tau(k)\right)+\widehat{\mathbf{h}}_{ij}^{\mathrm{T}}(k)P\widehat{\mathbf{h}}_{ij}(k)\\&\quad+2\mathbf{f}_{ij}^{\mathrm{T}}(k)P\mathbf{g}_{ij}\left(k-\tau(k)\right)\\&\quad+2\mathbf{f}_{ij}^{\mathrm{T}}(k)P\widehat{\mathbf{h}}_{ij}(k)-2\mathbf{f}_{ij}^{\mathrm{T}}(k)\\&\quad\times(Nw_{ij}P\Gamma)\mathbf{x}_{ij}+2\mathbf{g}_{ij}^{\mathrm{T}}\left(k-\tau(k)\right)\\&\quad\times P\widehat{\mathbf{h}}_{ij}(k)-2\mathbf{g}_{ij}^{\mathrm{T}}\left(k-\tau(k)\right)\\&\quad\times(Nw_{ij}P\Gamma)\mathbf{x}_{ij}(k)-\widehat{\mathbf{h}}_{ij}^{\mathrm{T}}(k)\\&\quad\times(Nw_{ij}P\Gamma)\mathbf{x}_{ij}(k)+(1+\tau_M-\tau_m)\\&\quad\times\mathbf{g}_{ij}^{\mathrm{T}}(k)Q\mathbf{g}_{ij}(k)-\mathbf{g}_{ij}\left(k-\tau(k)\right)\\&\quad\times Q\mathbf{g}_{ij}\left(k-\tau(k)\right)+\bar{\mu}\mathbf{h}_{ij}^{\mathrm{T}}(k)R\mathbf{h}_{ij}(k)\\&\quad-\frac{1}{\bar{\mu}}\widehat{\mathbf{h}}_{ij}^{\mathrm{T}}(k)R\widehat{\mathbf{h}}_{ij}(k)\Bigg\}\\=&\sum_{1\le i<j\le N}\xi_{ij}^{\mathrm{T}}(k)\Phi_{ij}^{(1)}\xi_{ij}(k)\end{aligned}\tag{32}$$

where the expressions for $\xi_{ij}(k)$ and $\Phi_{ij}^{(1)}$ are shown at the bottom of the next page.

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Therefore, from (32) along with (18)–(20), we obtain

$$\begin{aligned}\Delta V(k) \leq & \sum_{1\leq i<j\leq N} \left\{ \xi_{ij}^{\mathrm{T}}(k)\Phi_{ij}^{(1)}\xi_{ij}(k) - \delta_1 \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{f}_{ij}(k) \end{bmatrix}^{\mathrm{T}} \right. \\ & \times \begin{bmatrix} \breve{B}_1 & -\breve{B}_2 \\ -\breve{B}_2^{\mathrm{T}} & I \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{f}_{ij}(k) \end{bmatrix} \\ & - \delta_2 \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{g}_{ij}(k) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \breve{D}_1 & -\breve{D}_2 \\ -\breve{D}_2^{\mathrm{T}} & I \end{bmatrix} \\ & \times \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{g}_{ij}(k) \end{bmatrix} - \delta_3 \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{h}_{ij}(k) \end{bmatrix}^{\mathrm{T}} \\ & \left. \times \begin{bmatrix} \breve{V}_1 & -\breve{V}_2 \\ -\breve{V}_2^{\mathrm{T}} & I \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ij}(k) \\ \mathbf{h}_{ij}(k) \end{bmatrix} \right\} \\ = & \sum_{1\leq i<j\leq N} \xi_{ij}^{\mathrm{T}}(k)\Phi_{ij}\xi_{ij}(k) \\ \leq & \sum_{1\leq i<j\leq N} \lambda_{\max}(\Phi_{ij})|\xi_{ij}|^2. \end{aligned} \tag{33}$$

Noticing that $\lambda_{\max}(\Phi_{ij}) < 0$ and letting $\lambda_0 = \max_{1\leq i<j\leq N}\{\lambda_{\max}(\Phi_{ij})\}$, we have $\lambda_0 < 0$, and then, it follows readily from (33) that

$$\Delta V(k) \leq \lambda_0 \sum_{1\leq i<j\leq N} |\mathbf{x}_{ij}(k)|^2 . \tag{34}$$

Letting m be a positive integer, one has from (34)

$$V(m+1) - V(1) = \sum_{k=1}^{m} \Delta V(k) = \lambda_0 \sum_{1\leq i<j\leq N} \sum_{k=1}^{m} |\mathbf{x}_{ij}(k)|^2$$

which implies that

$$-\lambda_0 \sum_{1\leq i<j\leq N} \sum_{k=1}^{m} |\mathbf{x}_{ij}(k)|^2 \leq V(1).$$

By letting $m \to +\infty$, we can deduce that the series $\sum_{k=1}^{+\infty} |\mathbf{x}_{ij}(k)|^2$ is convergent for $1 \leq i < j \leq N$, and therefore, $|\mathbf{x}_{ij}(k)|^2 \to 0$, namely

$$\lim_{k\to+\infty} |x_i(k) - x_j(k)| = 0$$

which completes the proof of Theorem 1. ∎

The complex network (10) is a quite general model that includes both the discrete and distributed time delays in the discrete-time domain. For example, if we consider the discrete time delay only, i.e., $h = 0$, then, the complex network (10) reduces to

$$x(k+1) = F(x(t)) + G(x(k-\tau(k))) + (W\otimes\Gamma)x(k). \tag{35}$$

For the complex network (35), it is straightforward to have the following synchronization result from Theorem 1.

Corollary 1: Suppose that the conditions (7) and (8) hold in Assumption 1. The discrete complex network (35) is synchronized if there exist two scalar constants δ_1 and δ_2 and two positive definite matrices P and Q such that the following LMIs hold:

$$\Phi_{ij} = \begin{bmatrix} \Pi_{ij} & -Nw_{ij}\Gamma P + \delta_1\breve{B}_2 & \delta_2\breve{D}_2 & -Nw_{ij}\Gamma P \\ * & P - \delta_1 I & 0 & P \\ * & * & \Xi & 0 \\ * & * & * & P - Q \end{bmatrix} < 0$$

$$1 \leq i < j \leq N \tag{36}$$

where $\Pi_{ij} = -P - Nw_{ij}^{(2)}\Gamma P\Gamma - \delta_1\breve{B}_1 - \delta_2\breve{D}_1$ and $\breve{B}_1$, $\breve{B}_2$, $\breve{D}_1$, $\breve{D}_2$, and Ξ are defined in Theorem 1.

Remark 5: In Theorem 1, the synchronization problem is studied for an array of discrete-time neural networks with mixed time delays. Note that the obtained criteria in (12) are dependent not only on the upper and lower bounds of the discrete time delays but also on the distributed delays. Therefore, the criteria are less conservative than the traditional delay-independent ones. Also, the LMI-based criteria can be checked efficiently via the Matlab LMI Toolbox.

IV. State Estimation of Complex Networks

As discussed in Section I, for relatively high-order and large-scale complex networks, sometimes, we can only know the partial information about the states of the key network nodes from the network outputs (measurements). Therefore, in order to make use of the networks in practice, it becomes necessary to estimate the node states through available network output.

Suppose that the output from the ith node of the complex network (10) is of the form

$$y_i(k) = C_i x_i(k), \qquad i = 1, 2, \ldots, N \tag{37}$$

where $y_i(k) = (y_{i1}(k), y_{i2}(k), \ldots, y_{im}(k)) \in \mathbb{R}^m$ is the measurement output of the ith node and $C_i \in \mathbb{R}^{m\times n}$ is a known constant matrix.

$$\xi_{ij}(k) = \begin{bmatrix} \mathbf{x}_{ij}^{\mathrm{T}}(k) & \mathbf{f}_{ij}^{\mathrm{T}}(k) & \mathbf{g}_{ij}^{\mathrm{T}}(k) & \mathbf{g}_{ij}^{\mathrm{T}}(k-\tau(k)) & \mathbf{h}_{ij}^{\mathrm{T}}(k) & \widehat{\mathbf{h}}_{ij}^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}$$

$$\Phi_{ij}^{(1)} = \begin{bmatrix} -P - Nw_{ij}^{(2)}\Gamma P\Gamma & -Nw_{ij}\Gamma P & 0 & -Nw_{ij}\Gamma P & 0 & -Nw_{ij}\Gamma P \\ * & P & 0 & P & 0 & P \\ * & * & (\tau_M - \tau_m + 1)Q & 0 & 0 & 0 \\ * & * & * & P - Q & 0 & P \\ * & * & * & * & \bar{\mu}R & 0 \\ * & * & * & * & * & P - \frac{1}{\bar{\mu}}R \end{bmatrix}$$

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Remark 6: Usually, we have $m < n$ which means that the network output is a linear combination (although partial) of the information about the network nodes. We wish to design an easy-to-implement estimator/observer to estimate the network states through the available network output.

In order to estimate the states of the complex network (10), we construct the following state estimator:

$$\hat{x}(k+1)=F(\hat{x}(k))+G(\hat{x}(k-\tau(k)))+\sum_{m=1}^{+\infty}\mu_m H(\hat{x}(k-m)) + (W\otimes\Gamma)\hat{x}(k)+K[y(k)-C\hat{x}(k)] \quad (38)$$

where

$$y(k)=\begin{bmatrix} y_1(k)\\ y_2(k)\\ \vdots \\ y_N(k)\end{bmatrix} \quad C=\begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_N\end{bmatrix}$$

$$K=\begin{bmatrix} K_1 & & & \\ & K_2 & & \\ & & \ddots & \\ & & & K_N\end{bmatrix} \quad (K_i\in\mathbb{R}^{n\times m})$$

with K being the estimator gain matrix. Our goal hereafter is to choose a suitable K_i such that $\hat{x}(k)$ asymptotically approaches $x(k)$.

To this end, we let $\varepsilon(k)=(\varepsilon_1^{\mathrm{T}}(k),\varepsilon_2^{\mathrm{T}}(k),\ldots,\varepsilon_N^{\mathrm{T}}(k))^{\mathrm{T}}:=\hat{x}(k)-x(k)$ with $\varepsilon_i(k)=\hat{x}_i(k)-x_i(k)$ being the state estimator error and denote

$$\hat{F}(\varepsilon(k))=\left[\hat{f}^{\mathrm{T}}(\varepsilon_1(k)),\ \hat{f}^{\mathrm{T}}(\varepsilon_2(k)),\ldots,\hat{f}^{\mathrm{T}}(\varepsilon_N(k))\right]^{\mathrm{T}} := F(\hat{x}(k))-F(x(k)) \quad (39)$$

$$\hat{G}(\varepsilon(k))=\left[\hat{g}^{\mathrm{T}}(\varepsilon_1(k)),\ \hat{g}^{\mathrm{T}}(\varepsilon_2(k)),\ldots,\hat{g}^{\mathrm{T}}(\varepsilon_N(k))\right]^{\mathrm{T}} := G(\hat{x}(k))-G(x(k)) \quad (40)$$

$$\hat{H}(\varepsilon(k))=\left[\hat{h}^{\mathrm{T}}(\varepsilon_1(k)),\ \hat{h}^{\mathrm{T}}(\varepsilon_2(k)),\ldots,\hat{h}^{\mathrm{T}}(\varepsilon_N(k))\right]^{\mathrm{T}} := H(\hat{x}(k))-G(x(k)). \quad (41)$$

Then, from (10) and (38), we obtain the following system governing the state error dynamics:

$$\varepsilon(k+1)=-KC\varepsilon(k)+\hat{F}(\varepsilon(k))+\hat{G}(\varepsilon(k-\tau(k))) +\sum_{m=1}^{+\infty}\mu_m\hat{H}(\varepsilon(k-m))+(W\otimes\Gamma)\varepsilon(k). \quad (42)$$

It is easy to verify that

$$\begin{bmatrix}\varepsilon\\ \hat{F}(\varepsilon)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\breve{\mathbf{B}}_1 & -\breve{\mathbf{B}}_2\\ -\breve{\mathbf{B}}_2^{\mathrm{T}} & I\end{bmatrix}\begin{bmatrix}\varepsilon\\ \hat{F}(\varepsilon)\end{bmatrix}\le 0 \quad \forall\varepsilon\in\mathbb{R}^{n\times N} \quad (43)$$

$$\begin{bmatrix}\varepsilon\\ \hat{G}(\varepsilon)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\breve{\mathbf{D}}_1 & -\breve{\mathbf{D}}_2\\ -\breve{\mathbf{D}}_2^{\mathrm{T}} & I\end{bmatrix}\begin{bmatrix}\varepsilon\\ \hat{G}(\varepsilon)\end{bmatrix}\le 0 \quad \forall\varepsilon\in\mathbb{R}^{n\times N} \quad (44)$$

$$\begin{bmatrix}\varepsilon\\ \hat{H}(\varepsilon)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\breve{\mathbf{V}}_1 & -\breve{\mathbf{V}}_2\\ -\breve{\mathbf{V}}_2^{\mathrm{T}} & I\end{bmatrix}\begin{bmatrix}\varepsilon\\ \hat{H}(\varepsilon)\end{bmatrix}\le 0 \quad \forall\varepsilon\in\mathbb{R}^{n\times N} \quad (45)$$

where

$$\breve{\mathbf{B}}_1=\mathrm{diag}\{\overbrace{\breve{B}_1,\breve{B}_1,\ldots,\breve{B}_1}^{N}\} \quad \breve{\mathbf{B}}_2=\mathrm{diag}\{\overbrace{\breve{B}_2,\breve{B}_2,\ldots,\breve{B}_2}^{N}\}$$

$$\breve{\mathbf{D}}_1=\mathrm{diag}\{\overbrace{\breve{D}_1,\breve{D}_1,\ldots,\breve{D}_1}^{N}\} \quad \breve{\mathbf{D}}_2=\mathrm{diag}\{\overbrace{\breve{D}_2,\breve{D}_2,\ldots,\breve{D}_2}^{N}\}$$

$$\breve{\mathbf{V}}_1=\mathrm{diag}\{\overbrace{\breve{V}_1,\breve{V}_1,\ldots,\breve{V}_1}^{N}\} \quad \breve{\mathbf{V}}_2=\mathrm{diag}\{\overbrace{\breve{V}_2,\breve{V}_2,\ldots,\breve{V}_2}^{N}\}.$$

By following a similar line as in the proof of Theorem 1, we can obtain the following result without proof.

Theorem 2: Let K be a given constant matrix. Then, under Assumption 1, the error system (42) is globally asymptotically stable if there exist three scalar constants $\delta_1>0$, $\delta_2>0$, and $\delta_3>0$ and three positive definite diagonal block matrices $\mathbf{P}=\mathrm{diag}\{P_1,P_2,\ldots,P_N\}$, $\mathbf{Q}=\mathrm{diag}\{Q_1,Q_2,\ldots,Q_N\}$, and $\mathbf{R}=\mathrm{diag}\{R_1,R_2,\ldots,R_N\}$ such that the LMI holds

$$\begin{bmatrix}\mathbf{Z}_1 & \mathbf{S}^{\mathrm{T}}\mathbf{P}+\delta_1\breve{\mathbf{B}}_2 & \delta_2\breve{\mathbf{D}}_2 & \mathbf{S}^{\mathrm{T}}\mathbf{P} & \delta_3\breve{\mathbf{V}}_2 & \mathbf{S}^{\mathrm{T}}\mathbf{P}\\ * & \mathbf{P}-\delta_1 I & 0 & \mathbf{P} & 0 & \mathbf{P}\\ * & * & \mathbf{Z}_2 & 0 & 0 & 0\\ * & * & * & \mathbf{P}-\mathbf{Q} & 0 & \mathbf{P}\\ * & * & * & * & \bar{\mu}\mathbf{R}-\delta_3 I & 0\\ * & * & * & * & * & \mathbf{P}-\bar{\mu}^{-1}\mathbf{R}\end{bmatrix}<0$$

where $\mathbf{S}=W\otimes\Gamma-KC$, $\mathbf{Z}_1=\mathbf{S}^{\mathrm{T}}\mathbf{P}\mathbf{S}-\mathbf{P}-\delta_1\breve{\mathbf{B}}_1-\delta_2\breve{\mathbf{D}}_1-\delta_3\breve{\mathbf{V}}$, and $\mathbf{Z}_2=(\tau_M-\tau_m+1)\mathbf{Q}-\delta_2 I$.

Having obtained the analysis results in Theorem 2, we are now ready to consider the design problem of the state estimator (10). From Theorem 2 and Lemma 3, the following result can be derived easily.

Theorem 3: Under Assumption 1, the system (38) becomes a state estimator of the discrete-time complex network (10) if there exist three scalar constants δ_1, δ_2, and δ_3 and three positive definite diagonal block matrices $\mathbf{P}=\mathrm{diag}\{P_1,P_2,\ldots,P_N\}$, $\mathbf{Q}=\mathrm{diag}\{Q_1,Q_2,\ldots,Q_N\}$, and $\mathbf{R}=\mathrm{diag}\{R_1,R_2,\ldots,R_N\}$ such that the LMI shown at the bottom of the page holds, where $\hat{\mathbf{W}}=W\otimes\Gamma$, $\mathbf{Z}_0=-\mathbf{P}-\delta_1\breve{\mathbf{B}}_1-\delta_2\breve{\mathbf{D}}_1-\delta_3\breve{\mathbf{V}}$,

$$\begin{bmatrix}\mathbf{Z}_0 & \mathbf{Z}_3 & \delta_2\breve{\mathbf{D}}_2 & \hat{\mathbf{W}}^{\mathrm{T}}\mathbf{P}-C^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}} & \delta_3\breve{\mathbf{V}}_2 & \hat{\mathbf{W}}^{\mathrm{T}}\mathbf{P}-C^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}} & \hat{\mathbf{W}}^{\mathrm{T}}\mathbf{P}-C^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}}\\ * & \mathbf{P}-\delta_1 I & 0 & \mathbf{P} & 0 & \mathbf{P} & 0\\ * & * & \mathbf{Z}_2 & 0 & 0 & 0 & 0\\ * & * & * & \mathbf{P}-\mathbf{Q} & 0 & \mathbf{P} & 0\\ * & * & * & * & \bar{\mu}\mathbf{R}-\delta_3 I & 0 & 0\\ * & * & * & * & * & \mathbf{P}-\bar{\mu}^{-1}\mathbf{R} & 0\\ * & * & * & * & * & * & -\mathbf{P}\end{bmatrix}<0 \quad (46)$$

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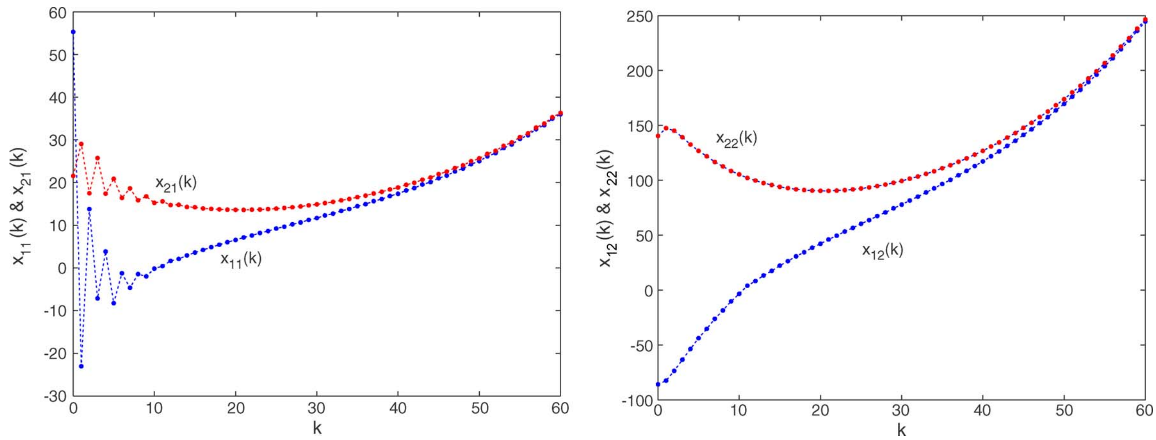

Fig. 1. Comparison of state trajectories of $x_1(k)$ and $x_2(k)$.

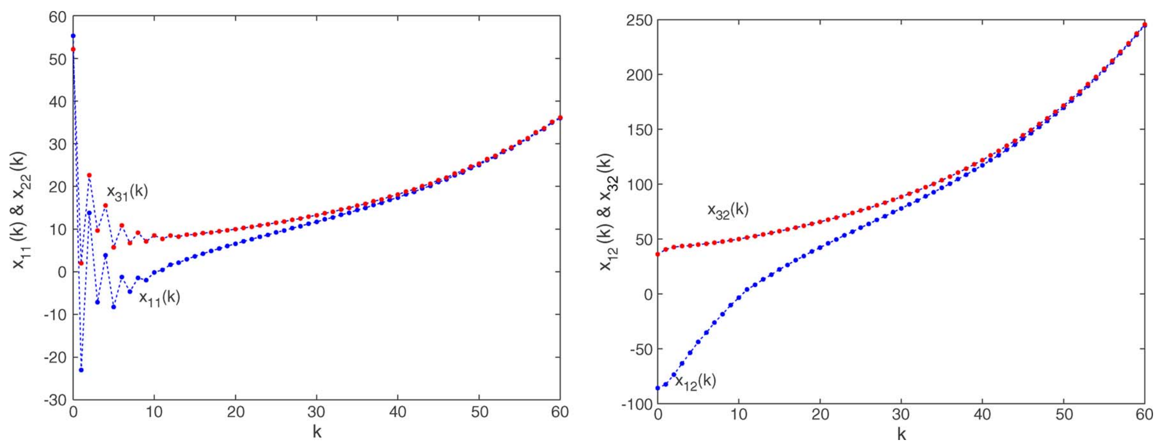

Fig. 2. Comparison of state trajectories of $x_1(k)$ and $x_3(k)$.

$\mathbf{Z}_3 = \hat{\mathbf{W}}^{\mathrm{T}}\mathbf{P} - C^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}} + \delta_1\breve{\mathbf{B}}_2$, and $\mathbf{Z}_2$ is defined as in Theorem 2. In this case, the estimator gain matrix K can be chosen as $K = \mathbf{P}^{-1}\mathbf{Y}$.

V. Numerical Simulation

In this section, two numerical examples are presented to demonstrate the usefulness of the developed designs on the synchronization as well as state estimation problems for the complex network (10). In order to show the validity of our results, we select an unstable complex network on which the proposed synchronization and estimation schemes still work well.

Example 1: For simplicity, let us consider the system (10) of three nodes. Suppose that

$$n = 2 \quad \tau(k) = 3 + \left(1 + (-1)^k\right)/2 \quad \mu_m = 2^{-(m+3)}$$
$$\Gamma = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} \quad w_{ij} = \begin{cases} 0.1, & i \neq j \\ -0.2, & i = j. \end{cases}$$

Let the nonlinear vector-valued functions be given by

$$\begin{aligned} f\left(x_i(k)\right) = &\left(-0.5x_{i1}(k) + \tanh\left(0.2x_{i1}(k)\right)\right. \\ &+ 0.2x_{i2}(k),\ 0.95x_{i2}(k) \\ &\left.- \tanh\left(0.75x_{i2}(k)\right)\right)^{\mathrm{T}}, \qquad i = 1,2,3 \\ g\left(x_i(k)\right) = &\, h\left(x_i(k)\right) \\ = &\left(0.2x_{i1}(k) - \tanh\left(0.1x_{i1}(k)\right),\ 0.1x_{i2}(k)\right)^{\mathrm{T}} \\ & \qquad i = 1,2,3. \end{aligned}$$

Then, it can be verified that $\tau_m = 3$, $\tau_M = 4$, $\overline{\mu} = 1/8$, and

$$B_1 = \begin{pmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{pmatrix} \quad B_2 = \begin{pmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{pmatrix}$$
$$D_1 = V_1 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.1 \end{pmatrix} \ D_2 = V_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}.$$

By using the Matlab LMI Toolbox, we solve LMI (12) and obtain a feasible solution as follows:

$$P = \begin{pmatrix} 1.5314 & -0.2020 \\ -0.2020 & 2.9295 \end{pmatrix} \quad Q = \begin{pmatrix} 6.1702 & -0.7353 \\ -0.7353 & 13.2959 \end{pmatrix}$$
$$R = \begin{pmatrix} 5.5543 & -0.4468 \\ -0.4468 & 9.8905 \end{pmatrix}$$
$$\delta_1 = 8.6589 \quad \delta_2 = 73.7843 \quad \delta_3 = 16.4907.$$

Then, it follows from Theorem 1 that the system (10) with given parameters reaches synchronization, which is further verified by the simulation result shown in Figs. 1 and 2. Fig. 1 shows that the state of $x_2(k)$ asymptotically approaches that of $x_1(k)$, whereas Fig. 2 shows that the state of $x_3(k)$ asymptotically tends to that of $x_1(k)$.

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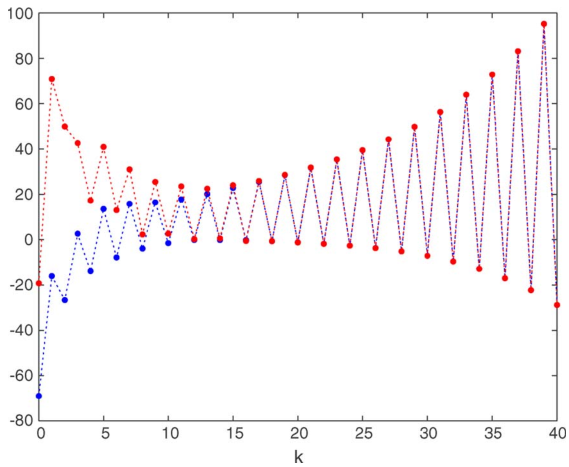

Fig. 3. State trajectories of x_{11} and $\hat{x}_{11}$.

Example 2: Consider the system (10) of three nodes. Suppose that

$$n=3 \quad \tau(k)=3+\left(1+(-1)^k\right)/2 \quad \mu_m=2^{-(m+1)}$$
$$\Gamma=I_3 \quad w_{ij}=\begin{cases}0.1, & i\neq j\\ -0.2, & i=j.\end{cases} \qquad C_i=\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}.$$

Let the nonlinear vector-valued functions be given by

$$\begin{aligned} f(x_i(k)) = {} & (-0.8x_{i1}(k)+\tanh(0.5x_{i1}(k))+0.4x_{i2}(k),\\ & 0.9x_{i2}(k)-\tanh(0.6x_{i2}(k)),\\ & 0.6x_{i3}-\tanh(0.4x_{i3}(k)))^{\mathrm{T}}, \qquad i=1,2,3\\ g(x_i(k)) = {} & h(x_i(k))\\ = {} & (0.2x_{i1}(k)-\tanh(0.1x_{i1}(k)),\\ & 0.3x_{i2}(k)-\tanh(0.2x_{i2}(k)),\\ & 0.3x_{i3}(k)-\tanh(0.2x_{i3}(k)))^{\mathrm{T}}, \quad i=1,2,3.\end{aligned}$$

It can also be checked that $\tau_m=3$, $\tau_M=4$, $\overline{\mu}=1/2$, and

$$B_1=\begin{pmatrix}-0.8 & 0.4 & 0\\ 0 & 0.8 & 0\\ 0 & 0 & 0.6\end{pmatrix} \quad B_2=\begin{pmatrix}-0.3 & 0.4 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.2\end{pmatrix}$$
$$D_1=V_1=\mathrm{diag}\{0.2,\ 0.3,\ 0.3\}$$
$$D_2=V_2=\mathrm{diag}\{0.1,\ 0.1,\ 0.1\}.$$

By using the Matlab LMI Toolbox, the LMI (46) can be solved and the estimator gain matrices are given by

$$K_i=\begin{pmatrix}-0.4958 & 0.4003\\ 0.0001 & 0.3405\\ 0.1114 & 0.0002\end{pmatrix}, \qquad (i=1,\ 2,\ 3).$$

Then, according to Theorem 3, the system (38) becomes a state estimator of the discrete-time complex network (10). That is, the state of the system (38) asymptotically approaches that of (10). The numerical simulation perfectly supports the theoretical results. Specifically, in Figs. 3–6, we show the evolution of the state $x_1(k)$ and its estimator $\hat{x}_1(k)$ of node 1, as well as the magnitude $|\varepsilon(k)|$ of the estimate error for the whole complex network. It is noticed from Fig. 6 that the magnitude $|\varepsilon(k)|$ of the error between the states of the whole network and its estimator approaches zero asymptotically.

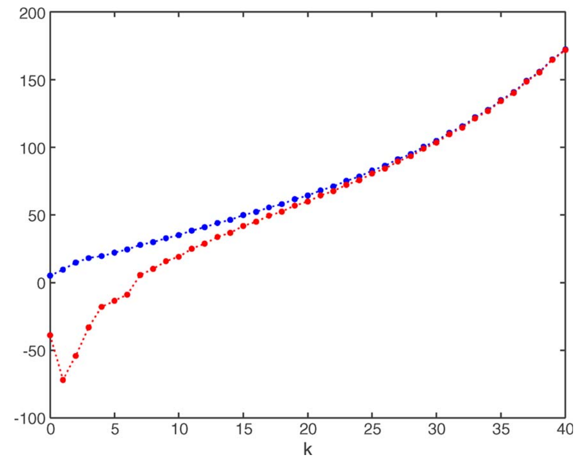

Fig. 4. State trajectories of x_{12} and $\hat{x}_{12}$.

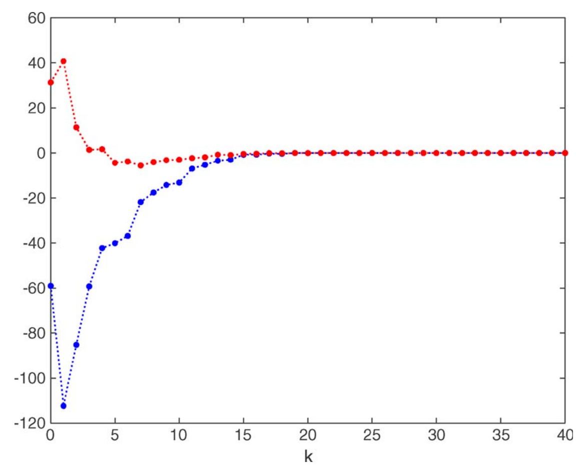

Fig. 5. State trajectories of x_{13} and $\hat{x}_{13}$

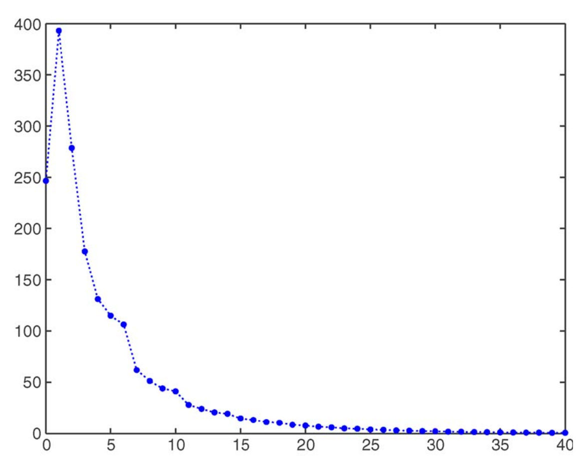

Fig. 6. Magnitude $|\varepsilon(k)|$ of estimate error.

Remark 7: It is trivial to consider the synchronization between the nodes for a stable network. To validate our analysis results, in Example 1, the given network is unstable which is of more significance to be used to test the theoretical results. Similarly, in Example 2, we again employ an unstable network to test the performance of our designed state estimator. In other words, both the two examples are nontrivial in evaluating the designed synchronizer and estimator.

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VI. Conclusion

In this paper, we have investigated the synchronization problem for an array of coupled complex discrete-time networks with the simultaneous presence of both the discrete and distributed time delays. Rather than the commonly used Lipschitz-type function, a more general sectorlike nonlinear function has been employed to describe the nonlinearities existing in the network. We have first defined the distributed time delays for the complex networks in the discrete-time domain. By utilizing a novel Lyapunov–Krasovskii functional and the Kronecker product, we have shown that the addressed synchronization problem can be converted into the feasibility problem of a set of LMIs. We have then tackled the state estimation problem for the same complex networks. Through available output measurements, we have developed an LMI approach to design a state estimator in order to estimate the network states such that the dynamics of the estimation error is guaranteed to be globally asymptotically stable. Two simulation examples have been provided to show the effectiveness of the proposed approach. It has been confirmed through simulation that our main results are valid even if the nominal subsystems within the network are unstable.

References

[1] J. Cao and J. Lu, "Adaptive synchronization of neural networks with or without time-varying delays," *Chaos*, vol. 16, no. 1, p. 013 133, Mar. 2006.

[2] J. Cao, P. Li, and W. W. Wang, "Global synchronization in arrays of delayed neural networks with constant and delayed coupling," *Phys. Lett. A*, vol. 353, no. 4, pp. 318–325, May 2006.

[3] M. P. Casey, "The dynamics of discrete-time computation, with application to recurrent neural networks and finite state machine extraction," *Neural Comput.*, vol. 8, no. 6, pp. 1135–1178, Aug. 1996.

[4] C. J. Cheng, T. L. Liao, J. J. Yan, and C. C. Hwang, "Exponential synchronization of a class of neural networks with time-varying delays," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 36, no. 1, pp. 209–215, Feb. 2006.

[5] L. O. Chua, C. W. Wu, A. Huang, and G.-Q. Zhong, "A universal circuit for studying and generating chaos—Part I: Routes to chaos," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 40, no. 10, pp. 732–742, Oct. 1993.

[6] L. Crocco, "Aspects of combustion stability in liquid propellant rocket motors, Part I: Fundamentals—low frequency instability with monopropellants," *J. Amer. Rocket Soc.*, vol. 21, pp. 163–178, 1951.

[7] C. Cruz-Hernández and N. Romero-Haros, "Communicating via synchronized time-delay Chua's circuits," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 13, no. 3, pp. 645–659, Jun. 2008.

[8] H. Gao and T. Chen, "New results on stability of discrete-time systems with time-varying state delay," *IEEE Trans. Autom. Control*, vol. 52, no. 2, pp. 328–334, Feb. 2007.

[9] H. Gao, J. Lam, and G. Chen, "New criteria for synchronization stability of general complex dynamical networks with coupling delays," *Phys. Lett. A*, vol. 360, no. 2, pp. 263–273, Dec. 2006.

[10] K. Q. Gu, "An integral inequality in the stability problem of time-delay systems," in *Proc. 39th IEEE Conf. Decision Control*, Sydney, Australia, Dec. 2000, pp. 2805–2810.

[11] Y. He, Q. Wang, M. Wu, and C. Lin, "Delay-dependent state estimation for delayed neural networks," *IEEE Trans. Neural Netw.*, vol. 17, no. 4, pp. 1077–1081, Jul. 2006.

[12] Y. He, G. Liu, and D. Rees, "New delay-dependent stability criteria for neural networks with time-varying delay," *IEEE Trans. Neural Netw.*, vol. 18, no. 1, pp. 310–314, Jan. 2007.

[13] Y. Y. Hou, T. L. Liao, and J. J. Yan, "Stability analysis of Takagi–Sugeno fuzzy cellular neural networks with time-varying delays," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 37, no. 3, pp. 720–726, Jun. 2007.

[14] X. Hu and J. Wang, "Design of general projection neural networks for solving monotone linear variational inequalities and linear and quadratic optimization problems," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 37, no. 5, pp. 1414–1421, Oct. 2007.

[15] J. Jost and M. Joy, "Special properties and synchronization in coupled map lattices," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 65, no. 1, p. 061 201, Jan. 2002.

[16] R. Z. Khasminskii, *Stochastic Stability of Differential Equations*. Alphen aan den Rijn, The Netherlands: Sijthoff & Noorhoff, 1980.

[17] C. G. Li and G. Chen, "Synchronization in general complex dynamical networks with coupling delays," *Physica A*, vol. 343, pp. 263–278, Nov. 2004.

[18] Z. Li and G. Chen, "Global synchronization and asymptotic stability of complex dynamical networks," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 53, no. 1, pp. 28–33, Jan. 2006.

[19] J. Liang, Z. Wang, and X. Liu, "Exponential synchronization of stochastic delayed discrete-time complex networks," in *Nonlinear Dynamics*, Jul. 2008, vol. 53, pp. 153–165.

[20] Y. Liu, Z. Wang, and X. Liu, "Global exponential stability of generalized recurrent neural networks with discrete and distributed delays," *Neural Netw.*, vol. 19, no. 5, pp. 667–675, Jun. 2006.

[21] Y. Liu, Z. Wang, and X. Liu, "Design of exponential state estimators for neural networks with mixed time delays," *Phys. Lett. A*, vol. 364, no. 5, pp. 401–412, May 2007.

[22] Y. Liu, Z. Wang, and X. Liu, "Exponential synchronization of complex networks with Markovian jump and mixed delays," *Phys. Lett. A*, vol. 372, no. 22, pp. 3986–3998, May 2008.

[23] W. L. Lu and T. P. Chen, "Synchronization of coupled connected neural networks with delays," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 51, no. 12, pp. 2491–2503, Dec. 2004.

[24] W. L. Lu and T. P. Chen, "Synchronization analysis of linearly coupled networks of discrete time systems," *Phys. D: Nonlinear Phenomena*, vol. 198, no. 1/2, pp. 148–168, 2004.

[25] W. L. Lu and T. P. Chen, "Global synchronization of discrete-time dynamical network with a directed graph," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 54, no. 2, pp. 136–140, Feb. 2007.

[26] R. Palm, "Synchronization of decentralized multiple-model systems by market-based optimization," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 34, no. 1, pp. 665–672, Feb. 2004.

[27] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, vol. 64, no. 8, pp. 821–824, Feb. 1990.

[28] V. Perez-Munuzuri, V. Perez-Villar, and L. O. Chua, "Autowaves for image processing on a two-dimensional CNN array of excitable nonlinear circuits: Flat and Wrinkled labyrinths," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 40, no. 3, pp. 174–181, Mar. 1993.

[29] S. Senan and S. Arik, "Global robust stability of bidirectional associative memory neural networks with multiple time delays," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 37, no. 5, pp. 1375–1381, Oct. 2007.

[30] Q. Song and J. Cao, "Impulsive effects on stability of fuzzy Cohen–Grossberg neural networks with time-varying delays," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 37, no. 3, pp. 733–741, Jun. 2007.

[31] Z. Toroczkai, "Complex networks: The challenge of interaction topology," *Los Alamos Sci.*, vol. 29, pp. 94–109, 2005.

[32] X.-F. Wang and G. Chen, "Synchronization in small-world dynamical networks," *Int. J. Bifurc. Chaos*, vol. 12, no. 1, pp. 187–192, 2002.

[33] Z. Wang, Y. Liu, M. Li, and X. Liu, "Stability analysis for stochastic Cohen–Grossberg neural networks with mixed time delays," *IEEE Trans. Neural Netw.*, vol. 17, no. 3, pp. 814–820, May 2006.

[34] Z. Wang, D. W. C. Ho, and X. Liu, "State estimation for delayed neural networks," *IEEE Trans. Neural Netw.*, vol. 16, no. 1, pp. 279–284, Jan. 2005.

[35] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, no. 6684, pp. 440–442, Jun. 1998.

[36] C. W. Wu, "Synchronization in coupled arrays of chaotic oscillators with nonreciprocal coupling," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 50, no. 2, pp. 294–297, Feb. 2003.

[37] C. W. Wu, "Synchronization in arrays of coupled nonlinear systems with delay and nonreciprocal time-varying coupling," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 52, no. 5, pp. 282–286, May 2005.

[38] K. Yuan, J. Cao, and H. Li, "Robust stability of switched Cohen–Grossberg neural networks with mixed time-varying delays," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 36, no. 6, pp. 1356–1363, Dec. 2006.

[39] A. Zheleznyak and L. O. Chua, "Coexistence of low- and high-dimensional spatio-temporal chaos in a chain of dissipatively coupled Chua's circuits," *Int. J. Bifurc. Chaos*, vol. 4, no. 3, pp. 639–674, 1994.

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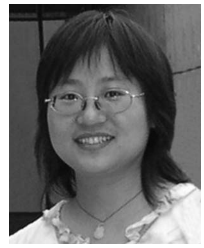

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