Robust H_∞ Control for Discrete-Time Fuzzy Systems With Infinite-Distributed Delays

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***Abstract*—This paper is concerned with the robust H_∞ control problem for a class of discrete-time Takagi–Sugeno (T–S) fuzzy systems with time delays and uncertain parameters. The time delay is assumed to be infinitely distributed in the discrete-time domain, and the uncertain parameters are norm-bounded. By using the linear matrix inequality (LMI) technique, sufficient conditions are derived for ensuring the exponential stability as well as the H_∞ performance for the closed-loop fuzzy control system. It is also shown that the controller gain can be characterized in terms of the solution to a set of LMIs, which can be easily solved by using standard software packages. A simulation example is exploited in order to illustrate the effectiveness of the proposed design procedures.**

***Index Terms*—Fuzzy systems, H_∞ control, infinite-distributed delays, linear matrix inequality (LMI), parameter uncertainties.**

I. Introduction

MANY mathematical models for real-world phenomena are inherently nonlinear, and the stability analysis and synthesis problems for nonlinear systems are generally difficult. To facilitate the mathematical analysis, in the literature, some stringent assumptions have been imposed on the nonlinearities, such as smoothness and Lipschitz continuity (see, e.g., [16] and [25]), which have inevitably led to considerable conservatism. As an alternative approach, in the past few decades, the fuzzy logic theory has been demonstrated to be effective in dealing with a variety of complex nonlinear systems, which has therefore received a great deal of attention in the literature (see, e.g., [11]–[19] and [21]). Among various fuzzy systems, one of the most popular models is the Takagi–Sugeno (T–S) model (see [2], [4], [10], [20], and [23] for some recent publications). In this type of fuzzy model, a nonlinear system is represented by a set of local linear models smoothly connected by nonlinear membership functions, which has a convenient and simple dynamic structure such that the existing results for linear systems theory can be readily extended for this class of nonlinear systems. A rich body of literature has appeared on the stability analysis and synthesis problems for T–S fuzzy systems (see, e.g., [5]–[7] and [27]).

When modeling real-time plants, the parameter uncertainties are unavoidable, which would lead to perturbations of the elements of a system matrix in a state-space model. These uncertainties may arise from variations of the operating point, aging of the devices, identification errors, etc. Therefore, in the past decade, considerable attention has been devoted to different issues for linear or nonlinear uncertain systems, and a large number of papers have been published (see [3], [8], [9], and [24] for some recent results). Recently, the uncertain parameters have also been taken into account in T–S fuzzy systems. For example, in [4] and [22], the stability issue has been considered for a class of T–S fuzzy dynamical systems with time delays and uncertain parameters. The robust H_∞ control problem for T–S fuzzy systems with parameter uncertainties has also been addressed by many researchers (for instance, see [4], [6], [7], [9], and [10]).

On another active research frontier, owing to the fact that time delays commonly reside in practical systems and constitute a main source for system performance degradation or even instability, the past decade has witnessed significant progress on analysis and synthesis for linear/nonlinear systems with various types of delays, and a large amount of literature has appeared on the general topic of time-delay systems (see, e.g., [3], [8], [13], [16], [26], and [28]). In particular, the linear matrix inequality (LMI) technique has been extensively used because of its computational efficiency, and it is not surprising that a great number of LMI-based results have been published (see, e.g., [3], [8], and [28]). It is worth pointing out that the distributed delay occurs very often in reality and it has been drawning increasing attention (see, e.g., [12], [13], and [26]). However, *almost all existing works* on distributed delays have focused on continuous-time systems that are described in the form of either finite or infinite integral. It is known that the discrete-time system is in a better position to model digitally transmitted signals in a dynamical way than its continuous-time analogue. Discrete-time systems have already been applied in a wide range of areas, such as image processing, time-series analysis, quadratic optimization problems, and system identification. On the other hand, a large-scale system network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of subsystem sizes and lengths, which gives rise to possible distributed delays for discrete-time systems. With the increasing application of digitalization, distributed delays may emerge in a discrete-time manner, and therefore, it becomes desirable to study the discrete-time systems with distributed

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delays. Some pioneering research has been carried out [14] on the formulation of discrete-time-distributed delays. However, to the best of the authors' knowledge, the research on discrete-time fuzzy systems with distributed delays has not been addressed yet and remains as an open topic for further investigation.

In this paper, the H_∞ control problem is addressed for a class of discrete-time T–S systems with infinite-distributed delays and uncertain parameters, where the parameter uncertainties are assumed to be a norm-bounded. The objective is to design a fuzzy controller such that in the presence of time delays as well as parameter uncertainties, the closed-loop fuzzy control system is exponentially stable and also satisfies a prescribed H_∞ disturbance attenuation index. By using the LMI technique, sufficient conditions are first established that guarantee the desired stability and H_∞ performance, and the controller gain is then characterized in terms of the solution to a set of LMIs. A simulation example is finally presented to illustrate the effectiveness of the proposed design procedures.

Notation: In this paper, $\mathbb{R}^n, \mathbb{R}^{n\times m}$, and $\mathbb{Z}(\mathbb{Z}^+, \mathbb{Z}^-)$ denote, respectively, the n-dimensional Euclidean space, the set of all $n\times m$ real matrices, and the set of integers (nonnegative integers, negative integers). $|\cdot|$ refers to the Euclidean norm in $\mathbb{R}^n$. Letting $\tau \in \mathbb{Z}^+, C([-\tau, 0]; \mathbb{R}^n)$ denotes the family of continuous functions μ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the norm $\|\mu\| = \sup_{-\tau\le k\le 0} |\mu(k)|$, and I denotes the identity matrix of compatible dimension. The notation $X \ge Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). For a matrix M, M^T represents its transpose and $\|M\|$ denotes its spectral norm. The shorthand $\mathrm{diag}\{M_1,\ M_2, \ldots,\ M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices $M_1,\ M_2,\ \ldots,\ M_n$. In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

II. Problem Formulation

In this paper, we consider the following discrete-time fuzzy systems with infinite-distributed delay and uncertain parameters:

`Plant Rule` i:

IF $\theta_1(k)$ is η_{i1} and $\cdots$ and $\theta_p(k)$ is η_{ip},

THEN

$$x(k+1) = A_i(k)x(k) + A_{di}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{1i}v(k) + D_{1i}u(k) \tag{1}$$

$$z(k) = C_i(k)x(k) + B_{2i}v(k) + D_{2i}u(k) \tag{2}$$

$$x(k) = \phi(k) \qquad \forall\, k \in \mathbb{Z}^-, \qquad i = 1, \ldots, r.$$

where η_{ij} is the fuzzy set, $x(k) \in \mathbb{R}^n$ is the state, $z(k) \in \mathbb{R}^q$ is the controlled output vector, $u(k) \in \mathbb{R}^m$ is the control input, $v(k) \in l_2[0, \infty)$ is the disturbance input, $\phi(k)(\forall\, k \in \mathbb{Z}^-)$ is the initial state, $A_i(k) = A_i + \Delta A_i(k), C_i(k) = C_i + \Delta C_i(k)$, and $A_i, A_{di}, B_{1i}, B_{2i}, C_i, D_{1i}$, and D_{2i} are all constant matrices with appropriate dimensions.

The matrices $\Delta A_i(k)$ and $\Delta C_i(k)$ represent time-varying norm-bounded parameter uncertainties that satisfy

$$\begin{bmatrix}\Delta A_i(k)\\ \Delta C_i(k)\end{bmatrix} = \begin{bmatrix}E_{ai}\\ E_{ci}\end{bmatrix} F(k)N$$

where E_{ai}, E_{ci}, and N are the constant matrices of appropriate dimensions, and $F(k)$ is an unknown matrix function satisfying

$$F^T(k)F(k) \le I \qquad \forall\, k. \tag{3}$$

The constants $\mu_d \ge 0 (d = 1, 2, \ldots)$ satisfy the following convergence conditions:

$$\bar{\mu} := \sum_{d=1}^{\infty}\mu_d \le \sum_{d=1}^{\infty} d\mu_d < +\infty. \tag{4}$$

Remark 1: The delay term $\sum_{d=1}^{\infty}\mu_d x(k-d)$ in the fuzzy system (1), (2) is the so-called infinitely distributed delay in the discrete-time setting, which can be regarded as the discretization of the infinite integral form $\int_{-\infty}^{t} k(t-s)x(s)ds$ for the continuous-time system. The importance of distributed delays has been widely recognized and intensively studied (see, e.g., [12], [13], and [26]). However, almost all existing references concerning distributed delays are concerned with the continuous-time systems, where the distributed delays are described in the form of a finite or infinite integral. The description of the discrete-time-distributed delays has been proposed in [14], and we aim to study the control problem for discrete-time fuzzy systems with such kind of distributed delays in this paper.

Remark 2: In this paper, similar to the convergence restriction on the delay kernels of the infinite-distributed delays for continuous-time systems, the constants $\mu_d (d = 1, 2, \ldots)$ are assumed to satisfy the convergence conditions (4), which can guarantee the convergence of the terms of infinite delays as well as the Lyapunov–Krasovskii functional defined later.

The compact form of the T–S fuzzy system (1), (2) is represented as

$$x(k+1) = \sum_{i=1}^{r} h_i(\theta(k))\Bigg[A_i(k)x(k) + A_{di}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{1i}v(k) + D_{1i}u(k)\Bigg] \tag{5}$$

and

$$z(k) = \sum_{i=1}^{r} h_i(\theta(k))[C_i(k)x(k) + B_{2i}v(k) + D_{2i}u(k)] \tag{6}$$

where

$$h_i(\theta(k)) = \frac{\vartheta_i(\theta(k))}{\sum_{j=1}^{r}\vartheta_j(\theta(k))}, \qquad \vartheta_i(\theta(k)) = \prod_{j=1}^{r}\eta_{ij}(\theta_j(k))$$

$$\theta(k) = [\theta_1(k),\ \theta_2(k),\ \ldots,\ \theta_r(k)]$$

with $\eta_{ij}(\theta_j(k))$ being the grade of membership of $\theta_j(k)$ in η_{ij}. Here, $\vartheta_i(\theta(k))$ has the following basic property:

$$\vartheta_i(\theta(k)) \geq 0, \qquad i = 1, \ldots, r, \qquad \sum_{i=1}^{r} \vartheta_i(\theta(k)) \geq 0 \quad \forall k \tag{7}$$

and therefore

$$h_i(\theta(k)) \geq 0, \qquad i = 1, \ldots, r, \qquad \sum_{i=1}^{r} h_i(\theta(k)) = 1 \qquad \forall k. \tag{8}$$

We consider the following fuzzy control law for the fuzzy system (1), (2):

Controller Rule i:
IF $\theta_1(k)$ is η_{i1} and $\cdots$ and $\theta_p(k)$ is η_{ip},
THEN

$$u(k) = -K_i x(k), \qquad i = 1, \ldots, r. \tag{9}$$

The control law can also be given by

$$u(k) = -\sum_{i=1}^{r} h_i(\theta(k)) K_i x(k) \tag{10}$$

and the closed-loop T–S fuzzy control system is governed by

$$\begin{aligned} x(k+1) = {} & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k)) h_j(\theta(k)) \\ & \times \left[\mathcal{A}_{ij}(k)x(k) + A_{di}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{1i}v(k)\right] \end{aligned} \tag{11}$$

$$z(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k)) h_j(\theta(k)) \times [\mathcal{C}_{ij}(k)x(k) + B_{2i}v(k)] \tag{12}$$

where $\mathcal{A}_{ij}(k) := A_i(k) - D_{1i}K_j$ and $\mathcal{C}_{ij}(k) := C_i(k) - D_{2i}K_j$.

Before formulating the problem to be investigated, we first introduce the following stability concept for the fuzzy system (11), (12).

Definition 1: For the system (11), (12) and every initial conditions ϕ, the trivial solution is said to be *exponentially stable* if there exist scalars α ($\alpha > 0$) and β ($0 < \beta < 1$) such that

$$|x(k, \phi)|^2 \leq \alpha\beta^k \sup_{s \in \mathbb{Z}^-} |\phi(s)|^2. \tag{13}$$

Definition 2: Given a scalar $\gamma > 0$, the fuzzy system (11), (12) is said to be *exponentially stable with disturbance attenuation level* γ if it is exponentially stable and under zero initial conditions

$$\|z(k)\|_2 < \gamma^2 \|v(k)\|_2 \tag{14}$$

holds for all nonzero $v(k) \in l_2[0, \infty)$.

The objective of this paper is to design a controller for the discrete-time fuzzy system (11), (12) such that for all admissible infinite-distributed delays and uncertain parameters, the fuzzy system (11), (12) is exponentially stable with disturbance attenuation level γ.

III. MAIN RESULTS

First, we give the following lemmas that will be used in the proofs of our main results in this paper.

Lemma 1 [1] (Schur complement): Given constant matrices $\Sigma_1, \Sigma_2, \Sigma_3$, where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$. Then, $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \qquad \text{or} \qquad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Lemma 2: Let $x \in \mathbb{R}^n, y \in \mathbb{R}^n$ and matrix $Q > 0$. Then, we have $x^T Q y + y^T Q x \leq x^T Q x + y^T Q y$.

Lemma 3 (S-procedure) [1]: Let $\Upsilon = \Upsilon^T$, $\mathcal{M}$ and $\mathcal{N}$ be real matrices of appropriate dimensions with F satisfying $F^T F \leq I$. Then

$$\Upsilon + \mathcal{M}F\mathcal{N} + \mathcal{N}^T F^T \mathcal{M}^T < 0$$

if and only if there exists a positive scalar $\delta > 0$ such that

$$\Upsilon + \delta\mathcal{M}\mathcal{M}^T + \frac{1}{\delta}\mathcal{N}^T\mathcal{N} < 0$$

or equivalently

$$\begin{bmatrix} \Upsilon & \delta\mathcal{M} & \mathcal{N}^T \\ \delta\mathcal{M}^T & -\delta I & 0 \\ \mathcal{N} & 0 & -\delta I \end{bmatrix} < 0.$$

Lemma 4 [14]: Let $M \in \mathbb{R}^{n \times n}$ be a positive semidefinite matrix, $x_i \in \mathbb{R}^n$, and constant $a_i > 0 (i = 1, 2, \ldots)$. If the series concerned is convergent, then we have

$$\left(\sum_{i=1}^{\infty} a_i x_i\right)^T M \left(\sum_{i=1}^{\infty} a_i x_i\right) \leq \left(\sum_{i=1}^{\infty} a_i\right) \sum_{i=1}^{\infty} a_i x_i M x_i. \tag{15}$$

Proof: Assuming m is a positive constant and from Lemma 2, we can easily know that

$$\begin{aligned} & \left(\sum_{i=1}^{m} a_i x_i\right)^T M \left(\sum_{i=1}^{m} a_i x_i\right) \\ & = \sum_{i=1}^{m}\sum_{j=1}^{m} a_i a_j x_i^T M x_j \leq \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{1}{2} a_i a_j (x_i^T M x_i + x_j^T M x_j) \\ & = \left(\sum_{i=1}^{m} a_i\right) \sum_{i=1}^{m} a_i x_i M x_i \end{aligned}$$

then, the desired result follows directly by letting $m \to \infty$. ■

For convenience of presentation, we first discuss the nominal system of (11) and (12) (i.e., without parameter uncertainties) and will eventually extend our main results to the more general case. We have the following result.

Theorem 1: Consider the nominal system of (11) and (12) with given controller parameters and a prescribed H_∞ performance index $\gamma > 0$. If there exist matrices $P > 0$ and $Q > 0$ such that the following LMIs, (16) and (17), as shown at the bottom of the next page, hold for $j = 2, \ldots, r$, then the nominal fuzzy system of (11) and (12) is exponentially stable with disturbance attenuation level γ.

Proof: In order to show that system (11), (12) is exponentially stable with disturbance attenuation level γ under conditions (16) and (17), we define the following Lyapunov–Krasovskii functional candidate:

$$V(k) = x^T(k)Px(k) + \sum_{d=1}^{\infty}\mu_d \sum_{\tau=k-d}^{k-1} x^T(\tau)Qx(\tau). \qquad (18)$$

Calculating the difference of $V(k)$, we have

$$\begin{aligned}\Delta V(k) &= V(k+1) - V(k)\\ &= x^T(k+1)Px(k+1) - x^T(k)Px(k)\\ &\quad + \sum_{d=1}^{\infty}\mu_d \sum_{\tau=k+1-d}^{k} x^T(\tau)Qx(\tau)\\ &\quad - \sum_{d=1}^{\infty}\mu_d \sum_{\tau=k-d}^{k-1} x^T(\tau)Qx(\tau)\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{l=1}^{r} h_i(\theta)h_j(\theta)h_s(\theta)h_l(\theta)\\ &\quad \times \left[\mathcal{A}_{ij}x(k) + A_{di}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{1i}v(k)\right]^T P\\ &\quad \times \left[\mathcal{A}_{sl}x(k) + A_{ds}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{1s}v(k)\right]\\ &\quad - x^T(k)Px(k) + \bar{\mu}x^T(k)Qx(k)\\ &\quad - \sum_{d=1}^{\infty}\mu_d x^T(k-d)Qx(k-d) \end{aligned} \qquad (19)$$

where $\mathcal{A}_{ij} = A_i - D_{1i}K_j$.

From Lemma 4, it can be easily seen that

$$\begin{aligned}&-\sum_{d=1}^{\infty}\mu_d x^T(k-d)Qx(k-d)\\ &\quad \le -\frac{1}{\bar{\mu}}\left(\sum_{d=1}^{\infty}\mu_d x(k-d)\right)^T Q\left(\sum_{d=1}^{\infty}\mu_d x(k-d)\right)\end{aligned} \qquad (20)$$

where $\bar{\mu}$ is defined in (4).

Substituting (20) into (19), we can obtain

$$\begin{aligned}\Delta V(k) &\le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{l=1}^{r} h_i(\theta)h_j(\theta)h_s(\theta)h_l(\theta)\\ &\quad \times \left[\mathcal{A}_{ij}x(k) + A_{di}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{1i}v(k)\right]^T P\\ &\quad \times \left[\mathcal{A}_{sl}x(k) + A_{ds}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{1s}v(k)\right]\\ &\quad - x^T(k)Px(k) + \bar{\mu}x^T(k)Qx(k)\\ &\quad - \frac{1}{\bar{\mu}}\left(\sum_{d=1}^{\infty}\mu_d x(k-d)\right)^T Q\left(\sum_{d=1}^{\infty}\mu_d x(k-d)\right). \end{aligned} \qquad (21)$$

For notational convenience, we denote the following matrix variables:

$$\xi(k) = \left[x^T(k)\ \ \sum_{d=1}^{\infty}\mu_d x^T(k-d)\ \ v^T(k)\right]^T$$

$$\zeta(k) = \left[x^T(k)\ \ \sum_{d=1}^{\infty}\mu_d x^T(k-d)\right]^T$$

$$\bar{A}_{ij} = [A_i - D_{1i}K_j\ \ A_{di}\ \ B_{1i}] \quad \tilde{A}_{ij} = [A_i - D_{1i}K_j\ \ A_{di}] \qquad (22)$$

$$\bar{C}_{ij} = [C_i - D_{2i}K_j\ \ 0\ \ B_{2i}] \qquad \Xi_{ij} = [\bar{A}_{ij}^T\ \ \bar{C}_{ij}^T]^T$$

$$\bar{P} = \mathrm{diag}\{P,\ I\} \qquad \tilde{P} = \mathrm{diag}\left\{P - \bar{\mu}Q,\ \frac{1}{\bar{\mu}}Q, \gamma^2 I\right\}$$

$$\hat{P} = \mathrm{diag}\left\{P - \bar{\mu}Q, \frac{1}{\bar{\mu}}Q\right\}. \qquad (23)$$

In the following, we first prove the exponential stability of the resulting fuzzy system (11) with $v(k) = 0$. It follows from Lemma 2 and (21) that

$$\begin{aligned}\Delta V(k) &\le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{l=1}^{r} h_i(\theta)h_j(\theta)h_s(\theta)h_l(\theta)\zeta^T(k)\\ &\quad \times \left(\tilde{A}_{ij}^T P\tilde{A}_{sl} - \hat{P}\right)\zeta(k)\end{aligned}$$

$$\begin{bmatrix} -P+\bar{\mu}Q & * & * & * & * \\ 0 & -\frac{1}{\bar{\mu}}Q & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ PA_i - PD_{1i}K_i & PA_{di} & PB_{1i} & -P & * \\ C_i - D_{2i}K_i & 0 & B_{2i} & 0 & -I \end{bmatrix} < 0, \qquad i = 1,\ldots,r \qquad (16)$$

$$\begin{bmatrix} -4P+4\bar{\mu}Q & * & * & * & * \\ 0 & -4\frac{1}{\bar{\mu}}Q & * & * & * \\ 0 & 0 & -4\gamma^2 I & * & * \\ PA_i - PD_{1i}K_j + PA_j - PD_{1j}K_i & PA_{di} + PA_{dj} & PB_{1i} + PB_{1j} & -P & * \\ C_i - D_{2i}K_j + C_j - D_{2j}K_i & 0 & B_{2i} + B_{2j} & 0 & -I \end{bmatrix} < 0, \qquad i < j \qquad (17)$$

$$
\begin{aligned}
&= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{l=1}^{r} h_i(\theta)h_j(\theta)h_s(\theta)h_l(\theta)\\
&\quad \times \zeta^T(k)\left[\left(\frac{\tilde{A}_{ij}+\tilde{A}_{ji}}{2}\right)^T P\left(\frac{\tilde{A}_{sl}+\tilde{A}_{ls}}{2}\right)-\hat{P}\right]\zeta(k)\\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta)h_j(\theta)\zeta^T(k)\\
&\quad \times \left[\left(\frac{\tilde{A}_{ij}+\tilde{A}_{ji}}{2}\right)^T P\left(\frac{\tilde{A}_{ij}+\tilde{A}_{ji}}{2}\right)-\hat{P}\right]\zeta(k)\\
&= \sum_{i=1}^{r} h_i^2(\theta)\zeta^T(k)\left(\tilde{A}_{ii}^T P\tilde{A}_{ii}-\hat{P}\right)\zeta(k)\\
&\quad + \frac{1}{2}\sum_{i,j=1,i<j}^{r} h_i(\theta)h_j(\theta)\zeta^T(k)\\
&\quad \times \left[(\tilde{A}_{ij}+\tilde{A}_{ji})^T P(\tilde{A}_{ij}+\tilde{A}_{ji})-4\hat{P}\right]\zeta(k). \qquad (24)
\end{aligned}
$$

From Schur complement lemma, we know that $\Delta V(k) < 0$ if and only if (16) and (17) are true. From [24, Lemma 1], it can be concluded that the discrete-time fuzzy system (11), (12) with $v(k) = 0$ is exponentially stable.

For the proof of attainment of the H_∞ performance, we assume zero initial condition and consider the following index:

$$
\begin{aligned}
J_N &= \sum_{k=0}^{N}\left[z^T(k)z(k)-\gamma^2 v^T(k)v(k)\right]\\
&= \sum_{k=0}^{N}\{z^T(k)z(k)-\gamma^2 v^T(k)v(k)\\
&\quad + [V(k+1)-V(k)]\} - V(N+1)\\
&\le \sum_{k=0}^{N}\left[z^T(k)z(k)-\gamma^2 v^T(k)v(k)+\Delta V(k)\right]. \qquad (25)
\end{aligned}
$$

Similarly, from (21), we have

$$
\begin{aligned}
J_N &\le \sum_{k=0}^{N}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{l=1}^{r} h_i(\theta)h_j(\theta)h_s(\theta)h_l(\theta)\\
&\quad \times \left[\mathcal{A}_{ij}x(k)+A_{di}\sum_{d=1}^{\infty}\mu_d x(k-d)+B_{1i}v(k)\right]^T\\
&\quad \times P\left[\mathcal{A}_{sl}x(k)+A_{ds}\sum_{d=1}^{\infty}\mu_d x(k-d)+B_{1s}v(k)\right]\\
&\quad + [\mathcal{C}_{ij}x(k)+B_{2i}v(k)]^T[\mathcal{C}_{sl}x(k)+B_{2s}v(k)]\\
&\quad - x^T(k)Px(k)+\bar{\mu}x^T(k)Qx(k)\\
&\quad - \frac{1}{\bar{\mu}}\left(\sum_{d=1}^{\infty}\mu_d x(k-d)\right)^T Q\left(\sum_{d=1}^{\infty}\mu_d x(k-d)\right)\\
&\quad - \gamma^2 v^T(k)v(k)\\
&= \sum_{k=0}^{N}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{l=1}^{r} h_i(\theta)h_j(\theta)h_s(\theta)h_l(\theta)\xi^T(k)\\
&\quad \times \left(\Xi_{ij}^T\bar{P}\Xi_{sl}-\tilde{P}\right)\xi(k) \qquad (26)
\end{aligned}
$$

where $\mathcal{C}_{ij} = C_i - D_{2i}K_j$.

Again applying Lemma 2, we have

$$
\begin{aligned}
J_N &\le \sum_{k=0}^{N}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{l=1}^{r} h_i(\theta)h_j(\theta)h_s(\theta)h_l(\theta)\xi^T(k)\\
&\quad \times \left(\Xi_{ij}^T\bar{P}\Xi_{sl}-\tilde{P}\right)\xi(k)\\
&\le \sum_{k=0}^{N}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{l=1}^{r} h_i(\theta)h_j(\theta)h_s(\theta)h_l(\theta)\xi^T(k)\\
&\quad \times \left[\left(\frac{\Xi_{ij}+\Xi_{ji}}{2}\right)^T\bar{P}\left(\frac{\Xi_{sl}+\Xi_{ls}}{2}\right)-\tilde{P}\right]\xi(k)\\
&\le \sum_{k=0}^{N}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta)h_j(\theta)\xi^T
\end{aligned}
$$

$$
\Upsilon_1 := \begin{bmatrix} -X+\bar{\mu}\mathcal{Q} & * & * & * & * \\ 0 & -\frac{1}{\bar{\mu}}\mathcal{Q} & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ A_iX-D_{1i}Y_i & A_{di}X & B_{1i} & -X & * \\ C_iX-D_{2i}Y_i & 0 & B_{2i} & 0 & -I \end{bmatrix} < 0 \qquad i=1,\ldots,r \qquad (28)
$$

$$
\Upsilon_2 := \begin{bmatrix} -4X+4\bar{\mu}\mathcal{Q} & * & * & * & * \\ 0 & -4\frac{1}{\bar{\mu}}\mathcal{Q} & * & * & * \\ 0 & 0 & -4\gamma^2 I & * & * \\ A_iX-D_{1i}Y_j+A_jX-D_{1j}Y_i & A_{di}X+A_{dj}X & B_{1i}+B_{1j} & -X & * \\ C_iX-D_{2i}Y_j+C_jX-D_{2j}Y_i & 0 & B_{2i}+B_{2j} & 0 & -I \end{bmatrix} < 0 \qquad i<j \qquad (29)
$$

$$
\begin{aligned}
&= \sum_{k=0}^{N}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i^2(\theta)\xi^T\left(\Xi_{ii}^T\bar{P}\Xi_{ii}-\tilde{P}\right)\xi(k)\\
&\quad+\frac{1}{2}\sum_{i,j=1,i<j}^{r} h_i(\theta)h_j(\theta)\xi^T\\
&\quad\times\left[(\Xi_{ij}+\Xi_{ji})^T\bar{P}(\Xi_{ij}+\Xi_{ji})-4\tilde{P}\right]\xi(k). \qquad (27)
\end{aligned}
$$

By Schur complement, we can conclude from (16) and (17) that $J_N < 0$, and therefore, the inequality (14) holds. The proof of this theorem is thus completed. ∎

Remark 3: In Theorem 1, with given controller gain and disturbance attenuation level γ, we obtain the exponential stability conditions of the nominal fuzzy system of (11) and (12), which are represented via a set of LMIs in (16) and (17) and can be easily checked by using the Matlab LMI toolbox.

The following theorem shows that the desired controller parameters can be determined by solving a set of LMIs.

Theorem 2: Consider the nominal fuzzy system of (11) and (12). For a prescribed constant $\gamma > 0$, if there exist positive-definite matrices $X > 0$ and $\mathcal{Q} > 0$, and matrix Y_i such that the following LMIs, (28) and (29), as shown at the bottom of previous page, hold for $j = 2,\ldots,r$, then the nominal system of (11) and (12) is exponentially stable with disturbance attenuation γ. In this case, the parameters of the desired controller are given as follows:

$$K_i = Y_i X^{-1}. \qquad (30)$$

Proof: Let $X = P^{-1}$ and $\mathcal{Q} = XQX$. Pre- and postmultiplying the LMIs in (28) and (29) by $\mathrm{diag}\{P,P,I,P,I\}$, we have (16) and (17), and therefore, we can know from Theorem 1, (30), and Schur complement that system (11), (12) is exponentially stable with the prescribed disturbance attenuation level γ and the given controller parameters in (30). ∎

In the following theorem, we show that the robust H_∞ controller parameters can be determined based on the results of Theorem 2 for the nominal fuzzy system. This theorem can be easily proved along the lines of Theorem 2, and we, therefore, only keep necessary details in order to avoid unnecessary duplication.

Theorem 3: Consider the *uncertain* fuzzy system (11), (12). For a prescribed constant $\gamma > 0$, if there exist positive-definite matrices $X > 0$ and $\mathcal{Q} > 0$, and matrix Y_i such that the following LMIs, (31) and (32), as shown at the bottom of this page, hold for $j = 2,\ldots,r$, where $\mathbb{A}_{ij} = A_iX - D_{1i}Y_j + A_jX - D_{1j}Y_i, \mathbb{C}_{ij} = C_iX - D_{2i}Y_j + C_jX - D_{2j}Y_i$, then the system (11), (12) is exponentially stable with disturbance attenuation γ. In this case, the parameters of the desired controller are given as follows:

$$K_i = Y_i X^{-1}. \qquad (33)$$

Proof: In (28) and (29), replace A_i, A_j, C_i, and C_j with $A_i + \Delta A_i(k), A_j + \Delta A_j(k), C_i + \Delta C_i(k)$, and $C_j + \Delta C_j(k)$, respectively, and then rewrite (28) and (29) in terms of Lemma 3 in the form of the inequalities

$$
\begin{aligned}
\Upsilon_1 + \mathcal{M}_1 F \mathcal{N} + \mathcal{N}^T F^T \mathcal{M}_1^T < 0\\
\Upsilon_2 + \mathcal{M}_2 F \mathcal{N} + \mathcal{N}^T F^T \mathcal{M}_2^T < 0
\end{aligned}
$$

where Υ_1 and Υ_2 have been defined in (28) and (29), respectively, and

$$
\begin{aligned}
\mathcal{M}_1 &= [0 \quad 0 \quad 0 \quad E_{ai}^T \quad E_{ci}^T]^T,\\
\mathcal{M}_2 &= [0 \quad 0 \quad 0 \quad (E_{ai}+E_{aj})^T \quad (E_{ci}+E_{cj})^T]^T,\\
\mathcal{N} &= [NX \quad 0 \quad 0 \quad 0 \quad 0].
\end{aligned}
$$

From Lemmas 1 and 3, we can easily obtain the results of this theorem, and the details are thus omitted. ∎

IV. Illustrative Example

In this section, a simulation example is presented to illustrate the controller design method developed in this paper.

$$
\begin{bmatrix}
-X+\bar{\mu}\mathcal{Q} & * & * & * & * & * & *\\
0 & -\frac{1}{\bar{\mu}}\mathcal{Q} & * & * & * & * & *\\
0 & 0 & -\gamma^2 I & * & * & & \\
A_iX - D_{1i}Y_i & A_{di}X & B_{1i} & -X & * & * & *\\
C_iX - D_{2i}Y_i & 0 & B_{2i} & 0 & -I & * & *\\
0 & 0 & 0 & \delta E_{ai}^T & \delta E_{ci}^T & -\delta I & *\\
NX & 0 & 0 & 0 & 0 & 0 & -\delta I
\end{bmatrix} < 0 \qquad i = 1,\ldots,r \qquad (31)
$$

$$
\begin{bmatrix}
-4X+4\bar{\mu}\mathcal{Q} & * & * & * & * & * & *\\
0 & -4\frac{1}{\bar{\mu}}\mathcal{Q} & * & * & * & * & *\\
0 & 0 & -4\gamma^2 I & * & * & & \\
\mathbb{A}_{ij} & A_{di}X + A_{dj}X & B_{1i}+B_{1j} & -X & * & * & *\\
\mathbb{C}_{ij} & 0 & B_{2i}+B_{2j} & 0 & -I & * & *\\
0 & 0 & 0 & \delta(E_{ai}^T+E_{aj}^T) & \delta(E_{ci}^T+E_{cj}^T) & -\delta I & *\\
NX & 0 & 0 & 0 & 0 & 0 & -\delta I
\end{bmatrix} < 0 \qquad i < j \qquad (32)
$$

Consider a T–S fuzzy model (1), (2) with infinite-distributed delays and uncertain parameters. The rules are given as follows:

`Plant Rule` 1: IF $x_1(k)$ is $h_1(x_1(k))$, THEN

$$x(k+1) = A_1(k)x(k) + A_{d1}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{11}v(k) + D_{11}u(k) \tag{34}$$

$$z(k) = C_1(k)x(k) + B_{21}v(k) + D_{21}u(k). \tag{35}$$

`Plant Rule` 2: IF $x_1(k)$ is $h_2(x_1(k))$, THEN

$$x(k+1) = A_2(k)x(k) + A_{d2}\sum_{d=1}^{\infty}\mu_d x(k-d) + B_{12}v(k) + D_{12}u(k) \tag{36}$$

$$z(k) = C_2(k)x(k) + B_{22}v(k) + D_{22}u(k). \tag{37}$$

The model parameters are given as follows:

$$A_1 = \begin{bmatrix} 1.0 & 0.31 & 0 \\ 0 & 0.33 & 0.21 \\ 0 & 0 & -0.52 \end{bmatrix} \quad A_{d1} = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & -0.1 & 0 \\ -0 & 0.2 & -0.1 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix} \quad B_{21} = \begin{bmatrix} 0.15 \\ 0 \\ 0 \end{bmatrix} \quad D_{11} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.02 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.1 \end{bmatrix} \quad D_{21} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$E_{a1} = \begin{bmatrix} 0.1 \\ 0.02 \\ 0 \end{bmatrix} \quad E_{c1} = \begin{bmatrix} 0.08 \\ 0 \\ 0 \end{bmatrix} \quad N = [0.12 \;\; 0 \;\; 0]$$

$$A_2 = \begin{bmatrix} 0.8 & -0.38 & 0 \\ -0.2 & 0 & 0.21 \\ 0.1 & 0 & -0.55 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} 0 & -0.21 & 0 \\ 0.31 & 0.1 & 0 \\ 0 & -0.22 & 0.1 \end{bmatrix} \quad B_{12} = \begin{bmatrix} 0 \\ 0.12 \\ 0 \end{bmatrix}$$

$$B_{22} = \begin{bmatrix} 0 \\ 0 \\ 0.22 \end{bmatrix} \quad D_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -0.12 & 0 & 0.1 \\ 0 & -0.31 & 0.1 \\ 0 & 0.2 & -0.1 \end{bmatrix} \quad D_{22} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$E_{a2} = \begin{bmatrix} 0.05 \\ 0 \\ 0 \end{bmatrix} \quad E_{c2} = \begin{bmatrix} 0.18 \\ 0.2 \\ 0 \end{bmatrix} \quad \gamma^2 = 2$$

where the membership function is assumed to be

$$h_1(x_1(k)) = \frac{1-\sin^2(x_1(k))}{2} \quad h_2(x_1(k)) = \frac{1+\sin^2(x_1(k))}{2} \tag{38}$$

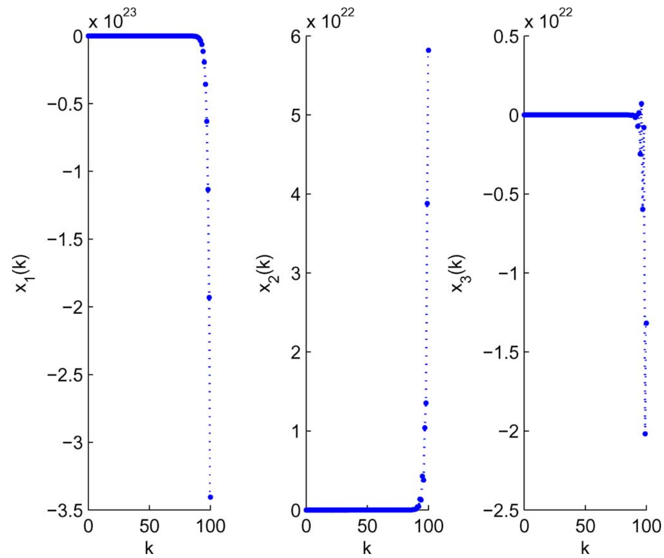

Fig. 1. State evolution $x(k)$ of uncontrolled systems.

and choosing the constants $\mu_d = 2^{-3-d}$, we easily find that $\bar{\mu} = \sum_{d=1}^{\infty}\mu_d = 2^{-3} < \sum_{d=1}^{\infty} d\mu_d = 2 < +\infty$, which satisfies the convergence condition (4).

By using the LMI toolbox, we solve (28) and obtain $\delta = 3.7319$ and

$$X = \begin{bmatrix} 1.3928 & 0.0081 & 0.0555 \\ 0.0081 & 1.6648 & -0.0328 \\ 0.0555 & -0.0328 & 1.4598 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.1671 & 0.0000 & 0.0001 \\ 0.0000 & 0.1676 & -0.0001 \\ 0.0001 & -0.0001 & 0.1672 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} 0.9582 & 0.1610 & 0.1049 \\ -0.1355 & 0.1118 & -0.0760 \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} 0.4854 & -0.3314 & 0.1390 \\ -0.0497 & 0.0522 & -0.0804 \end{bmatrix}.$$

According to Theorem 3, the controller parameters can be calculated as follows:

$$K_1 = \begin{bmatrix} 0.6855 & 0.0943 & 0.0479 \\ -0.0958 & 0.0667 & -0.0469 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 0.3465 & -0.1992 & 0.0776 \\ -0.0338 & 0.0305 & -0.0531 \end{bmatrix}.$$

Fig. 1 gives the state responses for the uncontrolled fuzzy systems, which are apparently unstable. Fig. 2 gives the simulation results of the responses of the closed-loop fuzzy systems, which confirms that the closed-loop system is indeed stable. Fig. 3 shows the controller output. The disturbance input $v(k)$ and controlled output $z(k)$ are depicted in Fig. 4. By simple computation, it is found that $\|z(k)\|_2 = 0.2109$ and $\|v(k)\|_2 = 1.0500$; therefore, $\|z(k)\|_2/\|v(k)\|_2 = 0.2009$, which is less than $\gamma^2 = 2$.

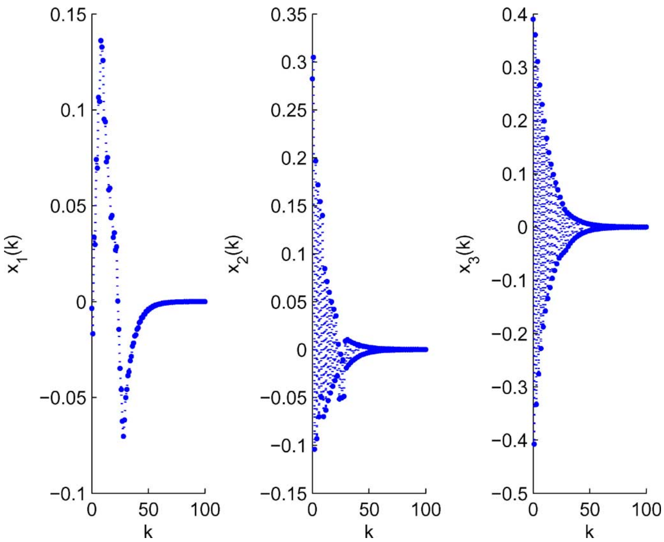

Fig. 2. State evolution $x(k)$ of controlled systems.

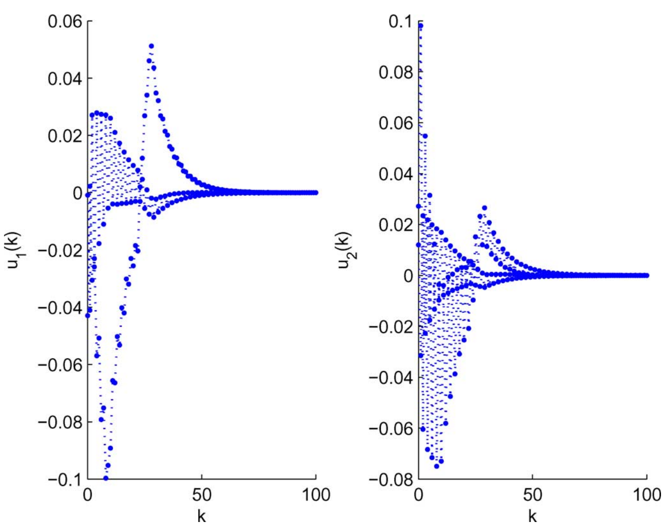

Fig. 3. Controllers $u(k)$.

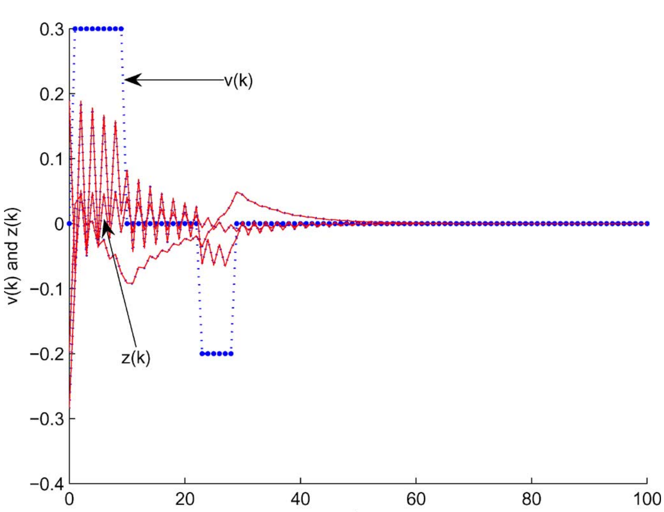

Fig. 4. Controlled output $z(k)$ and disturbance input $v(k)$.

V. Conclusion

In this paper, we have investigated the robust H_∞ control problem for a class of uncertain time-delay T–S fuzzy systems, where the delays are assumed as infinite-distributed delays and the uncertain parameters are norm-bounded. By using the Lyapunov stability theory and LMI technique, sufficient conditions have been developed so that the closed-loop fuzzy control system is guaranteed to be exponentially stable with H_∞ performance. It is also shown that the controller gains can be obtained by solving a set of LMIs. It should be pointed out that it is not difficult to extend the main results in this paper to more complex cases, such as control and filtering problems for a fuzzy system with infinite-distributed delays based on piecewise Lyapunov–Krasovskii functions.

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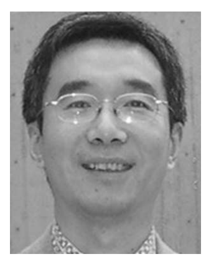

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