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A Sensor Activation Approach to Energy-Conserving Distributed Kalman Filtering for Wireless Sensor Networks

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***Abstract*—In this paper, the distributed Kalman filtering problem is investigated for a class of wireless sensor networks. Apart from achieving the desired estimation performance, enhancing energy efficiency is also considered as one of the design objectives. A novel sensor activation scheme is proposed to determine when to temporarily shut down the sensors to conserve energy. The influence of the sensor activation scheme is analyzed while designing the filter gains. A greedy strategy for sensor activation is designed to guarantee that error covariances are restricted below covariance levels. Finally, the effectiveness of the proposed approach is verified through a simulation experiment.**

***Index Terms*—Distributed Kalman filtering, State estimation, Recursive estimation, Wireless sensor network, Sensor activation**

I. Introduction

The past few decades have witnessed a surge in application demands for wireless sensor networks (WSNs) in various scenarios, including environmental monitoring, power monitoring, process automation, target tracking, and so on [1]–[5]. WSNs are generally deployed with distributed architectures, where sensors can not only acquire measurement data and obtain local estimations but also communicate with each other. Applying WSNs in control systems brings the advantages of reducing overall costs, alleviating the computational burden, and enhancing robustness [6], [7], thus improving the performance and efficiency of the entire system.

Distributed state estimation over WSNs is a fundamental research topic that has attracted much attention [8], [9]. So far, most previous studies have focused only on estimation accuracy, regardless of the energy efficiency of sensors, which inevitably increases energy costs and accelerates the depletion of the remaining useful life of sensors. To tackle this problem, recent studies have attempted to reduce the amount of transmitted data by neglecting unimportant measurement data. This line of work includes state estimation with event-triggered transmission [10], [11], state estimation using sensors with energy harvesting mechanisms [12], [13], state estimation with random sensor activation [14]–[16], partial-nodes-based state estimation [17], and so on.

The purpose of energy conservation is to reduce energy costs and prolong the lifespan of sensors, where the key lies in reducing the energy consumption of the most energy-consuming part in WSNs. In some cases, the sensing process is the most energy-consuming. For instance, image capturing requires a significant amount of energy to convert light into digital data [18]. Radars and ultrasonic sensors emit signals with high energy before obtaining the measurement [19]. Compared with such sensing processes, the energy cost of data processing and data transmission is relatively low. Under such circumstances, deactivating the sensors from time to time while maintaining communication between sensors is a straightforward and effective way to conserve energy in WSNs.

Distributed Kalman filtering is one of the most popular state estimation approaches for minimum variance estimation. This approach is highly scalable and can be employed to address challenges arising from complex system dynamics and unconventional measurements [20]–[22]. Therefore, the distributed Kalman filtering method would be a promising choice to tackle the state estimation problem with sensor activation. Some preliminary studies have been conducted on stochastic sensor activation in recent years [14], [15]. Nevertheless, the design of sensor activation schemes for energy conservation has not been adequately investigated. To fill in this gap, there is an urgent need to design a sensor activation scheme concerning both estimation accuracy and energy efficiency.

Motivated by the above discussions, this paper aims to investigate the design of sensor activation schemes to reduce the energy consumption of the measurement process while guaranteeing the precision of state estimation. The contributions of this paper are highlighted as follows: *i) the influence of the sensor activation on the error covariance is analyzed theoretically; and ii) an energy-conserving distributed Kalman filtering method is designed to achieve a balance between energy consumption and estimation accuracy*.

The organization of the rest of this paper is as follows. The problem formulation is given in section II. In Section III, we design the optimal filter gains, the suboptimal filter gains, and the sensor activation scheme. A simulation example is given to verify the effectiveness of the proposed method in Section

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IV. Finally, conclusions are drawn.

II. Problem Formulation

Consider the following linear discrete time-invariant system observed by a WSN with N spatially distributed sensors:

$$x(s+1) = Fx(s) + w(s), \tag{1}$$

$$y_i(s) = q_i(s)(H_i x(s) + v_i(s)) \tag{2}$$

where s indicates the time instant in the discrete-time system, $x(s) \in \mathbb{R}^n$ indicates the state vector at time instant s, $y_i(s) \in \mathbb{R}^{m_i}$ ($i \in \{1, 2, \cdots, N\}$) represents the measurement provided by the ith sensor at time instant s, $w(s)$ is the process noise, and $v_i(s)$ is the measurement noise. $F \in \mathbb{R}^{n\times n}$ and $H_i \in \mathbb{R}^{m_i\times n}$ are known constant matrices.

In a WSN with a sensor activation scheme, some sensors are deactivated at each time instant to save energy and prolong the lifespan of sensing components. To model such a scenario, in (2), a scalar variable $q_i(s) \in \{0, 1\}$ is introduced, where $q_i(s) = 1$ indicates that the ith sensor is activated at time instant s, and vice versa [14], [15]. The value of q_i is dependent on engineering practice and is assumed to be known to the state estimator at each time instant.

For state estimation via the WSN, it is assumed that each sensor is equipped with a local state estimator. Communication between neighbors via the sensor network is possible for each local state estimator. To keep track of the system state, it is further assumed that all local state estimators are always working, unlike the sensors. A state estimator updates its local state estimation not only by measurements provided by the corresponding sensor but also by information about state estimation received from neighbors.

To address the state estimation problem, the distributed Kalman filtering method is considered in this paper. The local state estimator of the ith sensor can be constructed as follows:

$$\hat{x}_i(s|s-1) = F\hat{x}_i(s-1|s-1), \tag{3}$$

$$\begin{aligned}\hat{x}_i(s|s) =& \hat{x}_i(s|s-1) \\ &+ K_i(s)(y_i(s) - q_i(s)H_i\hat{x}_i(s|s-1)) \\ &+ \sum_{j\in\mathcal{N}_i} L_{ij}(s)(\hat{x}_j(s|s-1) - \hat{x}_i(s|s-1)),\end{aligned} \tag{4}$$

where $\hat{x}_i(s|s-1)$ and $\hat{x}_i(s|s)$ are the prediction and the estimation of $x(s)$ acquired at time instant s, respectively. $\mathcal{N}_i$ stands for the indices of all neighbors of the ith state estimator. Furthermore, we assume that the topology of the WSN is represented by an undirected graph $\mathcal{G}$ with an adjacency matrix $G \triangleq [g_{ij}]$, where $g_{ij} = 1$ if the ith state estimator and the jth state estimator can receive data from each other.

Assumption 1: $w(s)$ and $v_i(s)$ are mutually independent Gaussian noises, which have zero mean and covariances $\mathbb{E}\left[w(s)w^T(l)\right] = \delta_{sl}Q$ and $\mathbb{E}\left[v_i(s)v_j^T(l)\right] = \delta_{ij}\delta_{sl}R_i$. The initial state $x(0)$ with known mean μ_0 and covariance P_0 is uncorrelated with the noises $w(s)$ and $v_i(s)$.

Remark 1: Assumption 1 is typical for Kalman filtering. The information about the topology of $\mathcal{G}$ will be used when designing the state estimator without relying on any assumption on the connectivity.

The prediction error and the estimation error of the ith state estimator are defined as

$$\tilde{x}_i(s|s-1) \triangleq x(s) - \hat{x}_i(s|s-1), \tag{5}$$

$$\tilde{x}_i(s|s) \triangleq x(s) - \hat{x}_i(s|s) \tag{6}$$

and the error covariances of the ith state estimator are then defined as

$$P_i(s|s-1) \triangleq \mathbb{E}\left[\tilde{x}_i(s|s-1)\tilde{x}_i^T(s|s-1)\right], \tag{7}$$

$$P_i(s|s) \triangleq \mathbb{E}\left[\tilde{x}_i(s|s)\tilde{x}_i^T(s|s)\right]. \tag{8}$$

Since obtaining measurements is sometimes energy consuming, we aim to reduce energy consumption by setting $q_i(s) = 0$ for certain sensors at some time instants. The state estimation performance is ensured by restricting

$$\mathrm{tr}\{P_i(s|s)\} \leqslant \mathcal{J}_i,$$

where $\mathcal{J}_i$ ($i \in \{1, 2, \cdots, N\}$) are pre-defined covariance levels. Detailed discussion on appropriate choices of $\mathcal{J}_i$ will be made in Section III.

III. Main Results

A. Design of Optimal Filter Gains

Theorem 1: The optimal filter gains for the ith state estimator minimizing $\mathrm{tr}\{P_i(s|s)\}$ are as follows:

$$\begin{aligned}\bar{\mathcal{L}}_i^*(s) =& \left(\mathcal{I}_n - q_i(s)P_i(s|s-1)H_i^T M_i(s)H_i(s)\right) \\ &\times G_i(s)\left[\bar{\mathfrak{I}}_i P_i(s|s-1)\bar{\mathfrak{I}}_i^T - \bar{\mathfrak{I}}_i\bar{\mathcal{P}}_i(s|s-1)\right. \\ &- \bar{\mathcal{P}}_i^T(s|s-1)\bar{\mathfrak{I}}_i^T + \bar{\Xi}_i(s|s-1) \\ &\left.-q_i(s)G_i^T(s)H_i^T M_i(s)H_i(s)G_i(s)\right]^{-1},\end{aligned} \tag{9}$$

$$\begin{aligned}K_i^*(s) =& \left(P_i(s|s-1) - \bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_i P_i(s|s-1)\right. \\ &\left.+\bar{\mathcal{L}}_i(s)\bar{\mathcal{P}}_i^T(s|s-1)\right) \\ &\times H_i^T\left(H_i P_i(s|s-1)H_i^T + R_i\right)^{-1},\end{aligned} \tag{10}$$

where $\mathcal{N}_i \triangleq \{i_1, i_2, \cdots, i_{r_i}\}$ and

$$\bar{\mathcal{P}}_i(s|s-1) \triangleq \begin{bmatrix} P_{ii_1} & P_{ii_2} & \cdots & P_{ii_{r_i}} \end{bmatrix}, \tag{11}$$

$$\bar{\mathcal{L}}_i(s) \triangleq \begin{bmatrix} L_{ii_1}(s) & \cdots & L_{ii_{r_i}}(s) \end{bmatrix}, \tag{12}$$

$$M_i(s) \triangleq \left(H_i P_i(s|s-1)H_i^T + R_i\right)^{-1}, \tag{13}$$

$$\bar{\Xi}_i(s|s-1) \triangleq \begin{bmatrix} P_{i_1} & P_{i_1 i_2} & \cdots & P_{i_1 i_{r_i}} \\ P_{i_2 i_1} & P_{i_2} & \cdots & P_{i_2 i_{r_i}} \\ \vdots & \vdots & \ddots & \vdots \\ P_{i_{r_i} i_1} & P_{i_{r_i} i_2} & \cdots & P_{i_{r_i} i_{r_i}} \end{bmatrix}, \tag{14}$$

$$G_i(s) \triangleq P_i(s|s-1)\bar{\mathfrak{I}}_i^T - \bar{\mathcal{P}}_i(s|s-1), \tag{15}$$

$$\bar{\mathfrak{I}}_i \triangleq \big[\underbrace{\mathcal{I}_n \quad \mathcal{I}_n \quad \cdots \quad \mathcal{I}_n}_{r_i}\big]^T. \tag{16}$$

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Note that, for the sake of simplicity, in (11) and (14), the indicator of time $(s|s-1)$ after P_{ii_p} or $P_{i_p i_q}$ is omitted above, where $p,q \in \{1,2,\cdots,r_i\}$.

Proof: According to (1) to (6), we have

$$\tilde{x}_i(s|s-1) = F\tilde{x}_i(s-1|s-1) + w(s-1), \tag{17}$$

$$\begin{aligned}\tilde{x}_i(s|s) =& \tilde{x}_i(s|s-1) - q_i(s)K_i(s) \\ &\times (H_i\tilde{x}_i(s|s-1) + v_i(s)) \\ &+ \sum_{j\in\mathcal{N}_i} L_{ij}(s)(\tilde{x}_j(s|s-1) - \tilde{x}_i(s|s-1)) \\ =& \bar{\mathcal{F}}_i(s)\tilde{x}_i(s|s-1) - q_i(s)K_i(s)v_i(s) \\ &+ \sum_{j\in\mathcal{N}_i} L_{ij}(s)\tilde{x}_j(s|s-1),\end{aligned} \tag{18}$$

where $\bar{\mathcal{F}}_i(s) \triangleq I - q_i(s)K_i(s)H_i - \sum_{j\in\mathcal{N}_i} L_{ij}(s)$. According to (7), we have

$$P_i(s|s-1) = FP_i(s-1|s-1)F^T + Q. \tag{19}$$

If the ith sensor is connected to the jth sensor in the sensor network, the cross item $P_{ij}(s|s-1)$ can be defined by

$$P_{ij}(s|s-1) \triangleq \mathbb{E}\left[\tilde{x}(s|s-1)\tilde{x}^T(s|s-1)\right]$$

and therefore we obtain

$$P_{ij}(s|s-1) = FP_{ij}(s-1|s-1)F^T + Q. \tag{20}$$

According to (8), (11)–(16), and Assumption 1, $P_i(s|s)$ can be expressed in a vectorized form as follows:

$$\begin{aligned}P_i(s|s) =& \bar{\mathcal{F}}_i(s)P_i(s|s-1)\bar{\mathcal{F}}_i^T(s) \\ &+ q_i(s)K_i(s)R_iK_i(s) \\ &+ \bar{\mathcal{F}}_i(s)\bar{\mathcal{P}}_i(s|s-1)\bar{\mathcal{L}}_i^T(s) \\ &+ \bar{\mathcal{L}}_i(s)\bar{\mathcal{P}}_i^T(s|s-1)\bar{\mathcal{F}}_i^T(s) \\ &+ \bar{\mathcal{L}}_i(s)\bar{\Xi}_i(s|s-1)\bar{\mathcal{L}}_i^T(s),\end{aligned} \tag{21}$$

where $\bar{\mathcal{F}}_i(s) = \mathcal{I}_n - q_i(s)K_i(s)H_i - \bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_i$.

By considering the partial derivatives of $\operatorname{tr}\{P_i(s|s)\}$, the optimal filter gains minimizing $\operatorname{tr}\{P_i(s|s)\}$ can be written as follows.

$$\begin{aligned}\bar{\mathcal{L}}_i^*(s) =& (\mathcal{I}_n - q_i(s)K_i(s)H_i)G_i(s) \\ &\times \big(\bar{\mathfrak{I}}_iP_i(s|s-1)\bar{\mathfrak{I}}_i^T - \bar{\mathfrak{I}}_i\bar{\mathcal{P}}_i(s|s-1) \\ &-\bar{\mathcal{P}}_i^T(s|s-1)\bar{\mathfrak{I}}_i^T + \bar{\Xi}_i(s|s-1)\big)^{-1},\end{aligned} \tag{22}$$

$$\begin{aligned}K_i^*(s) =& \big(P_i(s|s-1) - \bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_iP_i(s|s-1) \\ &+\bar{\mathcal{L}}_i(s)\bar{\mathcal{P}}_i^T(s|s-1)\big)H_i^TM_i(s).\end{aligned} \tag{23}$$

By substituting (23) into (22), (9) can be obtained, which completes the proof. □

After determining the filter gains, the error covariances of different sensors can be acquired. If the ith sensor is connected to the jth one in the sensor network, $P_{ij}(s|s)$ can be updated as follows:

$$\begin{aligned}P_{ij}(s|s) =& \bar{\mathcal{F}}_i(s)P_{ij}(s|s-1)\bar{F}_j^T(s) \\ &+ \bar{\mathcal{F}}_i(s)\sum_{h\in\mathcal{N}_j} P_{ih}(s|s-1)L_{jh}^T \\ &+ \sum_{g\in\mathcal{N}_i} L_{ig}(s)P_{gj}(s|s-1)\bar{F}_j^T(s) \\ &+ \sum_{g\in\mathcal{N}_i}\sum_{h\in\mathcal{N}_j} L_{ig}(s)P_{gh}(s|s-1)L_{jh}^T(s).\end{aligned} \tag{24}$$

Remark 2: Note that the optimal filter gains $K_i^*(s)$ and $\bar{\mathcal{L}}_i^*(s)$ are dependent on $q_i(s)$. Hence, it is necessary to consider how to design $q_i(s)$, and this will be addressed later.

Remark 3: The computation of the optimal filter gains is computationally costly since it involves computing the inverses of high-dimensional matrices. In the following subsection, the design of suboptimal filter gains that are easier to compute in distributed filtering will be discussed.

B. Design of Suboptimal Filter Gains

Lemma 1: For an arbitrary positive scalar $\rho > 0$ and real matrices X, Y, the following inequality holds:

$$XY^T + YX^T \leqslant \rho XX^T + \rho^{-1}YY^T.$$

The following theorem presents the way of obtaining an upper bound (UB) on $P_i(s|s-1)$ and an UB on $P_i(s|s)$.

Theorem 2: For any given positive constants $\beta_{ii_1},\cdots,\beta_{ii_{r_i}}$ and β_i, an UB on $P_i(s|s-1)$ and an UB on $P_i(s|s)$ can be calculated by

$$\mathfrak{P}_i(s|s-1) = F\mathfrak{P}_i(s-1|s-1)F^T + Q, \tag{25}$$

$$\begin{aligned}\mathfrak{P}_i(s|s) =& (1+\beta_i)\bar{\mathcal{F}}_i(s)\mathfrak{P}_i(s|s-1)\bar{\mathcal{F}}_i(s) \\ &+ \bar{\mathcal{L}}_i(s)\Lambda_i(s|s-1)\bar{\mathcal{L}}_i^T(s) \\ &+ q_i(s)K_i(s)R_iK_i^T(s),\end{aligned} \tag{26}$$

$$\mathfrak{P}_i(0|0) \triangleq P_i(0|0), \tag{27}$$

where $\beta_{ii_{r_i}} = 0$ and, for $j = 1,2,\cdots,r_i$,

$$\eta_{i,i_j} \triangleq \left(1+\beta_i^{-1}\right)\left[\prod_{l=1}^{j-1}\left(1+\beta_{ii_l}^{-1}\right)\right]\left(1+\beta_{ii_j}\right), \tag{28}$$

$$\Lambda_i(s|s-1) \triangleq \operatorname{diag}\left\{\eta_{i,i_j}\mathfrak{P}_{i_j}(s|s-1)\right\}. \tag{29}$$

In other words, we have $\mathfrak{P}_i(s|s-1) \geqslant P_i(s|s-1)$ and $\mathfrak{P}_i(s|s) \geqslant P_i(s|s)$.

Proof: Assuming that $\mathfrak{P}_i(s-1|s-1) \geqslant P_i(s-1|s-1)$ holds, we can easily obtain

$$F(\mathfrak{P}_i(s-1|s-1) - P_i(s-1|s-1))F^T \geqslant 0 \tag{30}$$

and

$$\mathfrak{P}_i(s|s-1) \geqslant P_i(s|s-1). \tag{31}$$

Next, we consider the estimation error $P_i(s|s)$. Applying Lemma 1 to the cross terms in (21), we have

$$P_i(s|s) \leqslant (1+\beta_i)\bar{\mathcal{F}}_i(s)P_i(s|s-1)\bar{\mathcal{F}}_i^T(s)$$

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$$
\begin{aligned}
&+\left(1+\beta_i^{-1}\right)\mathbb{E}\left[\left(\sum_{l=1}^{r_i}L_{ii_l}(s)\tilde{x}_{i_l}(s|s-1)\right)\right.\\
&\quad\left.\times\left(\sum_{l=1}^{r_i}L_{ii_l}(s)\tilde{x}_{i_l}(s|s-1)\right)^T\right]\\
&+q_i(s)K_i(s)R_iK_i^T(s)\\
\leqslant&(1+\beta_i)\bar{\mathcal{F}}_i(s)P_i(s|s-1)\bar{\mathcal{F}}_i^T(s)\\
&+\left(1+\beta_i^{-1}\right)(1+\beta_{ii_1})\\
&\times L_{ii_1}(s)P_{i_1}(s|s-1)L_{ii_1}^T(s)\\
&+\left(1+\beta_i^{-1}\right)\left(1+\beta_{ii_1}^{-1}\right)\\
&\times\mathbb{E}\left[\left(\sum_{l=2}^{r_i}L_{ii_l}(s)\tilde{x}_{i_l}(s|s-1)\right)\right.\\
&\quad\left.\times\left(\sum_{l=2}^{r_i}L_{ii_l}(s)\tilde{x}_{i_l}(s|s-1)\right)^T\right]\\
&+q_i(s)K_i(s)R_iK_i^T(s)\\
\leqslant&(1+\beta_i)\bar{\mathcal{F}}_i(s)P_i(s|s-1)\bar{\mathcal{F}}_i(s)\\
&+\sum_{l=1}^{r_i}\eta_{i,l}L_{ii_l}(s)P_{i_l}(s|s-1)L_{ii_l}^T(s)\\
&+q_i(s)K_i(s)R_iK_i^T(s). \qquad (32)
\end{aligned}
$$

According to (29) and (31), we have

$$\mathfrak{P}_i(s|s)\geqslant P_i(s|s). \qquad (33)$$

Since the initial condition $P_i(0|0)\leqslant\mathfrak{P}_i(0|0)$ is satisfied, the relations $P_i(s|s-1)\leqslant\mathfrak{P}_i(s|s-1)$ and $P_i(s|s)\leqslant\mathfrak{P}_i(s|s)$ hold for any $s\geqslant 1$, and this completes the proof. □

Theorem 3: Suboptimal filter gains minimizing the trace of $\mathfrak{P}_i(s|s)$ can be expressed as

$$
\begin{aligned}
K_i^{(d)}(s)=&(1+\beta_i)\left(\mathcal{I}_n-\bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_i\right)\mathfrak{P}_i(s|s-1)\\
&\times H_i^T\mathcal{M}_i(s), \qquad (34)\\
\bar{\mathcal{L}}_i^{(d)}(s)=&(1+\beta_i)\left[\mathcal{I}_n-q_i(s)(1+\beta_i)\mathfrak{P}_i(s|s-1)H_i^T\right.\\
&\left.\times\mathcal{M}_i(s)H_iH_i^T\mathcal{M}_i(s)H_i\right]\mathfrak{P}_i(s|s-1)\bar{\mathfrak{I}}_i^T\\
&\times\left[(1+\beta_i)\bar{\mathfrak{I}}_i\mathfrak{P}_i(s|s-1)\bar{\mathfrak{I}}_i^T+\Lambda_i(s|s-1)\right.\\
&-q_i(s)(1+\beta_i)^2\bar{\mathfrak{I}}_i\mathfrak{P}_i(s|s-1)\\
&\left.\times H_i^T\mathcal{M}_i(s)H_i\mathfrak{P}_i(s|s-1)\bar{\mathfrak{I}}_i^T\right]^{-1}, \qquad (35)
\end{aligned}
$$

where

$$\mathcal{M}_i(s)=\left((1+\beta_i)H_i\mathfrak{P}_i(s|s-1)H_i^T+R_i\right)^{-1}.$$

Proof: This proof is similar to the proof of Theorem 1 and is skipped here. □

C. Design of the Sensor Activation Scheme

Theorem 4: Compared with the case where $q_i(s)=0$, the trace of $\mathfrak{P}_i(s|s)$ is always smaller when $q_i(s)=1$.

Proof: According to (26), we have

$$
\begin{aligned}
\mathfrak{P}_i(s|s)=&(1+\beta_i)\left(\mathcal{I}_n-\bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_i\right)\mathfrak{P}_i(s|s-1)\\
&\times\left(\mathcal{I}_n-\bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_i\right)^T+\bar{\mathcal{L}}_i(s)\Lambda_i(s|s-1)\bar{\mathcal{L}}_i^T(s)\\
&+q_i(s)\left(K_i(s)M_i(s)K_i^T(s)\right.\\
&\left.-N_i(s)K_i^T(s)-K_i(s)N_i^T(s)\right)\\
\geqslant&(1+\beta_i)\left(\mathcal{I}_n-\bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_i\right)\mathfrak{P}_i(s|s-1)\\
&\times\left(\mathcal{I}_n-\bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_i\right)^T+\bar{\mathcal{L}}_i(s)\Lambda_i(s|s-1)\bar{\mathcal{L}}_i^T(s)\\
&-q_i(s)N_i(s)M_i^{-1}(s)N_i^T(s), \qquad (36)
\end{aligned}
$$

where

$$
\begin{aligned}
M_i(s)&\triangleq(1+\beta_i)H_i\mathfrak{P}_i(s|s-1)H_i^T+R_i,\\
N_i(s)&\triangleq(1+\beta_i)\left(\mathcal{I}_n-\bar{\mathcal{L}}_i(s)\bar{\mathfrak{I}}_i\right)\mathfrak{P}_i(s|s-1)H_i^T.
\end{aligned}
$$

The equality in (36) holds when $K_i(s)=K_i^{(d)}(s)$. Since $M_i(s)>0$, it follows from the utilization of $K_i^{(d)}(s)$ and $q_i(s)=1$ that $q_i(s)N_i(s)M_i^{-1}(s)N_i^T(s)>0$. The proof is now complete. □

According to (25) and (26), the updating process of $\mathfrak{P}_i(s|s-1)$ can be compactly rewritten as follows:

$$
\begin{aligned}
\mathfrak{P}(s+1|s)=&\Psi(s)\mathfrak{P}(s|s-1)\Psi(s)\\
&+\operatorname{diag}\left\{q_iFK_i(s)R_iK_i^T(s)F^T+Q\right\}\\
&+F\bar{\Gamma}(s)\mathfrak{P}(s|s-1)\bar{\Gamma}^TF^T, \qquad (37)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathfrak{P}(s|s)&\triangleq\operatorname{diag}\left\{\mathfrak{P}_1(s|s),\cdots,\mathfrak{P}_N(s|s)\right\},\\
\mathfrak{P}(s|s-1)&\triangleq\operatorname{diag}\left\{\mathfrak{P}_1(s|s-1),\cdots,\mathfrak{P}_N(s|s-1)\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\Psi(s)\triangleq\operatorname{diag}\left\{\sqrt{1+\beta_i}F\bar{\mathcal{F}}_i(s)\right\},\\
&\bar{\Gamma}(s)\triangleq\left[\Gamma_{i,j}(s)\right],\Gamma_{i,j}(s)\triangleq\begin{cases}\mathbf{0}, & \text{if } g_{ij}=0\\ \sqrt{\eta_{i,j}}L_{ij}(s), & \text{if } g_{i,j}=1.\end{cases}
\end{aligned}
$$

According to Theorem 4, by setting $q_i(s)=1$, the UB $\mathfrak{P}(s|s)$ of error covariance can be reduced compared with the case where $q_i(s)=0$. Furthermore, if $q_i(s)\equiv 1$ ($\forall 1\leqslant i\leqslant N, s\geqslant 1$), an infimum of $\mathfrak{P}(s|s)$ can be acquired. Let $\mathfrak{P}^*(s|s)$ denote the infimum, and we have

$$\mathfrak{P}(s|s)\geqslant\mathfrak{P}^*(s|s),\forall s\geqslant 1. \qquad (38)$$

Assumption 2: When $q_i(s)\equiv 1,\forall i,s$, there exist constant filter gains $\underline{K}_i$, $\underline{\bar{L}}_{ij}$, $i,j\in\{1,2,\cdots,N\}$ such that $\rho\left(\bar{J}\right)<1$ where

$$
\begin{aligned}
\bar{J}\triangleq&\left[\operatorname{diag}\left\{\sqrt{1+\beta_i}F\left(\mathcal{I}_n-\underline{K}_iH_i-\underline{\bar{L}}_i\bar{\mathfrak{I}}_i\right)\right\}\right]\\
&\otimes\left[\operatorname{diag}\left\{\sqrt{1+\beta_i}F\left(\mathcal{I}_n-\underline{K}_iH_i-\underline{\bar{L}}_i\bar{\mathfrak{I}}_i\right)\right\}\right]^T\\
&+\left(F\underline{\bar{\Gamma}}\right)\otimes\left(F\underline{\bar{\Gamma}}\right)^T,\\
\underline{\bar{\Gamma}}\triangleq&\left[\underline{\Gamma}_{i,j}\right],\underline{\Gamma}_{i,j}\triangleq\begin{cases}\mathbf{0}, & \text{if } g_{ij}=0\\ \sqrt{\eta_{i,j}}\underline{L}_{ij}, & \text{if } g_{i,j}=1.\end{cases} \qquad (39)
\end{aligned}
$$

Assumption 3: $\left(\sqrt{1+\beta_i}A,Q^{1/2}\right)$ is reachable.

Theorem 5: Under Assumptions 1 to 3, $\mathfrak{P}^*(s+1|s)$ (with any positive definite initial value) converges to the unique positive-definite solution Ω of the following equation:

$$\Omega=\Psi\Omega(\Psi)^T+F\bar{\Gamma}\Omega\left(\bar{\Gamma}\right)^TF^T+\Delta, \qquad (40)$$

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where

$$\Psi \triangleq \text{diag}\left\{\sqrt{1+\beta_i}F\left(\mathcal{I}_n - K_i H_i - \bar{\mathcal{L}}_i \bar{\mathfrak{I}}_i\right)\right\},$$

$$\bar{\Gamma} \triangleq \left[\Gamma_{i,j}\right], \Gamma_{i,j} \triangleq \begin{cases} \mathbf{0}, & \text{if } g_{ij} = 0 \\ \sqrt{\eta_{i,j}} L_{ij}, & \text{if } g_{i,j} = 1, \end{cases}$$

$\Omega = \lim_{s\to\infty} \mathfrak{P}^*(s+1|s)$, $K_i = \lim_{s\to\infty} K_i$, and $\bar{\mathcal{L}}_i = \lim_{s\to\infty} \bar{\mathcal{L}}_i(s)$.

Proof: The proof of the convergence of $\mathfrak{P}(s+1|s)$ is similar to the proof of Theorem 6 in [14]. Due to space limitations, the proof is skipped here. □

In case that $\Omega = \text{diag}\{\Omega_1, \cdots, \Omega_N\}$ and that a series of expected covariance levels $\mathcal{J}_i$ satisfies

$$\mathcal{J}_i > \text{tr}\{\Omega_i\}, i = 1, 2, \cdots, N.$$

It is obvious that, when $q_i(s) \equiv 1$, we have $\lim_{s\to\infty} \text{tr}\{\mathfrak{P}_i(s|s)\} = \text{tr}\{\Omega_i\} < \mathcal{J}_i$. Therefore, there always exists a series of $q_i(s)$ which restricts $\mathfrak{P}_i(s|s)$ below $\mathcal{J}_i$ when $s \to \infty$.

According to (25), (26), (34), and (35), $\mathfrak{P}_i(s|s)$ can be determined by $\mathfrak{P}_j(s|s-1)$, $\mathfrak{P}_i(s|s-1)$, and $q_i(s)$, where $j \in \mathcal{N}_i$. Furthermore, when $\mathfrak{P}_i(s|s) = \mathfrak{P}_i^{(0)}(s|s)$ (if $q_i(s) = 0$), a greedy strategy for sensor activation can be designed as follows:

$$q_i(s) = \begin{cases} 0, & \text{tr}\left\{\mathfrak{P}_i^{(0)}(s|s)\right\} < \mathcal{J}_i \\ 1, & \text{tr}\left\{\mathfrak{P}_i^{(0)}(s|s)\right\} \geqslant \mathcal{J}_i. \end{cases} \quad (41)$$

Remark 4: In (41), a feasible and easy-to-implement scheme is proposed to design $q_i(s)$. Such a sensor activation scheme not only guarantees that the expected covariance levels are reached but also efficiently reduces the energy consumption caused by measurement acquisitions.

IV. Numerical Simulations

In this section, a simulation example is proposed to verify the effectiveness of the proposed method.

Consider the following linear tracking system:

$$x(s+1) = \begin{bmatrix} 1 & 0.1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(s) + w(s), \quad (42)$$

$$y_i(s) = \begin{bmatrix} 1.0+0.5i & 0 & 0 & 0 \\ 0 & 0 & 1.1-0.4i & 0 \end{bmatrix} x(s) + v_i(s), \quad (43)$$

where $w(s) \sim \mathcal{N}(0, 0.01I_4)$ and $v_i(s) \sim \mathcal{N}(0, 0.01I_2)$. Fig. 1 illustrates the topology of the sensor network. The expected covariance levels are set as $\mathcal{J}_1 = 0.22$, $\mathcal{J}_2 = 0.10$, $\mathcal{J}_3 = 0.24$, $\mathcal{J}_4 = 0.14$, and $\mathcal{J}_5 = 0.21$.

The true states and the state estimation acquired by the distributed Kalman filter developed in Section III-B are illustrated in Fig. 2. In Fig. 3, $\text{tr}\{\mathfrak{P}(s|s)\}$, $\text{tr}\{\mathfrak{P}^*(s|s)\}$, and the mean-square error of the estimation are shown. Indicators of measurements $q_i(s)$ are shown in Fig. 4.

It is observed that, at the beginning of the state estimation process, measurements are always required to quickly reduce

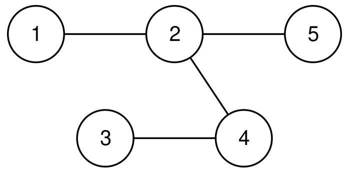

Fig. 1. The topology of the sensor network

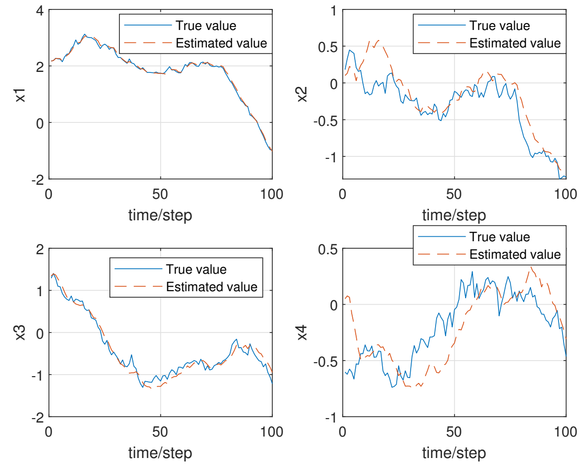

Fig. 2. State estimation

$\text{tr}\{\mathfrak{P}(s|s)\}$. Measurements are obtained less frequently when $\text{tr}\{\mathfrak{P}(s|s)\}$ is close to the expected covariance level. It is worth noting that $q_2(s) = 0$ when $s > 7$. The reason is that sensor 2 can receive information from sensors 1, 4, and 5. This information is enough for sensor 2 to obtain a state estimation that is accurate enough.

It is revealed that after applying the sensor activation scheme, the number of measurements obtained by sensors is

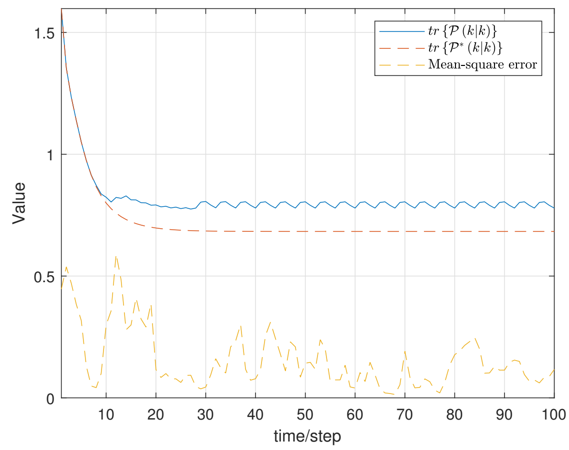

Fig. 3. Estimation error covariances

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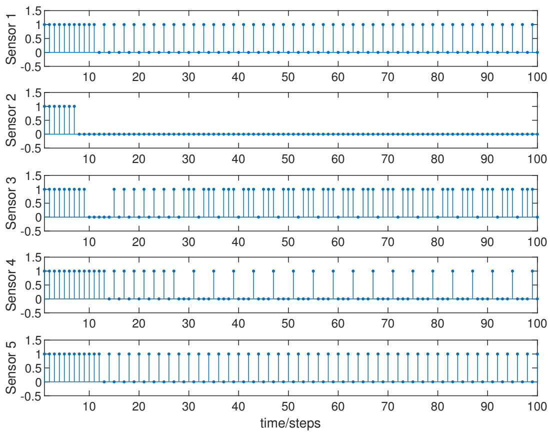

Fig. 4. Indicators of measurements

reduced by 54.8% while $\operatorname{tr}\{\mathfrak{P}(s|s)\}$ is around 16% larger than $\operatorname{tr}\{\mathfrak{P}^*(s|s)\}$. Moreover, the actual mean-square error is much smaller than $\operatorname{tr}\{\mathfrak{P}(s|s)\}$. Therefore, the proposed sensor activation scheme can effectively reduce energy consumption without losing too much accuracy.

V. Conclusions

In this paper, the reduction of energy consumption for a class of WSNs has been addressed in the context of the distributed Kalman filtering problem. A filter model has been proposed to describe the influence of the sensor activation scheme. Optimal filter gains have been designed to minimize the covariance of the filtering error. Suboptimal filter gains for distributed Kalman filtering have also been designed to minimize the UBs of the error covariances to reduce computational complexity. A sensor activation scheme has been designed to reduce the required amount of measurement data and restrict the UBs of the error covariances of filtering below expected covariance levels. Finally, the effectiveness of the proposed method has been verified via a simulation example.

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