

Rapid aerodynamic approximation of rotating blades using AI and automation logic

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Abstract. The aerodynamic performance of rotorcraft blades critically impacts rotor system efficiency, directly influencing lift, fuel consumption, and aircraft endurance. Conventional fixed-blade designs constrain aerodynamic optimisation across varying flight conditions. Refining techniques applied to rotational aerodynamics presents significant challenges: (a) high complexity, (b) time consumption, and (c) susceptibility to errors. In parallel with advances in non-conventional rotor blade designs, artificial intelligence (AI) has emerged as a transformative technology in aerodynamic modelling, offering enhanced computational capabilities and efficiency. This paper demonstrates how integrating AI-driven aerodynamic modelling facilitates rapid approximation of performance parameters. Specifically, the study addresses three objectives: 1) streamlining methodology while maintaining accuracy, 2) substantially reducing calculation time, and 3) minimising or eliminating errors in manual processing. A Python-based Automation Logic (PAL) algorithm is employed to automate estimation of aerodynamic parameters, reducing reliance on iterative, labour-intensive techniques. Processing time decreased from approximately 200 hours to under 7; a 97% reduction, while preserving computational fidelity and eliminating the ~1.4% rounding error found in manual integration. The findings underscore the transformative potential of AI-driven methodologies in rotorcraft aerodynamics, enabling faster, more reliable, and computationally efficient analyses. Ultimately, the study illustrates how accuracy, speed, and innovation can co-exist rather than be mutually exclusive.

1 Introduction

Over the past few years, Artificial Intelligence (AI) has asserted itself not as a fleeting novelty but as a transformative pillar within the evolving landscape of aerospace engineering, particularly in the realm of aerodynamics. When judiciously trained; balancing the pitfalls of overfitting and underfitting, AI methodologies have shown remarkable efficacy in augmenting and, in select cases, rivalling traditional approaches such as Computational Fluid Dynamics (CFD) and wind-tunnel experimentation. While these classical techniques remain benchmarks for fidelity and physical accuracy, they are undeniably constrained by prohibitive computational and temporal demands. What has become increasingly evident is that data-driven AI frameworks, particularly those grounded in machine learning, afford an unprecedented combination of computational expediency, predictive reliability, and adaptability. These systems can resolve complex aerodynamic flow features in a fraction of the time required by conventional solvers, and more importantly, facilitate real-time optimisation cycles, a capability previously inaccessible at this scale. Moreover, they enable exploration of aerodynamic regimes that would otherwise remain impractical due to the sheer cost or intricacy of exhaustive simulations or experimental investigations. Their capacity to interpolate and, in some instances, extrapolate (performance parameters and rated engine power, to name but a few) with a high degree of fidelity renders them an invaluable asset in both

preliminary design and detailed analysis phases. The maturation of AI (an on-going process, expected to accelerate significantly over the next decade) as a methodological counterpart, if not successor, to legacy aerodynamic tools marks a paradigm shift in the design and analysis of aerospace systems. This study does not suggest the obsolescence of classical aerodynamic methodologies; rather, it demonstrates that Artificial Intelligence fundamentally enhances and expands the aerodynamicist's analytical arsenal. We are witnessing the advent of a methodological convergence; one in which accuracy, computational velocity, and design innovation are no longer opposing forces to be balanced, but synergistic attributes to be simultaneously realised. In this light, AI does not replace tradition; it elevates it, signalling a transformative chapter in the evolution of aerodynamic science. Aulich et al. [1] developed a transformer-based AI model for rapid 3D flow prediction in turbomachinery compressors, bypassing CFD mesh dependencies by directly mapping geometry and boundary conditions to flow fields. The model, with ~47 million parameters, was trained on 1,500 samples derived from 500 RANS simulations and achieved a 20-90× speed-up over traditional CFD (10–15 s vs. 5-15 min per case) whilst overpredicted isentropic efficiency by ~1%, a significant margin in compressor optimisation. Despite this, it preserved relative performance rankings, making it suitable for early design filtering. However, only 8 of 100 AI-optimised designs converged in CFD, underscoring limitations in physical reliability when extrapolating

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beyond the training set. The authors suggest its strength lies in multi-fidelity frameworks where speed outweighs strict accuracy. Similarly, another AI-enhanced Blade Element Momentum (BEM) approach was introduced by Prajapat et al. [2]. Their method integrated neural network algorithms directly into traditional BEM formulations for wind turbine performance prediction, enabling significant gains in computational speed and predictive accuracy. Despite these advantages, this hybrid approach remained sensitive to the quality and size of training datasets, limiting its robustness in certain operational scenarios. Further extending the potential of AI in aerospace simulations, Axios (2025) described advancements utilising AI-powered simulation environments such as Altair's HyperWorks® and PhysicsAI™. These platforms substantially reduced traditional aerodynamic simulation times from weeks to mere seconds (i.e. up to 1,000x faster than traditional solver-driven simulations), profoundly accelerating iterative design processes. However, the interpretability of these AI algorithms posed challenges, particularly concerning validation, transparency, and regulatory acceptance in practical aerospace environments [3]. Bauersfeld et al. [4] introduced NeuroBEM, a hybrid quadrotor dynamics model combining Blade Element Momentum (BEM) theory with a neural network to capture residual aerodynamic forces and torques. Trained on 1.8 million datapoints from 96 flights (up to 65 km/h, 46.8 m/s²), the model learns from 50 ms of state history using a TCN-medium architecture. The hybrid model achieved a 50% reduction in force/torque RMSE (to 0.335 N and 0.012 Nm) versus existing methods and lowered positional RMSE in closed-loop simulations from ~0.8 m to <0.3 m. It also demonstrated only ~20% degradation when trained on low-speed data, outperforming models like PolyFit which failed entirely under the same conditions. However, limitations include increased computational cost ~100 μ s per step for BEM and NN components, versus 1 μ s for simple parametric models. Purely learned models also occasionally introduced unstable feedback loops. While NeuroBEM showed strong generalisation, its advantage diminished at low speeds (<5 m/s), where traditional models remained competitive at lower computational cost.

Wei et al. [5] also used Neural Networks (NNs) when introducing DeepGeo, a neural network-based framework that simultaneously optimised shape and mesh deformation. This model was found to simplify the parameterization process, allowing for faster convergence to optimal aerodynamic designs. Their study also highlighted the potential of AI to directly handle high-dimensional geometric and aerodynamic variables, making it a valuable tool for complex design scenarios. Yan et al. [6] developed an optimisation framework for aerodynamic shape design that combined reinforcement learning (RL) and transfer learning (TL) to enhance efficiency and accuracy. The framework used the deep deterministic policy gradient (DDPG) algorithm and reduced CFD calls by over 62.5%. Compared to traditional methods such as Multi-Objective Particle Swarm Optimisation (MOPSO) and Non-Dominated Sorting Genetic Algorithm II (NSGA-II), the RL-TL approach achieved an 18.67% improvement in lift-to-drag

ratio and faster convergence, demonstrating its effectiveness for efficient aerodynamic optimisation.

Dabaghians et al. [7] proposed NN-BET, a hybrid surrogate model that integrates Neural Networks (NN) with Blade Element Theory (BET) to predict rotor aerodynamic loads. The approach utilises a neural network to estimate mean inflow, which is then passed into the BET framework to compute aerodynamic forces, effectively embedding physical knowledge into the learning pipeline. To evaluate the effectiveness of this physics-informed structure, NN-BET was benchmarked against a conventional NN trained with the same data and architecture. NN-BET achieved higher predictive accuracy, validating the advantage of incorporating domain-specific physics. Additionally, it delivered results at ~10 \times faster speeds compared to the conventional Blade Element Momentum Theory (BEMT), while maintaining comparable fidelity. The key strengths of NN-BET include its computational efficiency, enhanced generalisation, and suitability for design optimisation and multidisciplinary analysis workflows, where both speed and accuracy are critical. The study also underscores the value of physics-based hybrid modelling, showing that embedding intermediate aerodynamic variables improves model interpretability and reduces reliance on large datasets. While no major limitations were reported, the approach assumes that the predicted mean inflow adequately captures unsteady effects, which may not generalise to all rotorcraft regimes.

Phillips et al. [8] proposed an efficient Uncertainty Quantification (UQ) framework for Design Under Uncertainty (DUU) that integrates analytical derivatives into multidisciplinary design optimisation using NASA's OpenMDAO platform. The approach replaces conventional gradient estimation techniques (e.g., finite differences) with non-intrusive Polynomial Chaos Expansion (PCE), enabling direct differentiation of confidence intervals. This significantly reduces the computational burden of UQ, cutting function calls by 87%, from 30,952 to 3,816, in a representative case study. The method was demonstrated on aerodynamic wing design problems. In a range-optimisation task, the uncertain-optimal design achieved a 10.2% higher lower-bound range compared to the nominal-optimal design, despite a modest 1.2% reduction in mean range and a trade-off in structural mass (wing weight increased from 8,338 kg to 9,291 kg). In a separate lift-to-drag ratio (L/D) optimisation, the robust solution yielded a significantly better worst-case L/D (12.71 vs. 8.03) while maintaining an acceptable central value (15.73 vs. 16.11). Key advantages include the framework's modularity, compatibility with gradient-based optimisers, and suitability for incorporating confidence-bound objectives or constraints directly into the optimisation. However, the method relies on reasonably accurate prior statistical estimates (e.g., mean and variance) and may struggle with vanishing gradients when confidence bounds lie far from nominal predictions. The reviewed literature reveals a broad and rapidly evolving landscape of advanced AI-driven and automated algorithmic techniques that are reshaping methodologies within aerospace engineering; particularly in aerodynamics. From hybrid neural-BET models and transformer-based surrogates to

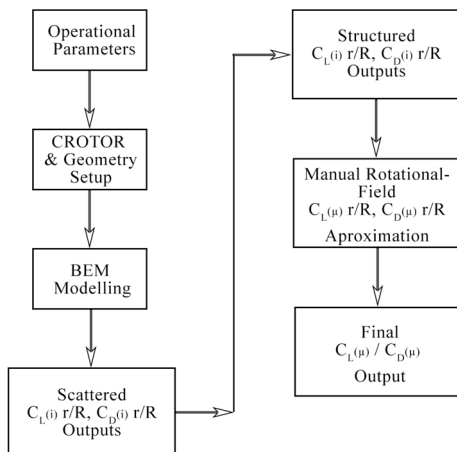
reinforcement learning optimisers and uncertainty-aware frameworks, these innovations consistently demonstrate substantial gains in speed, scalability, and predictive capability. In most instances, the benefits decisively outweigh the limitations, which primarily stem from two persistent challenges: (a) the need for extensive training time, and (b) the risks of overfitting or underfitting. Encouragingly, recent studies show that such limitations, especially prolonged training times, are becoming increasingly manageable with the emergence of more efficient, physics-informed, and data-efficient architectures. Included within this investigation is an automated, AI-based study that demonstrates how artificial intelligence can serve as a powerful extension of the aerodynamicist's toolkit; enhancing accuracy, predictability, and supporting or even streamlining traditionally manual tasks such as Blade Element Method (BEM) calculations. This integration underscores and supports the argument that AI's emerging role is not as a replacement, but as a valuable augmentative force in advancing aerodynamic analysis and design.

2 Methodology and setup

In this investigation, we examine the aerodynamic behaviour of a morphing rotor blade based on the geometry of the Sea King helicopter. The study focuses on evaluating key performance parameters; lift coefficient (C_L), drag coefficient (C_D), and the lift-to-drag ratio (C_L/C_D), using a combination of three tools: CROTOR, Blade Element Method (BEM) calculations, and a custom-built Python-based Automation Logic AI algorithm. The methodology begins with the rotor blade being designed to precise specifications (Table 1) within the CROTOR environment. CROTOR is then used to simulate the blade's performance under defined operating conditions, generating the initial raw aerodynamic output data. From this point, the process diverges into two analysis paths:

- (a) **A manual route**, wherein the CROTOR-generated outputs are processed using traditional BEM equations to compute the C_L/C_D values numerically.

Manual Processing



- (b) **An automated route**, which employs a AI-assisted Python-based Automation Logic algorithm to process the raw output data, perform real-time calculations, and iterate through the processing in a looped structure; thereby streamlining what would otherwise be a labour-intensive computation.

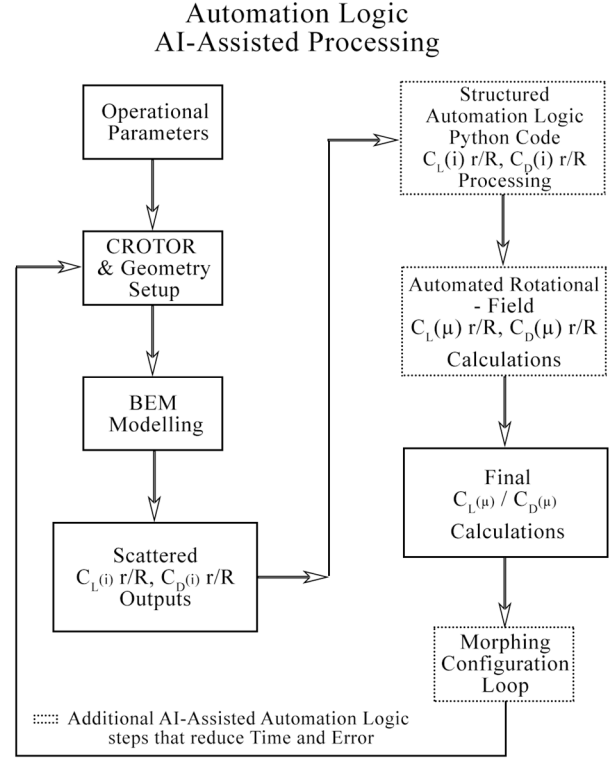


Fig. 2. Automation Logic AI-Assisted Processing to calculate C_L/C_D

These two approaches are then compared across three critical dimensions: **1)** streamlining of the methodology, **2)** time required to obtain results, and **3)** susceptibility to human or computational error. The aim is to evaluate whether AI-driven automation can meaningfully enhance aerodynamic performance analysis, particularly in the context of morphing rotor configurations.

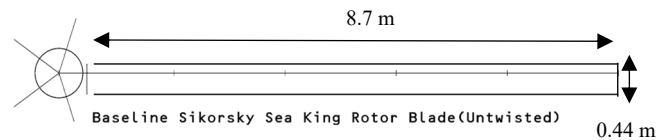


Fig. 3. Baseline Sea King Rotor Blade in a 5-Blade Configuration as Defined by CROTOR

#blad= 5	R m = 8.700	$c_{N0} = 0.0844$	$\beta_{twist} = 0.000$	$\eta_{ideal} = 0.0966$
$V_{in}/s = 1.000$	$V/DR = 0.0054$	$P_C =$	$C_p = 0.0099$	$\eta = 0.0753$
h km= 10.000	J = 0.0170	$T_C =$	$C_T = 0.0438$	
T kN= 19.0000	P kW= 252.2397	RPM = 203.0	$\beta_{tip} = 0.000$	
Helicopter	$C_{TH} = 0.005656$	$C_{PH} = 0.000406$	$C_{TH}/\sigma = 0.0671$	FOM = 0.7409

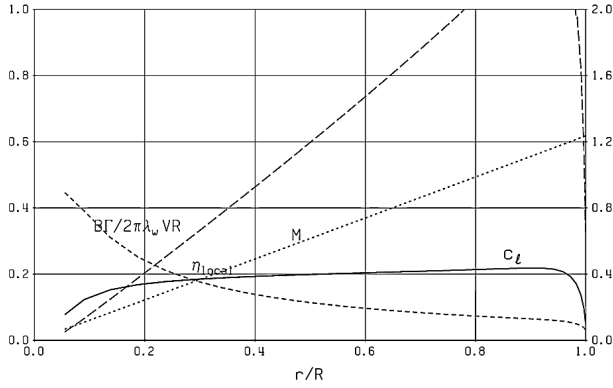


Fig. 4. Baseline outputs for Sea King rotor blade in a 5-blade configuration(untwisted) as defined by CROTOR

Figure 4. presents the baseline outputs for the Sea King rotor blade(untwisted). The input parameters are from Table 1. Additionally, the local C_l and C_d values along the baseline rotor blade's span (Radii) are presented in Table 2.

Table 1. Characteristics for Sea King Rotor Blade and operational environment

Parameter	Symbol	Unit
in-flow velocity	v_i	1 m/s
rotor radius	R	8.7 m
hub radius	HR	0.375 m
angle of attack	AOA	°
positive twist angle	$+\Phi$	°
rotations per minute	RPM	203
chord length	c	0.053R (0.44 m)
flight segment	Hover	
NACA	0012	
environmental conditions	SLS	Sea Level Standard

Table 2. Baseline segment-specific lift (dL) and drag (dD) outputs for the baseline at specific radii positions

Radii	Local $C_l(r_i)$	Local $C_d(r_i)$
0.09348	0.01647	0.01324
0.11797	1.00268	0.01206
0.15665	1.42647	0.01276
0.20123	1.49432	0.01446
0.24812	1.50350	0.01741
0.29580	1.50649	0.02033
0.34350	1.50816	0.02308
0.39079	1.50940	0.02577
0.43737	1.51047	0.02853
0.48299	1.51147	0.03145
0.52748	1.51247	0.03462
0.57066	1.51350	0.03810

0.61239	1.51457	0.04194
0.65253	1.51570	0.04621
0.69096	1.51691	0.05094
0.72756	1.51820	0.05620
0.76221	1.51958	0.06210
0.79482	1.52105	0.06926
0.82529	1.52261	0.07837
0.85354	1.52428	0.08985
0.87948	1.52605	0.10395
0.90304	1.52792	0.12075
0.92416	1.52990	0.14016
0.94276	1.53197	0.16200
0.95881	1.53414	0.18599
0.97225	1.53639	0.21180
0.98305	1.53873	0.23907
0.99117	1.54115	0.26745
0.99660	1.54363	0.29662
0.99932	1.54617	0.32630

Now that the raw data has been extracted from CROTOR, the next step involves applying the Blade Element Method (BEM) to evaluate rotor performance. Firstly, the **(a) manual route** is undertaken, whereby the extracted data is processed using classical BEM formulations to numerically compute the C_L/C_D values. This involves calculating the local lift and drag forces across discrete blade elements, integrating these contributions spanwise, and deriving the overall aerodynamic performance metrics under the defined flight conditions.

For a differential blade element segment at radius (r) and width (dr), the segment-specific lift(dL) and drag (dD) in Hover($\mu \approx 0$) are given by the equations (1) and (2).

$$dL = \frac{1}{2} \rho (V_{rel})^2 c(r) c_l(\alpha(r)) dr \quad (1)$$

$$dD = \frac{1}{2} \rho (V_{rel})^2 c(r) c_d(\alpha(r)) dr \quad (2)$$

The momentum theory determines the induced velocity (v_i) which is essential for calculating (V_{rel}). In its most basic application(hover), equations (3) and (4) apply.

$$v_i = \sqrt{\frac{T}{2 \rho A}} \quad (3)$$

$$V_{rel}(r) = (v_{tip})^2 + (v_i)^2 \quad (4)$$

These quantities are then integrated along the span of the rotor blade. The total lift coefficient (C_L) for the rotor blade at hover condition can be estimated using BEMT (5).

$$\begin{aligned} C_L &= \frac{1}{S} \int_{r_h}^{r_t} \frac{1}{2} \rho \left[(v_{tip})^2 + \left(\sqrt{\frac{T}{2 \rho A}} \right)^2 \right]^2 c(r) c_l(\alpha(r)) dr \\ &= \frac{1}{S} \int_{r_h}^{r_t} \frac{1}{2} \rho \left[(v_{tip})^2 + (v_i)^2 \right]^2 c(r) c_l(\alpha(r)) dr \\ &= \frac{1}{S} \int_{r_h}^{r_t} \frac{1}{2} \rho (V_{rel}^2) c(r) c_l(\alpha(r)) dr \end{aligned} \quad (5)$$

Solving this format is notably time-consuming, not only for the baseline case but especially for hundreds of iterations involving different blade spanwise locations and twist angles. To mitigate this, the integral is reformulated using the trapezoidal rule, allowing for its approximation as a summation as presented in equation (6).

$$C_L \approx \frac{1}{S} \sum_{i=0}^{n-1} \left(\frac{1}{2} \rho [V_{\text{rel}\{i\}}^2 c_{l,i} + V_{\text{rel}\{i+1\}}^2 c_{l,i+1}] \frac{r_{i+1} - r_i}{2} \right) \quad (6)$$

The derived formula is the modified trapezoidal method when applied to BEMT for a helicopter rotor blade in hover mode. The summation form allows to calculate the (C_L) value by iterating over the blade's span. Similarly, the (C_D) can be obtained (equation 7).

$$C_D \approx \frac{1}{S} \sum_{i=0}^{n-1} \left(\frac{1}{2} \rho [V_{\text{rel}\{i\}}^2 c_{d,i} + V_{\text{rel}\{i+1\}}^2 c_{d,i+1}] \frac{r_{i+1} - r_i}{2} \right) \quad (7)$$

Equations 6 and 7 represent the modified-trapezoidal BEM summation approximation of the total lift and drag coefficients for a rotor blade in hover. The following is the numerical process used to solve for both of these equations.

$$\frac{C_L}{C_D} \approx \frac{\sum_{i=0}^{29} \frac{\text{local } c_{l,i} + \text{local } c_{l,i+1}}{2} \times \frac{r_{i+1} - r_i}{2}}{\sum_{i=0}^{29} \frac{\text{local } c_{d,i} + \text{local } c_{d,i+1}}{2} \times \frac{r_{i+1} - r_i}{2}}$$

$$C_L/C_D = 17.6127$$

This finalises the extraction of the total lift-to-drag ratio (C_L / C_D) based on the segment-specific lift (dL) and drag (dD) values for the manual numerical approach.

For the current baseline configuration, a total of 58 radial blade segments were defined. Local aerodynamic parameters; namely, the lift coefficient, drag coefficient and segmental area, were carefully retrieved and tabulated. Each value was independently processed to calculate elemental lift and drag, followed by numerical integration across the entire blade. This resulted in a final total lift-to-drag ratio of (17.6127), marking the successful completion of the manual method. However, it is important to note that this full calculation, across all 58 points, required approximately 2 hours of manual effort to complete. For the problem at hand, this numerical approach is intended to be extended to a morphing rotor blade configuration, where multiple fixed and actively variable geometries are introduced along the blade span. Specifically, morphing is applied to five delineated sections at relative spanwise positions of $r/R=0.75, 0.80, 0.85, 0.90$, and 0.95 . For each section, 20 discrete twist angle variations (ranging from 0° to $+40^\circ$ in 2° increments) are considered, yielding a total

of 100 morphing configurations. Given the previous timing benchmark, this would result in an estimated 200 hours of empirical numerical work to complete; making this approach prohibitively time-intensive for detailed optimisation studies and heavily susceptible to errors. To address this, the same computational process has been implemented in a fully automated Python loop, which reproduces the numerical BEM summation with identical methodology and accuracy, but at a fraction of the computational time. The use of such an automated framework enables rapid iteration over all morphing scenarios and provides a scalable, repeatable, and robust tool for aerodynamic evaluation in morphing rotor blade design.

3 Discussions

Figure 2 illustrates the automation logic with its loop-based architecture, designed to systematically process new morphing blade data sets. This procedure involved an initial training phase of approximately 1 hour, during which the AI-based algorithm was calibrated and tested against validated inputs. Once fully operational, the loop architecture allowed for each new configuration to be processed in roughly 3 minutes, with an additional 30-second verification step included at the end of each iteration to ensure no runtime or logical errors had occurred. Given the total of 100 morphing configurations under study, the complete automated analysis required:

1 hour (initial training) + 100×3.5 minutes = 410 minutes ~6.83 hours, rounded to approximately 7 hours for streamlined reporting. This represents a substantial reduction in total analysis time from the original 200 hours required for manual empirical calculations, bringing the total burden down to just 3.5% of the original workload. While this 7-hour figure is based on an idealised pipeline benchmarked for this study, even accounting for operational inefficiencies, the actual effort rarely exceeded 10 hours in practice; still a dramatic improvement over the exhaustive manual route, which could have easily surpassed the original estimate due to fatigue, transcription delays, or rework from accumulated numerical rounding. Moreover, by comparing the manually obtained results with those produced by the automated loop, it was found that the manual method exhibited an average error of approximately 1.4% relative to the automated output (i.e. $C_L/C_D = 17.61$ manual vs $C_L/C_D = 17.85$ automated). This discrepancy is attributed primarily to cumulative rounding errors, typically from truncating intermediate values at 3 or 4 decimal places. In contrast, the automated approach maintained full floating-point precision across all operations, effectively eliminating this source of error and improving result fidelity. Taken together, the application of an AI-based Python automation framework has not only streamlined the process, but also improved computational efficiency, accuracy, and

practical scalability, making it an indispensable tool for rotor blade aerodynamic studies involving high-dimensional morphing parameter spaces.

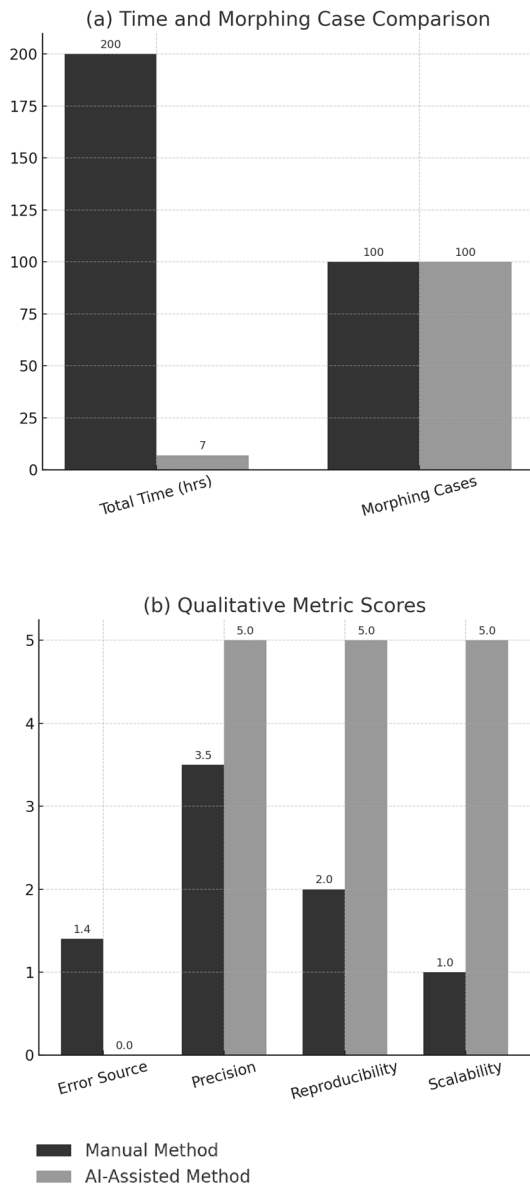


Fig. 5. AI-assisted automation logic processing vs manual method metrics

4 Conclusions

This study demonstrates the feasibility and effectiveness of integrating AI-driven automation within the aerodynamic analysis of morphing rotor blades. By combining traditional blade element theory with a Python-based Automation Logic loop architecture, the methodology significantly reduced the computational burden; from over 200 hours of manual processing to less than 10 hours of automated runtime, while preserving accuracy and improving consistency. The automated approach eliminated cumulative rounding errors observed in manual calculations (1.4%). These improvements not only streamline the process for high-resolution aerodynamic evaluation but

also provide a scalable framework adaptable to a broad range of rotor geometries and mission profiles. As demonstrated, AI can be used as a tool to supplement the classical aerodynamic methods; offering a hybrid pathway to greater efficiency, precision, and innovation in rotorcraft design. Future applications may include real-time optimisation, adaptive morphing controls, and integration with digital platforms for morphing rotor systems.

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