A NOVEL DIFFERENTIAL EVOLUTION ALGORITHMIC APPROACH TO TRANSMISSION EXPANSION PLANNING

A thesis submitted for the degree of Doctor of Philosophy

by

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March 2009

ABSTRACT

Nowadays modern electric power systems consist of large-scale and highly complex interconnected transmission systems, thus transmission expansion planning (TEP) is now a significant power system optimisation problem. The TEP problem is a large-scale, complex and nonlinear combinatorial problem of mixed integer nature where the number of candidate solutions to be evaluated increases exponentially with system size. The accurate solution of the TEP problem is essential in order to plan power systems in both an economic and efficient manner. Therefore, applied optimisation methods should be sufficiently efficient when solving such problems. In recent years a number of computational techniques have been proposed to solve this efficiency issue. Such methods include algorithms inspired by observations of natural phenomena for solving complex combinatorial optimisation problems. These algorithms have been successfully applied to a wide variety of electrical power system optimisation problems. In recent years differential evolution algorithm (DEA) procedures have been attracting significant attention from the researchers as such procedures have been found to be extremely effective in solving power system optimisation problems.

The aim of this research is to develop and apply a novel DEA procedure directly to a DC power flow based model in order to efficiently solve the TEP problem. In this thesis, the TEP problem has been investigated in both static and dynamic form. In addition, two cases of the static TEP problem, with and without generation resizing, have also been investigated. The proposed method has achieved solutions with good accuracy, stable convergence characteristics, simple implementation and satisfactory computation time. The analyses have been performed within the mathematical programming environment of MATLAB using both DEA and conventional genetic algorithm (CGA) procedures and a detailed comparison has also been presented. Finally, the sensitivity of DEA control parameters has also been investigated.

ACKNOWLEDGEMENTS

I would like to express my deep gratitude to Dr. Gareth A. Taylor for initiating this exciting research topic and for his invaluable guidance and encouragement throughout the duration of my research. I am also grateful to Professor Malcolm R. Irving and Professor Yong H. Song for their valuable comments and suggestions throughout my research.

My deep appreciation is to Dr. Thanawat Nakawiro, Dr. Pathomthat Chiradeja and Dr. Namkhun Srisanit for their valuable discussions, suggestions and great help during my PhD study. I would like to thank Dr. Jeremy Daniel for his technical support on computer hardware and software during my research. I am also thankful to the Brunel Institute of Power Systems at Brunel University for providing me the opportunity and resources to carry out this research work.

I gratefully acknowledge financial support from the Royal Thai Government and Srinakharinwirot University.

Finally, I would like to thank my family, especially my mother, my father and my lovely wife, for their love, support, patience and understanding.

DECLARATION

The work described in this thesis has not been previously submitted for a degree in this or any other university and unless otherwise referenced it is the author's own work.

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LIST OF ABBREVIATIONS

TEP	: Transmission Expansion Planning
DEA	: Differential Evolution Algorithm
GA	: Genetic Algorithm
CGA	: Conventional Genetic Algorithm
ILP	: Integer Linear Programming
CHA	: Constructive Heuristic Algorithm
TM	: Transportation Model
TS	: Tabu Search
SA	: Simulated Annealing
STEP	: Short Term Expansion Planning
FDLF	: Fast Decoupled Load Flow
IGA	: Improved Genetic Algorithm
TNEP	: Transmission Network Expansion Planning
ACS	: Ant Colony Search
PSO	: Particle Swarm Optimisation
AI	: Artificial Intelligence
EAs	: Evolutionary Algorithms
EP	: Evolutionary Programming
ESs	: Evolution Strategies
AEC-GA	: Advanced Engineered-Conditioning Genetic Algorithm
UC	: Unit Commitment
PGA	: Parallel Genetic Algorithm
CPF	: Continuation Power Flow
SQP	: Sequential Quadratic Programming
HDE	: Hybrid Differential Evolution

VSHDE : Variable Scaling Hybrid Differential Evolution

MDE : Modified Differential Evolution

DES : Differential Evolution Strategy

ORPF : Optimal Reactive Power Flow

TSCOPF : Transient Stability Constrained Optimal Power Flow

LIST OF NOMENCLATURE

P_i	: Real power of bus *i*
Q_i	: Reactive power of bus *i*
$\|V_i\|$	: Voltage magnitude of bus *i*
θ_i	: Voltage phase angle of bus *i*
$\|V_k\|$	: Voltage magnitude at bus *k*.
G_{ik} and B_{ik}	: Real and imaginary parts of element (*i*,*k*) of bus admittance matrix
N	: Total number of buses in the system
v	: Transmission investment cost
c_{ij}	: Cost of a circuit which is a candidate for addition to the branch *i-j*
n_{ij}	: Number of circuits added to the branch *i-j*
Ω	: Set of all candidate branches for expansion
g	: Real power generation vector in the existing power plants
d	: Real load demand vector in all network nodes
B	: Susceptance matrix of the existing and added lines in the network
θ	: Bus voltage phase angle vector.
f_{ij}	: Total branch power flow in the branch *i-j*
$f_{ij}^{\max}$	: Maximum branch power flow in the branch *i-j*
n_{ij}^{0}	: Number of circuits in the original base system
x_{ij}	: Reactance in the branch *i-j*
θ_i and θ_j	: Voltage phase angle of the terminal buses *i* and *j*
g_i	: Real power generation at node *i*
$g_i^{\min}$	: Lower real power generation limits at node *i*
$g_i^{\max}$	: Upper real power generation limits at node *i*
n_{ij}	: Total integer number of circuits added to the branch *i-j*
$n_{ij}^{\max}$	: Maximum number of added circuits in the branch *i-j*

c^t_{ij}	: The cost of a candidate circuit added to branch *i*-*j* at stage *t*
n^t_{ij}	: Number of circuits added to branch *i*-*j* at stage *t*.
δ^t_{inv}	: The discount factor used to calculate the present value of an investment cost at stage *t*
X_i	: A candidate solution, which is a *D*-dimensional vector, containing as many integer-valued parameters as the problem decision parameters *D*
$x_{j,i}^{(G=0)}$	: The initial value ($G = 0$) of the j^{th} parameter of the i^{th} individual vector.
$x_j^{\min}$	: Lower bounds of the j^{th} decision parameter
$x_j^{\max}$	: Upper bounds of the j^{th} decision parameter
N_P	: Population size
F	: A scaling mutation factor
CR	: A crossover constant or crossover probability
U_i	: The trial vectors
V_i	: The mutant vectors
ε	: Convergence criterion
D	: Number of problem variables
$G^{\max}$	: Maximum number of iterations or generations
gbest	: The best fitness value of the current iteration
pbest	: The best fitness value of the previous iteration
$F_s(X)$	: The fitness functions of the static TEP problem
$O_s(X)$	: The objective functions of the static TEP problem
$P_1(X)$	: The equality constraint penalty functions
$P_2(X)$	: The inequality constraint penalty functions
X	: The individual vector of decision variables
ω_1 and ω_2	: Penalty weighting factors that are set to “0.5” in this research
μ_l	: The penalty coefficient of the l^{th} inequality constraint

c : An inequality constraint constant that is used when an individual violates the inequality constraint.

nb : The number of buses in the transmission system

nc : The number of considered inequality constraints in TEP problem

$F_D(X)$: The fitness functions of the dynamic TEP problem

$O_D(X)$: The objective functions of the dynamic TEP problem

T : A horizon of time stage planning used in dynamic TEP problem

I : An annual interest rate value used in dynamic TEP problem

CHAPTER 1
INTRODUCTION

1.1 Research Motivation

Electric energy is the most popular form of energy because it can be transported easily at high efficiency and reasonable cost. Nowadays the real-world electric power systems are large-scale and highly complex interconnected transmission systems. An electric power system can be subdivided into four major parts that are generation, transmission, distribution and load. The purpose of a transmission system is to transfer electric energy from generating units at various locations to the distribution systems that ultimately supply the load. Transmission lines that also interconnect neighbouring utilities permit economic power dispatch across regions during normal conditions as well as the transfer of power between regions during emergency.

Over the past few decades, the amount of electric power energy to be transferred from generation sites to major load areas has been growing dramatically. Due to increasing costs and the essential need for reliable electric power systems, suitable and optimal design methods for different sections of the power system are required. Transmission systems are a major part of any power system therefore they have to be accurately and efficiently planned. In this research, electric power transmission systems are studied with regard to optimising the transmission expansion planning (TEP) problem.

Electric power transmission lines are initially built to link remote generating power plants to load centres, thus allowing power plants to be located in regions that are more economical and environmentally suitable. As systems grew, meshed networks of transmission lines have emerged, providing alternative paths for power flows from generators to loads that enhance the reliability of continuous supply. In regions where generation resources or load patterns are imbalanced, transmission interconnection eases the requirement for additional generation. Additional transmission capability is justified whenever there is a need to connect cheaper generation to meet growing load demand or enhance system reliability or both.

1.2 Problem Statement and Rationale

Transmission expansion planning has always been a rather complicated task especially for large-scale real-world transmission networks. First of all, electricity demand changes across both area and time. The change in demand is met by the appropriate dispatching of generation resources. As an electric power system must obey physical laws, the effect of any change in one part of network (e.g. changing the load at a node, raising the output of a generator, switching on/off a transmission line or a transformer) will spread instantaneously to other parts of the interconnected network, hence altering the loading conditions on all transmission lines. The ensuing consequences may be more marked on some transmission lines than others, depending on electrical characteristics of the lines and interconnection.

The electric transmission expansion planning problem involves determining the least investment cost of the power system expansion and operation through the timely addition of electric transmission facilities in order to guarantee that the constraints of the transmission system are satisfied over the defined planning horizon. The transmission system planner is entrusted with ensuring the above-stated goals are best met whilst utilising all the available resources. Therefore the purpose of transmission system planning is to determine the timing and type of new transmission facilities. The facilities are required in order to provide adequate transmission capacity to cope with future additional generation and power flow requirements. The transmission plans may require the introduction of higher voltage levels, the installation of new transmission elements and new substations. Transmission system planners tend to use many techniques to solve the transmission expansion planning problem. Planners utilise automatic expansion models to determine an optimum expansion system by minimising the mathematical objective function subject to a number of constraints.

In general, transmission expansion planning can be categorised as static or dynamic according to the treatment of the study period [1]. In static planning; the planner considers only one planning horizon and determines the number of suitable circuits that should be installed to each branch of the transmission network system. Investment is carried out at the beginning of the planning horizon time. In dynamic or multistage planning; the planner considers not only the optimal number and location of added lines and type of investments but also the most appropriate times to carry out such expansion investments. Therefore the continuing growth of the demand and generation is always assimilated by the system in an optimised way. The planning horizon is divided into various stages and the transmission lines must be installed at each stage of the planning horizon.

Many optimisation methods have been applied when solving the transmission expansion planning problem. The techniques range from expert engineering judgements to powerful mathematical programming methods. The engineering judgements depend upon human expertise and knowledge of the system. The most applied approaches in the transmission expansion planning problem can be classified into three groups that are mathematical optimisation methods (linear programming, nonlinear programming, dynamic programming, integer and mixed integer programming, benders decomposition and branch and bound, etc.), heuristic methods (mostly constructive heuristics) and meta-heuristic methods (genetic algorithms, tabu search, simulated annealing, particle swarm, evolutionary algorithms, etc).

Over the past decade, algorithms inspired by the observation of natural phenomena when solving complex combinatorial problems have been gaining increasing interest because they have been shown to have good performance and efficiency when solving optimisation problems [2]. Such algorithms have successfully applied to many power system problems [3, 4], for example power system scheduling, power system planning and power system control. In this research, a differential evolution algorithm (DEA) and genetic algorithm (GA) will be proposed and developed to solve both static and dynamic transmission expansion planning problems by direct application to the DC power flow based model.

1.3 Contributions of the Thesis

The major contribution of this thesis is the research and development of a novel DEA procedure and the investigation of the applicability of DEA method when applied to both static and dynamic TEP problems. In addition a detailed comparison of various DEA strategies used for solving these two electrical power system optimisation problems is presented. The most significant original contributions presented and investigated in this thesis are outlined as follows:

- Firstly, this thesis proposes the methodology where a novel DEA procedure is developed and improved by applying several DEA mutation strategies. In order to validate its searching capability and reliability, the proposed methodology has been tested with some selected mathematical benchmark test functions that are as follows: Sphere, Rosenbrock1, Rosenbrock2, Absolute, Salomon, Schwefel and Rastrigin functions, respectively. Regarding the obtained results, the proposed method performs effectively and gives better solutions in all cases when compared with a conventional genetic algorithm (CGA) procedure.
- Regarding the effectiveness of DEA method as tested on several numerical benchmark test functions. The proposed methodology has been successfully implemented to solve a real-world optimisation problem that is the static TEP problem. For this research, two different scenarios of the static TEP problem, with and without generation resizing, have been investigated and reported in this thesis. In addition, a heuristic search method has been adopted in order to deal with the static TEP when considering the DC power flow based model constraints.
- In addition, this research utilises the proposed effective methodology to deal with the dynamic or multistage TEP problem, which is more complex and difficult when compared with the static TEP problem. In this thesis, the dynamic TEP problem considering the DC power flow based model constraints has been analysed and considered in the separation of the planning horizon into multiple stages, which is an especially difficult task with regard to large-scale real-world transmission systems. A novel DEA method as applied to solve the dynamic TEP problem is tested on a realistically complex transmission system the Colombian 93-bus system.
- Finally, the influence of control parameter variation on the novel DEA method when applied to static and dynamic TEP problems has been investigated in this thesis. The simulation results clearly illustrate that the proposed algorithm provides higher robustness and reliability of approaching optimal solutions in both applications when compared to the CGA procedure.

1.4 List of Publications

Arising from this research project, a journal paper and a book chapter have been submitted. In addition, three conference papers have been presented and published in conference proceedings. The papers are listed as follows:

1.4.1 Refereed Journal Paper: Accepted

- T. Sum-Im, G. A. Taylor, M. R. Irving and Y. H. Song, "Differential evolution algorithm for static and multistage transmission expansion planning," *IET Proc. Gener. Transm. Distrib.,* (Accepted 2009).

1.4.2 Refereed Book Chapter: Submitted

- T. Sum-Im, G. A. Taylor, M. R. Irving and Y. H. Song, "Differential evolution algorithm for transmission expansion planning," in *Intelligent techniques for power system transmission*, G. K. Venayagamoorthy, R. Harley and N. G Hingorani, Ed., Wiley, (Submitted 2008).

1.4.3 Refereed Conference Papers: Published

- T. Sum-Im, G. A. Taylor, M. R. Irving and Y. H. Song, "A comparative study of state-of-the-art transmission expansion planning tools," *Proc. the 41st International Universities Power Engineering Conference* (*UPEC 2006*), Newcastle upon Tyne, United Kingdom, pp. 267-271, 6th-8th Sep. 2006.
- T. Sum-Im, G. A. Taylor, M. R. Irving and Y. H. Song, "A differential evolution algorithm for multistage transmission expansion planning," *Proc. the 42nd International Universities Power Engineering Conference* (*UPEC 2007*), Brighton, United Kingdom, pp. 357-364, 4th-6th Sep. 2007.
- T. Sum-Im, G. A. Taylor, M. R. Irving and Y. H. Song, "Transmission expansion planning using the DC model and a differential evolution algorithm," *Proc. the 1st School of Engineering and Design Research Student Conference* (*RESCon 2008*), Brunel University, United Kingdom, pp. 43-44, 25th-26th Jun. 2008.

1.5 Thesis Outline

Chapter 1 provides an introduction to the transmission expansion planning problem. In addition, the research contributions of applying the novel differential evolution algorithm to transmission expansion planning problems are presented.

Chapter 2 presents an overview of static and dynamic transmission expansion planning problems including problem formulation, treatment of the planning horizon and available literature.

Chapter 3 provides a review of DEA and genetic algorithms. The optimisation process and constraint handing techniques of the proposed algorithm are presented.

Chapter 4 presents the DEA optimisation procedure and program, which is tested on various numerical benchmark functions. The numerical test results and discussion are explained in this chapter.

Chapter 5 provides the implementation and development of the novel differential evolution algorithm for solving the static transmission expansion planning problem. Moreover the experimental results and comments are discussed in this chapter.

Chapter 6 presents the implementation of the novel DEA for solving the dynamic transmission expansion planning problem. In addition, the numerical test results for realistic transmission systems and comments are included in this chapter.

Chapter 7 gives the interpretations of results from chapter 5 and 6 with regard to sensitivity and convergence analysis of the DEA on static and dynamic transmission expansion planning problems.

Chapter 8 presents the overall conclusions of the research reported in this thesis and indicates further possible research directions.

CHAPTER 2
FUNDAMENTALS OF TRANSMISSION EXPANSION PLANNING PROBLEM

2.1 Introduction

In general, the objective of electric transmission expansion planning (TEP) is to specify addition of transmission facilities that provide adequate capacity and in the mean time maintain operating performance of electric transmission system [5]. To achieve effective plan, exact location, capacity, timing and type of new transmission equipment must be thoroughly determined to meet demand growth, generation addition and increased power flow. However, cost-effective transmission expansion planning becomes one of the major challenges in power system optimisation due to the nature of the problem that is complex, large-scale, difficult and nonlinear. Meanwhile, mixed integer nature of TEP results in an exponentially increased number of possible solutions when system size is enlarged.

To find an optimal solution of TEP over a planning horizon, extensive parameters are required; for instance topology of the base year, candidate circuits, electricity demand and generation forecast, investment constraints, etc. This would consequently impose more complexity to solving TEP problem. Given the above information, in–depth knowledge on problem formulation and computation techniques for TEP is crucial and therefore, this chapter aims essentially at presenting fundamental information of these issues.

The organisation of this chapter is as follows: section 2.2 presents the overview of treatment of the transmission expansion planning horizon, while in section 2.3 the overview and formulation of DC power flow model is introduced. Section 2.4 and 2.5 present the problem formulation and the mathematical model of static and dynamic transmission expansion planning, respectively. Section 2.6 presents the review of solution methods for transmission expansion planning found in the international technical literature. Finally, a summary of this chapter is made in section 2.7.

2.2 Treatment of the Transmission Expansion Planning Horizon

Based on the treatment of planning horizon, transmission expansion planning can be traditionally classified into two categories, namely static (single-stage) and dynamic (multi-stage) planning. In static planning, only a single time period is considered as a planning horizon. In contrast, dynamic planning considers the planning horizon by separating the period of study into multiple stages [1].

For static planning, the planner searches for an appropriate number of new circuits that should be added into each branch of the transmission system and in this case, the planner is not interested in scheduling when the new lines should be constructed and the total expansion investment is carried out at the beginning of the planning horizon [6]. Many research works regarding the static TEP are presented in [5, 8, 11, 14, 15, 19, 21, 22, 25, 67, 68, 74] that are solved using a variety of the optimisation techniques.

In contrast, time-phased or various stages are considered in dynamic planning while an optimal expansion schedule or strategy is considered for the entire planning period. Thus, multi-stage transmission expansion planning is a larger-scale and more complex problem as it deals with not only the optimal quantity, placement and type of transmission expansion investments but also the most suitable times to carry out such investments. Therefore, the dynamic transmission expansion planning inevitably considers a great number of variables and constraints that consequently require enormous computational effort to achieve an optimal solution, especially for large-scale real-world transmission systems. Many research works regarding the dynamic TEP [6, 12, 13, 19, 68, 73] are presented some of the dynamic models that have been developed.

2.3 DC Power Flow

For a long-term TEP study, some assumptions are invented and introduced for solving such planning problem, for example, a consideration of the reactive power allocation is neglected in the first moment of the planning. In this stage, the main concern is to identify the principal power corridors that probably will become part of the expanded system. There are several types of the mathematical model employed for representing the transmission network in the TEP study; AC power flow model, DC power flow model, transportation model, hybrid model, and disjunctive model [8].

Basically, the DC power flow model is widely employed to the TEP problem and it is frequently considered as a reference because in general, networks synthesized by this model satisfy the basic conditions stated by operation planning studies. The planning results found in this phase will be further investigated by operation planning tools such as AC power flow analysis, transient and dynamic stability analysis and short-circuit analysis [3]. In the simulation of this research, the DC power flow model is considered as it is widely used in transmission expansion planning [5, 8, 25, 66, 67].

The formulation of DC power flow is obtained from the modification of a general representation of AC power flow, which can be illustrated by the following equations.

$$P_i = |V_i| \sum_{k=1}^{N} |V_k| [G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)] \tag{2.1}$$

$$Q_i = |V_i| \sum_{k=1}^{N} |V_k| [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)] \tag{2.2}$$

where P_i and Q_i are real and reactive power of bus i respectively. $|V_i|$ and θ_i are voltage magnitude and voltage phase angle of bus i respectively. $|V_k|$ is voltage magnitude at bus k. G_{ik} and B_{ik} are real and imaginary parts of element (i,k) of bus admittance matrix respectively. N is total number of buses in the system.

To modify AC power flow model to the DC power flow based model, the following assumptions are normally considered [7]:

- Bus voltage magnitude at each bus bar is approximate one per unit ($|V_i| = 1$ p.u. for all i buses);
- Line conductance at each path is neglected ($G_{ik} = 0$), or on the other hand only line susceptance (B_{ik}) is considered in the DC model;
- Some trigonometric terms of AC model in equations (2.1) and (2.2) can be approximated as following terms:

  $\sin(\theta_i - \theta_k) \approx \theta_i - \theta_k$ and $\cos(\theta_i - \theta_k) \approx 1$

Given these assumptions, the AC power flow equation in (2.1) is therefore simplified to yield the DC power flow equation as follows:

$$P_i = \sum_{k=1}^{N} B_{ik}(\theta_i - \theta_k) \quad , i = 1, \ldots, N \tag{2.3}$$

where B_{ik} is the line susceptance between bus i and k.

2.4 Overview of the Static Transmission Expansion Planning

In this section, the static transmission expansion planning is formulated as a mathematical problem. The objective of solving this problem is typically to fulfil the required planning function in terms of investment and operation restriction. The detailed discussion is as follows.

2.4.1 Problem Statement

In general, transmission expansion planning problem can be mathematically formulated by using DC power flow model, which is a nonlinear mixed-integer problem with high complexity, especially for large-scale real-world transmission networks. There are several alternatives to the DC model such as the transportation, hybrid and disjunctive models. Detailed reviews of the main mathematical models for transmission expansion planning were presented in [8, 9].

2.4.2 The Objective Function

The objective of transmission expansion planning is to minimise investment cost while satisfying operational and economic constraints. In this research, the classical DC power flow model is applied to solve the TEP problem. Mathematically, the problem can be formulated as follows.

$$\min \quad v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} \tag{2.4}$$

where v, c_{ij} and n_{ij} represent, respectively, transmission investment cost, cost of a candidate circuit for addition to the branch i-j and the number of circuits added to the branch i-j. Here Ω is the set of all candidate branches for expansion.

2.4.3 Problem Constraints

The objective function (2.4) represents the capital cost of the newly installed transmission lines, which has some restrictions. These constraints must be included into mathematical model to ensure that the optimal solution satisfies transmission planning requirements. These constraints are described as following:

2.4.3.1 DC Power Flow Node Balance Constraint

This linear equality constraint represents the conservation of power at each node.

$$g = d + B\theta \qquad (2.5)$$

where g, d and B is real power generation vector in existing power plants, real load demand vector in all network nodes, and susceptance matrix of the existing and added lines in the network, respectively. Here θ is the bus voltage phase angle vector.

2.4.3.2 Power Flow Limit on Transmission Lines Constraint

The following inequality constraint is applied to transmission expansion planning in order to limit the power flow for each path.

$$\left| f_{ij} \right| \le (n_{ij}^0 + n_{ij})\, f_{ij}^{\max} \qquad (2.6)$$

In DC power flow model, each element of the branch power flow in constraint (2.6) can be calculated by using equation (2.7):

$$f_{ij} = \frac{(n_{ij}^0 + n_{ij})}{x_{ij}} \times (\theta_i - \theta_j) \qquad (2.7)$$

where f_{ij}, f_{ij}^{max}, n_{ij}, n_{ij}^{0} and x_{ij} represents, respectively, total branch power flow in branch *i-j*, maximum branch power flow in branch *i-j*, number of circuits added to branch *i-j*, number of circuits in original base system, and reactance of the branch *i-j*. Here θ_i and θ_j is voltage phase angle of the terminal buses *i* and *j* respectively.

2.4.3.3 Power Generation Limit Constraint

In transmission expansion planning problem, power generation limit must be included into the problem constraints. This can be mathematically represented as follows:

$$g_i^{\min} \le g_i \le g_i^{\max} \qquad (2.8)$$

where g_i, $g_i^{\min}$ and $g_i^{\max}$ is real power generation at node i, the lower and upper real power generation limits at node i, respectively.

2.4.3.4 Right-of-way Constraint

It is significant for an accurate transmission expansion planning that planners need to know the exact location and capacity of the newly required circuits. Therefore this constraint must be included into the consideration of planning problem. Mathematically, this constraint defines the new circuit location and the maximum number of circuits that can be installed in a specified location. It can be represented as follows.

$$0 \le n_{ij} \le n_{ij}^{\max} \qquad (2.9)$$

where n_{ij} and $n_{ij}^{\max}$ represents the total integer number of circuits added to the branch i-j and the maximum number of added circuits in the branch i-j, respectively.

2.4.3.5 Bus Voltage Phase Angle Limit Constraint

The bus voltage magnitude is not a factor in this analysis since a DC power flow model is used for transmission planning. The voltage phase angle is included as a transmission expansion planning constraint and the calculated phase angle (θ_{ij}^{cal}) should be less than the predefined maximum phase angle ($\theta_{ij}^{\max}$). This can be represented as the following mathematical expression.

$$\left|\theta_{ij}^{cal}\right| \le \left|\theta_{ij}^{\max}\right| \qquad (2.10)$$

2.5 Overview of the Dynamic Transmission Expansion Planning

In this section, a mathematical representation of the dynamic transmission expansion planning problem is discussed as following details.

2.5.1 Problem Statement

The purpose of dynamic transmission expansion planning is to minimise the present value of investment cost for transmission expansion over an entire planning periods. Normally, the problem of dynamic TEP requires a huge computational effort to search for an optimal solution. The DC power flow model was applied to the static TEP problem in the previous section and it can be extended to more complex dynamic transmission expansion planning as well. The dynamic planning problem is a mixed integer nonlinear programming problem that is difficult for solving especially medium-scale and large-scale transmission systems.

2.5.2 The Objective Function

A DC model can be applied to the dynamic planning in order to determine the financial investment for the most economical schedule [6]. The investment plan of transmission expansion is generally obtained with reference to the base year. Considering an annual rate *I*, the present values of the transmission expansion planning investment costs in the base year t_0 with a horizon of *T* stages are as follows:

$$\begin{aligned} c(x) &= (1-I)^{t_1-t_0} c_1(x) + (1-I)^{t_2-t_0} c_2(x) \\ &\quad + \ldots + (1-I)^{t_T-t_0} c_T(x) \\ &= \delta^1_{inv} c_1(x) + \delta^2_{inv} c_2(x) + \ldots + \delta^T_{inv} c_T(x) \end{aligned} \tag{2.11}$$

where

$$\delta^t_{inv} = 1-I^{\;t_t-t_0}$$

Using the above relations, the dynamic planning for the DC model assumes the following form:

$$\min \quad v = \sum_{t=1}^{T} \left[\delta^t_{inv} \sum_{(i,j)\in\Omega} c^t_{ij} n^t_{ij} \right] \tag{2.12}$$

where v, c^t_{ij}, and n^t_{ij} represents, respectively, the present value of the expansion investment cost of the added transmission system, the cost of a candidate circuit added to branch *i-j* at stage *t* and the number of circuits added to branch *i-j* at stage *t*. Here Ω is the set of all candidate right-of-ways for expansion. δ^t_{inv} is the discount factor used to find the present value of an investment at stage *t*.

2.5.3 Problem Constraints

The objective function (2.12) represents the present value of the dynamic expansion planning investment costs of the new transmission lines subject to the restrictions as described in the previous section. Therefore, these planning constraints must also be considered in the multi-stage mathematical formulation in order to guarantee that the achieved solutions satisfy transmission planning requirements. The constraints of dynamic transmission expansion planning can be formulated in a similar fashion to those of the static model and are presented as follows:

$$d^t + B^t\theta^t = g^t \tag{2.13}$$

$$\left| f_{ij}^t \right| \leq (n_{ij}^0 + \sum_{s=1}^{t} n_{ij}^s)\, f_{ij}^{\max} \tag{2.14}$$

$$f_{ij}^t = \frac{(n_{ij}^0 + \sum_{s=1}^{t} n_{ij}^s)}{x_{ij}} \times (\theta_i^t - \theta_j^t) \tag{2.15}$$

$$g_i^{t,\min} \leq g_i^t \leq g_i^{t,\max} \tag{2.16}$$

$$0 \leq n_{ij}^t \leq n_{ij}^{t,\max} \tag{2.17}$$

$$\sum_{t=1}^{T} n_{ij}^t \leq n_{ij}^{\max} \tag{2.18}$$

$$\left|\theta_{ij}^{\text{cal}}\right| \leq \left|\theta_{ij}^{\max}\right| \tag{2.19}$$

The variables of the dynamic transmission expansion planning constraints in (2.13)-(2.19) are similar to those of static transmission expansion planning except the addition of the index *t*, which indicates the specific stage of planning involved.

2.6 Review of Solution Methods for Transmission Expansion Planning

Over past few decades, many optimisation techniques have been proposed to solve the transmission expansion planning problem in regulated power systems. These techniques can be generally classified into mathematical, heuristic and meta-heuristic optimisation methods. A review of these methods is discussed in this section.

2.6.1 Mathematical Optimisation Methods

Mathematical optimisation methods search for an optimal expansion plan by using a calculation procedure that solves a mathematical formulation of the planning problem. In the problem formulation, the transmission expansion planning is converted into an optimisation problem with an objective function subject to a set of constraints. So far, there have been a number of applications of mathematical optimisation methods to solve the transmission expansion planning problem, for instance, linear programming [10], nonlinear programming [11] and [12], dynamic programming [13], branch and bound [14] and [15], mixed-integer programming [16] and [17] and Benders decomposition [18].

2.6.1.1 Linear Programming

In 1970, Garver proposed a linear programming method to solve the transmission expansion planning problem [10]. This original method was applied to long-term planning of electrical power systems and produced a feasible transmission network with near-minimum circuit miles using as input any existing network plus a load forecast and generation schedule. Two main steps of the method, in which the planning problem was formulated as load flow estimation and new circuit selection could be searched based on the system overloads, were presented in [10]. The linear programming was used to solve the minimisation problem for the needed power movements, whereas the result was called “linear flow estimate”. A circuit addition was selected based on the location of the largest overload in this flow estimate. These two steps were repeated until no overloads remain in the system.

2.6.1.2 Nonlinear Programming

In 1984, an interactive method was proposed and applied in order to optimise the transmission expansion planning by Ekwue and Cory [11]. The method was based upon a single-stage optimisation procedure using sensitivity analysis and the adjoint network approach to transmit power from a new generating station to a loaded AC power system. The nonlinear programming technique of gradient projection followed by a round-off procedure was used for this optimisation method.

2.6.1.3 Dynamic Programming

Discrete dynamic optimising (DDO) was proposed to solve the transmission planning problem by Dusonchet and El-Abiad [13]. The basic idea of this method was to combine deterministic search procedure of dynamic programming with discrete optimising a probabilistic search coupled with a heuristic stopping criterion. The proposed method provides a way of dealing with two problems, which are size and complexity of the procedures for evaluating the performance of alternate strategies, through the use of a probabilistic search procedure and dynamic programming. Another advantage of this method is the probability through the neighbourhood concept to take into account in solution process the planner's experience.

2.6.1.4 Integer and Mixed-Integer Programming

In 2003, Alguacil et al. [17] proposed a mixed-integer linear programming approach to solve the static transmission expansion planning that includes line losses consideration. The proposed mixed-integer linear formulation offers accurate optimal solution. Meanwhile, it is flexible enough to build new networks and to reinforce existing ones. The proposed technique was tested to Graver's 6-bus system, the IEEE reliability test system and a realistic Brazilian system whereas the results confirm the accuracy and efficiency of this computation approach.

2.6.1.5 Branch and Bound

Haffner et al. [15] presented a new specialised branch and bound algorithm to solve the transmission network expansion planning problem. Optimality was obtained at a cost, however: that was the use of a transportation model for representing the transmission network. The expansion problem then became an integer linear programming (ILP) which was solved by the proposed branch and bound method. To control combinatorial explosion, the branch and bound algorithm was specialised using specific knowledge about the problem for both the selection of candidate problems and the selection of the next variable to be used for branching. Special constraints were also used to reduce the gap between the optimal integer solution (ILP program) and the solution obtained by relaxing the integrality constraints.

2.6.1.6 Benders and Hierarchical Decomposition

A new Benders decomposition approach was applied to solve the real-world power transmission network design problems by Binato et al. [18]. This approach was characterised by using a mixed linear (0-1) disjunctive model, which ensures the optimality of the solution found by using additional constraints, iteratively evaluated, besides the traditional Benders cuts. In [18], the use of Gomory cuts iteratively evaluated from *master* sub-problem and the use of Benders cuts evaluated from relaxed versions of the *slave* sub-problem. Gomory cuts within Benders decomposition was used to improve the practical convergence to the optimal solution of the Benders approach.

2.6.2 Heuristic and Meta-heuristic Methods

In addition to mathematical optimisation methods, heuristic and meta-heuristic methods become the current alternative to solve the transmission expansion planning problem. These heuristic and meta-heuristic techniques are efficient algorithms to optimise the transmission planning problem. There have been many applications of heuristic and meta-heuristic optimisation methods to solve transmission expansion planning problem, for example heuristic algorithms [5, 19], tabu search [20], simulated annealing [21], genetic algorithms [6, 22, 23, 24], artificial neural networks [25], particle swarm [31] and hybrid artificial intelligent techniques [25]. The detail of these methods is as discussed below.

2.6.2.1 Heuristic Algorithms

Constructive heuristic algorithm (CHA) is the most-widely used heuristic algorithms in transmission expansion planning. A constructive heuristic algorithm is an iterative process that searches a good quality solution in a step-by-step process. Romero et al. [19] presented and analysed heuristic algorithms for the transportation model in static and multistage transmission expansion planning. A constructive heuristic algorithm for the transportation model (TM) of Garver’s work [10] was extensively analysed and excellent results were obtained in [19]. Furthermore, the Garver algorithm was extended to accommodate multistage planning, which is especially important to define financial investment according to the most economical scheduling. The CHA, which was proposed for the generalised transportation model, reaches quality topologies for all test systems even though its efficiency decreased as the complexity of system increased [19]. In 2005, Romero et al. [5] proposed constructive heuristic algorithm for the DC model in network transmission expansion planning. A novel constructive heuristic algorithm worked directly with the DC power flow model in [5]. This proposed algorithm was developed from Garver's works [10] that was applied to the transportation model. The algorithm presented excellent performance for systems with low complexity in Garver's 6-bus and medium complexity in IEEE 24-bus. The principal advantage of the algorithm was that it worked directly with the solution given by the DC model with relaxed integer variables.

2.6.2.2 Tabu Search

Tabu search (TS) is an iterative improvement procedure that starts from some initial feasible solution and attempts to determine a better solution in the manner of a 'greatest descent neighbourhood' search algorithm [2]. The basic components of the TS are the moves, tabu list and aspiration level (criterion). Silva et al. [20] presented transmission network expansion planning under a tabu search approach. The implementation of tabu search to cope with long-term transmission network expansion planning problem was proposed in [20]. Two real-world case studies were tested and the results obtained by this approach were a robust and promising technique to be applied to this planning problem. The good quality of results produced by the intensification phase in both case studies qualifies the strategy used, i.e. to look for consistent candidate circuits (those that appear in different plans) to build a consistent transmission expansion plan. The principal improvement of this approach, comparing with classical methods of optimisation, was related to its ability in avoiding local optimum solutions, consequently having a greater chance to find the global optimum solution.

2.6.2.3 Simulated Annealing

Simulated annealing (SA) approach based on thermodynamics was originally inspired by the formulation of crystals in solids during cooling [2]. Simulated annealing technique has been successfully applied to a number of engineering optimisation problems including power system optimisation problems. Romero et al. [21] proposed a simulated annealing approach for solving the long-term transmission system expansion planning problem. The proposed method [21] was compared with a conventional optimisation method based on mathematical decomposition with a zero-one implicit enumeration procedure. In [21], two small test systems were used for tuning the main parameters of the simulated annealing process and then the proposed technique was applied to a large test system for which no optimal solution had been known: a number of interesting solutions was obtained with costs about 7% less than the best solutions known for that particular example system obtained by optimisation and heuristic methods.

2.6.2.4 Expert Systems

Expert system is a knowledge-based or rule-based system, which uses the knowledge and interface procedure to solve problems. The state of the field of expert systems and knowledge engineering in transmission planning was reviewed by Galiana et al. [26]. The details of that review were the principal elements of transmission planning, including its aim, the principal activities that constituted transmission planning, the constraints and prerequisites that must be met by the planner, a general planning methodology, and a selection to justify the use of expert systems in transmission planning and to indicate area of potential. Moreover, an expert system approach for multi-year short-term expansion planning (STEP) was presented in [27] where the reactive power management issues were addressed in the multi-year STEP to ensure adequate quality of voltage supply and efficiency of transmission system, which could be measured by network congestion and percentage losses in the system. An expert system approach to STEP using enhanced fast decoupled load flow (FDLF) was proposed to address these reactive power issues.

2.6.2.5 Evolutionary Algorithms

Evolutionary algorithm is based on the Darwin’s principle of ‘survival of the fittest strategy’. An evolutionary algorithm begins with initialising a population of candidate solutions to a problem and then new solutions are generated by randomly varying those of initial population. All solutions are evaluated with respect to how well they address the task. Finally, a selection operation is applied to eliminate bad solutions. An evolutionary programming approach for transmission network planning in electric power systems was presented in [28]. The proposed evolutionary programming algorithm was tested in two electric power systems, including Graver 6-bus system and the Mexican electric power system.

2.6.2.6 Genetic Algorithms

Genetic algorithm (GA) is a global search approach based on mechanics of natural selection and genetics. GA is different from conventional optimisation techniques as it uses the concept of population genetics to guide the optimisation search. GA searches from population to population instead of point-to-point search. In 1998, Gallego et al. [22] presented an extended genetic algorithm for solving the optimal transmission network expansion planning problem. Two main improvements of GA, which are an initial population obtained by conventional optimisation based methods and the mutation approach inspired in the simulated annealing technique, was introduced in [22].

The application of an improved genetic algorithm (IGA) was also proposed to solve the transmission network expansion planning problem by Silva et al. [23]. Genetic algorithms (GAs) had demonstrated the ability to deal with non-convex, nonlinear, integer-mixed optimisation problems, which include transmission network expansion planning (TNEP) problem, as it generates better performance than a number of other mathematical methodologies. Some special features had been added to the basic GAs to improve its performance in solving the TNEP problem for three real large-scale transmission systems. Results in [23] showed that the proposed approach was not only suitable but a promising technique for solving such a problem.

In 2001, Gil and Silva presented a reliable approach for solving the transmission network expansion planning problem using genetic algorithms [24]. The procedure to find the solution was based on the 'loss of load limit curve' of the transmission system under study, which was produced utilising unfeasible solutions found by the GA. A modified procedure made GA more robust to solve the different large-scale transmission expansion problems and this proposed method was proved to be efficient for solving in two real large-scale power systems [24].

In 2004, Escobar et al. [6] proposed an efficient genetic algorithm to solve the multistage and coordinated transmission planning problem, which was a mixed integer nonlinear programming problem. The proposed GA had a set of specialised genetic operators and utilised an efficient form of generation for the initial population that found high quality suboptimal topologies for large size and high complexity transmission systems. The achieved results illustrated that an efficient GA was effectively and efficiently implemented for multistage planning on medium and large size systems.

2.6.2.7 Ant Colony System Algorithm

Ant colony search (ACS) system was initially introduced by Dorigo in 1992 [32]. ACS technique was originally inspired by the behaviour of real ant colonies and it was applied to solve function or combinatorial optimisation problems. Gomez et al. [29] presented ant colony system algorithm for the planning of primary distribution circuits. The planning problem of electrical power distribution networks, stated as a mixed nonlinear integer optimisation problem, was solved using the ant colony system algorithm. In [29], the ant colony system methodology was coupled with a conventional load flow algorithm for distribution system and adapted to solve the primary distribution system planning problem. Furthermore, this technique [29] was very flexible and it could calculate location and the characteristics of the circuits minimising the investment and operation costs while enforcing the technical constraints, such as the transmission capabilities, the limits on the voltage magnitudes, allowing the consideration of a very complete and detailed model for the electric system.

2.6.2.8 Particle Swarm

Particle swarm optimisation (PSO), using an analogy of swarm behaviour of natural creatures, was started in the early of the 1990s. Kennedy and Eberhart developed PSO based on the analogy of swarms of birds and fish schooling [30], which achieved efficient search by remembrance and feedback mechanisms. By imitating the behaviours of biome, PSO is highly fit for parallel calculation and good performance for optimisation problems. A new discrete method for particle swarm optimisation was applied for transmission network expansion planning (TNEP) in [31]. The principle of PSO was introduced and an efficient discrete PSO method for TNEP according to its characters was developed by researchers [31]. Moreover, parameter selection, convergence judgment, optimisation fitness function construction and PSO characters were also analysed in [31]. Numerical results demonstrated that the proposed discrete method was feasible and efficient for small test systems.

2.6.2.9 Hybrid Artificial Intelligent Techniques

Al-Saba and El-Amin [25] proposed the application of artificial intelligent (AI) tools, such as genetic algorithm, tabu search and artificial neural networks (ANNs) with linear and quadratic programming models, to solve transmission expansion problem. The effectiveness of these AI methods in dealing with small-scale and large-scale systems was tested through their applications to the Graver six-bus system, the IEEE-24 bus network and the Saudi Arabian network [25]. The planning work [25] aimed to obtain the optimal design using a fast automatic decision-maker. An intelligent tool started from a random state and it proceeded to allocate the calculated cost recursively until the stage of the negotiation point was reached.

2.7 Conclusions

This chapter has covered the basis of transmission expansion planning problem, problem formulation and literature survey on a variety of solution techniques application to the planning problem. Over several past decades, researchers have worked on transmission expansion planning and set their interest mostly on static planning models. Unfortunately, the dynamic and pseudo-dynamic planning models are still in an undeveloped status as dynamic planning models have some limitations for their application to real-world transmission systems. The transmission expansion planning models can be developed and used several different tools, from spreadsheets to custom-written programs.

CHAPTER 3
FUNDAMENTALS OF DIFFERENTIAL EVOLUTION ALGORITHM AND GENETIC ALGORITHMS

3.1 Introduction

Evolutionary algorithms (EAs) are heuristic and stochastic optimisation techniques based on the principles of natural evolution theory. The field of investigation, concerning all EAs, is known as “evolutionary computation”. The origin of evolutionary computation can be traced back to the late 1950’s and since then a variety of EAs have been developed independently by many researchers. The most popular algorithms are genetic algorithms (GAs), evolutionary programming (EP), evolution strategies (ESs) and differential evolution algorithm (DEA). These approaches attempt to search the optimal solution of an optimisation problem via a simplified model of the evolutionary processes observed in nature and they are based on the concept of a population of individuals that evolve and improve their fitness through probabilistic operators via processes of recombination, mutation and selection. The individuals are evaluated with regard to their fitness and the individual with superior fitness is selected to compose the population in the next generation. After several iterations of the optimisation procedure, the fitness of individuals should be improved while current individuals explore the solution space for the optimal value.

In this research, a novel differential evolution algorithm is proposed to be applied directly to DC power flow based model of transmission expansion planning problem. In addition, conventional genetic algorithm is employed to compare its achieved results with that of the proposed method. These two optimisation techniques are introduced and discussed in this chapter. Moreover, the optimisation process and constraint handing techniques of the proposed algorithm are also included in this chapter.

3.2 Genetic Algorithms

3.2.1 Background and Literature Review

Genetic algorithm (GA) was first introduced in the book "Adaptation in Natural and Artificial Systems" in 1975 and was mainly developed in the USA by J. H. Holland [33]. In addition, genetic algorithm was put into practical applications in the late 1980s and it has been continuously used until now.

Genetic algorithm is a mechanism that mimics the process observed in natural evolution. It is a general-purpose optimisation method that is distinguished from conventional optimisation techniques by the use of concepts of population genetics to guide the optimisation search. A population of individuals, representing a potential candidate solution to a given problem, is maintained through optimisation process. A fitness value of each individual is assigned according to the fitness function to indicate the quality of a candidate solution. The individuals then must compete with others in the population to generate their offspring. The highly fit individuals that are those with higher fitness value have more opportunities to reproduce through recombination operation. The offspring inherits genes of their highly fit parents and will become even fitter, which represent a better solution to the problem concerned. The lowest fit individuals have few opportunities to reproduce and the trace of their genes will eventually disappear in the population. Comparison between the newly generated offspring and their parents, the best individuals are selected regard to their fitness values to form the population of the next generation. By repeating the GA optimisation process, the population of individuals will develop into an optimal solution of the problem.

Over past 20 years, genetic algorithm has been applied to solve various engineering optimisation problems, especially electrical power system problems such as economic dispatch [34], unit commitment [35, 36], generator/hydrothermal scheduling [37, 38], optimal power flow [39], voltage/reactive power control [40], capacitor placement [41, 42], generation expansion planning [43], transmission expansion planning [22, 23, 24, 44].

An advanced engineered-conditioning genetic algorithm hybrid (AEC-GA) with applications in power economic dispatch was proposed by Song and Chou in

[34]. It was a combined strategy involving local search algorithms and genetic algorithms. Moreover, several advanced techniques, which enhanced program efficiency and accuracy such as elite policy, adaptive mutation prediction, non-linear fitness mapping, different crossover techniques, were also explored in [34]. The combination of the nonlinear fitness mapping and the sigma truncation scaling was highly beneficial. Overall, the improved efficiency, accuracy and reliability achieved by the proposed AEC-GA hybrid demonstrated its advantages in power system optimisations in [34].

According to [35], a genetic algorithm was applied to solve the unit commitment problem. It was necessary to enhance a standard GA implementation with the addition of problem specific operators and the Varying Quality Function technique in order to obtain satisfactory unit commitment solutions. The proposed GA-UC was tested in the systems up to 100 units and the obtained results of the proposed method were compared with Lagrangian relaxation and dynamic programming in [35].

A genetic algorithm based approach to the scheduling of generators in a power system was presented in [37]. An enhanced genetic algorithm incorporating a sequential decomposition logic was employed to provide a faster search mechanism. The power of the GA presented in [37] relied on the selection and grading of the penalty functions to allow the fitness function that differentiates between good and bad solutions. This method guarantees the production of solutions that did not violate system or unit constraints. The proposed approach demonstrated a good ability to provide accurate and feasible solutions for a medium-scale power system within reasonable computational times.

According to [38], the problem of determining the optimal hourly schedule of power generation in a hydrothermal power system was solved by applying a genetic algorithm. In [38], a multi-reservoir cascaded hydro-electric system with a nonlinear relationship between water discharge rate, net head and power generation was investigated. In addition, the water transport delay between connected reservoirs was also included in the problem. The proposed method provided a good solution to the short-term hydrothermal scheduling problem and was able to take into account the variation in net head and water transport delay factors.

An application of parallel genetic algorithm (PGA) to optimal long-range generation expansion planning was presented in [43]. This planning problem was formulated as a combinatorial optimisation problem that determined the number of newly generation units at each time interval under different scenarios. The PGA developed in [43] belonged to the class of coarse-grain PGA in order to achieve the trade-off between computational speed and hardware cost.

In general, genetic algorithm is a global search method based on the mechanics of natural selection and genetics. Its characteristics make GA a robust algorithm to adaptively search the global optimal point of certain class of engineering problems. There are a number of significant advantages of genetic algorithm over traditional optimisation techniques have been described in [45].

- GA searches the solution from a population of points that is not a single point. Therefore GA can discover a globally optimal point because each individual in the population computes independently of each other. GA has inherent parallel processing nature.
- GA evaluates the fitness of each string to guide its search instead of the optimisation function. GA only needs to evaluate objective function (fitness) to guide its search. Derivatives or other auxiliary knowledge are not required by GA. Therefore GA can deal with non-smooth, non-continuous and non-differentiable functions that are the realistic optimisation problems.
- GA employs the probabilistic transition rules to select generations, which are not deterministic rules. Therefore GA has the ability to search a complicated and uncertain area to find the global optimum.

Although GA has many advantages as above explanation, there are also a number of disadvantages of GA that are as follows:

- GA does not always produce an exact global optimum, which may give the local minima (premature convergence).
- GA requires tremendously high computational time since a great number of complicated fitness evaluations.

3.2.2 Basis of Genetic Algorithms and Optimisation Process

Genetic algorithms are the most popular form of EAs and belong to the class of population-based search strategies. They work in a particular way on a population of strings (chromosomes), in which each string represents a possible candidate solution to the problem being optimised and each bit (or group of bits) represents a value for a decision variable of the problem. Firstly, each candidate solution is encoded and each encoding represents an individual in the GA population. The population is initialised to random individuals (random chromosomes) at the beginning of the GA optimisation process and GA then explores the search space of possible chromosomes for better individuals. The GA search is guided regard to the fitness value return by an environment, which provides a measure of how well each adapted individual in term of the problem solving. Therefore, the fitness value of each individual determines its probability of appearing or surviving in future generations. Codification is an essential process of GA and binary encoding of the parameters is traditionally employed. It has been mathematically proven that the cardinality of the binary alphabet maximises the number of similarity template (schemata) in which GA operates and hence enhances the search mechanism. The main concept of GA optimisation process is illustrated in figure 3.1 and a simple GA involves the following steps:

- **Encoding**: Code parameters of the problem as binary strings of fixed length;
- **Initialisation**: Randomly generate initial population strings, which evolve to the next generation by genetic optimisation operators;
- **Fitness Evaluation**: Compute and evaluate each string's fitness, which measures the quality of solutions coded by strings;
- **Selection**: Permit highly-fit strings as parents and produce offsprings according to their fitness in the next generation;
- **Crossover**: Crossover is the main genetic operator and combines two selected parents by swapping chromosome parts between their strings, starting from a randomly selected crossover point. This leads to new strings inheriting desirable qualities from both chosen parents;

- **Mutation**: Mutation works as a kind of 'life insurance' and flips single bits in a string, which prevents GA from premature convergence by exploiting new regions in the search space;
- **Termination**: The new strings replace the existing ones and optimisation process continues until the predetermined termination criterion is satisfied.

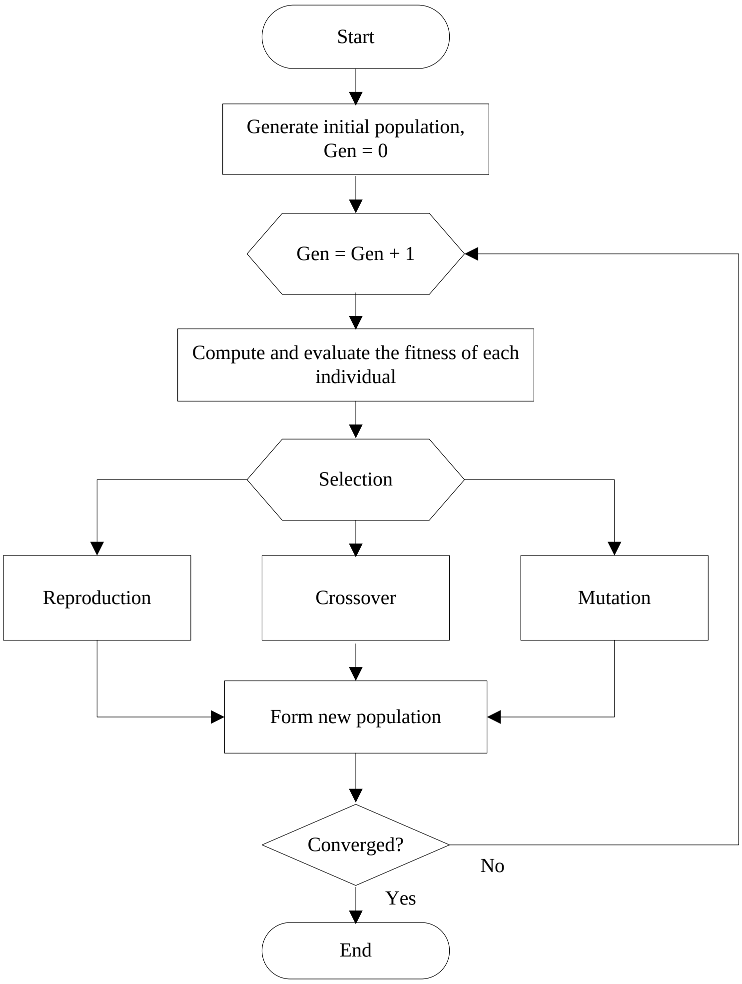


Figure 3.1 The main flowchart of the typical GA optimisation process

3.3 Differential Evolution Algorithm

3.3.1 Background and Literature Review

A differential evolution algorithm (DEA) is an evolutionary computation method that was originally introduced by Storn and Price in 1995 [46]. Furthermore, they developed DEA to be a reliable and versatile function optimiser that is also readily applicable to a wide range of optimisation problems [47]. DEA uses rather greedy selection and less stochastic approach to solve optimisation problems than other classical EAs. There are also a number of significant advantages when using DEA, which were summarised by Price in [48].

- Ability to find the true global minimum regardless of the initial parameter values;
- Fast and simple with regard to application and modification;
- Requires few control parameters;
- Parallel processing nature and fast convergence;
- Capable of providing multiple solutions in a single run;
- Effective on integer, discrete and mixed parameter optimisation;
- Ability to find the optimal solution for a nonlinear constrained optimisation problem with penalty functions.

Most of the initial researches were conducted by the differential evolution algorithm inventors (Storn and Price) with several papers [46, 49 50, 51], which explained the basis of differential evolution algorithm and how the optimisation process is carried out. A constraint handling approach for the differential evolution algorithm was proposed by Lampinen [52]. An extension of the differential evolution algorithm for handling multiple constraint functions was performed and demonstrated with a set of ten well-known test functions. Only the replacement operation of the original DEA was modified by applying a new replacement criterion for handling the constraint function.

A parameter study for differential evolution was presented by Gamperle et al. [53]. In this work, differential evolution was applied to several uni-modal and multi-modal test functions to find appropriate strategy parameters. The original differential evolution algorithm was analysed with respect to its performance depending on the choice of strategy parameters. The appropriate control parameters were guided and provided in this article. According to [54], Vesterstrom and Thomsen presented a comparative study of differential evolution, particle swarm optimisation and evolutionary algorithms on numerical benchmark problems. The performances of differential evolution, particle swarm optimisation and evolutionary algorithms were evaluated regarding their general applicability as numerical optimisation techniques.

A modified differential evolution for constrained optimisation was proposed by Mezura-Montes et al. [55].

Over last decade, differential evolution algorithm has been attracting increasing attention for a wide variety of engineering applications including electrical power system engineering. There have been many researches that applied DEA for solving electrical power system problems such as power system transfer capability assessment [56], power system planning [57], economic power dispatch [58, 59, 71], distribution network reconfiguration problem [60], short-term hydrothermal scheduling problem [61], design of a gas circuit breaker [62], optimal reactive power flow [63, 72] and optimal power flow [64].

According to [56], Wong and Dong proposed differential evolution (DE) as an alternative approach to evolutionary algorithms with two application examples in power systems that were power system transfer capability assessment problem and other power engineering problems. In [56], differential evolution was used to calculate the value of available transfer capability (ATC) comparing with the traditional continuation power flow (CPF) based approach. The final solution achieved by DE and CPF were verified with PowerWorld to compare the accuracy. Regarding obtained results, DE based approach was able to generate very accuracy results however without the need to perform complex CPF repeatedly. In addition, DE could reach the close vicinity of the final solution within the first 500 iterations and the calculation process was illustrated to be robustness over different trials on two test systems in both solution accuracy and computational efficiency.

A differential evolution based method for power system planning problem was presented by Dong et al. [57]. The planning aimed at locating the minimum cost of additional transmission lines that must be added to satisfy the forecasted load in a power system. The planning in [57] considered several objectives including expansion investment cost, the reliability objective-expected energy not supplied, the social welfare objective-expected economical losses and the system expansion flexibility objective. Differential evolution could show its capability on handling integer variables and non-linear constrained multi-objective optimisation problem.

Perez-Guerrero and Cedeno-Maldonado applied DEA to solve economic power dispatch problem that features non-smooth cost functions [58]. The non-smooth cost functions arose in economic power dispatch studies due to valve point loading effects, quadratic cost functions and prohibited operating zone, which were solved using DEA in [58]. The achieved results demonstrated the ability of the proposed DE-based methodology to solve efficiently economic dispatch problems with non-smooth cost functions.

Coelho and Mariani performed and proposed the combination of chaotic differential evolution and quadratic programming for solving economic load dispatch problem with valve-point effect [59]. A new approach method combined DEA with the generator of chaos sequences and sequential quadratic programming (SQP) technique to optimise the performance of economic dispatch problem. In [59], differential evolution with chaos sequences was the global optimiser to obtain a nearly global solution and the SQP was used to determine the optimal solution. The combined methods could be shown very effective in solving economic dispatch problems with valve-point effect in [59].

Chiou et al. [60] proposed an effective method which was variable scaling hybrid differential evolution (VSHDE) to solve the network reconfiguration for power loss reduction and voltage profit enhancement of distribution systems. The VSHDE technique utilised the 1/5 success rule of evolution strategies to adjust the variable scaling factor to accelerate searching the global solution. The variable scaling factor was applied to overcome the drawback of fixed and random scaling factor used in hybrid differential evolution (HDE).

According to [61], a novel approach based on modified differential evolution (MDE) algorithm to solve short-term hydrothermal scheduling problem was presented by Lakshminarasimman and Subramanian. The DEA was modified in order to handle the reservoir end volume constraints in the hydrothermal scheduling. The algorithm modifications were carried out at the initialisation and mutation steps in the main DEA to efficiently deal with the final reservoir storage volume constraints. In addition, the transmission losses were also accounted through the use of loss coefficients in [61].

Kim et al. presented an improved differential evolution strategy (DES) for constrained global optimisation and application to practical engineering problems [62]. The modified method was used to solve the engineering design problems and the robust design of a gas circuit breaker to reduce the variation of the performance and improve the small current interruption capability. The main DES modifications were the choice of scaling factor, which was varied randomly within some range and an auxiliary set was employed to enhance the diversity of the population.

A differential evolution was studied and presented in detail for solving optimal reactive power flow (ORPF) problem by Liang et al. [63]. The objective of ORPF was to find out the optimal settings of the voltage/reactive power control variables, mainly considering the generator voltages, the transformer tap ratios and the susceptances of shunt reactive power compensators therefore the real power loss could be minimised by the proposed method.

Cai et al. proposed differential evolution algorithm application for transient stability constrained optimal power flow (TSCOPF) [64]. A robust and efficient technique was developed for solving TSCOPF problem based on differential evolution. According to the flexible properties of differential evolution mechanism, the hybrid method for transient stability assessment, which combined time-domain simulation and transient energy function method, could be employed in differential evolution. Several strategies were used to reduce the computation burden so that these strategies were proposed for the initialisation, assessment and selection of solution individuals in evolution process of differential evolution.

3.3.2 Basis of Differential Evolution Algorithm

A DEA is a novel evolution algorithm as it employs real-coded variables and typically relies on mutation as the search operator. More recently DEA has evolved to share many features with conventional genetic algorithm (CGA) [45]. The major similarity between these two types of algorithm is that they both maintain populations of potential solutions and use a selection mechanism for choosing the best individuals from the population. The main differences are as follows [50]:

- DEA operates directly on floating point vectors while CGA relies mainly on binary strings;
- CGA relies mainly on recombination to explore the search space, while DEA uses a special form of mutation as the dominant operator;
- DEA is an abstraction of evolution at individual behavioural level, stressing the behavioural link between an individual and its offspring, while CGA maintains the genetic link.

DEA is a parallel direct search method that employs a population P of size N_P, consisted of floating point encoded individuals or candidate solutions as shown in equation (3.1). At every generation G during the optimisation process, DEA maintains a population $P^{(G)}$ of N_P vectors of candidate solutions to the problem at hand.

$$P^{(G)} = [X_1^{(G)}, ..., X_i^{(G)}, ..., X_{N_P}^{(G)}] \tag{3.1}$$

Each candidate solution X_i is a D-dimensional vector, containing as many integer-valued parameters (3.2) as the problem decision parameters D.

$$X_i^{(G)} = [x_{1,i}^{(G)}, ..., x_{j,i}^{(G)}, ..., x_{D,i}^{(G)}], \;\; i = 1, ..., N_P, \;\; j = 1, ..., D \tag{3.2}$$

3.3.3 Differential Evolution Algorithm Optimisation Process

3.3.3.1 Initialisation

In the first step of the DEA optimisation process, the population of candidate solutions must be initialised. Typically, each decision parameter in every vector of the initial population is assigned a randomly chosen value from within its corresponding feasible bounds.

$$x_{j,i}^{(G=0)} = x_j^{\min} + \text{rand}_j[0,1].(x_j^{\max} - x_j^{\min}) \tag{3.3}$$

where $i = 1,…,N_P$ and $j = 1,…,D$. $x_{j,i}^{(G=0)}$ is the initial value (G=0) of the j^{th} parameter of the i^{th} individual vector. $x_j^{\min}$ and $x_j^{\max}$ are the lower and upper bounds of the j^{th} decision parameter, respectively. Once every vector of the population has been initialised, its corresponding fitness value is calculated and stored for future reference.

3.3.3.2 Mutation

The DEA optimisation process is carried out by applying the following three basic genetic operations; mutation, recombination (also known as crossover) and selection. After the population is initialised, the operators of mutation, crossover and selection create the population of the next generation $P^{(G+1)}$ by using the current population $P^{(G)}$. At every generation G, each vector in the population has to serve once as a target vector $X_i^{(G)}$, the parameter vector has index i, and is compared with a mutant vector. The mutation operator generates mutant vectors ($V_i^{(G)}$) by perturbing a randomly selected vector (X_{r1}) with the difference of two other randomly selected vectors (X_{r2} and X_{r3}).

$$V_i^{(G)} = X_{r1}^{(G)} + F\left(X_{r2}^{(G)} - X_{r3}^{(G)}\right), \quad i = 1,\dots,N_P \tag{3.4}$$

Vector indices $r1$, $r2$ and $r3$ are randomly chosen, which $r1$, $r2$ and $r3 \in \{1,\dots,N_P\}$ and $r1 \neq r2 \neq r3 \neq i$. X_{r1}, X_{r2} and X_{r3} are selected anew for each parent vector. F is a user-defined constant known as the "scaling mutation factor", which is typically chosen from within the range $[0,1^+]$.

3.3.3.3 Crossover

In this step, crossover operation is applied in DEA because it helps to increase the diversity among the mutant parameter vectors. At the generation G, the crossover operation creates trial vectors (U_i) by mixing the parameters of the mutant vectors (V_i) with the target vectors (X_i) according to a selected probability distribution.

$$U_i^{(G)} = u_{j,i}^{(G)} = \begin{cases} v_{j,i}^{(G)} & \text{if } \text{rand}_j(0,1) \leq CR \;\; or \;\; j = s \\ x_{j,i}^{(G)} & \text{otherwise} \end{cases} \tag{3.5}$$

The crossover constant CR is a user-defined value (known as the "crossover probability"), which is usually selected from within the range [0,1]. The crossover constant controls the diversity of the population and aids the algorithm to escape from local optima. rand_j is a uniformly distributed random number within the range (0,1) generated anew for each value of j. s is the trial parameter with randomly chosen index $\in \{1,\dots,D\}$, which ensures that the trial vector gets at least one parameter from the mutant vector.

3.3.3.4 Selection

Finally, the selection operator is applied in the last stage of the DEA procedure. The selection operator chooses the vectors that are going to compose the population in the next generation. This operator compares the fitness of the trial vector and the corresponding target vector and selects the one that provides the best solution. The fitter of the two vectors is then allowed to advance into the next generation according to equation (3.6).

$$X_i^{(G+1)} = \begin{cases} U_i^{(G)} & \text{if} \quad f(U_i^{(G)}) \leq f(X_i^{(G)}) \\ X_i^{(G)} & \text{otherwise} \end{cases} \tag{3.6}$$

The DEA optimisation process is repeated across generations to improve the fitness of individuals. The overall optimisation process is stopped whenever maximum number of generations is reached or other predetermined convergence criterion is satisfied. The main concept of DEA optimisation process is illustrated in figure 3.2.

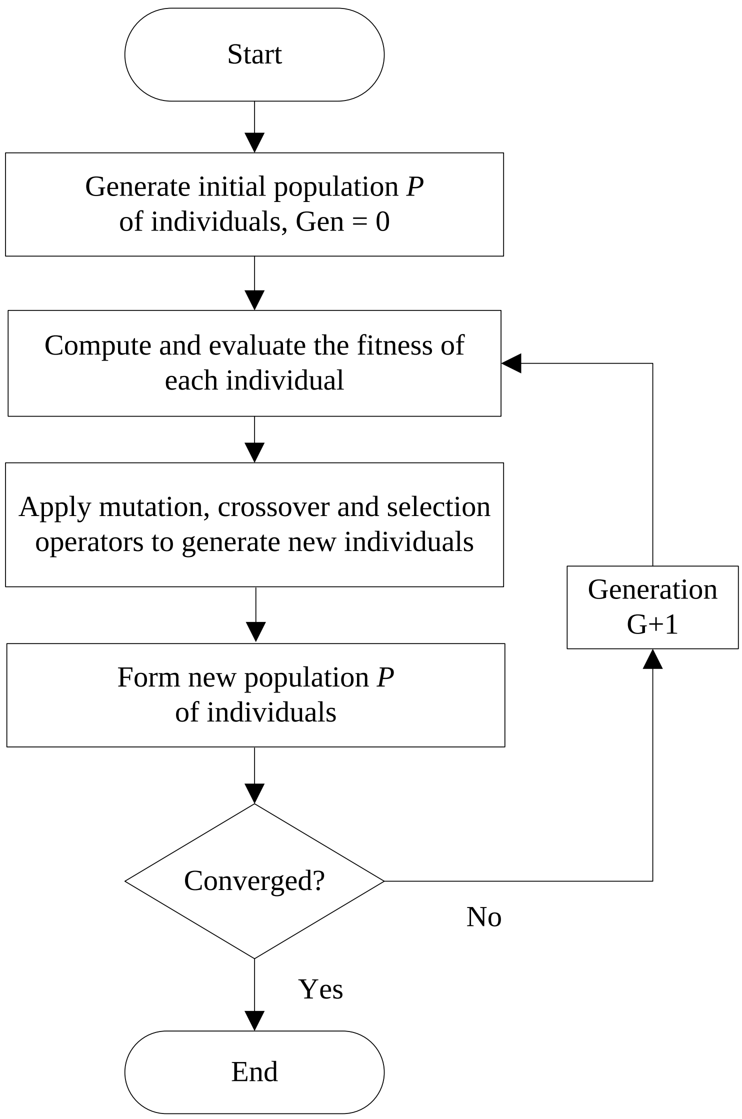


Figure 3.2 The main flowchart of the typical DEA optimisation process

3.3.3.5 DEA Strategies

There are several variations of DEA strategies to be employed for optimisation. Five variations, originally proposed by Storn in [51], are used to solve the TEP problem. In this thesis, these five DEA strategies are defined as DEA1-DEA5. In addition, the author of this thesis proposes further five DEA variations, which are defined as DEA6-DEA10.

DEA1

In the first DEA strategy, the mutant vector can be generated according to the following equation:

$$V_i^{(G)} = X_{best}^{(G)} + F \quad X_{r2}^{(G)} - X_{r3}^{(G)} \quad , \quad i = 1, \ldots, N_P \tag{3.7}$$

where $X_{\text{best}}^{(G)}$ is the best performing vector of the current generation.

DEA2

Basically, this scheme works in a similar way as DEA1 except that it generates the mutant vector from the randomly chosen base vector $X_{\text{r1}}^{(G)}$.

$$V_i^{(G)} = X_{r1}^{(G)} + F \quad X_{r2}^{(G)} - X_{r3}^{(G)} \quad , \quad i = 1, \ldots, N_P \tag{3.8}$$

DEA3

In this scheme, the perturbation is applied at a location between the best performing vector and a randomly selected population vector.

$$V_i^{(G)} = X_i^{(G)} + \lambda(X_{best}^{(G)} - X_i^{(G)}) + F \quad X_{r1}^{(G)} - X_{r2}^{(G)} \quad , \quad i = 1, \ldots, N_P \tag{3.9}$$

λ is applied to control the greediness of the scheme, which usually it is set equally to F to reduce the number of control variables.

DEA4

Two different vectors are used as a perturbation in this strategy.

$$V_i^{(G)} = X_{best}^{(G)} + F \quad X_{r1}^{(G)} + X_{r2}^{(G)} - X_{r3}^{(G)} - X_{r4}^{(G)} \quad , \quad i = 1, \ldots, N_P \tag{3.10}$$

DEA5

This scheme works in a similar way as DEA4 but it replaces the best performing vector $X_{best}^{(G)}$ by a randomly selected vector $X_{r5}^{(G)}$.

$$V_i^{(G)} = X_{r5}^{(G)} + F\left(X_{r1}^{(G)} + X_{r2}^{(G)} - X_{r3}^{(G)} - X_{r4}^{(G)}\right), \quad i = 1, \ldots, N_P \tag{3.11}$$

DEA6

This strategy works in a similar way as DEA1 to create the mutant vector but the randomly selected vectors $X_{r2}^{(G)}$ and $X_{r3}^{(G)}$ are substituted by $X_{best}^{(G)}$ and $X_i^{(G)}$ respectively.

$$V_i^{(G)} = X_{best}^{(G)} + F\left(X_{best}^{(G)} - X_i^{(G)}\right), \quad i = 1, \ldots, N_P \tag{3.12}$$

DEA7

This scheme follows the similar idea of DEA4 but it uses three different vectors for perturbation.

$$V_i^{(G)} = X_{best}^{(G)} + F\left(X_{best}^{(G)} - X_i^{(G)} - X_{r1}^{(G)} - X_{r2}^{(G)}\right), \quad i = 1, \ldots, N_P \tag{3.13}$$

DEA8

This scheme follows the similar idea of DEA3 except that $X_i^{(G)}$ is replaced by $X_{best}^{(G)}$ to generate the mutant vector.

$$V_i^{(G)} = X_{best}^{(G)} + \lambda(X_{best}^{(G)} - X_i^{(G)}) + F\left(X_{r1}^{(G)} - X_{r2}^{(G)}\right), \quad i = 1, \ldots, N_P \tag{3.14}$$

DEA9

This strategy follows the similar idea of DEA5 but it uses $X_{r1}^{(G)}$ and $X_{r2}^{(G)}$ for perturbation.

$$V_i^{(G)} = X_{best}^{(G)} + F\left(X_{best}^{(G)} + X_i^{(G)} - X_{r1}^{(G)} - X_{r2}^{(G)}\right), \quad i = 1, \ldots, N_P \tag{3.15}$$

DEA10

This strategy is performed as similar to DEA6 but $X_i^{(G)}$ is replaced by $X_{best}^{(G-1)}$ from the previous generation G-1 in order to create the mutant vector.

$$V_i^{(G)} = X_{best}^{(G)} + F \quad X_{best}^{(G)} - X_{best}^{(G-1)} \quad , \quad i = 1, ..., N_P \qquad (3.16)$$

3.3.3.6 The Example of DEA Optimisation Process

The DEA optimisation process is described in the following example.

Objective function $f(X) = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$

where X represents an encoded individual or a candidate solution. x_1, x_2, x_3, x_4, x_5, x_6, x_7 and x_8 are the parameters of the individual.

Step 1: Select control parameters of the DEA optimisation process;

Decision parameters *(D)*	*8*
Population size (N_P)	5
Scaling mutation factor (F)	0.7
Crossover constant (CR)	0.6

Step 2: Initialise population P of individuals according to equation (3.3);

Parameters/Individuals	Individual 1	Individual 2	Individual 3	Individual 4	Individual 5
Parameter 1 (x_1)	0.95	0.57	0.18	0.92	0.6
Parameter 2 (x_2)	0.43	0.88	0.29	0.87	0.79
Parameter 3 (x_3)	0.38	0.93	0.99	0.65	0.28
Parameter 4 (x_4)	0.78	0.74	0.86	0.47	0.34
Parameter 5 (x_5)	0.64	0.81	0.39	0.38	0.56
Parameter 6 (x_6)	0.55	0.66	0.42	0.82	0.93
Parameter 7 (x_7)	0.71	0.59	0.56	0.21	0.86
Parameter 8 (x_8)	0.82	0.28	0.8	0.33	0.33
Fitness $f(X)$	5.26	5.46	4.49	4.65	4.69

Step 3: Select a target vector index (i) and random vector indices ($r1$, $r2$ and $r3$) from the current population, which i, $r1$, $r2$ and $r3 \in \{1,\ldots,N_P\}$ and $r1 \neq r2 \neq r3 \neq i$;

i	1
$r1$	5
$r2$	3
$r3$	4

Step 4: Generate mutant vectors (V_i) by perturbing a randomly selected vector (X_{r1}) with the difference of two other randomly selected vectors (X_{r2} and X_{r3}) according to equation (3.4);

	X_{r1}	X_{r2}	X_{r3}	X_{r2} - X_{r3}	$F(X_{r2}$-$X_{r3})$	X_{r1}+$F(X_{r2}$ - $X_{r3})$
Parameter 1 (x_1)	0.6	0.18	0.92	-0.74	-0.518	0.082
Parameter 2 (x_2)	0.79	0.29	0.87	-0.58	-0.406	0.384
Parameter 3 (x_3)	0.28	0.99	0.65	0.34	0.238	0.518
Parameter 4 (x_4)	0.34	0.86	0.47	0.39	0.273	0.613
Parameter 5 (x_5)	0.56	0.39	0.38	0.01	0.007	0.567
Parameter 6 (x_6)	0.93	0.42	0.82	-0.4	-0.28	0.65
Parameter 7 (x_7)	0.86	0.56	0.21	0.35	0.245	1.105
Parameter 8 (x_8)	0.33	0.8	0.33	0.47	0.329	0.659
Fitness $f(X)$	4.69	4.49	4.65	-	-	4.578

Step 5: Create trial vectors (U_i) by mixing the parameters of the mutant vectors (V_i) with the target vectors (X_i) according to equation (3.5);

	Target vector	Mutant vector	Trial vector	Random (0,1)
Parameter 1 (x_1)	0.95	0.082	0.082	0.43
Parameter 2 (x_2)	0.43	0.384	0.384	0.15
Parameter 3 (x_3)	0.38	0.518	0.38	0.78
Parameter 4 (x_4)	0.78	0.613	0.613	0.44
Parameter 5 (x_5)	0.64	0.567	0.64	0.91
Parameter 6 (x_6)	0.55	0.65	0.65	0.27
Parameter 7 (x_7)	0.71	1.105	0.71	0.66
Parameter 8 (x_8)	0.82	0.659	0.659	0.35
Fitness $f(X)$	5.26	4.578	4.118	-

Step 6: Select the vector that is going to compose the population in the next generation;

Parameters/Individuals	Individual 1	Individual 2	Individual 3	Individual 4	Individual 5
Parameter 1 (x_1)	0.082				
Parameter 2 (x_2)	0.384				
Parameter 3 (x_3)	0.38				
Parameter 4 (x_4)	0.613				
Parameter 5 (x_5)	0.64				
Parameter 6 (x_6)	0.65				
Parameter 7 (x_7)	0.71				
Parameter 8 (x_8)	0.659				
Fitness $f(X)$	4.118				

At the same time, the individual 2-5 in this table are fully filled in step 6.

Step 7: Return to step 3, which the DEA optimisation process is repeated across generations to improve the fitness of individuals. Repeat until the maximum number of generations is reached or other predetermined convergence criterion is satisfied.

3.3.4 Constraint Handling Techniques

Evolutionary algorithms, for example evolutionary programming, genetic algorithms and differential evolution algorithm were originally proposed to solve unconstrained optimisation problem. However, most optimisation problems in the real world involve finding a solution that not only is optimal but also satisfies one or more constraints. Over the last few decades, various techniques have therefore been applied to handle constraints in EAs. A literature review in EAs for constrained parameter optimisation problems with a classification of the methods to handle constraints was surveyed by Michalewicz and Schoenauer in [65]. These methods could be categorised into four groups that are: methods based on preserving feasibility of solutions, methods based on penalty functions, methods which clearly distinguish between feasible and infeasible solutions and other hybrid methods.

Two main groups of constrained optimisation methods in EAs, which are methods based on preserving feasibility of solutions and methods based on penalty function, are selected to solve the transmission expansion planning problem in this research. Feasible solution can be obtained through the use of specialised operators or feasible region boundary search. The first strategy used to explore only the feasible solution space is to create and retain candidate solutions within the feasible region as presented in [52]. This technique can be implemented as follows:

$$x_{j,i}^{(G)} = \begin{cases} x_{j,i}^{\min} & \text{if} \quad x_{j,i}^{(G)} \le x_{j,i}^{\min} \\ x_{j,i}^{\max} & \text{if} \quad x_{j,i}^{(G)} \ge x_{j,i}^{\max} \\ x_{j,i}^{(G)} & \text{otherwise} \end{cases} \qquad , i = 1,...,N_P, \quad j = 1,...,D \tag{3.17}$$

According to equation (3.17), the candidate solutions that fall outside boundary limit are essentially adjusted to reconcile their values within the feasible bound. This is to ensure that only feasible solutions will be tested in next process. These solutions can be achieved by fixing the values to the nearest bound violated or creating new values within the feasible bound.

The second method is based on penalty functions that are used whenever any equality and/or inequality constraints have been violated. This method modifies the objective function providing information terms of the feasible and infeasible bounds aiding the algorithms to search the required optimal solution. In the simple form, the fitness function value $F^{/}(X)$ to be minimised by EAs can be computed by penalising the objective function value $F(X)$ with penalty function value whenever the parameters at candidate solution violate the problem constraints (3.18). Penalty functions can be classified as exterior or interior penalty functions depending upon whether they penalise infeasible or feasible solutions respectively.

$$F^{/}(X) = F(X) + \text{Penalty}(X) \tag{3.18}$$

Michalewicz and Schoenauer [65] presented constrained optimisation and constraint handling technique in evolutionary algorithms. A DEA for handling nonlinear constraint functions was proposed by Lampinen [52].

3.4 Conclusions

This chapter presents two artificial intelligence (AI) techniques, which are genetic algorithms and differential evolution algorithm, employed to solve transmission expansion planning problem in this thesis. These methods have much more potential and efficient for applying a wide variety of practical engineering problems, especially electrical power system problems. DEA is also latest entry into the AI fields, which has been increasing in current attention. According to the DEA's characteristics as explained in this chapter, the DEA is particularly fast and simple with regard to application and modification. In addition, it requires few control variables and is robust, parallel processing nature and fast convergence. DEA has ability to handle nonlinear and multimodal cost function including the TEP problem. Therefore, DEA is reasonably selected to apply with TEP problem regard to above its advantages. This technique needs to be understood in relation to the computation requirement and convergence property before its application. The fundamentals of the DEA method mentioned in this chapter such as DEA optimisation process, DEA strategies and DEA constraint handing techniques will be applied to TEP in this research.

CHAPTER 4
DESIGN AND TESTING OF DIFFERENTIAL EVOLUTION ALGORITHM PROGRAM

4.1 Introduction

Based on background information as discussed in chapter 3, a basic design of DEA optimisation program has been performed in this chapter whereas a computer program, MATLAB, is deployed. Meanwhile, a number of strategies in mutation operation of DEA have also been considered into this analysis as they have significant impact to the accuracy of the optimisation result. To examine this impact as well as to evaluate performance of the proposed method, the computer program formulated in this chapter has also been tested with seven numerical benchmark test functions, which are classified into both unimodal and multimodal schemes. A comparison of the results between DEA method and conventional genetic algorithm (CGA) has also been stated in this chapter.

The organisation of this chapter is as follows: section 4.2 presents the design of DEA optimisation program while in section 4.3 some selected numerical benchmark test functions are introduced. Section 4.4 provides the details of experimental setup and control parameters setting. Section 4.5 presents the experimental results and discussion for each test function. Section 4.6 provides overall analysis and discussion on all test results. Finally, a summary of this experiment is made in section 4.7.

4.2 Design of the Differential Evolution Algorithm Optimisation Program

Given the basic optimisation process of DEA and several variations of mutation operator strategies, DEA optimisation program has been designed in this chapter using MATLAB. The proposed optimisation program is expected to be able to solve a number of mathematical and engineering problems, such as economic power dispatch, unit commitment, optimal power flow, power system planning, transmission expansion planning, etc. The overall procedure of the DEA optimisation program has been described as follows:

Step 1: Set up all required parameters of the DEA optimisation process by the user;

- Set up control parameters of the DEA optimisation process that are population size (N_P), scaling mutaion factor (F), crossover probability (CR), convergence criterion (ε), number of problem variables (D), lower and upper bounds of initial population ($x_j^{\min}$ and $x_j^{\max}$) and maximum number of iterations or generations ($G^{\max}$);
- Select a DEA mutation operator strategy;

Step 2: Set generation $G = 0$ for initialisation step of DEA optimisation process;

Step 3: Initialisation step;

- Initialise population P of individuals according to equation (3.3) where each decision parameter in every vector of the initial population is assigned a randomly selected value from within its corresponding feasible bounds;

Step 4: Calculate and evaluate the fitness values of the initial individuals according to the problem's fitness function;

Step 5: Rank the initial individuals according to their fitness;

Step 6: Set iteration $G = 1$ for optimisation step of DEA optimisation process;

Step 7: Apply mutation, crossover and selection operators to generate new individuals;

- Apply mutation operator to generate mutant vectors ($V_i^{(G)}$) according to equation (3.4) with a selected DEA mutation operator strategy in step 1;
- Apply crossover operator to generate trial vectors ($U_i^{(G)}$) according to equation (3.5);
- Apply selection operator according to equation (3.6) by comparing the fitness of the trial vector ($U_i^{(G)}$) and the corresponding target vector ($X_i^{(G)}$) and then select one that provides the best solution;

Step 8: Calculate and evaluate the fitness values of new individuals according to the problem's fitness function;

Step 9: Rank new individuals by their fitness;

Step 10: Update the best fitness value of the current iteration (*gbest*) and the best fitness value of the previous iteration (*pbest*)

Step 11: Check the termination criteria;

- If $|X_i^{best} - X_i| > \varepsilon$ or $|pbest - gbest| > \varepsilon$ but the number of current generation remains not over the maximum number of generations $G < G^{max}$, set $G = G + 1$ and return to step 7 for repeating to search the solution. Otherwise, stop to calculate and go to step 12;

Step 12: Output *gbest* of the last iteration as the best solution of the problem.

4.3 Numerical Benchmark Test Functions

To evaluate DEA optimisation program, ten variant DEA schemes have been studied and tested whereas their performances are compared with CGA procedure. A test suite of benchmark functions, previously introduced in [47, 50, 54], with a varying number of dimensions have been employed to test the performance of the proposed algorithm. The suite of benchmark functions contains a diverse set of mathematical problems, including unimodal as well as multimodal functions that are with correlated and uncorrelated variables. In this experiment, seven test functions are selected from the benchmark function class, which appears to be very difficult class of problems for many optimisation methods.

The details of the used benchmark functions with a varying number of dimensions from 2 to 100, the ranges of their search spaces and their global minimum fitness are tabulated in the table 4.1. As previously mentioned, the selected seven test functions are Sphere function (f_1), Rosenbrock1 function (f_2), Rosenbrock2 function (f_3), Absolute function (f_4), Salomon function (f_5), Schwefel function (f_6) and Rastrigin function (f_7). These test functions range from simple to difficult challenge depending on dimension of the problem because the number of local minima for each test function increases exponentially. Therefore, the increasing local minima affect the difficulty for approaching the optimal solution of the problem.

Table 4.1 Numerical benchmark test functions

Function name	Expression and condition
1. Sphere function	$f_1(x) = \sum_{j=0}^{n-1} x_j^2; \quad -5.12 \le x_j \le 5.12$ n = 3 dimensions $f_1(\vec{0}) = 0$
2. Rosenbrock1 function	$f_2(x) = 100 \cdot (x_0^2 - x_1)^2 + (1 - x_0)^2; \quad -2.048 \le x_j \le 2.048$ n = 2 dimensions $f_2(\vec{1}) = 0$
3. Rosenbrock2 function	$f_3(x) = \sum_{j=0}^{n-1} (100 \cdot (x_{j+1} - x_j^2)^2 + (x_j - 1)^2); \quad -30 \le x_j \le 30$ n = 30 dimensions $f_3(\vec{1}) = 0$
4. Absolute function	$f_4(x) = \sum_{j=0}^{n-1} \left(\left\lvert x_j + \frac{1}{2} \right\rvert \right)^2; \quad -100 \le x_j \le 100$ n = 30 dimensions $f_4(\vec{p}) = 0, \quad -\frac{1}{2} \le p_i < \frac{1}{2}$
5. Salomon function	$f_5(x) = -\cos(2\pi \cdot \lVert x \rVert) + 0.1 \cdot \lVert x \rVert + 1$ $\lVert x \rVert = \sqrt{\sum_{j=0}^{n-1} x_j^2} \quad ; \quad -100 \le x_j \le 100$ n = 30 dimensions $f_5(\vec{0}) = 0$

Table 4.1 Numerical benchmark test functions (Contd.)

Function name	Expression and condition
6. Schwefel function	$f_6(x) = \sum_{j=0}^{n-1} -x_j \cdot \sin(\sqrt{\lvert x_j \rvert});\quad -500 \le x_j \le 500$ n = 100 dimensions $f_6(\overrightarrow{420.97}) = -4.18983 \times 10^4$
7. Rastrigin function	$f_7(x) = \sum_{j=0}^{n-1} \left[x_j^2 - 10\cos(2\pi \cdot x_j) + 10 \right];\quad -5.12 \le x_j \le 5.12$ n = 100 dimensions $f_7(\vec{0}) = 0$

4.4 Experimental Setup and Control Parameters Setting

In this experiment, DEA as well as CGA are implemented in MATLAB and evaluated the performance regarding their general applicability as numerical optimisation techniques. As the suitable parameters of DEA and CGA are crucial to the accurate result, therefore these parameters should be selected carefully by the user. The details of these parameters setting have been discussed and included in this section.

4.4.1 DEA Control Parameters and Their Effect

The convergence of DEA is normally affected by a number of control parameters, which include the population size (N_P), mutaion factor (F) and crossover probability (CR). Proper selection of these parameters is required to obtain the reliable result with fewer function evaluations. The DEA parameters setting is not the difficult task for a simple objective function problem, whereas it is the difficult task for parameters adjustment in a complex problem.

4.4.1.1 Population Size (N_P)

The population size of DEA should be moderate. As DEA may converge to local optimum if population size is very small due to its less diversity of discovery. On the other hand, if the population size is very large, DEA would require huge numbers of function evaluations for convergence, which needs tremendously high computation time. The population size should relate to the number of a problem decision papameter or variable *D*. Storn and Price [49] remarked how to choose the proper control variables N_P, *F* and *CR* for real-world optimisation problems. According to their experience, a reasonable choice for N_P setting is between 5*D and 10*D but N_P must not be less than 4*D to guarantee that DEA will have enough mutually different vectors with which to work.

4.4.1.2 Mutation Factor (*F*)

Mutation factor is a real and constant factor that controls the amplification of the differential variation ($X_{r2}^{(G)}$ - $X_{r3}^{(G)}$) in equation (3.4) and it affects the DEA convergence. Mutation factor shoud not be less than a certain value to prevent premature convergence. The suitable mutation factor value depends upon the problem function. A larger mutation factor value increases the probability for escaping a local minimum. However, if the mutation factor value is more than 1 [53], the convergence speed decreases. The selection of proper mutation factor value is a dfficult task for the user, and therefore it should be chosen carefully with the user's experience. For DEA optimistation, the mutation factor is much more sensitive than crossover probability, which is more similar to a fine tuning element. This will be discussed in the next section.

4.4.1.3 Crossover Probability (*CR*)

Crossover probability affects the number of variables to be changed in the trial vectors ($U_i^{(G)}$) compare to the target vectors ($X_i^{(G)}$). If the value of crossover probability is large, more variables are taken from the mutant vectors ($V_i^{(G)}$) than the target vectors ($X_i^{(G)}$). A large crossover probability often speeds up convergence but the population may converge prematurely. On the other hand, more variables are taken from the target vectors ($X_i^{(G)}$) than the mutant vectors ($V_i^{(G)}$) if the value of crossover probability is small. If the crossover probability equal to 0 then all variables in the trial vectors ($U_i^{(G)}$) remain as same as the member in target vectors ($X_i^{(G)}$) and there is no improvement in the result. If the crossover probability equal to 1 then all variables in the trial vectors ($U_i^{(G)}$) are taken from the mutant vectors ($V_i^{(G)}$). This case means there is no shuffle of components between mutant vectors ($V_i^{(G)}$) and target vectors ($X_i^{(G)}$) and it decreases the diversity of population. Therefore the user should select the proper value of crossover probability carefully between 0 and 1.

4.4.1.4 Number of Problem Variables (*D*)

The number of variables in the objective function depends on the problem size and affects the convergence speed of DEA. If the problem comprises many variables, they will increase the region of solution and take longer time to converge. The increasing number of problem variables affects the difficulty for approaching the optimal solution of the problem.

4.4.1.5 Convergence Criterion (ε)

Convergence criterion compares two differences of the candidate solution population that are the difference between fitness function values of other members and the best member in the same iteration or the difference between fitness values of the best solution in present iteration and previous iteration. Convergence criterion affects an accuracy of the problem result. If convergence criterion value is very small then DEA gives more accurate result but DEA requires more computational time for convergence. However, small convergence criterion value may not give the accurate solution if other control parameters of DEA are not chosen appropriately.

4.4.2 Control Parameters Setting

For DEA testing of this chapter, the control parameter settings are manually tuned based on a few preliminary experiments. In these experiments, the DEA parameter settings are as following ranges: F = [0.5,0.9], CR = [0.55,0.95] and N_P = [5*D,10*D]. The predetermined convergence criterion (ε) is set to $1\text{x}10^{-50}$ and the maximum number of iterations or generations ($G^{\max}$) is set to 1500 for each run. The defined control parameters of DEA and CGA, which are implemented in this experiment, are listed in table 4.2.

Table 4.2 Parameters used in the implementation

Methods	F	CR	N_p	D	ε	G^{max}
All DEA strategies	0.6	0.8	10*D	2, 3, 30 and 100 for each problem's dimensions	1×10^{-50}	1500
CGA	0.05	0.8				

4.5 Experimental Results and Discussion

Each algorithm has been tested with all of the numerical benchmark functions f_1-f_7 as stated in table 4.1. In order to reduce the random effect of results, therefore each of the numerical benchmark experiment is run at least 50 times with different random seeds and the average fitness value of the best solutions throughout the optimisation run is recorded. A Pentium IV 3 GHz personal computer with 496 MB RAM is used in this experiment.

The experimental fitness value results, which consist of the best results, the worst results, the standard deviation and average values of the obtained results of all algorithms on benchmark problems f_1-f_7, are tabulated in table 4.3-4.9 respectively. In addition, the computational times are also included in table 4.3-4.9. The convergence graphs of two DEA strategies, which show the best and the worst performance DEA schemes in each benchmark problem, are selected to present their convergences. Meanwhile, the convergence graphs of CGA procedure for all benchmark problems f_1-f_7 are also illustrated in figures 4.1-4.7.

4.5.1 Sphere Function Test Results

In this experiment, the first mathematical benchmark test function is Sphere (f_1) that is a unimodal function and the simplest function compared to other test functions. The achieved results are illustrated in table 4.3 and figure 4.1. From these results, the discussion can be made as follows:

- For Sphere function, all methods except DEA10 and CGA perform well to find problem solution. They found the best function values less than 1×10^{-50}.

- Although CGA performs not as well as DEA1-DEA9 to approach the problem solution, CGA is not the worst method for this test function. As an achieved best function value of CGA is equal to 3×10^{-13} that is an acceptable value for the solution of this test case.
- DEA10 shows the poorest performance for approaching the problem solution compared to other DEA strategies and CGA procedure because the best function value of DEA10, which is equal to 6.47×10^{-2}, is a huge value for this case. In addition, DEA10 has some large values of an average result, the worst result and a standard deviation that are greater than "1". Therefore, these obtained results of DEA10 are unacceptable values for the solution of this test case.
- DEA6 performs well in searching the problem solution and is more robust than other methods. This is shown by the smallest values of the best result, an average result and the worst result respectively.
- For the calculation time comparison, DEA1 takes the lowest an average CPU time. In contrast, CGA takes the highest CPU time in this case. Among DEA schemes, DEA10 requires more an average CPU time for calculation than other schemes while generating a poor convergence rate for all range of search process in this test function study.

Table 4.3 Comparison of simulation results for Sphere function (f_1)

Results	Methods										
	DEA6	DEA4	DEA3	DEA7	DEA1	DEA8	DEA9	DEA2	DEA5	CGA	DEA10
Best	1.95E-52	2.53E-52	3.05E-52	3.18E-52	3.51E-52	4.20E-52	6.58E-52	1.39E-51	2.05E-51	3.00E-13	6.47E-02
Average	4.00E-51	5.43E-51	5.17E-51	5.33E-51	4.72E-51	4.69E-51	5.61E-51	6.38E-51	6.45E-51	4.46E-11	2.23E+00
Worst	9.37E-51	9.83E-51	9.95E-51	9.88E-51	9.94E-51	9.40E-51	9.83E-51	9.72E-51	9.82E-51	2.06E-10	8.77E+00
Std Dev	2.63E-51	2.81E-51	2.35E-51	2.51E-51	2.78E-51	2.77E-51	2.50E-51	2.46E-51	2.37E-51	5.75E-11	1.78E+00
Average CPU time	0.16	0.17	0.16	0.27	0.12	0.27	0.26	0.28	0.33	2.78	0.35

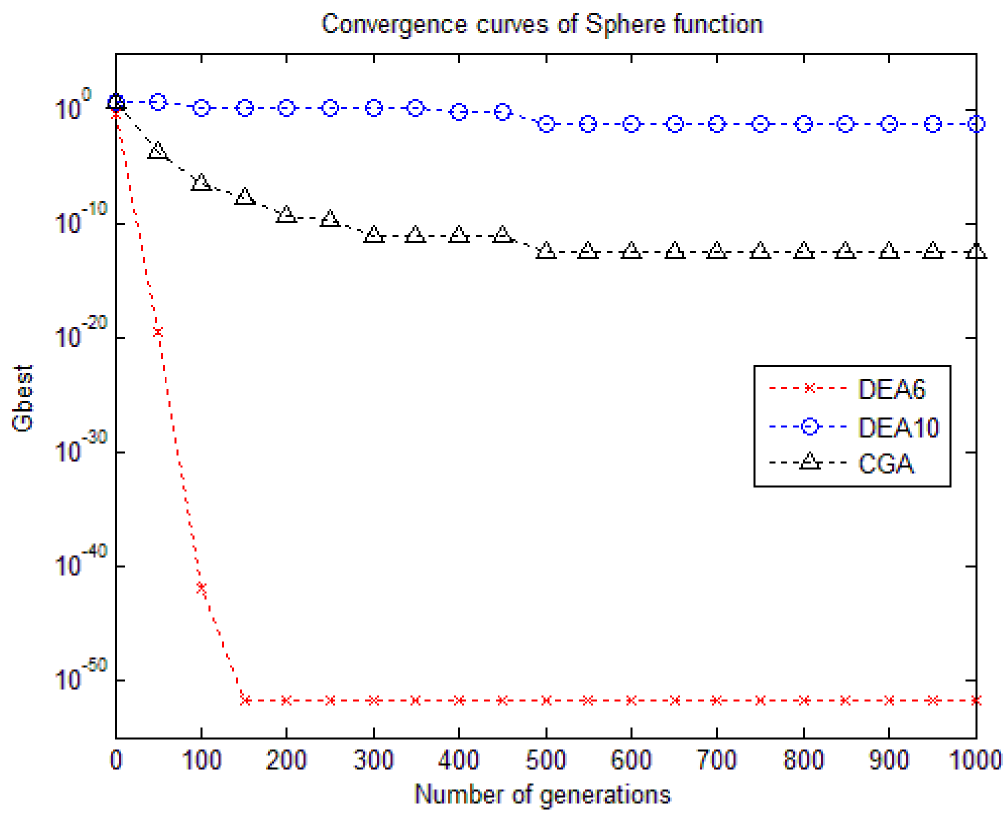


Figure 4.1 Convergence curves of DEA strategies and CGA procedure on mathematical benchmark function 1

4.5.2 Rosenbrock1 Function Test Results

The second mathematical benchmark test function is Rosenbrock1 (f_2), which is a unimodal function and has the lowest dimensions in this experiment. This test function is more complex than Sphere function but having fewer dimensions. The achieved results are illustrated in table 4.4 and figure 4.2. Based on these results, the discussion is as follows:

- For Rosenbrock1 function, all DEA schemes except DEA7 and DEA10 perform successfully to find the problem solution. They found the function solution that is equal to "0" in this test case.
- According to the previous successful DEA schemes, only DEA4, DEA5, DEA8 and DEA9 perform the best performance for approaching the optimal solution because they yield the outstanding values of the best result, an average result and the worst result. These values are equal to "0" for this test

function.

- DEA7, DEA10 and CGA are not successful to approach the optimal solution because they could not find the fitness optimum for this test function.
- Similar to Sphere function, DEA10 remains the poorest performance for finding the problem solution and least robust compared with other methods because it has the greatest values of the best result, an average result, the worst result and a standard deviation.
- In this case, DEA1 and DEA4 require less average CPU time for calculation than other DEA strategies and the CGA procedure.
- Regarding convergence curves presented in figure 4.2, DEA4 gives the best convergence rate comparing with DEA10 and CGA procedure, which has the poorest convergence rate in this test function. DEA10 converges quicker than CGA in the early stage of searching process but it gets into the stagnation state at around 100 iterations and is trapped into the local optimal solution.

Table 4.4 Comparison of simulation results for Rosenbrock1 function (f_2)

Results	Methods										
	DEA4	DEA9	DEA5	DEA8	DEA2	DEA3	DEA1	DEA6	CGA	DEA7	DEA10
Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.60E-08	7.00E-04	2.48E-02
Average	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.28E-03	2.15E-03	1.09E-02	3.86E-02	1.05E-02	6.14E-02	3.27E+00
Worst	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.57E-02	6.48E-02	5.24E-01	6.20E-01	6.63E-02	3.29E-01	1.90E+01
Std Dev	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.72E-03	9.61E-03	7.41E-02	1.07E-01	1.58E-02	8.02E-02	4.35E+00
Average CPU time	0.09	0.11	0.13	0.14	0.14	0.15	0.09	0.62	4.67	0.72	0.74

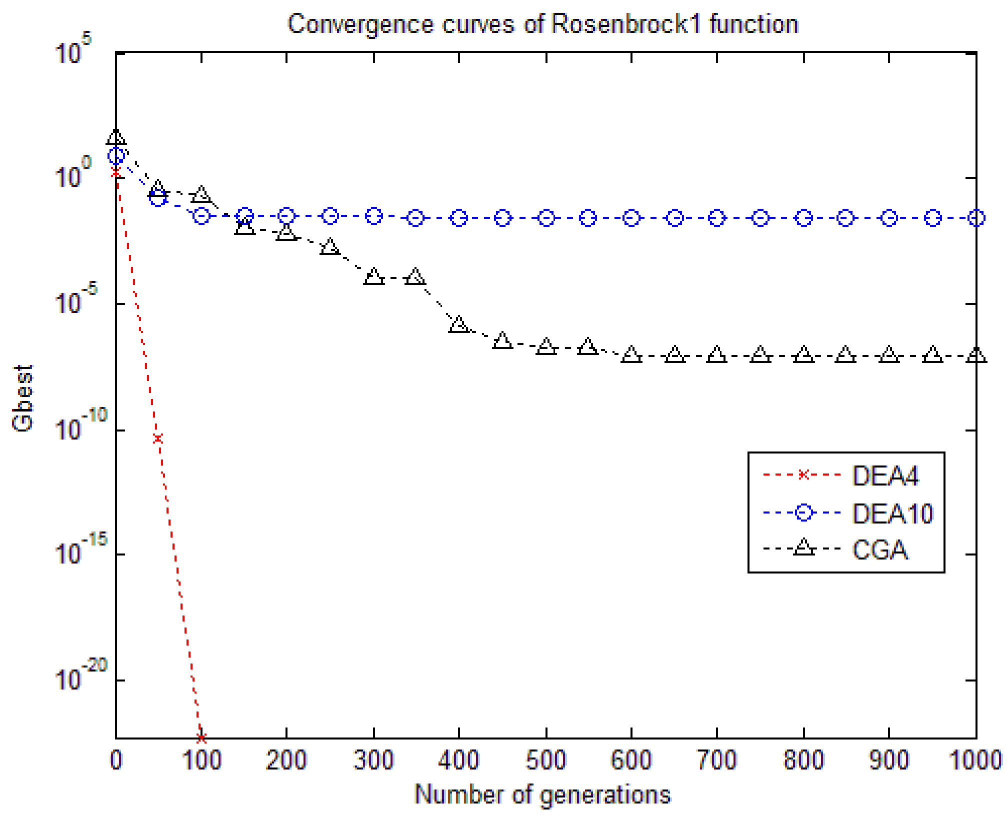


Figure 4.2 Convergence curves of DEA strategies and CGA procedure on mathematical benchmark function 2

4.5.3 Rosenbrock2 Function Test Results

The third mathematical benchmark test function is Rosenbrock2 (f_3), which is a unimodal function. This test function is almost similar to Rosenbrock1 function except that it has more dimensions. The achieved results are illustrated in table 4.5 and figure 4.3 and the result discussion is as follows:

- DEA1 and DEA3 perform better than other DEA schemes and CGA as their best function values are very smaller than that of other methods.
- For Rosenbrock2 function, DEA3 is superior to other DEA schemes and CGA procedure. It shows better performance for seeking the function solution as shown by the smallest values of the best result, an average result and the worst result respectively.
- DEA2, DEA4, DEA5, DEA7, DEA8, DEA10 are not successful for finding the problem solution in this test case because they found the best function

values that are greater than the problem solution.

- DEA5 performs poorest compared with other DEA strategies and CGA procedure because it has the highest values of the best result, an average result and the worst result.
- Similar to Sphere and Rosenbrock1 functions, DEA1 is faster than other methods for calculation, whereas CGA is the slowest method in this case.
- In figure 4.3, DEA3 gives the best convergence rate comparing with DEA5 and CGA procedure, whereas DEA5 gives the poorest convergence rate in this test function. Moreover, DEA5 gets into the stagnation state at around 700 iterations and is finally trapped into the local optimal solution.

Table 4.5 Comparison of simulation results for Rosenbrock2 function (f_3)

Results	Methods										
	DEA3	DEA1	DEA9	CGA	DEA6	DEA8	DEA2	DEA4	DEA7	DEA10	DEA5
Best	7.47E-11	8.06E-11	1.36E-02	3.80E-02	5.48E-01	1.11E+01	2.84E+01	2.84E+01	1.43E+02	2.29E+02	2.74E+02
Average	9.15E-11	9.36E-11	9.44E-02	4.68E-02	1.72E+00	5.79E+01	3.01E+01	4.45E+01	2.70E+02	3.03E+02	3.20E+02
Worst	9.98E-11	9.98E-11	2.40E-01	7.83E-02	4.28E+00	1.14E+02	3.17E+01	1.20E+02	3.61E+02	3.56E+02	3.75E+02
Std Dev	7.43E-12	6.61E-12	7.96E-02	7.98E-03	1.15E+00	3.61E+01	8.65E-01	2.64E+01	7.11E+01	4.25E+01	2.96E+01
Average CPU time	1.63	1.51	7.18	11.25	7.47	7.08	4.60	4.55	7.16	7.34	4.85

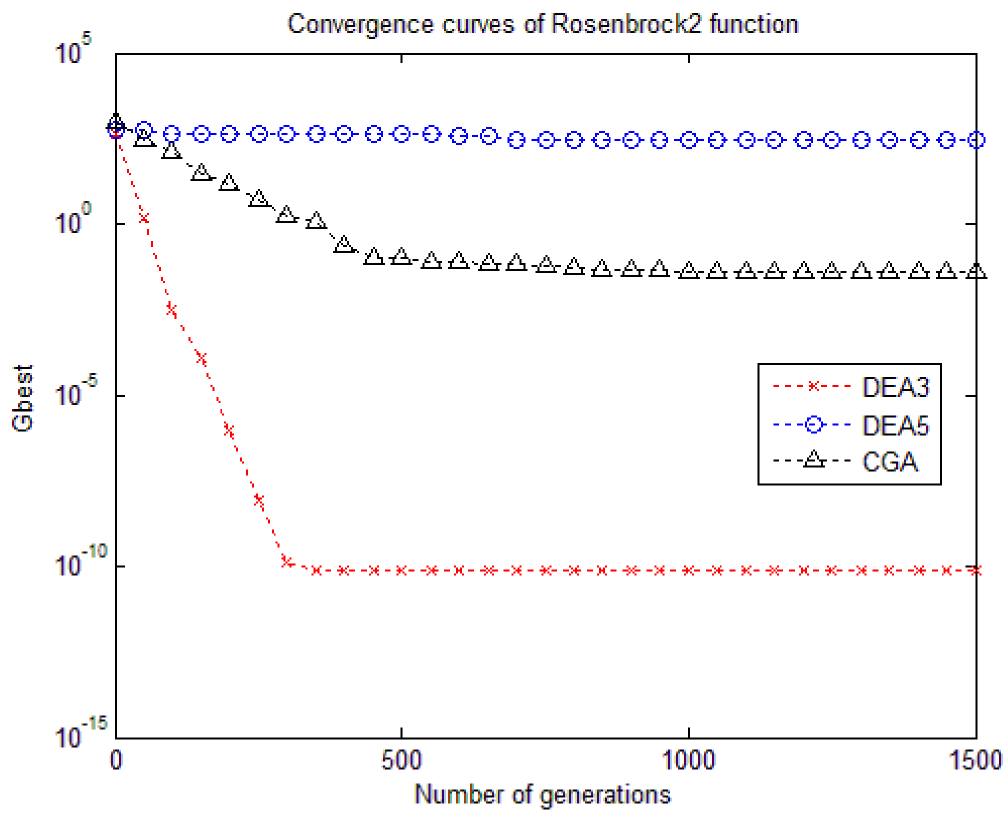


Figure 4.3 Convergence curves of DEA strategies and CGA procedure on mathematical benchmark function 3

4.5.4 Absolute Function Test Results

Absolute function (f_4) is a unimodal function and consists of the summation of absolute terms. The variables of this function range from -100 to +100, which have wider than those previous three test functions. The achieved results are illustrated in table 4.6 and figure 4.4 and the result discussion is as follows:

- Similar to Rosenbrock2 function, DEA1 and DEA3 perform better than other methods because the best function values of DEA1 and DEA3 are very smaller than those of other methods.
- Although DEA9 does not perform as well as DEA1 and DEA3 for searching the problem solution, it found an acceptable value of the best result that is equal to 1.62×10^{-7}.
- DEA3 is superior to other algorithms for finding the solution. It gives the lowest values of the best result, an average result and the worst result. In

addition, it converges faster than other DEA strategies and CGA procedure in this test case.

- On the other hand, DEA5 is inferior to other methods for its performance of finding the fitness optimal solution because it has the highest values of the best result, an average value and the worst result.
- Similar to all previous test functions, DEA1 is the fastest method compared to other strategies, whereas CGA takes more an average computational time than all DEA strategies in this test case.
- In figure 4.4, DEA3 gives the best convergence rate comparing with DEA5 and CGA procedure.

Table 4.6 Comparison of simulation results for Absolute function (f_4)

Results	Methods										
	DEA3	DEA1	DEA9	DEA6	DEA8	DEA2	DEA4	CGA	DEA10	DEA7	DEA5
Best	5.21E-11	5.88E-11	1.62E-07	5.78E-05	4.12E-03	5.06E-03	5.10E-03	2.24E+00	2.79E+00	3.27E+00	4.15E+00
Average	8.76E-11	8.82E-11	1.29E-05	1.96E-02	5.59E-02	7.98E-03	1.03E-02	4.36E+00	4.41E+00	4.80E+00	5.38E+00
Worst	9.92E-11	9.96E-11	8.60E-05	2.22E-01	1.58E-01	1.16E-02	1.92E-02	5.42E+00	5.45E+00	6.44E+00	6.57E+00
Std Dev	1.04E-11	9.36E-12	1.75E-05	3.54E-02	3.75E-02	1.48E-03	3.63E-03	1.03E+00	7.40E-01	7.49E-01	5.34E-01
Average CPU time	1.23	1.12	8.70	8.47	7.88	4.74	4.76	10.92	9.06	8.15	4.77

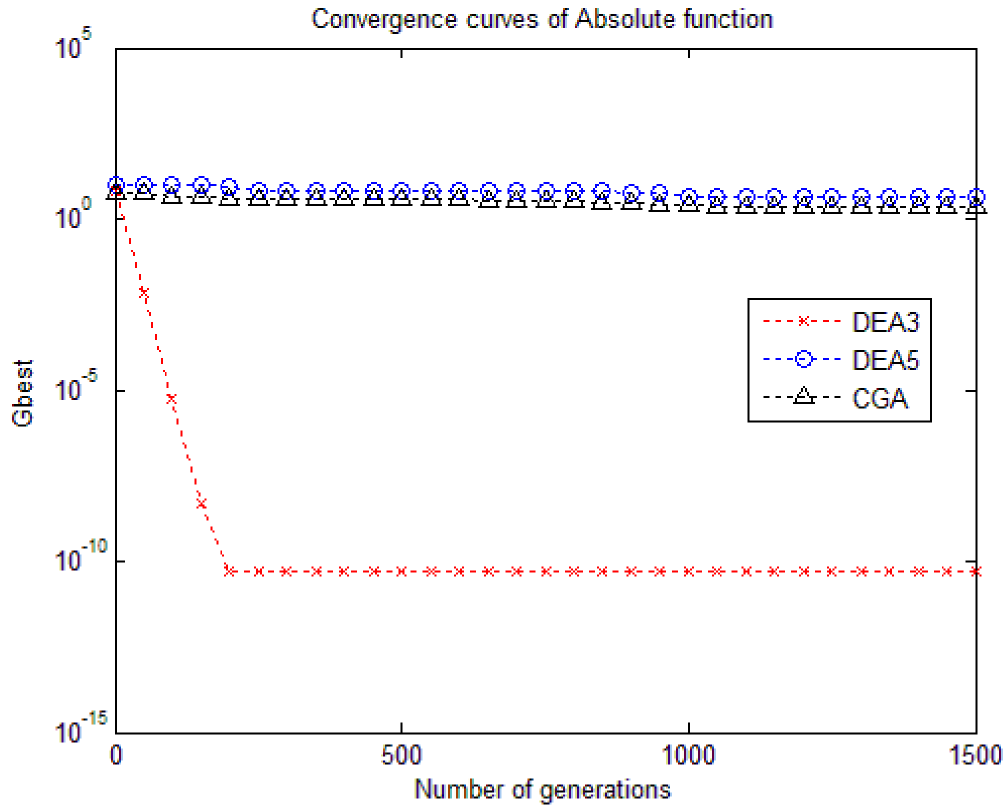


Figure 4.4 Convergence curves of DEA strategies and CGA procedure on mathematical benchmark function 4

4.5.5 Salomon Function Test Results

Salomon function (f_5) is a highly multimodal function that comprises terms of cosine function and square root function. This benchmark test function is more difficult for solving than the previous four test functions. The achieved results are illustrated in table 4.7 and figure 4.5. The discussion of these results can be presented as follows:

- For Salomon function, DEA1 and DEA3 still perform better than other methods. In contrast, DEA2, DEA4, DEA5, DEA7, DEA8 and DEA10 could not successfully discover any solution of this function.
- DEA1 and DEA3 perform outstandingly to find the problem solution compared with other methods. They reach the similar value of the best function result that is 9.99×10^{-2}. Between these two methods, DEA3 has less value of an average function result but more for standard deviation than DEA1.

- DEA6, DEA9 and CGA are almost as good methods for finding the optimal solution of this test case. These methods perform moderately better than DEA2, DEA4, DEA5, DEA7, DEA8 and DEA10 because their results are better in terms of both the best and average function values.
- Similar to Absolute function, DEA5 is poorer than other methods for its performance of finding the fitness optimal solution because it has the highest values of the best, an average, and worst function results. Moreover, DEA5 is least robust for finding the solution because it has the smallest value of a standard deviation.
- In term of computational time, DEA1 is the fastest while the slowest belongs to CGA procedure.
- In figure 4.5, DEA3 gives the fastest convergence rate compared to DEA5 and CGA, which are very slow. Moreover, DEA5 gets into the stagnation state after around 650 iterations and ultimately trapped in the local optimal solution.
- Comparing convergence rates between DEA5 and CGA procedure, CGA is faster than DEA5 in the early stage of search process but its convergence rate deteriorates dramatically after around 200 iterations, which finally leads to the stagnation state.

Table 4.7 Comparison of simulation results for Salomon function (f_5)

Results	Methods										
	DEA3	DEA1	DEA6	CGA	DEA9	DEA7	DEA4	DEA2	DEA8	DEA10	DEA5
Best	9.99E-02	9.99E-02	2.00E-01	3.00E-01	8.01E-01	1.00E+00	2.61E+00	3.74E+00	4.06E+00	1.35E+01	1.68E+01
Average	1.20E-01	1.90E-01	4.90E-01	4.93E-01	1.23E+00	1.60E+00	3.13E+00	4.13E+00	5.08E+00	1.52E+01	1.90E+01
Worst	2.00E-01	2.00E-01	1.50E+00	6.00E-01	1.60E+00	2.20E+00	3.72E+00	4.61E+00	5.75E+00	1.74E+01	2.08E+01
Std Dev	4.04E-02	3.03E-02	3.51E-01	8.47E-02	2.45E-01	3.91E-01	3.13E-01	2.32E-01	6.56E-01	1.07E+00	1.21E+00
Average CPU time	55.18	53.88	85.29	94.27	83.86	84.71	54.53	54.56	84.2	87.86	54.18

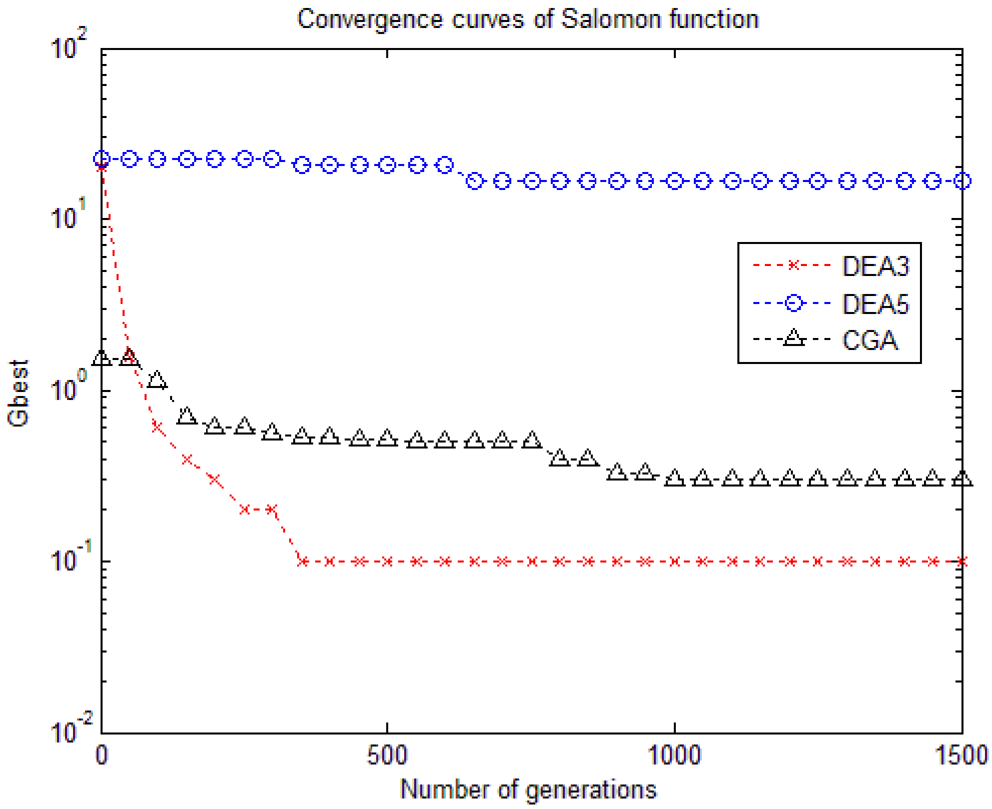


Figure 4.5 Convergence curves of DEA strategies and CGA procedure on mathematical benchmark function 5

4.5.6 Schwefel Function Test Results

Schwefel function (f_6) is a highly multimodal function, which comprises term of sine function of square root. The solution of this test function is a negative value that is distinguished from other test function solutions. The achieved results are illustrated in table 4.8 and figure 4.6. These results can be discussed as follows:

- For Schwefel function, all methods perform consistently well for approaching the fitness optimum of the problem. All techniques found their best solutions that are nearly to the fitness optimal solution of the problem.
- However both DEA1 and DEA3 perform better than other methods for finding the fitness optimum of this test function because they found the lowest value of the best function result that is equal to -3.15×10^4.
- DEA1 is superior to other methods for finding the problem solution. It gives the lowest values of the best, an average, and worst function results.

- DEA10 is inferior to other methods for its performance of searching the fitness optimum in this test case because it has the highest values of the best and average function results. However DEA10 is not poor for finding the optimal solution in this case as its best function result is close to the problem solution.
- DEA4 requires the least average computational time whereas CGA still requires the longest average calculation time in this test case.
- In figure 4.6, DEA1 gives the fastest convergence rate comparing with DEA10 and CGA procedure whereas CGA gives the slowest convergence rate in this test function.

Table 4.8 Comparison of simulation results for Schwefel function (f_6)

Results	Methods										
	DEA1	DEA3	DEA9	CGA	DEA6	DEA7	DEA4	DEA8	DEA2	DEA5	DEA10
Best	-3.15E+04	-3.15E+04	-3.14E+04	-3.13E+04	-3.13E+04	-3.12E+04	-3.11E+04	-3.08E+04	-3.06E+04	-3.06E+04	-3.04E+04
Average	-2.98E+04	-2.96E+04	-2.96E+04	-2.97E+04	-2.95E+04	-2.96E+04	-2.95E+04	-2.95E+04	-2.94E+04	-2.94E+04	-2.94E+04
Worst	-2.87E+04	-2.86E+04	-2.86E+04	-2.81E+04	-2.85E+04	-2.86E+04	-2.86E+04	-2.82E+04	-2.85E+04	-2.83E+04	-2.84E+04
Std Dev	7.80E+02	7.35E+02	5.37E+02	8.45E+02	6.00E+02	6.04E+02	5.13E+02	5.89E+02	4.68E+02	5.65E+02	4.15E+02
Average CPU time	0.69	1.52	0.56	4.70	0.86	0.85	0.51	0.56	1.36	0.66	1.75

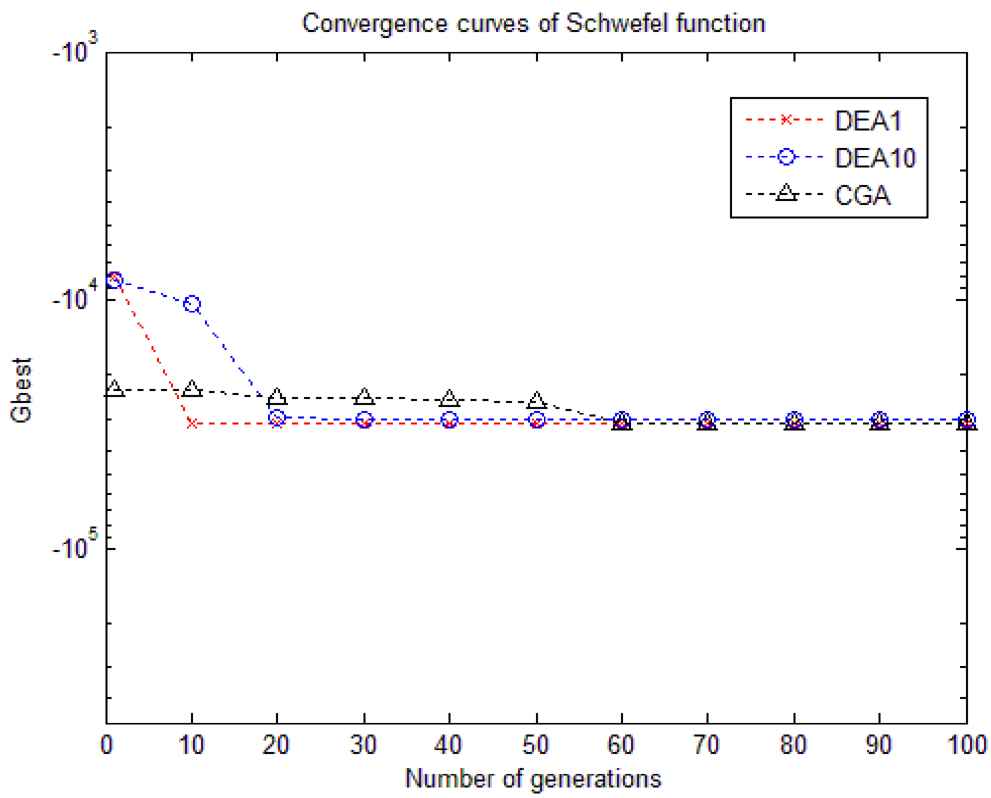


Figure 4.6 Convergence curves of DEA strategies and CGA procedure on mathematical benchmark function 6

4.5.7 Rastrigin Function Test Results

The last mathematical benchmark test function is Rastrigin (f_7), which is a highly multimodal function. This test function is a highly complex problem. It is also difficult to approach the optimal solution. There are many local optima arrayed on the side of a larger bowl-shaped depression in Rastrigin function that is symmetric about its solution [47]. The achieved results are illustrated in table 4.9 and figure 4.7. The discussion of these results is as follows:

- For Rastrigin function, DEA3 performs better than other methods to find the problem solution. It found the smallest values of the best, an average, and worst function results compared to other techniques. In addition, DEA3 is also more robust than others as it has the smallest values of an average function result and a standard deviation.
- However, all methods have no good performance for approaching the fitness

optimal solution because the achieved best function values of all algorithms are not near the problem solution. As mentioned earlier, this test function has many local optima where the population of all algorithms may not escape.

- DEA5 performs poorer than other methods because it found the greatest value of the best function result in this case.
- CGA is less robust to find the problem solution as shown by the largest values of an average function result and a standard deviation.
- Similar to all previous test functions, DEA1 takes least average computational time whereas CGA still has the largest one in this test case.
- In figure 4.7, DEA3 gives the fastest convergence rate comparing with DEA5 and CGA procedure. DEA3 converges quickly in the early stage of the search process. Then its convergence rate decreases significantly after 200 iterations and approaches to the stagnation stage. Similar to other DEA schemes and CGA procedure, DEA3 is trapped in the local minimum solution and obtains the best solution that is equal to 9.95×10^{-1}.
- Finally, all methods get into the stagnation state and are trapped in the local optimal solution.

Table 4.9 Comparison of simulation results for Rastrigin function (f_7)

Results	Methods										
	DEA3	DEA1	DEA6	CGA	DEA4	DEA2	DEA7	DEA9	DEA10	DEA8	DEA5
Best	9.95E-01	1.31E+00	1.02E+01	3.76E+02	4.71E+02	5.23E+02	5.52E+02	5.55E+02	5.62E+02	6.06E+02	6.61E+02
Average	1.89E+00	7.46E+00	4.12E+01	7.79E+02	5.54E+02	5.66E+02	6.34E+02	6.28E+02	6.17E+02	6.46E+02	6.72E+02
Worst	4.97E+00	1.19E+01	1.13E+02	1.20E+03	6.13E+02	5.85E+02	6.79E+02	7.27E+02	6.47E+02	6.83E+02	6.84E+02
Std Dev	1.18E+00	2.65E+00	2.08E+01	3.64E+02	4.50E+01	1.68E+01	3.61E+01	5.33E+01	2.82E+01	2.04E+01	8.20E+00
Average CPU time	80.47	77.81	109.37	167.82	83.88	83.14	125.78	131.62	121.61	124.51	84.78

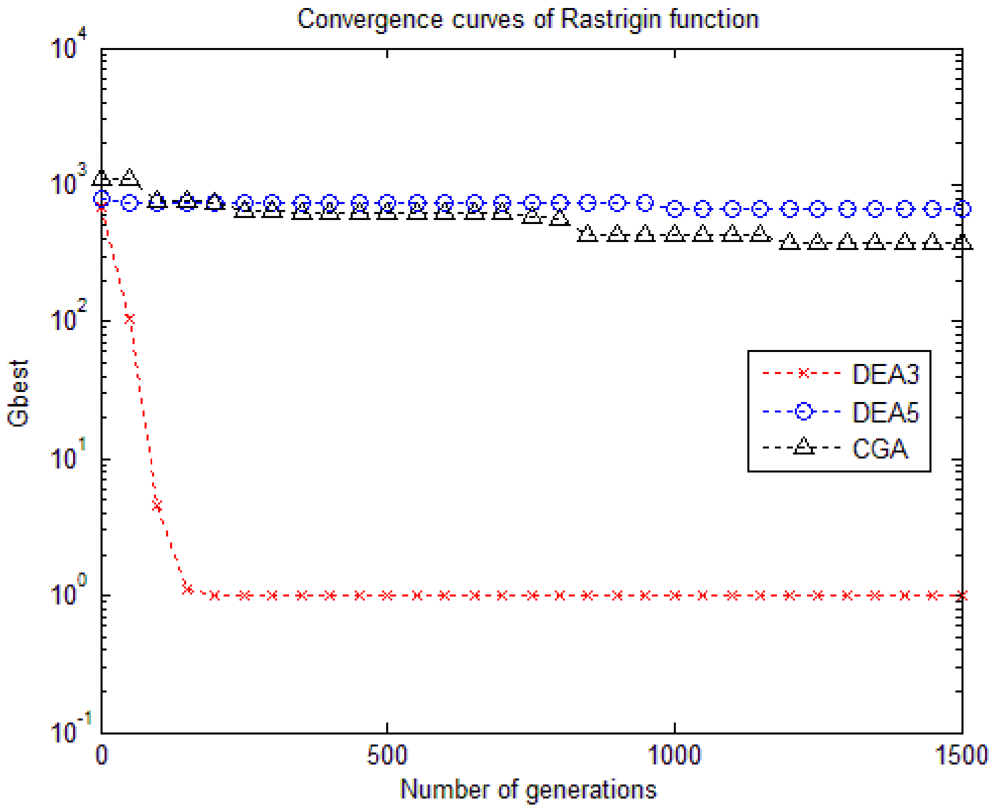


Figure 4.7 Convergence curves of DEA strategies and CGA procedure on mathematical benchmark function 7

4.6 Overall Analysis and Discussion on Test Results

Initially, the numerical benchmark test functions $f_1 - f_4$, which are unconstrained unimodal test functions, have been implemented for testing the algorithms performance. There is a consistent performance pattern across DEA1, DEA3 DEA6 and DEA9 that perform well to find the optimum solution of these four test functions. In contrast, DEA10 is not successful in approaching the fitness optimum of functions $f_1 - f_4$ and CGA procedure is not good for finding the optimal solution of functions f_2 and f_4. Under test function f_2, the optimal solution has been easily found by all methods except DEA7, DEA10 and CGA.

Last three test functions, f_5 - f_7, are highly multimodal functions and also more difficult than the previous functions $f_1 - f_4$. Three DEA schemes, DEA1, DEA3, DEA6, still perform well on solving these test functions f_5 - f_7 and CGA performs better than previous function $f_1 - f_4$. On the other hand, DEA5 and DEA 10 are not successful to find the optimal solution on functions f_5 and f_7.

Regarding the achieved results, DEA1, DEA3 and DEA6 show the outstanding performance to optimise all test functions f_1-f_7 because they found the best function value nearly the problem solution of each case. In contrast, DEA10 is the poorest technique because it obtained larger values of the best and an average function results than other methods on almost test functions. In terms of calculation time comparison, DEA1 requires the smallest an average computational time on all test function except for f_6, whereas CGA requires the largest computational time on all test case.

4.7 Conclusions

In this chapter, a novel DEA method and CGA procedure have been tested on seven numerical benchmark functions and a detailed comparative study is presented. The achieved results on all test functions illustrate that some DEA strategies, DEA1, DEA3, DE6, perform effectively to solve these selected benchmark problems. The advantages of DEA method are as follows: simple, robust, fast convergence and capable of finding the optimum in almost every run. In addition, it requires few control parameters to set and the same parameter settings can be applied to many different problems. In this study, DEA1 and DEA3 outperform CGA procedure on the majority of the numerical benchmark test functions. As indicated by the numerical test results, these proposed DEA strategies can obtain the best solution with lower computational fitness value than CGA procedure on all test functions. The most attractive feature of the proposed method is the good computational performance that is faster than CGA for all test functions investigated in this experiment. As a consequence of these successful results, the DEA method will be implemented to solve the transmission expansion planning problem as the next chapter of this thesis.

CHAPTER 5
APPLICATION OF DIFFERENTIAL EVOLUTION ALGORITHM TO STATIC TRANSMISSION EXPANSION PLANNING

5.1 Introduction

In chapter 4, DEA method has been applied to some selected numerical benchmark test functions whereas the results show clearly that this algorithm can successfully solve mathematical optimisation problem. However, in the real world, optimisation for engineering problems is significantly different from those mathematical functions. Most real world optimisation problems are considered not only optimal solution but also satisfying the problem constraints. Therefore, this chapter aims at applying DEA method to transmission expansion planning (TEP) problem, which is one of the complex optimisation problems in engineering.

As previously discussed in chapter 2, TEP can generally be classified into the static or dynamic planning depending upon how period of study is considered. However, the analysis of this chapter covers only the static TEP problem that is investigated in two different scenarios, with and without generation resizing. Then the dynamic TEP problem is studied and reported in next chapter. To solve the static TEP problem, ten variant DEA schemes and a conventional genetic algorithm (CGA) procedure have been adopted for searching optimal solution. The main differences between DEA and CGA procedures are discussed in chapter 3.

The organisation of this chapter is as follows: Section 5.2 presents the formulation of the static TEP problem. Section 5.3 states the implementation of DEA method for solving the static TEP problem that consists of cases, with and without generation resizing consideration. Section 5.4 shows significant data of three selected electrical transmission systems to be tested as static TEP problem. Meanwhile, the experimental results of these test systems are also presented in the same section. Subsequently, these results are discussed and further analysed in section 5.5. Finally, section 5.6 provides summary of this chapter.

5.2 Primal Static Transmission Expansion Planning – Problem Formulation

Generally, the objective of fitness function is to find optimal solution, measure performance of candidate solutions and check for violation of the planning problem constraints. Fitness function of the static TEP problem is basically a combination between objective function and penalty functions. In this chapter, the objective function of the static TEP problem is referred to as formulated in equation (2.4). The purpose of applying penalty functions to the fitness function is to represent violations of equality and inequality constraints. In this static TEP problem, there is only one equality constraint, which is node balance of DC power flow. In contrast, there are several inequality constraints to be considered, namely power flow limit on transmission lines constraint, power generation limit, right of way constraint and bus voltage phase angle limit. The general fitness function of the static TEP problem can be formulated as follows:

$$F_s(X) = \frac{1}{O_s(X) + \omega_1 P_1(X) + \omega_2 P_2(X)} \tag{5.1}$$

$F_s(X)$ and $O_s(X)$ are fitness and objective functions of the static TEP problem, respectively. $P_1(X)$ and $P_2(X)$ are equality and inequality constraint penalty functions respectively. X denotes individual vector of decision variables. $\omega_1$and ω_2 are penalty weighting factors, which are set to “0.5” in this research.

For the static TEP problem, the objective and penalty functions are as follows:

$$O_s(X) = V(X) = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} \tag{5.2}$$

$$P_1(X) = \sum_{k=1}^{nb} \left| d_k + B_k \theta_k - g_k \right| \tag{5.3}$$

$$P_2(X) = \sum_{l=1}^{nc} \mu_l \tag{5.4}$$

where

$$\mu_1 = \begin{cases} 0 & \text{if} \quad \left| f_{ij} \right| \le (n_{ij}^0 + n_{ij}) f_{ij}^{\max} \\ c & \text{otherwise} \end{cases} \tag{5.5}$$

$$\mu_2 = \begin{cases} 0 & \text{if} \quad g_i^{\min} \le g_i \le g_i^{\max} \\ c & \text{otherwise} \end{cases} \tag{5.6}$$

$$\mu_3 = \begin{cases} 0 & \text{if} \quad 0 \le n_{ij} \le n_{ij}^{\max} \\ c & \text{otherwise} \end{cases} \tag{5.7}$$

$$\mu_4 = \begin{cases} 0 & \text{if} \quad \left| \theta_{ij}^{\text{cal}} \right| \le \left| \theta_{ij}^{\max} \right| \\ c & \text{otherwise} \end{cases} \tag{5.8}$$

μ_l is the penalty coefficient of the l^{th} inequality constraint. c is an inequality constraint constant that is used if an individual violates that inequality constraint. In this research, c is set to “0.5” for applying in equations (5.5-5.8). *nb* and *nc* represent the number of buses in transmission system and the number of considered inequality constraints, respectively. The variables as used in the research presented in this chapter and defined in equations (5.5 and 5.7-5.8) are valid for all *ij* and the variables in equation (5.6) are valid for all *i*. Full details of these variables as used for solving static TEP problem are described in section 2.4.

5.3 Implementation of DEA for Static Transmission Expansion Planning Problem

Given the advantages of DEA performance and basic optimisation process as discussed in chapter 3, the DEA method can be adapted and applied to optimise static TEP problem. The objective of DEA is to find an individual X_i that optimises the fitness function. The DEA optimisation process comprises 4 main steps that are initialisation, mutation, crossover and selection. These optimisation operations are presented as follows:

5.3.1 Initialisation Step

The first step of DEA optimisation process is that an initial population is created based on equation (3.3). In the static TEP problem formulation, each individual vector X_i in (5.9) contains many integer-valued parameters n (5.10), where $n_{j,i}$ represents the number of candidate lines in the possible branch j of the individual i. The problem decision parameter D in (5.10) is the number of possible branches for expansion.

$$P^{(G)} = [X_1^{(G)},...,X_i^{(G)},...,X_{N_P}^{(G)}] \quad (5.9)$$

$$X_i^{(G)} = [n_{1,i}^{(G)},...,n_{j,i}^{(G)},...,n_{D,i}^{(G)}], \quad i = 1,...,N_p \quad (5.10)$$

5.3.2 Optimisation Step

New individuals are then created by applying mutation (3.4), crossover (3.5) and selection (3.6) operators. Ten variations of the DEA schemes in (3.7)-(3.16) are applied directly to mutation process. To search for final solution, optimisation step is repeated until the maximum number of generations ($G^{\max}$) is reached or predetermined convergence criterion (ε) is satisfied. In this optimisation process, the convergence criterion compares two differences of the candidate solution population. The first one is the difference between fitness function values of the best member and other members in the same iteration. The second one is the difference between fitness function values of the best solution in present iteration and previous iteration.

5.3.3 Control Parameter Settings

For DEA, a suitable selection of control parameters is very significant for algorithm performance and success to reach optimal solution. As the optimal control parameters of DEA are problem-specific [53], the control parameters should be carefully selected for each optimisation problem. Storn and Price [49] remarked how to choose the proper control variables N_P, F and CR for real-world optimisation problems that a reasonable choice for N_P setting is between 5*D and 10*D but N_P must not be less than 4*D to guarantee that DEA will substantially have mutual different vectors to work. In addition, they recommended that a good initial setting of control variable F is "0.5" and if the population converges prematurely, then F and/or

N_P should be increased by the user's discretion. A good initial setting of *CR* is "1" or "0.9", whereas a large *CR* returns faster convergence if it occurs. However, if convergence has not been reached, then the user should decrease value of *CR* to make DEA robust enough for a particular problem. In this research, control parameters are adjusted through extensive tests until the best settings have been found. The suitable DEA parameter settings for the static TEP problem are as follows: F = [0.5,0.9], CR = [0.55,0.95] and N_P = [4*D,10*D]. The maximum predetermined convergence criterion (ε) is set to "1×10^{-3}" and the maximum number of generations (G^{max}) is set to "1×10^{3}" or "1×10^{4}" depending on test system size.

5.3.4 DEA Optimisation Program for Static TEP problem - Overall Procedure

The major steps of DEA optimisation program for solving static TEP problem can be summarised as follows:

Step 1: Read all required transmission system data from database for the static TEP calculation, including;

- The data of actual power generation, load demand and transmission line system (for the case without power generation resizing consideration) ;
- The data of minimum and maximum sizes of power generation, load demand and transmission line system (for the case with power generation resizing consideration);

Step 2: Set up all required parameters of DEA optimisation process by the user;

- These control parameters are population size (N_P), scaling mutaion factor (F), crossover probability (CR), convergence criterion (ε), number of problem variables (D), lower and upper bounds of initial population (x_j^{min} and x_j^{max}) and maximum number of iterations or generations (G^{max});
- Select a DEA mutation operator strategy;

Step 3: The user selects a type of static TEP problem, which is either the case with or without power generation resizing consideration;

- If the user selects a case of without power generation resizing consideration, DEA programme will use the given actual power generation values from step 1 for DC power flow calculation;

- If the user selects a case of with power generation resizing consideration, DEA programme will random and attempt to search the proper power generation value, which must be within the given bound from step1, for each generation unit in the network;

Step 4: Set iteration $G = 0$ for initialisation step of DEA optimisation process;

Step 5: Initialise population P of individuals according to equation (3.3);

Step 6: Calculate and evaluate fitness values of initial individuals according to the problem fitness function (5.1) and check constraints for each initial individual by using DC model static TEP method;

Step 7: Rank the initial individuals according to their fitness;

Step 8: Set iteration $G = 1$ for optimisation step of DEA optimisation process;

Step 9: Apply mutation, crossover and selection operators to generate new individuals;

- Apply mutation operator to generate mutant vectors ($V_i^{(G)}$) according to equation (3.4) with a selected DEA mutation operator strategy in step 2;
- Apply crossover operator to generate trial vectors ($U_i^{(G)}$) according to equation (3.5);
- Apply selection operator according to equation (3.6) by comparing the fitness of trial vector ($U_i^{(G)}$) and the corresponding target vector ($X_i^{(G)}$) and then select one that provides the best solution;

Step 10: Calculate and evaluate the fitness values of new individuals according to the problem fitness function (5.1) and check constraints for each new individual by using DC model of static TEP method;

Step 11: Rank new individuals according to their fitness;

Step 12: Update the best fitness value of the current iteration (*gbest*) and the best fitness value of the previous iteration (*pbest*)

Step 13: Check the termination criteria;

- If $|X_i^{best} - X_i| > \varepsilon$ or $|pbest - gbest| > \varepsilon$ when the number of current generation is not over the maximum number of generations $G < G^{\max}$, set $G = G + 1$ and return to step 9 for repeating to search the solution. Otherwise, stop to calculate and go to step 14;

Step 14: Calculate and print out the final solution that is the best investment cost of static TEP problem and the number of new transmission lines added to each candidate right-of-way;

Step 15: Run and display the DC load flow of the obtained final result.

5.4 Test Systems and Numerical Test Results

The proposed DEA method has been implemented in Matlab7 and tested on three electrical transmission networks, which are as reported in [11, 14, 66]. In addition, the results of these methods are also compared with those of CGA procedure. In these analyses, static TEP procedure is tested on the following three test systems; 6-bus system originally proposed by Garver [10], IEEE 25-bus system and Brazilian 46-bus system. The static TEP problem has been investigated in two cases that are with and without power generation resizing consideration. In case of with generation resizing consideration, the generated MW power at each generator varies between $g_i^{\min}$ and $g_i^{\max}$, of which the details have been explained in section 2.4. In this experiment, the values of $g_i^{\min}$ are set to “0” MW for all generating units in three test systems. Meanwhile, setting data of $g_i^{\max}$ are referred to as in [8] for 6-bus system, [11] for IEEE 25-bus system and [14] for Brazilian 46-bus system. Ten different DEA strategies, as described in chapter 3, have been employed to test the static TEP procedure. In this research, the initial control parameters of DEA procedure are set for approaching the static TEP problem for each test system as following details in section 5.3.3. The proper DEA control parameters of the best solution are then found through a great number of tests for each system.

5.4.1 Garver 6-Bus System

The first test system adopted in this research is the well-known Garver’s system as shown in figure A1. Generally, it comprises of 6 buses, 9 possible branches and 760 MW of demand. The data of this electrical system, which includes transmission line, load and generation data with resizing range in MW, are available in [8, 66]. In this test system, bus 6 is a new generation bus that needs to be connected to the existing network. The dotted lines represent new possible line additions and solid lines are the existing lines. In this research, static TEP problem is analysed in both cases, with and without power generation resizing. The maximum number of permitted parallel lines is four for each branch. The simulation results of Garver's case are presented in tables 5.1 and 5.2, whereas the proposed method was run 100 times in order to determine appropriate values for DEA parameters. In order to obtain the best investment costs using DEA in tables 5.1 and 5.2, the DEA parameter settings had been initiated as described in section 5.3.3. The proper DEA parameter settings were then achieved as following values: F = 0.7, CR = 0.6, D = 9 and N_p = 5*D = 45 respectively.

5.4.1.1 Without Generation Resizing - Garver's System

The achieved results of Graver's system in case of without power generation resizing consideration are presented in table 5.1 and figure 5.1. These results are then discussed as follows:

- In this case, the optimal solution of the static TEP problem was found by all of DEA strategies and CGA procedure. The investment cost of this optimal solution equals to v = US$ 200,000 with the following topology: $n_{2\text{-}6}$ = 4, $n_{3\text{-}5}$ = 1 and $n_{4\text{-}6}$ = 2.
- The convergence curves of DEA1 and CGA to obtain the optimal solution are illustrated in figure 5.1, whereas the optimal solution was found by DEA1 at the 6th iteration and was found by CGA at the 14th iteration.
- The optimal solution of this planning case was previously found in [44, 67]. The configuration of this optimal expansion plan was illustrated in [44], whereas a specialised genetic algorithm was applied to solve the static TEP problem.
- Given the results presented in table 5.1, even though all DEA strategies and CGA found the optimal solution, the performance of DEA1 is more robust than other strategies as shown by the smallest values of an average investment cost and a standard deviation.
- DEA1 requires less an average CPU time for calculation than other strategies in this test case.
- Overall, the best algorithmic procedure for this case is DEA1 based on all above mentioned reasons.

Table 5.1 Summary results of Garver 6-bus system without generation resizing case

Results of static TEP (without power gen resizing)	DEA1	DEA6	DEA8	DEA9	DEA3	CGA	DEA4	DEA7	DEA10	DEA2	DEA5
	Methods										
Best, 10^3 US$	200	200	200	200	200	200	200	200	200	200	200
Average, 10^3 US$	210.58	218.04	219.11	220.62	222.52	224.76	226.82	256.82	257.44	262.79	271.06
Worst, 10^3 US$	271	292	302	292	292	300	302	360	352	322	341
Diff. between best and worst, %	35.5	46	51	46	46	50	51	80	76	61	70.5
Standard deviation, 10^3 US$	19.10	25.82	26.99	25.80	27.70	26.74	26.58	37.99	36.28	27.81	35.69
Average CPU time, second	2.54	2.56	2.56	2.57	2.56	6.29	2.56	2.58	2.58	2.57	2.57
Line additions for the best result	$n_{2\text{-}6} = 4$, $n_{3\text{-}5} = 1$ and $n_{4\text{-}6} = 2$										

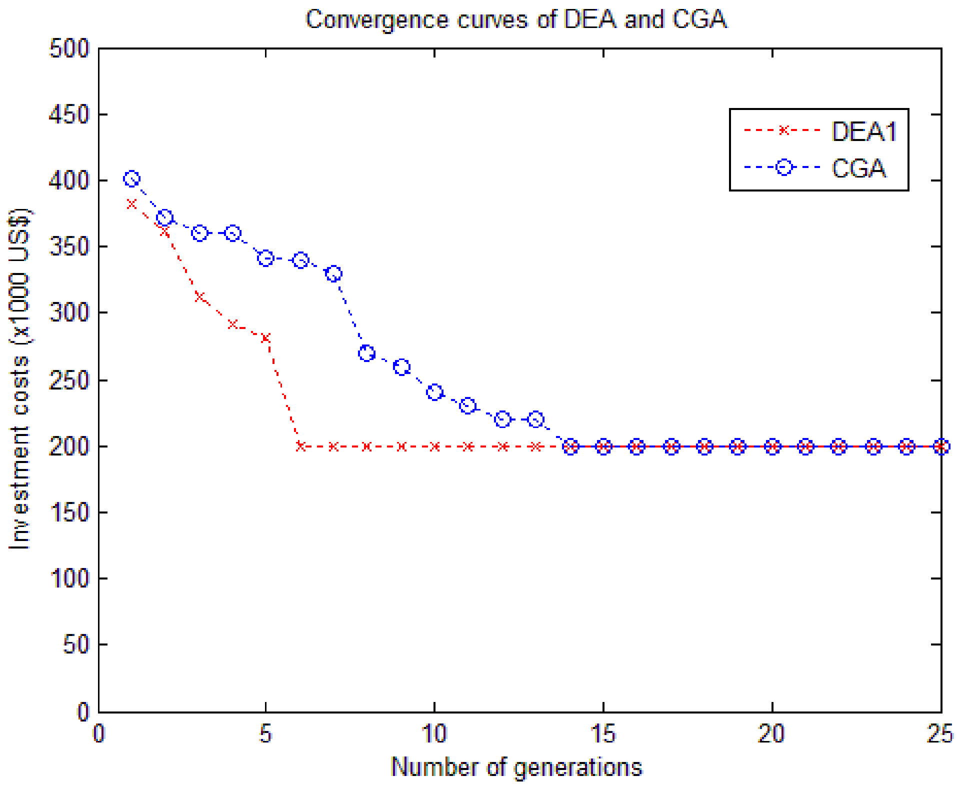


Figure 5.1 Convergence curves of DEA1and CGA for Garver 6-bus system without generation resizing case

5.4.1.2 With Generation Resizing - Garver's System

In case that power generation resizing is considered, the test results of Graver's system can be shown in table 5.2 and figure 5.2. The discussion on these results is as follows:

- The optimal solution of static TEP problem with generation resizing was found by all DEA strategies and CGA procedure whereas an investment cost equals to v = US$ 110,000 at the following topology: $n_{3\text{-}5}$ = 1 and $n_{4\text{-}6}$ = 3.
- The convergence curves of DEA3 and CGA to obtain the optimal solution are illustrated in figure 5.2. In this case, the optimal solution was found by DEA3 at the 10th iteration and by CGA at the 21th iteration.
- The optimal solution was previously found in [44, 67], whereas the configuration of this optimal expansion plan was illustrated in [44].
- According to results in table 5.2, the performance of DEA3 is very robust to find the solution, as suggested by the least values of a standard deviation and an average investment cost.
- In addition, DEA3 requires less an average computational time than any other strategies.
- Overall, the best algorithmic procedure for this case is DEA3 based on all above mentioned reasons.

Table 5.2 Summary results of Garver 6-bus system with generation resizing case

Results of static TEP	Methods										
(with power gen resizing)	DEA3	DEA1	DEA6	DEA8	CGA	DEA9	DEA4	DEA7	DEA2	DEA10	DEA5
Best, 10^3 US$	110	110	110	110	110	110	110	110	110	110	110
Average, 10^3 US$	112.60	113.40	118.50	120.40	122.20	123.50	124	127.60	140	149.53	151.60
Worst, 10^3 US$	150	160	180	180	190	190	190	190	190	202	210
Diff. between best and worst, %	36.36	45.45	63.64	63.64	72.73	72.73	72.73	72.73	72.73	83.64	90.91
Standard deviation, 10^3 US$	8.95	10.56	15.20	19.22	21.06	20.07	24.33	24.33	21.56	28.35	24.97
Average CPU time, second	4.92	4.93	4.93	4.95	8.94	4.96	4.96	4.96	4.98	4.97	4.98
Line additions for best result	$n_{3\text{-}5}$ = 1 and $n_{4\text{-}6}$ = 3										

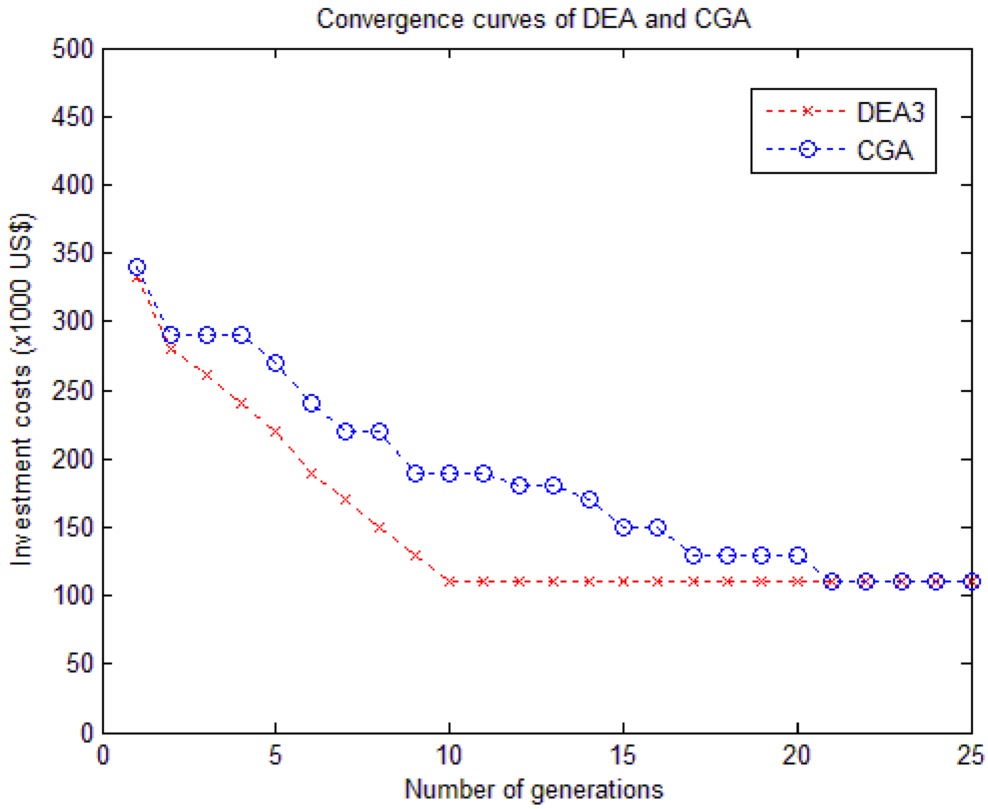


Figure 5.2 Convergence curves of DEA3and CGA for Garver 6-bus system with generation resizing case

5.4.2 IEEE 25-Bus System

The second test system is the IEEE 25-bus system as illustrated in figure A2. It has 25 buses, 36 possible branches and 2750 MW of total demand. These electrical system data consist of transmission line data, load data and generation data that includes generation resizing range in MW. These data are available in [11]. The new bus is bus 25, connected between buses 5 and 24. Two cases of the static TEP problem have been analysed for this system, with and without generation resizing. The maximum number of permitted parallel lines is four for each branch. The simulation results of this case are presented in tables 5.3 and 5.4 whereas the proposed method was again run 100 times in order to determine appropriate values for the DEA parameters. To obtain the best investment costs using the DEA method in tables 5.3 and 5.4, the DEA parameter settings had been initiated as described in section 5.3.3, and then the proper DEA parameter settings were achieved as follows:

F = 0.7, CR = 0.6, D = 36 and N_p = 5*D = 180 respectively.

5.4.2.1 Without Generation Resizing - IEEE 25-Bus System

Without generation resizing consideration, the results of testing all proposed algorithms to IEEE 25-bus system can be shown in table 5.3 and figure 5.3. These results can be discussed as follows:

- In this case, the best solution of the static TEP problem without generation resizing consideration, as shown in table 5.3, was found by DEA1, DEA3 and DEA6 whereas an investment cost is v = US$ 114.383 million, with the addition of the following lines to the base topology: $n_{1\text{-}2}$ = 3, $n_{5\text{-}25}$ = 1, $n_{7\text{-}13}$ = 1, $n_{8\text{-}22}$ = 3, $n_{12\text{-}14}$ = 2, $n_{12\text{-}23}$ = 3, $n_{13\text{-}18}$ = 3, $n_{13\text{-}20}$ = 3, $n_{17\text{-}19}$ = 1 and $n_{24\text{-}25}$ = 1.
- The convergence curve of DEA3 to obtain the best solution is illustrated in figure 5.3, whereas the best solution was found at the 23rd iteration. On the other hand, the investment cost v = US$ 114.526 million was found by CGA at 38th iteration as shown in figure 5.3.
- In [25], the best result of the static TEP problem was found by hybrid methods of ANN, GA and TS, of which an investment cost was v = US$ 143.56 million. However, in this case the best optimal result achieved by DEA1, DEA 3 and DEA6 is less than that of those hybrid methods.
- As results indicated in table 5.3, although DEA1, DEA3 and DEA6 found the best solution in this case, DEA3 showed its best performance in robustness to search the solution, as indicated by the least value of an average investment cost and much less value of a standard deviation comparing with other DEA strategies.
- DEA3 is the fastest strategy to approach the solution because it provides the best convergence rate compared with all other strategies. Moreover, on average it requires less computational time than other techniques.
- Overall, the best algorithmic procedure for this case is DEA3 based on all above mentioned reasons.
- DEA7 shows the poorest performance for finding the problem solution and the least robust performance compared with other methods because it has the greatest values of the best, average, and worst results and a standard deviation.

Table 5.3 Summary results of IEEE 25-bus system without generation resizing case

Results of static TEP	Methods										
(without power gen resizing)	DEA3	DEA1	DEA6	DEA8	CGA	DEA2	DEA4	DEA9	DEA5	DEA10	DEA7
Best, 10^3 US$	114383	114383	114383	114526	114526	114526	114526	114526	114526	115201	116551
Average, 10^3 US$	114426	115356	115438	115480	115554	118770	120122	120246	121233	124178	126587
Worst, 10^3 US$	114526	118562	119237	120688	131503	131954	133304	135965	138640	143040	157930
Diff. between best and worst, %	0.13	3.65	4.24	5.38	14.82	15.22	16.40	18.72	21.06	24.17	35.50
Standard deviation, 10^3 US$	69.08	1379.43	1548.61	1940.41	3406.35	5153.05	6603.58	7273.02	7730.18	9348.26	13286
Average CPU time, second	26.57	26.59	26.58	26.59	52.37	26.61	26.65	26.66	26.66	26.68	26.68
Line additions for best result	$n_{1\text{-}2} = 3$, $n_{5\text{-}25} = 1$, $n_{7\text{-}13} = 1$, $n_{8\text{-}22} = 3$, $n_{12\text{-}14} = 2$, $n_{12\text{-}23} = 3$, $n_{13\text{-}18} = 3$, $n_{13\text{-}20} = 3$, $n_{17\text{-}19} = 1$ and $n_{24\text{-}25} = 1$										

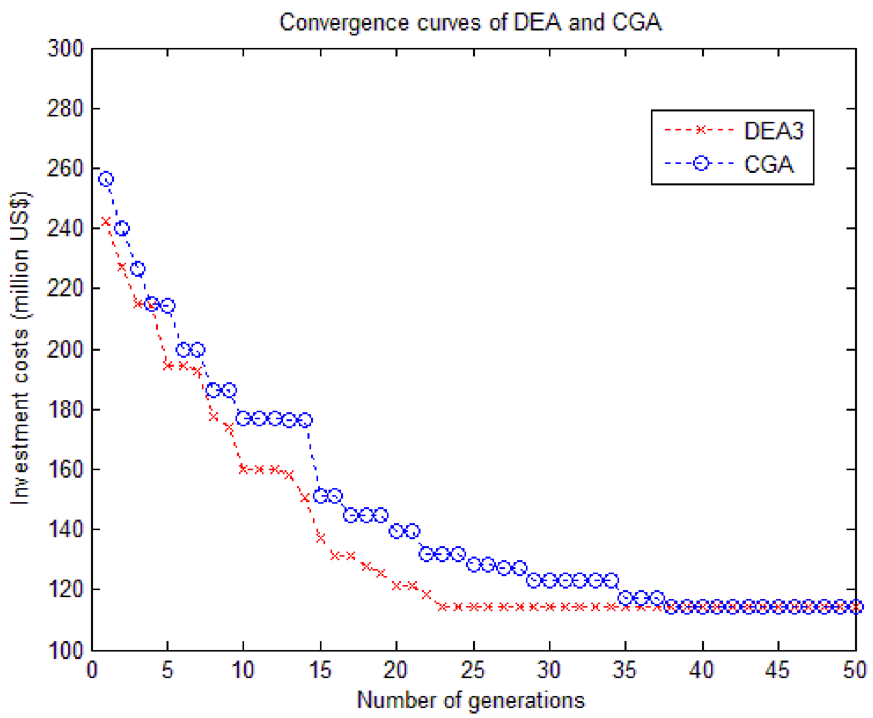


Figure 5.3 Convergence curves of DEA3 and CGA for IEEE 25-bus system without generation resizing case

5.4.2.2 With Generation Resizing - IEEE 25-Bus System

The obtained results of IEEE 25-bus system in case of with power generation resizing consideration can be shown in table 5.4 and figure 5.4 including the result discussion as follows:

- The necessary investment to solve the static TEP problem with generation resizing consideration for this test system is v = US$ 41.803 million and the following lines are added: $n_{1\text{-}2} = 2$, $n_{2\text{-}3} = 1$, $n_{5\text{-}20} = 1$, $n_{5\text{-}25} = 3$, $n_{8\text{-}22} = 1$, $n_{12\text{-}23} = 1$, $n_{13\text{-}18} = 3$ and $n_{16\text{-}20} = 2$.

- The convergence curve of DEA3 to obtain the best solution is illustrated in figure 5.4, where the best solution was found at the 34th iteration. On the other hand, the investment cost v = US$ 42.478 million was found by CGA at 47th iteration as shown in figure 5.4.
- In this case, no data of the optimal solution has been found in previous researches, especially in [25] where static TEP problem was investigated in case of without generation resizing consideration only.
- As the results presented in table 5.4, DEA1, DEA3 and DEA6 found the best solution for this case but only DEA3 showed more robust performance than other strategies for searching the solution, given its least value of average investment cost and standard deviation.
- However, DEA1 and DEA6 are proved that they are good enough to find the best solution as same as DEA3 in this case, as shown by the least values of the best investment cost, the worst investment cost and different between the best and the worst cost.
- Overall, DEA3 is the best algorithmic procedure for this case based on all previous mentioned reasons.
- Similar to a case of IEEE 25-bus system without power generation resizing, DEA7 still shows the poorest performance for finding the problem solution and the least robust performance compared with other methods because it gives the greatest values of the best result, an average result, the worst result and a standard deviation.
- In this case, DEA3 is faster than other methods for calculation while CGA is the slowest one.

Table 5.4 Summary results of IEEE 25-bus system with generation resizing case

Results of static TEP	Methods										
(with power gen resizing)	DEA3	DEA1	DEA6	DEA8	CGA	DEA2	DEA4	DEA9	DEA5	DEA10	DEA7
Best, 10^3 US$	41803	41803	41803	42478	42478	42478	44477	44477	44477	44477	45827
Average, 10^3 US$	42675	43545	43748	44771	45542	46441	51204	51321	52178	53605	55350
Worst, 10^3 US$	45827	45827	45827	52716	53538	53127	54858	54858	56499	56499	59199
Diff. between best and worst, %	9.63	9.63	9.63	24.10	26.04	25.07	23.34	23.34	27.03	27.03	29.18
Standard deviation, 10^3 US$	1378.43	1646.60	1713.81	3102.76	4169.18	3670.17	4383.19	4196.05	4734.52	4580.62	4023.80
Average CPU time, second	50.01	50.08	50.03	50.09	80.33	50.11	50.13	50.15	50.11	50.15	50.17
Line additions for best result	$n_{1\text{-}2} = 2$, $n_{2\text{-}3} = 1$, $n_{5\text{-}20} = 1$, $n_{5\text{-}25} = 3$, $n_{8\text{-}22} = 1$, $n_{12\text{-}23} = 1$, $n_{13\text{-}18} = 3$ and $n_{16\text{-}20} = 2$										

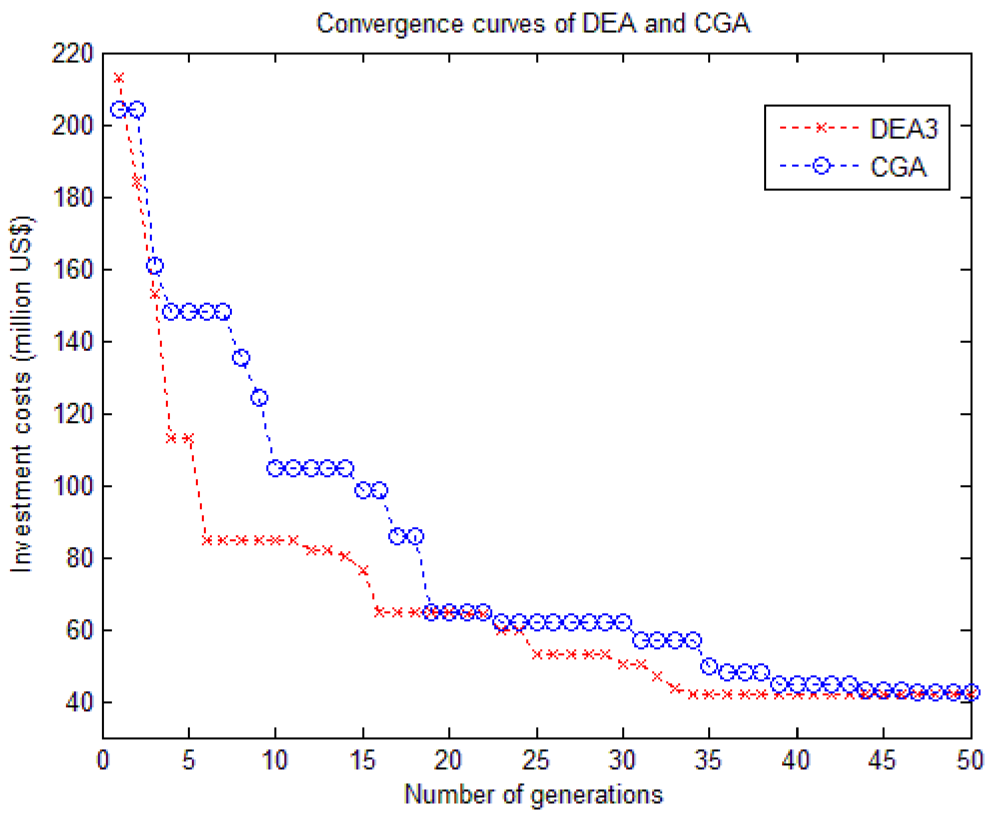


Figure 5.4 Convergence curves of DEA3 and CGA for IEEE 25-bus system with generation resizing case

5.4.3 Brazilian 46-Bus System

The third test system is the Brazilian 46-bus system as depicted in figure A3. The system comprises 46 buses, 79 circuits, 6880 MW of total demand. The electrical system data, which consist of transmission line, load and generation data including generation resizing range in MW, are available in [14]. This system represents a good test to the proposed approach because it is a real-world transmission system. In figure A3, the solid lines represent existing circuits in the base case topology and the dotted lines represent the possible addition of new transmission lines. The addition of parallel transmission lines to existing lines is again allowed in this case with a limit of 4 lines for each branch. The simulation results of this case are shown in tables 5.5 and 5.6, whereas the proposed method was run 100 times in order to determine appropriate values for the DEA parameters. The DEA parameter settings are as follows: F = 0.7, CR = 0.55, D = 79 and N_p = 5*D = 395 respectively.

5.4.3.1 Without Generation Resizing - Brazilian 46 Bus System

The obtained results of Brazilian 46-bus system without power generation resizing consideration can be shown in table 5.5 and figure 5.5. The discussion on this simulation result is as follows:

- In this case, the optimal solution was found by branch and bound algorithm in [67] where an investment cost of this expansion equals to v = US$ 154.42 million. In this experiment, the optimum was also found by only DEA3 at the following topology: $n_{5\text{-}6}$ = 2, $n_{6\text{-}46}$ = 1, $n_{19\text{-}25}$ = 1, $n_{20\text{-}21}$ = 1, $n_{24\text{-}25}$ = 2, $n_{26\text{-}29}$ = 3, $n_{28\text{-}30}$ = 1, $n_{29\text{-}30}$ = 2, $n_{31\text{-}32}$ = 1 and $n_{42\text{-}43}$ = 2.
- The convergence curve of DEA3 to reach the optimal solution is illustrated in figure 5.5, whereas the optimum solution was found at the 110th iteration. On the other hand, the investment cost v = US$ 162.598 million was found by CGA procedure at 155th iteration as shown in figure 5.5.
- In this case, DEA3 shows the best performance in ability and robustness for searching the solution, as shown by the least figures of the best and average investment costs and a standard deviation value. The robust feature is a significant performance for algorithm to show that it can find a reliable result in a single run.
- DEA3 requires less average computational time than any other strategies. In contrast, CGA requires the largest computational CPU time in this test case.

Table 5.5 Summary results of Brazilian 46-bus system without generation resizing case

Results of static TEP (without power gen resizing)	Methods										
	DEA3	DEA1	DEA6	DEA8	CGA	DEA2	DEA4	DEA9	DEA5	DEA10	DEA7
Best, 10^3 US$	154420	158314	158314	162598	162598	162598	166492	170532	170532	173674	173674
Average, 10^3 US$	156017	160458	163970	173220	173434	177905	184265	185080	188518	189291	192359
Worst, 10^3 US$	162598	166492	173674	185915	196183	196319	199535	200213	209229	212613	227123
Diff. between best and worst, %	5.30	5.17	9.70	14.34	20.66	20.74	19.85	17.40	22.69	22.42	30.78
Standard deviation, 10^3 US$	2822.82	2972.65	6418.39	9949.75	13339.41	14588.77	13433.63	14741.47	16482.30	17956.46	21583.62
Average CPU time, second	489	492	491	493	897	494	495	497	498	497	502
Line additions for best result	$n_{5\text{-}6} = 2$, $n_{6\text{-}46} = 1$, $n_{19\text{-}25} = 1$, $n_{20\text{-}21} = 1$, $n_{24\text{-}25} = 2$, $n_{26\text{-}29} = 3$, $n_{28\text{-}30} = 1$, $n_{29\text{-}30} = 2$, $n_{31\text{-}32} = 1$ and $n_{42\text{-}43} = 2$										

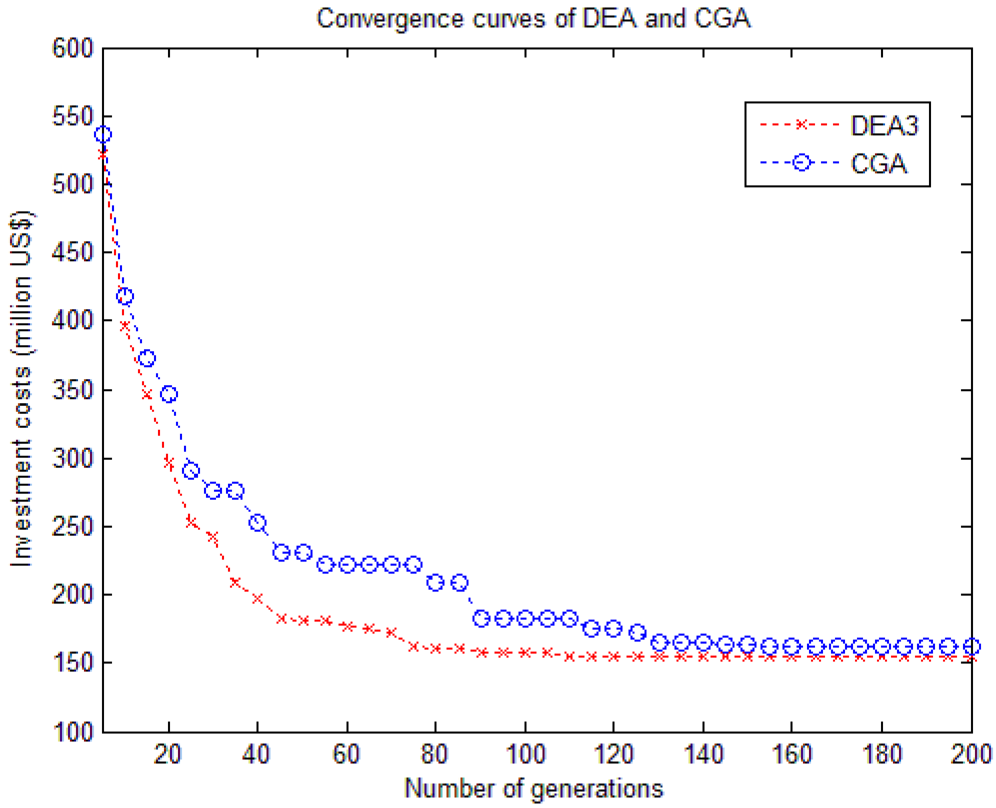


Figure 5.5 Convergence curves of DEA3 and CGA for Brazilian 46-bus system without generation resizing case

5.4.3.2 With Generation Resizing - Brazilian 46-Bus System

The achieved results of Brazilian 46-bus system with power generation resizing consideration can be shown in table 5.6 and figure 5.6. The discussion on these results is as follows:

- In this case, branch and bound algorithm found the optimum solution in [67] and an expansion investment cost is v = US$ 72.87 million, with the topology: $n_{2-5} = 1$, $n_{5-6} = 2$, $n_{13-20} = 1$, $n_{20-21} = 2$, $n_{20-23} = 1$, $n_{42-43} = 1$ and $n_{6-46} = 1$.
- In this experiment, all DEA strategies and CGA cannot successfully find the optimal solution, previously obtained by branch and bound algorithm.
- DEA3 could find only the best solution compared with other DEA strategies and CGA procedure, which an investment cost is v = US$ 74.733 million, with the added lines topology: $n_{5-6} = 2$, $n_{6-46} = 1$, $n_{13-18} = 1$, $n_{20-21} = 2$, $n_{20-23} = 1$ and $n_{42-43} = 1$.
- The convergence curve of DEA3 to obtain the best solution is illustrated in figure 5.6, whereas the best solution was found at the 145th iteration. On the other hand, the investment cost v = US$ 89.179 million was found by CGA at 230th iteration as shown in figure 5.6.
- All DEA strategies and CGA procedure are not good enough to find the optimal solution in this case. The premature convergence may be a cause for the failure of finding the global optimum because there are many local optimums in this problem case.
- However, DEA3 remains superior to other algorithms for finding the solution in this case as it generates the least values of the best, average, and the worst results. In addition, it converges faster than all other strategies in this test case.
- In contrast, DEA7 is inferior to other methods for its performance of finding the fitness optimal solution in this test case because it has the highest values of the best, average, the worst and a standard deviation results.
- DEA3 requires less average computational time than all other strategies while CGA procedure still requires the longest computational time in this test case.

Table 5.6 Summary results of Brazilian 46-bus system with generation resizing case

Results of static TEP	Methods										
(with power gen resizing)	DEA3	DEA1	DEA6	DEA8	CGA	DEA2	DEA4	DEA9	DEA5	DEA10	DEA7
Best, 10^3 US$	74733	82911	82911	89179	89179	89179	97357	97357	97357	98745	98745
Average, 10^3 US$	75551	84695	86576	93268	94363	94502	100632	101770	102631	104747	105587
Worst, 10^3 US$	82911	94487	94487	97357	98745	98745	107349	107349	107349	115747	115747
Diff. between best and worst, %	10.94	13.96	13.96	9.17	10.73	10.73	10.26	10.26	10.26	17.22	17.22
Standard deviation, 10^3 US$	2586.11	3964.44	4174.63	4310.18	4493.89	4616.39	4667.90	4834.79	4999.39	5763.37	6717.49
Average CPU time, second	868	870	872	873	1422	876	875	878	880	879	883
Line additions for best result	$n_{5\text{-}6} = 2$, $n_{6\text{-}46} = 1$, $n_{13\text{-}18} = 1$, $n_{20\text{-}21} = 2$, $n_{20\text{-}23} = 1$ and $n_{42\text{-}43} = 1$										

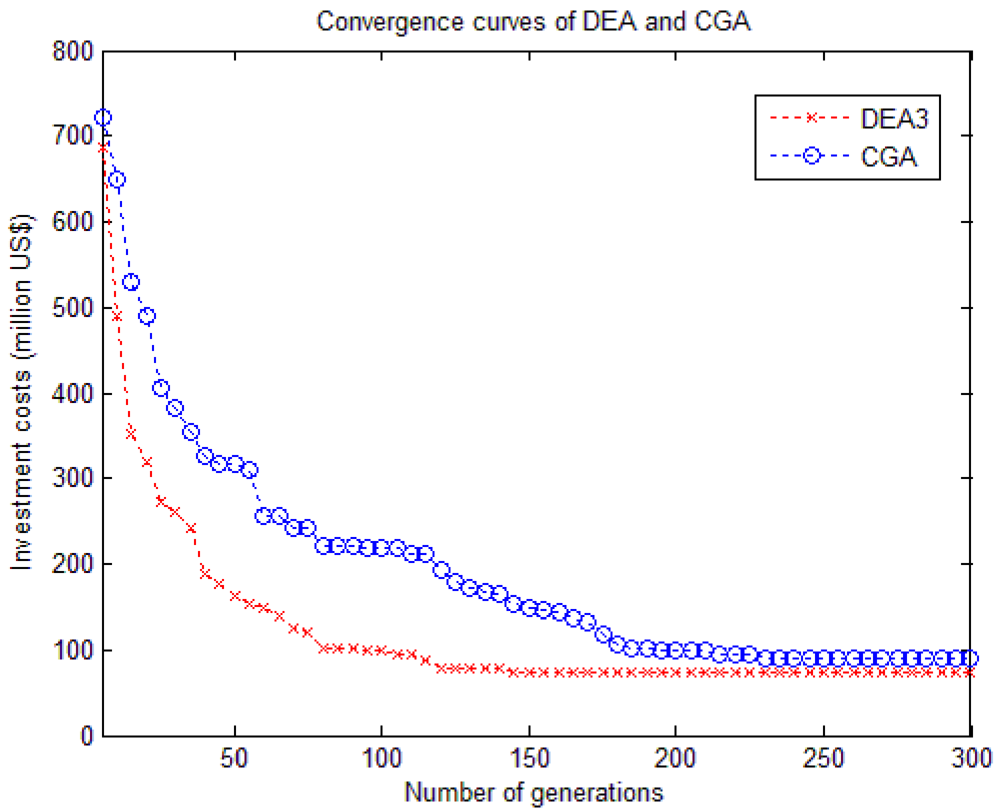


Figure 5.6 Convergence curves of DEA3 and CGA for Brazilian 46-bus system with generation resizing case

The results of static TEP problem in both cases of with and without generation resizing consideration have been summarised in table 5.7 whereas the best investment costs of expansion corresponding to the proposed method are compared to those of other algorithms. As indicated by the results in table 5.7, all methods found the optimal solution in both cases of the static TEP problem on Gaver 6-bus system except the hybrid of ANN, GA and TS algorithms in [25] where the static TEP problem with generation resizing consideration case was not investigated. For IEEE 25-bus system, DEA3 performed the best method to find the best solution in both cases of the static TEP problem, compared to CGA procedure and the hybrid of ANN, GA and TS algorithms. For Brazilian 46-bus system, DEA3 and branch & bound [67] performed the best performance to find the optimal solution in a case without generation resizing consideration of the static TEP problem, as shown by the cheapest investment cost. On the other hand, branch & bound [67] and Chu-Beasley genetic algorithm (CBGA) [68] showed the best performance for finding the optimal solution in case that power generation resizing is considered.

Table 5.7 Results of static transmission expansion planning problem

Methods	Best cost (10^3 US$)					
	Garver 6-bus system		IEEE 25-bus system		Brazilian 46-bus system	
	without power gen resizing	with power gen resizing	without power gen resizing	with power gen resizing	without power gen resizing	with power gen resizing
DEA3	200	110	114383	41803	154420	74733
CGA	200	110	114526	42478	162598	89179
Hybrid of ANN, GA and TS [25]	200	-	143560	-	-	-
Branch & Bound [67]	200	110	-	-	154420	72870
Chu-Beasley GA (CBGA) [68]	200	110	-	-	-	72870

Table 5.8 Computational effort of static transmission expansion planning problem

Methods	Case of results	Garver 6-bus system		IEEE 25-bus system		Brazilian 46-bus system	
		without power gen resizing	with power gen resizing	without power gen resizing	with power gen resizing	without power gen resizing	with power gen resizing
DEA3	No. of iteration	5-15	8-25	15-40	20-55	80-150	120-250
	Cal. Time (sec)	1.29-3.8	2.74-8.54	18.32-48.76	29.3-79.53	393.65-740.5	720.6-1495.7
CGA	No. of iteration	9-25	15-35	20-50	30-70	120-220	200-400
	Cal. Time (sec)	3.51-9.55	6.46-14.94	32.37-77.93	52.8-121.8	780.4-1378.3	1392.3-2724

The comparisons of computational effort between the proposed method and CGA procedure have been shown in table 5.8. As the obtained results indicated, DEA3 is better performance than CGA procedure to converge the best solution in all cases of the static TEP problem, as shown by its smaller number of iterations for seeking the solution in each case. Furthermore, the proposed method requires less computational time than CGA in each case of the static TEP problem.

5.5 Overall Analysis and Discussion on the Results

The obtained results clearly indicate that DEA method can be efficiently applied to static TEP problem. Several DEA strategies, which are DEA1, DEA3, DEA6 and DEA8, show better overall performance especially in robustness than CGA procedure in the optimisation of the static TEP problem both with and without generation resizing. In addition, all DEA schemes require less computational time than the CGA for all cases. In table 5.8, the computational efficiency of the best performing DEA method, DEA3, is compared directly with the CGA with regard to number of iterations and calculation time. From the test results in table 5.8, DEA3 can find the best solution faster than CGA in all cases. The proposed algorithm and CGA were tested 100 times to find the best results in each case the parameters were set as follows: F = 0.7, CR = 0.55 and N_p = 5*D respectively.

The performance of DEA depends upon the selection of suitable control parameters. In this research, the parameter settings of DEA procedures were manually tuned based upon preliminary experiments. The specific settings for each case are described in section 5.3.3. According to the experiments, the scaling mutation factor F is much more sensitive than crossover probability CR. Therefore, CR is more useful as a fine tuning parameter. In this work, the researcher proposed five novel DEA schemes that are DEA6, DEA7, DEA8, DEA9 and DEA10. Only DEA6 has shown comparable performance with DEA3 when finding the optimal solutions for the two cases of static TEP planning on the Garver 6-bus system and the IEEE 25-bus system. Only DEA3 could find an optimal solution on the Brazilian 46-bus system for the static TEP problem without generation resizing consideration case.

5.6 Conclusions

In this chapter, a novel DEA has been applied to solve static TEP problem in two cases, with and without generation resizing consideration whereas these algorithms have been tested on three selected electrical systems. The results indicate that a few DEA schemes, especially DEA1, DEA3 and DEA6 are efficient to solve the static TEP problem. As the numerical test results indicated, the proposed method can obtain the best investment with lower computational cost than CGA procedure for the static TEP on IEEE 25-bus system and the Brazilian 46-bus system in cases of both with and without generation resizing. The most attractive feature of the proposed algorithm is its good computational performance that is faster than CGA procedure for all the static TEP problems investigated in this chapter. The accuracy of these results is in very good agreement with those obtained by other researchers. As a consequence of these successful results, the dynamic TEP problem will be investigated in the next chapter.

CHAPTER 6

APPLICATION OF DIFFERENTIAL EVOLUTION ALGORITHM TO DYNAMIC TRANSMISSION EXPANSION PLANNING

6.1 Introduction

In chapter 5, a novel differential evolution algorithm (DEA) has been directly applied to DC based power flow model in order to solve the static transmission expansion planning (TEP) problem. The DEA performed well with regard to both low and medium complex transmission systems as demonstrated by Garver six-bus system, IEEE 25-bus system and Brazilian 46-bus system, respectively. As a consequence of the successful results obtained from solving static TEP problem, DEA is then re-implemented to solve dynamic TEP problem with DC power flow model, which is classed as a mixed integer nonlinear optimisation problem. Dynamic TEP problem is more complex and difficult to be solved than the static one as not only the optimal number of new transmission lines and their locations but also the most appropriate times to carry out the investment must be considered (as stated in chapter 2). In this research, the effectiveness of the proposed enhancement is initially demonstrated by the analysis of a highly complex transmission test system, as described in figures A.4. The analysis is performed within the mathematical programming environment of MATLAB using both DEA and CGA procedures and a detailed comparison of accuracy and performance is also presented in this chapter. An outline of this chapter is as follows: Section 6.2 states the problem formulation that describes how to perform the fitness function of the dynamic TEP problem. Section 6.3 describes the implementation of DEA procedure for solving the dynamic TEP problem. In addition, all details of DEA optimisation programme for approaching this planning problem are also included in this section. The data required for this test system is illustrated in section 6.4 and the achieved experimental results are also reported in the similar section. Finally, the discussion and conclusion of test results are given in section 6.5

and 6.6, respectively.

6.2 Primal Dynamic Transmission Expansion Planning - Problem Formulation

Similar to static TEP problem, the fitness function of dynamic TEP problem is a combination of objective and penalty functions but there are few different details of these functions between the static and dynamic TEP problem. The objective function of the dynamic TEP problem as formulated in equation (2.12) is employed to find the minimum investment cost for this planning problem. The fitness function is implemented to find optimal solution, measure performance of candidate solutions and check for violation of the planning problem constraints. In this dynamic TEP problem, there is only one equality constraint, which is node balance of DC power flow. In contrast, there are several inequality constraints to be considered, namely power flow limit on transmission lines constraint, power generation limit, right of way constraint and bus voltage phase angle limit. The general fitness function of the dynamic TEP problem can be formulated as follows:

$$F_D(X) = \frac{1}{O_D(X) + \omega_1 P_1(X) + \omega_2 P_2(X)} \tag{6.1}$$

$F_D(X)$ and $O_D(X)$ are fitness and objective functions of the dynamic TEP problem, respectively. $P_1(X)$ and $P_2(X)$ are equality and inequality constraint penalty functions respectively. X denotes individual vector of decision variables. $\omega_1$and ω_2 are penalty weighting factors, which are set to “0.5” in this research.

For the dynamic TEP problem, the objective function and penalty functions are as follows:

$$O_D(X) = V(X) = \sum_{t=1}^{T}\left[\delta_{inv}^{t} \sum_{(i,j)\in\Omega} c_{ij}^{t} n_{ij}^{t}\right] \tag{6.2}$$

$$P_1(X) = \sum_{\substack{k=1\\t=1}}^{\substack{nb\\T}} \left| d_k^t + B_k^t \theta_k^t - g_k^t \right| \tag{6.3}$$

$$P_2(X) = \sum_{l=1}^{nc} \mu_l \tag{6.4}$$

where

$$\mu_1 = \begin{cases} 0 & \text{if} \quad \left| f_{ij}^t \right| \le (n_{ij}^0 + \sum_{s=1}^{t} n_{ij}^s)\, f_{ij}^{\max} \\ c & \text{otherwise} \end{cases} \tag{6.5}$$

$$\mu_2 = \begin{cases} 0 & \text{if} \quad g_i^{t,\min} \le g_i^t \le g_i^{t,\max} \\ c & \text{otherwise} \end{cases} \tag{6.6}$$

$$\mu_3 = \begin{cases} 0 & \text{if} \quad 0 \le n_{ij}^t \le n_{ij}^{t,\max} \\ c & \text{otherwise} \end{cases} \tag{6.7}$$

$$\mu_4 = \begin{cases} 0 & \text{if} \quad \sum_{t=1}^{T} n_{ij}^t \le n_{ij}^{\max} \\ c & \text{otherwise} \end{cases} \tag{6.8}$$

$$\mu_5 = \begin{cases} 0 & \text{if} \quad \left| \theta_{ij}^{\text{cal}} \right| \le \left| \theta_{ij}^{\max} \right| \\ c & \text{otherwise} \end{cases} \tag{6.9}$$

In the dynamic TEP problem formulation, an index t indicates specific stage of planning involved with a horizon of T stages planning. μ_l is the penalty coefficient of the l^{th} inequality constraint. c is an inequality constraint constant that is used when an individual violates the inequality constraint. In this research, c is set to “0.5” for applying in eqs. (6.5-6.9). nb and nc represent the number of buses in the transmission system and the number of considered inequality constraints, respectively. Full details of these variables as used for solving dynamic TEP problems are described in section 2.5.

6.3 Implementation of DEA for Dynamic Transmission Expansion Planning Problem

Given the advantages of DEA performance and basic optimisation process as discussed in chapter 3, the DEA method can be applied to optimise dynamic TEP problem. The objective of DEA procedure is to find an individual X_i that optimises the fitness function. The DEA optimisation process comprises 4 main steps that are initialisation, mutation, crossover and selection. These optimisation operations are presented as follows:

6.3.1 Initialisation Step

Normally, an initial population of candidate solution must be generated according to equation (3.3) in the first step of DEA optimisation process. In the dynamic TEP problem formulation, each individual vector (X_i) comprises many integer-valued parameters n, where $n^t_{j,i}$ represents the number of candidate lines in the possible branch j of the individual i at time stage planning t. The problem decision parameter D is number of possible branches for expansion.

$$P^{(G)} = [X_1^{(G)}, ..., X_i^{(G)}, ..., X_{N_p}^{(G)}] \quad (6.10)$$

$$X_i^{(G)} = [n_{1,i}^{1,(G)}, ..., n_{j,i}^{t,(G)}, ..., n_{D,i}^{T,(G)}], \quad i = 1, ..., N_p \quad (6.11)$$

6.3.2 Optimisation Step

After the initial population is generated as stated in the previous section then new individuals are created by applying mutation (3.4), crossover (3.5) and selection (3.6) operators. Ten variant schemes of DEA procedure that are implemented directly to mutation process for generating the mutant parameter vectors. Three basic optimisation steps of the DEA method are repeated to enhance the fitness value of the candidate solution until the maximum number of generations ($G^{\max}$) is reached or other predetermined convergence criterion (ε) is satisfied.

6.3.3 Control Parameter Settings

Similar to static TEP problem, a proper selection of DEA control parameters is a key to approach the optimal solution. The control parameters should be selected carefully by the user for each optimisation problem. A guideline paper [49] is employed to initially set the DEA control parameters in this research. In this study, parameter tuning adjusts the control parameters through extensive testing until the best settings are found. The suitable DEA parameters settings for the dynamic TEP problem are as follows: F = [0.5,0.9], CR = [0.55,0.95] and N_P = [4*D*T,8*D*T]. The maximum predetermined convergence criterion (ε) is set to "1×10^{-3}" and the maximum number of generations (G^{max}) is set to "1×10^{3}".

6.3.4 DEA Optimisation Program for Dynamic TEP problem - Overall Procedure

The major steps of the DEA optimisation program for solving the dynamic TEP problem can be summarised as follows:

Step 1: Read all required transmission system data from database for the dynamic TEP calculation;

- The data of power generation, load demand and transmission line system at each time stage planning
- A horizon of time stage planning (T) and an annual interest rate value (I);

Step 2: Set up all required parameters of the DEA optimisation process by the user;

- Set up control parameters of the DEA optimisation process that are population size (N_P), scaling mutaion factor (F), crossover probability (CR), convergence criterion (ε), number of problem variables (D*T), lower and upper bounds of initial population (x_j^{min} and x_j^{max}) and maximum number of iterations or generations (G^{max});
- Select a DEA mutation operator strategy;

Step 3: Set iteration G = 0 for initialisation step of DEA optimisation process;

Step 4: Initialise population P of individuals according to equation (3.3);

Step 5: Calculate and evaluate fitness values of initial individuals according to the problem fitness function (6.1) and check constraints for each initial individual by using DC model dynamic TEP method;

Step 6: Rank the initial individuals according to their fitness;

Step 7: Set iteration G = 1 for optimisation step of DEA optimisation process;

Step 8: Apply mutation, crossover and selection operators to generate new individuals;

- Apply mutation operator to generate mutant vectors ($V_i^{(G)}$) according to equation (3.4) with a selected DEA mutation operator strategy in step 2;

- Apply crossover operator to generate trial vectors ($U_i^{(G)}$) according to equation (3.5);
- Apply selection operator according to equation (3.6) by comparing the fitness of the trial vector ($U_i^{(G)}$) and the corresponding target vector ($X_i^{(G)}$) and then select one that provides the best solution;

Step 9: Calculate and evaluate the fitness values of new individuals according to the problem fitness function (6.1) and check constraints for each new individual by using DC model dynamic TEP method;

Step 10: Rank new individuals according to their fitness;

Step 11: Update the best fitness value of the current iteration (*gbest*) and the best fitness value of the previous iteration (*pbest*)

Step 12: Check the termination criterion;

- If $|X_i^{best} - X_i| > \varepsilon$ or $|pbest - gbest| > \varepsilon$ when the number of current generation is not over the maximum number of generations $G < G^{\max}$, set $G = G + 1$ and return to step 8 for repeating to search the solution. Otherwise, stop to calculate and go to step 13;

Step 13: Calculate and output the final solution that are the best investment cost of the dynamic TEP problem and the number of added transmission lines in each candidate right-of-way at each stage;

Step 14: Run and display the DC load flow of the obtained final result.

6.4 Test Systems and Numerical Test Results

The proposed DEA method has been implemented in Matlab 7 and tested on an electrical transmission network as reported in [6] comparing the results with CGA procedure. In this study, the Colombian 93-bus system has been selected to test dynamic TEP procedure. Ten DEA strategies have been implemented to solve the dynamic TEP problem. In the experiment, initial control parameters of the DEA method had been set to solve the dynamic TEP problem as following details in section 6.3.3 then the appropriate DEA control parameters of the best solution were found through a great number of testing for each test system.

Colombian 93-Bus System

The transmission network is selected to test the dynamic TEP procedure that is the Colombian system as shown in figure A.4. The solid lines represent existing circuits in the base case topology and the dotted lines represent the possible addition of new transmission lines. The system consists of 93 buses, 155 possible right-of-ways and 14559 MW of total demand for the entire planning horizon. The required electrical system data, which consist of transmission line, generation and load data including the load growth along the study horizon, are available in [6, 69]. The addition of parallel transmission lines to existing lines is allowed in this case with a limit of 4 lines in each branch. Three planning stages P_1, P_2 and P_3 are considered in this case. The P_1 stage is the first stage that is the period from 2002 until 2005 and 2002 is the base year for this stage. The P_2 stage is the period from 2005 until 2009 and 2005 is the base year for the second stage. The P_3 stage is the period from 2009 until 2012 and 2009 is the base year for the third stage. Furthermore, the total transmission expansion investment plan is obtained with reference to the base year 2002 and an annual interest rate value I = 10 %. Hence, the total investment cost can again be calculated by using equation (2.11).

The achieved results of DEA and CGA procedures on the Colombian 93-bus system can be tabulated in table 6.1 including the discussion of these results as follows:

- In this case, the best solution of dynamic TEP problem was found by DEA3 whereas the present value of investment cost projected to the base year 2002 is v = US$ 505.8 million.
- DEA3 shows the best performance approach in ability and robustness for searching the problem solution, as shown by the smallest of the best investment cost, an average investment cost and a standard deviation value.
- In contrast, DEA10 shows the poorest performance for finding the problem solution compared to other DEA strategies and CGA procedure because it gives the largest of the best investment cost and an average investment cost in this test case.
- In this test case, DEA3 requires less an average computational time than other DEA strategies and CGA procedure. On the other hand, CGA takes larger an average calculation time than all DEA strategies.

- Overall, the best algorithmic procedure for this case is DEA3 based on all above mentioned reasons.

Table 6.1 Summary results of Colombian 93-bus system

Results of dynamic TEP	Methods										
	DEA3	DEA1	DEA6	DEA8	CGA	DEA2	DEA4	DEA9	DEA5	DEA7	DEA10
Best, 10^6 US$	505.8	520.62	530.81	530.81	590.38	618.3	646.41	681.88	734.61	734.61	739.21
Average, 10^6 US$	515	532	539	570	627	654	677	729	786	810	830
Worst, 10^6 US$	524.63	547.07	551.77	618.3	661.87	681.88	734.61	823.2	860.71	913.23	930.23
Diff. between best and worst, %	3.72	5.08	3.95	16.48	12.11	10.28	13.64	20.73	17.17	24.31	25.83
Standard deviation, 10^6 US$	6.85	9.94	7.34	23.38	31.82	28.1	32.44	42.09	63.96	78.48	78.17
Average CPU time, minute	156	159	165	168	229	170	173	177	179	185	188

In order to obtain the results in table 6.1, the DEA control parameters setting is as follows: F = 0.8, CR = 0.6, $D*T$ = 155*3 stages = 465 and $N_p = 5*D*T$ = 2325 respectively. In the dynamic TEP, only the best performing of DEA strategies is selected to present full details of the achieved result, which is DEA3. The proposed DEA3 scheme could find the best solution of the dynamic TEP problem for this test system. The number of additional transmission lines determined by the proposed DEA scheme is as follows:

- Stage P_1 : $n_{45\text{-}81} = 1$, $n_{55\text{-}57} = 1$, $n_{55\text{-}62} = 1$, $n_{57\text{-}81} = 2$ and $n_{82\text{-}85} = 1$
- Stage P_2 : $n_{19\text{-}82} = 1$, $n_{27\text{-}29} = 1$, $n_{62\text{-}73} = 1$ and $n_{72\text{-}73} = 1$
- Stage P_3 : $n_{15\text{-}18} = 1$, $n_{29\text{-}31} = 1$, $n_{29\text{-}64} = 2$, $n_{52\text{-}88} = 1$, $n_{55\text{-}62} = 1$, $n_{55\text{-}84} = 1$ and $n_{68\text{-}86} = 1$

The best solution with the present value of the expansion investment cost projected to the base year 2002 is v = US$ 505.8 million, which can be calculated as follows:

Table 6.2 The expansion investment cost calculation of Colombian 93-bus system

Planning stage	Added lines	Investment cost of an additional line, x10^3 US$	Investment Cost, x10^3 US$
Stage P_1	$n_{45\text{-}81} = 1$	13270	13270
	$n_{55\text{-}57} = 1$	46808	46808
	$n_{55\text{-}62} = 1$	70988	70988
	$n_{57\text{-}81} = 2$	58890	117780
	$n_{82\text{-}85} = 1$	89898	89898
	Total investment cost in stage P_1		**338744**
Stage P_2	$n_{19\text{-}82} = 1$	13270	13270
	$n_{27\text{-}29} = 1$	5052	5052
	$n_{62\text{-}73} = 1$	73158	73158
	$n_{72\text{-}73} = 1$	13270	13270
	Total investment cost in stage P_2		**104750**
Stage P_3	$n_{15\text{-}18} = 1$	7927	7927
	$n_{29\text{-}31} = 1$	32981	32981
	$n_{29\text{-}64} = 2$	4362	8724
	$n_{52\text{-}88} = 1$	34190	34190
	$n_{55\text{-}62} = 1$	70988	70988
	$n_{55\text{-}84} = 1$	26658	26658
	$n_{68\text{-}86} = 1$	8272	8272
	Total investment cost in stage P_3		**189740**

- Investment cost of stage P_1 with reference to base year 2002:

$$v_1 = \delta_{inv}^1 c_1(x) = (1-I)^{2002-2002} \times c_1$$
$$= (1-0.1)^0 \times 338.74 \times 10^6$$
$$= 1 \times 338.74 \times 10^6 = 338.74 \times 10^6$$

- Investment cost of stage P_2 with reference to base year 2005:

$$v_2 = \delta_{inv}^2 c_2(x) = (1-I)^{2005-2002} \times c_2$$
$$= (1-0.1)^3 \times 104.75 \times 10^6$$
$$= 0.729 \times 104.75 \times 10^6 = 76.36 \times 10^6$$

- Investment cost of stage P_3 with reference to base year 2009:

$$v_3 = \delta_{inv}^3 c_3(x) = (1-I)^{2009-2002} \times c_3$$
$$= (1-0.1)^7 \times 189.74 \times 10^6$$
$$= 0.478 \times 189.74 \times 10^6 = 90.7 \times 10^6$$

Summation of the investment costs v_1, v_2 and v_3, gives the total investment cost of the Colombian 93-bus system and is $v = v_1 + v_2 + v_3$ = US$ 505.8 million.

Table 6.3 The best results comparison of Colombian 93-bus system

Methods	Best cost million US$	Average CPU Times
DEA3	505.8	156 min
CGA	590.38	229 min
Efficient GA [6]	514.4	-
GA of Chu and Beasley [68]	503.8	-
Specialised GA [76]	505.8	-

6.5 Discussion on the Results

The results of the dynamic TEP problem for the Colombian 93-bus system are summarised in table 6.3 where the best expansion investment cost of the proposed methodology is compared directly to other algorithms. In addition, the computational time for the DEA method is also presented and compared to CGA procedure in this table. The results in table 6.3 clearly indicate that the best expansion investment cost found by the proposed DEA3 is 505.8 million US$ and the same value as the investment cost found by specialised GA [76] whereas the best expansion investment cost found by GA of Chu and Beasley (GACB) is 503.8 million US$. The number of additional transmission lines determined by GACB in [68] is as follows: stage P_1: $n_{57\text{-}81} = 2$, $n_{55\text{-}57} = 1$, $n_{55\text{-}62} = 1$, $n_{45\text{-}81} = 1$ and $n_{82\text{-}85} = 1$; stage P_2: $n_{27\text{-}29} = 1$, $n_{62\text{-}73} = 1$, $n_{72\text{-}73} = 1$ and $n_{19\text{-}82} = 1$; stage P_3: $n_{52\text{-}88} = 1$, $n_{15\text{-}18} = 1$, $n_{55\text{-}84} = 1$, $n_{55\text{-}62} = 1$, $n_{29\text{-}31} = 1$, $n_{29\text{-}64} = 2$ and $n_{68\text{-}86} = 1$. According to the obtained results in [68], the expansion plan as found by GACB is the same topology as the expansion plan found by the DEA method presented in this chapter. Although the expansion plan found by GACB and DEA are the same topology, the investment cost calculated in this chapter is not same value due to the investment cost provided by GABC in [68] where the full details of investment cost calculation are not presented. It is important to note that the transmission system data of the Colombian 93-bus system, especially the additional line cost data, used in this chapter is also in agreement with the data available in reference [69].

For dynamic planning, the proposed method shows a good performance to find the optimal solution but it has a drawback in slow computation. Even though all DEA strategies require less computational times than CGA procedure as shown in table 6.1, they still require a great number of minutes to calculate. Regarding the disadvantage of slow computation, DEA optimisation program for the dynamic TEP problem should be improved its searching performance.

6.6 Conclusions

In this research, a novel DEA method is proposed to solve dynamic TEP problem without generation resizing consideration. The obtained results of the Colombian 93-bus system illustrate that the DEA3 is good efficient and effectively minimises the total investment cost of the dynamic TEP problem on a realistically complex transmission system. As the empirical solution of this test case indicates, the total investment cost of the DEA method is less expensive than the CGA procedure on the Colombian 93-bus system. In addition, the all DEA strategies require less computational CPU time than CGA procedure in test case. The interpretations of obtained results from chapter 5 and 6 with regard to sensitivity and convergence analysis of the DEA on static and TEP problems are presented in next chapter.

CHAPTER 7
INTERPRETATION OF TEST RESULTS IN TRANSMISSION EXPANSION PLANNING PROBLEM

7.1 Introduction

This chapter aims at studying and testing the influence of the variant control parameters setting of DEA method in transmission expansion planning (TEP) problem. Over past few years, a number of researches have investigated the sensitivity analysis of DEA control parameters as shown in [46, 49, 53, 54, 70, 71, 72]. These research papers can be classified into two main categorises: (1) sensitivity analysis in mathematical optimisation problems [46, 49, 53, 54] and (2) sensitivity analysis in real-world optimisation problems [70, 71, 72]. According to these two categories, the reports in testing values that are the best solutions are achieved for the particular problem and an applied method. In this chapter, the study focuses directly to the sensitivity analysis that investigates the influence of DEA control parameters variation in both static and dynamic TEP problems.

The organisation of this chapter is as follows: The sensitivity analysis of DEA control parameters on static TEP problem is presented in section 7.2 while both with and without generation resizing consideration cases have been investigated in this section. Section 7.3 presents the sensitivity analysis of DEA control parameters on dynamic TEP problem. Subsequently, these results are discussed and further analysed in section 7.4. Finally, section 7.5 provides summary of this chapter.

7.2 Sensitivity Analysis of DEA Control Parameters on Static Transmission Expansion Planning

To study the effect of varying DEA control parameters on static TEP problem, a Graver 6-bus test system with and without generation resizing consideration cases has been investigated by applying several DEA mutation strategies that are DEA1-DEA10, respectively. The system data used in this experiment are available in [8, 66] and full details of this test system are illustrated in appendix A1.

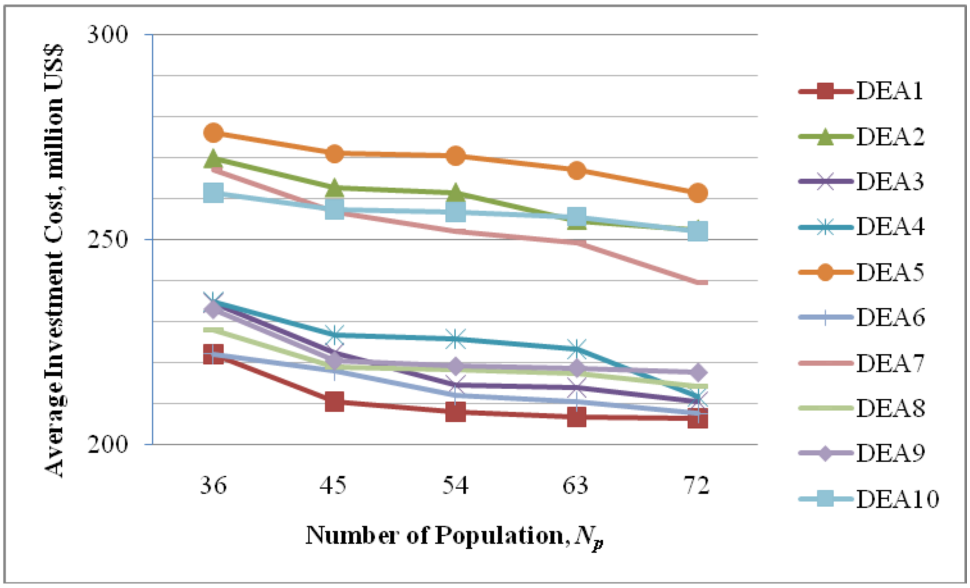
The investigation in the sensitivity of DEA control parameters on the static TEP problem is categorised into three scenarios regard to the control parameter settings. The first group focuses on the sensitivity of the population size (N_P) while the second group focuses on sensitivity of scaling mutation factor (F) and the last group focuses on sensitivity of crossover probability (CR). In addition, each previous study group is analysed and compared in three aspects that are (1) an average expansion investment cost, (2) a standard deviation of results and (3) an average computational time, respectively. Due to the randomness of the simulation results, each point on the graphs is achieved through an average value of final results 50 different runs.

7.2.1 Sensitivity of Population Size (N_P)

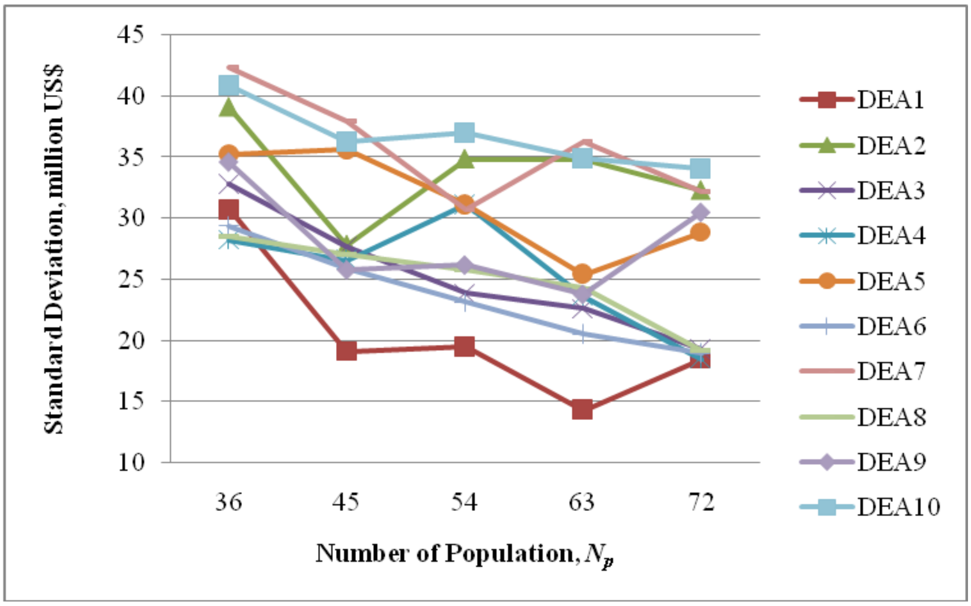
In this section, the sensitivity of population size of DEA method is considered as with and without generation resizing cases for static TEP problem. The population size varying of DEA method is analysed and discussed in this section where the population size is varied from 4*D to 8*D. The problem decision parameters D of Garver 6-bus system are equal to 9 and 12 for without and with generation resizing consideration cases, respectively. The other DEA parameters setting used in this simulation are as follows: F = 0.7 and CR = 0.6.

7.2.1.1 Static TEP Problem - without Generation Resizing

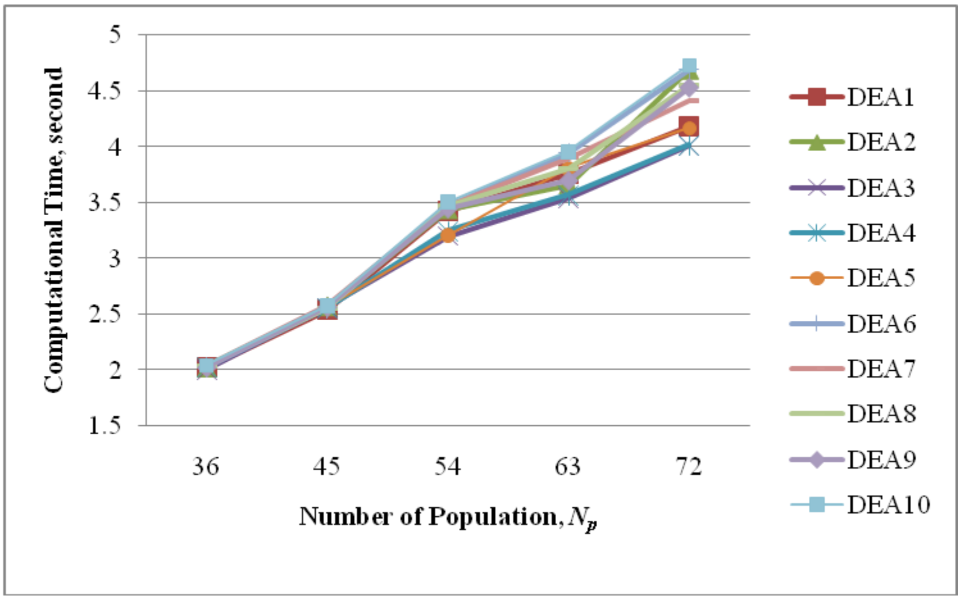
According to the achieved results on graphs as shown in figure 7.1(a) indicate, DEA1 provides the smallest average expansion investment cost whereas DEA5 provides the largest average expansion investment cost compared to other DEA strategies for all various population sizes in this case. All DEA strategies yield lower average investment cost while their population sizes increase. The simulation results can be clearly separated into two groups regard to performance of the DEA method to provide average investment cost.



(a) Average investment costs versus population sizes



(b) Standard deviations of the investment costs versus population sizes



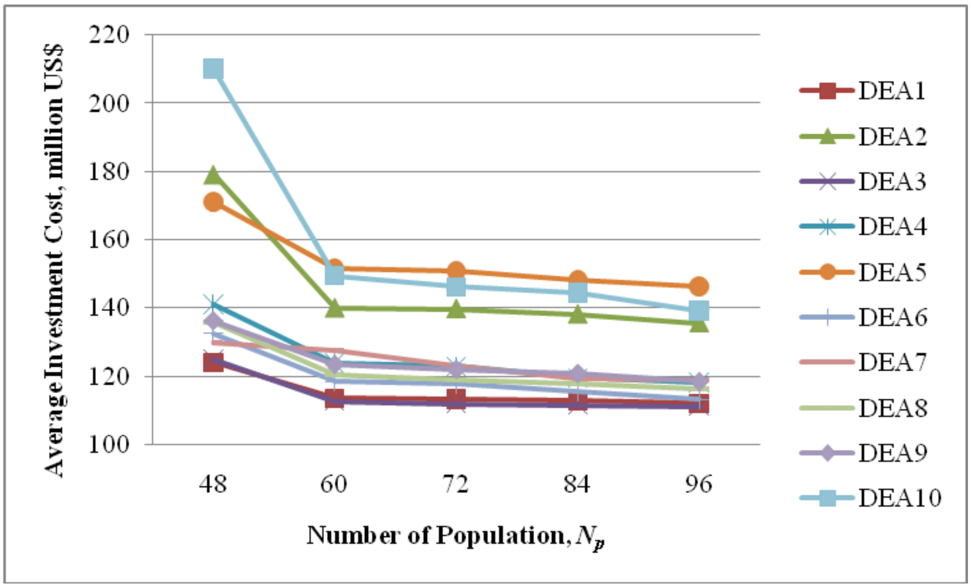
(c) Average computational times versus population sizes

Figure 7.1 Comparison of various population sizes obtained from DEA1-DEA10 for static TEP problem without generation resizing on Garver 6-bus system

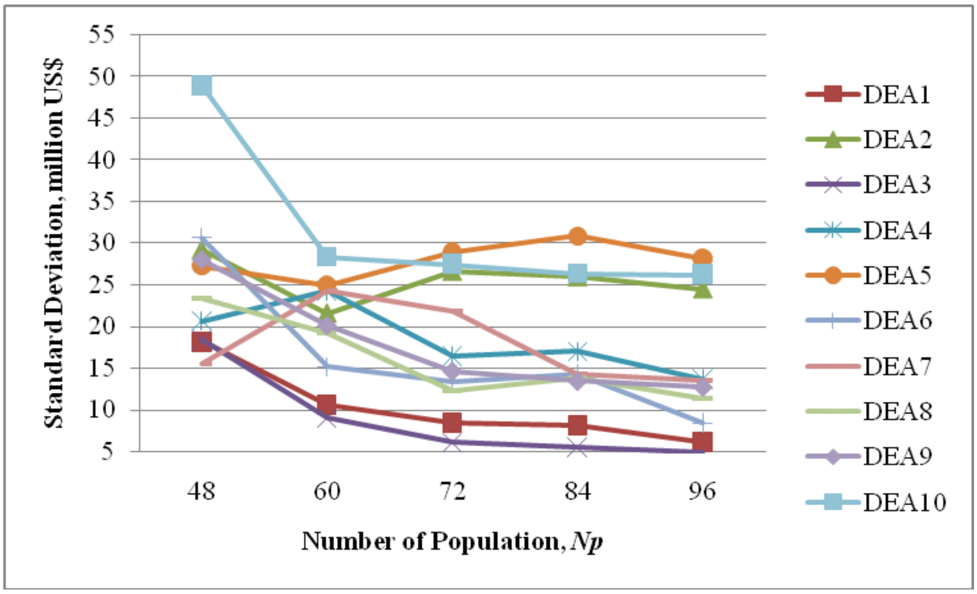
From obtained results in figure 7.1(b), DEA1 provides smaller standard deviation value of investment cost than other DEA strategies in this case when population size is more than 36. In addition, DEA1 gives the least value of standard deviation at population size N_P = 63 for this case.

For the calculation time comparison as shown in figure 7.1(c), DEA3 takes the smallest average computational time, whereas DEA10 takes the highest average calculation time for this case. Although, all DEA strategies require more calculation time while their population sizes increase, they perform well in calculation time consideration for this test case.

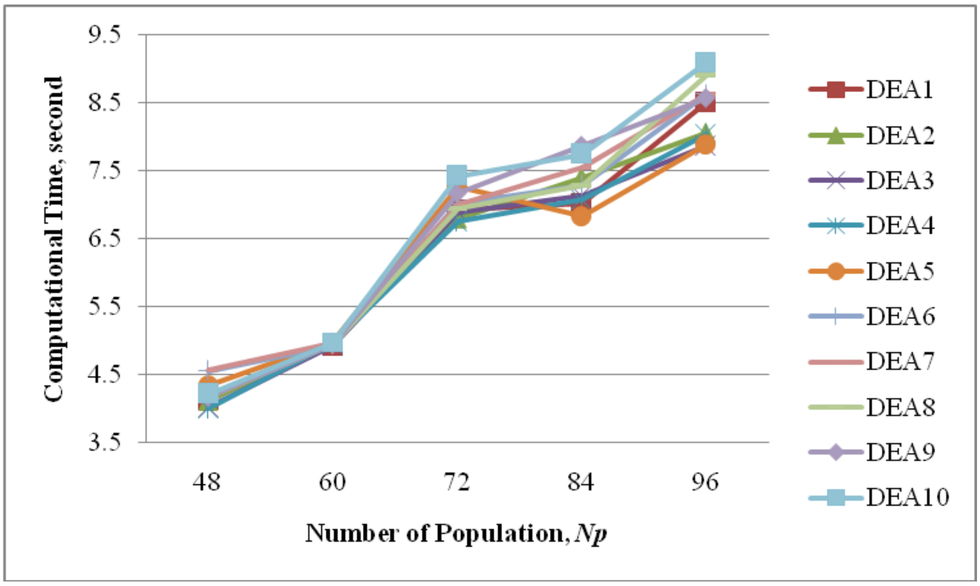
7.2.1.2 Static TEP Problem - with Generation Resizing



(a) Average investment costs versus population sizes



(b) Standard deviations of the investment costs versus population sizes



(c) Average computational times versus population sizes

Figure 7.2 Comparison of various population sizes obtained from DEA1-DEA10 for static TEP problem with generation resizing on Garver 6-bus system

According to the obtained results on graphs as shown in figure 7.2(a) indicate, DEA3 yields the smallest average expansion investment cost compared to other DEA strategies for all various population sizes except its population size N_P = 48, which DEA1 gives lower average cost than DEA3. On the other hand, DEA5 provides the largest average expansion investment cost compared to other DEA strategies for all various population sizes except its population size N_P = 48, which DEA2 and DEA10 give higher average cost than DEA5. The simulation results can be clearly separated into two groups regard to the performance of DEA method to provide average investment cost in this case. The lowest average investment cost for this case is provided by DEA at population size N_P = 96.

As results in figure 7.2(b), DEA3 yields less standard deviation values of investment cost than other DEA strategies except at population sizes N_P = 48, which DEA7 gives smaller value than DEA3. On the other hand, DEA5 and DEA10 are not robust to find the solution compared to other DEA strategies, as shown the largest values of standard deviation and average investment cost.

For the calculation time comparison as shown in figure 7.2(c), DEA3 and

DEA4 require smaller computational time than other DEA strategies but it is tiny difference among these computational times. Similar to the previous case of Graver 6-bus system, All DEA strategies take larger computation time while their population sizes rise up.

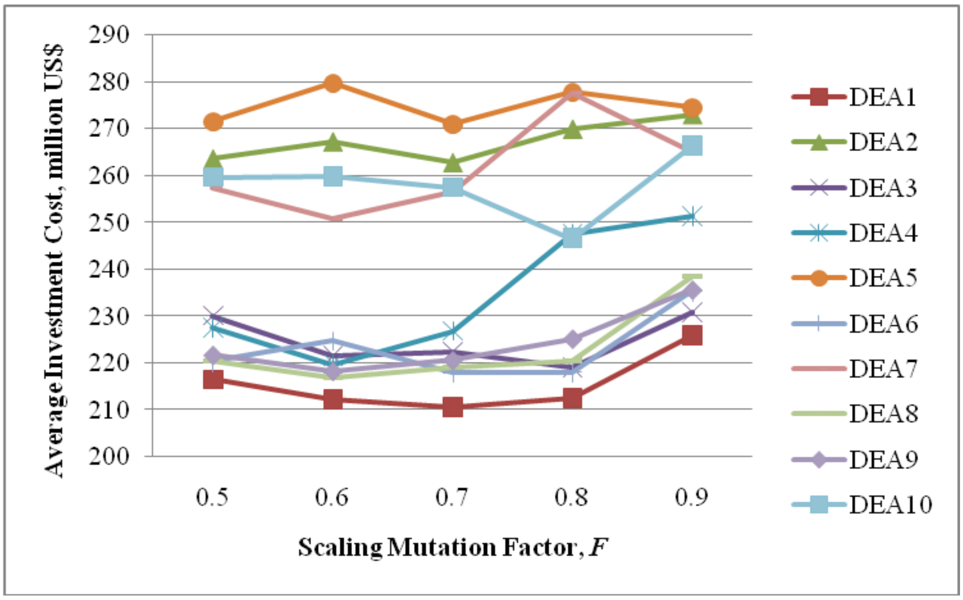
7.2.2 Sensitivity of Scaling Mutation Factor (*F*)

In this section, the sensitivity of scaling mutation factor of DEA method is considered as with and without generation resizing cases for static TEP problem. The scaling mutation factor of DEA method is analysed and discussed in this section where this factor is varied from 0.5 to 0.9. The other DEA parameters setting used in this simulation are as follows: $N_p = 5*D$ and $CR = 0.6$, respectively.

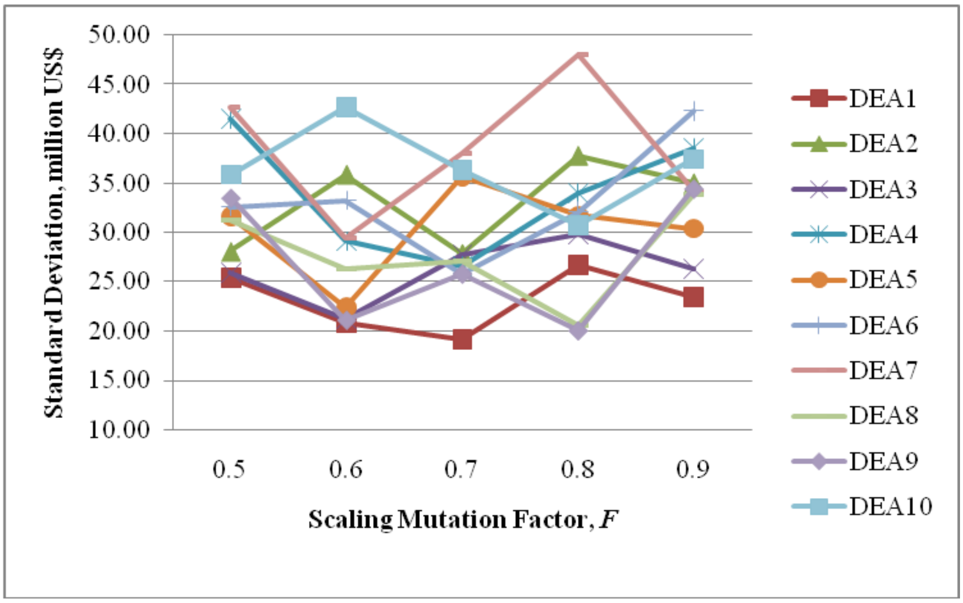
7.2.2.1 Static TEP Problem – without Generation Resizing

According to obtained results in figure 7.3(a) indicate, DEA1 clearly shows the best performance to provide the smallest average investment cost for all various scaling mutation factor values, whereas DEA5 gives the largest value of investment cost. The lowest average investment cost is provided by DEA1 at mutation factor $F = 0.7$ for this case. The simulation results can be classified into two groups regard to the average investment cost except DEA4 performs worse when its mutation factor value is more than 0.7.

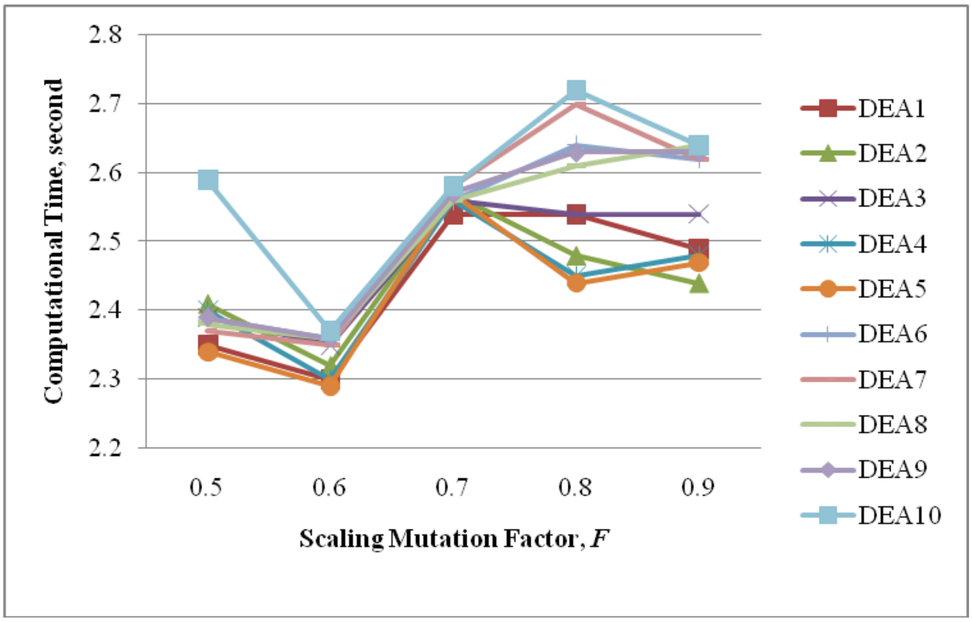
From figure 7.3(b), DEA1 yields the smallest standard deviation value than other DEA strategies for all mutation factor values except $F = 0.8$ where DEA9 gives smaller standard deviation value than DEA1. In addition, DEA1 provides the smallest standard deviation value of this case at the mutation factor $F = 0.7$ whereas DEA7 gives the highest standard deviation value at mutation factor $F = 0.8$.



(a) Average investment costs versus scaling mutation factors (*F*)



(b) Standard deviation of the investment costs versus scaling mutation factors (*F*)



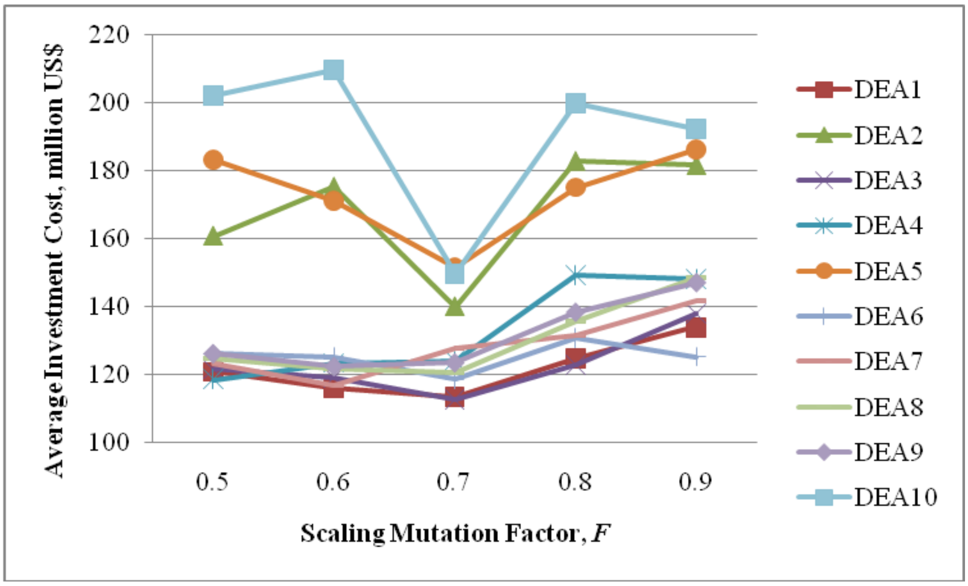
(c) Average computational times versus scaling mutation factors (*F*)

Figure 7.3 Comparison of various scaling mutation factors (*F*) obtained from DEA1-DEA10 for static TEP problem without generation resizing on Garver 6-bus system

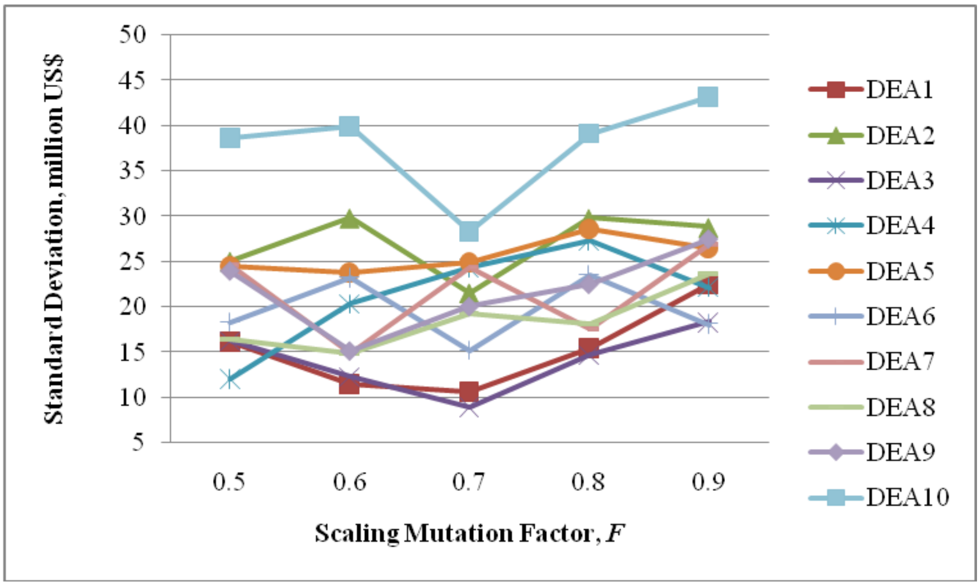
For calculation time comparison as shown in figure 7.3(c), DEA5 requires the lowest calculation time compared to other DEA strategies at three mutation factor values F = 0.5, 0.6 and 0.8, respectively. In contrast, DEA10 is the slowest strategy in computation for this case.

7.2.2.2 Static TEP Problem - with Generation Resizing

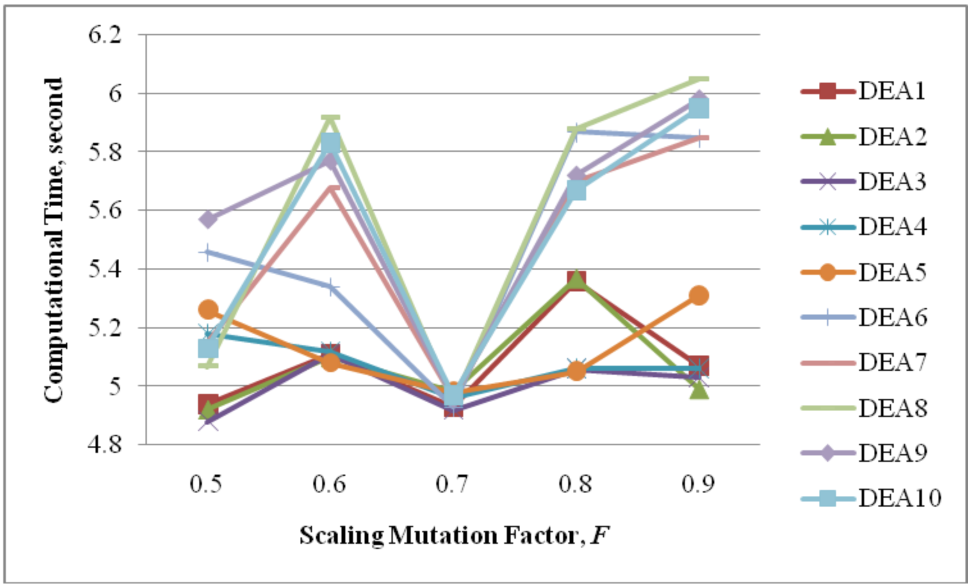
According to figure 7.4(a), the simulation results can be clearly separated into two groups. The first group comprises DEA2, DEA5 and DEA10 that perform poor to provide average investment cost whereas the second group consists of DEA1, DEA3, DEA4, DEA6, DEA7, DEA8, and DEA9 that give smaller values of average investment cost than the first group. The smallest average investment cost is provided by DEA3 at mutation factor F = 0.7, whereas DEA10 gives the highest average investment cost at the mutation factor F = 0.6 for this case.



(a) Average investment costs versus scaling mutation factors (*F*)



(b) Standard deviation of the investment costs versus scaling mutation factors (*F*)



(c) Average computational times versus scaling mutation factors (*F*)

Figure 7.4 Comparison of various scaling mutation factors (*F*) obtained from DEA1-DEA10 for static TEP problem with generation resizing on Garver 6-bus system

From figure 7.4(b), DEA1 and DEA3 perform well in robustness to find the problem solution. They yield smaller standard deviation value than other DEA strategies for all mutation factor values except F = 0.5 where DEA4 gives smaller standard deviation value than DEA1 and DEA3. On the other hand, DEA10 performs poorer in robustness to find the solution than other DEA strategies as shown the largest standard deviation value for all mutation factor values. The smallest standard deviation value of this case is yielded by DEA3 at mutation factor F = 0.7, whereas DEA10 gives the largest standard deviation value at mutation factor F = 0.9.

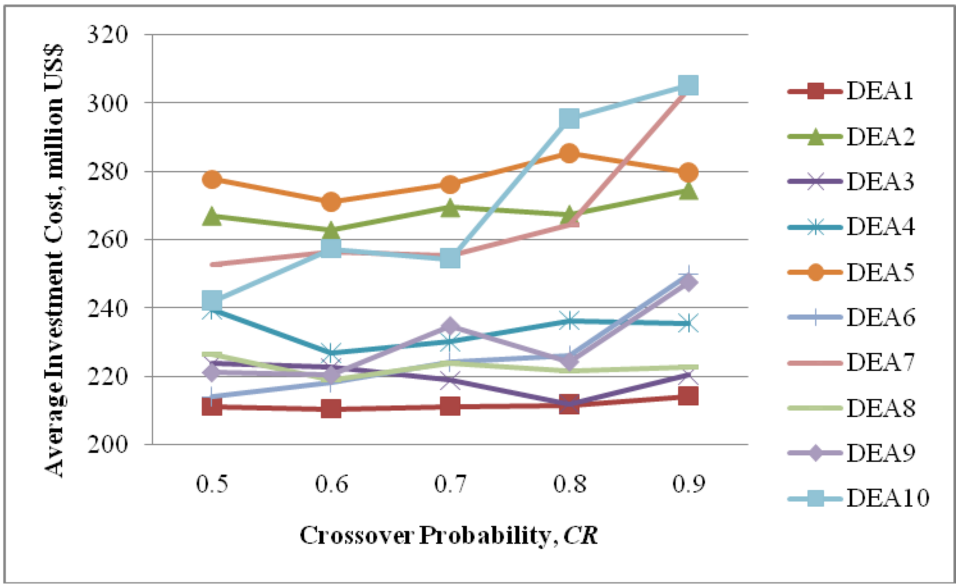
For calculation time comparison shown in figure 7.3(c), all DEA strategies perform well in computational time when the mutation factor is 0.7. In this case, DEA3 requires computational time smaller than other DEA strategies at F = 0.5 and 0.7, whereas DEA5 requires computational time smaller than other DEA strategies at F = 0.6 and 0.8.

7.2.3 Sensitivity of Crossover Probability (*CR*)

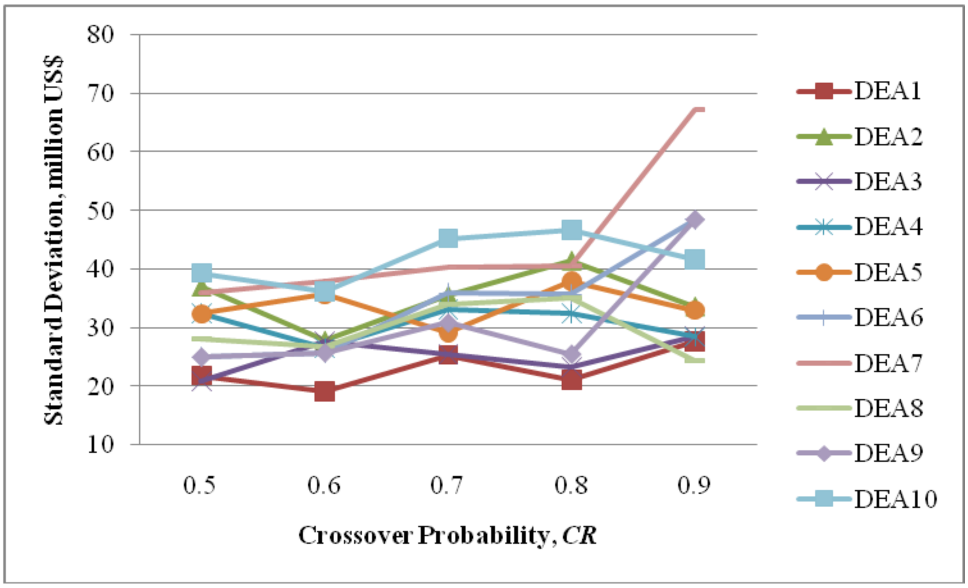
In this section, the sensitivity of crossover probability of DEA method is considered as with and without generation resizing cases for static TEP problem. The crossover probability of the DEA method is analysed and discussed in this section where the crossover constant is varied from 0.5 to 0.9. The other DEA parameters setting used in this simulation are as follows: $N_p = 5*D$ and $F = 0.7$, respectively.

7.2.3.1 Static TEP Problem - without Generation Resizing

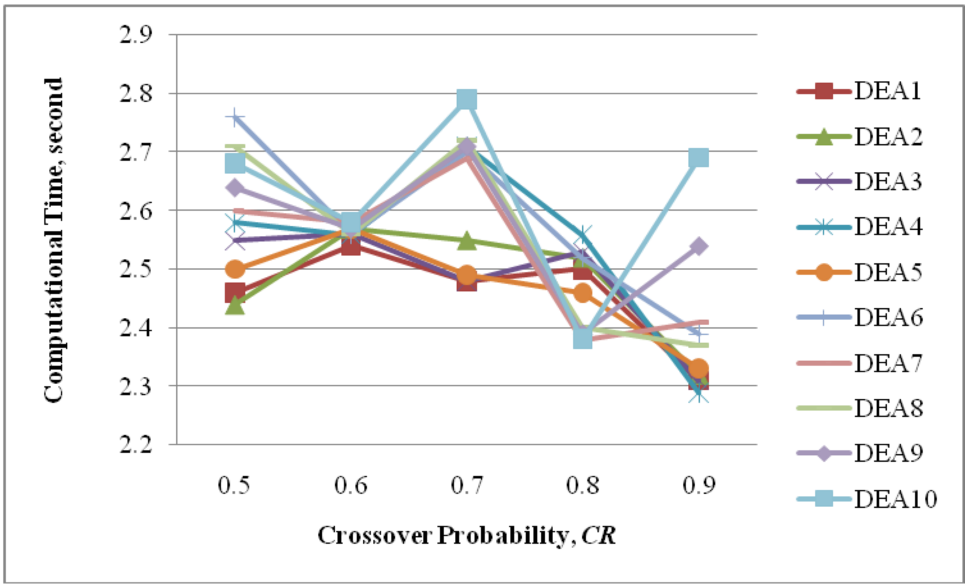
According to the obtained results on graphs as shown in figure 7.5(a) indicate, DEA1 provides the cheapest average expansion investment cost compared to other DEA strategies for all values of crossover probability. On the other hand, DEA5 provides the most expensive average expansion investment cost at *CR* = 0.5, 0.6 and 0.7 and DEA10 provides the most expensive average investment cost at *CR* = 0.8 and 0.9 in this case. The simulation results can be clearly separated into two groups regard to the performance of DEA method to provide the average investment cost.



(a) Average investment costs versus crossover probabilities (*CR*)



(b) Standard deviation of the investment costs versus crossover probabilities (*CR*)



(c) Average computational times versus crossover probabilities (*CR*)

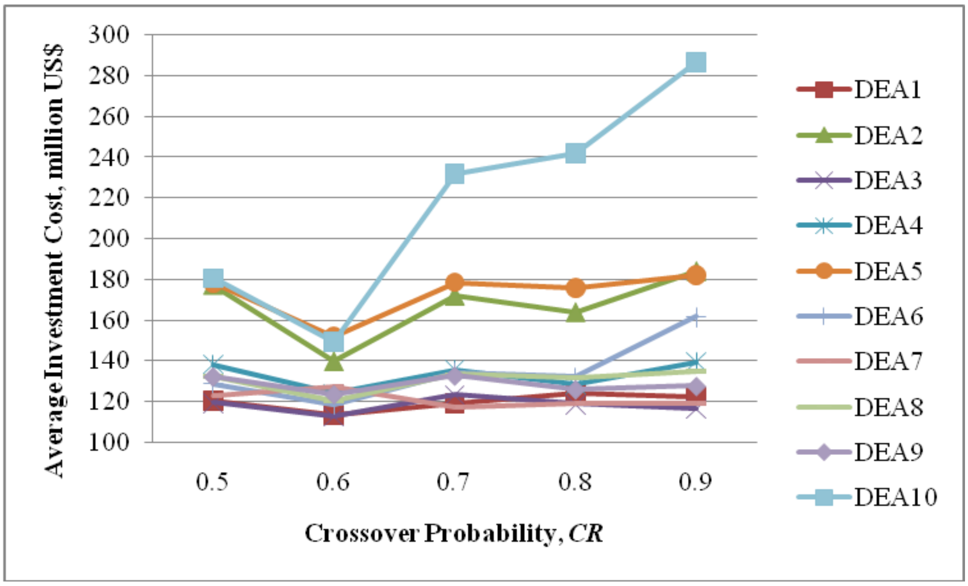
Figure 7.5 Comparison of various crossover probabilities (*CR*) obtained from DEA1-DEA10 for static TEP problem without generation resizing on Garver 6-bus system

From figure 7.5(b), DEA1 performs well in robustness to find the problem solution because it yields smaller standard deviation values than other DEA strategies at *CR* = 0.6, 0.7 and 0.8, respectively. In addition, the smallest standard deviation value of this case is yielded by DEA1 at *CR* = 0.6, whereas DEA7 gives the largest standard deviation value at *CR* = 0.9.

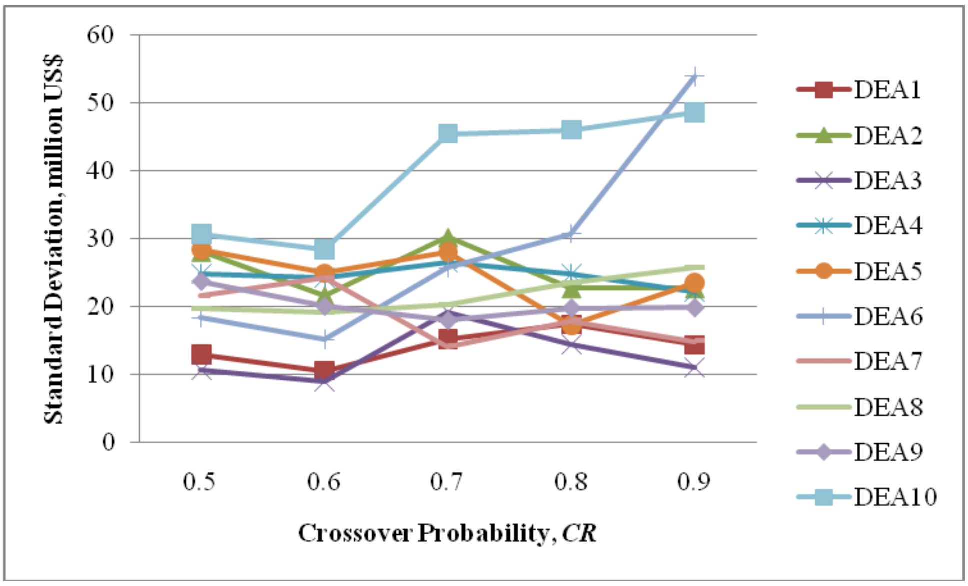
For calculation time comparison as shown in figure 7.5(c), DEA1 requires smaller calculation time compared to other DEA strategies at *CR* = 0.6 and 0.7, whereas DEA4 provides the smallest computational time at *CR* = 0.9 for this case.

7.2.3.2 Static TEP Problem - with Generation Resizing

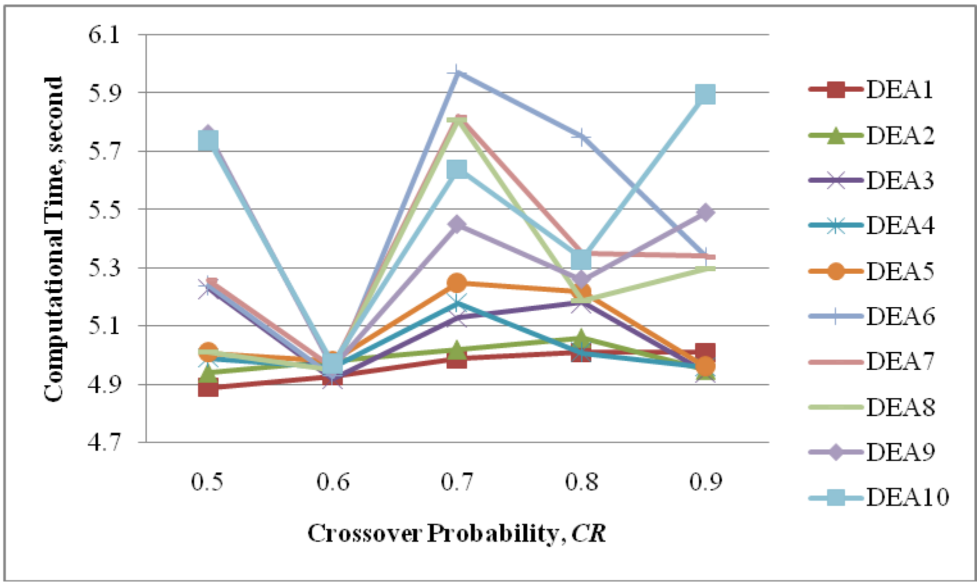
As the obtained results on graphs as illustrated in figure 7.6(a) indicate, DEA3 yields average expansion investment cost cheaper than other DEA strategies for all values of crossover probability except *CR* = 0.7 where DEA1 and DEA7 provide the average investment cost cheaper than DEA3. On the other hand, DEA10 performs poor to find the problem solution because it gives the average expansion cost more expensive than other DEA strategies for all values of the crossover probability except *CR* = 0.6 where DEA5 provides the average investment cost more expensive than DEA10.



(a) Average investment costs versus crossover probabilities (*CR*)



(b) Standard deviation of the investment costs versus crossover probabilities (*CR*)



(c) Average computational times versus crossover probabilities (*CR*)

Figure 7.6 Comparison of various crossover probabilities (*CR*) obtained from DEA1-DEA10 for static TEP problem with generation resizing on Garver 6-bus system

From figure 7.6(b), DEA3 performs well in robustness to find the problem solution because it yields smaller standard deviation values than other DEA strategies for all values of crossover probability except CR = 0.7 where DEA1 and DEA7 yield the standard deviation values smaller than DEA3. On the other hand, DEA10 performs poor in robustness to find the solution because it gives larger standard deviation value than other DEA schemes for all values of crossover probability except CR = 0.9. The smallest standard deviation value of this case is yielded by DEA3 at crossover probability CR = 0.6, whereas DEA6 gives the largest standard deviation value at CR = 0.9.

For calculation time comparison as shown in figure 7.6(c), DEA1 requires smaller calculation time compared to other DEA strategies at crossover probability CR = 0.5, 0.7 and 0.8 whereas DEA3 provides the computational time smaller than other DEA strategies at crossover probability CR = 0.6 and 0.9 for this case.

7.3 Sensitivity Analysis of DEA Control Parameters on Dynamic Transmission Expansion Planning

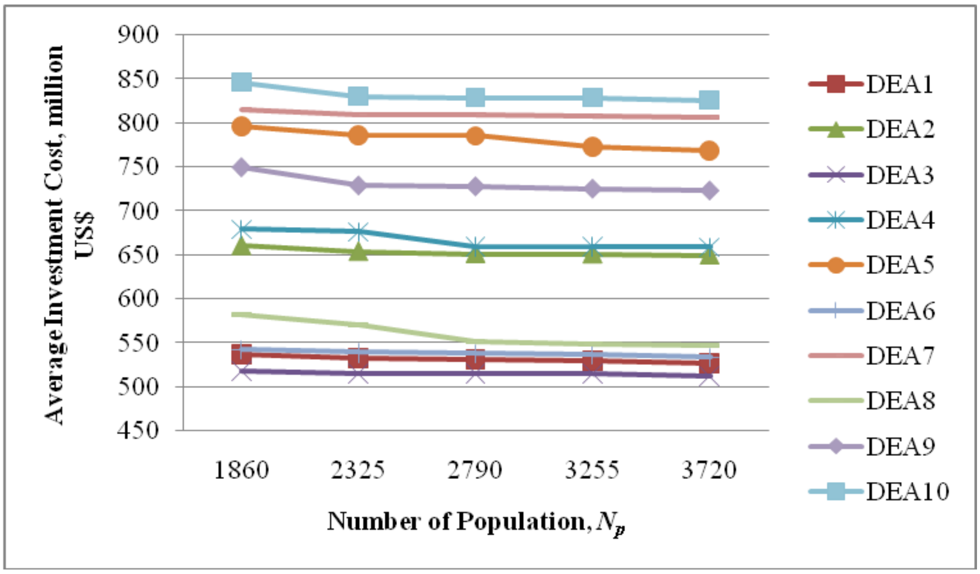
To study the influence of varying DEA control parameters on dynamic TEP problem, the Colombian 93-bus test system without generation resizing consideration case has been investigated by applying several DEA mutation strategies that are DEA1-DEA10, respectively. The system data used in this experiment are available in [6, 69] and the system details are illustrated in appendix A4.

Similar to the sensitivity analysis of DEA control parameters on static TEP, the investigation in sensitivity of DEA control parameters on the dynamic TEP problem are classified into three main groups regard to control parameter settings that are the sensitivity of population size (N_P), the sensitivity of mutation factor (F) and the sensitivity of crossover probability (CR). In addition, each previous study group is analysed and compared in three aspects that are (1) an average expansion investment cost, (2) a standard deviation of results and (3) an average computational time, respectively. Due to the randomness of the simulation results, each point on the graphs is achieved through an average value of final results 25 different runs.

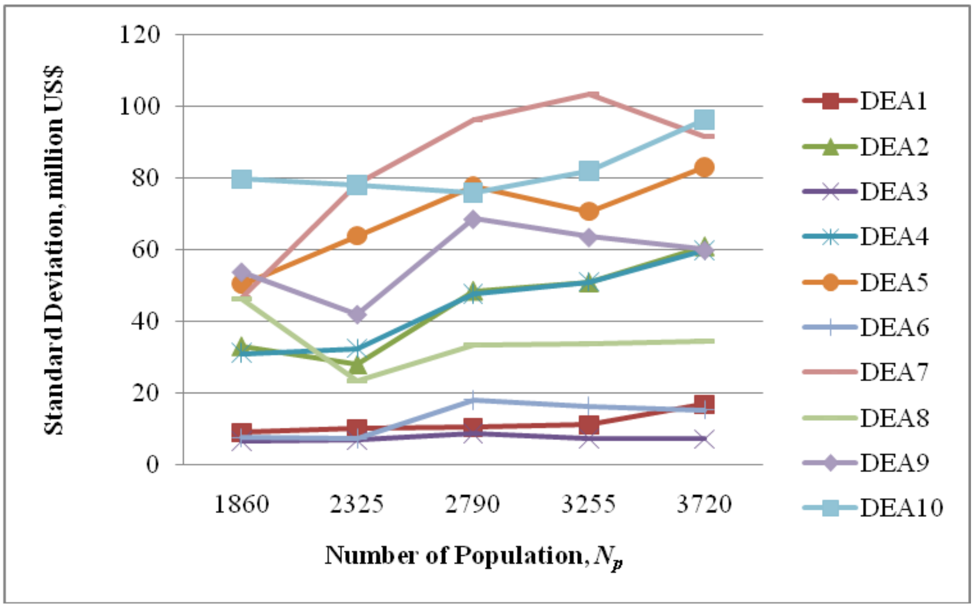
7.3.1 Sensitivity of Population Size (N_P)

In this section, the sensitivity of population size of DEA method is considered only without generation resizing case for dynamic TEP problem. The population size variation of DEA method is analysed and discussed in this section, which the population size is varied from 4**D***T* to 8**D***T*. The problem decision parameter *D* of the Colombian 93-bus test system is equal to 155 for without generation resizing consideration case and the whole planning period study is 3 stages, so the population size is varied from 1860 to 3720 for this experiment. The settings of other DEA parameters used in this simulation are as follows: F = 0.8 and CR = 0.6, respectively.

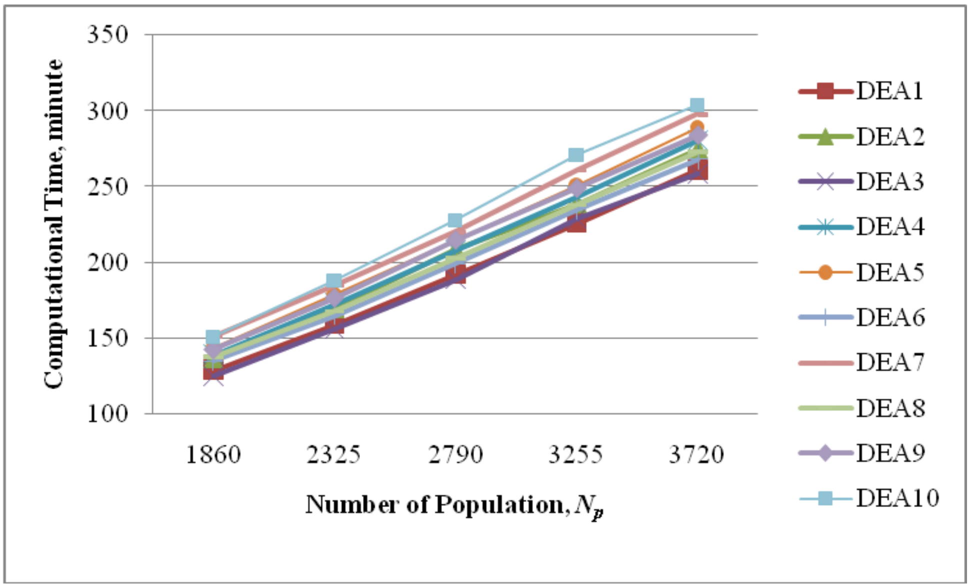
According to the obtained results on graphs as shown in figure 7.7(a) indicate, DEA3 performs well to find the problem solution as shown the smallest average expansion investment cost compared to other DEA strategies for all population sizes. On the other hand, DEA10 performs poor to find the problem solution as shown the largest average expansion investment cost for all population sizes in this case. The graphs of average investment cost as shown in figure 7.7(a) go down very slightly while the population sizes increase.



(a) Average investment costs versus population sizes



(b) Standard deviations of the investment costs versus population sizes



(c) Average computational times versus population sizes

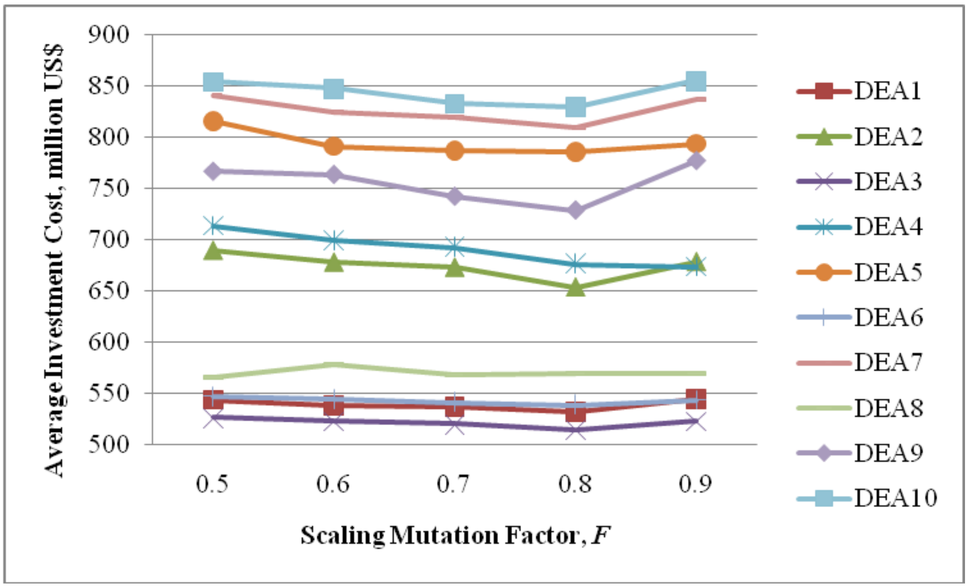
Figure 7.7 Comparison of various population sizes obtained from DEA1-DEA10 for dynamic TEP problem without generation resizing on Colombian 93-bus system

As results in figure 7.7(b), DEA3 performs well in robustness to find the solution as shown smaller standard deviation value of investment cost than other DEA strategies for all population sizes. On the other hand, DEA7 and DEA10 are not robustness to find the problem solution as shown the values of standard deviation larger than others.

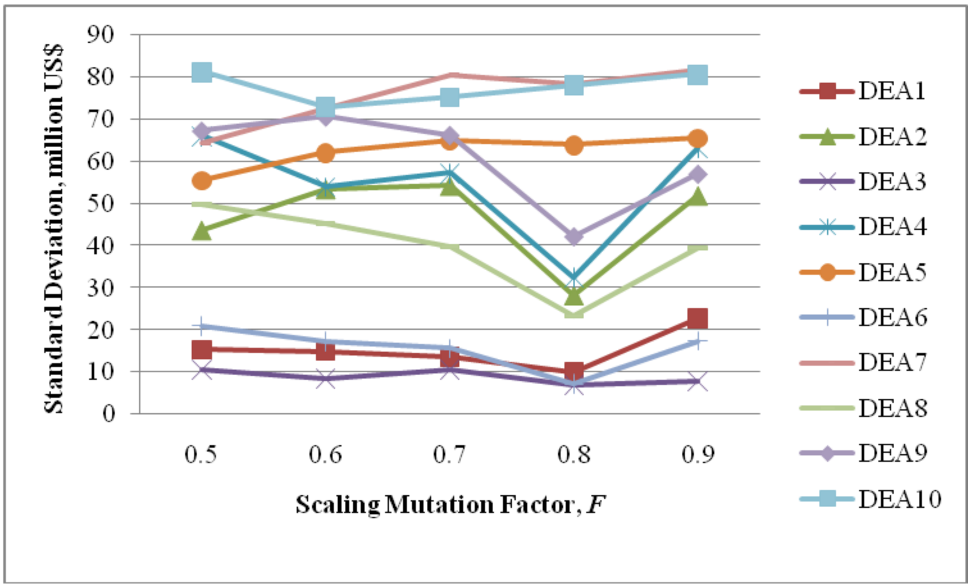
For calculation time comparison as shown in figure 7.7(c), DEA3 requires lower calculation time than other DEA strategies. All DEA strategies require larger computation time while their population sizes rise up.

7.3.2 Sensitivity of Scaling Mutation Factor (*F*)

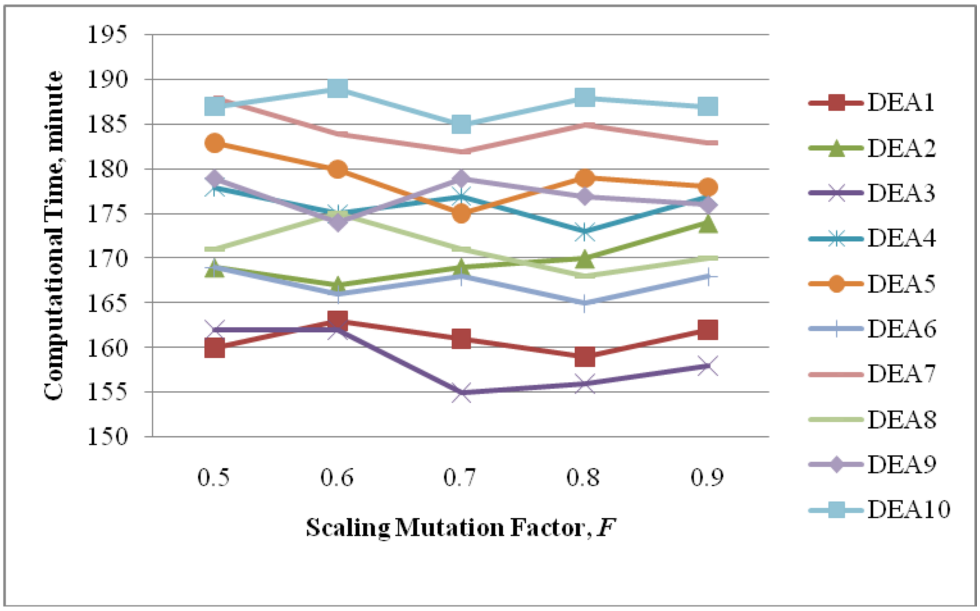
In this section, the sensitivity of scaling mutation factor of DEA method is considered as without generation resizing case for dynamic TEP problem. The scaling mutation factor of DEA method is analysed and discussed in this section. This factor is varied from 0.5 to 0.9. The settings of other DEA parameters used in this simulation are as follows: $N_p = 5*D = 2325$ and $CR = 0.6$, respectively.



(a) Average investment costs versus scaling mutation factors (*F*)



(b) Standard deviation of the investment costs versus scaling mutation factors (*F*)



(c) Average computational times versus scaling mutation factors (*F*)

Figure 7.8 Comparison of various scaling mutation factors (*F*) obtained from DEA1-DEA10 for dynamic TEP problem without generation resizing on Colombian 93-bus system

According to the obtained results in figure 7.8(a) indicate, DEA3 clearly shows the best performance to find the problem solution as shown the cheapest average investment cost for all scaling mutation factor values. On the other hand, DEA10 performs poor to find the problem solution as shown its average investment cost more expensive than other DEA strategies for all mutation factor values. The cheapest average investment cost of this case is provided by DEA3 at mutation factor $F = 0.8$, whereas the most expensive average investment cost is provided by DEA10 at mutation factor $F = 0.9$.

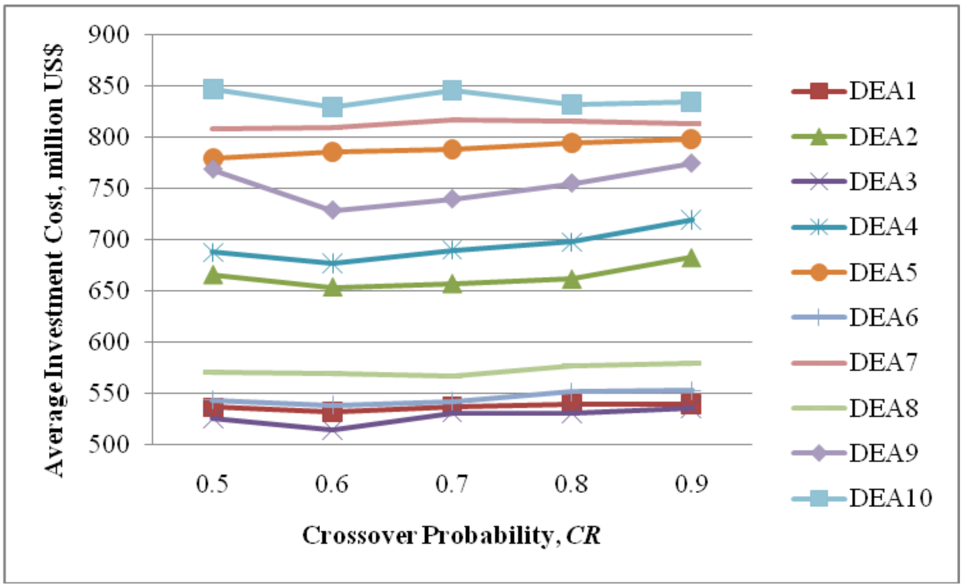
From figure 7.8(b), DEA1, DEA3 and DEA6 perform well in robustness to find the problem solution as shown small values of standard deviation for all mutation factor values. On the other hand, DA7 and DEA10 are not robust to find the problem solution as shown larger standard deviation values than other DEA strategies for all mutation factor values. In addition, the lowest standard deviation value of this case is yielded by DEA3 at mutation factor $F = 0.8$, whereas DEA7 gives the highest standard deviation value at mutation factor $F = 0.9$.

For calculation time comparison as shown in figure 7.8(c), DEA3 is faster than other DEA strategies for all mutation factor values except $F = 0.5$ where DEA1 is faster than DEA3. On the other hand, DEA10 requires computational time longer than other DEA strategies in this case.

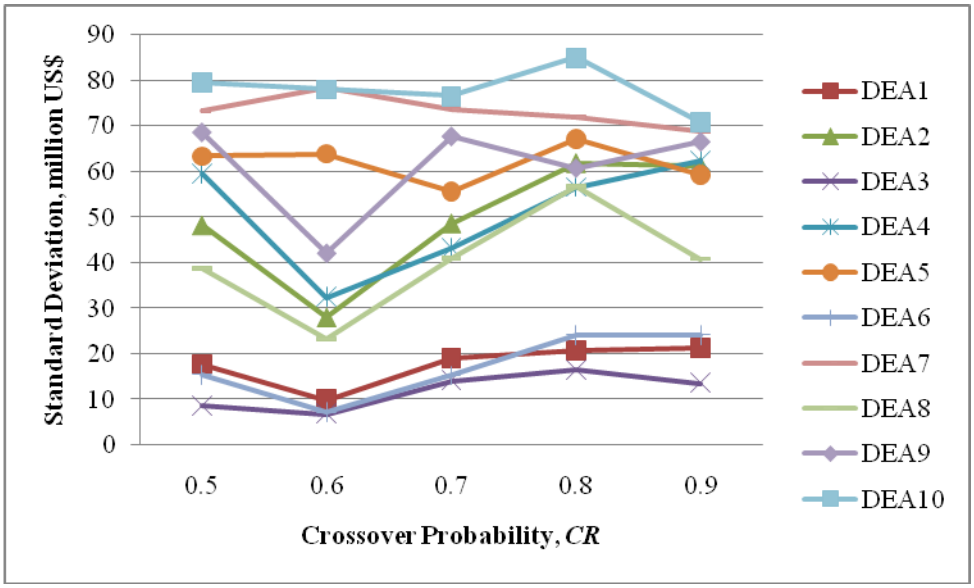
7.3.3 Sensitivity of Crossover Probability (*CR*)

In this section, the sensitivity of crossover probability of DEA method is considered only without generation resizing cases for dynamic TEP problem. The crossover probability of DEA method is analysed and discussed in this section where it is varied from 0.5 to 0.9. The settings of other DEA parameters used in this simulation are as follows: $N_p = 5*D = 2325$ and $F = 0.8$, respectively.

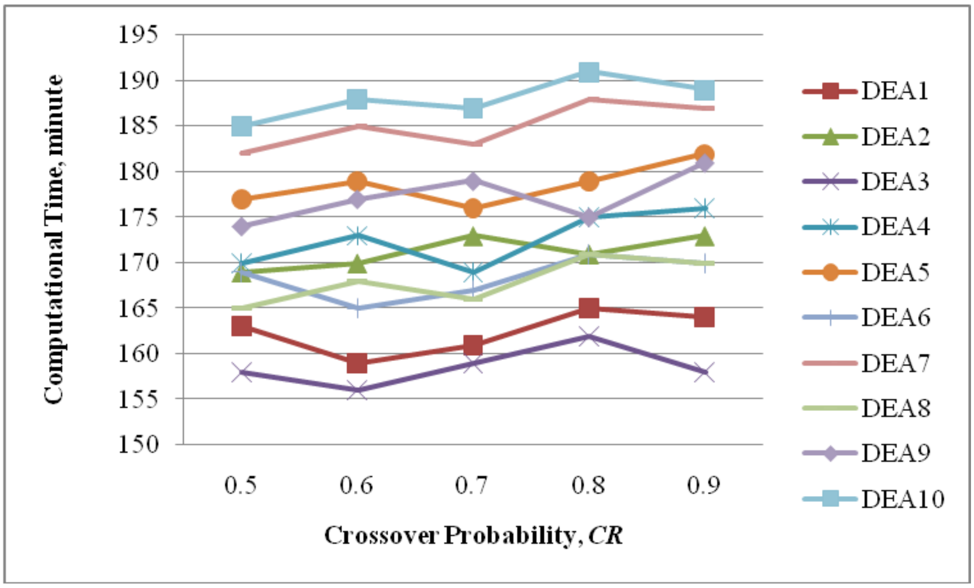
As the obtained results on graphs as illustrated in figure 7.9(a) indicate, DEA3 perform well to find the problem solution as shown its average expansion investment cost cheaper than other DEA strategies for all values of crossover probability. On the other hand, DEA10 performs poor to find the problem solution as shown its average expansion cost more expensive than other DEA strategies for all crossover probability values. In addition, the cheapest average investment cost of this case is provided by DEA3 at crossover probability $CR = 0.6$, whereas the most expensive average investment cost is provided by DEA10 at crossover probability $CR = 0.5$.



(a) Average investment costs versus crossover probabilities (*CR*)



(b) Standard deviation of the investment costs versus crossover probabilities (*CR*)



(c) Average computational times versus crossover probabilities (*CR*)

Figure 7.9 Comparison of various crossover probabilities (*CR*) obtained from DEA1-DEA10 for dynamic TEP problem without generation resizing on Colombian 93-bus system

From figure 7.9(b), DEA1, DEA3 and DEA6 perform well in robustness to find the problem solution because they yield smaller standard deviation values than other DEA strategies for all values of the crossover probability. In addition, DEA3 has the highest robust performance to find the solution in this case as shown the smallest standard deviation values for all crossover probability values.

For the calculation time comparison as shown in figure 7.9(c), DEA3 requires the least calculation time compared to other DEA strategies for all crossover probability values, whereas DEA10 takes the largest computational time for this case.

7.4 Overall Discussions

The overall discussions of the achieved results on the influence of DEA control parameters variation for TEP problem are presented in this section. Initially, the sensitivity of DEA control parameters variation is investigated as with and without generation resizing consideration cases for static TEP problem. The average investment costs of the static TEP problem decline while the population sizes increase for all DEA strategies in both cases of with and without generation resizing consideration. In contrast, there are marked rise in the computational times of all DEA strategies while the population sizes increase for both cases of the static TEP problem. DEA1 shows the best performance to find the problem solution in case of without generation resizing consideration as shown smaller the average and standard deviation of expansion investment cost than other DEA strategies for all analyses in the sensitivity of DEA control parameter variation. In case of with generation resizing consideration, DEA3 is the best strategy to find the solution as shown the smallest the average and standard deviation of the expansion investment cost. For the static TEP problem in both cases of with and without generation resizing consideration, the suitable values of scaling mutation factor and crossover probability for DEA1 and DEA3 are 0.7 and 0.6, which provide the smallest values of the average and standard deviation of investment cost.

Finally, the sensitivity of DEA control parameters variation is investigated only without generation resizing consideration case for dynamic TEP problem. As the obtained results, average investments costs of the dynamic TEP problem reduce very slightly while the population sizes of all DEA strategies increase. Similar to the static TEP problem, there are marked rise in the calculation times of all DEA strategies while the population sizes increase for the static TEP problem. DEA3 is superior to other strategies to find the problem solution for the dynamic TEP problem because it yields the smallest values of average and standard deviation of expansion investment cost for all sensitivity analyses in the DEA control parameters variation. For the dynamic TEP problem, the suitable values of scaling mutation factor and crossover probability for DEA3 are 0.8 and 0.6, which provide the smallest values of the average and standard deviation of investment cost.

7.5 Conclusions

In this chapter, the sensitivity analysis of DEA control parameters is carried out so as to study the effect of parameters variation on both the static and dynamic TEP problems. In addition, the sensitivity with respect to the population size, scaling mutation factor and crossover probability has been extensively investigated. The investigation of each control parameter is classified into three main aspects that are (1) average investment cost, (2) standard deviation, and (3) average computation time. For the static TEP problem analysis, DEA1 and DEA3 perform outstandingly to find the problem solution in both cases of without and with generation resizing consideration, respectively. They provide the smallest values of average expansion investment cost and standard deviation. For the dynamic TEP problem, DEA3 is efficient and robust to find the problem solution under consideration since it is less sensitivity to the DEA control parameters. In addition, it yields the smallest values of average expansion investment cost and standard deviation for the dynamic TEP problem. For the selection of control parameters of the DEA method, the suitable settings F = 0.7 and CR = 0.6 are recommended for the static TEP problem and the suitable settings F = 0.8 and CR = 0.6 are recommended for the dynamic TEP problem.

CHAPTER 8
CONCLUSIONS AND FUTURE WORK

8.1 Conclusions

Cost-effective transmission expansion planning (TEP) is a major challenge for electrical power system optimisation as its main objective is to obtain the optimal expansion plan that meets technical requirements while offering economical investment. Over past few decades, a number of conventional methods for optimisation have been applied to solve the TEP problem; for instance, linear programming, branch and bound, dynamic programming, interactive method, nonlinear programming, mixed integer programming and interior point method. More recently, other optimisation methods based on artificial intelligence (AI) techniques have been also proposed to solve the TEP problem. These AI techniques include genetic algorithms, simulated annealing, tabu search, particle swarm optimisation, evolutionary programming and artificial neural networks. The detail of each method has been as also provided in chapter 2 of this thesis.

A novel differential evolution algorithm (DEA) is an artificial intelligence technique that was firstly introduced by Storn and Price in year 1995. The DEA becomes a reliable and versatile function optimiser that is also readily applicable to a wide range of optimisation problems. Several variations of DEA mutation strategies were proposed and implemented successfully to a real-world problem that is the design of a howling removal unit for audio communications by Storn in [51]. In addition, the DEA method has been applied to optimise a wide variety of problems in electrical power system, such as economic dispatch, short-term scheduling of hydrothermal power system, power system planning and optimal reactive power flow, as stated in chapter 3. In a number of cases, DEA has proved to be more accurate, reliable as it can provide optimum solutions within acceptable computational times. Given its success, DEA has never been employed to solve any TEP problems, it has been therefore studied and applied to solve TEP problem in this research, whereas the implementation consists of five variations of DEA mutation schemes as proposed by Storn in [51]. Moreover, five additional DEA variations have also been proposed in this research.

The main contribution of the thesis is the development of a novel DEA procedure and the application of proposed DEA method to TEP problem. First of all, chapter 4 presents the methodology where a novel DEA procedure is developed by applying several DEA mutation strategies. In order to validate its searching capability and reliability, the proposed methodology has been tested with some selected mathematical benchmark functions, namely Sphere, Rosenbrock1, Rosenbrock2, Absolute, Salomon, Schwefel and Rastrigin functions. Given the achieved results of chapter 4, some DEA strategies that are DEA1, DEA3 and DEA6 perform effectively to solve these selected benchmark test functions f_1-f_7 because they found the best function value nearly the problem solution of each case. In addition, DEA1 and DEA3 provide better results in all cases when compared to a conventional genetic algorithm (CGA) procedure, whereas all DEA strategies require smaller computational times than CGA for all cases.

Based on the results of DEA application to selected mathematical functions, as indicated in chapter 4, the proposed DEA methodology is subsequently implemented in chapter 5 to solve static TEP, which is a real-world optimisation problem. In this chapter, the simulation comprises two different scenarios of static TEP problem, with and without generation resizing. In addition, a heuristic search method has been adopted in order to deal with static TEP considering DC based power flow model constraints. The proposed method has been implemented in Matlab7 and tested on three electrical transmission networks as shown in appendix A1-A3. The obtained results indicate that a few DEA schemes, DEA1, DEA3 and DEA6 perform effectively to solve the static TEP problem for Graver 6-bus system and IEEE 25-bus system. In addition, DEA3 performs outstandingly to find the optimal solution compared to other DEA strategies and CGA procedure as shown by the least values of the best investment cost and an average result. On the other hand, all DEA strategies and CGA are not successful for finding the optimal solution in case of with generation resizing for the Brazilian 46-bus system. The most attractive feature of the proposed algorithm is its good computational performance that is faster than the CGA procedure for all cases of the static TEP problems, as presented in chapter 5.

Given its effectiveness to solve static TEP, the proposed methodology is then applied to deal with dynamic TEP problem, which is more complex and difficult. In this thesis, dynamic TEP problem based on DC power flow model has been analysed. However, the key difficulty of dynamic TEP in large-scale real-world power system is that the planning horizon has to be separated into multiple stages. The proposed method as applied to solve the dynamic TEP problem is tested on a realistically complex transmission system, the Colombian 93-bus system, as shown in appendix A4. The obtained results of the Colombian 93-bus system illustrate that DEA3 is the best algorithmic procedure to minimise the total investment cost of dynamic TEP problem on the selected real-world transmission system. In addition, DEA3 is a robust procedure for approaching problem solution as shown by the least standard deviation value and average investment cost. For the dynamic TEP study, all DEA strategies require less calculation time than CGA procedure in this test case.

Overall, a novel DEA method performs superior to other classical EAs in terms of simple implementation with high quality of solution. Meanwhile, it requires less control parameters while being independent from initialisation. In addition, its convergence is stable and robust as DEA procedure uses rather greedy selection and less stochastic approach to solve optimisation problems than other classical EAs. Unfortunately, there remains a drawback of DEA procedure that is a tedious task of the DEA control parameters tuning due to complex relationship among problem's parameters. The optimal parameter settings of the DEA method may not be found and the final result may be trapped in a local minimum.

The accuracy of the results obtained in these TEP studies is in a very good agreement with those obtained by other researchers. According to the empirical results, it can be concluded that DEA3 is the best algorithmic procedure to find optimal solution in both cases of TEP problems because it provides global convergence property, accurate solution, and efficient and robust computation compared to other DEA strategies and CGA procedure under investigation. Despite some DEA strategies show good performance to solve these selected optimisation problems in this thesis, the further research directions are also proposed in order to enhance the quality of this algorithm.

8.2 Future Work

The proposed method of this research can be further extended and performed in two major categorises, which are as follows.

8.2.1 Further Work Concerning in the Modified DEA Procedure

It is important to note that at present few algorithms have been practically applied to solve the TEP problem [1]. Although the method proposed in this thesis has been successfully solved many cases of TEP problem, it is not yet sufficiently robust for practical use for industry. The novel DEA method proposed in this thesis has the notable limitation of DEA control parameter tuning due to a complex interaction of parameters as mentioned in section 8.1. Therefore, a further improvement of the novel DEA method is essentially required before it can be generally adopted for practical use in industry.

Therefore, a self-adaptive DEA should be proposed to enhance the performance of DEA method. Meanwhile the modified version should integrate mutation factor (*F*) and crossover probability (*CR*) as additional decision variables of the problem. These two DEA control parameters are embedded as additional control variables in the first and second positions of the *D*-dimensional parent vector X_i as illustrated in figure 8.1.

	1	2	3				D+2
X_i	F_i	CR_i	x_1	....	x_j	....	x_D

Figure 8.1 Chromosome structure of self-adaptive DEA method

8.2.2 Further Work Concerning in Power System Problems

The transmission expansion planning as studied in this thesis is called basic planning, in which the security constraints are not considered. In other words, the optimal expansion plan is determined without considering the *n*-1 contingencies caused by a transmission line or generator outage. The *n*-1 security criterion is an important index in power system reliability study as it states that the system should be expanded in such a way that, if a single line or generator is withdrawn, the expanded system should still operate adequately. Moreover, the TEP with system loss consideration is a significant issue that should be included in the planning problem for enhancing the result accuracy.

Given these important issues, DEA method for TEP problem should be improved to consider real power losses and n-1 contingencies, such as single line or generator outage. This should be a future work of this research. It is also important to note that alternative solution methods such as Branch and Bound have some valuable attributes and a detailed comparison between DEA and such traditional methods can also be investigated in the future.

Finally, the economic solution of the TEP problem under the current deregulatory environment remains an important issue in electrical power system analysis, therefore this topic can be further investigated. Some issues for market-based transmission expansion planning, i.e. the losses of social welfare and the expansion flexibility in the system should be considered and included in the TEP problem.

REFERENCES

[1] G. Latorre, R. D. Cruz, J. M. Areiza and A. Villegas, "Classification of publications and models on transmission expansion planning," *IEEE Trans. Power Syst.*, vol. 18, no. 2, pp. 938-946, May 2003.

[2] Y. H. Song and M. R. Irving, "Optimisation techniques for electrical power systems: Part 2 Heuristic optimisation methods," *Power Engineering Journal*, vol. 15, no. 3, pp. 151-160, Jun. 2001.

[3] K. Y. Lee and M. A. El-Sharkawi [Ed.], *Modern heuristic optimization techniques: Theory and applications to power systems*, Wiley-IEEE Press, 2008.

[4] L. L. Lai, *Intelligent System Applications in Power Engineering: Evolutionary Programming and Neural Networks*, Wiley, 1998.

[5] R. Romero, C. Rocha, J. R. S. Mantovani and I. G. Sanchez, "Constructive heuristic algorithm for the DC model in network transmission expansion planning," *IEE Proc. Gener. Transm. Distrib.*, vol. 152, no. 2, pp. 277-282, Mar. 2005.

[6] A. H. Escobar, R. A. Gallego and R. Romero, "Multistage and coordinated planning of the expansion of transmission systems," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 735-744, May 2004.

[7] A. J. Wood and B. F. Wollenberg, *Power generation operation and control*, Wiley & Sons, 2nd edition, 1996.

[8] R. Romero, A. Monticelli, A. Garcia and S. Haffner, "Test systems and mathematical models for transmission network expansion planning," *IEE Proc. Gener. Transm. Distrib.*, vol. 149, no.1, pp. 27-36, Jan. 2002.

[9] T. Sum-Im, G. A. Taylor, M. R. Irving, M. R. and Y. H. Song, “A comparative study of state-of-the-art transmission expansion planning tools,” in *Proc. the 41st International Universities Power Engineering Conference* (*UPEC 2006*), Newcastle upon Tyne, United Kingdom, pp. 267-271, 6th–8th Sep. 2006.

[10] L. L. Garver, “Transmission network estimation using linear programming,” *IEEE Trans. Power App. Syst.*, vol. PAS-89, no.7, pp.1688-1697, Sep./Oct. 1970.

[11] A. O. Ekwue and B. J. Cory, “Transmission system expansion planning by interactive methods,” *IEEE Trans. Power App. Syst.*, vol. PAS-103, no.7, pp.1583-1591, Jul. 1984.

[12] H. K. Youssef and R. Hackam, “New transmission planning model,” *IEEE Trans. Power Syst.*, vol. 4, pp. 9–18, Feb. 1989.

[13] Y. P. Dusonchet and A. H. El-Abiad, “Transmission planning using discrete dynamic optimization,” *IEEE Trans. Power App. Syst.*, vol. PAS-92, pp. 1358–1371, Jul. 1973.

[14] S. Haffner, A. Monticelli, A. Garcia, J. Mantovani and R. Romero, “Branch and bound algorithm for transmission system expansion planning using transportation model,” *IEE Proc. Gener. Transm. Distrib.*, vol. 147, no.3, pp. 149-156, May 2000.

[15] S. Haffner, A. Monticelli, A. Garcia and R. Romero, “Specialised branch and bound algorithm for transmission network expansion planning,” *IEE Proc. Gener. Transm. Distrib.*, vol. 148, no. 5, pp. 482-488, Sep. 2001.

[16] L. Bahiense, G. C. Oliveira, M. Pereira, and S. Granville, “A mixed integer disjunctive model for transmission network expansion,” *IEEE Trans. Power Syst.*, vol. 16, pp. 560–565, Aug. 2001.

[17] N. Alguacil, A. L. Motto and A. J. Conejo, “Transmission expansion planning: A

mixed-integer LP approach," *IEEE Trans. Power Syst.*, vol. 18, no. 3, pp. 1070–1077, Aug. 2003.

[18] S. Binato, M. V. F. Pereira and S. Granville, "A new Benders decomposition approach to solve power transmission network design problems," *IEEE Trans. Power Syst.*, vol. 16, no. 2, pp. 235-240, May 2001.

[19] R. Romero, C. Rocha, M. Mantovani and J. R. S. Mantovani, "Analysis of heuristic algorithms for the transportation model in static and multistage planning in network expansion systems," *IEE Proc. Gener. Transm. Distrib.*, vol. 150, no. 5, pp. 521-526, Sep. 2003.

[20] E. L. Silva, J. M. A. Ortiz, G. C. Oliveira and S. Binato, "Transmission network expansion planning under a tabu search approach," *IEEE Trans. Power Systems*, vol.16, no.1, pp. 62-68, Feb. 2001.

[21] R. Romero, R. A. Gallego and A. Monticelli, "Transmission system expansion planning by simulated annealing," *IEEE Trans. Power Systems*, vol.11, no.1, pp. 364-369, Feb. 1996.

[22] R. A. Gallego, A. Monticelli and R. Romero, "Transmission system expansion planning by an extended genetic algorithm," *IEE Proc. Gener. Transm. Distrib.*, vol. 145, no.3, pp. 329-335, May 1998.

[23] E. L. Silva, H. A. Gil and J. M. Areiza, "Transmission network expansion planning under an improved genetic algorithm," *IEEE Trans. Power Systems*, vol.15, no.3, pp. 1168-1175, Aug. 2000.

[24] H. A. Gil and E. L. Silva, "A reliable approach for solving the transmission network expansion planning problem using genetic algorithms," *Elsevier Science, Electric Power Systems Research*, vol. 58, pp.45-51, 2001.

[25] T. Al-Saba and I. El-Amin, "The application of artificial intelligent tools to the

transmission expansion problem," *Elsevier Science, Electric Power Systems Research*, vol. 62, pp.117-126, 2002.

[26] F. D. Galiana, D. T. McGillis and M. A. Marin, "Expert systems in transmission planning," *Proc. the IEEE*, vol. 80, no. 5, pp. 712-726, May 1992.

[27] R. K. Gajbhiye, D. Naik, S. Dambhare and S. A. Soman, "An expert system approach for multi-year short-term transmission system expansion planning: An Indian experience," *IEEE Trans. Power Systems*, vol.23, no.1, pp. 226-237, Feb. 2008.

[28] J. L. Ceciliano and R. Nieva, "Transmission network planning using evolutionary programming," in *Proc. the 1999 Congress on Evolution Computation* (*CEC 1999*), Washington DC, U.S.A., vol. 3, pp. 1796-1803.

[29] J. F. Gomez, H. M. Khodr, P. M. De Oliveira, L. Ocque, J. M. Yusta, R. Villasana, and A. J. Urdaneta, "Ant colony system algorithm for the planning of primary distribution circuits," *IEEE Trans. Power Systems*, vol.19, no.2, pp. 996-1004, May 2004.

[30] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proc. IEEE International Conference on Neural Networks* (*ICNN 1995*), Perth, Australia, vol. 4, pp. 1942-1948, 27th Nov. – 1th Dec. 1995.

[31] Y. X. Jin, H. Z. Cheng, J. Y. Yan and L. Zhang, "New discrete method for particle swarm optimization and its application in transmission network expansion planning," *Elsevier Science, Electric Power Systems Research*, vol. 77, pp.227-233, 2007.

[32] M. Dorigo, "Optimization, Learning and Natural Algorithms," *PhD thesis*, Politecnico di Milano, Italy, 1992.

[33] J. H. Holland, "Adaptation in natural and artificial systems: an introductory

analysis with applications to biology, control and artificial intelligence," The University of Michigan Press, Ann Arbor, U.S.A., 1975.

[34] Y. H. Song and C. S. V. Chou, "Advanced engineered-conditioning genetic approach to power economic dispatch," *IEE Proc. Gener. Transm. Distrib.*, vol. 144, no. 3, pp. 285-292, May 1997.

[35] S. A. Kazarlis, A. G. Bakirtzis and V. Petridis, "A genetic algorithm solution to the unit commitment problem," *IEEE Trans. Power Syst.*, vol. 11, no. 1, pp. 83-92, Feb. 1996.

[36] T. T. Maifeld and G. B. Sheble, "Genetic-based unit commitment algorithm," *IEEE Trans. Power Syst.*, vol. 11, no. 3, pp. 1359-1370, Aug. 1996.

[37] S. O. Orero and M. R. Irving, "A genetic algorithm for generator scheduling in power system," *Int. Journal of Electrical Power and Energy Systems*, vol. 18, no. 1, pp. 19-26, 1996.

[38] S. O. Orero and M. R. Irving, "A genetic algorithm modelling framework and solution technique for short term optimal hydrothermal scheduling," *IEEE Trans. Power Syst.*, vol. 13, no. 2, pp. 501-518, May 1998.

[39] A. G. Bakirtzis, P. N. Biskas, C. E. Zoumas and V. Petridis, "Optimal power flow by enhanced genetic algorithm," *IEEE Trans. Power Syst.*, vol. 17, no. 2, pp. 229-236, May 2002.

[40] R. Yokoyama, T. Niimura and Y. Nakanishi, "A coordinated control of voltage and reactive power by heuristic modeling and approximate reasoning," *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 636-645, May 1993.

[41] S. Sundhararajan and A. Pahwa, "Optimal selection of capacitors for radial distribution systems using a genetic algorithm," *IEEE Trans. Power Syst.*, vol. 9, no. 3, pp. 1499-1507, Aug. 1994.

[42] M. Delfanti, G. P. Granelli, P. Marannino and M. Montagna, "Optimal capacitor placement using deterministic and genetic algorithms," *IEEE Trans. Power Syst.*, vol. 15, no. 3, pp. 1041-1046, Aug. 2000.

[43] Y. Fukuyama and H. D. Chiang, " A parallel genetic algorithm for generation expansion planning," *IEEE Trans. Power Syst.*, vol. 11, no. 2, pp. 955-961, May 1996.

[44] I. J. Silva, M. J. Rider, R. Romero, A. V. Garcia and C. A. Murari, "Transmission network expansion planning with security constraints," *IEE Proc. Gener. Transm. Distrib.*, vol. 152, no. 6, pp. 828-836, Nov. 2005.

[45] D. E.Goldberg, *Genetic algorithm in search, optimization and machine learning*, Reading M. A. Addison Wesley, 1989.

[46] R. Storn and K. Price, "Differential evolution: a simple and efficient adaptive scheme for global optimization over continuous spaces," *Technical Report,* International Computer Science Institute, Berkley, 1995.

[47] K. V. Price, R. M. Storn and J. A. Lampinen, *Differential evolution*: *a practical approach to global optimization*, Springer, 2005.

[48] K. V. Price, "An introduction to differential evolution," in *New ideas in optimization,* D. Corne, M. Dorigo and F. Glover, Ed., McGraw-Hill, 1999.

[49] R. Storn and K. Price, "Differential evolution: a simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, Kluwer Academic Publishers, vol. 11, pp. 341-359, 1997.

[50] K. Price and R. Storn, "Differential evolution: a simple evolution strategy for fast optimization," *Dr. Dobb's Journal,* vol. 264, pp. 18-24, Apr. 1997.

[51] R. Storn, "On the usage of differential evolution for function optimization," in *Proc. the 1996 Biennial Conference of the North American Fuzzy Information*

Processing Society, (*NAFIPS 1996*), pp. 519 – 523, 19th-22nd Jun. 1996.

[52] J. Lampinen, "A constraint handling approach for the differential evolution algorithm," in *Proc. the 2002 Congress on Evolutionary Computation* (*CEC'02*), vol. 2, pp. 1468-1473, 2002.

[53] R. Gamperle, S. D. Muller and P. Koumoutsakos, "A parameter study for differential evolution," *Advances in Intelligent Systems*, *Fuzzy Systems*, *Evolution Computation*, WSEAS Press, pp. 293-298, 2002.

[54] J. Vesterstrom and R. Thomsen, "A comparative study of differential evolution, particle swarm optimization and evolutionary algorithms on numerical benchmark problems," in *Proc. the 2004 IEEE Congress on Evolutionary Computation* (*CEC'04*), Jun. 2004.

[55] E. Mezura-Montes, J. Velazquez-Reyes and C. Coello Coello, "Modified differential evolution for constrained optimization," in *Proc. the 2006 IEEE Congress on Evolutionary Computation* (*CEC'06*), pp. 25-32, Jul. 2006.

[56] K. P. Wong and Z. Y. Dong, "Differential evolution, an alternative approach to evolutionary algorithm," in *Proc. 2005 International Symposium on Antenna and Propagation* (*ISAP 2005*), pp. 73-83, 6th-10th Nov. 2005.

[57] Z. Y. Dong, M. Lu, Z. Lu and K. P. Wong, "A differential evolution based method for power system planning," in *Proc. 2006 IEEE Congress on Evolution Computation Conf.*, Vancouver, BC, Canada, pp. 2699-2706, 16th-21th Jul. 2006.

[58] R. E. Perez-Guerrero and J. R. Cedeno-Maldonado, "Economic power dispatch with non-smooth cost functions using differential evolution," in *Proc. the 37th Annual North American Power Symposium* 2005, pp. 183-190, 23rd-25th Oct. 2005.

[59] L. dos Santos Coelho and V. C. Mariani, "Combining of Chaotic Differential

Evolution and Quadratic Programming for Economic Dispatch Optimization with Valve-Point Effect," *IEEE Trans. Power Syst.*, vol. 21, no. 2, pp. 989-996, May 2006.

[60] J. P. Chiou, C. F. Chang and C. T. Su, "Variable scaling hybrid differential evolution for solving network reconfiguration of distribution systems," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 668-674, May 2005.

[61] L. Lakshminarasimman and S. Subramanian, "Short-term scheduling of hydrothermal power system with cascaded reservoirs by using modified differential evolution," *IEE Proc. Gener. Transm. Distrib.*, vol. 153, no. 6, pp. 693-700, Nov. 2006.

[62] H. K. Kim, J. K. Chong, K. Y. Park and D. A. Lowther, "Differential evolution strategy for constrained global optimization and application to practical engineering problems," *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1565-1568, Apr. 2007.

[63] C. H. Liang, C. Y. Chung, K. P. Wong, X. Z. Duan, and C. T Tse, "Study of differential evolution for optimal reactive power flow," *IET Proc. Gener. Transm. Distrib.*, vol. 1, no. 2, pp. 253-260, Mar. 2007.

[64] H. R. Cai, C. Y. Chung and K. P. Wong, "Application of differential evolution algorithm for transient stability constrained optimal power flow," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 719-728, May 2008.

[65] Z. Michalewicz and M. Schoenauer, "Evolutionary algorithms for constrained parameter optimization problems," *Evolutionary Computation*, vol. 4, no. 1, pp. 1-32, 1996.

[66] R. Villasana, L. L. Garver, and S. J. Salon, "Transmission network planning using linear programming," *IEEE Trans. Power App. Syst.*, vol. PAS-104, no. 2,

pp. 349-356, 1985.

[67] M. J. Rider, A.V. Garcia and R. Romero, “Transmission system expansion planning by a branch and bound algorithm,” *IET Proc. Gener. Transm. Distrib.*, vol. 2, no. 1, pp. 90-99, 2008.

[68] I. J. Silva, M. J. Rider, R. Romero and C. A. Murari, “Genetic algorithm of Chu and Beasley for static and multistage transmission expansion planning,” in *Proc. IEEE Power Engineering Society General Meeting* (*PES* 2006), Montreal, Canada, 7 pages, 18th-22nd June 2006.

[69] A. Escobar, “Planeamiento Dinamico de la expansion de sistemas de transmision usando algoritmos combinatoriales,” *PhD thesis*, Universidad Technologica Depereira, 2002.

[70] K. Zielinski, P. Weitkemper, R. Laur and K. D. Kammeyer, “Parameter study for differential evolution using a power allocation problem including interference cancellation,” in *Proc. 2006 IEEE Congress on Evolution Computation Conf.*, Vancouver, BC, Canada, pp. 6748-6754, 16th-21th Jul. 2006.

[71] S. K. Wang, J. P. Chiou and C. W. Liu, “Non-smooth/ non-convex economic dispatch by a novel hybrid differential evolution algorithm,” *IET Proc. Gener. Transm. Distrib.*, vol. 1, no. 5, pp. 793-803, Sep. 2007.

[72] C. H. Liang, C. Y. Chung, K. P. Wong and Z. Y. Duan, “Parallel optimal reactive power flow based on cooperative co-evolutionary differential evolution and power system decomposition,” *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 249-257, Feb. 2007.

[73] E. N. Asada, E. Carreno, R. Romero and A. V. Garcia, “A branch-and-bound algorithm for the multi-stage transmission expansion planning,” in *Proc. IEEE Power Engineering Society General Meeting* (*PES* 2005), San Francisco, U.S.A.,

vol. 1, pp. 171-176, 12th-16th Jun. 2005.

[74] S. H. M. Hashimoto, R. Romero and J. R. S. Mantovani, “Efficient linear programming algorithm for the transmission network expansion planning problem,” *IEE Proc. Gener. Transm. Distrib*., vol. 150, no. 5, pp. 536-542, Sep. 2003.

[75] T. Sum-Im, G. A. Taylor, M. R. Irving and Y. H. Song, “A differential evolution algorithm for multistage transmission expansion planning,” in *Proc. the 42nd International Universities Power Engineering Conference* (*UPEC 2007*), Brighton, United Kingdom, pp. 357-364, 4th-6th Sep. 2007.

[76] R. Romero, M. J Rider and I. J. Silva, “A metaheuristic to solve the transmission expansion planning,” *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 2289-2291, Nov. 2007.

APPENDIX A

TEST SYSTEMS DATA

A1 Garver 6-Bus System

Table A1.1 Generation and load data for Garver 6-bus system

Bus No.	Generation, MW		Load, MW	Bus No.	Generation, MW		Load, MW
	Maximum	Level			Maximum	Level	
1	150	50	80	4	0	0	160
2	0	0	240	5	0	0	240
3	360	165	40	6	600	545	0

Table A1.2 Branch data for Garver 6-bus system

From-To	n_{ij}^{0}	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, $\times 10^3$ US$
1-2	1	0.4	100	40
1-4	1	0.6	80	60
1-5	1	0.2	100	20
2-3	1	0.2	100	20
2-4	1	0.4	100	40
2-6	0	0.3	100	30
3-5	1	0.2	100	20
4-6	0	0.3	100	30
5-6	0	0.61	78	61

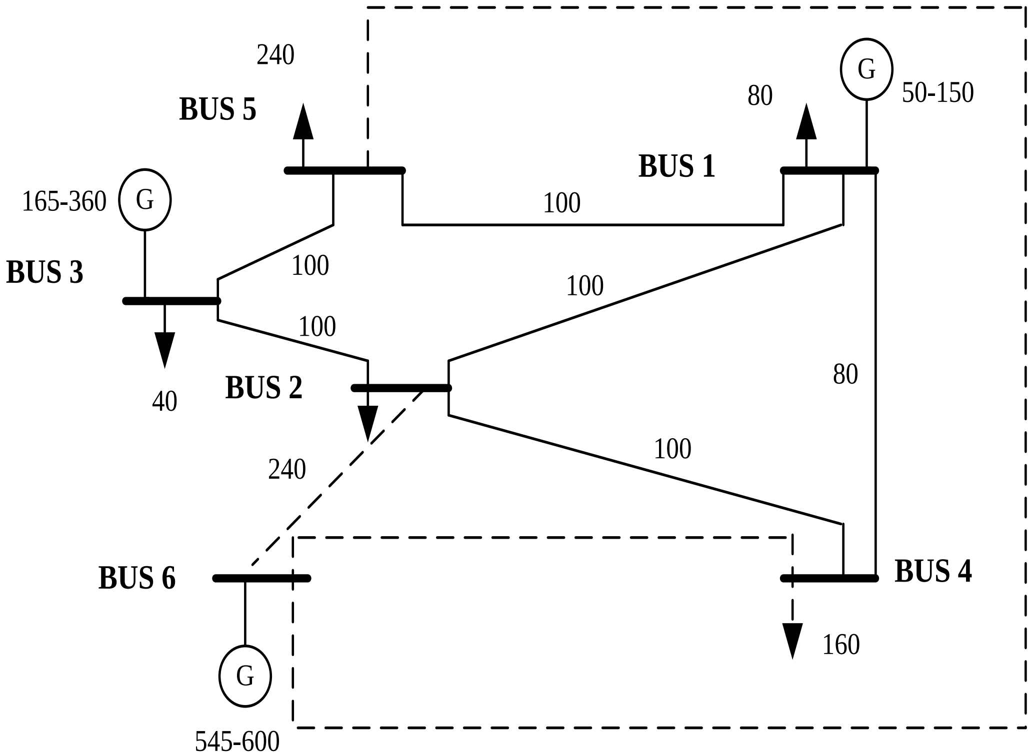


Figure A1 Garver 6-Bus System

A2 IEEE 25-Bus System

Table A2.1 Generation and load data for IEEE 25-bus system

Bus No.	Generation, MW		Load, MW	Bus No.	Generation, MW		Load, MW
	Maximum	Level			Maximum	Level	
1	660	530	0	14	215	43	317
2	0	0	128	15	0	0	0
3	0	0	181	16	0	0	0
4	0	0	74	17	192	40	108
5	0	0	71	18	0	0	175
6	0	0	71	19	192	40	97
7	595	594	265	20	0	0	195
8	0	0	194	21	0	0	136
9	400	400	333	22	155	155	100
10	300	300	0	23	0	0	180
11	400	400	0	24	300	60	125
12	0	0	0	25	660	330	0
13	0	0	0				

Table A2.2 Branch data for IEEE 25-bus system

From-To	n_{ij}^0	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, $\times 10^3$ US$
1-2	1	0.0108	800	3760
1-7	1	0.0865	65	27808
1-13	1	0.0966	100	30968
2-3	1	0.0198	500	7109
3-22	1	0.0231	200	8187
4-18	1	0.1037	1000	4907
4-19	1	0.1267	250	5973
5-17	1	0.0854	800	3987
5-20	1	0.0883	940	4171
5-25	0	0.0902	220	1731
6-18	1	0.1651	440	7776
6-20	1	0.1651	280	7776
6-24	1	0.0614	1080	2944
7-13	1	0.0476	250	16627
7-16	1	0.0476	90	16627
8-16	1	0.0418	490	14792
8-22	1	0.0389	65	13760
9-11	1	0.0129	260	4587
9-15	1	0.0144	250	5112
10-11	1	0.0678	800	21909
10-15	1	0.1053	250	33920
11-14	1	0.0245	700	8507
12-14	1	0.0519	100	16915
12-23	1	0.0839	70	675
13-18	1	0.0839	100	675
13-20	1	0.0839	250	675
14-22	1	0.0173	200	5963
15-22	1	0.0259	360	9243
16-18	1	0.0839	250	675
16-20	1	0.0839	564	675
17-19	1	0.0139	400	493
17-23	1	0.2112	350	8880
18-23	1	0.1190	150	5605
19-21	1	0.1920	110	9045
20-21	1	0.0605	180	2245
24-25	0	0.1805	220	3067

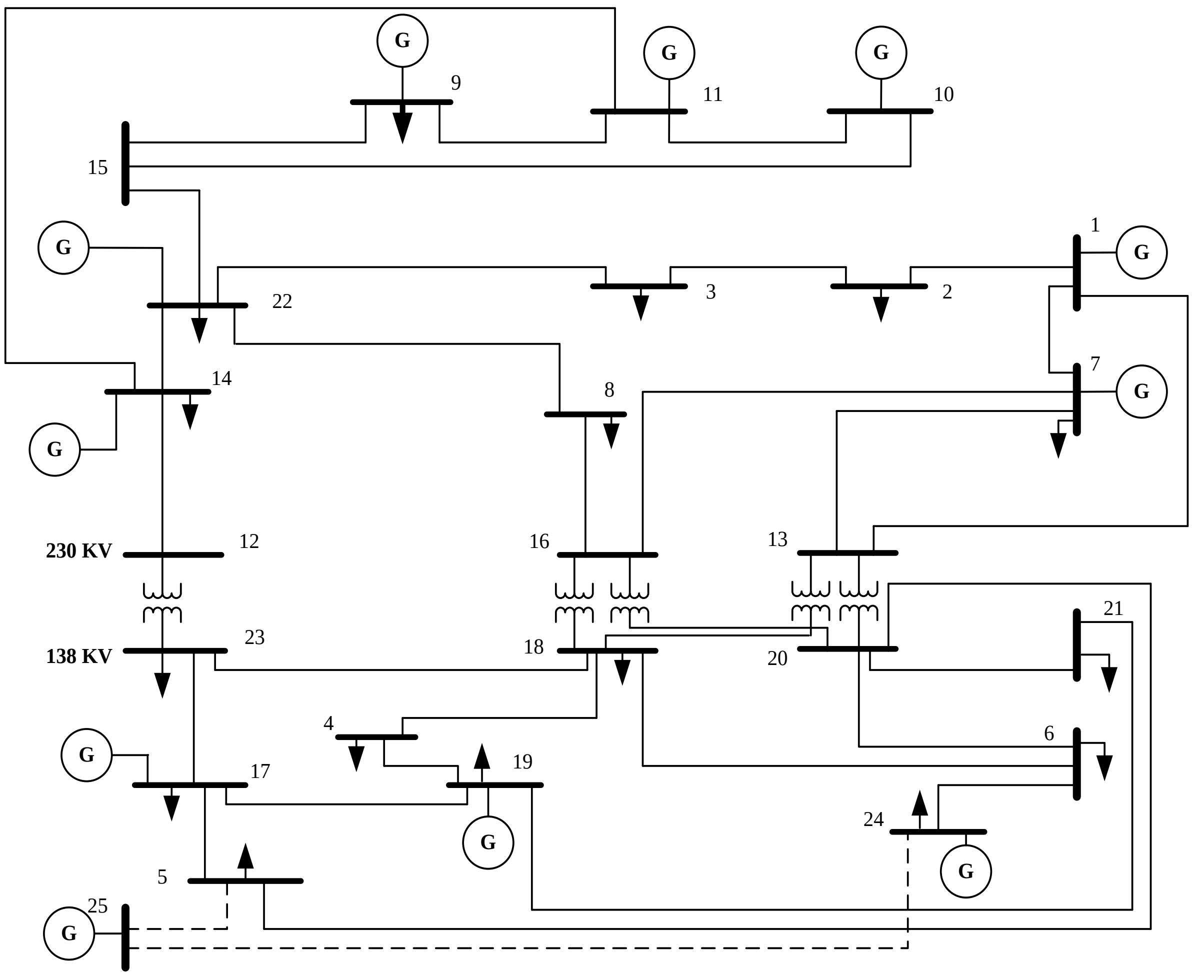


Figure A2 IEEE 25-Bus System

A3 Brazilian 46-Bus System

Table A3.1 Generation and load data for Brazilian 46-bus system

Bus No.	Generation, MW		Load, MW	Bus No.	Generation, MW		Load, MW
	Maximum	Level			Maximum	Level	
1	0	0	0	24	0	0	478.2
2	0	0	443.1	25	0	0	0
3	0	0	0	26	0	0	231.9
4	0	0	300.7	27	220	54	0
5	0	0	238	28	800	730	0
6	0	0	0	29	0	0	0
7	0	0	0	30	0	0	0
8	0	0	72.2	31	700	310	0
9	0	0	0	32	500	450	0
10	0	0	0	33	0	0	229.1
11	0	0	0	34	748	221	0
12	0	0	511.9	35	0	0	216
13	0	0	185.8	36	0	0	90.1
14	1257	944	0	37	300	212	0
15	0	0	0	38	0	0	216
16	2000	1366	0	39	600	221	0
17	1050	1000	0	40	0	0	262.1
18	0	0	0	41	0	0	0
19	1670	773	0	42	0	0	1607.9
20	0	0	1091.2	43	0	0	0
21	0	0	0	44	0	0	79.1
22	0	0	81.9	45	0	0	86.7
23	0	0	458.1	46	700	599	0

Table A3.2 Branch data for Brazilian 46-bus system

From-To	n_{ij}^0	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost,×10^3 US$	From-To	n_{ij}^0	Reactanc e x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, ×10^3 US$
1-2	2	0.1065	270	7076	20-21	1	0.0125	600	8178
1-7	1	0.0616	270	4349	20-23	2	0.0932	270	6268
2-3	0	0.0125	600	8178	21-25	0	0.0174	2000	21121
2-4	0	0.0882	270	5965	22-26	1	0.0790	270	5409
2-5	2	0.0324	270	2581	23-24	2	0.0774	270	5308
3-46	0	0.0203	1800	24319	24-25	0	0.0125	600	8178
4-5	2	0.0566	270	4046	24-33	1	0.1448	240	9399
4-9	1	0.0924	270	6217	24-34	1	0.1647	220	10611
4-11	0	0.2246	240	14247	25-32	0	0.0319	1400	37109
5-6	0	0.0125	600	8178	26-27	2	0.0832	270	5662
5-8	1	0.1132	270	7480	26-29	0	0.0541	270	3894
5-9	1	0.1173	270	7732	27-29	0	0.0998	270	6672
5-11	0	0.0915	270	6167	27-36	1	0.0915	270	6167
6-46	0	0.0128	2000	16005	27-38	2	0.2080	200	13237
7-8	1	0.1023	270	6823	28-30	0	0.0058	2000	8331
8-13	1	0.1348	240	8793	28-31	0	0.0053	2000	7819
9-10	0	0.0125	600	8178	28-41	0	0.0339	1300	39283
9-14	2	0.1756	220	11267	28-43	0	0.0406	1200	46701
10-46	0	0.0081	2000	10889	29-30	0	0.0125	600	8178
11-46	0	0.0125	600	8178	31-32	0	0.0046	2000	7052
12-14	2	0.0740	270	5106	31-41	0	0.0278	1500	32632
13-18	1	0.1805	220	11570	32-41	0	0.0309	1400	35957
13-20	1	0.1073	270	7126	32-43	1	0.0309	1400	35957

Table A3.2 Branch data for Brazilian 46-bus system (Contd.)

From-To	n_{ij}^0	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, $\times 10^3$ US$	From-To	n_{ij}^0	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, $\times 10^3$ US$
14-15	0	0.0374	270	2884	33-34	1	0.1265	270	8288
14-18	2	0.1514	240	9803	34-35	2	0.0491	270	3591
14-22	1	0.0840	270	5712	35-38	1	0.1980	200	12631
14-26	1	0.1614	220	10409	36-37	1	0.1057	270	7025
15-16	0	0.0125	600	8178	37-39	1	0.0283	270	2329
16-17	1	0.0078	2000	10505	37-40	1	0.1281	270	8389
16-28	0	0.0222	1800	26365	37-42	1	0.2105	200	13388
16-32	0	0.0311	1400	36213	38-42	3	0.0907	270	6116
16-46	1	0.0203	1800	24319	39-42	3	0.2030	200	12934
17-19	1	0.0061	2000	8715	40-41	0	0.0125	600	8178
17-32	0	0.0232	1700	27516	40-42	1	0.0932	270	6268
18-19	1	0.0125	600	8178	40-45	0	0.2205	180	13994
18-20	1	0.1997	200	12732	41-43	0	0.0139	2000	17284
19-21	1	0.0278	1500	32632	42-43	1	0.0125	600	8178
19-25	0	0.0325	1400	37748	42-44	1	0.1206	270	7934
19-32	1	0.0195	1800	23423	44-45	1	0.1864	200	11924
19-46	1	0.0222	1800	26365					

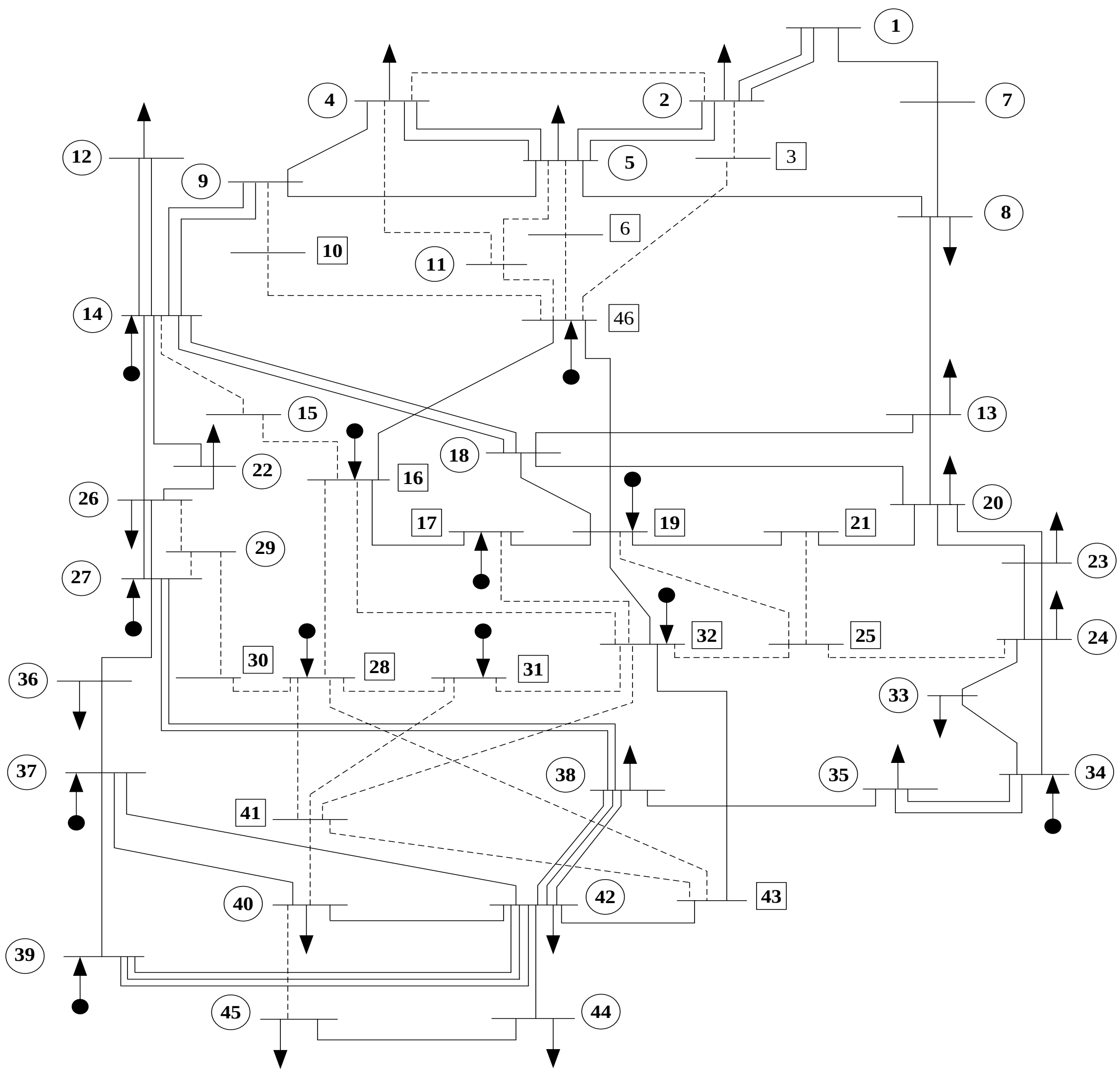


Figure A3 Brazilian 46-Bus System

A4 Colombian 93-Bus System

Table A4.1 Generation and load data for Colombian 93-bus system

Bus No.	Year 2005		Year 2009		Year 2012	
	Gen, MW	Load, MW	Gen, MW	Load, MW	Gen, MW	Load, MW
1	240	0	240	0	240	0
2	0	352.9	165	406.53	165	486.66
3	0	393	0	490.5	0	587.08
4	0	0	0	0	0	0
5	40	235	40	293.56	40	351.42
6	34	0	34	0	34	0
7	0	300	0	374.26	136	448.03
8	100	339	230	423	230	505.87
9	0	348	0	434.12	0	519.69
10	0	60	0	74.21	0	88.84
11	80	147	108	183.9	108	220.15
12	47	0	47	0	47	0
13	0	174	0	217.26	0	260.08
14	0	0	0	0	0	0
15	0	377	0	470.17	0	562.84
16	0	236	0	294	0	351.9
17	35	136	35	169.57	35	203
18	480	36.2	540	45.2	540	54.1
19	900	19.6	1340	24.46	1340	29.28
20	0	202.4	0	252.5	45	302.27
21	0	186	0	231.7	0	277.44
22	200	53	200	66.13	200	79.17
23	0	203	0	252.5	0	302.27
24	120	0	150	0	150	0
25	86	0	86	0	86	0
26	70	0	70	0	70	0
27	0	266	0	331.4	0	396.71
28	0	326	0	406.3	14	486.39
29	618	339	618	422.6	618	505.96
30	0	137	0	166.7	0	199.55
31	189	234	189	327.3	189	391.88
32	0	126	0	157.3	0	188.33
33	0	165	0	206.53	0	247.24
34	0	77.5	0	96.7	0	115.81
35	200	172	200	214.6	200	256.86
36	0	112	0	140	44	167.29
37	138	118	138	147.3	138	176.3
38	0	86	15	108.4	15	129.72
39	0	180	0	224	15	268.19
40	305	0	305	0	305	0

Table A4.1 Generation and load data for Colombian 93-bus system (Contd.)

Bus No.	Year 2005		Year 2009		Year 2012	
	Gen, MW	Load, MW	Gen, MW	Load, MW	Gen, MW	Load, MW
41	70	54.8	100	68.4	100	81.85
42	0	102	0	127.3	0	152.39
43	0	35.4	0	44.2	0	52.9
44	23	257	23	321.3	23	384.64
45	950	0	1208	0	1208	0
46	150	121	150	151.7	150	181.62
47	0	41.15	0	51.5	0	61.6
48	775	600	885	750	885	896.26
49	0	130	0	162	0	193.27
50	240	424	240	528	240	632.75
51	0	128	0	159	0	190.45
52	0	38	0	46.5	0	55.6
53	280	0	320	0	320	0
54	0	76	0	95.3	0	114.19
55	40	223	40	279	40	333.59
56	0	0	0	0	0	0
57	0	226	130	281	130	336.94
58	190	0	190	0	190	0
59	160	0	160	0	160	0
60	1191	0	1216	0	1216	0
61	155	0	155	0	155	0
62	0	0	0	0	0	0
63	900	35	1090	44	1090	52.77
64	0	88	0	110.55	280	132.35
65	0	132	0	165	0	197.58
66	200	0	300	0	300	0
67	474	266	474	332.45	474	397.98
68	0	0	0	0	0	0
69	0	71.4	0	89	0	106.61
70	30	0	180	0	180	0
71	0	315	211	393	424	471.21
72	0	0	0	0	0	0
73	0	0	0	0	0	0
74	0	0	0	0	0	0
75	0	0	0	0	0	0
76	40	0	40	0	40	0
77	0	55	0	70	0	82.85
78	0	36.65	0	45.1	0	54.07
79	0	98	0	123	300	146.87
80	0	60	0	72	0	88.34

Table A4.1 Generation and load data for Colombian 93-bus system (Contd.)

Bus No.	Year 2005		Year 2009		Year 2012	
	Gen, MW	Load, MW	Gen, MW		Gen, MW	Load, MW
81	0	0	0	0	0	0
82	0	0	0	0	0	0
83	0	0	0	0	0	0
84	0	0	0	0	500	0
85	0	0	0	0	0	0
86	0	0	300	0	850	0
87	0	0	0	0	0	0
88	0	0	0	0	300	0
89	0	0	0	0	0	0
90	0	0	0	0	0	0
91	0	0	0	0	0	0
92	0	0	0	0	0	0
93	0	0	0	0	0	0

Table A4.2 Branch data for Colombian 93-bus system

From-To	n_{ij}^0	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, $\times 10^6$ US$	From-To	n_{ij}^0	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, $\times 10^6$ US$
1-3	1	0.1040	250	15.862	30-72	2	0.0173	350	5.512
1-8	1	0.0810	250	13.217	31-32	1	0.0259	350	6.547
1-11	1	0.0799	250	12.527	31-33	2	0.0248	350	6.432
1-59	2	0.0232	350	6.202	31-34	1	0.0792	250	12.412
1-71	2	0.0841	250	14.367	31-60	2	0.1944	250	25.982
1-93	1	0.0267	450	13.270	31-72	2	0.0244	350	6.317
2-4	2	0.0271	350	6.662	32-34	1	0.0540	350	9.767
2-9	1	0.0122	350	5.282	33-34	1	0.1139	320	16.322
2-83	1	0.0200	570	5.972	33-72	1	0.0228	350	6.202
3-6	1	0.0497	350	9.422	34-70	2	0.0415	350	8.272
3-71	1	0.0136	450	5.167	35-36	1	0.2074	250	27.362
3-90	1	0.0074	350	4.592	35-44	2	0.1358	250	20.347
4-5	3	0.0049	350	4.247	37-61	1	0.0139	350	4.937
4-34	2	0.1016	270	14.942	37-68	1	0.0544	320	9.652
4-36	2	0.0850	250	13.562	38-39	1	0.0300	350	6.317
5-6	2	0.0074	350	4.477	38-68	1	0.0389	350	7.927
6-10	1	0.0337	350	7.582	39-40	2	0.1020	250	16.207
7-78	1	0.0043	350	4.132	39-43	1	0.1163	250	16.552
7-90	2	0.0050	350	4.247	39-68	1	0.0145	350	5.282
8-9	1	0.0168	350	5.972	39-86	0	0.0545	350	9.880
8-59	2	0.1056	250	15.402	40-41	1	0.0186	350	5.742
8-67	0	0.2240	250	29.200	40-42	1	0.0153	350	5.167
8-71	1	0.0075	400	4.477	40-68	1	0.1320	320	18.162
8-87	1	0.0132	350	5.167	41-42	1	0.0094	350	4.707
9-69	2	0.1098	350	15.747	41-43	1	0.1142	250	16.322
9-77	1	0.0190	350	5.857	43-88	0	0.1816	250	39.560
9-83	1	0.0200	400	5.972	44-80	1	0.1014	250	17.587
10-78	1	0.0102	350	4.937	45-50	2	0.0070	350	4.362
11-92	1	0.0267	450	13.270	45-54	1	0.0946	320	13.562
12-17	1	0.0086	350	4.707	45-81	1	0.0267	450	13.270
12-75	1	0.0641	320	11.492	46-51	1	0.1141	250	16.322
12-76	1	0.0081	350	4.707	46-53	2	0.1041	250	14.597
13-14	2	0.0009	350	3.902	47-49	2	0.0942	250	13.562
13-20	1	0.0178	350	5.742	47-52	1	0.0644	350	10.572
13-23	1	0.0277	350	7.007	47-54	2	0.1003	250	14.252
14-18	2	0.1494	250	20.232	48-54	3	0.0396	350	8.042
14-31	2	0.1307	250	18.622	48-63	1	0.0238	350	6.317
14-60	2	0.1067	300	15.977	49-53	2	0.1008	250	14.252

Table A4.2 Branch data for Colombian 93-bus system (Contd.)

From-To	n_{ij}^0	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, $\times 10^6$ US$	From-To	n_{ij}^0	Reactance x_{ij}, p.u.	f_{ij}^{max}, MW	Cost, $\times 10^6$ US$
15-17	1	0.0483	320	9.422	50-54	2	0.0876	250	12.872
15-18	1	0.0365	450	7.927	51-52	1	0.0859	250	12.872
15-20	1	0.0513	320	9.652	52-88	0	0.0980	300	34.190
15-24	1	0.0145	350	5.282	54-56	3	0.0267	450	13.270
15-76	1	0.0414	320	9.882	54-63	3	0.0495	320	9.077
16-18	1	0.0625	350	10.917	55-57	1	0.0174	600	46.808
16-21	1	0.0282	350	6.892	55-62	1	0.0281	550	70.988
16-23	1	0.0238	350	6.892	55-82	1	0.0290	550	77.498
17-23	1	0.0913	250	12.987	55-84	1	0.0087	600	26.658
17-76	1	0.0020	350	3.902	56-57	2	0.0240	600	62.618
18-20	1	0.0504	350	9.537	56-81	1	0.0114	550	32.858
18-21	1	0.0348	350	7.467	57-81	0	0.0219	550	58.890
18-22	1	0.0209	350	6.432	57-84	1	0.0087	600	26.658
18-58	2	0.0212	350	5.742	59-67	2	0.1180	250	16.667
18-66	2	0.0664	350	11.377	60-62	3	0.0257	450	13.270
19-22	1	0.0691	350	11.722	60-69	2	0.0906	350	13.677
19-58	1	0.0826	320	11.722	61-68	1	0.0789	250	12.412
19-61	2	0.1105	250	16.092	62-73	1	0.0272	750	73.158
19-66	1	0.0516	350	9.307	62-82	1	0.0101	600	30.998
19-82	1	0.0267	450	13.270	64-65	1	0.0741	350	11.837
19-86	1	0.1513	300	20.922	64-74	1	0.0267	500	13.270
21-22	1	0.0549	350	9.882	66-69	2	0.1217	250	17.127
23-24	1	0.0255	350	6.317	67-68	2	0.1660	250	22.072
24-75	1	0.0161	350	5.512	68-86	1	0.0404	350	8.272
25-28	1	0.0565	320	9.767	69-70	2	0.0228	350	6.202
25-29	1	0.0570	320	9.882	72-73	2	0.0267	500	13.270
26-27	1	0.0657	350	10.917	73-74	1	0.0214	600	58.278
26-28	1	0.0512	350	9.307	73-82	0	0.0374	550	97.960
27-28	1	0.0238	350	6.202	73-89	0	0.0246	550	66.650
27-29	1	0.0166	350	5.052	74-89	0	0.0034	550	14.570
27-35	1	0.1498	250	22.072	77-79	1	0.0097	350	5.167
27-44	1	0.0893	250	16.322	79-83	0	0.0457	350	15.400
27-64	1	0.0280	350	6.777	79-87	1	0.0071	350	4.477
27-80	1	0.0242	350	7.007	82-85	1	0.0341	700	89.898
27-89	0	0.0267	450	13.270	83-85	2	0.0267	450	13.270
28-29	1	0.0281	350	6.777	85-91	1	0.0139	600	40.298
29-31	2	0.1042	250	32.981	90-91	1	0.0267	550	13.270
29-64	1	0.0063	350	4.362	91-92	1	0.0088	600	27.588
30-64	1	0.1533	250	20.577	92-93	1	0.0097	600	30.068
30-65	1	0.0910	250	13.677					

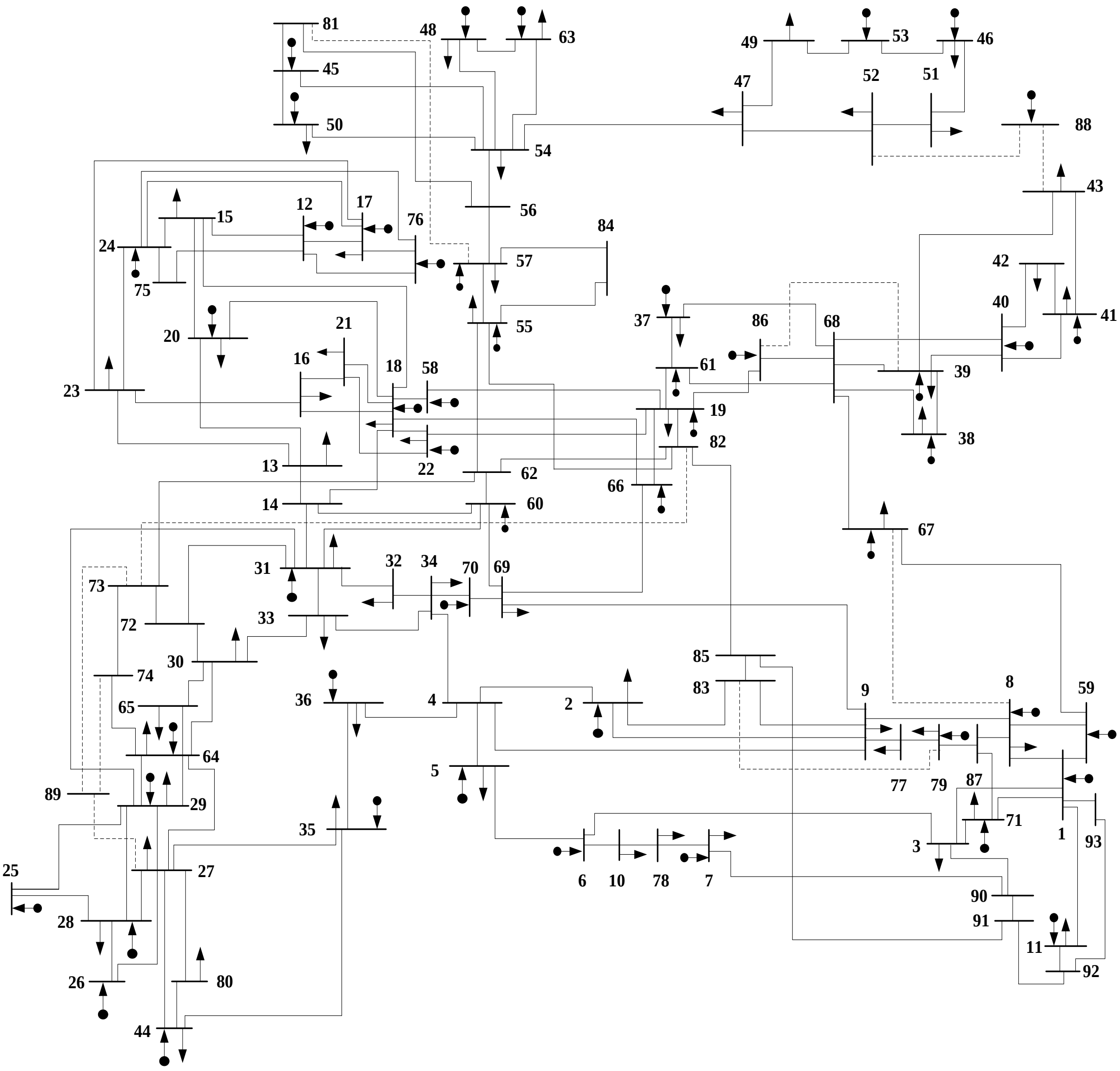


Figure A.4 Colombian 93-Bus System