Size corrected significance tests in Seemingly Unrelated Regressions with autocorrelated errors

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Abstract

Refined asymptotic methods are used to produce degrees-of-freedom-adjusted Edgeworth and Cornish-Fisher size corrections of the t and F testing procedures for the parameters of a S.U.R. model with serially correlated errors. The corrected tests follow the Student-t and F distributions, respectively, with an approximation error of order $O(\tau^3)$, where $\tau = 1/\sqrt{T}$ and T is the number of time observations. Monte Carlo simulations provide evidence that the size corrections suggested hereby have better finite sample properties, compared to the asymptotic testing procedures (either standard or Edgeworth corrected), which do not adjust for the degrees of freedom.

Key words: Linear regression; S.U.R. models; stochastic expansions; asymptotic approximations; AR(1) errors.

JEL classification: C10, C12, D24.

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1 Introduction

The use of refined asymptotic techniques can considerably improve the finite-sample performance of testing procedures in applied econometric research (see, e.g., Ullah (2004), for a survey). These techniques involve the use of Edgeworth expansions which effectively provide higher-order asymptotic approximations of the finite-sample distributions of well known econometric test statistics (see Magdalinos and Symeonides (1995), Magee (1985), Rothenberg (1984b), Symeonides et al. (2007), *inter alia*). In finite samples, there are considerable discrepancies between the actual (sample) and nominal size of many standard testing procedures, employed in econometric literature. These discrepancies are found to be very severe, especially for the generalized linear regression model with a non-scalar covariance matrix of the error terms estimated by the feasible generalized least squares (FGLS), or maximum likelihood (see, e.g., Kiviet and Phillips (1996), Ullah (2004)).

Despite the substantial amount of work on refined asymptotic bias expansions of alternative estimators for the linear regression model or simultaneous equations systems (see, e.g., Iglesias and Phillips (2010, 2012), Kiviet and Phillips (1996), Kiviet et al. (1995), Phillips (2000, 2007), *inter alia*), there are only a few papers applying these methods to conventional tests, like the F and t. Rothenberg (1984b, 1988) used Edgeworth expansions in terms of the chi-square and normal distributions to derive general formulas of corrected critical values of the Wald (or F) and t tests, respectively.

In this paper, we derive size corrections of the t and F tests for the system of Seemingly Unrelated Regression (S.U.R.) equations with first-order autoregressive error terms, introduced by Parks (1967). The fact that these tests are over-sized in finite samples can be attributed to two sources: (i) the non-zero cross-correlations of the error terms of the S.U.R. equations, and (ii) the specific dynamic structure of these error terms, i.e., the existence of serial correlation (with possibly distinct autocorrelation coefficients) in the S.U.R. equations.

Since the Edgeworth expansions are not well-defined distribution functions and they may assign negative 'probabilities' to the tails of the approximated distributions, the paper suggests using the Cornish-Fisher expansion of the tests rather than the Edgeworth expansion of their distribution functions (see Cornish and Fisher (1937), Fisher and Cornish (1960), Hill and Davis (1968), Magdalinos (1985), Ogasawara (2012), *inter alia*). The above suggested corrections are asymptotically equivalent, but there are arguments—both theoretical and practical—in favor of the Cornish-Fisher correction: First, the Cornish-Fisher corrected test statistics are theoretically superior because they are proper random variables and their distributions have well-behaved tails; second, since they do not require the calculation of new critical values, they can be readily implemented in applied research based on the publicly available tables of standard distributions.

The paper proposes the use of degrees-of-freedom-adjusted Edgeworth corrected critical values and Cornish-Fisher corrected statistics of the t and F tests when the S.U.R. model with serially correlated errors is estimated using the Parks' estimator (see Parks (1967)). These corrections follow the Student-t and F distributions, respectively, with an approximation error of order $O(\tau^3)$, where $\tau = 1/\sqrt{T}$ and T is the number of time observations of the sample. The use of degree-of-freedom-adjusted forms of the above tests lead to approximations that are 'locally exact' (see Magdalinos (1985)), which means that the approximate distributions reduce to the exact ones, when the model is sufficiently simplified. These approximations are found to improve the small-sample performance of the tests (see Magdalinos and Symeonides (1995), Symeonides et al. (2007)). To our knowledge, this is the first attempt in the literature to develop analytic size corrected testing procedures for the S.U.R. model with serially correlated errors.

The analytic size corrections suggested by the paper take into account the magnitude of the various nuisance parameters, as well as the way in which they influence the elements of the disturbance covariance matrix. They can be implemented separately to correct for the non-zero cross-correlations of the error terms, or their serial correlation effects, or the combination of the above.

The paper is organized as follows. Section 2 provides some preliminary notations. Section 3 presents the S.U.R. model and the assumptions needed in our expansions. Analytic formulas for the locally exact Edgeworth and Cornish-Fisher second-order size corrections of the t and F test statistics are derived in Section 4. Section 5 conducts out a Monte Carlo simulation evaluating the performance of the suggested corrected tests. Finally, Section 6 concludes the paper. Proofs of the results of the paper are given in the Appendix.

2 Preliminary notation

Throughout the paper, we use the tr, vec, $\otimes$, and matrix differentiation notation as defined in Dhrymes (1978, pages 518–540), and for any two indexes i and j, we denote Kronecker's delta as δ_{ij}. Moreover, any $(n \times m)$ matrix L with elements l_{ij} is denoted as

$$L = [(l_{ij})_{i=1,\,\ldots,\,n;\,j=1,\,\ldots,\,m}],$$

with obvious modifications for vectors and square matrices. If l_{ij} are $(n_i \times m_j)$ matrices, then L is the $(\sum n_i \times \sum m_j)$ partitioned matrix with sub-matrices l_{ij}. The following matrices:

$$P_X = X(X'X)^{-1}X', \quad \overline{P}_X = I - P_X = I - X(X'X)^{-1}X'$$

denote the orthogonal projectors into the spaces spanned by the columns of the matrix X and its orthogonal complement, respectively. Finally, for any stochastic quantity (scalar, vector, or matrix) we use the symbol $\mathcal{E}(\cdot)$ to denote the expectation operator.

3 The model

Consider a S.U.R. system of M contemporaneously correlated regression equations of the form

$$y_\mu = X_\mu \beta_\mu + u_\mu \quad (\mu = 1,\, \ldots,\, M), \tag{1}$$

where y_μ are $(T \times 1)$ vectors of observations on the dependent variables, X_μ are $(T \times n_\mu)$ matrices of observations on sets of n_μ non-stochastic regressors, β_μ are $(n_\mu \times 1)$ vectors of parameters to be estimated and u_μ are $(T \times 1)$ vectors of non-observable serially correlated stochastic error terms of the μ-th equation, defined as $u_{t\mu}$ $(t = 1,\, \ldots,\, T)$. These terms are generated by the following stationary first-order autoregressive (AR(1)) process:

$$u_{t\mu} = \rho_\mu u_{(t-1)\mu} + \varepsilon_{t\mu}, \quad -1 < \rho_\mu < 1 \quad (t = 1,\, \ldots,\, T;\, \mu = 1,\, \ldots,\, M), \tag{2}$$

where $\varepsilon_{t\mu}$ are normally distributed innovations (see Turkington (2000)). For any two indexes μ, $\mu' = 1, \ldots, M$, we have $\mathcal{E}(\varepsilon_{t\mu}) = 0$, for all t. Moreover, for $t \neq 1$ or $t' \neq 1$, the covariance between two innovations $\varepsilon_{t\mu}$ and $\varepsilon_{t'\mu'}$ is given as $\mathcal{E}(\varepsilon_{t\mu}\varepsilon_{t'\mu'}) = \delta_{tt'}\sigma_{\mu\mu'}$. For $t = t' = 1$ and μ, $\mu' = 1, \ldots, M$, $\mathcal{E}(\varepsilon_{t\mu}\varepsilon_{t'\mu'})$ becomes

$$\mathcal{E}(\varepsilon_{1\mu}\varepsilon_{1\mu'}) = \sigma_{\mu\mu'}(1-\rho_\mu^2)^{1/2}(1-\rho_{\mu'}^2)^{1/2}/(1-\rho_\mu\rho_{\mu'}) \tag{3}$$

(see Parks (1967, pages 507–508)). In addition to assumption $\rho_\mu \in (-1, 1)$, stationarity of AR(1) processes (2) implies the following relationships on the initial conditions of the error terms of the S.U.R. equations:

$$u_{1\mu} = (1-\rho_\mu^2)^{-1/2}\varepsilon_{1\mu} \quad (t = 1;\ \mu = 1, \ldots, M). \tag{4}$$

These relationships imply that, for all $t = 1, \ldots, T$ and $\mu, \mu' = 1, \ldots, M$, the error terms $u_{t\mu}$ satisfy the following conditions:

$$\mathcal{E}(u_{t\mu}) = 0, \quad \mathcal{E}(u_{t\mu}^2) = \sigma_{\mu\mu}/(1-\rho_\mu^2), \quad \mathcal{E}(u_{t\mu}u_{t\mu'}) = \sigma_{\mu\mu'}/(1-\rho_\mu\rho_{\mu'}). \tag{5}$$

Let $n = \sum_{\mu=1}^{M} n_\mu$, and define the $(MT \times 1)$ vectors y and u, the $(n \times 1)$ vector β and the $(MT \times n)$ block diagonal matrix X as follows:

$$\begin{gathered} y = [(y_\mu)_{\mu=1,\ldots,M}], \quad u = [(u_\mu)_{\mu=1,\ldots,M}], \\ \beta = [(\beta_\mu)_{\mu=1,\ldots,M}], \\ X = [(\delta_{\mu\mu'}X_\mu)_{\mu,\mu'=1,\ldots,M}]. \end{gathered} \tag{6}$$

Then, the system of equations (1) can be written in a matrix form as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_M \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{bmatrix}, \tag{7}$$

or more compactly as

$$y = X\beta + u. \tag{8}$$

To derive size corrected significance tests for the elements of the vector β, the above representations of the S.U.R. system will be written in an autocorrelation-free form, after applying appropriate transformations on y, X and u. Following

Parks (1967), define the $(T \times T)$ matrices P_μ and $R^{\mu\mu'}$ as follows:

$$P_\mu = \begin{bmatrix} (1-\rho_\mu^2)^{-\frac{1}{2}} & 0 & 0 & \cdots & 0 \\ (1-\rho_\mu^2)^{-\frac{1}{2}}\rho_\mu & 1 & 0 & \cdots & 0 \\ (1-\rho_\mu^2)^{-\frac{1}{2}}\rho_\mu^2 & \rho_\mu & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (1-\rho_\mu^2)^{-\frac{1}{2}}\rho_\mu^{T-1} & \rho_\mu^{T-2} & \rho_\mu^{T-3} & \cdots & 1 \end{bmatrix}, \quad R^{\mu\mu'} = P_\mu^{-1\prime} P_{\mu'}^{-1}, \tag{9}$$

and the following $(MT \times MT)$ block diagonal matrix

$$P = [(\delta_{\mu\mu'} P_\mu)_{\mu,\mu'=1,\,\ldots,\,M}]. \tag{10}$$

Then, (2) implies that the $(T \times 1)$ random vectors u_μ can be written as

$$u_\mu = P_\mu \varepsilon_\mu \quad (\mu = 1,\,\ldots,\,M), \tag{11}$$

where ε_μ are $(T \times 1)$ random vectors with non-autocorrelated elements $\varepsilon_{t\mu}$, i.e.,

$$\varepsilon_\mu = [(\varepsilon_{t\mu})_{t=1,\,\ldots,\,T;\,\mu=1,\,\ldots,\,M}]. \tag{12}$$

As in (11), consider the $(T \times 1)$ vectors $y_{\mu *}$ and $(T \times n_\mu)$ matrices $X_{\mu *}$, with non-autocorrelated elements, satisfying the following relations:

$$y_{\mu *} = P_\mu^{-1} y_\mu, \quad X_{\mu *} = P_\mu^{-1} X_\mu, \tag{13}$$

and define the $(MT \times 1)$ vector y_* and $(MT \times n)$ block diagonal matrix X_* as follows:

$$y_* = [(y_{\mu *})_{\mu=1,\,\ldots,\,M}], \quad X_* = [(\delta_{\mu\mu'} X_{\mu *})_{\mu,\mu'=1,\,\ldots,\,M}]. \tag{14}$$

Then, pre-multiplying the μ-th equation of (7) by P_μ^{-1}, we can derive the following S.U.R. model with non-autocorrelated error terms:

$$\begin{bmatrix} y_{1*} \\ y_{2*} \\ \vdots \\ y_{M*} \end{bmatrix} = \begin{bmatrix} X_{1*} & 0 & \cdots & 0 \\ 0 & X_{2*} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_{M*} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_M \end{bmatrix} \tag{15}$$

(see Zellner (1962, 1963), Zellner and Huang (1962), Zellner and Theil (1962)). In more compact form, this model can be written as

$$y_* = X_* \beta + \varepsilon, \tag{16}$$

where $y_* = P^{-1}y$, $X_* = P^{-1}X$ and $\varepsilon = P^{-1}u$. The above representation of the S.U.R. system implies that the $(MT \times 1)$ error vector u in (8) is normally distributed with mean and variance-covariance matrix given as follows:

$$\mathcal{E}(u) = 0, \quad \mathcal{E}(uu') = \Omega^{-1} = P\mathcal{E}(\varepsilon\varepsilon')P' = P(\Sigma \otimes I_T)P', \tag{17}$$

where

$$\Sigma = [(\sigma_{\mu\mu'})_{\mu,\mu'=1,\,\ldots,\,M}]. \tag{18}$$

The last relationship implies that

$$\Omega = P'^{-1}(\Sigma^{-1} \otimes I_T)P^{-1} \tag{19}$$

is a function of the $((M + M^2) \times 1)$ parameter vector $\gamma = (\varrho', \varsigma')'$, where $\varrho = (\rho_1,\, \ldots,\, \rho_M)'$ is the $(M \times 1)$ vector of autocorrelation coefficients in (2) and the $(M^2 \times 1)$ vector $\varsigma = vec(\Sigma^{-1}) \in \pounds = \mathbb{R}^{M^2} - \mho$, where $\mho$ is the subspace of $\mathbb{R}^{M^2}$ in which Σ is not positive definite. After defining the composite index

$$(\mu\mu') = \mu + M(\mu' - 1) \quad ((\mu\mu') = 1,\, \ldots,\, M^2), \tag{20}$$

for any two indexes $\mu, \mu' = 1,\, \ldots,\, M$, it can be easily seen that the $(\mu\mu')$-th element of vector ς, denoted as $\varsigma_{(\mu\mu')}$, is actually the (μ, μ')-th element of matrix Σ^{-1}, denoted as $\sigma^{\mu\mu'}$.

The system of equations (16) (or (15)) can be seen as the vectorization outcome of the following form of the S.U.R. model of M equations:

$$Y_* = ZB + E, \tag{21}$$

where Y_* and E are $(T \times M)$ random matrices defined as

$$y_* = vec(Y_*), \quad \varepsilon = vec(E), \tag{22}$$

respectively, where the rows of matrix E are $\mathcal{N}_M(0, \Sigma)$ random vectors and B is a $(K \times M)$ matrix whose columns, denoted as b_μ, are defined as

$$b_\mu = \Psi_\mu \beta_\mu \quad (\mu = 1,\, \ldots,\, M), \tag{23}$$

where Ψ_μ are $(K \times n_\mu)$ known sub-matrices of the $(MK \times n)$ block diagonal matrix

$$\Psi = [(\delta_{\mu\mu'}\Psi_\mu)_{\mu,\mu'=1,\,\ldots,\,M}]. \tag{24}$$

Finally, Z is a $(T \times K)$ matrix with non-autocorrelated columns, defined by the following relationship:

$$\begin{aligned} X_* &= [(\delta_{\mu\mu'} X_{\mu *})_{\mu,\mu'=1,\,\dots,\,M}] = [(\delta_{\mu\mu'} Z \Psi_\mu)_{\mu,\mu'=1,\,\dots,\,M}] \\ &= [(\delta_{\mu\mu'} Z)_{\mu,\mu'=1,\,\dots,\,M}][(\delta_{\mu\mu'} \Psi_\mu)_{\mu,\mu'=1,\,\dots,\,M}] \\ &= (I_M \otimes Z)\Psi. \end{aligned} \tag{25}$$

The above representation of the S.U.R. model, given by (21), will facilitate the expansions needed in our derivations of the size corrected tests suggested in the paper.

3.1 Assumptions

To carry out our expansions, it would be theoretically convenient to introduce a re-parameterization of the error covariance matrix of model (8) as follows:

$$y = X\beta + \sigma u, \quad \sigma > 0, \quad u \sim \mathcal{N}_{MT}(0, \Omega^{-1}), \tag{26}$$

assuming that parameter σ^2 can be estimated separately from the rest terms of the covariance matrix Ω^{-1} of vector u.[1]

For the derivation of our size corrected tests, we need to make a number of assumptions on the elements of matrix Ω, which is the inverse of the variance-covariance matrix of the error vector u. To this end, we denote as Ω_i, Ω_{ij}, etc., the $(MT \times MT)$ matrices of first-, second- and higher-order derivatives, respectively, of the elements of matrix Ω with respect to the elements of the $((M + M^2) \times 1)$ vector of nuisance parameters $\gamma = (\varrho', \varsigma')'$. For any estimator of γ, define the $((1 + M + M^2) \times 1)$ vector δ, with elements

$$\delta_0 = \frac{\hat{\sigma}^2 - 1}{\tau}, \quad \delta_{\rho_\mu} = \frac{\hat{\rho}_\mu - \rho_\mu}{\tau}, \quad \delta_{\varsigma_{(\mu\mu')}} = \frac{\hat{\varsigma}_{(\mu\mu')} - \varsigma_{(\mu\mu')}}{\tau}, \tag{27}$$

where $\mu = 1, \dots, M$, $(\mu\mu') = 1, \dots, M^2$ and $\tau = 1/\sqrt{T}$ is the 'asymptotic

[1]The nuisance parameters σ and γ can be simultaneously identified under the restriction $\sigma = 1$, which implies that the estimate of matrix Σ, denoted as $\hat{\Sigma}$, is accurate, up to a multiplicative factor. This is not true in samples with small time dimension. A convenient method to estimate σ is through the following feasible generalized least squares (GL) estimator

$$\hat{\sigma}_{GL} = \left[(y - X\hat{\beta})' \left(\hat{P}'^{-1}_{GL}(\hat{\Sigma}^{-1}_{GL} \otimes I_T)\hat{P}^{-1}_{GL}\right)(y - X\hat{\beta})/(MT - n)\right]^{1/2},$$

where $\hat{\beta}$ is the feasible GL estimator based on any consistent estimators of Σ^{-1} and P^{-1}.

scale' of our second-order stochastic expansions. Then, our size corrected tests can be derived based on the following assumption.

Assumption 1:

(i) The elements of matrices Ω and Ω^{-1} are bounded for all T, all vectors ϱ with elements $\rho_\mu \in (-1, 1)$, and all vectors $\varsigma \in \pounds$. Moreover, the following matrices:

$$A = X'\Omega X/T, \quad F = X'X/T, \quad \Gamma = Z'Z/T \tag{28}$$

converge to non-singular limits, as $T \to \infty$.

(ii) Up to the fourth order, the partial derivatives of the elements of Ω with respect to the elements of ϱ and ς, are bounded for all T, all vectors ϱ with elements in the interval $(-1, 1)$, and all vectors $\varsigma \in \pounds$.

(iii) The estimators $\hat{\varrho}$ and $\hat{\varsigma}$ are even functions of u, and they are functionally unrelated to the parameter vector β. As a result, they can be written as functions of X, Z, and u only.

(iv) The vector of nuisance parameters δ admits a stochastic expansion of the form

$$\begin{aligned} \delta &= \left[\delta_0, \ [(\delta_{\rho_\mu})_{\mu=1, \dots, M}]', \ [(\delta_{\varsigma_{(\mu\mu')}})_{(\mu\mu')=1, \dots, M^2}]' \ \right]' \\ &= d_1 + \tau d_2 + \omega(\tau^2), \end{aligned} \tag{29}$$

where the order of magnitude $\omega(\cdot)$, defined in the Appendix, has the same operational properties as order $O(\cdot)$. Moreover, the expectations

$$\mathcal{E}(d_1 d_1'), \quad \mathcal{E}(\sqrt{T} d_1 + d_2) \tag{30}$$

exist and have finite limits, as $T \to \infty$.

The first two conditions of Assumption 1 imply that the following matrices:

$$A_i = X'\Omega_i X/T, \quad A_{ij} = X'\Omega_{ij} X/T, \quad A_{ij}^* = X'\Omega_i \Omega^{-1} \Omega_j X/T \tag{31}$$

are bounded. Thus, according to Magdalinos (1992), the Taylor series expansion of β constitutes a stochastic expansion. Since the vectors of nuisance parameters ϱ and ς are functionally unrelated to β, condition (iii) of Assumption 1 is satisfied for a wide class of estimators $\hat{\varrho}$ and $\hat{\varsigma}$, including the maximum likelihood estimators and the simple or iterative estimators based on the regression residuals (see Breusch (1980), Rothenberg (1984a)). Note that we *need not* assume that estimators $\hat{\varrho}$ and $\hat{\varsigma}$ are asymptotically efficient.

Further, conditions (i)–(iv) of Assumption 1 should be satisfied by all the estimators of vectors ϱ and ς, considered in the paper. The estimators of the elements of vector ϱ, i.e., ρ_μ ($\mu = 1, ..., M$) include the following: the least squares (LS), Durbin-Watson (DW), generalized least squares (GL), Prais-Winsten (PW) and maximum likelihood (ML).[2] The elements of vector $\varsigma = vec(\Sigma^{-1})$ can be estimated by

$$\hat{\varsigma} = vec\left[(Y_* - Z\hat{B})'(Y_* - Z\hat{B})/T\right]^{-1}, \tag{32}$$

where $\hat{B}$ is any consistent estimator of the matrix of parameters B of regression model (21). Consistent estimators of B include the unrestricted and restricted least squares (denoted as UL and RL, respectively), the simple and iterative generalized least squares (denoted as GL and IG, respectively) and the maximum likelihood (ML) estimators.[3]

To present the expansions suggested in the paper, expectations $\mathcal{E}(d_1 d_1')$ and

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[2]The closed forms of these estimators of ρ_μ, for all μ, are given as follows:

(i) LS:

$$\tilde{\rho}_\mu = \sum\nolimits_{t=2}^{T} \tilde{u}_{t\mu}\tilde{u}_{(t-1)\mu} \Big/ \sum\nolimits_{t=1}^{T} \tilde{u}_{t\mu}^2,$$

where $\tilde{u}_{t\mu}$ are the LS residuals of regression model (1).

(ii) DW:

$$\hat{\rho}_\mu^{(DW)} = 1 - (DW/2),$$

where the DW is the Durbin-Watson statistic.

(iii) GL:

$$\hat{\rho}_\mu = \sum\nolimits_{t=2}^{T} \hat{u}_{t\mu}\hat{u}_{(t-1)\mu} \Big/ \sum\nolimits_{t=1}^{T} \hat{u}_{t\mu}^2,$$

where $\hat{u}_{t\mu}$ denote the GL estimates of $u_{t\mu}$, based on the autocorrelation-correction of regression model (1), for all μ, using any asymptotically efficient estimator of ρ_μ.

(iv) PW: This estimator of ρ_μ, denoted as $\hat{\rho}_\mu^{(PW)}$, together with the PW estimator of β, denoted as $\hat{\beta}_\mu^{(PW)}$, minimize the sum of squared GL residuals (Prais and Winsten (1954)).

(v) ML: This estimator, denoted as $\hat{\rho}_\mu^{(ML)}$, satisfies a cubic equation with coefficients defined in terms of the ML residuals (Beach and MacKinnon (1978)).

[3]The closed forms of these estimators of B are given as follows:

(i) UL:

$$\hat{B}_{(UL)} = (Z'Z)^{-1}Z'Y_*.$$

$\mathcal{E}(\sqrt{T}d_1 + d_2)$ will be defined as follows:

$$\lim_{T\to\infty} \mathcal{E}(d_1 d_1') = \begin{bmatrix} \lambda_0 & \lambda_\varrho' & \lambda_\varsigma' \\ \lambda_\varrho & \Lambda_\varrho & \Lambda_{\varrho\varsigma}' \\ \lambda_\varsigma & \Lambda_{\varrho\varsigma} & \Lambda_\varsigma \end{bmatrix} \quad \text{and} \quad \lim_{T\to\infty} \mathcal{E}(\sqrt{T}d_1 + d_2) = \begin{bmatrix} \kappa_0 \\ \kappa_\varrho \\ \kappa_\varsigma \end{bmatrix}, \tag{33}$$

respectively, where λ_0 and κ_0 are scalars, λ_ϱ and κ_ϱ are $(M \times 1)$ vectors, λ_ς and κ_ς are $(M^2 \times 1)$ vectors, Λ_ϱ is a $(M \times M)$ matrix, Λ_ς is a $(M^2 \times M^2)$ matrix and $\Lambda_{\varrho\varsigma}$ is a $(M^2 \times M)$ matrix. The following partitions of the above matrix and vector will be of use in the paper:

$$\begin{bmatrix} \lambda_0 & \lambda' \\ \lambda & \Lambda \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \kappa_0 \\ \kappa \end{bmatrix}, \tag{34}$$

where

$$\Lambda = \begin{bmatrix} \Lambda_\varrho & \Lambda_{\varrho\varsigma}' \\ \Lambda_{\varrho\varsigma} & \Lambda_\varsigma \end{bmatrix}, \quad \lambda = \begin{bmatrix} \lambda_\varrho \\ \lambda_\varsigma \end{bmatrix} \quad \text{and} \quad \kappa = \begin{bmatrix} \kappa_\varrho \\ \kappa_\varsigma \end{bmatrix}, \tag{35}$$

where Λ is a $((M + M^2) \times (M + M^2))$ matrix, and λ and κ are $((M + M^2) \times 1)$ vectors. The elements of the vectors and matrices in (33), (34) and (35) can be interpreted as 'measures' of the accuracy of the expansions of estimators $\hat{\sigma}^2$, $\hat{\rho}_\mu$ and $\hat{\varsigma}_{(\mu\mu')}$ around the true values of the corresponding parameters.

4 Size corrected test statistics

In this section, we derive size corrected t, Wald and F test statistics, as well as the second-order approximations of their distributions based on the conditions of Assumption 1. The versions of the test statistics which adjust for the degrees

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(ii) RL:

$$vec(\hat{B}_{(RL)}) = \Psi(X_*' X_*)^{-1} X_*' y_*.$$

(iii) GL:

$$vec(\hat{B}_{(GL)}) = \Psi \left[X_*' (\hat{\Sigma}_I^{-1} \otimes I_T) X_* \right]^{-1} X_*' (\hat{\Sigma}_I^{-1} \otimes I_T) y_*,$$

where $\hat{\Sigma}_I^{-1}$ is the UL or RL estimator of Σ.

(iv) IG: This estimator, denoted as $\hat{B}_{(IG)}$, is computed by iterative implementation of the GL estimator.

(v) ML: This estimator, denoted as $\hat{B}_{(ML)}$, can be computed by iterating the GL estimation process up to convergence (Dhrymes (1971)).

of freedom, namely the Student-t and F, are locally exact. That is, if the vector of parameters $\gamma = (\varrho', \varsigma')'$ is known to belong to a ball of radius ϑ, then the approximate distributions of these test statistics become exact, as $\vartheta \to 0$.

The analytic size corrections developed in this section can provide size corrections to either the non-zero cross-correlations of the error terms or their serial correlation effects. The part of the size corrections corresponding to the serial correlation effects constitutes a extension of the results in Magdalinos and Symeonides (1995) to the multiple equation framework. On the other hand, the part of the size corrections due to the non-zero cross-correlations constitutes a completely genuine contribution to the literature, which can be readily implemented to correct the size of the t and F tests in the standard Zellner's S.U.R. model (see Zellner (1962)) alone.

4.1 The t test

Let the elements of the $(n \times 1)$ vector e and scalar e_0 be known quantities. Testing any null hypothesis of the form

$$H_0 : \; e'\beta = e_0 \tag{36}$$

against its one-sided alternatives, can be based upon the following t statistic:

$$t = (e'\beta - e_0) / \left[\hat{\sigma}^2 e'(X'\hat{\Omega}X)^{-1} e \right]^{1/2}, \tag{37}$$

which is adjusted for the degrees of freedom of the Student-t distribution.

For the derivation of the suggested asymptotic expansions, we define the $((M+M^2)\times 1)$ vector l and the $((M+M^2)\times(M+M^2))$ matrix L as follows:

$$l = \left[\; [(l_{\rho_\mu})_{\mu=1,\,\ldots,\,M}]', \quad [(l_{\varsigma_{(\mu\mu')}})_{(\mu\mu')=1,\,\ldots,\,M^2}]' \; \right]', \tag{38}$$

$$L = \begin{bmatrix} [(l_{\rho_\mu \rho_{\mu'}})_{\mu,\mu'=1,\ldots,M;}] & [(l_{\rho_\mu \varsigma_{(\nu\nu')}})_{\substack{\mu=1,\,\ldots,\,M; \\ (\nu\nu')=1,\ldots,M^2}}] \\ [(l_{\varsigma_{(\nu\nu')} \rho_\mu})_{\substack{(\nu\nu')=1,\,\ldots,\,M^2; \\ \mu=1,\,\ldots,\,M}}] & [(l_{\varsigma_{(\mu\mu')} \varsigma_{(\nu\nu')}})_{\substack{(\mu\mu')=1,\,\ldots,\,M^2; \\ (\nu\nu')=1,\,\ldots,\,M^2}}] \end{bmatrix}, \tag{39}$$

where the elements of vector l and matrix L are defined below:

$$\begin{aligned} l_{\rho_\mu} &= h'GA_{\rho_\mu}Gh, \quad l_{\varsigma_{(\mu\mu')}} = h'GA_{\varsigma_{(\mu\mu')}}Gh, \\ l_{\rho_\mu \rho_{\mu'}} &= h'GC_{\rho_\mu \rho_{\mu'}}Gh, \quad l_{\rho_\mu \varsigma_{(\nu\nu')}} = h'GC_{\rho_\mu \varsigma_{(\nu\nu')}}Gh, \\ l_{\varsigma_{(\nu\nu')}\rho_\mu} &= h'GC_{\varsigma_{(\nu\nu')}\rho_\mu}Gh, \quad l_{\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}} = h'GC_{\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}}Gh, \end{aligned} \tag{40}$$

where $G = A^{-1} = (X'\Omega X/T)^{-1}$ is a $(n \times n)$ matrix, $h = e/(e'Ge)^{1/2}$ is a $(n \times 1)$ vector and

$$\begin{aligned} C_{\rho_\mu \rho_{\mu'}} &= A^*_{\rho_\mu \rho_{\mu'}} - 2A_{\rho_\mu} G A_{\rho_{\mu'}} + A_{\rho_\mu \rho_{\mu'}}/2, \\ C_{\rho_\mu \varsigma_{(\nu\nu')}} &= A^*_{\rho_\mu \varsigma_{(\nu\nu')}} - 2A_{\rho_\mu} G A_{\varsigma_{(\nu\nu')}} + A_{\rho_\mu \varsigma_{(\nu\nu')}}/2, \\ C_{\varsigma_{(\mu\mu')} \varsigma_{(\nu\nu')}} &= A^*_{\varsigma_{(\mu\mu')} \varsigma_{(\nu\nu')}} - 2A_{\varsigma_{(\mu\mu')}} G A_{\varsigma_{(\nu\nu')}} + A_{\varsigma_{(\mu\mu')} \varsigma_{(\nu\nu')}}/2, \end{aligned} \tag{41}$$

with obvious modifications for $C_{\varsigma_{(\nu\nu')} \rho_\mu}$.

The next two theorems give alternative Edgeworth approximations of the distribution function of the t statistic, given in (37), in terms of the normal and Student-t distributions, respectively.

Theorem 1. *The distribution function of the t statistic (37), under the null hypothesis (36), admits the Edgeworth expansion*

$$\Pr\{t \leq x\} = I(x) - \frac{\tau^2}{2} \left[\left(p_1 + \tfrac{1}{2}\right) + \left(p_2 + \tfrac{1}{2}\right) x^2 \right] x i(x) + O(\tau^3), \tag{42}$$

where $I(\cdot)$ and $i(\cdot)$ are the standard normal distribution and density functions, respectively, and scalars p_1 and p_2 can be calculated as follows:

$$p_1 = tr(\Lambda L) + \frac{l'\Lambda l}{4} + l'(\kappa + \frac{\lambda}{2}) - \kappa_0 + \frac{\lambda_0 - 2}{4}, \quad p_2 = \frac{l'\Lambda l - 2l'\lambda + \lambda_0 - 2}{4}. \tag{43}$$

Analytic formulas for the computation of scalars λ_0, κ_0, and the elements of λ, κ, Λ, l and L are given in the Technical Appendix (see Lemmas A.15 and A.17).

Instead of using the Edgeworth expansion (42), we can approximate the distribution function of the t statistic in terms of the Student-t distribution as follows:

Theorem 2. *The distribution function of the t statistic (37), under the null hypothesis (36), admits the Edgeworth expansion*

$$\Pr\{t \leq x\} = I_{MT-n}(x) - \frac{\tau^2}{2} \left[p_1 + p_2 x^2 \right] x i_{MT-n}(x) + O(\tau^3), \tag{44}$$

where $I_{MT-n}(\cdot)$ and $i_{MT-n}(\cdot)$ are the Student-t distribution and density functions, respectively, with $MT - n$ degrees of freedom, and scalars p_1 and p_2 are defined in (43).

Theorem 1 implies that we can calculate the Edgeworth corrected $\alpha\%$ critical value of the t statistic (37) as

$$n^*_\alpha = n_\alpha + \frac{\tau^2}{2} \left[\left(p_1 + \tfrac{1}{2}\right) + \left(p_2 + \tfrac{1}{2}\right) n_\alpha^2 \right] n_\alpha, \tag{45}$$

based on the $\alpha\%$ significant point of the standard normal distribution, denoted as n_α. Similarly, based on Theorem 2, we can calculate the Edgeworth corrected $\alpha\%$ critical value of the t statistic (37) as

$$t_\alpha^* = t_\alpha + \frac{\tau^2}{2}\left[p_1 + p_2 t_\alpha^2\right] t_\alpha, \tag{46}$$

using the $\alpha\%$ significant point of the Student-t distribution, denoted as t_α.

The Edgeworth approximation employed by Theorems 1 and 2 to obtain the size corrected critical values n_α^* and t_α^* is not a proper distribution function, as it may assign negative 'probabilities' in the tails of the approximate distribution. To overcome this problem, we can use a Cornish-Fisher expansion. This corrects the test statistics of interest, instead of their critical values. The Cornish-Fisher expansion is simply the inversion of the Edgeworth correction of the critical values and, thus, it is expected to have very similar properties around the mean of the approximate distribution. However, at the tails of this distribution, which are important for inference, the properties of the Cornish-Fisher expansion are different. In fact, the Cornish-Fisher size corrected statistics constitute random variables with well-behaved tails, and thus they *do not* assign negative 'probabilities' at the tails of their distributions.

The Cornish-Fisher corrected t statistic for testing null hypothesis (36) is given in the following theorem.

Theorem 3. *The Cornish-Fisher size corrected t statistic*

$$t_* = t - \frac{\tau^2}{2}\left[p_1 + p_2 t^2\right] t \tag{47}$$

is distributed, under the null hypothesis (36), as a Student-t random variable with $MT - n$ degrees of freedom, with an approximation error of order $O(\tau^3)$.

The Cornish-Fisher size corrected t statistic t_*, given by equation (47), can be readily used, in practice, to test null hypothesis (36) against its one-sided alternatives. This can be done by using the standard tables of the Student-t distribution with $MT - n$ degrees of freedom.

4.2 The Wald and F tests

Let H be a $(m \times n)$ matrix of rank m with known elements and h_0 be a known $(m \times 1)$ vector. Testing any null hypothesis of the form

$$H_0 : \; H\beta = h_0 \tag{48}$$

against all possible alternatives, can be based upon the Wald statistic

$$w = (H\hat{\beta} - h_0)' \left[H(X'\hat{\Omega}X)^{-1}H' \right]^{-1} (H\hat{\beta} - h_0)/\hat{\sigma}^2, \tag{49}$$

or the familiar F statistic

$$F = (H\hat{\beta} - h_0)' \left[H(X'\hat{\Omega}X)^{-1}H' \right]^{-1} (H\hat{\beta} - h_0)/m\hat{\sigma}^2, \tag{50}$$

which is adjusted for the degrees of freedom of the F distribution.

For the derivation of the suggested asymptotic expansions, we define the $(n \times n)$ matrix

$$Q = H'(HGH')^{-1}H, \tag{51}$$

and we partition the $(n \times n)$ matrices $G = A^{-1} = (X'\Omega X/T)^{-1}$ and $\Xi = GQG$ and the $(n \times 1)$ vector h as follows:

$$G = [(G_{ij})_{i,j=1,\,\dots,\,M}], \quad \Xi = [(\Xi_{ij})_{i,j=1,\,\dots,\,M}], \quad h = [(h_i)_{i=1,\,\dots,\,M}], \tag{52}$$

where G_{ij} and Ξ_{ij} are the (i,j)-th $(n_i \times n_j)$ sub-matrices of G and Ξ, respectively, and $h_i = e_i/(e'Ge)^{1/2}$ is the i-th $(n_i \times 1)$ sub-vector of h, where e_i is the corresponding i-th $(n_i \times 1)$ sub-vector of the $(n \times 1)$ vector e.

Next, define the $((M + M^2) \times 1)$ vector c, and the $((M + M^2) \times (M + M^2))$ matrices C and D_* as follows:

$$c = \left[\; [(c_{\rho_\mu})_{\mu=1,\,\dots,\,M}]', \quad [(c_{\varsigma_{(\mu\mu')}})_{(\mu\mu')=1,\,\dots,\,M^2}]' \; \right]', \tag{53}$$

$$C = \begin{bmatrix} [(c_{\rho_\mu \rho_{\mu'}})_{\mu,\mu'=1,\,\dots,\,M}] & [(c_{\rho_\mu \varsigma_{(\nu\nu')}})_{\substack{\mu=1,\,\dots,\,M; \\ (\nu\nu')=1,\,\dots,\,M^2}}] \\ [(c_{\varsigma_{(\nu\nu')}\rho_\mu})_{\substack{(\nu\nu')=1,\,\dots,\,M^2; \\ \mu=1,\,\dots,\,M}}] & [(c_{\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}})_{\substack{(\mu\mu')=1,\,\dots,\,M^2; \\ (\nu\nu')=1,\,\dots,\,M^2}}] \end{bmatrix} \tag{54}$$

and

$$D_* = \begin{bmatrix} [(d_{\rho_\mu \rho_{\mu'}})_{\mu,\mu'=1,\,\dots,\,M}] & [(d_{\rho_\mu \varsigma_{(\nu\nu')}})_{\substack{\mu=1,\,\dots,\,M; \\ (\nu\nu')=1,\,\dots,\,M^2}}] \\ [(d_{\varsigma_{(\nu\nu')}\rho_\mu})_{\substack{(\nu\nu')=1,\,\dots,\,M^2; \\ \mu=1,\,\dots,\,M}}] & [(d_{\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}})_{\substack{(\mu\mu')=1,\,\dots,\,M^2; \\ (\nu\nu')=1,\,\dots,\,M^2}}] \end{bmatrix}, \tag{55}$$

where the elements of vector c and matrices C and D_* are defined as follows:

$$\begin{gathered}
c_{\rho_\mu} = tr(A_{\rho_\mu}\Xi), \quad c_{\rho_\mu\rho_{\mu'}} = tr(C_{\rho_\mu\rho_{\mu'}}\Xi), \\
c_{\rho_\mu\varsigma_{(\nu\nu')}} = tr(C_{\rho_\mu\varsigma_{(\nu\nu')}}\Xi), \\
c_{\varsigma_{(\mu\mu')}} = tr(A_{\varsigma_{(\mu\mu')}}\Xi), \quad c_{\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}} = tr(C_{\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}}\Xi), \\
d_{\rho_\mu\rho_{\mu'}} = tr(D_{*\rho_\mu\rho_{\mu'}}\Xi), \quad d_{\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}} = tr(D_{*\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}}\Xi), \\
d_{\rho_\mu\varsigma_{(\nu\nu')}} = tr(D_{*\rho_\mu\varsigma_{(\nu\nu')}}\Xi),
\end{gathered} \tag{56}$$

where

$$\begin{gathered}
D_{*\rho_\mu\rho_{\mu'}} = \frac{A_{\rho_\mu}\Xi A_{\rho_{\mu'}}}{2}, \quad D_{*\rho_\mu\varsigma_{(\nu\nu')}} = \frac{A_{\rho_\mu}\Xi A_{\varsigma_{(\nu\nu')}}}{2}, \\
D_{*\varsigma_{(\mu\mu')}\varsigma_{(\nu\nu')}} = \frac{A_{\varsigma_{(\mu\mu')}}\Xi A_{\varsigma_{(\nu\nu')}}}{2},
\end{gathered} \tag{57}$$

with obvious modifications for $c_{\varsigma_{(\nu\nu')}\rho_\mu}$, $d_{\varsigma_{(\nu\nu')}\rho_\mu}$ and $D_{*\varsigma_{(\nu\nu')}\rho_\mu}$.

The next two theorems give Edgeworth approximations of the distribution functions of the Wald (w) and F statistics, given by (49) and (50), respectively.

Theorem 4. *The distribution function of the Wald statistic (49), under the null hypothesis (48), admits the Edgeworth expansion*

$$\Pr\{w \le x\} = F_m(x) - \tau^2\left[\xi_1 + (\xi_2/(m+2))x\right]\frac{x}{m}f_m(x) + O(\tau^3), \tag{58}$$

where $F_m(\cdot)$ and $f_m(\cdot)$ are the chi-square distribution and density functions, respectively, and scalars ξ_1 and ξ_2 can be calculated as follows:

$$\begin{gathered}
\xi_1 = tr[\Lambda(C + D_*)] - c'\Lambda c/4 + c'\kappa + m[c'\lambda/2 - \kappa_0 - (m-2)\lambda_0/4], \\
\xi_2 = tr(\Lambda D_*) + [c'\Lambda c - (m+2)(2c'\lambda - m\lambda_0)]/4.
\end{gathered} \tag{59}$$

Analytic formulas for the computation of scalars λ_0 and κ_0, and the elements of λ, κ, Λ, c, C and D_* are given in the Technical Appendix (see Lemmas A.16 and A.17).

Instead of using the Wald statistic (49) and the Edgeworth expansion of its distribution, given in (58), we can use the F statistic, given by (50), and approximate its distribution function in terms of the F distribution as follows:

Theorem 5. *The distribution function of the F statistic (50), under null hypothesis (48), admits the Edgeworth expansion*

$$\Pr\{F \le x\} = F^m_{MT-n}(x) - \tau^2 \left[q_1 + q_2 x\right] x f^m_{MT-n}(x) + O(\tau^3), \tag{60}$$

where $F^m_{MT-n}(\cdot)$ and $f^m_{MT-n}(\cdot)$ are the F distribution and density functions, respectively, with m and $MT-n$ degrees of freedom, and scalars q_1 and q_2 can be calculated as follows:

$$q_1 = \xi_1/m + (m-2)/2, \quad q_2 = \xi_2/(m+2) - m/2, \tag{61}$$

where scalars ξ_1 and ξ_2 are defined in (59).

Theorem 4 implies that the Edgeworth corrected $\alpha\%$ critical value of the Wald statistic (49) is given as

$$\chi^*_\alpha = \chi_\alpha + \tau^2 \left[\frac{\xi_1}{m} + \frac{\xi_2}{m(m+2)}\chi_\alpha\right]\chi_\alpha, \tag{62}$$

based on the $\alpha\%$ significant point of the chi-square distribution, denoted as χ_α. Theorem 5 enables us to calculate the Edgeworth corrected $\alpha\%$ critical value of F statistic (50) as

$$F^*_\alpha = F_\alpha + \tau^2 \left[q_1 + q_2 F_\alpha\right] F_\alpha, \tag{63}$$

based on the $\alpha\%$ significant point of the F distribution, denoted as F_α.

The Cornish-Fisher size corrected F statistic for testing null hypothesis (48) is given in the next theorem.

Theorem 6. *The Cornish-Fisher size corrected F statistic*

$$F_* = F - \tau^2 \left[q_1 + q_2 F\right] F \tag{64}$$

is distributed, under null hypothesis (48), as an F random variable with m and $MT-n$ degrees of freedom, with an approximation error of order $O(\tau^3)$.

Unlike the Edgeworth approximation, the Cornish-Fisher corrected F statistic, denoted as F_* in equation (64), is a proper random variable and it does not assign negative 'probabilities' in the tails of its distribution. Thus, the Cornish-Fisher corrected F statistic can be be readily implemented, in applied research, to test null hypothesis (48). This can be done by using the standard tables of the F distribution, with m and $MT-n$ degrees of freedom.

5 Monte-Carlo simulations

In this section, we evaluate the small-sample performance of the size corrected tests suggested in the previous section, compared to their corresponding standard (first-order asymptotic approximation) versions. To this end, we rely on a Monte Carlo simulation based on 5000 iterations and we consider small-smaples of $T = 15, 20, 40$ observations.

In our simulation, we consider the S.U.R. model of $M = 2$ seemingly unrelated equations (see, e.g., Zellner (1962)), i.e.,

$$\begin{aligned} y_{t,1} &= \beta_{0,1} + \beta_{1,1}x_{t1,1} + \beta_{2,1}x_{t2,1} + u_{t,1} \\ y_{t,2} &= \beta_{0,2} + \beta_{1,2}x_{t1,2} + \beta_{2,2}x_{t2,2} + u_{t,2} \end{aligned} \quad (t = 1, \ldots, T), \tag{65}$$

where the error terms, $u_{t,1}$ and $u_{t,2}$, are contemporaneously correlated with covariance σ_{12}. Both of these error terms follow AR(1) process (2), with normally distributed innovations (see Turkington (2000)). The autoregressive coefficients of this process ρ_1 and ρ_2 are assumed to be equal, i.e., $\rho_1 = \rho_2 = \rho = \pm 0.5$, ± 0.8. To ensure stationarity of error terms $u_{t,1}$ and $u_{t,2}$, conditions (3) are satisfied. For $t = 0$, these conditions require that

$$\begin{aligned} y_{0,1} &\sim \mathcal{N}(0, \sigma_{11}/(1-\rho_1^2)) \\ y_{0,2} &\sim \mathcal{N}(0, \sigma_{22}/(1-\rho_2^2)) \end{aligned} \quad \text{and} \quad \mathcal{E}(y_{0,1}y_{0,2}) = \sigma_{12}\frac{(1-\rho_1^2)^{1/2}(1-\rho_2^2)^{1/2}}{1-\rho_1\rho_2}.$$

In our analysis, we assume $\sigma_{11} = \sigma_{22} = 1$ and we are focused on investigating the consequences of the different sign and magnitude of covariances σ_{12} on our tests, for the following cases: $\sigma_{12} = \pm 0.5, \pm 0.75, \pm 0.9$. Since $\sigma_{11} = \sigma_{22} = 1$, σ_{12} is the correlation coefficient between $u_{t,1}$ and $u_{t,2}$.

According to (15) (or (16)), the above S.U.R. model can be written in terms of the following transformed equations, with non-autocorrelated errors:

$$y_{1*} = X_{1*}\beta_1 + \varepsilon_1; \quad y_{2*} = X_{2*}\beta_2 + \varepsilon_2,$$

where y_{1*} and y_{2*} are $(TX1)$ vectors of observations on the dependent variables, with $P_\mu y_{\mu*} = y_\mu$, for $\mu = 1, 2$, where P_μ is defined by (9), X_{1*} and X_{2*} are $(T \times 3)$ matrices of regressors, with $P_\mu X_{\mu*} = X_\mu$ and $\beta_1 = (\beta_{0,1},\ \beta_{1,1},\ \beta_{2,1})'$, $\beta_2 = (\beta_{0,2},\ \beta_{1,2},\ \beta_{2,2})'$ are (3×1) vectors of parameters, including the constant. In terms of the S.U.R. representation (21), the above equations can be written as

$$Y_* = ZB + E,$$

where Y_* is a $(T \times 2)$ matrix of observations on vectors y_{1*} and y_{2*}, E is a $(T \times 2)$ matrix whose rows are vectors of normally distributed innovations with variance-covariance $\Sigma = [(\sigma_{\mu\mu'})_{\mu,\mu'=1,2}]$, B is a (3×2)-dimension matrix whose columns, β_1 and β_2, are vectors of parameters, and Z is a $(T \times 6)$ matrix whose columns are vectors of possibly collinear variables defined as

$$\begin{aligned} z_{t1} &\equiv z_{t6} \equiv (1-\rho^2)^{1/2} & (t=1), \\ z_{t1} &\equiv z_{t6} \equiv (1-\rho) & (t=2,3,...,T), \\ z_{tj} &= \alpha^{1/2}\zeta_{t1} + (1-\alpha)^{1/2}\zeta_{tj} & (j=2,3,4,5), \end{aligned}$$

where ζ_{tj} $(j = 2,3,4,5)$ are $\mathcal{N}(0,1)$ random variables and α stands for the common correlation coefficient between any two non-constant columns of Z (see also McDonald and Galarneau (1975)). This captures the same degree of multicollinearity between regressors $x_{t1,\mu}$ and $x_{t2,\mu}$ of S.U.R. model (65). In our simulation, we consider the following two values of the collinearity coefficient: $\alpha = 0.5, 0.9$. According to (25), sub-matrices X_{1*} and X_{2*} (collected in matrix X_*) can be obtained from Z by assuming that sub-matrices Ψ_1 and Ψ_2, of the block diagonal matrix Ψ are given as follows:

$$\Psi_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \Psi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

In all iterations of our simulation, the two equations of S.U.R. model (65) were estimated by LS. The residuals of these equations were used to compute the LS estimates of autoregressive coefficients ρ_1 and ρ_2, denoted as $\tilde{\rho}_1$ and $\tilde{\rho}_2$. Then, the transformed variables $y^*_{1,\mu}$ and $x^*_{tj,\mu}$, for $j = 0,1,2$ (where '0' stands for the constant), are calculated as follows:

$$\begin{aligned} y^*_{1,\mu} &= (1-\tilde{\rho}^2_\mu)^{1/2} y_{1,\mu} & x^*_{1j,\mu} &= (1-\tilde{\rho}^2_\mu)^{1/2} x_{1j,\mu} & (t=1), \\ y^*_{t,\mu} &= y_{t,\mu} - \tilde{\rho}_\mu y_{(t-1),\mu} & x^*_{tj,\mu} &= x_{tj,\mu} - \tilde{\rho}_\mu x_{(t-1)j,\mu} & (t \neq 1). \end{aligned} \tag{66}$$

These variables were then used to compute the feasible GL estimates of $\beta_{j,\mu}$ ($j = 0, 1, 2;\ \mu = 1, 2$), denoted as $\hat{\beta}_{j,\mu}$. The columns of matrix Z were obtained as $z_1 = x^*_{0,1}$, $z_2 = x^*_{1,1}$, $z_3 = x^*_{2,1}$, $z_6 = x^*_{0,2}$, $z_4 = x^*_{1,2}$, $z_5 = x^*_{2,2}$, while the unrestricted estimates of matrix B were based on the GL estimates $\hat{\beta}_{j,\mu}$. The unrestricted estimates of the inverse covariance matrix Σ^{-1} were estimated based on (32) and the feasible GL estimate $\hat{\sigma}_{GL}$ which is calculated by using the following formula:

$$\hat{\sigma}_{GL} = \left[(y - X\hat{\beta})' \left(\hat{P}_I'^{-1}(\hat{\Sigma}_I^{-1} \otimes I_T)\hat{P}_I^{-1}\right)(y - X\hat{\beta})/(MT - n)\right]^{1/2},$$

where I denotes any consistent estimators of matrices Σ^{-1} and P^{-1} (see Appendix), used to obtain a feasible GL estimator of β.

The results of these simulations are presented in Tables 1a, 1b and 2. The actual sizes of our size corrected tests of the following null hypothesis:

$$H_0 : \beta_{2,1} = 0, \tag{67}$$

against its one-sided alternatives, are reported in Tables 1a and 1b. In particular, Table 1a presents results against alternative $H_A : \beta_{2,1} > 0$, while Table 1b against $H_A : \beta_{2,1} < 0$. The table presents the actual sizes (i.e., the rejection probabilities) at the 5% significance level of the following: the standard normal and Student-t tests (denoted as z and t, respectively), their finite-sample size corrected versions based on the Edgeworth corrected critical values of the standard normal and Student-t distributions (denoted as E-z and E-t, respectively) and the Cornish-Fisher finite-sample size corrected Student-t test (denoted as CF-t). Note that we do not examine the performance of the above t tests for the null hypothesis (67) against its two-sided alternatives, since this is a special case of the F test examined in Table 2.

Table 2 presents the actual sizes of our size corrected tests of the following joint null hypothesis on the slope coefficients of S.U.R. model (65), across its two equations:

$$H_0 : \beta_{1,1} = \beta_{2,1} = \beta_{1,2} = \beta_{2,2} = 0. \tag{68}$$

This is done against the alternative hypothesis that at least one of these coefficients are different from zero, i.e., at least one $\beta_{j,\mu} \neq 0$ ($j = 1, 2;\ \mu = 1, 2$). The table presents the actual sizes at the 5% significance level of the following: the standard Wald (chi-square) and F tests (denoted as χ^2 and F, respectively), their finite-sample size corrected versions based on the Edgeworth corrected critical values of the chi-square and F distributions (denoted by E-χ^2 and E-F, respectively) and the Cornish-Fisher finite-sample size corrected F test (denoted as CF-F).

Turning now into the discussion of the results in this simulation, Tables 1a and 1b clearly indicate that the size corrected tests have better size performance in all reported sample sizes (T = 15, 20, 40), compared to the standard versions of these tests, based on first-order approximations. This is true for both the Edgeworth and Cornish-Fisher size corrections, and across all different values of ρ, σ_{12} and α examined.

Between the above different categories of size corrected tests, our results indicate that the CF-t test outperforms the E-z and E-t ones. This is true for almost all cases of α and σ_{12} considered, if ρ takes large values, i.e., $\rho = \pm 0.8$. The same is true for small samples (T = 15 or 20) and $\rho = \pm 0.5$.

Regarding the chi-square and F tests, the results of Table 2 indicate that, in most of the cases examined, the size corrected versions of these tests, i.e., E-χ^2, E-F and CF-F, perform better in small samples, compared to their standard versions. Between the Edgeworth and Cornish-Fisher size corrected versions of these tests (i.e., E-F (or E-χ^2) and CF-F), the latter is found to perform better than the former for all sample sizes considered, and across all values of ρ, σ_{12} and α examined. Notice that, for relatively large samples (T = 40), the E-χ^2 test outperforms the degrees-of-freedom-adjusted E-F test. This suggests that, for the model considered in our simulation, samples of 40 observations seem to be large enough to induce the reduction of the magnitude of the degrees-of-freedom-adjusted Edgeworth size corrections.

Summing up, this first set of simulations clearly indicate that the finite-sample size corrected tests E-χ^2, E-F and CF-F can considerably improve the performance of the standard (uncorrected) tests in small samples. This happens even for very high levels of autocorrelation and/or cross-correlation between the error terms of the equations of the S.U.R. model. Another interesting conclusion that can be drawn from the results of this exercise is that the adjusted for the degrees of freedom versions of the tests perform better than their unadjusted ones in most of the cases considered in our simulation. Note that this is also true for the standard (uncorrected) versions of the tests.

Table 1a: $H_0 : \beta_{2,1} = 0$ against $H_A : \beta_{2,1} > 0$ (Nominal size: 5%)

			Actual sizes (%)																			
Test:			z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t
α	σ_{12}	T	$\rho = -0.8$					$\rho = -0.5$					$\rho = 0.5$					$\rho = 0.8$				
0.5	-0.90	15	14.6	10.2	13.8	10.4	8.0	11.8	8.0	11.0	8.4	7.4	11.9	8.5	11.1	8.9	8.1	14.9	11.0	13.9	11.2	9.9
		20	12.4	8.6	11.8	8.8	7.3	9.7	6.7	9.1	6.9	6.6	10.5	7.5	9.6	7.7	7.5	12.9	9.5	12.4	9.9	8.8
		40	9.0	7.2	8.7	7.3	7.0	6.9	5.3	6.6	5.4	5.3	7.4	5.9	7.3	6.1	6.0	9.8	7.7	9.5	7.9	7.5
	-0.75	15	14.6	10.1	13.9	10.3	7.9	11.1	7.7	10.3	8.0	7.3	11.7	8.2	10.9	8.6	7.9	14.5	10.5	13.5	10.7	9.6
		20	12.5	9.0	11.9	9.2	7.6	9.0	6.3	8.3	6.5	6.1	10.2	7.6	9.7	7.9	7.5	13.4	9.8	12.7	10.1	9.2
		40	8.1	6.0	7.9	6.2	5.8	7.1	5.7	6.9	5.9	5.8	7.4	5.9	7.2	6.0	5.9	9.1	7.2	8.9	7.4	7.0
	-0.50	15	14.8	10.2	14.0	10.4	7.7	10.4	7.4	9.7	7.6	7.0	11.4	8.1	10.6	8.5	7.9	14.2	10.6	13.5	10.8	9.6
		20	12.4	7.8	11.8	9.0	7.3	9.0	6.6	8.4	6.8	6.6	9.4	6.9	9.0	7.2	6.8	12.9	9.5	12.3	9.7	8.6
		40	8.5	6.5	8.3	6.7	6.4	7.0	5.4	6.8	5.6	5.5	7.4	6.1	7.2	6.2	6.1	9.2	7.1	9.0	7.3	7.1
	0.50	15	14.0	9.7	13.2	9.9	7.7	10.5	7.2	9.8	7.4	6.9	11.5	8.1	10.6	8.5	7.9	14.9	10.7	14.0	11.1	9.9
		20	11.9	8.1	11.4	8.3	6.8	8.7	6.4	8.3	6.6	6.3	10.3	7.7	9.7	8.0	7.5	13.3	10.2	12.8	10.3	9.3
		40	8.1	6.3	7.9	6.4	6.2	6.8	5.4	6.5	5.5	5.4	7.1	5.7	6.9	5.9	5.7	9.0	6.9	8.8	7.0	6.8
	0.75	15	14.7	10.2	14.0	10.4	8.0	11.5	8.0	10.5	8.2	7.5	12.2	8.5	11.3	8.9	8.3	13.8	10.3	13.1	10.5	9.3
		20	12.2	8.8	11.6	8.9	7.4	9.3	6.7	8.8	6.9	6.5	10.2	7.3	9.6	7.7	7.3	12.5	9.4	11.9	9.6	8.6
		40	8.8	6.8	8.6	6.9	6.5	7.2	5.9	7.0	6.1	6.0	7.5	5.9	7.2	6.1	6.0	9.2	7.0	8.9	7.2	6.8
	0.90	15	13.8	9.7	13.0	9.8	7.5	11.2	7.7	10.3	8.0	7.3	12.2	8.7	11.6	9.0	8.4	15.0	11.0	14.1	11.2	10.1
		20	12.9	9.0	12.4	9.2	7.7	9.4	6.6	8.7	6.8	6.3	10.0	7.3	9.4	7.5	7.2	12.9	9.5	12.3	9.8	8.8
		40	9.1	6.9	8.7	7.1	6.7	7.0	5.4	6.8	5.6	5.4	7.2	5.7	7.0	5.8	5.7	9.4	7.3	9.2	7.5	7.2
0.9	-0.90	15	14.6	10.4	13.8	10.5	7.7	11.2	7.7	10.4	7.9	7.3	11.8	8.5	11.0	8.7	8.2	14.5	10.9	13.8	11.1	9.8
		20	12.7	9.3	12.2	9.5	7.8	9.8	6.8	9.2	7.1	6.7	10.4	7.6	9.9	7.8	7.5	13.2	10.0	12.8	10.2	9.3
		40	9.2	7.2	9.0	7.4	7.1	7.4	6.0	7.2	6.2	6.0	7.3	5.9	7.1	6.0	6.0	9.9	7.9	9.7	8.0	7.7
	-0.75	15	14.5	9.8	13.5	10.0	7.8	10.7	7.3	9.8	7.5	6.9	11.7	8.4	10.9	8.7	8.1	14.9	11.0	13.9	11.3	10.1
		20	11.9	8.3	11.4	8.5	7.0	9.9	7.2	9.4	7.4	7.0	9.7	6.9	9.1	7.2	6.8	13.0	9.9	12.5	10.1	9.0
		40	8.5	6.5	8.3	6.7	6.4	6.7	5.2	6.4	5.3	5.2	7.5	5.9	7.3	6.1	6.0	9.7	7.8	9.4	7.9	7.7
	-0.50	15	14.2	9.6	13.3	9.8	7.3	10.8	7.4	9.9	7.6	7.1	11.7	8.3	10.8	8.6	8.2	14.5	10.9	13.6	11.1	9.7
		20	11.5	8.0	11.0	8.2	6.8	9.3	6.8	8.8	7.1	6.7	10.2	7.4	9.6	7.7	7.2	12.6	9.7	11.9	9.9	9.0
		40	9.0	7.0	8.8	7.2	6.8	7.1	5.8	6.9	5.9	5.8	7.3	5.7	6.9	5.9	5.7	8.9	6.8	8.7	7.0	6.7
	0.50	15	14.6	10.3	13.8	10.4	7.9	10.6	7.5	9.8	7.7	7.1	11.9	8.3	11.0	8.6	8.0	14.9	11.2	14.2	11.4	10.1
		20	12.7	8.8	12.1	9.0	7.7	9.1	6.4	8.6	6.7	6.4	9.8	7.0	9.2	7.3	6.9	12.8	9.3	12.2	9.5	8.5
		40	8.5	6.5	8.3	6.7	6.3	6.9	5.4	6.7	5.6	5.4	7.3	5.8	7.1	5.9	5.9	9.3	7.0	9.1	7.1	6.8
	0.75	15	14.0	9.6	13.2	9.7	7.3	10.7	7.3	9.9	7.5	6.9	11.6	8.0	10.6	8.3	7.8	14.1	10.3	13.3	10.5	9.5
		20	12.2	8.8	11.7	9.0	7.4	9.3	6.5	8.7	6.8	6.4	9.8	7.0	9.2	7.4	6.9	12.8	9.7	12.3	9.9	8.9
		40	8.5	6.3	8.2	6.5	6.2	7.2	5.8	7.0	6.0	5.9	7.7	5.9	7.4	6.1	6.0	9.2	7.0	8.9	7.1	6.9
	0.90	15	14.3	10.0	13.5	10.2	7.8	11.1	7.8	10.2	8.0	7.3	12.3	8.7	11.5	9.1	8.3	15.3	11.3	14.3	11.5	10.1
		20	13.0	9.1	12.4	9.3	7.7	9.1	6.8	8.7	7.0	6.6	9.9	7.2	9.4	7.4	7.0	12.8	9.3	12.2	9.4	8.5
		40	8.8	6.9	8.6	7.0	6.8	7.1	5.5	6.8	5.7	5.6	7.2	5.6	6.9	5.8	5.7	9.6	7.6	9.4	7.7	7.4

Table 1b: $H_0 : \beta_{2,1} = 0$ against $H_A : \beta_{2,1} < 0$ (Nominal size: 5%)

			Actual sizes (%)																			
Test:			z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t
α	σ_{12}	T	$\rho = -0.8$					$\rho = -0.5$					$\rho = 0.5$					$\rho = 0.8$				
0.5	-0.90	15	15.0	10.5	14.1	10.6	8.2	11.2	7.7	10.3	8.0	7.2	12.0	8.5	11.1	8.8	8.2	14.7	10.9	13.9	11.1	9.9
		20	12.5	8.8	11.8	9.0	7.3	10.1	7.3	9.6	7.5	7.0	10.0	7.3	9.5	7.7	7.3	13.2	9.6	12.5	9.9	8.9
		40	8.8	6.9	8.6	7.0	6.5	7.5	5.9	7.3	6.0	6.0	7.2	5.8	7.1	6.0	5.9	9.1	7.1	8.8	7.2	6.8
	-0.75	15	14.4	10.0	13.5	10.1	8.1	11.2	7.9	10.4	8.1	7.4	11.7	8.1	10.9	8.5	7.9	14.7	10.7	13.8	11.0	9.8
		20	12.9	9.4	12.2	9.5	8.1	9.3	6.6	8.7	6.8	6.4	9.6	6.8	9.0	7.1	6.7	12.7	9.4	12.1	9.6	8.8
		40	8.7	6.8	8.4	6.9	6.7	7.3	6.0	7.1	6.1	6.1	7.6	5.9	7.3	6.2	6.0	8.9	7.1	8.7	7.2	6.9
	-0.50	15	14.5	10.2	13.7	10.3	7.6	10.7	7.4	10.0	7.7	7.2	11.7	8.1	10.8	8.4	7.8	14.6	10.9	13.7	11.1	9.9
		20	12.3	8.7	11.7	8.9	7.3	9.5	6.7	8.8	7.0	6.6	9.7	7.1	9.1	7.3	7.0	13.1	9.7	12.5	10.1	9.0
		40	7.9	6.1	7.5	6.2	5.9	7.1	5.9	6.8	6.0	5.9	6.8	5.5	6.6	5.7	5.5	9.0	7.0	8.8	7.2	6.9
	0.50	15	13.8	9.9	12.9	10.1	7.6	10.9	7.3	10.1	7.6	6.9	11.4	8.2	10.5	8.5	7.9	14.8	11.0	14.0	11.2	10.1
		20	12.1	8.3	11.5	8.5	6.8	9.1	6.2	8.5	6.4	6.1	9.8	7.1	9.1	7.4	7.0	13.0	9.6	12.4	9.8	8.8
		40	8.6	6.4	8.4	6.6	6.4	7.2	5.7	6.9	5.9	5.8	7.6	5.8	7.4	6.0	5.9	9.9	7.6	9.6	7.7	7.3
	0.75	15	14.5	10.0	13.6	10.2	7.8	11.4	7.8	10.5	8.0	7.3	11.6	8.5	10.9	8.8	8.2	14.2	10.6	13.4	10.9	9.7
		20	12.9	8.9	12.2	9.2	7.8	9.8	7.1	9.2	7.4	7.0	9.8	6.9	9.2	7.2	6.9	12.8	9.2	12.2	9.4	8.5
		40	8.9	6.6	8.5	6.8	6.5	7.0	5.5	6.8	5.6	5.6	7.2	5.8	7.0	6.0	5.9	9.4	7.3	9.1	7.4	7.1
	0.90	15	14.1	10.1	13.2	10.2	8.2	11.2	7.9	10.4	8.1	7.4	11.9	8.2	11.0	8.5	7.8	14.6	10.7	13.8	11.0	9.7
		20	12.3	8.6	11.7	8.7	7.3	9.6	6.8	9.1	7.0	6.7	9.8	7.2	9.3	7.4	7.1	13.5	10.0	12.8	10.3	9.4
		40	8.0	6.3	7.8	6.5	6.1	6.9	5.5	6.8	5.7	5.6	7.2	5.8	7.0	6.0	5.9	9.6	7.7	9.3	7.9	7.5
0.9	-0.90	15	14.4	10.0	13.7	10.1	7.7	11.6	8.2	10.9	8.5	7.6	12.0	8.5	11.3	8.8	8.2	15.4	11.2	14.5	11.5	10.3
		20	12.4	8.9	11.8	9.1	7.5	9.3	6.5	8.7	6.7	6.4	9.9	7.2	9.2	7.4	6.9	13.0	9.8	12.3	10.1	8.9
		40	8.7	6.1	8.5	6.8	6.4	7.1	5.6	6.8	5.8	5.7	7.3	5.8	7.1	6.0	5.9	9.7	7.7	9.4	7.8	7.6
	-0.75	15	14.5	10.4	13.8	10.5	8.2	11.0	7.6	10.2	7.9	7.4	11.7	8.4	11.0	8.7	8.1	14.7	10.6	13.9	10.9	9.4
		20	12.3	8.7	11.7	8.9	7.4	9.3	6.7	8.8	7.0	6.6	9.9	7.3	9.2	7.6	7.3	12.5	9.2	12.1	9.5	8.5
		40	8.7	6.5	8.5	6.7	6.3	6.9	5.6	6.7	5.8	5.7	7.0	5.6	6.8	5.8	5.7	9.1	7.1	8.7	7.2	7.0
	-0.50	15	14.7	9.8	13.5	10.0	7.7	10.6	7.3	9.8	7.6	7.0	11.6	8.2	10.8	8.5	7.9	14.3	10.6	13.4	10.8	9.4
		20	11.7	8.1	11.2	8.4	6.7	9.5	6.8	9.0	7.1	6.6	10.2	7.5	9.6	7.9	7.3	12.5	9.0	12.0	9.3	8.4
		40	8.9	6.8	8.7	7.0	6.6	6.9	5.6	6.7	5.7	5.6	6.7	5.4	6.4	5.6	5.5	9.0	6.9	8.7	7.1	6.8
	0.50	15	14.0	9.6	13.1	9.7	7.6	10.2	7.0	9.5	7.2	6.6	11.2	8.0	10.5	8.3	7.7	14.0	10.5	13.3	10.7	9.5
		20	11.5	8.2	11.0	8.3	7.0	9.6	6.9	9.0	7.3	6.8	9.9	7.3	9.4	7.6	7.2	12.5	9.2	12.0	9.5	8.7
		40	8.5	6.3	8.1	6.5	6.2	7.2	5.7	6.9	5.9	5.8	7.4	5.7	7.1	6.0	5.9	8.9	6.8	8.7	7.0	6.7
	0.75	15	14.2	9.9	13.3	10.0	7.6	11.4	7.9	10.6	8.2	7.3	12.0	8.6	11.2	8.8	8.3	14.5	10.7	13.7	10.9	9.8
		20	12.0	8.6	11.4	8.8	7.1	9.3	6.9	8.7	7.1	6.8	9.5	6.9	9.0	7.2	6.7	12.8	9.6	12.2	9.8	9.0
		40	8.4	6.4	8.2	6.6	6.3	7.3	5.8	7.1	6.0	5.9	10.0	5.7	6.8	5.8	5.7	9.2	7.3	9.0	7.5	7.1
	0.90	15	15.3	10.5	14.4	10.6	8.2	11.3	7.9	10.4	8.2	7.6	11.2	7.8	10.4	8.1	7.5	15.3	11.5	14.5	11.7	10.3
		20	13.0	9.2	12.4	9.3	7.7	9.4	6.7	8.7	7.0	6.5	10.6	8.0	10.1	8.3	7.8	13.2	9.9	12.5	10.2	9.2
		40	9.1	7.0	8.8	7.2	6.9	7.1	5.6	6.8	5.8	5.6	7.1	5.9	7.0	6.1	6.0	10.3	7.9	10.1	8.1	7.7

Table 2: $H_0 : \beta_{1,1} = \beta_{2,1} = \beta_{1,2} = \beta_{2,2} = 0$ (Nominal size: 5%)

			Actual sizes (%)																			
Test:			χ^2	E-χ^2	F	E-F	CF-F	χ^2	E-χ^2	F	E-F	CF-F	χ^2	E-χ^2	F	E-F	CF-F	χ^2	E-χ^2	F	E-F	CF-F
α	σ_{12}	T	$\rho = -0.8$					$\rho = -0.5$					$\rho = 0.5$					$\rho = 0.8$				
0.5	-0.90	15	46.1	31.4	40.2	27.9	4.7	30.5	18.2	24.7	16.5	9.8	33.3	20.7	26.7	19.3	14.0	47.6	33.8	41.4	31.6	16.1
		20	38.2	25.0	33.6	23.6	6.4	22.7	13.2	18.6	12.9	9.6	26.1	15.6	21.6	15.2	11.9	39.9	27.6	35.6	26.5	15.3
		40	20.5	13.3	18.7	13.5	10.7	12.6	7.6	11.1	8.2	7.4	12.9	8.0	11.3	8.4	7.7	23.0	15.4	21.2	15.6	13.1
	-0.75	15	45.8	31.5	39.9	28.4	5.8	28.4	16.6	22.4	15.4	10.9	33.2	21.4	27.1	20.2	15.9	47.0	33.7	40.8	31.6	18.2
		20	36.7	24.2	32.3	22.6	7.9	22.4	12.9	18.3	12.8	10.2	25.4	15.9	21.3	15.6	13.0	38.9	26.6	34.5	25.7	16.0
		40	20.2	12.8	18.2	13.0	10.2	12.4	7.6	10.9	8.0	7.4	13.2	8.4	11.7	8.8	8.0	22.6	15.1	20.7	15.4	13.0
	-0.50	15	46.2	31.6	39.7	28.3	7.3	28.9	17.6	23.2	16.5	12.4	33.0	21.0	26.7	20.1	16.6	46.9	33.6	40.8	31.4	19.3
		20	36.0	23.1	31.6	21.7	8.9	21.1	12.1	17.1	12.1	10.0	23.1	14.2	19.1	14.2	12.2	39.2	27.4	34.8	26.5	16.7
		40	17.6	11.4	16.2	11.7	9.7	11.9	7.5	10.4	7.9	7.5	12.8	8.1	11.2	8.5	7.8	21.3	14.0	19.4	14.4	12.2
	0.50	15	45.8	31.1	39.8	28.1	7.6	29.2	17.5	23.3	16.4	12.4	32.6	20.6	26.3	19.6	16.2	47.4	33.7	41.2	31.5	19.3
		20	35.9	23.4	31.5	21.9	8.7	21.2	12.5	17.6	12.4	10.4	24.4	14.6	20.0	14.5	12.2	39.2	26.9	34.5	26.0	17.1
		40	18.3	11.4	16.4	11.6	9.4	12.1	7.6	10.7	8.1	7.6	13.2	8.1	11.5	8.5	8.0	21.8	14.2	19.8	14.4	12.1
	0.75	15	45.5	31.1	39.4	28.1	6.2	30.3	18.5	24.1	17.2	11.6	33.9	21.8	27.8	20.6	16.2	48.5	34.7	42.3	32.5	18.3
		20	36.9	24.0	32.3	22.7	8.2	22.6	13.5	18.5	13.3	10.7	24.9	15.4	20.6	15.2	12.4	40.5	28.1	36.2	27.1	17.0
		40	19.2	12.6	17.5	12.8	10.2	12.9	7.9	11.4	8.4	7.7	13.3	8.2	11.8	8.8	8.0	21.9	14.5	20.0	14.8	12.1
	0.90	15	46.1	31.7	40.1	28.2	4.9	29.9	18.0	24.2	16.4	9.7	35.0	22.2	28.9	20.7	14.9	47.4	33.7	41.5	31.3	15.6
		20	37.8	24.5	33.3	23.0	7.2	23.1	13.2	18.9	12.9	9.6	25.0	15.5	20.7	15.2	12.2	40.7	28.0	36.2	26.7	15.4
		40	20.6	13.5	18.9	13.7	10.8	12.2	7.4	10.7	7.9	7.2	13.2	7.8	11.4	8.4	7.4	23.6	15.7	21.9	16.0	13.1
0.9	-0.90	15	46.23	32.0	40.1	28.9	5.4	29.8	18.2	23.8	17.0	11.2	34.4	22.4	28.3	21.2	16.6	48.2	34.7	42.1	32.2	17.7
		20	38.2	25.3	33.8	23.8	7.3	22.9	13.4	19.1	13.3	10.7	26.1	15.9	21.8	15.8	13.2	40.7	28.1	36.1	26.9	16.4
		40	20.6	13.7	19.0	14.0	11.3	12.2	7.8	10.8	8.2	7.8	14.2	9.2	12.7	9.8	9.0	22.9	15.7	20.9	16.0	13.0
	-0.75	15	45.7	32.0	39.8	29.0	6.8	29.1	17.4	22.8	16.4	11.7	33.4	21.0	26.9	19.9	16.1	47.5	34.4	41.2	32.2	18.3
		20	36.9	24.9	32.8	23.5	8.6	21.2	11.9	17.2	11.8	9.7	24.6	15.1	20.5	15.2	12.9	39.8	27.5	35.4	26.6	16.0
		40	18.8	11.8	17.1	12.1	9.7	12.3	7.6	10.5	8.0	7.5	13.0	8.2	11.4	8.6	7.9	22.7	15.7	21.0	15.9	13.6
	-0.50	15	44.5	30.4	38.3	27.6	7.4	27.7	16.1	21.8	15.2	11.7	32.4	21.0	26.5	20.2	16.9	47.2	33.6	40.5	31.5	19.0
		20	36.1	23.5	31.5	22.2	8.5	20.7	12.0	16.8	11.9	9.8	24.1	14.9	20.5	14.9	12.7	39.4	27.0	34.8	26.3	16.6
		40	18.1	11.4	16.3	11.7	9.4	11.6	7.3	10.1	7.7	7.3	12.3	7.8	10.9	8.2	7.8	21.3	13.9	19.4	14.2	11.9
	0.50	15	44.9	30.7	38.6	27.3	7.1	28.1	17.0	21.9	16.0	12.3	32.2	20.7	26.3	20.0	16.8	47.3	33.8	40.8	32.1	19.1
		20	35.4	23.4	31.0	22.2	8.9	20.7	11.9	16.8	11.8	9.9	23.8	14.5	19.6	14.5	12.3	38.6	26.5	34.4	25.5	15.9
		40	18.4	11.8	16.8	12.1	9.7	11.9	7.5	10.4	8.1	7.5	12.3	7.9	10.7	8.4	7.8	21.3	14.1	19.3	14.3	12.2
	0.75	15	46.4	32.2	40.3	29.0	6.4	29.2	17.4	22.9	16.3	11.5	33.1	20.7	26.6	19.6	15.9	48.8	35.2	42.5	33.0	18.7
		20	37.2	24.8	32.8	23.4	8.7	22.0	12.8	17.7	12.7	10.5	25.2	15.4	21.0	15.4	13.0	39.1	27.4	34.7	26.7	16.5
		40	19.4	12.8	17.9	13.2	10.6	12.0	7.4	10.4	7.9	7.4	13.1	8.1	11.5	8.6	8.0	22.3	15.0	20.5	15.3	12.8
	0.90	15	46.8	31.9	40.2	28.5	4.9	30.4	18.3	24.5	17.1	11.6	34.4	21.8	28.0	20.6	15.7	49.0	35.1	42.8	32.8	16.9
		20	38.8	25.8	34.3	24.2	7.9	22.6	13.3	18.7	13.1	10.3	26.2	16.3	22.0	16.0	13.1	41.0	27.9	36.5	27.1	15.8
		40	20.5	13.4	18.5	13.8	11.0	12.9	8.3	11.4	8.8	8.1	13.1	8.3	11.6	8.8	8.0	22.3	15.4	20.7	15.6	13.0

In all the simulations considered so far, the errors $\varepsilon_{t\mu}$ were drawn from the standard normal distribution. The assumption of normally distributed errors can be found in many theoretical papers (see, e.g., Turkington (2000)). However, it would be very interesting to investigate what happens when the $\varepsilon_{t\mu}$'s are drawn either from a Student-t distribution with various degrees of freedom, or from an asymmetric distribution. To this end, we present the results of the relevant simulations for a collinearity coefficient $\alpha = 0.5$.

Table 3 reports the actual sizes of the compared tests of the null hypothesis $H_0 : \beta_{2,1} = 0$ against the one-sided alternative $H_0 : \beta_{2,1} > 0$ for Student-t distributed errors. The two halves of the table correspond to $\varepsilon_{t\mu}$ drawn from the Student-t distribution with 2 and 15 degrees of freedom, respectively. These Student-t distributions measure the effect of the thick distribution-tails on the performance of the suggested size corrections. Notice that the Student-t distribution with 15 degrees of freedom is exactly in the middle of the distance between the Cauchy distribution and the normal distribution (i.e., the Student-t distribution with 30 degrees of freedom). The results in Table 3 are very similar to the results in Table 1a for $\alpha = 0.5$, and, therefore, we can conclude that the thick-distribution-tails departure from the normally distributed errors *does not* deteriorate the performance of the size corrected tests.

Table 4 reports the actual sizes of the compared tests of the null hypothesis $H_0 : \beta_{2,1} = 0$ against the one-sided alternative $H_0 : \beta_{2,1} > 0$ for chi-square distributed errors. The two halves of the table correspond to $\varepsilon_{t\mu}$ equal to $\chi^2_{(1)} - 1$ and $\chi^2_{(3)} - 3$, respectively, where $\chi^2_{(1)}$ and $\chi^2_{(3)}$ are pseudo-random numbers from chi-square distribution with 1 and 3 degree of freedom, respectively. These chi-square distributions measure the effect of the skewness of distribution on the performance of the suggested size corrections. Notice that, since the expected value of a chi-square distribution equals its degrees of freedom (d.o.f.), the subtraction of the d.o.f. ensures that the errors $\varepsilon_{t\mu} = \chi^2_{(d.o.f.)} - d.o.f.$ have zero mean. The results in Table 4 are again close to the results in Table 1a for $\alpha = 0.5$, and enable us to conclude that the distribution-skewness departure from the normally distributed errors *does not* deteriorate the performance of the suggested size corrections.

Finally, we would like to assess the performance of the size corrected tests when we test some more complicated hypotheses. In doing so, we present the results of two simulation experiments for a collinearity coefficient $\alpha = 0.5$ and normal errors just like in the first set of simulations.

Table 5a reports the actual sizes of the examined t tests of the null hypothesis $H_0 : \beta_{1,1} + \beta_{2,1} = 0$ against the one-sided alternative $H_0 : \beta_{1,1} + \beta_{2,1} > 0$. The results in Table 5a are almost identical to the results in Table 1a for $\alpha = 0.5$ and normal errors just like in the first set of simulations. This means that the relative performance of the size corrected t tests *does not* depend on how complicated the null hypothesis is.

Table 5b reports the actual sizes of the examined F tests of the joint null hypothesis $H_0 : \beta_{1,1} + \beta_{2,1} = 0$ and $\beta_{1,2} + \beta_{2,2} = 0$ against the alternative $H_A :$ H_0 is false. Table 5b is qualitatively similar to Table 2 but has smaller sizes both for the uncorrected and the corrected statistics. This comes from the new hypothesis that we test which is more complicated but comprises by two restrictions rather than the initial 4. Overall the intuition of Table 5a applies here as well: complicated hypotheses do not change the properties of the tests. However the number of hypotheses does.

Table 3: $H_0 : \beta_{2,1} = 0$ against $H_A : \beta_{2,1} > 0$ (Nominal size: 5%)

			Actual sizes (%)																			
Test:			z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t
α	σ_{12}	T	$\rho = -0.8$					$\rho = -0.5$					$\rho = 0.5$					$\rho = 0.8$				
$\varepsilon_{t\mu}$ are pseudo-random numbers from Student-t distribution with 2 degrees of freedom																						
0.5	-0.90	15	15.5	11.0	14.6	11.1	8.5	11.0	7.9	10.3	8.1	7.2	11.8	8.3	11.2	8.5	8.0	14.7	10.6	13.8	10.7	9.3
		20	12.1	9.1	11.7	9.2	7.5	9.7	7.0	9.2	7.1	6.5	10.2	7.6	9.7	7.9	7.5	12.6	9.0	11.8	9.3	8.4
		40	8.8	6.6	8.5	6.8	6.4	7.5	6.0	7.3	6.1	6.1	7.7	6.0	7.5	6.1	6.0	10.0	8.0	9.8	8.2	7.7
	-0.75	15	13.9	10.1	13.3	10.1	7.6	11.4	7.4	10.6	7.7	7.1	12.2	8.5	11.4	8.9	8.2	14.8	10.9	13.8	11.0	9.7
		20	12.3	8.5	11.8	8.7	7.3	9.5	6.9	9.0	7.2	6.8	10.1	7.4	9.6	7.8	7.3	12.9	9.7	12.4	9.8	9.0
		40	9.0	6.7	8.6	6.9	6.6	6.5	5.2	6.4	5.3	5.3	7.1	6.0	7.0	6.1	6.0	9.2	7.7	9.1	7.8	7.4
	-0.50	15	13.5	9.6	12.6	9.9	7.6	11.5	8.0	10.8	8.4	7.7	12.0	8.5	11.2	8.7	8.3	14.8	10.5	14.0	10.7	9.4
		20	12.2	8.8	11.6	9.0	7.5	9.3	6.6	8.8	7.0	6.7	10.0	7.5	9.6	7.8	7.4	13.3	9.7	12.6	10.0	9.0
		40	8.5	6.4	8.3	6.5	6.2	7.8	6.2	7.6	6.4	6.3	7.4	5.9	7.2	6.1	6.0	9.0	7.0	8.8	7.3	6.9
	0.50	15	15.2	10.7	14.3	11.0	8.3	11.5	8.1	10.8	8.4	7.6	11.5	8.6	10.8	8.7	8.2	15.4	11.5	14.8	11.8	10.2
		20	12.2	8.6	11.5	8.7	7.1	9.5	6.7	8.8	6.9	6.6	9.3	7.1	8.8	7.3	7.0	12.9	9.5	12.3	9.8	9.0
		40	8.7	6.3	8.4	6.5	6.1	7.1	5.7	6.8	6.0	5.8	7.3	5.9	7.0	6.1	5.9	9.1	7.3	8.8	7.4	7.2
	0.75	15	14.4	10.1	13.7	10.2	8.0	11.2	7.6	10.1	7.8	7.1	11.8	8.0	10.7	8.3	7.6	15.3	11.5	14.3	11.8	10.3
		20	12.6	9.4	12.1	9.5	7.8	8.3	5.9	7.9	6.1	5.7	10.4	7.7	9.9	8.0	7.6	12.7	9.5	12.0	9.8	8.8
		40	8.9	6.9	8.6	7.0	6.6	7.0	5.4	6.7	5.7	5.5	7.6	6.1	7.4	6.2	6.1	9.4	7.4	9.2	7.6	7.1
	0.90	15	14.5	10.1	13.6	10.2	8.0	10.9	7.7	10.2	7.9	7.2	12.0	9.1	11.3	9.5	8.7	14.8	10.9	14.2	11.0	9.7
		20	11.9	8.6	11.4	8.7	7.1	10.5	7.2	10.0	7.6	7.1	10.2	7.4	9.5	7.5	7.2	13.1	10.0	12.6	10.0	9.4
		40	8.9	6.9	8.6	7.1	6.8	7.0	5.6	6.7	5.7	5.6	7.5	6.2	7.3	6.4	6.2	10.0	8.0	9.7	8.1	7.9
$\varepsilon_{t\mu}$ are pseudo-random numbers from Student-t distribution with 15 degrees of freedom																						
0.5	-0.90	15	14.6	9.8	13.6	10.0	7.6	11.5	7.7	10.9	8.1	7.2	12.0	8.6	11.2	8.9	8.3	15.1	10.9	14.1	11.1	9.8
		20	12.4	8.6	11.9	8.8	7.3	9.6	6.6	9.0	6.8	6.5	10.4	7.6	9.8	7.8	7.3	13.1	10.1	12.6	10.4	9.5
		40	9.3	7.2	9.1	7.4	7.2	6.7	5.2	6.4	5.5	5.3	7.8	6.0	7.5	6.2	6.1	10.1	8.2	9.8	8.4	8.0
	-0.75	15	14.6	10.2	13.8	10.3	8.0	11.2	7.5	10.2	7.9	6.9	11.8	8.7	10.9	9.0	8.5	14.7	10.9	13.8	11.1	10.1
		20	12.5	8.8	12.0	9.0	7.4	9.5	6.6	8.8	6.8	6.5	9.5	6.7	9.0	7.0	6.5	13.4	10.6	12.9	10.7	9.8
		40	8.9	6.7	8.6	6.9	6.5	7.1	5.6	6.8	5.7	5.7	7.4	6.0	7.3	6.1	6.0	8.6	6.6	8.5	6.8	6.5
	-0.50	15	14.2	9.4	13.3	9.5	7.4	10.6	7.3	9.7	7.5	6.7	11.2	7.6	10.2	8.0	7.4	14.2	10.4	13.4	10.7	9.3
		20	11.0	8.1	10.5	8.3	7.1	9.3	6.6	8.6	6.8	6.4	10.4	7.7	10.1	8.1	7.6	12.9	9.7	12.2	9.8	8.9
		40	8.5	6.6	8.4	6.7	6.4	6.9	5.6	6.7	5.8	5.7	7.1	5.7	6.9	5.9	5.7	8.6	7.0	8.4	7.1	6.8
	0.50	15	13.9	9.6	13.0	9.6	7.2	11.1	7.3	10.1	7.6	7.1	11.9	8.0	11.2	8.5	7.7	14.9	11.3	14.1	11.5	10.4
		20	12.4	8.9	11.8	9.2	7.6	8.6	6.0	8.2	6.4	5.9	9.5	7.0	9.1	7.3	7.0	13.5	9.9	13.0	10.2	9.3
		40	8.5	6.4	8.3	6.6	6.3	7.3	5.6	7.0	5.9	5.8	6.9	5.2	6.6	5.4	5.3	9.0	7.0	8.8	7.1	6.8
	0.75	15	15.2	10.7	14.4	10.8	8.2	11.1	8.1	10.4	8.4	7.6	11.8	8.4	11.0	8.8	8.1	15.5	11.6	14.7	11.9	10.5
		20	12.4	9.1	12.0	9.3	7.8	10.3	7.1	9.7	7.5	7.0	9.5	7.0	8.9	7.2	6.9	12.9	9.6	12.4	9.8	9.2
		40	8.8	6.5	8.6	6.6	6.4	7.2	5.7	6.9	5.9	5.8	7.8	6.4	7.6	6.5	6.4	9.1	7.1	8.8	7.4	6.9
	0.90	15	14.1	10.0	13.4	10.3	7.8	11.7	7.9	10.9	8.2	7.4	11.9	8.6	11.3	9.0	8.3	14.5	10.5	13.6	10.8	9.5
		20	12.9	8.8	12.3	9.0	7.5	9.9	7.1	9.6	7.3	7.0	10.3	7.3	9.7	7.5	7.0	12.8	9.4	12.3	9.6	8.5
		40	9.9	7.3	9.5	7.4	7.0	7.3	5.7	6.9	5.8	5.7	6.8	5.2	6.5	5.4	5.3	9.1	7.0	8.9	7.2	6.9

Table 4: $H_0 : \beta_{2,1} = 0$ against $H_A : \beta_{2,1} > 0$ (Nominal size: 5%)

			Actual sizes (%)																			
Test:			z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t
α	σ_{12}	T	$\rho = -0.8$					$\rho = -0.5$					$\rho = 0.5$					$\rho = 0.8$				
$\varepsilon_{t\mu} = \chi^2_{(1)} - 1$, where $\chi^2_{(1)}$ are pseudo-random numbers from chi-square distribution with 1 degree of freedom																						
0.5	-0.90	15	14.8	9.7	13.8	9.9	7.6	12.4	8.8	11.6	9.0	8.0	11.9	8.3	11.0	8.7	8.0	14.6	10.7	13.7	10.9	9.7
		20	13.1	9.3	12.3	9.5	7.9	9.2	6.7	8.5	6.8	6.6	9.8	7.2	9.3	7.5	7.1	13.3	10.1	12.6	10.3	9.3
		40	8.4	6.5	8.2	6.6	6.3	7.1	5.8	7.0	5.9	5.8	7.5	5.9	7.2	6.0	5.9	9.6	7.6	9.2	7.8	7.5
	-0.75	15	14.7	10.5	13.9	10.6	8.3	11.2	8.0	10.4	8.3	7.6	12.5	8.8	11.5	9.2	8.5	15.0	11.1	14.1	11.4	9.9
		20	12.8	9.2	12.3	9.5	7.7	10.1	7.1	9.7	7.5	7.0	9.2	6.8	8.7	7.0	6.6	13.1	9.6	12.5	9.9	8.8
		40	9.5	7.2	9.2	7.4	7.1	7.0	5.7	6.8	5.8	5.7	7.3	5.5	7.0	5.7	5.5	8.9	7.0	8.6	7.1	6.9
	-0.50	15	15.4	11.0	14.6	11.1	8.8	10.5	7.6	9.5	7.7	7.2	11.3	7.7	10.4	7.9	7.5	15.3	11.5	14.5	11.8	10.5
		20	11.8	8.4	11.2	8.5	7.1	9.2	6.8	8.8	7.0	6.6	9.9	7.1	9.4	7.5	7.0	13.4	10.0	13.0	10.2	9.2
		40	9.2	7.1	8.9	7.3	7.0	7.4	6.1	7.1	6.2	6.1	7.1	5.7	6.9	5.9	5.7	8.8	7.0	8.6	7.2	6.9
	0.50	15	14.1	9.7	13.2	9.9	7.4	11.0	7.7	10.4	7.9	7.3	11.6	8.3	10.8	8.6	8.0	14.8	10.8	13.9	11.0	10.1
		20	13.1	9.4	12.5	9.5	7.7	9.1	6.7	8.6	6.9	6.6	9.4	6.9	8.8	7.2	6.8	13.1	9.8	12.5	10.0	9.1
		40	8.7	6.5	8.6	6.6	6.4	6.5	4.9	6.3	5.2	5.0	6.9	5.5	6.7	5.6	5.6	9.3	7.0	9.2	7.2	6.7
	0.75	15	15.0	10.6	14.1	10.7	8.2	10.7	7.1	10.0	7.3	6.5	11.7	8.1	10.8	8.4	7.8	14.3	10.9	13.5	11.1	9.9
		20	13.0	9.4	12.3	9.4	7.7	9.3	6.6	8.8	6.9	6.7	9.8	7.1	9.4	7.3	7.0	13.3	9.8	12.7	10.1	9.1
		40	8.8	6.4	8.4	6.6	6.3	7.0	5.5	6.7	5.6	5.5	7.1	5.8	6.9	6.0	5.9	9.3	7.2	9.2	7.4	7.0
	0.90	15	14.5	10.2	13.6	10.4	7.8	11.2	7.6	10.1	8.0	7.1	12.2	8.2	11.3	8.4	7.7	15.2	11.0	14.4	11.2	10.0
		20	12.8	9.1	12.2	9.3	7.7	10.1	7.2	9.4	7.3	7.0	10.3	7.1	9.7	7.3	7.0	13.5	9.7	12.9	9.9	9.0
		40	9.1	7.3	8.9	7.6	7.2	7.1	5.6	6.9	5.7	5.6	7.0	5.7	6.8	5.8	5.7	9.1	7.3	8.8	7.5	7.2
$\varepsilon_{t\mu} = \chi^2_{(3)} - 3$, where $\chi^2_{(3)}$ are pseudo-random numbers from chi-square distribution with 3 degrees of freedom																						
0.5	-0.90	15	14.1	10.0	13.5	10.2	8.1	11.8	8.3	11.1	8.6	7.7	12.0	8.7	11.1	9.0	8.4	15.4	11.1	14.5	11.4	9.9
		20	12.4	9.1	11.8	9.3	7.7	9.1	6.5	8.6	6.7	6.4	10.8	7.9	10.2	8.1	7.6	13.2	10.1	12.9	10.5	9.2
		40	9.2	6.9	8.9	7.1	6.8	7.6	5.7	7.3	5.9	5.8	7.2	5.6	6.9	5.8	5.6	9.4	7.4	9.2	7.4	7.2
	-0.75	15	14.3	10.1	13.8	10.3	7.8	10.8	7.9	10.0	8.2	7.5	12.2	8.7	11.2	9.0	8.3	14.3	10.0	13.4	10.3	9.1
		20	12.1	8.4	11.5	8.6	6.9	9.6	6.9	9.1	7.2	6.7	10.5	7.5	9.9	7.9	7.5	13.2	9.9	12.6	10.2	9.2
		40	8.8	6.6	8.5	6.8	6.4	7.4	5.7	7.1	5.8	5.8	7.7	6.2	7.4	6.3	6.2	9.4	7.4	9.1	7.5	7.2
	-0.50	15	13.5	9.8	12.8	9.8	7.3	10.5	7.4	9.7	7.6	7.0	12.3	9.0	11.4	9.2	8.9	15.1	10.9	14.1	11.2	10.1
		20	12.0	8.5	11.5	8.6	7.2	8.9	6.5	8.4	6.7	6.5	9.6	6.9	9.0	7.1	6.9	13.5	10.0	12.8	10.1	9.2
		40	7.7	5.8	7.5	6.0	5.6	7.0	5.9	6.8	6.0	5.9	7.2	5.8	6.9	5.9	5.8	9.2	7.2	8.9	7.3	7.1
	0.50	15	13.9	9.1	13.0	9.2	6.9	11.4	7.7	10.6	8.0	7.4	11.7	8.4	11.0	8.7	8.1	14.6	10.6	13.9	10.8	9.6
		20	12.0	8.0	11.4	8.2	6.9	9.1	6.5	8.5	6.8	6.4	10.1	7.3	9.5	7.5	7.3	12.4	8.8	11.8	9.0	8.2
		40	8.0	6.2	7.7	6.4	6.0	7.3	5.8	7.1	6.0	5.8	7.1	5.7	6.9	5.9	5.7	9.2	7.1	9.0	7.3	6.9
	0.75	15	15.0	10.2	14.2	10.5	8.1	10.7	7.8	10.0	8.0	7.4	12.1	8.8	11.4	9.2	8.6	14.5	10.5	13.6	10.7	9.6
		20	11.9	7.9	11.2	8.2	6.7	9.5	6.3	8.9	6.7	6.3	10.4	7.2	9.5	7.5	7.1	12.4	9.0	11.8	9.4	8.4
		40	9.0	6.8	8.6	7.0	6.6	7.0	5.7	6.8	5.9	5.7	7.5	5.9	7.3	6.2	6.1	9.6	7.5	9.2	7.6	7.3
	0.90	15	14.2	10.2	13.2	10.3	8.1	11.7	8.1	10.9	8.4	7.7	12.4	8.7	11.3	9.0	8.3	15.3	11.0	14.5	11.3	10.0
		20	12.5	8.9	11.8	9.1	7.5	9.0	6.3	8.5	6.6	6.2	9.7	6.9	9.3	7.2	6.7	13.5	10.4	13.0	10.6	9.4
		40	8.5	6.1	8.3	6.3	6.0	7.4	6.2	7.2	6.3	6.2	8.0	6.5	7.8	6.7	6.6	9.3	7.5	9.1	7.6	7.1

Table 5a: $H_0 : \beta_{1,1} + \beta_{2,1} = 0$ against $H_A : \beta_{1,1} + \beta_{2,1} > 0$ (Nominal size: 5%)

			Actual sizes (%)																				
Test:			z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t	z	E-z	t	E-t	CF-t
α	σ_{12}	T	$\rho = -0.8$					$\rho = -0.5$					$\rho = 0.5$					$\rho = 0.8$				
0.5	-0.90	15	14.0	9.7	13.3	9.9	7.5	11.3	7.8	10.4	8.0	7.1	12.1	8.6	11.2	8.9	8.4	14.4	10.6	13.7	10.8	9.6
		20	12.3	8.2	11.6	8.4	6.7	10.1	7.3	9.5	7.6	7.0	9.6	7.0	9.1	7.3	6.9	12.3	9.0	11.8	9.3	8.2
		40	9.3	6.9	8.9	7.1	6.8	7.6	6.0	7.3	6.2	6.0	7.3	6.0	7.1	6.1	6.0	9.0	7.1	8.9	7.2	6.9
	-0.75	15	14.4	10.1	13.7	10.3	7.6	11.4	7.8	10.4	7.9	7.2	11.8	8.5	11.0	8.7	7.9	14.7	10.8	14.0	10.9	9.9
		20	12.4	8.7	11.7	8.9	7.1	8.5	6.3	8.0	6.6	6.2	10.4	7.2	9.7	7.5	7.0	12.9	9.4	12.4	9.7	8.5
		40	8.8	6.7	8.5	6.8	6.4	7.4	5.8	7.1	6.0	5.8	7.1	5.9	6.9	6.0	5.9	8.9	7.0	8.7	7.3	6.9
	-0.50	15	13.5	9.2	13.0	9.3	7.1	11.4	8.2	10.7	8.5	7.8	11.0	7.9	10.3	8.1	7.6	14.4	10.3	13.6	10.5	9.5
		20	12.6	8.7	12.0	8.7	7.2	9.5	6.4	8.6	6.7	6.3	10.2	7.6	9.6	7.9	7.3	13.0	9.5	12.4	9.7	8.7
		40	8.4	6.4	8.1	6.5	6.4	6.8	5.0	6.5	5.3	5.1	7.2	5.7	7.0	5.9	5.7	9.8	7.5	9.5	7.7	7.4
	0.50	15	13.8	9.8	13.1	9.9	7.1	10.4	6.8	9.6	7.1	6.4	11.9	8.1	10.8	8.3	7.9	14.5	10.4	13.5	10.7	9.6
		20	12.6	8.7	12.2	8.8	7.1	9.0	6.2	8.4	6.6	6.1	10.2	7.1	9.6	7.4	7.1	12.9	9.8	12.3	10.1	9.2
		40	8.9	6.6	8.6	6.8	6.3	7.1	5.7	6.9	5.7	5.7	7.7	6.1	7.5	6.2	6.1	9.3	7.1	9.0	7.4	6.8
	0.75	15	15.0	10.5	14.2	10.8	8.2	11.3	8.0	10.6	8.3	7.6	12.1	9.0	11.4	9.2	8.5	15.6	11.3	14.9	11.5	10.0
		20	12.4	8.7	12.0	8.9	7.7	9.0	6.3	8.5	6.5	6.1	9.4	6.8	8.8	7.0	6.7	13.0	9.4	12.5	9.7	8.4
		40	8.1	6.1	7.8	6.2	5.9	6.9	5.4	6.7	5.4	5.4	7.6	6.1	7.5	6.2	6.1	9.4	7.4	9.0	7.5	7.2
	0.90	15	13.8	9.7	13.0	9.8	7.2	10.1	7.1	9.5	7.3	6.6	12.5	8.6	11.5	9.0	8.4	14.9	10.8	13.9	11.0	10.0
		20	12.5	8.9	12.1	9.0	7.2	10.4	7.5	9.8	7.9	7.3	10.1	7.3	9.5	7.6	7.1	12.9	9.6	12.3	9.8	8.9
		40	8.7	6.6	8.3	6.7	6.2	7.7	5.9	7.3	6.1	6.1	7.3	5.9	7.1	6.1	6.0	9.1	7.0	9.0	7.2	6.9

Table 5b: $H_0 : \beta_{1,1} + \beta_{2,1} = 0$ and $\beta_{1,2} + \beta_{2,2} = 0$ against $H_A :$ H_0 is false (Nominal size: 5%)

			Actual sizes (%)																			
Test:			χ^2	E-χ^2	F	E-F	CF-F	χ^2	E-χ^2	F	E-F	CF-F	χ^2	E-χ^2	F	E-F	CF-F	χ^2	E-χ^2	F	E-F	CF-F
α	σ_{12}	T	$\rho = -0.8$					$\rho = -0.5$					$\rho = 0.5$					$\rho = 0.8$				
0.5	-0.90	15	30.6	20.6	27.1	19.4	7.6	22.1	14.0	18.7	13.9	9.5	23.3	15.3	19.9	15.1	11.7	33.3	22.1	29.1	21.5	14.6
		20	25.5	16.5	23.3	16.1	8.0	16.5	10.1	14.4	10.1	8.1	18.3	11.8	16.1	12.0	10.0	27.1	18.3	24.5	18.3	13.0
		40	14.9	10.2	14.2	10.5	8.8	9.8	6.5	8.9	6.8	6.4	10.2	6.9	9.4	7.2	6.7	16.1	11.1	15.2	11.4	9.9
	-0.75	15	30.2	19.7	26.8	18.9	8.4	20.3	11.8	17.1	11.8	9.1	23.3	14.9	20.2	14.7	12.1	32.5	21.4	28.5	21.1	14.6
		20	25.4	17.0	23.0	16.8	9.4	15.3	9.0	13.1	9.3	7.7	17.6	10.9	15.2	11.1	9.2	27.3	18.5	24.8	18.6	13.2
		40	14.2	9.8	13.3	10.0	8.5	9.7	6.8	9.1	7.1	6.7	10.5	7.4	9.7	7.8	7.4	15.7	10.9	14.8	11.2	9.4
	-0.50	15	31.7	20.3	27.6	19.2	8.2	20.6	12.6	17.6	12.3	10.0	22.9	14.9	19.7	14.7	12.2	33.0	23.1	29.3	22.7	16.3
		20	23.7	14.8	21.1	14.5	8.2	15.3	9.0	13.0	9.2	7.8	16.5	9.8	14.1	10.2	8.8	26.9	18.1	24.3	18.1	13.8
		40	13.0	8.7	11.9	8.9	7.5	10.2	6.9	9.4	7.3	6.9	10.5	7.1	9.8	7.5	7.0	16.2	11.3	15.4	11.6	10.1
	0.50	15	30.4	19.2	26.4	18.4	7.9	19.4	11.8	16.1	11.5	9.3	23.3	15.1	19.9	14.9	12.7	31.6	21.7	27.6	21.3	15.3
		20	24.6	15.8	22.1	15.5	8.9	15.2	9.4	13.3	9.7	8.3	18.0	10.1	15.7	11.2	9.7	26.1	17.8	24.0	17.9	13.1
		40	13.1	9.2	12.4	9.4	8.4	9.3	6.1	8.3	6.4	6.0	10.6	7.3	9.9	7.7	7.2	15.6	10.6	14.8	11.0	9.4
	0.75	15	31.8	20.8	27.8	19.9	9.0	21.1	12.8	18.0	12.5	9.0	24.1	15.3	20.7	15.3	12.4	32.3	22.2	28.7	21.8	15.7
		20	25.1	16.1	23.0	15.9	9.0	15.9	9.6	13.4	9.9	8.2	18.1	10.9	15.4	11.2	9.6	27.0	18.2	24.3	18.4	13.2
		40	13.5	8.7	12.5	9.0	7.6	9.4	6.4	8.7	6.7	6.2	10.6	7.1	9.7	7.5	7.0	16.3	11.1	15.0	11.5	9.8
	0.90	15	31.4	20.9	27.7	19.8	8.1	21.2	13.2	17.9	12.9	8.7	23.3	14.7	19.9	14.4	11.2	32.8	22.3	29.3	21.9	15.0
		20	26.0	16.4	23.3	16.2	7.9	16.2	9.0	14.0	9.1	7.4	18.6	11.3	16.1	11.6	9.7	26.5	17.3	23.6	17.3	12.5
		40	14.6	9.9	13.6	10.1	8.7	10.2	6.6	9.4	7.0	6.5	10.7	7.4	10.0	7.8	7.2	16.4	11.3	15.4	11.6	10.4

6 Conclusions

In this paper, we have employed Edgeworth expansions of the standard normal (or Student-t) and chi-square (or F) distributions to derive second-order size corrected testing procedures for the coefficient of the S.U.R. model with first-order autocorrelated errors. These procedures include (i) the Edgeworth corrected critical values of the well-known Wald (or F) and t tests and (ii) the Cornish-Fisher corrected F and t test statistics. Since the standard F and t tests are adjusted for the degrees of freedom, they are locally exact, which means that their approximate distributions become exact when the model is sufficiently simplified.

The Edgeworth and Cornish-Fisher expansions, employed by the paper, are equivalent to each other, since the latter constitutes an inversion of the former. However, in practice, the use of the Cornish-Fisher corrected test statistics is recommended, since they are proper random variables with well-behaved distribution tails. The Edgeworth approximation, on the other hand, may assign negative 'probabilities' in the tails of the approximate distributions. Furthermore, the Cornish-Fisher size corrected tests can be easily implemented, in practice, using the standard tables of the Student-t and the F distributions.

To evaluate the small-sample performance of the suggested tests, we have conducted a series of Monte Carlo simulations. The results of these simulations indicate that the size corrected t and F tests lead to substantial size improvements upon their standard versions, which assume first-order asymptotic approximations. This is true even for very small samples of 15 or 20 observations. Between the Edgeworth and Cornish-Fisher categories of the size corrected tests suggested in the paper, the second category is found to perform better than the first for almost all cases of serial and cross-equation correlation of the error terms of the S.U.R. model examined. This result is also robust across different degrees of multicollinearity between the explanatory variables of the model considered. In particular, both the t and F Cornish-Fisher size corrected tests are found to outperform their Edgeworth size corrected counterparts even when the degree of serial correlation of the error terms is very high. This is true even for a close-to-unity degree of correlation across the S.U.R equations.

Finally, the paper shows that the relative performance of the suggested size corrected t and F tests is not affected by two distinct departures from the normality of the errors, i.e. if the errors are distributed either with thick tails (as in the case of the Student-t distribution), or asymmetrically. Finally, the relative performance of the examined size corrections remains the same no matter how complicated the specific form of the null hypothesis is.

Appendix

In this appendix, we provide proofs of the main results of the paper. To prove these results, we rely on a number of lemmas, which are presented with their proofs in a Technical Appendix, available at the ... . This appendix is structured as follows: First, we introduce the stochastic order $\omega(\cdot)$, which measures the approximation error of the asymptotic expansions given in the paper. Then, using the lemmas in the Technical Appendix, we provide the proofs of the theorems.

The order $\omega(\cdot)$

Following Magdalinos (1992, page 344), let $\mathcal{I}$ be a given set of indexes which, without loss of generality, can be considered to belong to the open interval $(0, 1)$. For any collection of real-valued stochastic quantities (scalars, vectors, or matrices) Y_τ $(\tau \in \mathcal{I})$, we write $Y_\tau = \omega(\tau^i)$, if for any given $n > 0$, there exists a $0 < \epsilon < \infty$ such that

$$\Pr\left[\|Y_\tau/\tau^i\| > (-\ln\tau)^\epsilon\right] = o(\tau^n), \tag{A.1}$$

as $\tau \to 0$, where the $\|\cdot\|$ is the Euclidean norm. If (A.1) is valid for any $n > 0$, we write $Y_\tau = \omega(\infty)$. The use of this order of magnitude is motivated by the fact that, if two stochastic quantities differ by a quantity of order $\omega(\tau^i)$, then, under general conditions, the distribution function of the one provides an asymptotic approximation of the distribution function of the other, with an error of order $O(\tau^i)$. Furthermore, orders $\omega(\cdot)$ and $O(\cdot)$ have similar operational properties (Magdalinos (1992)).

Asymptotic expansions of size corrected tests: Proofs of theorems

Given the lemmas in the Technical Appendix, next we derive the proofs of the theorems presented in the main text. These are based on known expansions of standard normal and chi-square distributed tests. We derive new expansions of the degrees-of-freedom-adjusted versions of these tests, by inverting their characteristic functions. These degrees-of-freedom-adjusted approximations of distribution functions are proved to be locally exact.

Proof of Theorems 1 and 2. Approximation (42) of Theorem 1 can be proved following the steps of the proof in Rothenberg (1988). The quantities in (40) can be obtained by expanding the corresponding quantities given by Rothenberg and retaining the first term in each of these expansions. The approximation (44) of Theorem 2 follows from the approximation (42) and the following asymptotic approximations of the Student-t distribution and density functions, which are given in terms of the standard normal distribution and density functions, respectively (see Fisher (1925)):

$$\begin{aligned} I_{T-n}(x) &= I(x) - (\tau^2/4)(1+x^2)xi(x) + O(\tau^4), \\ i_{T-n}(x) &= i(x) + O(\tau^2). \end{aligned} \tag{A.2}$$

Note that approximation (44) of Theorem 2 is locally exact. This can be easily seen as follows: If parameter vector $\gamma = (\varrho', \varsigma')'$ is known to belong to a ball of radius ϑ, then, as $\vartheta \to 0$, γ becomes a fixed known vector. By using (27), (29), (33) and (35) we can prove that

$$\Lambda = 0, \quad \lambda = \kappa = 0, \quad \lambda_0 = 2, \quad \kappa_0 = 0. \tag{A.3}$$

Then, the analytic formulas of p_1 and p_2, given in (43), become

$$p_1 = p_2 = 0. \tag{A.4}$$

This result implies that, with an error of order $O(\tau^3)$, approximation (44) becomes the Student-t distribution function with $MT - n$ degrees of freedom. $\square$

Proof of Theorem 3. We begin the proof by noticing that, under null hypothesis (36), the t statistic, given by (37), admits a stochastic expansion of the form

$$t = t_0 + \tau t_1 + \tau^2 t_2 + \omega(\tau^3), \tag{A.5}$$

where the first term in the expansion is given as

$$t_0 = e'b/(e'Ge)^{1/2} = h'b, \quad \text{where} \quad b = GX'\Omega u/\sqrt{T}.$$

The result given by equation (A.5) implies that the Cornish-Fisher corrected statistic t_*, given by (47), admits a stochastic expansion of the form

$$t_* = t_0 + \tau t_1 + \tau^2(t_2 - t_3) + \omega(\tau^3), \tag{A.6}$$

where

$$t_3 = (p_1 + p_2 t_0^2) t_0 / 2.$$

Let s be an imaginary number, and $\psi(s)$ and $\phi(s)$ denote the characteristic functions of the t statistic, given by (37), and a standard normal random variable, respectively. Using (A.6) and the relationships:

$$E[\exp(st_0)t_0] = s\phi(s) \quad \text{and} \quad E[\exp(st_0)t_0^3] = (3s + s^3)\phi(s),$$

we can show that the characteristic function of the Cornish-Fisher corrected statistic t_*, denoted as $\psi_*(s)$, can be approximated as follows:

$$\begin{aligned}
\psi_*(s) &= \psi(s) - \tau^2 s \ E[\exp(st_0)t_3] + O(\tau^3) \\
&= \psi(s) - \frac{\tau^2}{2} s \ [p_1 s + p_2(3s + s^3)]\phi(s) + O(\tau^3).
\end{aligned}$$

Dividing $\psi_*(s)$ by $-s$, applying the inverse Fourier transform, and using Theorem 2, we can show that

$$\begin{aligned}
\Pr\{t_* \le x\} &= \Pr\{t \le x\} + \frac{\tau^2}{2}(p_1 + p_2 x^2) x i_{T-n}(x) + O(\tau^3) \\
&= I_{T-n}(x) - \frac{\tau^2}{2}(p_1 + p_2 x^2) x i_{T-n}(x) \\
&\quad + \frac{\tau^2}{2}(p_1 + p_2 x^2) x i_{T-n}(x) + O(\tau^3) \\
&= I_{T-n}(x) + O(\tau^3). \qquad \text{(A.7)}
\end{aligned}$$

The last result means that the Cornish-Fisher corrected statistic t_* is distributed as a Student-t random variable with $MT - n$ degrees of freedom. □

Proof of Theorems 4 and 5. Approximation (58) of Theorem 4 can be proved following the steps of the proof in Rothenberg (1984b). The quantities in (56) can be obtained by expanding the corresponding quantities given by Rothenberg and retaining the first term in each of these expansions. Approximation (60) of Theorem 5 follows from approximation (58) and the following asymptotic approximations of the F distribution and density functions, which are given in terms of the chi-square distribution and density functions, respectively:

$$F^m_{T-n}(x) = F_m(mx) + (\tau^2/2)(m - 2 - mx)mx f_m(mx) + O(\tau^4),$$

$$\text{(A.8)}$$

$$f^m_{T-n}(x) = m f_m(mx) + O(\tau^2).$$

Note that approximation (60) of Theorem 5 can be easily seen to be locally exact. By using (A.3), (59), and (61), we can show that

$$\xi_1 = -m(m-2)/2 \text{ and } \xi_2 = m(m+2)/2 \tag{A.9}$$

$$\Rightarrow q_1 = q_2 = 0. \tag{A.10}$$

This result means that, with an error of order $O(\tau^3)$, approximation (60) becomes the F distribution function with m and $MT-n$ degrees of freedom. □

Proof of Theorem 6. To begin the proof, we first notice that, under null hypothesis (48), the F statistic, given by (50), admits a stochastic expansion of the form

$$F = F_0 + \tau F_1 + \tau^2 F_2 + \omega(\tau^3), \tag{A.11}$$

where the first term in the expansion is

$$F_0 = b'Qb/m, \quad b = GX'\Omega u/\sqrt{T}.$$

Equation (A.11) implies that the Cornish-Fisher corrected statistic F_*, given by (64), admits a stochastic expansion of the form

$$F_* = F_0 + \tau F_1 + \tau^2 (F_2 - F_3) + \omega(\tau^3), \tag{A.12}$$

where

$$F_3 = (q_1 + q_2 F_0) F_0.$$

Let s be an imaginary number, and $\psi(s)$ and $\phi(s)$ now denote the characteristic functions of the F statistic, given by (50), and a chi-square random variable with m degrees of freedom, respectively. Using (A.12) and the following relationships:

$$E[\exp(sF_0)F_0] = \phi_{m+2}(s/m) \text{ and } E[\exp(sF_0)F_0^2] = \frac{m+2}{m}\phi_{m+4}(s/m),$$

we can show that the characteristic function of the Cornish-Fisher corrected statistic F_*, denoted as $\psi_*(s)$, can be approximated as follows:

$$\begin{aligned} \psi_*(s) &= \psi(s) - \tau^2 s\, E[\exp(sF_0)F_3] + O(\tau^3) \\ &= \psi(s) - \tau^2 s\, [q_1 \phi_{m+2}(s/m) + q_2 \frac{m+2}{m}\phi_{m+4}(s/m)] + O(\tau^3). \end{aligned} \tag{A.13}$$

For the chi-square density $f_m(x)$, the following results can be shown:

$$(mx)f_m(mx) = m f_{m+2}(mx) \text{ and } (mx)^2 f_m(mx) = m(m+2) f_{m+4}(mx). \tag{A.14}$$

Dividing (A.13) by $-s$, applying the inverse Fourier transform, and using Theorem 5 and the results of equations (A.8) and (A.14), we can show that

$$\begin{aligned}
\Pr\{F_* \le x\} &= \Pr\{F \le x\} + \tau^2[(q_1 m f_{m+2}(mx) + q_2 \frac{m+2}{m} m f_{m+4}(mx)] + O(\tau^3) \\
&= \Pr\{F \le x\} + \tau^2[(q_1 mx f_m(mx) + q_2 mx^2 f_m(mx)] + O(\tau^3) \\
&= \Pr\{F \le x\} + \tau^2(q_1 + q_2 x) mx f_m(mx) + O(\tau^3) \\
&= F^m_{T-n}(x) - \tau^2(q_1 + q_2 x) x f^m_{T-n}(x) \\
&\quad + \tau^2(q_1 + q_2 x) x f^m_{T-n}(x) + O(\tau^3) \\
&= F^m_{T-n}(x) + O(\tau^3). \qquad \text{(A.15)}
\end{aligned}$$

The last result implies that the Cornish-Fisher corrected statistic F_* is distributed as an F random variable with m and $MT - n$ degrees of freedom. □

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