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Privacy-Preserving Pinning Synchronization for Time-Delay Complex Networks: A Noise Injection Scheme

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***Abstract*—This paper is concerned with the privacy-preserving synchronization problem for a class of nonlinear time-delay complex networks under the pinning control approach. To synchronize the network nodes with the unforced target node and protect the private initial states of the underlying networks, a differentially private pinning synchronization control (DPPSC) scheme is designed based on the system outputs with injected noises. By utilizing Lyapunov stability theory and stochastic analysis technique, some sufficient conditions are derived to ensure the ultimate synchronization in the mean-square sense and achieve a specified level of differential privacy. Finally, numerical simulations are presented to substantiate the effectiveness of the proposed DPPSC scheme.**

***Index Terms*—Complex network, time-delay, pinning synchronization control, differential privacy mechanism.**

I. Introduction

Complex networks (CNs), consisting of numerous interconnected nodes, have demonstrated a remarkable ability to characterize the intricate systems with interdependent behaviors across natural, societal, and engineering domains. Owing to the complex interactions and dynamic behaviors of network nodes, CNs have sparked significant research interest from a wide range of disciplines, which gives rise to a wealth of valuable results in recent years. Among others, the synchronization problem of CNs has received particular research attention due to its clear engineering significance, see e.g., [1], [2].

In real-world applications, because of the large scale and extensive distribution of network nodes, it is both economically infeasible and technically impractical to achieve the synchronization by managing all the nodes. A more preferred way is to use the so-called pinning control strategy, where only a subset of critical nodes needs to be controlled, and the entire network can be driven to achieve synchronization through the interactions among nodes. In comparison with the traditional synchronization control strategies [3], the pinning control schemes [4]–[6] have displayed significant advantages in terms of flexibility and low cost-efficiency, particularly in the resource-constrained network environments.

With the rapid proliferation of wireless communication technologies, privacy leakages have become increasingly common, and significant research efforts have been dedicated to investigating the issue of privacy preservation. In the context of CNs, the malicious attackers may intercept the messages transmitted over the wireless channels, and accordingly infer the sensitive node information. For example, in many practical scenarios, the eavesdroppers are likely to exploit the intercepted system outputs to infer the nodes' initial states, which typically represent the individual privacy to be protected.

In recent years, various privacy-preserving methods have been proposed which include, but are not limited to, differential privacy [7], homomorphic encryption [8], and output-mask-based privacy preservation [9]. Among them, owing to the strong mathematical rigor, the differential privacy mechanism (DPM) has gradually become one of the mainstream frameworks for quantifying privacy levels [10]. Up to now, the DPM has been widely applied to address the privacy-preserving consensus problems of multi-agent systems [11]–[13]. Nevertheless, the privacy-preserving pinning synchronization problem for nonlinear time-delay CNs has not received adequate research attention yet, which motivates this current study.

Inspired by the above discussions, this paper attempts to investigate the privacy-preserving pinning synchronization problem for a class of nonlinear time-delay CNs under the DPM. The main contributions of this paper are summarized as follow:

1) the privacy-preserving synchronization problem is, for the first time, studied for a class of nonlinear time-delay CNs under the DPM and the pinning control approach, which aims to achieve the network synchronization with preserved privacy and reduced control costs; and
2) some sufficient conditions are derived to guarantee the ultimate mean-square synchronization and preserve the private information of initial states with a certain privacy level.

II. Problem Formulation

A. System Dynamics

Consider a class of nonlinear time-delay CNs with the following dynamics:

$$x_\iota(s+1) = Ax_\iota(s) + f(x_\iota(s)) + \sum_{j=1}^{N} \ell_{\iota j}\Gamma x_j(s-\tau(s)) + u_\iota(s), \quad \iota \in \digamma \triangleq \{1,2,\ldots,N\}, \tag{1}$$

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where $x_\iota(s) \in \mathbb{R}^{n_x}$ and $u_\iota(s) \in \mathbb{R}^{n_x}$ represent, respectively, the internal state vector and the control input of node ι. $\Gamma \geq 0$ represents the inner-coupling matrix. $L \triangleq [\ell_{\iota j}]_{N\times N}$ is the coupling configuration matrix satisfying $\ell_{\iota\iota} \triangleq -\sum_{j=1,j\neq\iota}^{N} \ell_{\iota j}$ and $\ell_{\iota j} \geq 0$ for $\iota \neq j$. The time-delay $\tau(s)$ is a positive integer and satisfies $0 < \tau_1 \leq \tau(s) \leq \tau_2$, where τ_1 and τ_2 are known positive scalars.

The unforced target node is modeled as follows:

$$\begin{cases} \phi(s+1) = A\phi(s) + f(\phi(s)), \\ \phi(0) = s_0, \end{cases} \tag{2}$$

where $\phi(s) \in R^{n_x}$ is the state vector of the target node.

In this paper, we make the following assumption with respect to the nonlinear function $f : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$.

Assumption 1: [14] The nonlinear vector-valued function f satisfies:

$$\begin{aligned}&[f(x) - f(y) - B_1(x-y)]^T \\ &\quad\times[f(x) - f(y) - B_2(x-y)] \leq 0, \quad \forall x, y \in R^{n_x}, \end{aligned} \tag{3}$$

where B_1 and B_2 are known constant matrices.

B. Differentially Private Pinning Synchronization

Before proceeding further, let us briefly present some preliminaries on the Laplacian random variables. For a random variable $\eta_\iota(s) = [\eta_{\iota 1}(s), \eta_{\iota 2}(s), \ldots, \eta_{\iota n_x}(s)]^T$ obeying the Laplace distribution, the corresponding probability density function is given by [13]

$$f(\eta_{\iota j}) = \frac{1}{2b} \exp\left\{-\frac{|\eta_{\iota j} - \mu|}{b}\right\}, j \in \{1, 2, \cdots, n_x\},$$

which means that $\mathbb{E}\{\eta_{\iota j}\} = \mu$ and $\mathbb{V}\{\eta_{\iota j}\} = 2b^2$.

The differentially private pinning synchronization control (DPPSC) scheme is detailed as follows.

1) Differentially Private Noise Injection Scheme

The essence of noise injection is data fuzzification, which is one of the commonly used information protection methods. To protect the private information, similar to [15], each node will resort to the differential privacy mechanism to inject noise into the transmitted data. Specifically, one has

$$\zeta_\iota(s) = x_\iota(s) + \eta_\iota(s), \tag{4}$$

where $\zeta_\iota(s) \in \mathbb{R}_x^n$ is the message received by the remote controller, and $\eta_\iota(s)$ is the injected noise generated from a Laplace distribution with $\mathbb{E}\{\eta_{\iota j}(s)\} = 0$ and $\mathbb{V}\{\eta_{\iota j}(s)\} = c_\iota \rho_\iota^s$, where $c_\iota > 0$ and $0 < \rho_\iota < 1$. Note that $\eta_{\iota j}(s),\ j \in \{1, 2, \ldots, n_x\}$ are independent and identically distributed random variables.

2) Pinning Synchronization Controller

Pinning control is a kind of partial control strategy, where the control inputs are only applied to a selected subset of key nodes. Without loss of generality, the set of pinned nodes is denoted by $\digamma_0 \triangleq \{1, 2, \ldots, \kappa\}$. Accordingly, the pinning synchronization controller is designed by

$$u_\iota(s) = \begin{cases} G_\iota(\zeta_\iota(s) - \phi(s)), & \iota \in \digamma_0, \\ 0, & \iota \in \digamma \setminus \digamma_0, \end{cases} \tag{5}$$

where G_ι is the controller gain.

3) The Eavesdropper Attack Model

In this paper, the eavesdropper is assumed to know the system model, the transmitted message $\zeta_\iota(s)$, the noise injection scheme (4), and the statistical properties of the Laplacian noise.

C. Synchronization Error Dynamics

Combining (4) and (5), the closed-loop CN is represented by

$$\begin{cases} \begin{aligned} x_\iota(s+1) =& Ax_\iota(s) + f(x_\iota(s)) + \sum_{j=1}^{N} \ell_{\iota j}\Gamma x_j(s - \tau(s)) \\ &+ G_\iota(\zeta_\iota(s) - \phi(s)), \quad \iota \in \digamma_0, \end{aligned} \\ \begin{aligned} x_\iota(s+1) =& Ax_\iota(s) + f(x_\iota(s)) + \sum_{j=1}^{N} \ell_{\iota j}\Gamma x_j(s - \tau(s)), \\ &\iota \in \digamma \setminus \digamma_0. \end{aligned} \end{cases} \tag{6}$$

Define $e_\iota(s) \triangleq x_\iota(s) - \phi(s)$ as the synchronization error of node ι. For $\iota \in \digamma_0$, it follows from (2) and (6) that

$$\begin{aligned} e_\iota(s+1) =& (A + G_\iota)e_\iota(s) + \tilde{f}(e_\iota(s)) + \sum_{j=1}^{N} \ell_{\iota j}\Gamma e_j(s - \tau(s)) \\ &+ G_\iota\eta_\iota(s), \end{aligned} \tag{7}$$

where $\tilde{f}(e_\iota(s)) = f(x_\iota(s)) - f(\phi(s))$.

Similarly, for the uncontrolled nodes $i \in \digamma \setminus \digamma_0$, one has

$$e_\iota(s+1) = Ae_\iota(s) + \tilde{f}(e_\iota(s)) + \sum_{j=1}^{N} \ell_{\iota j}\Gamma e_j(s - \tau(s)). \tag{8}$$

For ease of presentation, let us define

$$\begin{aligned} e(s) &\triangleq \mathrm{col}_N\{e_\iota(s)\}, \quad \eta(s) \triangleq \mathrm{col}_N\{\eta_\iota(s)\}, \\ \tilde{F}(e(s)) &\triangleq \mathrm{col}_N\{\tilde{f}(e_\iota(s))\}, \quad \hat{A} \triangleq I_N \otimes A, \\ \bar{G} &\triangleq \mathrm{diag}\{G, 0\},\ G \triangleq \mathrm{diag}\{G_1, G_2, \cdots, G_\iota\}. \end{aligned}$$

By utilizing the Kronecter product, the synchronization error dynamics (7) and (8) can be rewritten as follows:

$$\begin{aligned} e(s+1) =& \Lambda e(s) + (L \otimes \Gamma)e(s - \tau(s)) + \tilde{F}(e(s)) \\ &+ \bar{G}\eta(s), \end{aligned} \tag{9}$$

where $\Lambda \triangleq \hat{A} + \bar{G}$.

It is worth noting that, unlike consensus problems in MASs, synchronization problems in CNs often involve more intricate dynamics, especially in the presence of time delays and nonlinearities. One of the key challenges lies in formulating a performance criterion that simultaneously accounts for synchronization behavior and the level of privacy preservation.

III. Main Results

In this section, we will first co-design the pinning controller and the DPM to ensure the synchronization of CN (1) and (2), and then discuss the level of privacy.

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A. Synchronization Analysis

In the following theorem, we will analyze the ultimate synchronization performance in the mean-square sense.

Theorem 1: Considering the given controller gain matrix G_ι ($i \in \digamma_0$), if there exist positive definite matrices $P \in \mathbb{R}^{n_x N \times n_x N}$ and $Q \in \mathbb{R}^{n_x N \times n_x N}$ and positive scalars $1 - \max_i\{\rho_\iota\} < \beta_1 < 1$, β_2 and ε such that the following conditions hold:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & -P^{-1} \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} -\beta_2 \underline{I} & \bar{G}^T \\ * & -P^{-1} \end{bmatrix} < 0, \tag{11}$$

where

$$\begin{aligned}
\Pi_{11} &\triangleq \begin{bmatrix} \Phi_{11} - \Omega_1 & -\Omega_2 & 0 \\ * & -\varepsilon \underline{I} & 0 \\ * & * & -Q \end{bmatrix}, \\
\Phi_{11} &\triangleq (\beta_1 - 1)P + (1 - \tau_1 + \tau_2)Q, \\
\Pi_{12} &\triangleq \begin{bmatrix} \Lambda & \underline{I} & L \otimes \Gamma \end{bmatrix}^T, \\
\Omega_1 &\triangleq \varepsilon(\bar{B}_1^T \bar{B}_2 + \bar{B}_2^T \bar{B}_1)/2, \quad \Omega_2 \triangleq -\varepsilon(\bar{B}_2 + \bar{B}_1)^T/2, \\
\bar{B}_1 &\triangleq I_N \otimes B_1, \quad \bar{B}_2 \triangleq I_N \otimes B_2, \quad \underline{I} \triangleq I_N \otimes I_{n_x},
\end{aligned}$$

then, under the DPM (4) and the pinning controller (5), the considered CN (1) can asymptotically achieve the mean-square synchronization.

Proof: Let us choose the following Lyapunov functional candidate:

$$\begin{aligned}
V(s) = e^T(s)Pe(s) &+ \sum_{m=s-\tau(s)}^{s-1} e^T(m)Qe(m) \\
&+ \sum_{n=s-\tau_2+1}^{s-\tau_1} \sum_{m=n}^{s-1} e^T(m)Qe(m).
\end{aligned} \tag{12}$$

Then, it follows from the linearity of expectation and the statistical property of noise $\eta(s)$ that

$$\begin{aligned}
&\Delta V(s) \\
&\triangleq \mathbb{E}\{V(s+1)|V(s)\} - V(s) \\
&\leq [\Lambda e(s) + (L \otimes \Gamma)e(s - \tau(s)) + \tilde{F}(e(s))]^T \\
&\quad \times P[\Lambda e(s) + (L \otimes \Gamma)e(s - \tau(s)) + \tilde{F}(e(s))] \\
&\quad + \mathbb{E}\{\eta^T(s)\bar{G}^T P \bar{G}\eta(s)\} + (1 - \tau_1 + \tau_2)e^T(s)Qe(s) \\
&\quad - e^T(s)Pe(s) - e^T(s - \tau(s))Qe(s - \tau(s)).
\end{aligned} \tag{13}$$

Defining the following augmented vector

$$\varsigma(s) \triangleq [e^T(s) \quad \tilde{F}^T(e(s)) \quad e^T(s - \tau(s))]^T,$$

one has

$$\begin{aligned}
\Delta V(s) = \varsigma^T(s)M\varsigma(s) &+ E\{\eta^T(s)\bar{G}^T P \bar{G}\eta(s)\} \\
&- \beta_1 e^T(s)Pe(s),
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
M &\triangleq \begin{bmatrix} \Pi_1 & \Lambda^T P & \Lambda^T \underline{L} \\ * & P & \underline{L} \\ * & * & \bar{L} - Q \end{bmatrix}, \\
\Pi_1 &\triangleq \Lambda^T P \Lambda + (1 - \tau_1 + \tau_2)Q + (\beta_1 - 1)P, \\
\underline{L} &\triangleq P(L \otimes \Gamma), \quad \bar{L} \triangleq (L \otimes \Gamma)^T P (L \otimes \Gamma).
\end{aligned}$$

From the definition of $\eta(s)$ and (11), it is clear that

$$\begin{aligned}
&\mathbb{E}\{\eta^T(s)\bar{G}^T P \bar{G}\eta(s)\} \\
=&\mathbb{E}\{\mathrm{tr}[\bar{G}^T P \bar{G}\eta(s)\eta^T(s)]\} \\
=&\mathrm{tr}\{\bar{G}^T P \bar{G}\mathbb{E}[\eta(s)\eta^T(s)]\} \\
\leq& n_x N \beta_2 \bar{c}\bar{\rho}^k,
\end{aligned} \tag{15}$$

where $\bar{c} \triangleq \max_\iota\{c_\iota\}$ and $\bar{\rho} \triangleq \max_\iota\{\rho_\iota\}$.

Based on Assumption 1, it is not difficult to obtain that for any $\epsilon > 0$, one has

$$\begin{bmatrix} e(s) \\ \tilde{F}(e(s)) \end{bmatrix}^T \begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \varepsilon \underline{I} \end{bmatrix} \begin{bmatrix} e(s) \\ \tilde{F}(e(s)) \end{bmatrix} \leq 0. \tag{16}$$

Substituting (15) and (16) into (14) yields

$$\begin{aligned}
\Delta V(s) <& \varsigma^T(s)\tilde{\Pi}\varsigma(s) \\
&- \beta_1 V(s) + n_x N \beta_2 \bar{c}\bar{\rho}^k,
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\tilde{\Pi} &\triangleq \begin{bmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \Lambda^T \underline{L} \\ * & P - \varepsilon \underline{I} & \underline{L} \\ * & * & \bar{L} - Q \end{bmatrix}, \\
\tilde{\Pi}_{11} &\triangleq \Pi_1 - \Omega_1, \quad \tilde{\Pi}_{12} \triangleq \Lambda^T P - \Omega_2.
\end{aligned}$$

According to Schur complement lemma [16] and (10), we have $\tilde{\Pi} < 0$, which implies that

$$\Delta V(s) < -\beta_1 V(s) + \mu(s), \tag{18}$$

where $\mu(s) \triangleq n_x N \beta_2 \bar{c}\bar{\rho}^s$.

Using the law of total expectation, we have

$$\mathbb{E}\{V(s+1)\} < (1 - \beta_1)\mathbb{E}\{V(s)\} + \mu(s). \tag{19}$$

Then, it follows from (19) that

$$\begin{aligned}
\mathbb{E}\{V(s)\} <& (1 - \beta_1)\mathbb{E}\{V(s-1)\} + \mu(s-1) \\
<& (1 - \beta_1)^2 \mathbb{E}\{V(s-2)\} + (1 - \beta_1)\mu(s-2) \\
&+ \mu(s-1) \\
<& \cdots \\
<& (1 - \beta_1)^s \mathbb{E}\{V(0)\} + \sum_{\tau=1}^{s} (1 - \beta_1)^{\tau-1} \mu(s - \tau),
\end{aligned} \tag{20}$$

which implies that

$$\begin{aligned}
\mathbb{E}\left\{\|e(s)\|_2^2\right\} <& \frac{(1 - \beta_1)^s \mathbb{E}\{V(0)\}}{\lambda_{\min}(P)} \\
&+ \frac{\sum_{\tau=1}^{s} (1 - \beta_1)^{\tau-1} \mu(s - \tau)}{\lambda_{\min}(P)}.
\end{aligned} \tag{21}$$

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Based on the preceding analysis, when $s \to +\infty$, we can arrive at

$$\mathbb{E}\left\{\|e(s)\|_2^2\right\} = 0. \tag{22}$$

The proof is now complete. ■

B. Pinning Controller Design

This subsection provides a solution to the design problem of the pinning controller gains.

Theorem 2: Considering the closed-loop system dynamics (6) and the DPM (4). Let a scalar $1-\max_i\{\rho_\iota\} < \beta_1 < 1$ be given. If there exist positive scalars β_2, ε, positive definite matrices $P \in \mathbb{R}^{n_x N \times n_x N}$, $Q \in \mathbb{R}^{n_x N \times n_x N}$, a nonsingular matrix $X \triangleq \text{diag}\{X_1, X_2, \cdots, X_N\} \in \mathbb{R}^{n_x N \times n_x N}$, and matrix $\mathscr{G}$ satisfying

$$\begin{bmatrix} \Pi_{11} & \tilde{\Pi}_{12} \\ * & -X^T - X + P \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} -\beta_2 \underline{I} & \mathscr{G}^T \\ * & -X^T - X + P \end{bmatrix} < 0, \tag{24}$$

where

$$\tilde{\Pi}_{12} \triangleq \begin{bmatrix} \tilde{\Lambda} & X & X(L \otimes \Gamma) \end{bmatrix}^T, \quad \tilde{\Lambda} \triangleq X\hat{A} + \mathscr{G},$$

then the considered CN (1) can asymptotically achieve the mean-square synchronization under the DPM (4) and the pinning controller (5) with $\bar{G} = X^{-1}\mathscr{G}$.

Proof: According to the positive definiteness of P, we have

$$(P - X)P^{-1}(P - X)^T \geq 0, \tag{25}$$

which implies that

$$-X^T - X + P \geq -XP^{-1}X^T. \tag{26}$$

It is evident from (23) and (26) that

$$\begin{bmatrix} \Pi_{11} & \tilde{\Pi}_{12} \\ * & -XP^{-1}X^T \end{bmatrix} < 0. \tag{27}$$

By pre- and post-multiplying the matrix in (23) with $\text{diag}\{\underline{I}, \underline{I}, \underline{I}, X^{-1}\}$ and its transpose, it is easy to deduce (10) from (25) and (27). An analogous derivation applied to (24) yields (11) in Theorem 1. ■

C. Analysis on Privacy

Before proceeding further, let us give the following definition.

Definition 1: [15] A randomized mechanism $\mathcal{M}$ is said to preserve ϵ-differential privacy, if for any pair of state vectors x and x', and any observation set $\mathsf{Obq} \subseteq \text{Range}(\mathcal{M})$, such that the following inequality holds

$$P\left\{\mathcal{M}(x) \in \mathsf{Obq}\right\} \leq e^{\epsilon\|x - x'\|_1} P\left\{\mathcal{M}(x') \in \mathsf{Obq}\right\}.$$

Next, we will analyze the differential privacy property of the proposed scheme.

Theorem 3: Given the scalar $\lambda > 0$ and the gain matrix $\bar{G}$, if there exists a scalar $0 < r < 1$ such that the following condition holds:

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} \\ * & -\underline{I} \end{bmatrix} < 0, \tag{28}$$

where

$$\bar{\Pi}_{11} \triangleq \begin{bmatrix} -\lambda\underline{I} - \Omega_1 & -\Omega_2 & 0 \\ * & -r\underline{I} & 0 \\ * & * & 0 \end{bmatrix},$$
$$\bar{\Pi}_{12} \triangleq \begin{bmatrix} \Lambda - \bar{G} & \underline{I} & L \otimes \Gamma \end{bmatrix}^T,$$
$$\Omega_1 \triangleq r(\bar{B}_1^T\bar{B}_2 + \bar{B}_2^T\bar{B}_1)/2, \quad \Omega_2 \triangleq -r(\bar{B}_2 + \bar{B}_1)^T/2,$$

then, under the DPM (4) with $\lambda < \rho_\iota < 1$, the ϵ-differentially privacy of initial state can be preserved with

$$\epsilon = \frac{\sqrt{2n_x N/\underline{c}}}{1 - \sqrt{\lambda/\underline{\rho}}}, \tag{29}$$

where $\underline{c} \triangleq \min_\iota\{c_\iota\}$ and $\underline{\rho} \triangleq \min_\iota\{\rho_\iota\}$.

Proof: Let $x(0)$ and $x'(0)$ denote a pair of φ-adjacent initial states. Clearly, one has

$$\begin{cases} x_q(0) = x'_q(0) + \varphi, & q \in \digamma, \\ x_\iota(0) = x'_\iota(0), & \iota \neq q, \end{cases}$$

where $\varphi \geq 0$.

Assume that the observational equivalence holds for released data under φ-adjacent initial conditions $x(0)$ and $x'(0)$, that is, $\zeta(s) \equiv \zeta'(s)$ hold for $\forall s \geq T_0$, which implies that

$$\eta'(s) - \eta(s) = x(s) - x'(s). \tag{30}$$

Next, denote $\delta(s) \triangleq \eta'(s) - \eta(s)$. It can be obtained from (6) and (30) that

$$\begin{aligned} \delta(s+1) &= x(s+1) - x'(s+1) \\ &= (\Lambda - \bar{G})\delta(s) + (L \otimes \Gamma)\delta(s - \tau(s)) \\ &\quad + \tilde{F}(\delta(s)). \end{aligned} \tag{31}$$

For any given $x(0)$, the observation sequence $\mathcal{M}(x(0), \eta(\mathcal{T}))$ is uniquely determined by the noise sequence η. Thus, for any sets of observation sequence $\mathcal{O}$, we obtain the joint probability density function of $\mathcal{M}(x(0), \eta(\mathcal{T}))$ over a time range T as

$$\begin{aligned} &f(\mathcal{M}(x(0), \eta(\mathcal{T})) \in \mathcal{O}) \\ =&\prod_{s=0}^{T}\prod_{\iota=1}^{N}\prod_{j=1}^{n_x} f(\eta_{\iota j}(s)) \\ =&\prod_{s=0}^{T}\prod_{\iota=1}^{N} \frac{\exp\left(-\|\eta_\iota(s)\|_1/\sqrt{c_\iota\rho_\iota^s/2}\right)}{(2c_\iota\rho_\iota^s)^{n_x/2}}. \end{aligned} \tag{32}$$

Furthermore, we obtain

$$\frac{f(\mathcal{M}(x(0), \eta(\mathcal{T})) \in \mathcal{O})}{f(\mathcal{M}(x'(0), \eta'(\mathcal{T})) \in \mathcal{O})}$$

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$$\leq \exp\left(\sum_{s=0}^{T} \|\delta(s)\|_1 / \sqrt{\underline{c}\underline{\rho}^s/2}\right). \tag{33}$$

It follows from the Cauchy-Schwarz inequality and (28) that

$$\begin{aligned}
&\frac{f(\mathcal{M}(x(0),\eta(\mathcal{T}))\in\mathcal{O})}{f(\mathcal{M}(x'(0),\eta'(\mathcal{T}))\in\mathcal{O})}\\
&\leq \exp\left(\sqrt{n_x N}\sum_{s=0}^{T}\|\delta(s)\|_2/\sqrt{\underline{c}\underline{\rho}^s/2}\right)\\
&\leq \exp\left(\epsilon\|\delta(0)\|_2\right),\ T\to\infty,
\end{aligned} \tag{34}$$

where $\epsilon = \frac{\sqrt{2n_x N/\underline{c}}}{1-\sqrt{\lambda/\underline{\rho}}}$.

Finally, we arrive at

$$\begin{aligned}
&\frac{P\{\mathcal{M}(x(0),\eta(\mathcal{T}))\in\mathcal{O}\}}{P\{\mathcal{M}(x'(0),\eta'(\mathcal{T})\in\mathcal{O}\}}\\
&=\frac{\int_{\eta(\mathcal{T})} f(\mathcal{M}(x(0),\eta(\mathcal{T})))\in\mathcal{O})d\eta}{\int_{\eta(\mathcal{T})} f(\mathcal{M}(x'(0),\eta'(\mathcal{T}))\in\mathcal{O})d\eta}\\
&\leq \exp\left(\epsilon\|x(0)-x'(0)\|_1\right).
\end{aligned} \tag{35}$$

The proof is now complete. ■

IV. An Illustrative Example

In this section, a numerical example is provided to demonstrate the effectiveness of the developed DPPSC scheme.

Consider a CN (1) with six coupled nodes, where the nodes 1-3 are selected as the pinned nodes. The parameters are set as follows:

$$A = \begin{bmatrix} 0.21 & 0.27 \\ 0.24 & 0.55 \end{bmatrix},$$

$$L = \begin{bmatrix} -0.5 & 0.2 & 0.1 & 0 & 0.1 & 0.1 \\ 0.1 & -0.5 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.15 & -0.5 & 0.1 & 0.05 & 0.1 \\ 0.1 & 0.1 & 0.1 & -0.5 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & -0.5 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & -0.5 \end{bmatrix},$$

and $\Gamma = 0.5I$. The time-delay is set as $\tau(s) = 2+\cos(s\pi)$, and the nonlinear function is given by

$$f(\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix}) = \begin{bmatrix} -\tanh(0.21\nu_1) \\ \tanh(1.02\nu_2)) \end{bmatrix}.$$

The initial states of node $x_\iota(s)$ and target node $\phi(s)$ are given as follows:

$$\begin{aligned}
&x_1(0) = \begin{bmatrix}4 & 2\end{bmatrix}^T,\ x_2(0) = \begin{bmatrix}2 & -2\end{bmatrix}^T,\ x_3(0) = \begin{bmatrix}1 & 3\end{bmatrix}^T,\\
&x_4(0) = \begin{bmatrix}2 & 3\end{bmatrix}^T,\ x_5(0) = \begin{bmatrix}4 & 5\end{bmatrix}^T,\ x_6(0) = \begin{bmatrix}3 & -1\end{bmatrix}^T,\\
&\phi(0) = \begin{bmatrix}-0.25 & 0.25\end{bmatrix}^T.
\end{aligned}$$

According to the proposed DPPSC scheme, the controller gains G_ι is designed by

$$G_1 = \begin{bmatrix} -0.0822 & -0.0787 \\ -0.0669 & -0.1810 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} -0.0836 & -0.0786 \\ -0.0664 & -0.1824 \end{bmatrix},$$

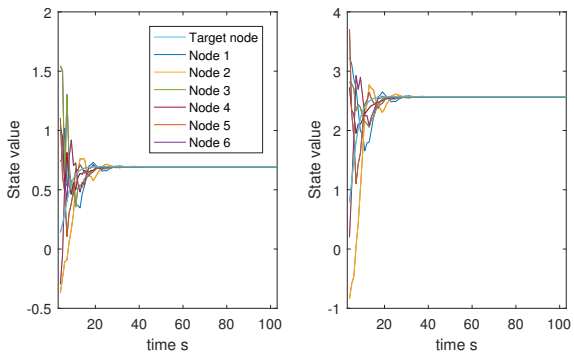

Fig. 1. Trajectories of the first state component and the second state component.

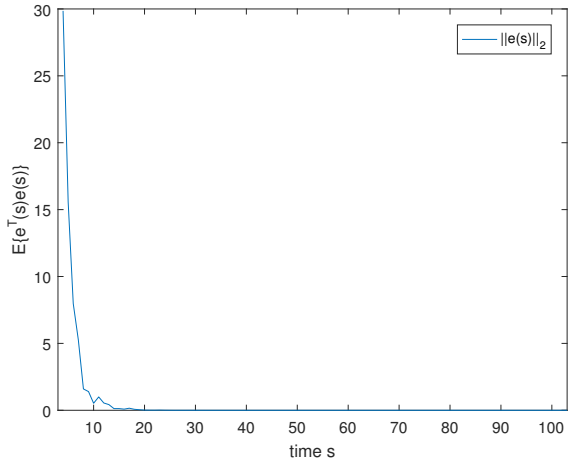

Fig. 2. Synchronization error with $\mathbb{V}\{\eta_{\iota j}(s)\} = 1\times 0.72^s$.

$$G_3 = \begin{bmatrix} -0.0820 & -0.0780 \\ -0.0662 & -0.1800 \end{bmatrix}.$$

The simulation results under the Laplacian noise injection scheme specified by $\mathbb{E}\{\eta_{\iota j}(s)\} = 0$ and $\mathbb{V}\{\eta_{\iota j}(s)\} = c\times 0.72^s$, where c=1, are presented in Figs. 1 and 2. Specifically, Figs. 1 show the trajectories of $\phi(s)$ and $x(s)$, and Fig. 2 displays the trajectory of synchronization error $e(s)$.

Figs. 3 and 4 show the behaviors of observation sequences $\zeta(s)$ and $\zeta'(s)$, which correspond to the φ-adjacent initial states $x(0)$ and $x'(0)$, respectively. It is clear that the larger the variance of injected noise, the better the performance of privacy preservation. On the other hand, increasing the variance will adversely affect the dynamic performance of synchronization, as shown in Fig. 5. Thus, the injected noise should be designed to achieve an optimal compromise between the synchronization performance and the privacy level.

V. Conclusion

In this paper, the privacy-preserving pinning synchronization control problem has been studied for a class of nonlinear time-delay CNs against eavesdropping attacks. By injecting noise into the output data, the initial state of the network node

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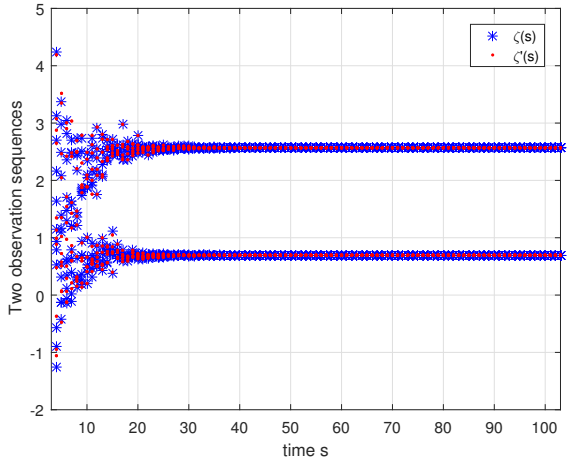

Fig. 3. Observation sequences $\zeta(s)$ and $\zeta'(s)$ with $\mathbb{V}\{\eta_{ij}(s)\} = 1 \times 0.72^s$.

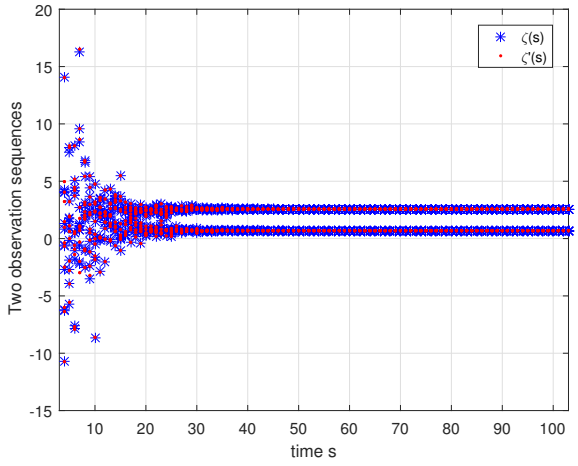

Fig. 4. Observation sequences $\zeta(s)$ and $\zeta'(s)$ with $\mathbb{V}\{\eta_{ij}(s)\} = 100 \times 0.72^s$.

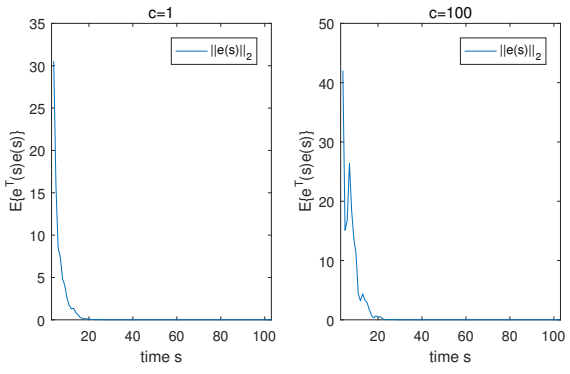

Fig. 5. Synchronization error with different variance of injected noise.

has been prevented from being inferred by the attacker. The desired pinning controller gains and the differentially private mechanism have been co-designed by exploiting the linear matrix inequality technique and the stochastic analysis method. Some sufficient conditions have been derived to guarantee the ultimate mean-square synchronization while preserving the private information. Finally, some simulation results have been given to demonstrate the effectiveness of the proposed DPPSC algorithm. One possible future research topic would be addressing the diverse security requirements by exploring other privacy-preserving schemes.

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