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Recursive Remote State Estimation for Stochastic Complex Networks with Degraded Measurements and Amplify-and-Forward Relays

Tong-Jian Liu, Zidong Wang, Yang Liu, and Rui Wang

***Abstract*—This paper is concerned with the remote state estimation problem for stochastic complex networks under the effects of degraded measurements and amplify-and-forward (AF) relays. Three sets of random variables are employed to describe the measurement degradation, the sensor transmission energy, and the relay transmission energy, respectively. The measurement from each node is transmitted to an AF relay and then forwarded to the remote estimator to facilitate the state estimation. A novel recursive estimator is constructed in the form of the extended Kalman filter. An upper bound of estimation error covariance is determined by solving Riccati-like difference equations based on the statistical information of the random variables, and such an upper bound is then minimized by choosing an appropriate estimator gain. Furthermore, sufficient conditions are established under which the estimation error is exponentially bounded in the sense of mean square. Finally, the effectiveness of the proposed estimation scheme is demonstrated by some numerical simulations.**

***Index Terms*—Complex networks, state estimation, amplify-and-forward relay, degraded measurements, variance constraints.**

I. Introduction

In the last few decades, complex networks have gained considerable research attention due to their wide range of practical applications such as smart grids [32], epidemic spread [38], biological networks [20], sensor networks [51], and failure propagation analysis in aero-engine [28]. Generally, a complex network is constituted by a group of nodes, and the dynamics of every node is affected by the neighboring nodes under a given topology [2]. So far, much research effort has been devoted to different issues in complex networks including modeling [7], stability analysis [16], and synchronization [53].

The availability of system state information is crucial for monitoring and comprehending the operational conditions of the considered systems. However, in the case of complex networks, obtaining system states is often challenging due to the complexities of the operating environments, limitations of physical devices, and high costs associated with measurements [5], [15], [27], [30], [47], [55]. So far, significant attention has been devoted to the problem of state estimation in complex networks [29], [39], [41], [56]. For instance, a mixed compensation method has been proposed in [26] to enhance the performance of set-membership filtering for complex networks with communication channel constraints. In [13], a minimum-variance estimator has been designed for complex networks under signal transmission attacks. An event-triggered recursive state estimation approach has been developed to handle time-varying nonlinear complex networks with the effects of quantization [34]. For complex networks with randomly varying topologies, a finite-horizon estimator has been constructed in [8] to simultaneously satisfy variance-constrained and H_∞ estimation requirements. Furthermore, in [23], a recursive estimator based on the extended Kalman filter (EKF) has been established for stochastic complex networks with switching topology.

Degraded measurements, a common network-induced phenomenon, frequently arise in signal transmissions due to factors such as multi-path propagation/shadowing caused by obstacles [4], limited bandwidth [11], and random fluctuations in network environments [52]. These degraded measurements effectively capture the non-ideal characteristics of measurements transmitted through imperfect channels, making them a topic of interest for many researchers [46]. For instance, the problem of recursive minimum-variance state estimation has been addressed in [25] for a class of two-dimensional shift-varying systems subject to degraded measurements. In the context of wireless localization, the target tracking problem has been investigated in [1] by considering the effects of degraded measurements and quantization. Nonetheless, to date, the consideration of degraded measurement in complex networks has not been given adequate attention yet, and this serves as one of the main motivations for our research.

It is important to note that most existing results on the state estimation problem may not directly apply to situations where measurements need to be transmitted over long-distance

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wireless channels. In such scenarios, the signal intensity of measurements may attenuate during long-distance transmission due to limitations in transmission capacity [9]. To enhance the quality of long-distance signal transmission, a common strategy is to incorporate a relay within the transmission link. Two widely adopted relay types are Decode-and-Forward (DF) relays [48] and Amplify-and-Forward (AF) relays [3]. In AF relays, the received signals from the source are amplified before being forwarded to the destination. The AF relay strategy is widely employed in practical applications, including bent-pipe satellites, fixed microwave links, and cooperative wireless communication systems [18], [31], [37]. Notably, state estimation becomes particularly crucial when signals are transmitted via AF relays, as the amplification process of the relay introduces additional noise into the measurements.

Till now, systems employing AF relays have recently gained considerable research attention [12], [40], [42], [45]. For example, the effects of noise correlation on AF relays have been investigated in [17] by focusing on optimizing the relay gains to maximize the rate. A robust recursive state estimation strategy has been proposed in [43] for estimating states in a linear stochastic system under the influence of AF relays with random transmission energy. However, to the best of the authors' knowledge, the state estimation problem for time-varying stochastic complex networks with AF relays and degraded measurements remains an ongoing research topic. Hence, the main objective of this paper is to address this research gap.

Motivated by the aforementioned considerations, the purpose of this paper is to explore the remote state estimation problem in a stochastic complex network with degraded measurements and AF relays. Addressing this problem is technically challenging due to the influence of AF relays and degraded measurements on the relationship between the system states and the received signal at the remote estimator. To tackle this challenge, we propose a remote estimator based on the EKF by taking into account the statistical characteristics of the random transmission energy and the measurement degradation. Furthermore, we analyze the estimation performance in terms of the exponential boundedness of the estimation error in the mean square sense.

This paper offers several key contributions, which can be summarized as follows.

1) A detailed and comprehensive model is developed to describe the dynamics of a time-varying complex network with degraded measurements and AF relays. This model provides a solid foundation for addressing the remote state estimation problem.
2) By solving Riccati-like difference equations, an upper bound for the estimation error covariance is derived. This bound serves as an important metric for evaluating the performance of the proposed estimator.
3) The gain of the EKF-based estimator is designed recursively in order to minimize the derived upper bound of the estimation error covariance. This recursive design approach enhances the efficiency and effectiveness of the estimation process.
4) Sufficient conditions are established to ensure the exponential boundedness of the estimation error in the mean square sense. These conditions provide theoretical guarantees for the stability and reliability of the proposed estimation scheme, an aspect not addressed in literature such as [43].

Overall, a comprehensive framework is established in this paper for addressing the remote state estimation problem in a time-varying complex network with degraded measurements and AF relays.

Notations. The notations used in the whole paper are standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{x\times y}$ denote the x-dimension of Euclidean space and the set of $x\times y$ real matrices, respectively. $\mathbb{Z}_+$ represents the non-negative integers. A^T and A^{-1} denote the transpose and the inverse of matrix A, respectively. $\|\cdot\|$ represents the spectral norm of matrices and the Euclidean norm of vectors. For the symmetrical matrices U and V, the notation $U \geq V$ (respectively, $U > V$) means that $U - V$ is positive semidefinite (respectively, positive definite). $\circ$ denotes the Hadamard product which is defined as $[A \circ B]_{ij} \triangleq a_{ij} \cdot b_{ij}$. The symbol $\otimes$ represents the Kronecker product defined as $A \otimes B = \begin{bmatrix} a_{1,1}B & \cdots & a_{1,n}B \\ \vdots & \ddots & \vdots \\ a_{m,1}B & \cdots & a_{m,n}B \end{bmatrix}$. I and 0 respectively represent the identity matrix and zero matrix with appropriate dimensions. $\mathbb{E}\{x\}$ stands for the mathematical expectation of the random variable x. $\mathrm{col}_N\{x_i\}$ denotes the column vector $\begin{bmatrix} x_1^T & x_2^T & \dots & x_N^T \end{bmatrix}$. $\mathbf{1}_n \in \mathbb{R}^{n\times n}$ is the all-ones matrix. $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix and $\mathrm{diag}_N\{A_i\}$ represents the matrix $\mathrm{diag}\{A_1, A_2, \dots, A_N\}$. $\mathrm{tr}\{\cdot\}$ represents the trace of a square matrix.

II. Problem Formulation

In this paper, we consider a class of discrete time-varying stochastic complex networks coupled with N nodes. The dynamics of the ith node is described as follows:

$$x_{i,t_z+1} = f(x_{i,t_z}) + \sum_{j=1}^{N} \omega_{ij}\Gamma x_{j,t_z} + B_{i,t_z}\varpi_{i,t_z}, \qquad (1)$$

$$y_{i,t_z} = \theta_{i,t_z} C_{i,t_z} x_{i,t_z} + \nu_{i,t_z}, \qquad (2)$$

where $t_z \in \mathbb{Z}_+$ is the time instant, $x_{i,t_z} \in \mathbb{R}^n$ is the system state of the ith node, $y_{i,t_z} \in \mathbb{R}^m$ is the measurement output, and ϖ_{i,t_z} and ν_{i,t_z} are zero-mean additive noises with covariances $Q_{i,t_z} > 0$ and $R_{i,t_z} > 0$, respectively. The mean value of the initial state $x_{i,0}$ is $\bar{x}_{i,0}$. $f(\cdot)$ is a known and continuously differentiable nonlinear function. B_{i,t_z} and $C_{i.t_z}$ are known matrices.

In the network (1)-(2), the phenomena of degraded measurement is characterized by the random variable $\theta_{i,t_z} \in [0,1]$ with mathematical expectation μ_{i,t_z} and variance σ_{i,t_z}^2. θ_{i,t_z} $(i = 1, 2, \dots, N)$ are independent of the noises ϖ_{i,t_z} and ν_{i,t_z}. $W = [\omega_{ij}] \in \mathbb{R}^{N\times N}$ is the coupled configuration matrix of the network with $\omega_{ij} \geq 0$ $(i \neq j)$. Specifically, $\omega_{ij} > 0$ if node j and node i are directly connected and $\omega_{ij} = 0$ $(i \neq j)$ otherwise. W is symmetric with $\sum_{j=1,j\neq i}^{N} \omega_{ij} = -\omega_{ii}$. $\Gamma = \mathrm{diag}\{g_1, g_2, \dots, g_n\}$ is an inner-coupling matrix.

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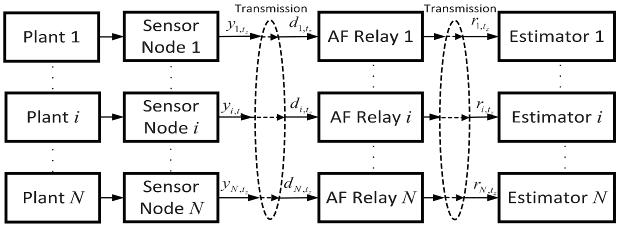

Fig. 1. The diagram of state estimation for complex networks with AF relays

For the considered complex network, all sensors need to send measurements to the remote estimator via the AF relay, which can be seen in Fig. 1. The random transmission energy of each sensor and AF relay are $p_{s,i,t_z} \in \mathbb{R}$ and $p_{r,i,t_z} \in \mathbb{R}$ ($i = 1, 2, \ldots, N$), respectively. p_{s,i,t_z} and p_{r,i,t_z} are independent of each other, the measurement degradation, and all the noises.

The transmission model for the AF relay is described as follows. The signals received by the ith AF relay at time t_z are denoted as $d_{i,t_z} \in \mathbb{R}^m$ with

$$d_{i,t_z} = \sqrt{p_{s,i,t_z}} H_{s,i,t_z} y_{i,t_z} + \zeta_{s,i,t_z}, \tag{3}$$

where the subscript s represents the sensor-to-relay channel, $H_{s,i,t_z} \in \mathbb{R}^{m\times m}$ is a diagonal matrix which characterizes the channel coefficient of sensor-to-relay channel, and ζ_{s,i,t_z} stands for the zero-mean noise with covariance $S_{s,i,t_z} > 0$.

The random transmission energy $p_{s,i,t_z} \in \mathbb{R}$ is subjected to the following probability distribution:

$$\text{Prob}\{p_{s,i,t_z} = \vartheta^{[\hbar]}_{s,i,t_z}\} = \chi^{[\hbar]}_{s,i,t_z}, \quad \hbar = 1, 2, \ldots, \varphi, \tag{4}$$

where $\vartheta^{[\hbar]}_{s,i,t_z}$ is the transmission energy, and $\chi^{[\hbar]}_{s,i,t_z} \in [0, 1]$ are known scalars with $\sum_{\hbar=1}^{\varphi} \chi^{[\hbar]}_{s,i,t_z} = 1$.

The signals received by the ith relay are amplified and forwarded to the destination for state estimation. The actual received signal at the destination $r_{i,t_z} \in \mathbb{R}^m$ is expressed as:

$$r_{i,t_z} = a_{i,t_z} \sqrt{p_{r,i,t_z}} H_{r,i,t_z} d_{i,t_z} + \zeta_{r,i,t_z}, \tag{5}$$

where the subscript r represents the relay-to-destination channel, $a_{i,t_z} > 0$ is the amplification factor, $H_{r,i,t_z} \in \mathbb{R}^{m\times m}$ is a diagonal matrix which characterizes the channel coefficient of relay-to-destination channel, and ζ_{r,i,t_z} stands for the zero-mean noise with covariance $S_{r,i,t_z} > 0$. The random transmission energy p_{r,i,t_z} is governed by

$$\text{Prob}\{p_{r,i,t_z} = \vartheta^{[\tau]}_{r,i,t_z}\} = \chi^{[\tau]}_{r,i,t_z}, \quad \tau = 1, 2, \ldots, \psi, \tag{6}$$

where $\vartheta^{[\tau]}_{r,i,t_z}$ is the transmission energy of the τth case and $0 \le \chi^{[\tau]}_{r,i,t_z} \le 1$ are known scalars with $\sum_{\tau=1}^{\psi} \chi^{[\tau]}_{r,i,t_z} = 1$.

Remark 1: The utilization of AF relays allows the transmission of measurements from each node to the destination through long-distance channels. The AF relays amplify the strength of the measurement signals, enabling their successful transmission. However, the presence of AF relays introduces additional complexities to the remote state estimation problem. Specifically, the AF relays introduce random transmission energies, denoted as p_{s,i,t_z} and p_{r,i,t_z}, associated with the source and relay nodes, respectively. These random transmission energies contribute to the variability in the received measurements and need to be considered in the estimation process. Additionally, the AF relays introduce extra transmission noises, denoted as ζ_{s,i,t_z} and ζ_{r,i,t_z}, which further affect the quality and reliability of the received measurements. Overall, the inclusion of AF relays in the system introduces random transmission energies and additional transmission noises, posing additional challenges to the remote state estimation problem. These challenges need to be effectively addressed to achieve accurate and reliable estimation of the system states.

The following estimator is constructed for node i:

$$\begin{aligned}\hat{x}_{i,t_z+1|t_z} =& f(\hat{x}_{i,t_z|t_z}) + \sum_{j=1}^{N} \omega_{ij} \Gamma \hat{x}_{j,t_z|t_z}, \qquad (7)\\ \hat{x}_{i,t_z+1|t_z+1} =& \hat{x}_{i,t_z+1|t_z} + K_{i,t_z+1}(r_{i,t_z+1} - a_{i,t_z+1} \\ & \times \bar{p}_{r,i,t_z+1} \bar{p}_{s,i,t_z+1} \mu_{i,t_z+1} H_{r,i,t_z+1} \\ & \times H_{s,i,t_z+1} C_{i,t_z+1} \hat{x}_{i,t_z+1|t_z}), \qquad (8)\end{aligned}$$

where $\hat{x}_{i,t_z|t_z}$ is the estimate of x_{i,t_z} at time t_z with $\hat{x}_{i,0|0} = \bar{x}_{i,0}$, and $\hat{x}_{i,t_z+1|t_z}$ is the one-step prediction at time t_z. $\bar{p}_{s,i,t_z+1} \triangleq \sum_{\hbar=1}^{\varphi} \chi^{[\hbar]}_{s,i,t_z+1} \sqrt{\vartheta^{[\hbar]}_{s,i,t_z+1}}$ and $\bar{p}_{r,i,t_z+1} \triangleq \sum_{\tau=1}^{\psi} \chi^{[\tau]}_{r,i,t_z+1} \sqrt{\vartheta^{[\tau]}_{r,i,t_z+1}}$. K_{i,t_z+1} is estimator gain that needs to be determined later.

Remark 2: The presence of degraded measurements and AF relays adds complexity to the design of the remote estimator. To effectively manage the stochastic nature arising from these factors, the proposed remote estimator, as detailed in (7) and (8), incorporates both the statistical information of the degraded measurements and the transmission energy of the sensors/relays. By leveraging statistical information, such as the mean and variance of the degraded measurements, the estimator can address the effects introduced by the measurement degradation. Similarly, incorporating the statistical information of the transmission energy of the sensors and relays enables the estimator to adapt to the varying energy levels in the system.

For node i, define the one-step prediction error and the estimation error as $\tilde{x}_{i,t_z+1|t_z} \triangleq x_{i,t_z+1} - \hat{x}_{i,t_z+1|t_z}$ and $\tilde{x}_{i,t_z+1|t_z+1} \triangleq x_{i,t_z+1} - \hat{x}_{i,t_z+1|t_z+1}$, respectively. Then, we have

$$\begin{aligned}\tilde{x}_{i,t_z+1|t_z} =& f(x_{i,t_z}) - f(\hat{x}_{i,t_z|t_z}) + \sum_{j=1}^{N} \omega_{ij} \Gamma (x_{j,t_z} - \hat{x}_{j,t_z|t_z}) \\ & + B_{i,t_z} \varpi_{i,t_z}. \end{aligned} \tag{9}$$

Using the Taylor series expansion around $\hat{x}_{i,t_z|t_z}$ for $f(x_{i,t_z})$, we have

$$f(x_{i,t_z}) = f(\hat{x}_{i,t_z|t_z}) + A_{i,t_z} \tilde{x}_{i,t_z|t_z} + o(|\tilde{x}_{i,t_z|t_z}|), \tag{10}$$

where

$$A_{i,t_z} = \frac{\partial f(x_{i,t_z})}{\partial x_{i,t_z}} |_{x_{i,t_z} = \hat{x}_{i,t_z|t_z}}$$

and $o(|\tilde{x}_{i,t_z|t_z}|)$ represents the high-order terms of Taylor series expansion. According to the analysis in [6] and [21], the high-order terms can be written as follows:

$$o(|\tilde{x}_{i,t_z|t_z}|) = L_{i,t_z} \aleph_{i,t_z} \tilde{x}_{i,t_z|t_z}, \tag{11}$$

where L_{i,t_z} is the problem-dependent scaling matrix, and $\aleph_{i,t_z}$ is the unknown time-varying matrix which represents the linearization errors of dynamical error and satisfies $\aleph_{i,t_z} \aleph^T_{i,t_z} \le$

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I. It follows from (9)-(11) that

$$\tilde{x}_{i,t_z+1|t_z} =(A_{i,t_z} + L_{i,t_z}\aleph_{i,t_z})\tilde{x}_{i,t_z|t_z} + \sum_{j=1}^{N}\omega_{ij}\Gamma(x_{j,t_z} - \hat{x}_{j,t_z|t_z}) + B_{i,t_z}\varpi_{i,t_z}. \quad (12)$$

According to (2), (3), (5) and (8), the estimation error of ith node is of the following form:

$$\begin{aligned}\tilde{x}_{i,t_z+1|t_z+1} =&(I - K_{i,t_z+1}a_{i,t_z+1}\bar{p}_{r,i,t_z+1}\bar{p}_{s,i,t_z+1}\mu_{i,t_z+1}\\ &\times H_{r,i,t_z+1}H_{s,i,t_z+1}C_{i,t_z+1})\tilde{x}_{i,t_z+1|t_z}\\ &+ K_{i,t_z+1}a_{i,t_z+1}(\bar{p}_{r,i,t_z+1}\bar{p}_{s,i,t_z+1}\mu_{i,t_z+1}\\ &- \sqrt{p_{r,i,t_z+1}}\sqrt{p_{s,i,t_z+1}}\theta_{i,t_z+1})H_{r,i,t_z+1}\\ &\times H_{s,i,t_z+1}C_{i,t_z+1}x_{i,t_z+1} - K_{i,t_z+1}a_{i,t_z+1}\\ &\times \sqrt{p_{r,i,t_z+1}}\sqrt{p_{s,i,t_z+1}}H_{r,i,t_z+1}H_{s,i,t_z+1}\\ &\times \nu_{i,t_z+1} - K_{i,t_z+1}a_{i,t_z+1}\sqrt{p_{r,i,t_z+1}}H_{r,i,t_z+1}\\ &\times \zeta_{s,i,t_z+1} - K_{i,t_z+1}\zeta_{r,i,t_z+1}. \end{aligned} \quad (13)$$

For brevity, let us denote

$$\begin{aligned}\tilde{x}_{t_z+1|t_z} &\triangleq \mathrm{col}_N\{\tilde{x}_{i,t_z+1|t_z}\},\ \tilde{x}_{t_z+1|t_z+1} \triangleq \mathrm{col}_N\{\tilde{x}_{i,t_z+1|t_z+1}\},\\ x_{t_z} &\triangleq \mathrm{col}_N\{x_{i,t_z}\},\ \hat{x}_{t_z+1|t_z+1} \triangleq \mathrm{col}_N\{\hat{x}_{i,t_z+1|t_z+1}\},\\ \hat{x}_{t_z+1|t_z} &\triangleq \mathrm{col}_N\{\hat{x}_{i,t_z+1|t_z}\},\ \nu_{t_z} \triangleq \mathrm{col}_N\{\nu_{i,t_z}\},\\ \varpi_{t_z} &\triangleq \mathrm{col}_N\{\varpi_{i,t_z}\},\ \zeta_{s,t_z} \triangleq \mathrm{col}_N\{\zeta_{s,i,t_z}\},\\ \zeta_{r,t_z} &\triangleq \mathrm{col}_N\{\zeta_{r,i,t_z}\},\ A_{t_z} \triangleq \mathrm{diag}_N\{A_{i,t_z}\},\\ B_{t_z} &\triangleq \mathrm{diag}_N\{B_{i,t_z}\},\ C_{t_z} \triangleq \mathrm{diag}_N\{C_{i,t_z}\},\\ K_{t_z} &\triangleq \mathrm{diag}_N\{K_{i,t_z}\},\ L_{t_z} \triangleq \mathrm{diag}_N\{L_{i,t_z}\},\\ Q_{t_z} &\triangleq \mathrm{diag}_N\{Q_{i,t_z}\},\ R_{t_z} \triangleq \mathrm{diag}_N\{R_{i,t_z}\},\\ \aleph_{t_z} &\triangleq \mathrm{diag}_N\{\aleph_{i,t_z}\},\ H_{r,t_z} \triangleq \mathrm{diag}_N\{H_{r,i,t_z}\},\\ H_{s,t_z} &\triangleq \mathrm{diag}_N\{H_{s,i,t_z}\},\\ \Lambda_{t_z} &\triangleq \mathrm{diag}\{a_{1,t_z}I, a_{2,t_z}I, \dots, a_{N,t_z}I\},\\ \Theta_{t_z} &\triangleq \mathrm{diag}\{\theta_{1,t_z}I, \theta_{2,t_z}I, \dots, \theta_{N,t_z}I\},\\ \mathfrak{A}_{t_z} &\triangleq \mathrm{diag}\{\mu_{1,t_z}I, \mu_{2,t_z}I, \dots, \mu_{N,t_z}I\},\\ \bar{\mathcal{P}}_{r,t_z+1} &\triangleq \mathrm{diag}\{\bar{p}_{r,1,t_z+1}I, \bar{p}_{r,2,t_z+1}I, \dots, \bar{p}_{r,N,t_z+1}I\},\\ \bar{\mathcal{P}}_{s,t_z+1} &\triangleq \mathrm{diag}\{\bar{p}_{s,1,t_z+1}I, \bar{p}_{s,2,t_z+1}I, \dots, \bar{p}_{s,N,t_z+1}I\},\\ \breve{\mathcal{P}}_{r,t_z+1} &\triangleq \mathrm{diag}\{\sqrt{p_{r,1,t_z+1}}I, \sqrt{p_{r,2,t_z+1}}I, \dots, \sqrt{p_{r,N,t_z+1}}I\},\\ \breve{\mathcal{P}}_{s,t_z+1} &\triangleq \mathrm{diag}\{\sqrt{p_{s,1,t_z+1}}I, \sqrt{p_{s,2,t_z+1}}I, \dots, \sqrt{p_{s,N,t_z+1}}I\}.\end{aligned}$$

Then, we have $\tilde{x}_{t_z+1|t_z} = x_{t_z+1} - \hat{x}_{t_z+1|t_z}$ and $\tilde{x}_{t_z+1|t_z+1} = x_{t_z+1} - \hat{x}_{t_z+1|t_z+1}$. With the definition of the Kronecker Product, (12) and (13) can be rewritten as follows:

$$\tilde{x}_{t_z+1|t_z} =(A_{t_z} + L_{t_z}\aleph_{t_z})\tilde{x}_{t_z|t_z} + (W \otimes \Gamma)\tilde{x}_{t_z|t_z} + B_{t_z}\varpi_{t_z}, \quad (14)$$

$$\begin{aligned}\tilde{x}_{t_z+1|t_z+1} =&(I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}\\ &\times H_{s,t_z+1}C_{t_z+1})\tilde{x}_{t_z+1|t_z} + K_{t_z+1}\Lambda_{t_z+1}\\ &\times (\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1} - \breve{\mathcal{P}}_{r,t_z+1}\breve{\mathcal{P}}_{s,t_z+1}\\ &\times \Theta_{t_z+1})H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}x_{t_z+1} - K_{t_z+1}\\ &\times \Lambda_{t_z+1}\breve{\mathcal{P}}_{r,t_z+1}\breve{\mathcal{P}}_{s,t_z+1}H_{r,t_z+1}H_{s,t_z+1}\nu_{t_z+1}\\ &- K_{t_z+1}\Lambda_{t_z+1}\breve{\mathcal{P}}_{r,t_z+1}H_{r,t_z+1}\zeta_{s,t_z+1}\\ &- K_{t_z+1}\zeta_{r,t_z+1}. \end{aligned} \quad (15)$$

Denote the one-step prediction error covariance and the estimation error covariance as $P_{t_z+1|t_z} \triangleq \mathbb{E}\{\tilde{x}_{t_z+1|t_z}\tilde{x}^T_{t_z+1|t_z}\}$ and $P_{t_z+1|t_z+1} \triangleq \mathbb{E}\{\tilde{x}_{t_z+1|t_z+1}\tilde{x}^T_{t_z+1|t_z+1}\}$, respectively.

The objective of this paper can be categorized into three main aspects.

1) The calculation of upper bounds for the prediction/estimation error covariances is pursued. These bounds provide valuable insights into the performance and accuracy of the estimation process.
2) The design of the estimator gain is crucial to minimize the upper bound of the estimation error at each time step, and this involves determining an appropriate gain that optimally balances the trade-off between estimation accuracy and stability.
3) The analysis of the boundedness of the estimation error obtained with the designed state estimator is conducted, which aims to establish conditions under which the estimation error remains bounded, ensuring the reliability and effectiveness of the proposed estimation scheme.

Some lemmas will be presented next, which will be exploited for further proceeding.

Lemma 1: [50] For the given matrices $\mathcal{U}$, $\mathcal{V}$, $\mathcal{W}$ and $\mathcal{R} > 0$ and an unknown matrix $\mathcal{F}$ which satisfies $\mathcal{F}\mathcal{F}^T \leq I$, if there exists an arbitrary positive constant $\gamma > 0$ such that $\gamma^{-1}I - \mathcal{W}\mathcal{R}\mathcal{W}^T > 0$, then the following inequality holds

$$(\mathcal{U} + \mathcal{V}\mathcal{F}\mathcal{W})\mathcal{R}(\mathcal{U} + \mathcal{V}\mathcal{F}\mathcal{W})^T \leq \mathcal{U}(\mathcal{R}^{-1} - \gamma\mathcal{W}^T\mathcal{W})^{-1}\mathcal{U}^T + \gamma^{-1}\mathcal{V}\mathcal{V}^T.$$

Lemma 2: [19] Let $\mathcal{A} = [a_{ij}]_{n\times n}$ be a real-valued matrix and $\mathcal{B} = \mathrm{diag}\{b_1, b_2, \dots, b_n\}$ be a diagonal random matrix. Then, we have

$$\mathbb{E}\{\mathcal{B}\mathcal{A}\mathcal{B}^T\} = \begin{bmatrix} \mathbb{E}\{b_1^2\} & \mathbb{E}\{b_1b_2\} & \dots & \mathbb{E}\{b_1b_n\} \\ \mathbb{E}\{b_2b_1\} & \mathbb{E}\{b_2^2\} & \dots & \mathbb{E}\{b_2b_n\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\{b_nb_1\} & \mathbb{E}\{b_nb_2\} & \dots & \mathbb{E}\{b_n^2\} \end{bmatrix} \circ \mathcal{A},$$

where $\circ$ is the Hadamard product.

Lemma 3: [22] Let $\mathcal{A}, \mathcal{B}, \mathcal{C} \in \mathbb{R}^{n\times n}$ and $\mathcal{B}, \mathcal{C}$ be symmetric positive definite matrices. If $\mathcal{C} - \mathcal{A}\mathcal{B}\mathcal{A}^T > 0$, then $\mathcal{B}^{-1} - \mathcal{A}^T\mathcal{C}^{-1}\mathcal{A} > 0$.

Lemma 4: For matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{M}$ and $\mathcal{N}$ with appropriate dimensions, the following equations hold:

$$\frac{\partial \mathrm{tr}\{\mathcal{A}\mathcal{M}\mathcal{B}\}}{\partial \mathcal{M}} = \mathcal{A}^T\mathcal{B}^T,$$

$$\frac{\partial \mathrm{tr}\{\mathcal{A}\mathcal{M}^T\mathcal{B}\}}{\partial \mathcal{M}} = \mathcal{B}\mathcal{A},$$

$$\frac{\partial \mathrm{tr}\{(\mathcal{A}\mathcal{M}\mathcal{B})\mathcal{N}(\mathcal{A}\mathcal{M}\mathcal{B})^T\}}{\partial \mathcal{M}} = 2\mathcal{A}^T\mathcal{A}\mathcal{M}\mathcal{B}\mathcal{N}\mathcal{B}^T.$$

Definition 1: [36] The stochastic process ϱ_n is said to be exponentially bounded in the sense of mean square if there exist real numbers $\vartheta_1, \vartheta_2 > 0$ and $0 < \vartheta_3 < 1$ such that

$$\mathrm{E}\{\|\varrho_n\|^2\} \leq \vartheta_1\|\varrho_0\|^2\vartheta_3^n + \vartheta_2$$

holds for every $n \geq 0$.

Lemma 5: [36] Let ϱ_n be a stochastic process. Assume that there is a stochastic process $V_n(\varrho_n)$ with $\bar{v}$, $\underline{v}$, $\beta > 0$ and $0 < \alpha \leq 1$ such that

$$\underline{v}\|\varrho_n\|^2 \leq V_n(\varrho_n) \leq \bar{v}\|\varrho_n\|^2$$

, and

$$\mathbb{E}\{V_{n+1}(\varrho_{n+1})|\varrho_n\} - V_n(\varrho_n) \leq \beta - \alpha V_n(\varrho_n).$$

Then, the stochastic process ϱ_n is exponentially bounded in the sense of mean square.

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III. Main Results

A. Estimator design

In this subsection, we will firstly calculate the covariances of the one-step prediction error and the estimation error. Then, the upper bound of the estimation error covariance will be minimized by selecting the proper estimator gain.

Let us investigate the one-step prediction error covariance now. From (14), it follows that

$$\begin{aligned}P_{t_z+1|t_z} =&\mathbb{E}\{\tilde{x}_{t_z+1|t_z}\tilde{x}^T_{t_z+1|t_z}\}\\ =&\left(A_{t_z}+L_{t_z}\aleph_{t_z}+W\otimes\Gamma\right)P_{t_z|t_z}\big(A_{t_z}+L_{t_z}\aleph_{t_z}\\ &+W\otimes\Gamma\big)^T+B_{t_z}Q_{t_z}B^T_{t_z}.\end{aligned}\tag{16}$$

Proposition 1: The estimation error covariance satisfies the following recursion:

$$\begin{aligned}&P_{t_z+1|t_z+1}\\ =&(I-K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}\\ &\times C_{t_z+1})P_{t_z+1|t_z}(I-K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}\\ &\times H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})^T+K_{t_z+1}\Lambda_{t_z+1}\big[\mathfrak{B}_{t_z+1}\\ &\circ(H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\mathbb{E}\{x_{t_z+1}x^T_{t_z+1}\}C^T_{t_z+1}H_{s,t_z+1}\\ &\times H_{r,t_z+1})\big]\Lambda_{t_z+1}K^T_{t_z+1}+K_{t_z+1}\Lambda_{t_z+1}\big[\mathfrak{C}_{t_z+1}\circ(H_{r,t_z+1}\\ &\times H_{s,t_z+1}R_{t_z+1}H_{s,t_z+1}H_{r,t_z+1})\big]\Lambda_{t_z+1}K^T_{t_z+1}+K_{t_z+1}\\ &\times\Lambda_{t_z+1}\big[\mathfrak{D}_{t_z+1}\circ(H_{r,t_z+1}S_{s,t_z+1}H_{r,t_z+1})\big]\Lambda_{t_z+1}K^T_{t_z+1}\\ &+K_{t_z+1}S_{r,t_z+1}K^T_{t_z+1},\end{aligned}\tag{17}$$

where

$$\begin{aligned}\mathfrak{B}_{t_z+1}\triangleq&\mathrm{diag}\big\{[\mathfrak{p}_{r,1,t_z+1}\mathfrak{p}_{s,1,t_z+1}(\sigma^2_{1,t_z+1}+\mu^2_{1,t_z+1})\\ &-\bar{p}^2_{r,1,t_z+1}\bar{p}^2_{s,1,t_z+1}\mu^2_{1,t_z+1}]\mathbf{1}_m,[\mathfrak{p}_{r,2,t_z+1}\\ &\times\mathfrak{p}_{s,2,t_z+1}(\sigma^2_{2,t_z+1}+\mu^2_{2,t_z+1})-\bar{p}^2_{r,2,t_z+1}\\ &\times\bar{p}^2_{s,2,t_z+1}\mu^2_{2,t_z+1}]\mathbf{1}_m,\dots,[\mathfrak{p}_{r,N,t_z+1}\\ &\times\mathfrak{p}_{s,N,t_z+1}(\sigma^2_{N,t_z+1}+\mu^2_{N,t_z+1})\\ &-\bar{p}^2_{r,N,t_z+1}\bar{p}^2_{s,N,t_z+1}\mu^2_{N,t_z+1}]\mathbf{1}_m\big\},\\ \mathfrak{p}_{r,i,t_z+1}\triangleq&\sum_{\tau=1}^{\psi}\chi^{[\tau]}_{r,i,t_z+1}\vartheta^{[\tau]}_{r,i,t_z+1},\quad i=1,2,\dots,N,\\ \mathfrak{p}_{s,i,t_z+1}\triangleq&\sum_{\hbar=1}^{\varphi}\chi^{[\hbar]}_{s,i,t_z+1}\vartheta^{[\hbar]}_{s,i,t_z+1},\quad i=1,2,\dots,N,\\ S_{s,t_z+1}\triangleq&\mathrm{diag}_N\{S_{s,i,t_z+1}\},\\ S_{r,t_z+1}\triangleq&\mathrm{diag}_N\{S_{r,i,t_z+1}\},\\ \mathfrak{C}_{t_z+1}\triangleq&\check{\mathfrak{C}}_{t_z+1}\otimes\mathbf{1}_m,\\ \mathfrak{D}_{t_z+1}\triangleq&\check{\mathfrak{D}}_{t_z+1}\otimes\mathbf{1}_m,\end{aligned}$$

$$\big[\check{\mathfrak{C}}_{t_z+1}\big]_{h,l}\triangleq\begin{cases}\mathfrak{p}_{r,l,t_z+1}\mathfrak{p}_{s,l,t_z+1}, & \text{if } h=l,\\ \bar{p}_{r,h,t_z+1}\bar{p}_{s,h,t_z+1}\bar{p}_{r,l,t_z+1}\\ \times\bar{p}_{s,l,t_z+1}, & \text{if } h\neq l,\end{cases}$$

$$\big[\check{\mathfrak{D}}_{t_z+1}\big]_{h,l}\triangleq\begin{cases}\mathfrak{p}_{r,l,t_z+1}, & \text{if } h=l,\\ \bar{p}_{r,h,t_z+1}\bar{p}_{r,l,t_z+1}, & \text{if } h\neq l.\end{cases}$$

Proof: According to (15), we have

$$\begin{aligned}&P_{t_z+1|t_z+1}\\ =&(I-K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}\\ &\times C_{t_z+1})P_{t_z+1|t_z}(I-K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\ &\times\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})^T+K_{t_z+1}\Lambda_{t_z+1}\\ &\times\mathbb{E}\{\mathfrak{Y}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}x_{t_z+1}x^T_{t_z+1}C^T_{t_z+1}\\ &\times H_{s,t_z+1}H_{r,t_z+1}\mathfrak{Y}^T_{t_z+1}\}\Lambda_{t_z+1}K^T_{t_z+1}+K_{t_z+1}\Lambda_{t_z+1}\\ &\times\mathbb{E}\{\check{\mathcal{P}}_{r,t_z+1}\check{\mathcal{P}}_{s,t_z+1}H_{r,t_z+1}H_{s,t_z+1}\nu_{t_z+1}\nu^T_{t_z+1}\\ &\times H_{s,t_z+1}H_{r,t_z+1}\check{\mathcal{P}}_{s,t_z+1}\check{\mathcal{P}}_{r,t_z+1}\}\Lambda_{t_z+1}K^T_{t_z+1}\\ &+K_{t_z+1}\Lambda_{t_z+1}\mathbb{E}\{\check{\mathcal{P}}_{r,t_z+1}H_{r,t_z+1}\zeta_{s,t_z+1}\zeta^T_{s,t_z+1}H_{r,t_z+1}\\ &\times\check{\mathcal{P}}_{r,t_z+1}\}\Lambda_{t_z+1}K^T_{t_z+1}+K_{t_z+1}S_{r,t_z+1}K^T_{t_z+1},\end{aligned}\tag{18}$$

where $\mathfrak{Y}_{t_z+1}\triangleq\check{\mathcal{P}}_{r,t_z+1}\check{\mathcal{P}}_{s,t_z+1}\Theta_{t_z+1}-\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}$.

Noting that the random variables p_{r,i,t_z+1}, p_{s,i,t_z+1} and θ_{i,t_z+1}, $i=1,2,\dots,N$ are independent of each other for each node i, we have

$$\begin{aligned}&\mathbb{E}\{(\sqrt{p_{r,i,t_z+1}}\sqrt{p_{s,i,t_z+1}}\theta_{i,t_z+1}-\bar{p}_{r,i,t_z+1}\bar{p}_{s,i,t_z+1}\\ &\times\mu_{i,t_z+1})^2\}\\ =&\mathfrak{p}_{r,1,t_z+1}\mathfrak{p}_{s,1,t_z+1}(\sigma^2_{i,t_z+1}+\mu^2_{i,t_z+1})\\ &-\bar{p}^2_{r,i,t_z+1}\bar{p}^2_{s,i,t_z+1}\mu^2_{i,t_z+1},\quad i=1,2,\dots,N.\end{aligned}$$

Similarly, for $i=1,2,\dots,N$, we have

$$\begin{aligned}\mathbb{E}\{p_{r,i,t_z+1}p_{s,i,t_z+1}\}&=\mathfrak{p}_{r,i,t_z+1}\mathfrak{p}_{s,i,t_z+1},\\ \mathbb{E}\{p^2_{r,i,t_z+1}\}&=\mathfrak{p}_{r,i,t_z+1}.\end{aligned}$$

Based on Lemma 2, it follows that

$$\begin{aligned}&\mathbb{E}\{\mathfrak{Y}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}x_{t_z+1}x^T_{t_z+1}C^T_{t_z+1}\\ &\times H_{s,t_z+1}H_{r,t_z+1}\mathfrak{Y}^T_{t_z+1}\}\\ =&\mathfrak{B}_{t_z+1}\circ\big(H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\mathbb{E}\{x_{t_z+1}x^T_{t_z+1}\}C^T_{t_z+1}\\ &\times H_{s,t_z+1}H_{r,t_z+1}\big),\end{aligned}\tag{19}$$

$$\begin{aligned}&\mathbb{E}\big\{\check{\mathcal{P}}_{r,t_z+1}\check{\mathcal{P}}_{s,t_z+1}H_{r,t_z+1}H_{s,t_z+1}\nu_{t_z+1}\nu^T_{t_z+1}\\ &\times H_{s,t_z+1}H_{r,t_z+1}\check{\mathcal{P}}_{s,t_z+1}\check{\mathcal{P}}_{r,t_z+1}\big\}\\ =&\mathfrak{C}_{t_z+1}\circ(H_{r,t_z+1}H_{s,t_z+1}R_{t_z+1}H_{s,t_z+1}H_{r,t_z+1}),\end{aligned}\tag{20}$$

$$\begin{aligned}&\mathbb{E}\left\{\check{\mathcal{P}}_{r,t_z+1}H_{r,t_z+1}\zeta_{s,t_z+1}\zeta^T_{s,t_z+1}H_{r,t_z+1}\check{\mathcal{P}}_{r,t_z+1}\right\}\\ =&\mathfrak{D}_{t_z+1}\circ(H_{r,t_z+1}S_{s,t_z+1}H_{r,t_z+1}).\end{aligned}\tag{21}$$

By substituting (19)-(21) into (18), we have (17), and the proof is complete. ■

With (16) and (17), we can determine the one-step prediction error covariance and the estimation error covariance in a recursive form. Unfortunately, it is challenging to directly utilize (16) and (17) to obtain the exact error covariances as the presence of unknown terms, namely $\aleph_{t_z}$ and $\mathbb{E}\{x_{t_z+1}x^T_{t_z+1}\}$, complicates the calculation process, where $\aleph_{t_z}$ arises from the linearization errors, while $\mathbb{E}\{x_{t_z+1}x^T_{t_z+1}\}$ is introduced by the stochastic degraded measurements. To address these difficulties and alleviate the computational burden, it is advisable to find and minimize an upper bound of the estimation error covariance by appropriately designing the estimator gain at each sampling time, which allows for a more practical and efficient implementation.

Theorem 1: Consider the one-step prediction error covariance $P_{t_z+1|t_z}$ in (16) and the estimation error covariance $P_{t_z+1|t_z+1}$ in (17). Let positive scalars ε_{t_z+1} and γ_{t_z} be given. Consider the following discrete-time Riccati-like difference equations:

$$\begin{aligned}\Sigma_{t_z+1|t_z}=&(A_{t_z}+W\otimes\Gamma)(\Sigma^{-1}_{t_z|t_z}-\gamma_{t_z}I)^{-1}(A_{t_z}+W\\ &\otimes\Gamma)^T+\gamma^{-1}_{t_z}L_{t_z}L^T_{t_z}+B_{t_z}Q_{t_z}B^T_{t_z}\end{aligned}\tag{22}$$

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and

$$\begin{aligned}\Sigma_{t_z+1|t_z+1} =&(I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}\\ &\times H_{s,t_z+1}C_{t_z+1})\Sigma_{t_z+1|t_z}(I - K_{t_z+1}\Lambda_{t_z+1}\\ &\times \bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}\\ &\times C_{t_z+1})^T + K_{t_z+1}\Lambda_{t_z+1}\big[\mathfrak{B}_{t_z+1}\circ(H_{r,t_z+1}\\ &\times H_{s,t_z+1}C_{t_z+1}\Omega_{t_z+1}C_{t_z+1}^T H_{s,t_z+1}H_{r,t_z+1})\big]\\ &\times \Lambda_{t_z+1}K_{t_z+1}^T + K_{t_z+1}\Lambda_{t_z+1}[\mathfrak{C}_{t_z+1}\\ &\circ(H_{r,t_z+1}H_{s,t_z+1}R_{t_z+1}H_{s,t_z+1}H_{r,t_z+1})]\\ &\times \Lambda_{t_z+1}K_{t_z+1}^T + K_{t_z+1}\Lambda_{t_z+1}[\mathfrak{D}_{t_z+1}\\ &\circ(H_{r,t_z+1}S_{s,t_z+1}H_{r,t_z+1})]\Lambda_{t_z+1}K_{t_z+1}^T\\ &+ K_{t_z+1}S_{r,t_z+1}K_{t_z+1}^T, \end{aligned}\tag{23}$$

with the initial condition $\Sigma_{0|0} \geq P_{0|0} > 0$, where $\Omega_{t_z+1} \triangleq (1+\varepsilon_{t_z+1})\Sigma_{t_z+1|t_z} + (1+\varepsilon_{t_z+1}^{-1})\hat{x}_{t_z+1|t_z}\hat{x}_{t_z+1|t_z}^T$. If $\Sigma_{t_z|t_z} < \gamma_{t_z}^{-1}I$ for all $t_z \geq 0$, then $\Sigma_{t_z+1|t_z+1}$ is an upper bound of the estimation error covariance $P_{t_z+1|t_z+1}$ Moreover, the trace of upper bound $\Sigma_{t_z+1|t_z+1}$ can be minimized by choosing the estimator gain K_{t_z+1} given as follows:

$$K_{t_z+1} \triangleq \mathrm{diag}\{K_{1,t_z+1}, K_{2,t_z+1}, \ldots, K_{N,t_z+1}\}, \tag{24}$$

where

$$\begin{aligned}K_{i,t_z+1} \triangleq&\Phi_{1,i}\Sigma_{t_z+1|t_z}C_{t_z+1}^T H_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\\ &\times \bar{\mathcal{P}}_{s,t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\Lambda_{t_z+1}\Phi_{2,i}^T(\Phi_{2,i}\Delta_{t_z+1}\Phi_{2,i}^T)^{-1},\\ \Phi_{1,i} \triangleq&[\underbrace{0\ \ 0\ \ \cdots\ \ 0}_{i-1}\ \ I_{n\times n}\ \ \underbrace{0\ \ 0\ \ \cdots\ \ 0}_{N-i}],\\ \Phi_{2,i} \triangleq&[\underbrace{0\ \ 0\ \ \cdots\ \ 0}_{i-1}\ \ I_{m\times m}\ \ \underbrace{0\ \ 0\ \ \cdots\ \ 0}_{N-i}],\\ \Delta_{t_z+1} \triangleq&\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}\\ &\times C_{t_z+1}\Sigma_{t_z+1|t_z}C_{t_z+1}^T H_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\\ &\times \bar{\mathcal{P}}_{s,t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\Lambda_{t_z+1} + \Lambda_{t_z+1}\big[\mathfrak{B}_{t_z+1}\\ &\circ(H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Omega_{t_z+1}C_{t_z+1}^T H_{s,t_z+1}\\ &\times H_{r,t_z+1})\big]\Lambda_{t_z+1} + \Lambda_{t_z+1}\big[\mathfrak{C}_{t_z+1}\circ(H_{r,t_z+1}\\ &\times H_{s,t_z+1}R_{t_z+1}H_{s,t_z+1}H_{r,t_z+1})\big]\Lambda_{t_z+1}\\ &+ \Lambda_{t_z+1}\big[\mathfrak{D}_{t_z+1}\circ(H_{r,t_z+1}S_{s,t_z+1}H_{r,t_z+1})\big]\Lambda_{t_z+1}\\ &+ S_{r,t_z+1}.\end{aligned}\tag{25}$$

Proof: For the initial condition, it can be seen that $P_{0|0} \leq \Sigma_{0|0}$. Based on mathematical induction, if we can prove $P_{t_z+1|t_z+1} \leq \Sigma_{t_z+1|t_z+1}$ when assuming $P_{t_z|t_z} \leq \Sigma_{t_z|t_z}$, then $\Sigma_{t_z+1|t_z+1}$ is an upper bound of $P_{t_z+1|t_z+1}$.

Firstly, from (16), Lemma 1 and the assumption that $P_{t_z|t_z} \leq \Sigma_{t_z|t_z}$, it follows that

$$\begin{aligned}P_{t_z+1|t_z} \leq&(A_{t_z} + L_{t_z}\aleph_{t_z} + W\otimes\Gamma)\,\Sigma_{t_z|t_z}\big(A_{t_z} + L_{t_z}\aleph_{t_z}\\ &+ W\otimes\Gamma\big)^T + B_{t_z}Q_{t_z}B_{t_z}^T\\ \leq&(A_{t_z} + W\otimes\Gamma)\,(\Sigma_{t_z|t_z}^{-1} - \gamma_{t_z}I)^{-1}(A_{t_z} + W\\ &\otimes\Gamma)^T + \gamma_{t_z}^{-1}L_{t_z}L_{t_z}^T + B_{t_z}Q_{t_z}B_{t_z}^T\\ =&\Sigma_{t_z+1|t_z},\end{aligned}\tag{26}$$

where γ_{t_z} is a positive scalar satisfying $\Sigma_{t_z|t_z} < \gamma_{t_z}^{-1}I$. Then, utilizing the elementary inequality

$$\begin{aligned}&\big(\varepsilon_{t_z+1}^{\frac{1}{2}}\tilde{x}_{t_z+1|t_z} - \varepsilon_{t_z+1}^{-\frac{1}{2}}\hat{x}_{t_z+1|t_z}\big)\big(\varepsilon_{t_z+1}^{\frac{1}{2}}\tilde{x}_{t_z+1|t_z}\\ &- \varepsilon_{t_z+1}^{-\frac{1}{2}}\hat{x}_{t_z+1|t_z}\big)^T \geq 0,\end{aligned}$$

we have

$$\begin{aligned}&\tilde{x}_{t_z+1|t_z}\hat{x}_{t_z+1|t_z}^T + \hat{x}_{t_z+1|t_z}\tilde{x}_{t_z+1|t_z}^T\\ \leq&\varepsilon_{t_z+1}\tilde{x}_{t_z+1|t_z}\tilde{x}_{t_z+1|t_z}^T + \varepsilon_{t_z+1}^{-1}\hat{x}_{t_z+1|t_z}\hat{x}_{t_z+1|t_z}^T,\end{aligned}$$

where ε_{t_z+1} is a positive scalar. It follows naturally that

$$\begin{aligned}&\mathbb{E}\{x_{t_z+1}x_{t_z+1}^T\}\\ \leq&(1+\varepsilon_{t_z+1})P_{t_z+1|t_z} + (1+\varepsilon_{t_z+1}^{-1})\hat{x}_{t_z+1|t_z}\hat{x}_{t_z+1|t_z}^T.\end{aligned}\tag{27}$$

Substituting (26) and (27) into (17) yields $P_{t_z+1|t_z+1} \leq \Sigma_{t_z+1|t_z+1}$.

Next, we aim to show that the trace of the upper bound $\Sigma_{t_z+1|t_z+1}$ is minimized with K_{t_z+1} chosen as (24). Rewrite $\Sigma_{t_z+1|t_z+1}$ in (23) as

$$\begin{aligned}\Sigma_{t_z+1|t_z+1} =&\Sigma_{t_z+1|t_z} - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}\\ &\times H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Sigma_{t_z+1|t_z} - \Sigma_{t_z+1|t_z}\\ &\times C_{t_z+1}^T H_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\\ &\times \Lambda_{t_z+1}K_{t_z+1}^T + K_{t_z+1}\Delta_{t_z+1}K_{t_z+1}^T,\end{aligned}\tag{28}$$

where Δ_{t_z+1} is shown in (25). Substituting $K_{t_z+1} = \sum_{i=1}^N(\Phi_{1,i}^T K_{i,t_z+1}\Phi_{2,i})$ into (28) gives rise to

$$\begin{aligned}&\mathrm{tr}\left\{\Sigma_{t_z+1|t_z+1}\right\}\\ =&\mathrm{tr}\Big\{\Sigma_{t_z+1|t_z} - \sum_{i=1}^N(\Phi_{1,i}^T K_{i,t_z+1}\Phi_{2,i})\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\ &\times \mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Sigma_{t_z+1|t_z} - \Sigma_{t_z+1|t_z}C_{t_z+1}^T\\ &\times H_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\Lambda_{t_z+1}\sum_{i=1}^N(\Phi_{1,i}^T\\ &\times K_{i,t_z+1}\Phi_{2,i})^T + \sum_{i=1}^N(\Phi_{1,i}^T K_{i,t_z+1}\Phi_{2,i})\Delta_{t_z+1}\\ &\times \sum_{j=1}^N(\Phi_{1,j}^T K_{j,t_z+1}\Phi_{2,j})^T\Big\}.\end{aligned}\tag{29}$$

It is noted that

$$\mathrm{tr}\left\{(\Phi_{1,i}^T K_{i,t_z+1}\Phi_{2,i})\Delta_{t_z+1}(\Phi_{1,j}^T K_{i,t_z+1}\Phi_{2,j})^T\right\} = 0$$

for $i \neq j$. According to Lemma 4 and taking the partial derivative of $\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}$ in (29) with respect to K_{i,t_z+1}, we have

$$\begin{aligned}&\frac{\partial\mathrm{tr}\left\{\Sigma_{t_z+1|t_z+1}\right\}}{\partial K_{i,t_z+1}}\\ =&-2\Phi_{1,i}\Sigma_{t_z+1|t_z}C_{t_z+1}^T H_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\\ &\times \bar{\mathcal{P}}_{s,t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\Lambda_{t_z+1}\Phi_{2,i}^T + 2\Phi_{1,i}\Phi_{1,i}^T\\ &\times K_{i,t_z+1}\Phi_{2,i}\Delta_{t_z+1}\Phi_{2,i}^T.\end{aligned}\tag{30}$$

Since $\Phi_{1,i}\Phi_{1,i}^T = I_{n\times n}$, it follows from $\dfrac{\partial\mathrm{tr}\left\{\Sigma_{t_z+1|t_z+1}\right\}}{\partial K_{i,t_z+1}} = 0$ that

$$\begin{aligned}K_{i,t_z+1} =&\Phi_{1,i}\Sigma_{t_z+1|t_z}C_{t_z+1}^T H_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\\ &\times \bar{\mathcal{P}}_{s,t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\Lambda_{t_z+1}\Phi_{2,i}^T(\Phi_{2,i}\Delta_{t_z+1}\Phi_{2,i}^T)^{-1},\end{aligned}$$

which is identical with (24). The proof is now complete. ■

Remark 3: Due to the presence of the unknown terms $\aleph_{t_z+1}$ and $\mathbb{E}\{x_{t_z+1}x_{t_z+1}^T\}$, we seek to calculate an upper bound of the estimation error covariance by employing the Riccati-like matrix difference equations presented in Theorem 1, thereby reducing the computational complexity. The estimator gain is designed to minimize the trace of this upper bound, and the

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recursive approach is well-suited for online implementation at each sampling instant.

The boundedness of the estimation error will be analyzed based on Lemma 5 in the next subsection.

B. The boundedness of the estimation error

Assumption 1: For any time step $t_z = 0, 1, 2, \ldots$, there are real positive constants $\bar{f}$, $\underline{f}$, $\bar{a}$, $\underline{a}$, $\bar{h}_s$, $\underline{h}_s$, $\bar{h}_r$, $\underline{h}_r$, $\bar{\eta}$, $\underline{\vartheta}_s$, $\bar{\vartheta}_s$, $\underline{\vartheta}_r$, $\bar{\vartheta}_r$, $\bar{c}$, $\bar{r}$, $\underline{r}$, $\bar{q}$, $\underline{q}$, $\bar{s}_r$, $\underline{s}_r$, $\bar{s}_s$, $\underline{s}_s$, $\bar{l}$, $\bar{w}$, $\bar{g}$, $\bar{\mu}$, and $\underline{\mu}$ such that

$$
\begin{aligned}
&\|A_{t_z}\| \leq \bar{\eta}, \|L_{t_z}\aleph_{t_z}\| \leq \bar{l}, \underline{a}I \leq \Lambda_{t_z+1} \leq \bar{a}I,\\
&\underline{c} \leq \|C_{t_z+1}\| \leq \bar{c}, \underline{h}_s I \leq H_{s,t_z+1} \leq \bar{h}_s I,\\
&\underline{h}_r I \leq H_{r,t_z+1} \leq \bar{h}_r I, \underline{s}_s I \leq S_{s,t_z+1} \leq \bar{s}_s I,\\
&\underline{s}_r I \leq S_{r,t_z+1} \leq \bar{s}_r I, \underline{q}I \leq B_{t_z} Q_{t_z} B_{t_z}^T \leq \bar{q}I,\\
&\underline{r}I \leq R_{t_z+1} \leq \bar{r}I, \|W\| \leq \bar{w}, \|\Gamma\| \leq \bar{g},\\
&\underline{\mu}I \leq \mathfrak{A}_{t_z+1} \leq \bar{\mu}I, \gamma_{t_z}^{-1} \leq \bar{f}, \underline{f}I \leq \Sigma_{t_z|t_z},\\
&\underline{\vartheta}_r \leq \vartheta_{r,i,t_z+1}^{[\tau]} \leq \bar{\vartheta}_r, \quad \tau = 1, 2, \ldots, \psi,\\
&\underline{\vartheta}_s \leq \vartheta_{s,i,t_z+1}^{[\hbar]} \leq \bar{\vartheta}_s, \quad \hbar = 1, 2, \ldots, \varphi.
\end{aligned}
$$

Theorem 2: Consider the discrete time-varying stochastic complex network in (1) with AF relays. Let the EKF-based estimator be constructed as (7)-(8) for each node ($i = 1, 2, \ldots, N$) with estimator gain K_{t_z+1} given in (25), and the initial estimation error $\tilde{x}_{0|0}$ be bounded. Then, the estimation error $\tilde{x}_{t_z|t_z}$ is exponentially bounded in the sense of mean square.

Proof: To prove the boundedness of the estimation error, we choose

$$
V_{t_z}(\tilde{x}_{t_z|t_z}) = \tilde{x}_{t_z|t_z}^T \Sigma_{t_z|t_z}^{-1} \tilde{x}_{t_z|t_z}. \tag{31}
$$

Since $\Sigma_{t_z|t_z} < \gamma_{t_z}^{-1} I$ for all $t_z \geq 0$, we have $\Sigma_{t_z|t_z} \leq \bar{f}I$ for all $t_z \geq 0$.

Based on Assumption 1, we have

$$
\begin{aligned}
\bar{f}^{-1}\|\tilde{x}_{t_z|t_z}\|^2 \leq& V_{t_z}(\tilde{x}_{t_z|t_z})\\
=& \tilde{x}_{t_z|t_z}^T \Sigma_{t_z|t_z}^{-1} \tilde{x}_{t_z|t_z} \leq \underline{f}^{-1}\|\tilde{x}_{t_z|t_z}\|^2.
\end{aligned} \tag{32}
$$

Combining (14) and (15), we obtain

$$
\begin{aligned}
&\tilde{x}_{t_z+1|t_z+1}\\
=&\big(I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}\\
&\times H_{s,t_z+1}C_{t_z+1}\big)\left(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma\right)\tilde{x}_{t_z|t_z}\\
&+ \mathfrak{T}_{t_z+1} + \mathfrak{G}_{t_z+1},
\end{aligned} \tag{33}
$$

where

$$
\begin{aligned}
\mathfrak{T}_{t_z+1} \triangleq& K_{t_z+1}\Lambda_{t_z+1}\mathfrak{Y}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}x_{t_z+1},\\
\mathfrak{G}_{t_z+1} \triangleq& \big(I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}\\
&\times H_{s,t_z+1}C_{t_z+1}\big)B_{t_z}\varpi_{t_z} - K_{t_z+1}(\Lambda_{t_z+1}\breve{\mathcal{P}}_{r,t_z+1}\\
&\times \breve{\mathcal{P}}_{s,t_z+1}H_{r,t_z+1}H_{s,t_z+1}\nu_{t_z+1} + \Lambda_{t_z+1}\breve{\mathcal{P}}_{r,t_z+1}\\
&\times H_{r,t_z+1}\zeta_{s,t_z+1} + \zeta_{r,t_z+1}).
\end{aligned}
$$

From (31) and (33), it follows that

$$
\begin{aligned}
&\mathbb{E}\{V_{t_z+1}(\tilde{x}_{t_z+1|t_z+1})|\tilde{x}_{t_z|t_z}\}\\
=&\tilde{x}_{t_z|t_z}^T(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma)^T\big(I - K_{t_z+1}\Lambda_{t_z+1}\\
&\times \bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\big)^T\\
&\times \Sigma_{t_z+1|t_z+1}^{-1}\big(I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\
&\times \mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\big)(A_{t_z} + L_{t_z}\aleph_{t_z}\\
&+ W \otimes \Gamma)\tilde{x}_{t_z|t_z} + \mathbb{E}\{\mathfrak{T}_{t_z+1}^T\Sigma_{t_z+1|t_z+1}^{-1}\big[2(I - K_{t_z+1}\\
&\times \Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})\\
&\times (A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma)\tilde{x}_{t_z|t_z} + \mathfrak{T}_{t_z+1}\big]|\tilde{x}_{t_z|t_z}\}\\
&+ \mathbb{E}\{\mathfrak{G}_{t_z+1}^T\Sigma_{t_z+1|t_z+1}^{-1}\big[2(I - K_{t_z+1}\Lambda_{t_z+1}\\
&\times \bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})\\
&\times (A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma)\tilde{x}_{t_z|t_z} + \mathfrak{G}_{t_z+1}\\
&+ 2\mathfrak{T}_{t_z+1}\big]|\tilde{x}_{t_z|t_z}\}.
\end{aligned} \tag{34}
$$

According to Assumption 1, the triangle inequality and the property of the Kronecker product, we have

$$
\begin{aligned}
\|A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma\| \leq& \|A_{t_z}\| + \|L_{t_z}\aleph_{t_z}\| + \|W \otimes \Gamma\|\\
\leq& \bar{\eta} + \bar{l} + \bar{w}\bar{g}.
\end{aligned} \tag{35}
$$

From (22), (26) and (35), it follows that

$$
\begin{aligned}
\Sigma_{t_z+1|t_z} =& \big(A_{t_z} + W \otimes \Gamma\big)(\Sigma_{t_z|t_z}^{-1} - \gamma_{t_z}I)^{-1}\big(A_{t_z}\\
&+ W \otimes \Gamma\big)^T + \gamma_{t_z}^{-1}L_{t_z}L_{t_z}^T + B_{t_z}Q_{t_z}B_{t_z}^T\\
\geq& \left(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma\right)\Sigma_{t_z|t_z}\big(A_{t_z}\\
&+ L_{t_z}\aleph_{t_z} + W \otimes \Gamma\big)^T + B_{t_z}Q_{t_z}B_{t_z}^T\\
\geq& \left[1 + \frac{\underline{q}}{(\bar{\eta} + \bar{l} + \bar{w}\bar{g})^2\bar{f}}\right]\big(A_{t_z} + L_{t_z}\aleph_{t_z}\\
&+ W \otimes \Gamma\big)\Sigma_{t_z|t_z}\big(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma\big)^T.
\end{aligned} \tag{36}
$$

Considering (23), we have

$$
\begin{aligned}
\Sigma_{t_z+1|t_z+1} >& (I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\
&\times \mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})\Sigma_{t_z+1|t_z}\\
&\times (I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}\\
&\times H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})^T.
\end{aligned} \tag{37}
$$

Substituting (36) into (37) yields

$$
\begin{aligned}
\Sigma_{t_z+1|t_z+1} >& \left[1 + \frac{\underline{q}}{(\bar{\eta} + \bar{l} + \bar{w}\bar{g})^2\bar{f}}\right](I - K_{t_z+1}\Lambda_{t_z+1}\\
&\times \bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})\\
&\times \left(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma\right)\Sigma_{t_z|t_z}\\
&\times \left(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma\right)^T\\
&\times (I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\
&\times \mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})^T.
\end{aligned} \tag{38}
$$

Based on Lemma 3 and (38), we have

$$
\begin{aligned}
\Sigma_{t_z|t_z}^{-1} >& \left[1 + \frac{\underline{q}}{(\bar{\eta} + \bar{l} + \bar{w}\bar{g})^2\bar{f}}\right]\left(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma\right)^T\\
&\times (I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}\\
&\times H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})^T\Sigma_{t_z+1|t_z+1}^{-1}\\
&\times (I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}\\
&\times H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma).
\end{aligned} \tag{39}
$$

It is straightforward to see that

$$
\begin{aligned}
&\mathbb{E}\{2\mathfrak{T}_{t_z+1}^T\Sigma_{t_z+1|t_z+1}^{-1}(I - K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}\\
&\times H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})(A_{t_z} + L_{t_z}\aleph_{t_z} + W \otimes \Gamma)\\
&\times \tilde{x}_{t_z|t_z}|\tilde{x}_{t_z|t_z}\} = 0.
\end{aligned} \tag{40}
$$

With (18), we have

$$
\begin{aligned}
&\Sigma_{t_z+1|t_z+1}\\
\geq& P_{t_z+1|t_z+1}
\end{aligned}
$$

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$$
\begin{aligned}
>&K_{t_z+1}\Lambda_{t_z+1}\mathbb{E}\{(\breve{\mathcal{P}}_{r,t_z+1}\breve{\mathcal{P}}_{s,t_z+1}\Theta_{t_z+1}-\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\
&\times \mathfrak{A}_{t_z+1})H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}x_{t_z+1}x_{t_z+1}^TC_{t_z+1}^TH_{s,t_z+1}\\
&\times H_{r,t_z+1}(\breve{\mathcal{P}}_{r,t_z+1}\breve{\mathcal{P}}_{s,t_z+1}\Theta_{t_z+1}-\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\
&\times \mathfrak{A}_{t_z+1})^T\}\Lambda_{t_z+1}K_{t_z+1}^T\\
=&\mathbb{E}\{\mathfrak{T}_{t_z+1}\mathfrak{T}_{t_z+1}^T\},
\end{aligned}
$$

and it follows readily that

$$
\begin{aligned}
&\mathbb{E}\{\mathfrak{T}_{t_z+1}^T\Sigma_{t_z+1|t_z+1}^{-1}\mathfrak{T}_{t_z+1}|\tilde{x}_{t_z|t_z}\}\\
<&\mathbb{E}\left\{\mathfrak{T}_{t_z+1}^T\left(\mathbb{E}\{\mathfrak{T}_{t_z+1}\mathfrak{T}_{t_z+1}^T\}\right)^{-1}\mathfrak{T}_{t_z+1}|\tilde{x}_{t_z|t_z}\right\}\\
=&nN. \qquad (41)
\end{aligned}
$$

Considering that ϖ_{t_z}, ν_{t_z}, ζ_{s,t_z}, and ζ_{r,t_z} are all zero-mean and independent of each other, we have

$$
\begin{aligned}
&\mathbb{E}\{2\mathfrak{G}_{t_z+1}^T\Sigma_{t_z+1|t_z+1}^{-1}(I-K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\
&\times \mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})(A_{t_z}+L_{t_z}\aleph_{t_z}\\
&+W\otimes\Gamma)\tilde{x}_{t_z|t_z}|\tilde{x}_{t_z|t_z}\}=0,\\
&\mathbb{E}\{2\mathfrak{G}_{t_z+1}^T\Sigma_{t_z+1|t_z+1}^{-1}\mathfrak{T}_{t_z+1}\}=0. \qquad (42)
\end{aligned}
$$

From Assumption 1 and (24), we have

$$
\begin{aligned}
&\|K_{t_z+1}\|\\
\le&\max_i\|K_{i,t_z+1}\|\\
<&\max_i\|\Phi_{1,i}\Sigma_{t_z+1|t_z}C_{t_z+1}^TH_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\\
&\times \bar{\mathcal{P}}_{r,t_z+1}\Lambda_{t_z+1}\Phi_{2,i}^T(\Phi_{2,i}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}\\
&\times H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Sigma_{t_z+1|t_z}C_{t_z+1}^TH_{s,t_z+1}H_{r,t_z+1}\\
&\times \mathfrak{A}_{t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\Lambda_{t_z+1}\Phi_{2,i}^T)^{-1}\|<\kappa,
\end{aligned}
$$

where $\kappa\triangleq\bar{c}\bar{h}_s\bar{h}_r\bar{\mu}\sqrt{\bar{\vartheta}_s}\sqrt{\bar{\vartheta}_r}\bar{a}(\underline{a}^2\underline{\vartheta}_r\underline{\vartheta}_s\underline{\mu}^2\underline{h}_r^2\underline{h}_s^2\underline{c}^2)^{-1}$.

Similarly, the following relationship can be obtained:

$$
\begin{aligned}
&\mathbb{E}\{\mathfrak{G}_{t_z+1}^T\Sigma_{t_z+1|t_z+1}^{-1}\mathfrak{G}_{t_z+1}\}\\
=&\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}^{-1}(I-K_{t_z+1}\Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}\\
&\times H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})B_{t_z}\mathbb{E}\{\varpi_{t_z}\varpi_{t_z}^T\}B_{t_z}^T(I-K_{t_z+1}\\
&\times \Lambda_{t_z+1}\bar{\mathcal{P}}_{r,t_z+1}\bar{\mathcal{P}}_{s,t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1})^T\}\\
&+\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}^{-1}K_{t_z+1}\Lambda_{t_z+1}\mathbb{E}\{\breve{\mathcal{P}}_{r,t_z+1}\breve{\mathcal{P}}_{s,t_z+1}H_{r,t_z+1}\\
&\times H_{s,t_z+1}\nu_{t_z+1}\nu_{t_z+1}^TH_{s,t_z+1}H_{r,t_z+1}\breve{\mathcal{P}}_{s,t_z+1}\breve{\mathcal{P}}_{r,t_z+1}\}\\
&\times \Lambda_{t_z+1}K_{t_z+1}^T\}+\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}^{-1}K_{t_z+1}\Lambda_{t_z+1}\mathbb{E}\{\breve{\mathcal{P}}_{r,t_z+1}\\
&\times H_{r,t_z+1}\zeta_{s,t_z+1}\zeta_{s,t_z+1}^TH_{r,t_z+1}\breve{\mathcal{P}}_{r,t_z+1}\}\Lambda_{t_z+1}K_{t_z+1}^T\}\\
&+\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}^{-1}K_{t_z+1}\mathbb{E}\{\zeta_{r,t_z+1}\zeta_{r,t_z+1}^T\}K_{t_z+1}^T\}\\
\le&nN\bar{q}\underline{f}^{-1}(1+\kappa^2\bar{a}^2\bar{\vartheta}_r\bar{\vartheta}_s\bar{h}_r^2\bar{h}_s^2\bar{c}^2\bar{\mu}^2)+mN\kappa^2(\bar{a}^2\bar{\vartheta}_r\bar{\vartheta}_s\bar{h}_r^2\\
&\times \bar{h}_s^2\bar{r}+\bar{a}^2\bar{\vartheta}_r\bar{h}_r^2\bar{s}_s+\bar{s}_r)\underline{f}^{-1}. \qquad (43)
\end{aligned}
$$

Combining (34) and (39)-(43), we obtain

$$
\begin{aligned}
&\mathbb{E}\{V_{t_z+1}(\tilde{x}_{t_z+1|t_z+1})|\tilde{x}_{t_z|t_z}\}-V_{t_z}(\tilde{x}_{t_z|t_z})\\
\le&-\xi\tilde{x}_{t_z|t_z}^T\Sigma_{t_z|t_z}^{-1}\tilde{x}_{t_z|t_z}+\varsigma\\
=&-\xi V_{t_z}(\tilde{x}_{t_z|t_z})+\varsigma, \qquad (44)
\end{aligned}
$$

where

$$
\begin{aligned}
\xi\triangleq&\underline{q}\big[(\bar{\eta}+\bar{l}+\bar{w}\bar{g})^2\bar{f}+\underline{q}\big]^{-1},\\
\varsigma\triangleq&nN+nN\bar{q}(1+\kappa^2\bar{a}^2\bar{\vartheta}_r\bar{\vartheta}_s\bar{h}_r^2\bar{h}_s^2\bar{c}^2\bar{\mu}^2)\underline{f}^{-1}+mN\kappa^2(\bar{a}^2\bar{\vartheta}_r\\
&\times\bar{\vartheta}_s\bar{h}_r^2\bar{h}_s^2\bar{r}+\bar{a}^2\bar{\vartheta}_r\bar{h}_r^2\bar{s}_s+\bar{s}_r)\underline{f}^{-1}.
\end{aligned}
$$

Obviously, $0<\xi<1$. Based on Lemma 5, (32), and (44), the estimation error $\tilde{x}_{t_z|t_z}$ is exponentially bounded in sense of mean square. The proof is complete. ■

Remark 4: It should be noted that the effects of AF relays have been imposed on the received signals at the remote state estimator. The influences from the AF relays, as well as the stochastic noises, probabilistic degraded measurements, and coupling dynamics, have been taken into consideration. The statistical characteristics of random parameters have been utilized in our proposed state estimators. Additionally, the estimator gain has been determined by employing Riccati-like matrix difference equations. Furthermore, an analysis has been conducted to examine the boundedness of the estimation error in the sense of mean square.

C. Effects of the sensor/relay transmission energies

Now, let us discuss the effects of the sensor/relay transmission energies on the estimation performance. For simplicity, we assume that p_{s,i,t_z+1} and p_{r,i,t_z+1} of every node obey the same probability distribution law, i.e.,

$$
\begin{aligned}
\bar{\mathcal{P}}_{r,t_z+1}=&\bar{p}_{r,t_z+1}I,\bar{\mathcal{P}}_{s,t_z+1}=\bar{p}_{s,t_z+1}I,\\
\mathfrak{C}_{t_z+1}=&[\mathfrak{p}_{r,t_z+1}\mathfrak{p}_{s,t_z+1}I+\bar{p}_{r,t_z+1}^2\bar{p}_{s,t_z+1}^2(\mathbf{1}_N-I)]\otimes\mathbf{1}_m,\\
\mathfrak{D}_{t_z+1}=&[\mathfrak{p}_{r,t_z+1}I+\bar{p}_{r,t_z+1}^2(\mathbf{1}_N-I)]\otimes\mathbf{1}_m. \qquad (45)
\end{aligned}
$$

for every time step t_z. Moreover, let us denote the variance of $\sqrt{p_{r,t_z+1}}$ ($\sqrt{p_{s,t_z+1}}$, respectively) as $\breve{p}_{r,t_z+1}$ ($\breve{p}_{s,t_z+1}$, respectively), i.e.,

$$
\begin{aligned}
\breve{p}_{r,t_z+1}\triangleq&\mathbb{E}\{(\sqrt{p_{r,t_z+1}}-\bar{p}_{r,t_z+1})^2\},\\
\breve{p}_{s,t_z+1}\triangleq&\mathbb{E}\{(\sqrt{p_{s,t_z+1}}-\bar{p}_{s,t_z+1})^2\}.
\end{aligned}
$$

In this case, substituting K_{t_z+1} into (28), we have

$$
\begin{aligned}
&\mathrm{tr}\Big\{\Sigma_{t_z+1|t_z+1}\Big\}\\
=&\mathrm{tr}\Big\{\Sigma_{t_z+1|t_z}-\sum_{i=1}^{N}\Sigma_{t_z+1|t_z}C_{t_z+1}^TH_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\\
&\times\Lambda_{t_z+1}\Phi_{2,i}^T(\Phi_{2,i}\mathfrak{W}_{t_z+1}\Phi_{2,i}^T)^{-1}\Phi_{2,i}\Lambda_{t_z+1}\\
&\times\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Sigma_{t_z+1|t_z}\Phi_{1,i}^T\Phi_{1,i}\Big\}, \qquad (46)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathfrak{W}_{t_z+1}\triangleq&\Lambda_{t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Sigma_{t_z+1|t_z}C_{t_z+1}^T\\
&\times H_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\Lambda_{t_z+1}+\Lambda_{t_z+1}\big[\mathcal{O}_{t_z+1}\\
&\circ(H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Omega_{t_z+1}C_{t_z+1}^TH_{s,t_z+1}\\
&\times H_{r,t_z+1})\big]\Lambda_{t_z+1}+(\bar{p}_{r,t_z+1}\bar{p}_{s,t_z+1})^{-2}\Big[\Lambda_{t_z+1}\\
&\times\big[[(\mathfrak{p}_{r,t_z+1}\mathfrak{p}_{s,t_z+1}I+\bar{p}_{r,t_z+1}^2\bar{p}_{s,t_z+1}^2(\mathbf{1}_N-I))\\
&\otimes\mathbf{1}_m]\circ(H_{r,t_z+1}H_{s,t_z+1}R_{t_z+1}H_{s,t_z+1}H_{r,t_z+1})\big]\\
&\times\Lambda_{t_z+1}+\Lambda_{t_z+1}\big[[(\mathfrak{p}_{r,t_z+1}I+\bar{p}_{r,t_z+1}^2(\mathbf{1}_N-I))\\
&\otimes\mathbf{1}_m]\circ(H_{r,t_z+1}S_{s,t_z+1}H_{r,t_z+1})\big]\Lambda_{t_z+1}\\
&+S_{r,t_z+1}\Big],\\
\mathcal{O}_{t_z+1}\triangleq&(\bar{p}_{r,t_z+1}\bar{p}_{s,t_z+1})^{-2}\mathfrak{p}_{r,t_z+1}\mathfrak{p}_{s,t_z+1}\mathrm{diag}\{(\sigma_{1,t_z+1}^2\\
&+\mu_{1,t_z+1}^2)\mathbf{1}_m,(\sigma_{2,t_z+1}^2+\mu_{2,t_z+1}^2)\mathbf{1}_m,\ldots,\\
&(\sigma_{N,t_z+1}^2+\mu_{N,t_z+1}^2)\mathbf{1}_m\}-\mathrm{diag}\{\mu_{1,t_z+1}^2\mathbf{1}_m,\\
&\mu_{2,t_z+1}^2\mathbf{1}_m,\ldots,\mu_{N,t_z+1}^2\mathbf{1}_m\}.
\end{aligned}
$$

Corollary 1: Assume that p_{s,i,t_z+1} and p_{r,i,t_z+1} of every node obey the same probability distribution law. If $\bar{p}_{r,t_z+1}$

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increases or $\check{p}_{r,t_z+1}$ decreases, then $\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}$ is non-increasing.

Proof: Taking the partial derivative of $\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}$ with respect to $\bar{p}_{r,t_z+1}$, we have

$$\begin{aligned}&\frac{\partial \mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}}{\partial \bar{p}_{r,t_z+1}}\\ =&\sum_{i=1}^{N}\mathrm{tr}\big\{\Sigma_{t_z+1|t_z}C_{t_z+1}^{T}H_{s,t_z+1}H_{r,t_z+1}\mathfrak{A}_{t_z+1}\Lambda_{t_z+1}\Phi_{2,i}^{T}\\ &\times(\Phi_{2,i}\mathfrak{W}_{t_z+1}\Phi_{2,i}^{T})^{-1}\Phi_{2,i}\frac{\partial \mathfrak{W}_{t_z+1}}{\partial \bar{p}_{r,t_z+1}}\Phi_{2,i}^{T}(\Phi_{2,i}\mathfrak{W}_{t_z+1}\\ &\times\Phi_{2,i}^{T})^{-1}\Phi_{2,i}\Lambda_{t_z+1}\mathfrak{A}_{t_z+1}H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Sigma_{t_z+1|t_z}\\ &\times\Phi_{1,i}^{T}\Phi_{1,i}\big\}. \end{aligned} \tag{47}$$

Note that

$$\begin{aligned}&\frac{\partial \mathfrak{W}_{t_z+1}}{\partial \bar{p}_{r,t_z+1}}\\ =&-2\frac{\check{p}_{r,t_z+1}\mathfrak{p}_{s,t_z+1}}{\bar{p}_{r,t_z+1}^{3}\bar{p}_{s,t_z+1}^{2}}\Lambda_{t_z+1}\bigg[\Big[\mathrm{diag}\big\{(\sigma_{1,t_z+1}^{2}+\mu_{1,t_z+1}^{2})\mathbf{1}_m,\\ &(\sigma_{2,t_z+1}^{2}+\mu_{2,t_z+1}^{2})\mathbf{1}_m,\ldots,(\sigma_{N,t_z+1}^{2}+\mu_{N,t_z+1}^{2})\mathbf{1}_m\big\}\\ &\circ(H_{r,t_z+1}H_{s,t_z+1}C_{t_z+1}\Omega_{t_z+1}C_{t_z+1}^{T}H_{s,t_z+1}H_{r,t_z+1})\Big]\\ &+\Big[(I\otimes\mathbf{1}_m)\circ(H_{r,t_z+1}H_{s,t_z+1}R_{t_z+1}H_{s,t_z+1}H_{r,t_z+1})\Big]\bigg]\\ &\times\Lambda_{t_z+1}-2\frac{\check{p}_{r,t_z+1}}{\bar{p}_{r,t_z+1}^{3}\bar{p}_{s,t_z+1}^{2}}\Lambda_{t_z+1}\big[(I\otimes\mathbf{1}_m)\circ(H_{r,t_z+1}\\ &\times S_{s,t_z+1}H_{r,t_z+1})\big]\Lambda_{t_z+1}-\frac{2}{\bar{p}_{r,t_z+1}^{3}\bar{p}_{s,t_z+1}^{2}}S_{r,t_z+1}<0. \end{aligned} \tag{48}$$

Based on (47) and (48), it follows that

$$\frac{\partial \mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}}{\partial \bar{p}_{r,t_z+1}}\le 0. \tag{49}$$

Similarly, taking the partial derivative of $\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}$ with respect to $\check{p}_{r,t_z+1}$, we have

$$\frac{\partial \mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}}{\partial \check{p}_{r,t_z+1}}\ge 0. \tag{50}$$

Based on (49) and (50), it can be seen that $\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}$ is non-increasing if $\bar{p}_{r,t_z+1}$ increases or $\check{p}_{r,t_z+1}$ decreases. The proof is complete. ■

Corollary 2: Assume that p_{s,i,t_z+1} and p_{r,i,t_z+1} of every node obey the same probability distribution law. Then, $\mathrm{tr}\{\Sigma_{t_z+1|t_z+1}\}$ is non-increasing when $\bar{p}_{s,t_z+1}$ increases or variance $\check{p}_{s,t_z+1}$ decreases.

Proof: The proof follows a similar approach to that of Corollary 1 and, for the sake of conciseness, it is omitted here. ■

IV. Illustration Example

In this section, a simulation example is presented to demonstrate the effectiveness of the proposed recursive EKF-based estimator in (8)-(9).

Consider a complex network with four nodes where the coupling matrices are shown as follows,

$$W=\begin{bmatrix}-0.3 & 0.1 & 0.1 & 0.1\\ 0.1 & -0.3 & 0.1 & 0.1\\ 0.1 & 0.1 & -0.3 & 0.1\\ 0.1 & 0.1 & 0.1 & -0.3\end{bmatrix},\quad \Gamma=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}.$$

For each node, the nonlinear function $f(x_{i,t_z})$ is chosen as

$$f(x_{i,t_z})=\begin{bmatrix}\tanh(0.25x_{i,t_z}^{[1]})-0.2x_{i,t_z}^{[2]}\\ \tanh(0.5x_{i,t_z}^{[2]})\end{bmatrix},$$

where $x_{i,t_z}=\begin{bmatrix}x_{i,t_z}^{[1]}, & x_{i,t_z}^{[2]}\end{bmatrix}^{T}$ is the system state of ith node. For $i=1,2,3,4$, $x_{i,0}^{[1]}$ and $x_{i,0}^{[2]}$ are uniform distribution over the following intervals: $x_{1,0}^{[1]}\in[1.3,2.3]$, $x_{1,0}^{[2]}\in[-0.75,0.25]$, $x_{2,0}^{[1]}\in[1.25,2.25]$, $x_{2,0}^{[2]}\in[-0.7,0.3]$, $x_{3,0}^{[1]}\in[1.25,2.25]$, $x_{3,0}^{[2]}\in[-0.75,0.25]$, $x_{4,0}^{[1]}\in[1.3,2.3]$, and $x_{4,0}^{[2]}\in[-0.75,0.25]$, respectively. The covariances of zero-mean Gaussian white noise ϖ_{i,t_z} $(i=1,2,3,4)$ are $Q_{1,t_z}=0.04$, $Q_{2,t_z}=0.05$, $Q_{3,t_z}=0.02$, and $Q_{4,t_z}=0.01$. The covariances of zero-mean Gaussian white noise ν_{i,t_z} $(i=1,2,3,4)$ are $R_{1,t_z}=0.02$, $R_{2,t_z}=0.03$, $R_{3,t_z}=0.01$, and $R_{4,t_z}=0.05$. The matrices B_{i,t_z} and C_{i,t_z} are set as

$$\begin{aligned}&B_{1,t_z}=\begin{bmatrix}0.7-0.2\sin(t_z)\\ 0.4\end{bmatrix},B_{2,t_z}=\begin{bmatrix}0.07\\ 0.21\end{bmatrix},\\ &B_{3,t_z}=\begin{bmatrix}-0.02\\ 0.03\end{bmatrix},B_{4,t_z}=\begin{bmatrix}0.01\\ -0.17\end{bmatrix},\\ &C_{1,t_z}=\begin{bmatrix}0.95 & 0.35\end{bmatrix},C_{2,t_z}=\begin{bmatrix}0.94 & 0.45\end{bmatrix},\\ &C_{3,t_z}=\begin{bmatrix}0.89 & 0.4\end{bmatrix},C_{4,t_z}=\begin{bmatrix}0.9 & 0.6\end{bmatrix}.\end{aligned}$$

To depict the estimation performance, the mean square estimation error of the ith node MSE_{i,t_z} is introduced as follows:

$$\mathrm{MSE}_{i,t_z}\triangleq\frac{1}{M}\sum_{\epsilon=1}^{M}\|x_{i,t_z}^{(\epsilon)}-\hat{x}_{i,t_z}^{(\epsilon)}\|^2,$$

where ϵ represents the ϵth simulation test. The sum of the MSE_{i,t_z} $(i=1,2,3,4)$ is denoted by SMSE_{t_z}, i.e., $\mathrm{SMSE}_{t_z}\triangleq\sum_{i=1}^{4}\mathrm{MSE}_{i,t_z}$.

The channel coefficient matrices are given as $H_{s,i,t_z}=0.36$ and $H_{r,i,t_z}=0.36$ $(i=1,2,3,4)$. Set the amplification factor as $a_{i,t_z}=1.2$ $(i=1,2,3,4)$. ζ_{r,i,t_z} and ζ_{r,i,t_z} are zero-mean Gaussian white noises with covariances $S_{r,i,t_z}=0.25$ and $S_{s,i,t_z}=0.25$ $(i=1,2,3,4)$, respectively. Other parameters are given as $\bar{x}_{1,0}=[1.8\ -0.25]^T$, $\bar{x}_{2,0}=[1.75\ -0.20]^T$, $\bar{x}_{3,0}=[1.75\ -0.25]^T$, $\bar{x}_{4,0}=[1.8\ -0.25]^T$, $\Sigma_{0|0}=\mathrm{diag}\{5,5,4,4,10,10,10,10\}$, $L_{1,t_z}=\mathrm{diag}\{0.03,0.03\}$, $L_{2,t_z}=\mathrm{diag}\{0.01,0.03\}$, $L_{3,t_z}=\mathrm{diag}\{0.03,0.01\}$, $L_{4,t_z}=\mathrm{diag}\{0.01,0.03\}$, $\gamma_{t_z}=0.02$, $\varepsilon_{t_z}=0.2$, $M=200$.

To verify the effectiveness of the proposed estimation algorithm and investigate the effects of the stochastic variables on the estimation performance, comparative simulations are presented in different cases. For the degraded measurement, the probability distribution laws of θ_{i,t_z} $(i=1,2,3,4)$ in four cases are listed in Table I.

For the random transmission energy of sensors and relays, the probability distribution laws of p_{s,i,t_z} and p_{r,i,t_z} $(i=1,2,3,4)$ in four different cases are presented in Tables II and III, respectively.

Now let us consider the probability distribution law for θ_{i,t_z} in Case D0, p_{s,i,t_z} in Case S0, and p_{r,i,t_z} in Case R0, respectively. According to (22) and (23) in Theorem 1, the upper bound of the error covariance and the estimator gain K_{i,t_z+1} $(i=1,2,3,4)$ can be obtained recursively. Figs. 2-5 show the actual states and their estimates achieved by using

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TABLE I
The probability distribution law for θ_{i,t_z} $(i = 1, 2, 3, 4)$

Case D0

θ_{i,t_z}	0	0.5	1
Prob$\{\theta_{1,t_z}\}$	0.05	0.05	0.9
Prob$\{\theta_{2,t_z}\}$	0.05	0.1	0.85
Prob$\{\theta_{3,t_z}\}$	0.03	0.17	0.8
Prob$\{\theta_{4,t_z}\}$	0.1	0.1	0.8

Case D1

θ_{i,t_z}	0	0.17	0.33
Prob$\{\theta_{1,t_z}\}$	0.05	0.05	0.9
Prob$\{\theta_{2,t_z}\}$	0.05	0.1	0.85
Prob$\{\theta_{3,t_z}\}$	0.03	0.17	0.8
Prob$\{\theta_{4,t_z}\}$	0.1	0.1	0.8

Case D2

θ_{i,t_z}	0.33	0.5	0.67
Prob$\{\theta_{1,t_z}\}$	0.05	0.05	0.9
Prob$\{\theta_{2,t_z}\}$	0.05	0.1	0.85
Prob$\{\theta_{3,t_z}\}$	0.03	0.17	0.8
Prob$\{\theta_{4,t_z}\}$	0.1	0.1	0.8

Case D3

θ_{i,t_z}	0.67	0.83	1
Prob$\{\theta_{1,t_z}\}$	0.05	0.05	0.9
Prob$\{\theta_{2,t_z}\}$	0.05	0.1	0.85
Prob$\{\theta_{3,t_z}\}$	0.03	0.17	0.8
Prob$\{\theta_{4,t_z}\}$	0.1	0.1	0.8

TABLE II
The probability distribution law for p_{s,i,t_z} $(i = 1, 2, 3, 4)$

p_{s,i,t_z}	1	1.5	2	2.5	3	6	6.5	7	12	12.5	13
Prob$\{p_{s,i,t_z}\}$ in S0	0.2	0.25	0.55	0	0	0	0	0	0	0	0
Prob$\{p_{s,i,t_z}\}$ in S1	0	0	0.2	0.25	0.55	0	0	0	0	0	0
Prob$\{p_{s,i,t_z}\}$ in S2	0	0	0	0	0	0.2	0.25	0.55	0	0	0
Prob$\{p_{s,i,t_z}\}$ in S3	0	0	0	0	0	0	0	0	0.2	0.25	0.55

the proposed EKF-based estimator for nodes $i = 1, 2, 3, 4$, respectively. The $\log(\text{MSE}_{i,t_z})$ of four nodes and their upper bounds are depicted in Fig. 6 under $M = 200$ iterations. It can be concluded that the proposed EKF-based estimator can track the actual states well, even when the received signals are influenced by the degraded measurements and the AF relays.

To examine the influences of degraded measurements, the estimation performances in Cases D1-D3 are compared in Fig. 7 with the sensor transmission energy in Case S0 and the relay transmission energy in Case R0. It can be seen that the estimation errors become smaller with a larger value of θ_{i,t_z}, which indicates less severe degraded measurements.

In order to show the effects of the random sensor transmission energy on the estimation performance, we compare Cases S0-S3 with the degraded measurements in Case D0 and the relay transmission energy in Case R0. From Fig. 8, we can see that the estimation error in case S3 is generally the smallest.

Similarly, to discuss the effects of the random relays transmission energy, we compare Cases R0-R3 with the degraded measurements in Case D0 and the sensor transmission energy in Case S0. From Fig. 9, it can be asserted that the estimation error is the smallest in case R3 with the biggest average transmission power at the relays. It means that the increase of the transmission energy can lead to more accurate estimation results, which is consistent with the analyses in Corollaries 1 and 2.

TABLE III
The probability distribution law for p_{r,i,t_z} $(i = 1, 2, 3, 4)$

p_{r,i,t_z}	1	1.5	2	2.5	3	6	6.5	7	12	12.5	13
Prob$\{p_{r,i,t_z}\}$ in R0	0.3	0.35	0.35	0	0	0	0	0	0	0	0
Prob$\{p_{r,i,t_z}\}$ in R1	0	0	0.3	0.35	0.35	0	0	0	0	0	0
Prob$\{p_{r,i,t_z}\}$ in R2	0	0	0	0	0	0.3	0.35	0.35	0	0	0
Prob$\{p_{r,i,t_z}\}$ in R3	0	0	0	0	0	0	0	0	0.3	0.35	0.35

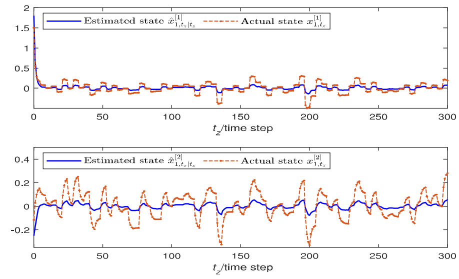

Fig. 2. State x_{1,t_z} and its estimate $\hat{x}_{1,t_z|t_z}$

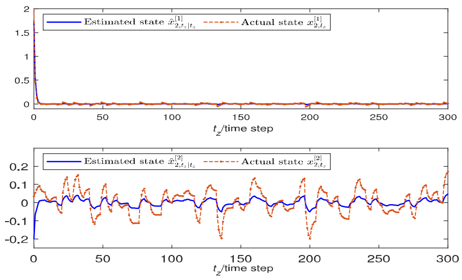

Fig. 3. State x_{2,t_z} and its estimate $\hat{x}_{2,t_z|t_z}$

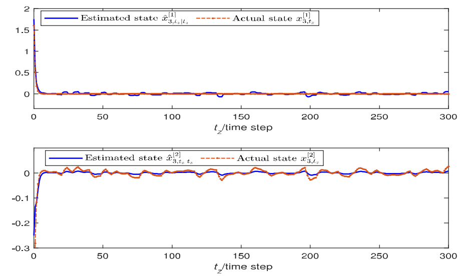

Fig. 4. State x_{3,t_z} and its estimate $\hat{x}_{3,t_z|t_z}$

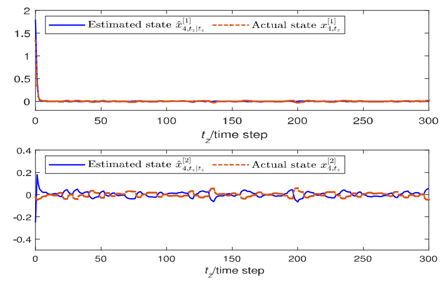

Fig. 5. State x_{4,t_z} and its estimate $\hat{x}_{4,t_z|t_z}$

V. CONCLUSIONS

This paper has explored the state estimation problem in complex networks with degraded measurements and AF relays. To address the challenges posed by degraded measurements and random transmission energy, EKF-based estimators have been developed for each node in the network. The upper bound of the estimation error covariance has been derived and

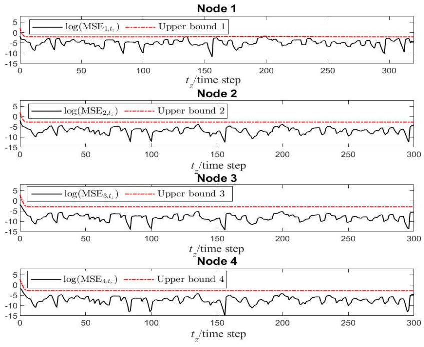

Fig. 6. $\log(\mathrm{MSE}_{i,t_z})$ and upper bounds

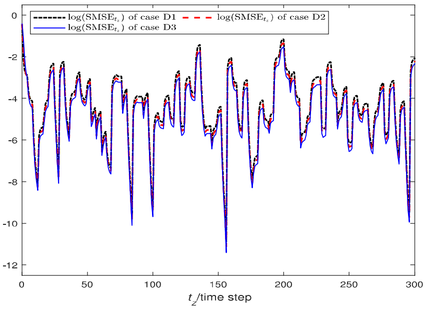

Fig. 7. $\log(\mathrm{SMSE}_{t_z})$ of Case D1, Case D2, and Case D3

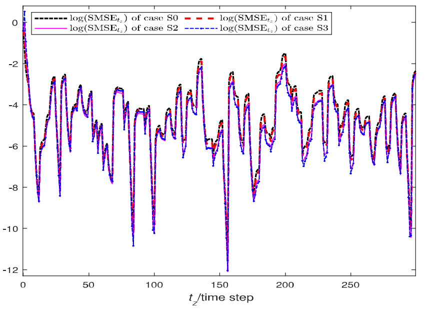

Fig. 8. $\log(\mathrm{SMSE}_{t_z})$ of Case S0, Case S1, Case S2, and Case S3

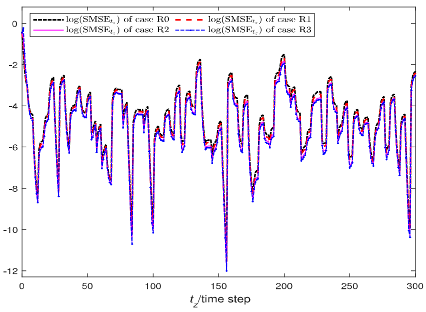

Fig. 9. $\log(\mathrm{SMSE}_{t_z})$ of Case R0, R1, Case R2, and Case R3

minimized through the selection of an appropriate estimator gain. Sufficient conditions have been established to ensure the exponential boundedness of the estimation error in the mean square sense. The effectiveness of the proposed estimator has been validated through numerical simulations. In future research, it would be valuable to extend the proposed methodology to more general systems that incorporate additional network-induced phenomena [10], [14], [24], [33], [35], [44], [49], [54].

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