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Networked State Estimation for Two-Dimensional Systems with Multiplicative Noises and Observation Delays: An Iterative Coupled Estimator Design

Wei Wang, Yu Chen, Zidong Wang, Chunyan Han, and Shuxin Du

***Abstract*—This paper addresses the state estimation problem for a class of networked two-dimensional systems subject to autocorrelated multiplicative noises and multi-step observation delays. The autocorrelated multiplicative noises are characterized as a linear system corrupted by additive white noises, and the multi-step observation delays are described by the instantaneous and delayed output equations. The globally coupled correlation terms are first introduced to deal with the autocorrelated multiplicative noises, and then a novel observation reconstruction approach is proposed to tackle the difficulty induced by multi-step observation delays. By utilizing the space equivalence verification, it is shown that the reconstructed delay-free observation sequence retains the same information as the original delayed observation sequence. The purpose of this paper is to design an iterative coupled estimator based on the globally coupled correlation terms and the reconstructed observation sequence. By means of the mathematical induction technique, it is demonstrated that the proposed estimator preserves the property of unbiasedness. Moreover, the estimator gains are obtained by minimizing the estimation error covariance, and some sufficient conditions are derived to guarantee the existence of an upper bound for the estimation error covariance in the mean-square sense. Finally, a numerical example is provided to verify the effectiveness of the proposed estimation scheme.**

***Index Terms*—Two-dimensional system, autocorrelated multiplicative noise, observation delay, iterative coupled estimator.**

I. Introduction

In the past several decades, two-dimensional (2-D) systems have garnered growing research interest owing to their applications in a variety of engineering realms which include, but are not restricted to, real-time network visualization, batch process modeling and control, and digital signal processing [1], [20], [26], [32], [33], [38]. In comparison with the one-dimensional (1-D) systems where the signals are only allowed to evolve along one direction, the 2-D systems are capable of characterizing many practical multi-variable systems exhibiting the inherent behavior of bidirectional signal evolution. Clearly, the resultant complex dynamic behaviors would pose additional challenges for the theoretical analysis. So far, a large body of literature has been reported on 2-D systems from various perspectives, mainly including the state-space representation [20], controllability and observability analysis [2], stability analysis [5], [6], [19], [45], state estimation [23], [39], [44], fault diagnosis [3], [48], [49], sliding mode control [29], and stabilization of fuzzy systems [17], [18].

With the ongoing advancement of manufacturing and sensing techniques, the sensor networks have been widely applied in a diversity of practical applications, such as guidance and navigation, space and ground detection, communication, target tracking, and underwater robot [35], [43], [47]. One of the active topics related to sensor networks is how to optimally estimate the system states based on the multi-source observation information, which is usually transmitted from different sensors to the remote estimator via a communication network. So far, considerable research attention has been devoted to the design of accurate, reliable, and suitable estimation algorithms, which give rises to a great deal of elegant estimation algorithms in the literature, see e.g. [11], [12], [34], [37], [50], [52] and the references therein.

It has been well recognized that in most real-world applications, the system dynamics and observations are inevitably affected by different kinds of random uncertainties. Among them, the multiplicative noise has attracted particular research attention due to its theoretical importance and practical significance. Generally speaking, the multiplicative noise can be categorized into two types, i.e, the white noise with known statistical properties and the autocorrelated multiplicative noise. Notably, the autocorrelated multiplicative noise can be represented as a linear system corrupted by white noises. Up to now, the state estimation problem for 1-D systems subject to time-correlated multiplicative noises has gradually become a hotshot, leading to a plethora of effective results [4], [7], [27], [28], [41]. Nevertheless, it should be pointed out that very little effort has been paid to the corresponding estimation problem for 2-D systems, despite its clear engineering insights. This constitutes the first motivation of our current research.

On the other hand, the communication delays usually behave as another important kind of random uncertainty due to various reasons such as network congestion, signal interference, environmental changes, and physical constraints [14], [40], [46]. For instance, during the data collection of industrial

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TABLE I: Comparisons between our work and existing relevant works.

Work	2-D system	Delays	Multiplicative noises	Filtering method
[42]	×	√ (multi-step, observation)	× (multiplicative, white)	Kalman-type filtering
[51]	×	√ (multi-step, state)	× (multiplicative, white)	Set-membership filtering
[28]	×	×	√ (multiplicative,autocorrelated)	Kalman-type filtering
[36]	√	×	√ (multiplicative, autocorrelated)	Kalman-type filtering
[39]	√	√ (one-step,observation)	√ (additive, white)	Kalman-type filtering via augmentation
This paper	√	√ (multi-step, observation)	√ (multiplicative, autocorrelated)	Kalman-type filtering via reconstruction

systems, the communication network may be congested if the incoming data flow is excessively high, thereby leading to the delayed reception of observation data. Similarly, insufficient network bandwidth can also impede data transmission in the network, causing delays in the transmission of observation data. There is no doubt that the communication delays, if not carefully tackled in designing the estimation algorithms, would result in substantial performance degradation [13], [15], [16], [25]. In most existing literature, the system augmentation method has been employed to address the problem of multi-step observation delays. However, such a method would increase the system dimension and further incur unnecessary computational overhead, which might be unacceptable in some scenarios. Consequently, it makes practical sense to explore new method to deal with the multi-step observation delays without introducing additional computational overhead, and this motivates us to investigate the state estimation problem for 2-D systems under the effects of multi-step observation delays and autocorrelated multiplicative noises.

Inspired by the above discussions, this paper makes one of the first few attempts to investigate the state estimation problem for a class of shift-varying 2-D systems subject to autocorrelated multiplicative noises and multi-step observation delays. Three key challenges of this research are presented as follows: 1) how to handle multi-step observation delays caused by the limited communication bandwidth? 2) how to compensate for the influence of autocorrelated multiplicative noises during the estimator design? and 3) how to construct an appropriate recursive estimator and derive the gain parameters to ensure the minimization of estimation error covariance?

In response to these identified challenges, the primary contributions of this paper are highlighted from the following four aspects.

(1) A novel observation reconstruction approach is proposed, for the first time, for 2-D systems with aim to tackle the multi-step observation delays, which is able to convert the original delayed observation sequence into a delay-free observation sequence.
(2) Some globally coupled correlation terms are newly introduced for 2-D systems in order to deal with the autocorrelated multiplicative noises.
(3) An iterative coupled estimator with four steps is proposed based on the reconstructed observations and the globally coupled correlation terms.
(4) Sufficient conditions are derived to guarantee the boundedness of the estimation error covariance.

A comparative analysis between our work and existing works is provided in Table I.

The notation utilized here is conventional.

TABLE II: Abbreviations and Notations

$\mathbb{R}^n$	The n-dimensional Euclidean space
I	The identity matrix with appropriate dimensions
0	The zero matrix with suitable dimensions
I_n	The identity vector with n-dimensions
$\mathbf{0}$	The zero vector with suitable dimensions
Z^{T}	The transpose of Z
Z^{-1}	The inverse matrix of Z
$\hat{\mathrm{E}}\{x\}$	The expectation of the variable x
$A > B$	$A - B > 0$ is positive definite
$A \geq B$	$A - B \geq 0$ is positive semi-definite
$[0, \Im]$	The set $\{0, 1, \cdots, \Im\}$
$\mathcal{L}\{a, b, c, \cdots\}$	The the linear space spanned by $a, b, c, \cdots$
$\mathrm{sym}\{A\}$	The matrix $A + A^{\mathsf{T}}$
$\delta(\cdot, \cdot)$	The Kronecker delta function

II. Problem Formulation

A. System setup

Consider the following 2-D discrete time-varying systems:

$$\begin{aligned} x(\tau_h, \tau_v) = {} & A_1(\tau_h, \tau_v - 1)x(\tau_h, \tau_v - 1) \\ & + A_2(\tau_h - 1, \tau_v)x(\tau_h - 1, \tau_v) \\ & + B_1(\tau_h, \tau_v - 1)w(\tau_h, \tau_v - 1) \\ & + B_2(\tau_h - 1, \tau_v)w(\tau_h - 1, \tau_v) \end{aligned} \tag{1}$$

$$\begin{aligned} y_{(1)}(\tau_h, \tau_v) = {} & [C_{(1)}(\tau_h, \tau_v) + \check{C}_{(1)}(\tau_h, \tau_v)\zeta_{(1)}(\tau_h, \tau_v)] \\ & \times x(\tau_h, \tau_v) + v_{(1)}(\tau_h, \tau_v) \\ y_{(2)}(\tau_h, \tau_v) = {} & [C_{(2)}(\tau_h, \tau_v) + \check{C}_{(2)}(\tau_h, \tau_v)\zeta_{(2)}(\tau_h - d, \tau_v - d)] \\ & \times x(\tau_h - d, \tau_v - d) + v_{(2)}(\tau_h, \tau_v) \end{aligned} \tag{2}$$

where $\tau_h, \tau_v \in [0, \Im]$ denote the horizontal and vertical coordinates with $\Im \in \mathbb{N}$; $x(\tau_h, \tau_v) \in \mathbb{R}^n$ is the state vector; $y_{(1)}(\tau_h, \tau_v) \in \mathbb{R}^{m_1}$ and $y_{(2)}(\tau_h, \tau_v) \in \mathbb{R}^{m_2}$ denote, respectively, the instantaneous and delayed observation outputs; $w(\tau_h, \tau_v) \in \mathbb{R}^p$ is the process noise and $v_{(c)}(\tau_h, \tau_v) \in \mathbb{R}^{m_c}$ (for $c \in [1, 2]$) are the observation noises, which obey the null mean Gaussian distributions with covariance $Q(\tau_h, \tau_v) \geq 0$ and $R_{(c)}(\tau_h, \tau_v) > 0$. $\zeta_{(c)}(\tau_h, \tau_v) \in \mathbb{R}$ is the autocorrelated multiplicative noise satisfying $\hat{\mathrm{E}}\{\zeta_{(c)}(\tau_h, 0)\} = \hat{\mathrm{E}}\{\zeta_{(c)}(0, \tau_v)\} = 0$ and

$$\begin{aligned} \zeta_{(c)}(\tau_h, \tau_v) = {} & F_{(c)}(\tau_h, \tau_v - 1)\zeta_{(c)}(\tau_h, \tau_v - 1) \\ & + E_{(c)}(\tau_h - 1, \tau_v)\zeta_{(c)}(\tau_h - 1, \tau_v) \\ & + \epsilon_{(c)}(\tau_h, \tau_v) \end{aligned} \tag{3}$$

where $\epsilon_{(c)}(\tau_h, \tau_v) \in \mathbb{R}$ denotes the Gaussian distributed additive noise with mean zero and variance $\sigma_{(c)}(\tau_h, \tau_v) > 0$, and the observation delay d is a known positive integer. $A_c(\tau_h, \tau_v)$, $B_c(\tau_h, \tau_v)$, $C_{(c)}(\tau_h, \tau_v)$, $\check{C}_{(c)}(\tau_h, \tau_v)$, $F_{(c)}(\tau_h, \tau_v)$, and $E_{(c)}(\tau_h, \tau_v)$ are known matrices with appropriate dimensions.

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Remark 1: In networked control systems (NCSs), control commands and sensor signals are transmitted over a shared network. When network load becomes excessive, data packets need to queue up in network devices for transmission opportunities, resulting in transmission delays. Such delays may cause sensor data or control commands to fail to reach target nodes in a timely manner. Meanwhile, clock desynchronization among devices will further exacerbate the impact of these delays. This multi-step delay phenomenon, in practical engineering applications, can significantly degrade system performance and even potentially compromise system reliability.

Assumption 1: For every $\tau_h, \tau_v \in [0, \Im]$ and $c \in [1, 2]$, the initial states $x(\tau_h, 0)$ and $x(0, \tau_v)$ as well as the autocorrelated noises $\zeta_{(c)}(\tau_h, 0)$ and $\zeta_{(c)}(0, \tau_v)$ are mutually independent, and also independent of the random variables $w(\tau_h, \tau_v)$, $v_{(c)}(\tau_h, \tau_v)$, and $\epsilon_{(c)}(\tau_h, \tau_v)$. Moreover, the following initial conditions are satisfied:

$$\begin{cases} \hat{\mathrm{E}}\{x(\tau_h, 0)\} = \hat{x}_1(\tau_h), \ \hat{\mathrm{E}}\{x(0, \tau_v)\} = \hat{x}_2(\tau_v) \\ \mathrm{Cov}\{x(\tau_h, 0), x(u, 0)\} = P_1(\tau_h)\delta(\tau_h, u) \\ \mathrm{Cov}\{x(0, \tau_v), x(0, s)\} = P_2(\tau_v)\delta(\tau_v, s) \\ \mathrm{Cov}\{\zeta_{(c)}(\tau_h, 0), \zeta_{(c)}(u, 0)\} = \tau_{(c),1}(\tau_h)\delta(\tau_h, u) \\ \mathrm{Cov}\{\zeta_{(c)}(0, \tau_v), \zeta_{(c)}(0, s)\} = \tau_{(c),2}(\tau_v)\delta(\tau_v, s) \end{cases}$$

where $\hat{x}_1(\tau_h)$, $\hat{x}_2(\tau_v)$, $P_1(\tau_h)$, $P_2(\tau_v)$, $\tau_{(c),1}(\tau_h)$, and $\tau_{(c),2}(\tau_v)$ are known parameters with appropriate dimensions.

Remark 2: As one of the important types of random uncertainty, the multiplicative noise is frequently encountered in practical engineering. For example, in a pulse radar system, the distance and orientation signals of a moving vehicle often suffer from the multiplicative noises due to the effects of signal interference, signal reflections, and radar fading. It should be noted that in most existing studies, the multiplicative noise is described as zero-mean white noise, which is incorporated into the process and observation noises and further forms new correlated noises or Gaussian noises with unknown covariance [41], [42].

Remark 3: In the context of global navigation satellite systems, the received signals are usually affected by various autocorrelated noises including satellite orbit noises, clock noises, and atmospheric noises [30], [31]. When considering the navigation and positioning under ultra-short baselines, the aforementioned noises can be ignored, and the autocorrelated noises, driven by the white noise $\epsilon_{(c)}(\tau_h, \tau_v)$, will be used to characterize the multi-path effect of satellite signal distortion. In this sense, the state estimation problem involving autocorrelated multiplicative noises, as considered in this paper, has significant practical implications.

B. *Original observation description*

Let $\mathscr{Y}(\varsigma_h, \varsigma_v)$ denote the original observations at the horizon $(\varsigma_h, \varsigma_v)$. For the sake of clarity, define the following sets

$$\begin{aligned} M_1 &= \{(\varsigma_h, \varsigma_v) | 0 \le \varsigma_h < d, 0 \le \varsigma_v < d\}, \\ M_2 &= \{(\varsigma_h, \varsigma_v) | 0 \le \varsigma_h < d, d \le \varsigma_v \le \tau_v\}, \\ M_3 &= \{(\varsigma_h, \varsigma_v) | d \le \varsigma_h \le \tau_h, 0 \le \varsigma_v < d\}, \\ M_4 &= \{(\varsigma_h, \varsigma_v) | d \le \varsigma_h \le \tau_h, d \le \varsigma_v \le \tau_v\}. \end{aligned}$$

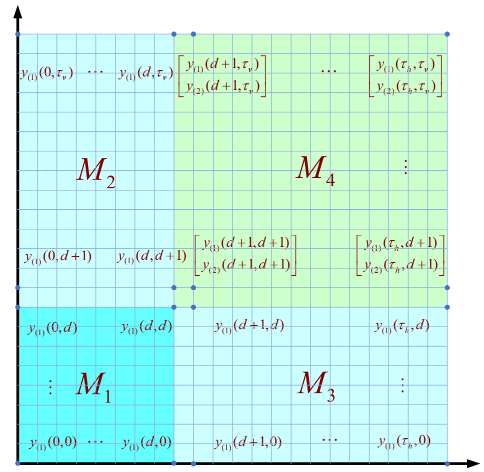

Fig. 1: The original observation sequence.

As shown in Fig. 1, the original observations can be described as

$$\mathscr{Y}(\varsigma_h, \varsigma_v) = \begin{cases} y_{(1)}(\varsigma_h, \varsigma_v), \ (\varsigma_h, \varsigma_v) \in M_1 \cup M_2 \cup M_3 \\ \begin{bmatrix} y_{(1)}(\varsigma_h, \varsigma_v) \\ y_{(2)}(\varsigma_h, \varsigma_v) \end{bmatrix}, \ (\varsigma_h, \varsigma_v) \in M_4. \end{cases} \tag{4}$$

It should be pointed out that the objective of this paper is to develop a recursive state estimation algorithm based on the observation sequence $\{\mathscr{Y}(\varsigma_h, \varsigma_v) | 0 \le \varsigma_h \le \tau_h, 0 \le \varsigma_v \le \tau_v\}$. Nevertheless, in view of (2) and (4), the original observations contain the multi-step delays, which renders it really difficult (if not impossible) to directly exploit the Kalman-type filter developed in [53]. In this sense, we need to reconstruct the original observation sequence and facilitate the utilization of delayed observation information, which will be discussed thoroughly in the subsequent section.

III. Main Results

A. *Reconstruction of observations*

Without loss of generality, we only consider the case of $(\tau_h, \tau_v) \in \{(s, t) | d \le s, d \le t\}$. Other cases can be discussed in the same manner. For brevity, let us define delay-compensated coordinates where

$$\tau_h^1 = \tau_h - d, \tau_v^1 = \tau_v - d,$$

and the following sets

$$\begin{aligned} N_1 &= \{(\varsigma_h, \varsigma_v) | 0 \le \varsigma_h \le \tau_h^1, 0 \le \varsigma_v \le \tau_v^1\}, \\ N_2 &= \{(\varsigma_h, \varsigma_v) | 0 \le \varsigma_h \le \tau_h^1, \tau_v^1 < \varsigma_v \le \tau_v\}, \\ N_3 &= \{(\varsigma_h, \varsigma_v) | \tau_h^1 < \varsigma_h \le \tau_h, 0 \le \varsigma_v \le \tau_v^1\}, \\ N_4 &= \{(\varsigma_h, \varsigma_v) | \tau_h^1 < \varsigma_h \le \tau_h, \tau_v^1 < \varsigma_v \le \tau_v\}. \end{aligned}$$

From the available information sets N_1, N_2, N_3, and N_4 defined above, we can collect all observations that reflect the state information of the current horizon, and then make full use of all multi-source observations to build a recursive filtering framework. This not only helps to extract state information

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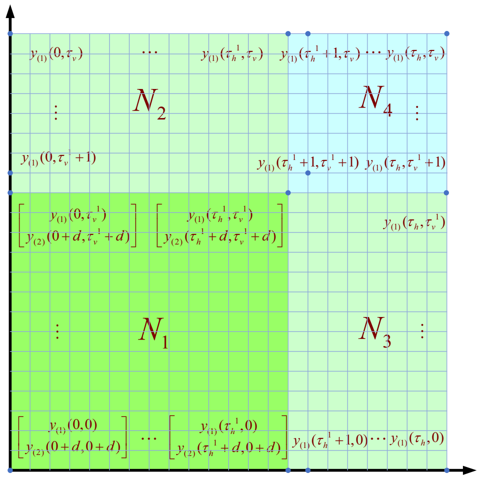

Fig. 2: The reconstructed observation sequence.

more efficiently, but also further improves the fault tolerance of the system.

From Fig. 2, it is clear that the original observation sequence can be reconstructed as follows:

$$y_1(\varsigma_h,\varsigma_v) = y_{(1)}(\varsigma_h,\varsigma_v) \quad (\varsigma_h,\varsigma_v) \in N_2 \cup N_3 \cup N_4, \tag{5a}$$

$$y_2(\varsigma_h,\varsigma_v) = \begin{bmatrix} y_{(1)}^{\mathsf{T}}(\varsigma_h,\varsigma_v) & y_{(2)}^{\mathsf{T}}(\varsigma_h+d,\varsigma_v+d) \end{bmatrix}^{\mathsf{T}} \quad (\varsigma_h,\varsigma_v) \in N_1. \tag{5b}$$

Obviously, the observations $y_1(\varsigma_h,\varsigma_v)$ and $y_2(\varsigma_h,\varsigma_v)$ satisfy

$$\begin{aligned} y_1(\varsigma_h,\varsigma_v) = {} & C_1(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v) + v_1(\varsigma_h,\varsigma_v) \\ & + \check{C}_{11}(\varsigma_h,\varsigma_v)\zeta_1(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v) \end{aligned} \tag{6a}$$

$$\begin{aligned} y_2(\varsigma_h,\varsigma_v) = {} & C_2(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v) + v_2(\varsigma_h,\varsigma_v) \\ & + \check{C}_{21}(\varsigma_h,\varsigma_v)\zeta_1(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v) \\ & + \check{C}_{22}(\varsigma_h,\varsigma_v)\zeta_2(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v) \end{aligned} \tag{6b}$$

with

$$\begin{aligned} & C_1(\varsigma_h,\varsigma_v) = C_{(1)}(\varsigma_h,\varsigma_v) \\ & C_2(\varsigma_h,\varsigma_v) = \begin{bmatrix} C_{(1)}^{\mathsf{T}}(\varsigma_h,\varsigma_v) \; C_{(2)}^{\mathsf{T}}(\varsigma_h+d,\varsigma_v+d) \end{bmatrix}^{\mathsf{T}} \\ & \check{C}_{11}(\varsigma_h,\varsigma_v) = \check{C}_{(1)}(\varsigma_h,\varsigma_v) \\ & \check{C}_{2c}(\varsigma_h,\varsigma_v) = \begin{bmatrix} \delta(1,c) & 0 \\ 0 & \delta(2,c) \end{bmatrix} \begin{bmatrix} \check{C}_{(1)}(\varsigma_h,\varsigma_v) \\ \check{C}_{(2)}(\varsigma_h+d,\varsigma_v+d) \end{bmatrix} \\ & \zeta_1(\varsigma_h,\varsigma_v) = \zeta_{(1)}(\varsigma_h,\varsigma_v), \; \zeta_2(\varsigma_h,\varsigma_v) = \zeta_{(2)}(\varsigma_h,\varsigma_v) \\ & v_1(\varsigma_h,\varsigma_v) = v_{(1)}(\varsigma_h,\varsigma_v) \\ & v_2(\varsigma_h,\varsigma_v) = \begin{bmatrix} v_{(1)}^{\mathsf{T}}(\varsigma_h,\varsigma_v) \; v_{(2)}^{\mathsf{T}}(\varsigma_h+d,\varsigma_v+d) \end{bmatrix}^{\mathsf{T}} \end{aligned}$$

where $v_1(\varsigma_h,\varsigma_v)$ and $v_2(\varsigma_h,\varsigma_v)$ are zero-mean white noises with covariance matrices $R_1(\varsigma_h,\varsigma_v) = R_{(1)}(\varsigma_h,\varsigma_v)$ and $R_2(\varsigma_h,\varsigma_v) = \mathrm{diag}\{R_{(1)}(\varsigma_h,\varsigma_v), R_{(2)}(\varsigma_h+d,\varsigma_v+d)\}$. $\zeta_1(\varsigma_h,\varsigma_v)$ and $\zeta_2(\varsigma_h,\varsigma_v)$ are the autocorrelated noises satisfying $\hat{\mathrm{E}}\{\zeta_c(\varsigma_h,0)\} = \hat{\mathrm{E}}\{\zeta_c(0,\varsigma_v)\} = 0$ (for $c \in [1,2]$) and the following recursive formula:

$$\begin{aligned} \zeta_c(\varsigma_h,\varsigma_v) = {} & F_c(\varsigma_h,\varsigma_v-1)\zeta_c(\varsigma_h,\varsigma_v-1) \\ & + E_c(\varsigma_h-1,\varsigma_v)\zeta_c(\varsigma_h-1,\varsigma_v) + \epsilon_c(\varsigma_h,\varsigma_v) \end{aligned} \tag{7}$$

with

$$\begin{aligned} & F_1(\varsigma_h,\varsigma_v) = F_{(1)}(\varsigma_h,\varsigma_v), \; F_2(\varsigma_h,\varsigma_v) = F_{(2)}(\varsigma_h,\varsigma_v) \\ & E_1(\varsigma_h,\varsigma_v) = E_{(1)}(\varsigma_h,\varsigma_v), \; E_2(\varsigma_h,\varsigma_v) = E_{(2)}(\varsigma_h,\varsigma_v) \\ & \epsilon_1(\varsigma_h,\varsigma_v) = \epsilon_{(1)}(\varsigma_h,\varsigma_v), \quad \epsilon_2(\varsigma_h,\varsigma_v) = \epsilon_{(2)}(\varsigma_h,\varsigma_v) \end{aligned}$$

where $\epsilon_1(\varsigma_h,\varsigma_v)$ and $\epsilon_2(\varsigma_h,\varsigma_v)$ are null mean white noises with covariance $\sigma_1(\varsigma_h,\varsigma_v) = \sigma_{(1)}(\varsigma_h,\varsigma_v)$ and $\sigma_2(\varsigma_h,\varsigma_v) = \sigma_{(2)}(\varsigma_h,\varsigma_v)$.

The following lemma reveals that the reconstructed observation sequence and the original observation sequence contain the same information.

Lemma 1: The linear space spanned by the reconstructed observation sequence is equivalent to the linear space spanned by the original observation sequence, i.e.,

$$\begin{aligned} & \mathcal{L}\{\{y_2(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in N_1\}; \{y_1(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in N_2\}; \\ & \{y_1(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in N_3\}; \{y_1(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in N_4\}\} \\ & = \mathcal{L}\{\mathscr{Y}(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in M_1 \cup M_2 \cup M_3 \cup M_4\} \end{aligned}$$

Proof: It is obvious from (4)-(6) that for $(\varsigma_h,\varsigma_v) \in M_4$, $\mathscr{Y}(\varsigma_h,\varsigma_v)$ is actually a linear combination of $y_s(\varsigma_h,\varsigma_v)$, $(\varsigma_h,\varsigma_v) \in N_1 \cup N_2 \cup N_3 \cup N_4$, namely,

$$\mathscr{Y}(\varsigma_h,\varsigma_v) = \begin{bmatrix} I_{m_1} \\ \mathbf{0} \end{bmatrix} y_1(\varsigma_h,\varsigma_v) + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_{m_2} \end{bmatrix} y_2(\varsigma_h-d,\varsigma_v-d).$$

On the other hand, $y_k(\varsigma_h,\varsigma_v)$ is also a linear combination of $\mathscr{Y}(\varsigma_h,\varsigma_v)$, i.e.,

$$y_1(\varsigma_h,\varsigma_v) = \begin{bmatrix} I_{m_1} & \mathbf{0} \end{bmatrix} \mathscr{Y}(\varsigma_h,\varsigma_v)$$

$$y_2(\varsigma_h,\varsigma_v) = \begin{bmatrix} I_{m_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathscr{Y}(\varsigma_h,\varsigma_v) + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_{m_2} \end{bmatrix} \mathscr{Y}(\varsigma_h+d,\varsigma_v+d).$$

The case of $(\varsigma_h,\varsigma_v) \in M_1 \cup M_2 \cup M_3$ can be discussed in the same way. The proof is now complete. ■

Remark 4: In this paper, the new observation sequence $\{\{y_2(\varsigma_h,\varsigma_v)|\ (\varsigma_h,\varsigma_v) \in N_1\};\ \{y_1(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in N_2\};\ \{y_1(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in N_3\};\ \{y_1(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in N_4\}\}$ is named as the reconstructed observation sequence of $\{\mathscr{Y}(\varsigma_h,\varsigma_v)|(\varsigma_h,\varsigma_v) \in M_1 \cup M_2 \cup M_3 \cup M_4\}$. It is clear from Lemma 1 that the reconstructed observation sequence contains the same information as the original observation sequence, which will facilitate the subsequent design of recursive estimator.

Motivated by the quadratic sensor problem in [8]–[10], in this paper, we newly introduce the following globally coupled correlation terms:

$$\begin{aligned} x^{\zeta_1}(\varsigma_h,\varsigma_v) & \triangleq \zeta_1(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v) \\ x^{\zeta_2}(\varsigma_h,\varsigma_v) & \triangleq \zeta_2(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v) \end{aligned}$$

which is actually the product of the autocorrelated multiplicative noise and the system state.

Remark 5: As shown in equation (3), the evolution of noise $\zeta_{(c)}(\tau_h,\tau_v)$ depends on its historical states $\zeta_{(c)}(\tau_h-1,\tau_v)$ and $\zeta_{(c)}(\tau_h,\tau_v-1)$. This characteristic destroys the fundamental

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whitening assumption (independence) in classical Kalman filter and introduces strong coupling between system state and multiplicative noise. This coupling effect poses a core challenge, significantly increasing the complexity of system modeling and analysis. To overcome this challenge, this paper proposes a novel global coupling correlation term and designs an iterative coupling estimator, which directly circumvents traditional decoupling strategies and achieves collaborative estimation of the system state and the coupled term between the system state and the multiplicative noise.

In the next subsection, we are going to propose an iterative coupled four-step estimator based on the globally coupled correlation terms and reconstructed observation sequence. Different from the system augmentation approach, the proposed scheme is able to avoid the increase of system dimension and effectively reduce the computational complexity. To begin with, the initial conditions with respect to the globally coupled correlation terms $x^{\zeta_c}(\varsigma_h, \varsigma_v)$ for $c \in [1, 2]$ are given as follows:

$$\begin{aligned}
&\hat{\mathrm{E}}\{x^{\zeta_c}(\varsigma_h, 0)\} = \hat{x}_1^{\zeta_c}(\varsigma_h),\ \hat{\mathrm{E}}\{x^{\zeta_c}(0, \varsigma_v)\} = \hat{x}_2^{\zeta_c}(\varsigma_v) \\
&\mathrm{Cov}\{x^{\zeta_c}(\varsigma_h, 0), x^{\zeta_c}(u, 0)\} = P_1^{\zeta_c}(\varsigma_h)\delta(\varsigma_h, u) \\
&\mathrm{Cov}\{x^{\zeta_c}(0, \varsigma_v), x^{\zeta_c}(0, v)\} = P_2^{\zeta_c}(\varsigma_v)\delta(\varsigma_v, v) \\
&\mathrm{Cov}\{x(\varsigma_h, 0), x^{\zeta_c}(u, 0)\} = M_1^{\zeta_c}(\varsigma_h)\delta(\varsigma_h, u) \\
&\mathrm{Cov}\{x(0, \varsigma_h), x^{\zeta_c}(0, v)\} = M_2^{\zeta_c}(\varsigma_h)\delta(\varsigma_v, v) \\
&\mathrm{Cov}\{x^{\zeta_1}(\varsigma_h, 0), x^{\zeta_2}(u, 0)\} = N_1^{\zeta_{12}}(\varsigma_h)\delta(\varsigma_h, u) \\
&\mathrm{Cov}\{x^{\zeta_1}(0, \varsigma_v), x^{\zeta_2}(0, v)\} = N_2^{\zeta_{12}}(\varsigma_v)\delta(\varsigma_v, v)
\end{aligned}$$

where $\hat{x}_1^{\zeta_c}(\varsigma_h)$, $\hat{x}_2^{\zeta_c}(\varsigma_v)$, $P_1^{\zeta_c}(\varsigma_h)$, $P_2^{\zeta_c}(\varsigma_v)$, $M_1^{\zeta_c}(\varsigma_h)$, $M_2^{\zeta_c}(\varsigma_v)$, $N_1^{\zeta_{12}}(\varsigma_h)$, and $N_2^{\zeta_{12}}(\varsigma_v)$ are known parameters with appropriate dimensions.

B. An iterative coupled four-step estimator

According to the divided areas of reconstructed observation, an iterative coupled four-step estimator is constructed with the following form.

Step 1: For $(\varsigma_h, \varsigma_v) \in N_1$,

$$\begin{aligned}
\hat{x}_p^1(\varsigma_h, \varsigma_v) = {}& A_1(\varsigma_h, \varsigma_v - 1)\hat{x}_u^1(\varsigma_h, \varsigma_v - 1) \\
&+ A_2(\varsigma_h - 1, \varsigma_v)\hat{x}_u^1(\varsigma_h - 1, \varsigma_v) \qquad (8a)
\end{aligned}$$

$$\begin{aligned}
\hat{x}_u^1(\varsigma_h, \varsigma_v) = {}& \hat{x}_p^1(\varsigma_h, \varsigma_v) + K_1(\varsigma_h, \varsigma_v)[y_2(\varsigma_h, \varsigma_v) - C_2(\varsigma_h, \varsigma_v) \\
&\times \hat{x}_p^1(\varsigma_h, \varsigma_v) - \check{C}_{21}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_1,1}(\varsigma_h, \varsigma_v) \\
&- \check{C}_{22}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)] \qquad (8b)
\end{aligned}$$

where $\hat{x}_p^1(\varsigma_h, \varsigma_v)$ is the one-step prediction of the state $x(\varsigma_h, \varsigma_v)$, and $\hat{x}_u^1(\varsigma_h, \varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^1(\varsigma_h, 0) = \hat{x}_1(\varsigma_h)$ and $\hat{x}_u^1(0, \varsigma_v) = \hat{x}_2(\varsigma_v)$. $\hat{x}_p^{\zeta_1,1}(\varsigma_h, \varsigma_v)$ is the one-step prediction of the globally coupled correlation term $x^{\zeta_1}(\varsigma_h, \varsigma_v)$, which is recursively calculated by

$$\begin{aligned}
\hat{x}_p^{\zeta_1,1}(\varsigma_h, \varsigma_v) = {}& F_1(\varsigma_h, \varsigma_v - 1)A_1(\varsigma_h, \varsigma_v - 1)\hat{x}_u^{\zeta_1,1}(\varsigma_h, \varsigma_v - 1) \\
&+ E_1(\varsigma_h - 1, \varsigma_v)A_2(\varsigma_h - 1, \varsigma_v)\hat{x}_u^{\zeta_1,1}(\varsigma_h - 1, \varsigma_v) \qquad (9a)
\end{aligned}$$

$$\begin{aligned}
\hat{x}_u^{\zeta_1,1}(\varsigma_h, \varsigma_v) = {}& \hat{x}_p^{\zeta_1,1}(\varsigma_h, \varsigma_v) + K_1^{\zeta_1}(\varsigma_h, \varsigma_v)[y_2(\varsigma_h, \varsigma_v) \\
&- C_2(\varsigma_h, \varsigma_v)\hat{x}_p^1(\varsigma_h, \varsigma_v) - \check{C}_{21}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_1,1}(\varsigma_h, \varsigma_v) \\
&- \check{C}_{22}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)] \qquad (9b)
\end{aligned}$$

where $\hat{x}_u^{\zeta_1,1}(\varsigma_h, \varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^{\zeta_1,1}(\varsigma_h, 0) = \hat{x}_1^{\zeta_1}(\varsigma_h)$ and $\hat{x}_u^{\zeta_1,1}(0, \varsigma_v) = \hat{x}_2^{\zeta_1}(\varsigma_v)$. Similarly, $\hat{x}_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)$ is the one-step prediction of the globally coupled correlation term $x^{\zeta_2}(\varsigma_h, \varsigma_v)$, which can be calculated by

$$\begin{aligned}
\hat{x}_p^{\zeta_2,1}(\varsigma_h, \varsigma_v) = {}& F_2(\varsigma_h, \varsigma_v - 1)A_1(\varsigma_h, \varsigma_v - 1)\hat{x}_u^{\zeta_2,1}(\varsigma_h, \varsigma_v - 1) \\
&+ E_2(\varsigma_h - 1, \varsigma_v)A_2(\varsigma_h - 1, \varsigma_v)\hat{x}_u^{\zeta_2,1}(\varsigma_h - 1, \varsigma_v) \qquad (10a)
\end{aligned}$$

$$\begin{aligned}
\hat{x}_u^{\zeta_2,1}(\varsigma_h, \varsigma_v) = {}& \hat{x}_p^{\zeta_2,1}(\varsigma_h, \varsigma_v) + K_1^{\zeta_2}(\varsigma_h, \varsigma_v)[y_2(\varsigma_h, \varsigma_v) \\
&- C_2(\varsigma_h, \varsigma_v)\hat{x}_p^1(\varsigma_h, \varsigma_v) - \check{C}_{21}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_1,1}(\varsigma_h, \varsigma_v) \\
&- \check{C}_{22}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)] \qquad (10b)
\end{aligned}$$

where $\hat{x}_u^{\zeta_2,1}(\varsigma_h, \varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^{\zeta_2,1}(\varsigma_h, 0) = \hat{x}_1^{\zeta_2}(\varsigma_h)$ and $\hat{x}_u^{\zeta_2,1}(0, \varsigma_v) = \hat{x}_2^{\zeta_2}(\varsigma_v)$.

Step 2: For $(\varsigma_h, \varsigma_v) \in N_2$,

$$\begin{aligned}
\hat{x}_p^2(\varsigma_h, \varsigma_v) = {}& A_1(\varsigma_h, \varsigma_v - 1)\hat{x}_u^2(\varsigma_h, \varsigma_v - 1) \\
&+ A_2(\varsigma_h - 1, \varsigma_v)\hat{x}_u^2(\varsigma_h - 1, \varsigma_v) \qquad (11a)
\end{aligned}$$

$$\begin{aligned}
\hat{x}_u^2(\varsigma_h, \varsigma_v) = {}& \hat{x}_p^2(\varsigma_h, \varsigma_v) + K_2(\varsigma_h, \varsigma_v)[y_1(\varsigma_h, \varsigma_v) - C_1(\varsigma_h, \varsigma_v) \\
&\times \hat{x}_p^2(\varsigma_h, \varsigma_v) - \check{C}_{11}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_1,2}(\varsigma_h, \varsigma_v)] \qquad (11b)
\end{aligned}$$

where $\hat{x}_p^2(\varsigma_h, \varsigma_v)$ is the one-step prediction of the state $x(\varsigma_h, \varsigma_v)$, and $\hat{x}_u^2(\varsigma_h, \varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^2(\varsigma_h, \tau_v^1) = \hat{x}_u^1(\varsigma_h, \tau_v^1)$ and $\hat{x}_u^2(0, \varsigma_v) = \hat{x}_2(\varsigma_v)$. $\hat{x}_p^{\zeta_1,2}(\varsigma_h, \varsigma_v)$ is the one-step prediction of the globally coupled correlation term $x^{\zeta_1}(\varsigma_h, \varsigma_v)$, which can be calculated by

$$\begin{aligned}
\hat{x}_p^{\zeta_1,2}(\varsigma_h, \varsigma_v) = {}& F_1(\varsigma_h, \varsigma_v - 1)A_1(\varsigma_h, \varsigma_v - 1)\hat{x}_u^{\zeta_1,2}(\varsigma_h, \varsigma_v - 1) \\
&+ E_1(\varsigma_h - 1, \varsigma_v)A_2(\varsigma_h - 1, \varsigma_v)\hat{x}_u^{\zeta_1,2}(\varsigma_h - 1, \varsigma_v) \qquad (12a)
\end{aligned}$$

$$\begin{aligned}
\hat{x}_u^{\zeta_1,2}(\varsigma_h, \varsigma_v) = {}& \hat{x}_p^{\zeta_1,2}(\varsigma_h, \varsigma_v) + K_2^{\zeta_1}(\varsigma_h, \varsigma_v)[y_1(\varsigma_h, \varsigma_v) \\
&- C_1(\varsigma_h, \varsigma_v)\hat{x}_p^2(\varsigma_h, \varsigma_v) - \check{C}_{11}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_1,2}(\varsigma_h, \varsigma_v)] \qquad (12b)
\end{aligned}$$

where $\hat{x}_u^{\zeta_1,2}(\varsigma_h, \varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^{\zeta_1,2}(\varsigma_h, \tau_v^1) = \hat{x}_u^{\zeta_2,1}(\varsigma_h, \tau_v^1)$ and $\hat{x}_u^{\zeta_1,2}(0, \varsigma_v) = \hat{x}_2^{\zeta_1}(\varsigma_v)$.

Step 3: For $(\varsigma_h, \varsigma_v) \in N_3$,

$$\begin{aligned}
\hat{x}_p^3(\varsigma_h, \varsigma_v) = {}& A_1(\varsigma_h, \varsigma_v - 1)\hat{x}_u^3(\varsigma_h, \varsigma_v - 1) \\
&+ A_2(\varsigma_h - 1, \varsigma_v)\hat{x}_u^3(\varsigma_h - 1, \varsigma_v) \qquad (13a)
\end{aligned}$$

$$\begin{aligned}
\hat{x}_u^3(\varsigma_h, \varsigma_v) = {}& \hat{x}_p^3(\varsigma_h, \varsigma_v) + K_3(\varsigma_h, \varsigma_v)[y_1(\varsigma_h, \varsigma_v) - C_1(\varsigma_h, \varsigma_v) \\
&\times \hat{x}_p^3(\varsigma_h, \varsigma_v) - \check{C}_{11}(\varsigma_h, \varsigma_v)\hat{x}_p^{\zeta_1,3}(\varsigma_h, \varsigma_v)] \qquad (13b)
\end{aligned}$$

where $\hat{x}_p^3(\varsigma_h, \varsigma_v)$ is the one-step prediction of the state $x(\varsigma_h, \varsigma_v)$, and $\hat{x}_u^3(\varsigma_h, \varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^3(\varsigma_h, 0) = \hat{x}_1(\varsigma_h)$ and $\hat{x}_u^3(\tau_h^1, \varsigma_v) = \hat{x}_u^1(\tau_h^1, \varsigma_v)$. $\hat{x}_p^{\zeta_1,3}(\varsigma_h, \varsigma_v)$ is the one-step prediction of the globally coupled correlation term $x^{\zeta_1}(\varsigma_h, \varsigma_v)$ satisfying the following recursion:

$$\hat{x}_p^{\zeta_1,3}(\varsigma_h, \varsigma_v) = F_1(\varsigma_h, \varsigma_v - 1)A_1(\varsigma_h, \varsigma_v - 1)\hat{x}_u^{\zeta_1,3}(\varsigma_h, \varsigma_v - 1)$$

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$$+ E_1(\varsigma_h-1,\varsigma_v)A_2(\varsigma_h-1,\varsigma_v)\hat{x}_u^{\zeta_1,3}(\varsigma_h-1,\varsigma_v) \tag{14a}$$

$$\hat{x}_u^{\zeta_1,3}(\varsigma_h,\varsigma_v) = \hat{x}_p^{\zeta_1,3}(\varsigma_h,\varsigma_v) + K_3^{\zeta_1}(\varsigma_h,\varsigma_v)[y_1(\varsigma_h,\varsigma_v) - C_1(\varsigma_h,\varsigma_v)\hat{x}_p^2(\varsigma_h,\varsigma_v) - \check{C}_{11}(\varsigma_h,\varsigma_v)\hat{x}_p^{\zeta_1,3}(\varsigma_h,\varsigma_v)] \tag{14b}$$

where $\hat{x}_u^{\zeta_1,3}(\varsigma_h,\varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^{\zeta_1,3}(\varsigma_h,0) = \hat{x}_1^{\zeta_1}(\varsigma_h)$ and $\hat{x}_u^{\zeta_1,3}(\tau_h^1,\varsigma_v) = \hat{x}_u^{\zeta_2,1}(\tau_h^1,\varsigma_v)$.

Step 4: For $(\varsigma_h,\varsigma_v) \in N_4$,

$$\hat{x}_p^4(\varsigma_h,\varsigma_v) = A_1(\varsigma_h,\varsigma_v-1)\hat{x}_u^4(\varsigma_h,\varsigma_v-1) + A_2(\varsigma_h-1,\varsigma_v)\hat{x}_u^4(\varsigma_h-1,\varsigma_v) \tag{15a}$$

$$\hat{x}_u^4(\varsigma_h,\varsigma_v) = \hat{x}_p^4(\varsigma_h,\varsigma_v) + K_4(\varsigma_h,\varsigma_v)[y_1(\varsigma_h,\varsigma_v) - C_1(\varsigma_h,\varsigma_v) \times \hat{x}_p^4(\varsigma_h,\varsigma_v) - \check{C}_{11}(\varsigma_h,\varsigma_v)\hat{x}_p^{\zeta_1,4}(\varsigma_h,\varsigma_v)] \tag{15b}$$

where $\hat{x}_p^4(\varsigma_h,\varsigma_v)$ is the one-step prediction of the state $x(\varsigma_h,\varsigma_v)$, and $\hat{x}_u^4(\varsigma_h,\varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^4(\tau_h^1,\varsigma_v) = \hat{x}_u^2(\tau_h^1,\varsigma_v)$ and $\hat{x}_u^4(\varsigma_h,\tau_v^1) = \hat{x}_u^3(\varsigma_h,\tau_v^1)$. $\hat{x}_p^{\zeta_1,4}(\varsigma_h,\varsigma_v)$ is the one-step prediction of the globally coupled correlation term $x^{\zeta_1}(\varsigma_h,\varsigma_v)$, which can be calculated as follows:

$$\hat{x}_p^{\zeta_1,4}(\varsigma_h,\varsigma_v) = F_1(\varsigma_h,\varsigma_v-1)A_1(\varsigma_h,\varsigma_v-1)\hat{x}_u^{\zeta_1,4}(\varsigma_h,\varsigma_v-1) + E_1(\varsigma_h-1,\varsigma_v)A_2(\varsigma_h-1,\varsigma_v)\hat{x}_u^{\zeta_1,4}(\varsigma_h-1,\varsigma_v) \tag{16a}$$

$$\hat{x}_u^{\zeta_1,4}(\varsigma_h,\varsigma_v) = \hat{x}_p^{\zeta_1,4}(\varsigma_h,\varsigma_v) + K_4^{\zeta_1}(\varsigma_h,\varsigma_v)[y_1(\varsigma_h,\varsigma_v) - C_1(\varsigma_h,\varsigma_v)\hat{x}_p^4(\varsigma_h,\varsigma_v) - \check{C}_{11}(\varsigma_h,\varsigma_v)\hat{x}_p^{\zeta_1,4}(\varsigma_h,\varsigma_v)] \tag{16b}$$

where $\hat{x}_u^{\zeta_1,4}(\varsigma_h,\varsigma_v)$ is the corresponding estimate with initial conditions $\hat{x}_u^{\zeta_1,4}(\tau_h^1,\varsigma_v) = \hat{x}_u^{\zeta_1,2}(\tau_h^1,\varsigma_v)$ and $\hat{x}_u^{\zeta_1,4}(\varsigma_h,\tau_v^1) = \hat{x}_u^{\zeta_1,3}(\varsigma_h,\tau_v^1)$.

It should be pointed out that in (8)-(16), $K_r(\varsigma_h,\varsigma_v), K_1^{\zeta_2}(\varsigma_h,\varsigma_h)$, and $K_r^{\zeta_1}(\varsigma_h,\varsigma_v)$, $r \in [1,4]$ denote the estimator gains to be designed. In what follows, we are going to show the unbiasedness of the proposed estimator.

Denote the one-step prediction errors as

$$\tilde{x}_p^r(\varsigma_h,\varsigma_v) \triangleq x(\varsigma_h,\varsigma_v) - \hat{x}_p^r(\varsigma_h,\varsigma_v)$$
$$\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v) \triangleq x^{\zeta_2}(\varsigma_h,\varsigma_v) - \hat{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)$$
$$\tilde{x}_p^{\zeta_1,r}(\varsigma_h,\varsigma_v) \triangleq x^{\zeta_1}(\varsigma_h,\varsigma_v) - \hat{x}_p^{\zeta_1,r}(\varsigma_h,\varsigma_v)$$

and the estimation errors as

$$\tilde{x}_u^r(\varsigma_h,\varsigma_v) \triangleq x(\varsigma_h,\varsigma_v) - \hat{x}_u^r(\varsigma_h,\varsigma_v)$$
$$\tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v) \triangleq x^{\zeta_2}(\varsigma_h,\varsigma_v) - \hat{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v)$$
$$\tilde{x}_u^{\zeta_1,r}(\varsigma_h,\varsigma_v) \triangleq x^{\zeta_1}(\varsigma_h,\varsigma_v) - \hat{x}_u^{\zeta_1,r}(\varsigma_h,\varsigma_v)$$

where $r \in [1,4]$. Then, it follows from (1)-(3) and (8)-(16) that:

(1) For $(\varsigma_h,\varsigma_v) \in N_1$,

$$\begin{aligned}&\tilde{x}_p^1(\varsigma_h,\varsigma_v)\\&= A_1(\varsigma_h,\varsigma_v-1)\tilde{x}_u^1(\varsigma_h,\varsigma_v-1) + A_2(\varsigma_h-1,\varsigma_v)\\&\times \tilde{x}_u^1(\varsigma_h-1,\varsigma_v) + B_1(\varsigma_h,\varsigma_v-1)w(\varsigma_h,\varsigma_v-1)\\&+ B_2(\varsigma_h-1,\varsigma_v)w(\varsigma_h-1,\varsigma_v)\end{aligned} \tag{17a}$$

$$\begin{aligned}&\tilde{x}_u^1(\varsigma_h,\varsigma_v)\\&= [I - K_1(\varsigma_h,\varsigma_v)C_2(\varsigma_h,\varsigma_v)]\,\tilde{x}_p^1(\varsigma_h,\varsigma_v)\\&- K_1(\varsigma_h,\varsigma_v)\check{C}_{21}(\varsigma_h,\varsigma_v)\tilde{x}_p^{\zeta_1,1}(\varsigma_h,\varsigma_v) - K_1(\varsigma_h,\varsigma_v)\\&\times \check{C}_{22}(\varsigma_h,\varsigma_v)\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v) - K_1(\varsigma_h,\varsigma_v)v_2(\varsigma_h,\varsigma_v)\end{aligned} \tag{17b}$$

$$\begin{aligned}&\tilde{x}_p^{\zeta_1,1}(\varsigma_h,\varsigma_v)\\&= F_1(\varsigma_h,\varsigma_v-1)A_1(\varsigma_h,\varsigma_v-1)\tilde{x}_u^{\zeta_1,1}(\varsigma_h,\varsigma_v-1)\\&+ E_1(\varsigma_h-1,\varsigma_v)A_2(\varsigma_h-1,\varsigma_v)\tilde{x}_u^{\zeta_1,1}(\varsigma_h-1,\varsigma_v)\\&+ F_1(\varsigma_h,\varsigma_v-1)A_2(\varsigma_h,\varsigma_v-1)\zeta_1(\varsigma_h,\varsigma_v-1)x(\varsigma_h-1,\varsigma_v)\\&+ F_1(\varsigma_h,\varsigma_v-1)B_1(\varsigma_h,\varsigma_v-1)\zeta_1(\varsigma_h,\varsigma_v-1)w(\varsigma_h,\varsigma_v-1)\\&+ F_1(\varsigma_h,\varsigma_v-1)B_2(\varsigma_h-1,\varsigma_v)\zeta_1(\varsigma_h,\varsigma_v-1)w(\varsigma_h-1,\varsigma_v)\\&+ E_1(\varsigma_h-1,\varsigma_v)A_1(\varsigma_h,\varsigma_v-1)\zeta_1(\varsigma_h-1,\varsigma_v)x(\varsigma_h,\varsigma_v-1)\\&+ E_1(\varsigma_h-1,\varsigma_v)B_1(\varsigma_h,\varsigma_v-1)\zeta_1(\varsigma_h-1,\varsigma_v)w(\varsigma_h,\varsigma_v-1)\\&+ E_1(\varsigma_h-1,\varsigma_v)B_2(\varsigma_h-1,\varsigma_v)\zeta_1(\varsigma_h-1,\varsigma_v)w(\varsigma_h-1,\varsigma_v)\\&+ A_1(\varsigma_h,\varsigma_v-1)\epsilon_1(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v-1) + A_2(\varsigma_h-1,\varsigma_v)\\&\times \epsilon_1(\varsigma_h,\varsigma_v)x(\varsigma_h-1,\varsigma_v) + B_1(\varsigma_h,\varsigma_v-1)\epsilon_1(\varsigma_h,\varsigma_v)\\&\times w(\varsigma_h,\varsigma_v-1) + B_2(\varsigma_h-1,\varsigma_v)\epsilon_1(\varsigma_h,\varsigma_v)w(\varsigma_h-1,\varsigma_v)\end{aligned} \tag{17c}$$

$$\begin{aligned}&\tilde{x}_u^{\zeta_1,1}(\varsigma_h,\varsigma_v)\\&= \left[I - K_1^{\zeta_1}(\varsigma_h,\varsigma_v)\check{C}_{21}(\varsigma_h,\varsigma_v)\right]\tilde{x}_p^{\zeta_1,1}(\varsigma_h,\varsigma_v)\\&- K_1^{\zeta_1}(\varsigma_h,\varsigma_v)C_2(\varsigma_h,\varsigma_v)\tilde{x}_p^1(\varsigma_h,\varsigma_v) - K_1^{\zeta_1}(\varsigma_h,\varsigma_v)\\&\times \check{C}_{22}(\varsigma_h,\varsigma_v)\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v) - K_1^{\zeta_1}(\varsigma_h,\varsigma_v)v_2(\varsigma_h,\varsigma_v)\end{aligned} \tag{17d}$$

$$\begin{aligned}&\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)\\&= F_2(\varsigma_h,\varsigma_v-1)A_1(\varsigma_h,\varsigma_v-1)\tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v-1)\\&+ E_2(\varsigma_h-1,\varsigma_v)A_2(\varsigma_h-1,\varsigma_v)\tilde{x}_u^{\zeta_2,1}(\varsigma_h-1,\varsigma_v)\\&+ F_2(\varsigma_h,\varsigma_v-1)A_2(\varsigma_h,\varsigma_v-1)\zeta_2(\varsigma_h,\varsigma_v-1)x(\varsigma_h-1,\varsigma_v)\\&+ F_2(\varsigma_h,\varsigma_v-1)B_1(\varsigma_h,\varsigma_v-1)\zeta_2(\varsigma_h,\varsigma_v-1)w(\varsigma_h,\varsigma_v-1)\\&+ F_2(\varsigma_h,\varsigma_v-1)B_2(\varsigma_h-1,\varsigma_v)\zeta_2(\varsigma_h,\varsigma_v-1)w(\varsigma_h-1,\varsigma_v)\\&+ E_2(\varsigma_h-1,\varsigma_v)A_1(\varsigma_h,\varsigma_v-1)\zeta_2(\varsigma_h-1,\varsigma_v)x(\varsigma_h,\varsigma_v-1)\\&+ E_2(\varsigma_h-1,\varsigma_v)B_1(\varsigma_h,\varsigma_v-1)\zeta_2(\varsigma_h-1,\varsigma_v)w(\varsigma_h,\varsigma_v-1)\\&+ E_2(\varsigma_h-1,\varsigma_v)B_2(\varsigma_h-1,\varsigma_v)\zeta_2(\varsigma_h-1,\varsigma_v)w(\varsigma_h-1,\varsigma_v)\\&+ A_1(\varsigma_h,\varsigma_v-1)\epsilon_2(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v-1) + A_2(\varsigma_h-1,\varsigma_v)\\&\times \epsilon_2(\varsigma_h,\varsigma_v)x(\varsigma_h-1,\varsigma_v) + B_1(\varsigma_h,\varsigma_v-1)\epsilon_2(\varsigma_h,\varsigma_v)\\&\times w(\varsigma_h,\varsigma_v-1) + B_2(\varsigma_h-1,\varsigma_v)\epsilon_2(\varsigma_h,\varsigma_v)w(\varsigma_h-1,\varsigma_v)\end{aligned} \tag{17e}$$

$$\begin{aligned}&\tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v)\\&= \left[I - K_1^{\zeta_2}(\varsigma_h,\varsigma_v)\check{C}_{22}(\varsigma_h,\varsigma_v)\right]\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)\\&- K_1^{\zeta_2}(\varsigma_h,\varsigma_v)C_2(\varsigma_h,\varsigma_v)\tilde{x}_p^1(\varsigma_h,\varsigma_v) - K_1^{\zeta_2}(\varsigma_h,\varsigma_v)\\&\times \check{C}_{21}(\varsigma_h,\varsigma_v)\tilde{x}_p^{\zeta_1,1}(\varsigma_h,\varsigma_v) - K_1^{\zeta_2}(\varsigma_h,\varsigma_v)v_2(\varsigma_h,\varsigma_v)\end{aligned} \tag{17f}$$

(2) For $(\varsigma_h,\varsigma_v) \in N_s$,

$$\begin{aligned}&\tilde{x}_p^s(\varsigma_h,\varsigma_v)\\&= A_1(\varsigma_h,\varsigma_v-1)\tilde{x}_u^s(\varsigma_h,\varsigma_v-1) + A_2(\varsigma_h-1,\varsigma_v)\\&\times \tilde{x}_u^s(\varsigma_h-1,\varsigma_v) + B_1(\varsigma_h,\varsigma_v-1)w(\varsigma_h,\varsigma_v-1)\\&+ B_2(\varsigma_h-1,\varsigma_v)w(\varsigma_h-1,\varsigma_v)\end{aligned} \tag{18a}$$

$$\begin{aligned}&\tilde{x}_u^s(\varsigma_h,\varsigma_v)\\&= \left[I - K_s(\varsigma_h,\varsigma_v)C_1(\varsigma_h,\varsigma_v)\right]\tilde{x}_p^s(\varsigma_h,\varsigma_v) - K_s(\varsigma_h,\varsigma_v)\\&\times \check{C}_{11}(\varsigma_h,\varsigma_v)\tilde{x}_p^{\zeta_1,s}(\varsigma_h,\varsigma_v) - K_s(\varsigma_h,\varsigma_v)v_1(\varsigma_h,\varsigma_v)\end{aligned} \tag{18b}$$

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$$\begin{aligned}&\tilde{x}_p^{\zeta_1,s}(\varsigma_h,\varsigma_v)\\&=F_1(\varsigma_h,\varsigma_v-1)A_1(\varsigma_h,\varsigma_v-1)\tilde{x}_u^{\zeta_1,s}(\varsigma_h,\varsigma_v-1)\\&+E_1(\varsigma_h-1,\varsigma_v)A_2(\varsigma_h-1,\varsigma_v)\tilde{x}_u^{\zeta_1,s}(\varsigma_h-1,\varsigma_v)\\&+F_1(\varsigma_h,\varsigma_v-1)A_2(\varsigma_h,\varsigma_v-1)\zeta_1(\varsigma_h,\varsigma_v-1)x(\varsigma_h-1,\varsigma_v)\\&+F_1(\varsigma_h,\varsigma_v-1)B_1(\varsigma_h,\varsigma_v-1)\zeta_1(\varsigma_h,\varsigma_v-1)w(\varsigma_h,\varsigma_v-1)\\&+F_1(\varsigma_h,\varsigma_v-1)B_2(\varsigma_h-1,\varsigma_v)\zeta_1(\varsigma_h,\varsigma_v-1)w(\varsigma_h-1,\varsigma_v)\\&+E_1(\varsigma_h-1,\varsigma_v)A_1(\varsigma_h,\varsigma_v-1)\zeta_1(\varsigma_h-1,\varsigma_v)x(\varsigma_h,\varsigma_v-1)\\&+E_1(\varsigma_h-1,\varsigma_v)B_1(\varsigma_h,\varsigma_v-1)\zeta_1(\varsigma_h-1,\varsigma_v)w(\varsigma_h,\varsigma_v-1)\\&+E_1(\varsigma_h-1,\varsigma_v)B_2(\varsigma_h-1,\varsigma_v)\zeta_1(\varsigma_h-1,\varsigma_v)w(\varsigma_h-1,\varsigma_v)\\&+A_1(\varsigma_h,\varsigma_v-1)\epsilon_1(\varsigma_h,\varsigma_v)x(\varsigma_h,\varsigma_v-1)+A_2(\varsigma_h-1,\varsigma_v)\\&\times\epsilon_1(\varsigma_h,\varsigma_v)x(\varsigma_h-1,\varsigma_v)+B_1(\varsigma_h,\varsigma_v-1)\epsilon_1(\varsigma_h,\varsigma_v)\\&\times w(\varsigma_h,\varsigma_v-1)+B_2(\varsigma_h-1,\varsigma_v)\epsilon_1(\varsigma_h,\varsigma_v)w(\varsigma_h-1,\varsigma_v)\end{aligned}\tag{18c}$$

$$\begin{aligned}&\tilde{x}_u^{\zeta_1,s}(\varsigma_h,\varsigma_v)\\&=\left[I-K_s^{\zeta_1}(\varsigma_h,\varsigma_v)\check{C}_{11}(\varsigma_h,\varsigma_v)\right]\tilde{x}_p^{\zeta_1,s}(\varsigma_h,\varsigma_v)-K_s^{\zeta_1}(\varsigma_h,\varsigma_v)\\&\times C_1(\varsigma_h,\varsigma_v)\tilde{x}_p^{s}(\varsigma_h,\varsigma_v)-K_s^{\zeta_1}(\varsigma_h,\varsigma_v)v_1(\varsigma_h,\varsigma_v)\end{aligned}\tag{18d}$$

for $s\in[2,4]$.

The unbiasedness of the proposed estimator is revealed in the following lemma.

Lemma 2: Considering the 2-D systems (1)-(3), the proposed estimator of the form (8)-(16) is unbiased. That is to say, the errors (17) and (18) have zero means.

Proof: The proof resembles that in [24], the detailed proof is omitted here for brevity. ■

C. Calculation of correlated second-order moments

In this subsection, we are going to derive recursive expressions for some correlated second-order moments. To begin with, let us define

$$X(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{x(\varsigma_h,\varsigma_v)x^{\mathsf{T}}(\varsigma_h,\varsigma_v)\}\tag{19}$$
$$\tau_c(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\zeta_c(\varsigma_h,\varsigma_v)\zeta_c^{\mathsf{T}}(\varsigma_h,\varsigma_v)\}\tag{20}$$
$$P_p^r(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_p^r(\varsigma_h,\varsigma_v)(\tilde{x}_p^r(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{21}$$
$$P_u^r(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_u^r(\varsigma_h,\varsigma_v)(\tilde{x}_u^r(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{22}$$
$$P_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)(\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{23}$$
$$P_p^{\zeta_1,r}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_p^{\zeta_1,r}(\varsigma_h,\varsigma_v)(\tilde{x}_p^{\zeta_1,r}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{24}$$
$$P_u^{\zeta_2,1}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v)(\tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{25}$$
$$P_u^{\zeta_1,r}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_u^{\zeta_1,r}(\varsigma_h,\varsigma_v)(\tilde{x}_u^{\zeta_1,r}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{26}$$
$$M_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_p^{1}(\varsigma_h,\varsigma_v)(\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{27}$$
$$M_p^{\zeta_1,r}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_p^{r}(\varsigma_h,\varsigma_v)(\tilde{x}_p^{\zeta_1,r}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{28}$$
$$M_u^{\zeta_2,1}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_u^{1}(\varsigma_h,\varsigma_v)(\tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{29}$$
$$M_u^{\zeta_1,r}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_u^{r}(\varsigma_h,\varsigma_v)(\tilde{x}_u^{\zeta_1,r}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{30}$$
$$N_p^{\zeta_{12},1}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_p^{\zeta_1,1}(\varsigma_h,\varsigma_v)(\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{31}$$
$$N_u^{\zeta_{12},1}(\varsigma_h,\varsigma_v)\triangleq\hat{\mathrm{E}}\{\tilde{x}_u^{\zeta_1,1}(\varsigma_h,\varsigma_v)(\tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\}\tag{32}$$

where $c\in[1,2]$ and $r\in[1,4]$.

For brevity, the notations $(\cdot,\cdot)$ in the coefficient matrices will be omitted in the subsequent calculations involving $X(\varsigma_h,\varsigma_v)$, $\tau_c(\varsigma_h,\varsigma_v)$, $P_p^r(\varsigma_h,\varsigma_v)$, $P_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)$, $P_p^{\zeta_1,r}(\varsigma_h,\varsigma_v)$, $M_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)$, $M_p^{\zeta_1,r}(\varsigma_h,\varsigma_v)$, and $N_p^{\zeta_{12},1}(\varsigma_h,\varsigma_v)$. Specifically,

$$\begin{aligned}&A_1=A_1(\varsigma_h,\varsigma_v-1),\ A_2=A_2(\varsigma_h-1,\varsigma_v)\\&B_1=B_1(\varsigma_h,\varsigma_v-1),\ B_2=B_2(\varsigma_h-1,\varsigma_v)\\&F_1=F_1(\varsigma_h,\varsigma_v-1),\ E_1=E_1(\varsigma_h-1,\varsigma_v)\\&F_2=F_2(\varsigma_h,\varsigma_v-1),\ E_2=E_2(\varsigma_h-1,\varsigma_v).\end{aligned}$$

Notably, the second-order moments of the autocorrelated multiplicative noises and the system state play a key role in the derivation of estimation error covariance matrix. As such, $X(\varsigma_h,\varsigma_v)$ and $\tau_c(\varsigma_h,\varsigma_v)$ are first summarized in the following lemma.

Lemma 3: Considering the 2-D systems (1)-(3) as well as the initial conditions $X(\varsigma_h,0)=P_1(\varsigma_h)$, $X(0,\varsigma_v)=P_2(\varsigma_v)$, $\tau_c(\varsigma_h,0)=\tau_{(c),1}(\varsigma_h)$, and $\tau_c(0,\varsigma_v)=\tau_{(c),2}(\varsigma_v)$, the recursive evolutions of the second-order moments of $x(\varsigma_h,\varsigma_v)$ and $\zeta_c(\varsigma_h,\varsigma_v)$ are given by

$$\begin{aligned}X(\varsigma_h,\varsigma_v)=&A_1X(\varsigma_h,\varsigma_v-1)A_1^{\mathsf{T}}+A_2X(\varsigma_h-1,\varsigma_v)A_2^{\mathsf{T}}\\&+B_1Q(\varsigma_h,\varsigma_v-1)B_1^{\mathsf{T}}+B_2Q(\varsigma_h-1,\varsigma_v)B_2^{\mathsf{T}}\\&+A_1\hat{\mathrm{E}}\{x(\varsigma_h,\varsigma_v-1)x^{\mathsf{T}}(\varsigma_h-1,\varsigma_v)\}A_2^{\mathsf{T}}\\&+A_2\hat{\mathrm{E}}\{x(\varsigma_h-1,\varsigma_v)x^{\mathsf{T}}(\varsigma_h,\varsigma_v-1)\}A_1^{\mathsf{T}}\end{aligned}\tag{33}$$

and

$$\begin{aligned}\tau_c(\varsigma_h,\varsigma_v)=&F_c\tau_c(\varsigma_h,\varsigma_v-1)F_c^{\mathsf{T}}+E_c\tau_c(\varsigma_h-1,\varsigma_v)E_c^{\mathsf{T}}\\&+F_c\hat{\mathrm{E}}\{\zeta_c(\varsigma_h,\varsigma_v-1)\zeta_c^{\mathsf{T}}(\varsigma_h-1,\varsigma_v)\}E_c^{\mathsf{T}}\\&+E_c\hat{\mathrm{E}}\{\zeta_c(\varsigma_h-1,\varsigma_v)\zeta_c^{\mathsf{T}}(\varsigma_h,\varsigma_v-1)\}F_c^{\mathsf{T}}\\&+\sigma_c(\varsigma_h,\varsigma_v).\end{aligned}\tag{34}$$

Proof: This lemma can be immediately proven by noting (1), (7) as well as the statistical properties of random variables $w(\varsigma_h,\varsigma_v)$ and $\epsilon_c(\varsigma_h,\varsigma_v)$. ■

Lemma 4: Considering the 2-D systems (1)-(3) and the proposed estimator of the form (8)-(16), the recursive evolution of the prediction error covariance $P_p^r(\varsigma_h,\varsigma_v)$ is given as follows:

Step 1: For $(\varsigma_h,\varsigma_v)\in N_1$, under the initial conditions $P_p^1(\varsigma_h,0)=P_1(\varsigma_h)$ and $P_p^1(0,\varsigma_v)=P_2(\varsigma_v)$, one has

$$\begin{aligned}P_p^1(\varsigma_h,\varsigma_v)=&A_1P_u^1(\varsigma_h,\varsigma_v-1)A_1^{\mathsf{T}}+A_2P_u^1(\varsigma_h-1,\varsigma_v)A_2^{\mathsf{T}}\\&+B_1Q(\varsigma_h,\varsigma_v-1)B_1^{\mathsf{T}}+B_2Q(\varsigma_h-1,\varsigma_v)B_2^{\mathsf{T}}\\&+A_1\hat{\mathrm{E}}\{\tilde{x}_u^1(\varsigma_h,\varsigma_v-1)(\tilde{x}_u^1(\varsigma_h-1,\varsigma_v))^{\mathsf{T}}\}A_2^{\mathsf{T}}\\&+A_2\hat{\mathrm{E}}\{\tilde{x}_u^1(\varsigma_h-1,\varsigma_v)(\tilde{x}_u^1(\varsigma_h,\varsigma_v-1))^{\mathsf{T}}\}A_1^{\mathsf{T}}.\end{aligned}\tag{35}$$

Step 2: For $(\varsigma_h,\varsigma_v)\in N_2$, under the initial conditions $P_p^2(\varsigma_h,\tau_v^1)=P(\varsigma_h,\tau_v^1)$ and $P_p^2(0,\varsigma_v)=P_2(\varsigma_v)$, one has

$$\begin{aligned}P_p^2(\varsigma_h,\varsigma_v)=&A_1P_u^2(\varsigma_h,\varsigma_v-1)A_1^{\mathsf{T}}+A_2P_u^2(\varsigma_h-1,\varsigma_v)A_2^{\mathsf{T}}\\&+B_1Q(\varsigma_h,\varsigma_v-1)B_1^{\mathsf{T}}+B_2Q(\varsigma_h-1,\varsigma_v)B_2^{\mathsf{T}}\\&+A_1\hat{\mathrm{E}}\{\tilde{x}_u^2(\varsigma_h,\varsigma_v-1)(\tilde{x}_u^2(\varsigma_h-1,\varsigma_v))^{\mathsf{T}}\}A_2^{\mathsf{T}}\\&+A_2\hat{\mathrm{E}}\{\tilde{x}_u^2(\varsigma_h-1,\varsigma_v)(\tilde{x}_u^2(\varsigma_h,\varsigma_v-1))^{\mathsf{T}}\}A_1^{\mathsf{T}}.\end{aligned}\tag{36}$$

Step 3: For $(\varsigma_h,\varsigma_v)\in N_3$, under the initial conditions $P_p^3(\varsigma_h,0)=P_1(\varsigma_h)$ and $P_p^3(\tau_h^1,\varsigma_v)=P_p^2(\tau_h^1,\varsigma_v)$, one has

$$P_p^3(\varsigma_h,\varsigma_v)=A_1P_u^3(\varsigma_h,\varsigma_v-1)A_1^{\mathsf{T}}+A_2P_u^3(\varsigma_h-1,\varsigma_v)A_2^{\mathsf{T}}$$

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$$
\begin{aligned}
&+ B_1 Q(\varsigma_h, \varsigma_v - 1) B_1^\top + B_2 Q(\varsigma_h - 1, \varsigma_v) B_2^\top \\
&+ A_1 \hat{\mathrm{E}}\{\tilde{x}_u^3(\varsigma_h, \varsigma_v - 1)(\tilde{x}_u^3(\varsigma_h - 1, \varsigma_v))^\top\} A_2^\top \\
&+ A_2 \hat{\mathrm{E}}\{\tilde{x}_u^3(\varsigma_h - 1, \varsigma_v)(\tilde{x}_u^3(\varsigma_h, \varsigma_v - 1))^\top\} A_1^\top .
\end{aligned} \tag{37}
$$

Step 4: For $(\varsigma_h, \varsigma_v) \in N_4$, under the initial conditions $P_p^4(\tau_h^1, \varsigma_v) = P_p^2(\tau_h^1, \varsigma_v)$ and $P_p^4(\varsigma_h, \tau_v^1) = P_p^3(\varsigma_h, \tau_v^1)$, one has

$$
\begin{aligned}
P_p^4(\varsigma_h, \varsigma_v) = {} & A_1 P_u^4(\varsigma_h, \varsigma_v - 1) A_1^\top + A_2 P_u^4(\varsigma_h - 1, \varsigma_v) A_2^\top \\
&+ B_1 Q(\varsigma_h, \varsigma_v - 1) B_1^\top + B_2 Q(\varsigma_h - 1, \varsigma_v) B_2^\top \\
&+ A_1 \hat{\mathrm{E}}\{\tilde{x}_u^4(\varsigma_h, \varsigma_v - 1)(\tilde{x}_u^4(\varsigma_h - 1, \varsigma_v))^\top\} A_2^\top \\
&+ A_2 \hat{\mathrm{E}}\{\tilde{x}_u^4(\varsigma_h - 1, \varsigma_v)(\tilde{x}_u^4(\varsigma_h, \varsigma_v - 1))^\top\} A_1^\top .
\end{aligned} \tag{38}
$$

Proof: Substituting (17a) and (18a) into (21), it is not difficult to obtain (35)-(38). The detailed proof is omitted here for brevity. ■

Lemma 5: Considering the 2-D systems (1)-(3) and the proposed estimator of the form (8)-(16), the recursive evolutions of the prediction error covariances $P_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)$ and $P_p^{\zeta_1,r}(\varsigma_h, \varsigma_v)$ are given as follows:

Step 1: For $(\varsigma_h, \varsigma_v) \in N_1$, under the initial conditions $P_p^{\zeta_1,1}(\varsigma_h, 0) = P_1^{\zeta_1}(\varsigma_h)$, $P_p^{\zeta_1,1}(0, \varsigma_v) = P_2^{\zeta_1}(\varsigma_v)$, $P_p^{\zeta_2,1}(\varsigma_h, 0) = P_1^{\zeta_2}(\varsigma_h)$, and $P_p^{\zeta_2,1}(0, \varsigma_v) = P_2^{\zeta_2}(\varsigma_v)$, one has

$$
\begin{aligned}
P_p^{\zeta_1,1}(\varsigma_h, \varsigma_v) = {} & F_1 A_1 P_u^{\zeta_1,1}(\varsigma_h, \varsigma_v - 1) A_1^\top F_1^\top \\
&+ E_1 A_2 P_u^{\zeta_1,1}(\varsigma_h - 1, \varsigma_v) A_2^\top E_1^\top \\
&+ F_1 A_2 \tau_1(\varsigma_h, \varsigma_v - 1) X(\varsigma_h - 1, \varsigma_v) A_2^\top F_1^\top \\
&+ E_1 A_1 \tau_1(\varsigma_h - 1, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top E_1^\top \\
&+ F_1 B_1 \tau_1(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h, \varsigma_v - 1) B_1^\top F_1^\top \\
&+ F_1 B_2 \tau_1(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h - 1, \varsigma_v) B_2^\top F_1^\top \\
&+ E_1 B_1 \tau_1(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\top E_1^\top \\
&+ E_1 B_2 \tau_1(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\top E_1^\top \\
&+ A_1 \sigma_1(\varsigma_h, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top \\
&+ A_2 \sigma_1(\varsigma_h, \varsigma_v) X(\varsigma_h - 1, \varsigma_v) A_2^\top \\
&+ B_1 \sigma_1(\varsigma_h, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\top \\
&+ B_2 \sigma_1(\varsigma_h, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\top \\
&+ \mathrm{sym}\{\beth_1^1 + \beth_2^1 + \beth_3^1 + \beth_4^1 + \beth_5^1 + \beth_6^1\}
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
P_p^{\zeta_2,1}(\varsigma_h, \varsigma_v) = {} & F_2 A_1 P_u^{\zeta_2,1}(\varsigma_h, \varsigma_v - 1) A_1^\top F_2^\top \\
&+ E_2 A_2 P_u^{\zeta_2,1}(\varsigma_h - 1, \varsigma_v) A_2^\top E_2^\top \\
&+ F_2 A_2 \tau_2(\varsigma_h, \varsigma_v - 1) X(\varsigma_h - 1, \varsigma_v) A_2^\top F_2^\top \\
&+ E_2 A_1 \tau_2(\varsigma_h - 1, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top E_2^\top \\
&+ F_2 B_1 \tau_2(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h, \varsigma_v - 1) B_1^\top F_2^\top \\
&+ F_2 B_2 \tau_2(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h - 1, \varsigma_v) B_2^\top F_2^\top \\
&+ E_2 B_1 \tau_2(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\top E_2^\top \\
&+ E_2 B_2 \tau_2(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\top E_2^\top \\
&+ A_1 \sigma_2(\varsigma_h, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top \\
&+ A_2 \sigma_2(\varsigma_h, \varsigma_v) X(\varsigma_h - 1, \varsigma_v) A_2^\top \\
&+ B_1 \sigma_2(\varsigma_h, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\top \\
&+ B_2 \sigma_2(\varsigma_h, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\top \\
&+ \mathrm{sym}\{\beth_1 + \beth_2 + \beth_3 + \beth_4 + \beth_5 + \beth_6\}.
\end{aligned} \tag{40}
$$

Step 2: For $(\varsigma_h, \varsigma_v) \in N_2$, under the initial conditions $P_p^{\zeta_1,2}(\varsigma_h, \tau_v^1) = P_p^{\zeta_1,1}(\varsigma_h, \tau_v^1)$ and $P_p^{\zeta_1,2}(0, \varsigma_v) = P_2^{\zeta_1}(\varsigma_v)$, one has

$$
\begin{aligned}
P_p^{\zeta_1,2}(\varsigma_h, \varsigma_v) = {} & F_1 A_1 P_u^{\zeta_1,2}(\varsigma_h, \varsigma_v - 1) A_1^\top F_1^\top \\
&+ E_1 A_2 P_u^{\zeta_1,2}(\varsigma_h - 1, \varsigma_v) A_2^\top E_1^\top \\
&+ F_1 A_2 \tau_1(\varsigma_h, \varsigma_v - 1) X(\varsigma_h - 1, \varsigma_v) A_2^\top F_1^\top \\
&+ E_1 A_1 \tau_1(\varsigma_h - 1, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top E_1^\top \\
&+ F_1 B_1 \tau_1(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h, \varsigma_v - 1) B_1^\top F_1^\top \\
&+ F_1 B_2 \tau_1(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h - 1, \varsigma_v) B_2^\top F_1^\top \\
&+ E_1 B_1 \tau_1(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\top E_1^\top \\
&+ E_1 B_2 \tau_1(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\top E_1^\top \\
&+ A_1 \sigma_1(\varsigma_h, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top \\
&+ A_2 \sigma_1(\varsigma_h, \varsigma_v) X(\varsigma_h - 1, \varsigma_v) A_2^\top \\
&+ B_1 \sigma_1(\varsigma_h, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\top \\
&+ B_2 \sigma_1(\varsigma_h, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\top \\
&+ \mathrm{sym}\{\beth_1^2 + \beth_2^2 + \beth_3^2 + \beth_4^2 + \beth_5^2 + \beth_6^2\}
\end{aligned} \tag{41}
$$

Step 3: For $(\varsigma_h, \varsigma_v) \in N_3$, under the initial conditions $P_p^{\zeta_1,3}(\varsigma_h, 0) = P_1^{\zeta_1}(\varsigma_h)$ and $P_p^{\zeta_1,3}(\tau_h^1, \varsigma_v) = P_p^{\zeta_1,2}(\tau_h^1, \varsigma_v)$, one has

$$
\begin{aligned}
P_p^{\zeta_1,3}(\varsigma_h, \varsigma_v) = {} & F_1 A_1 P_u^{\zeta_1,3}(\varsigma_h, \varsigma_v - 1) A_1^\top F_1^\top \\
&+ E_1 A_2 P_u^{\zeta_1,3}(\varsigma_h - 1, \varsigma_v) A_2^\top E_1^\top \\
&+ F_1 A_2 \tau_1(\varsigma_h, \varsigma_v - 1) X(\varsigma_h - 1, \varsigma_v) A_2^\top F_1^\top \\
&+ E_1 A_1 \tau_1(\varsigma_h - 1, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top E_1^\top \\
&+ F_1 B_1 \tau_1(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h, \varsigma_v - 1) B_1^\top F_1^\top \\
&+ F_1 B_2 \tau_1(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h - 1, \varsigma_v) B_2^\top F_1^\top \\
&+ E_1 B_1 \tau_1(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\top E_1^\top \\
&+ E_1 B_2 \tau_1(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\top E_1^\top \\
&+ A_1 \sigma_1(\varsigma_h, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top \\
&+ A_2 \sigma_1(\varsigma_h, \varsigma_v) X(\varsigma_h - 1, \varsigma_v) A_2^\top \\
&+ B_1 \sigma_1(\varsigma_h, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\top \\
&+ B_2 \sigma_1(\varsigma_h, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\top \\
&+ \mathrm{sym}\{\beth_1^3 + \beth_2^3 + \beth_3^3 + \beth_4^3 + \beth_5^3 + \beth_6^3\}
\end{aligned} \tag{42}
$$

Step 4: For $(\varsigma_h, \varsigma_v) \in N_4$, under the initial conditions $P_p^{\zeta_1,4}(\tau_h^1, \varsigma_v) = P_p^{\zeta_1,2}(\tau_h^1, \varsigma_v)$ and $P_p^{\zeta_1,4}(\varsigma_h, \tau_v^1) = P_p^{\zeta_1,3}(\varsigma_h, \tau_v^1)$, one has

$$
\begin{aligned}
P_p^{\zeta_1,4}(\varsigma_h, \varsigma_v) = {} & F_1 A_1 P_u^{\zeta_1,4}(\varsigma_h, \varsigma_v - 1) A_1^\top F_1^\top \\
&+ E_1 A_2 P_u^{\zeta_1,4}(\varsigma_h - 1, \varsigma_v) A_2^\top E_1^\top \\
&+ F_1 A_2 \tau_1(\varsigma_h, \varsigma_v - 1) X(\varsigma_h - 1, \varsigma_v) A_2^\top F_1^\top \\
&+ E_1 A_1 \tau_1(\varsigma_h - 1, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\top E_1^\top \\
&+ F_1 B_1 \tau_1(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h, \varsigma_v - 1) B_1^\top F_1^\top
\end{aligned}
$$

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$$
\begin{aligned}
&+ F_1 B_2 \tau_1(\varsigma_h, \varsigma_v - 1) Q(\varsigma_h - 1, \varsigma_v) B_2^\mathsf{T} F_1^\mathsf{T} \\
&+ E_1 B_1 \tau_1(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\mathsf{T} E_1^\mathsf{T} \\
&+ E_1 B_2 \tau_1(\varsigma_h - 1, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\mathsf{T} E_1^\mathsf{T} \\
&+ A_1 \sigma_1(\varsigma_h, \varsigma_v) X(\varsigma_h, \varsigma_v - 1) A_1^\mathsf{T} \\
&+ A_2 \sigma_1(\varsigma_h, \varsigma_v) X(\varsigma_h - 1, \varsigma_v) A_2^\mathsf{T} \\
&+ B_1 \sigma_1(\varsigma_h, \varsigma_v) Q(\varsigma_h, \varsigma_v - 1) B_1^\mathsf{T} \\
&+ B_2 \sigma_1(\varsigma_h, \varsigma_v) Q(\varsigma_h - 1, \varsigma_v) B_2^\mathsf{T} \\
&+ \mathrm{sym}\{\beth_1^4 + \beth_2^4 + \beth_3^4 + \beth_4^4 + \beth_5^4 + \beth_6^4\}
\end{aligned} \tag{43}
$$

where

$$
\begin{aligned}
\beth_1^r =\ & \hat{\mathrm{E}}\Big\{\Big(F_1 A_1 \tilde{x}_u^{\zeta_1, r}(\varsigma_h, \varsigma_v - 1)\Big)\Big(E_1 A_2 \tilde{x}_u^{\zeta_1, r}(\varsigma_h - 1, \varsigma_v) \\
& + F_1 A_2 \zeta_1(\varsigma_h, \varsigma_v - 1) x(\varsigma_h - 1, \varsigma_v) + E_1 A_1 \\
& \times \zeta_1(\varsigma_h - 1, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\beth_2^r =\ & \hat{\mathrm{E}}\Big\{\Big(E_1 A_2 \tilde{x}_u^{\zeta_1, r}(\varsigma_h - 1, \varsigma_v)\Big)\Big(F_1 A_2 \zeta_1(\varsigma_h, \varsigma_v - 1) \\
& \times x(\varsigma_h - 1, \varsigma_v) + E_1 A_1 \zeta_1(\varsigma_h - 1, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\beth_3^r =\ & \hat{\mathrm{E}}\Big\{\Big(F_1 A_2 \zeta_1(\varsigma_h, \varsigma_v - 1) x(\varsigma_h - 1, \varsigma_v)\Big)\Big(E_1 A_1 \\
& \times \zeta_1(\varsigma_h - 1, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\beth_4^r =\ & \hat{\mathrm{E}}\Big\{\Big(F_1 B_1 \zeta_1(\varsigma_h, \varsigma_v - 1) w(\varsigma_h, \varsigma_v - 1)\Big)\Big(E_1 B_1 \\
& \times \zeta_1(\varsigma_h - 1, \varsigma_v) w(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\beth_5^r =\ & \hat{\mathrm{E}}\Big\{\Big(F_1 B_2 \zeta_1(\varsigma_h, \varsigma_v - 1) w(\varsigma_h - 1, \varsigma_v)\Big)\Big(E_1 B_2 \\
& \times \zeta_1(\varsigma_h - 1, \varsigma_v) w(\varsigma_h - 1, \varsigma_v)\Big)^\mathsf{T}\Big\} \\
\beth_6^r =\ & \hat{\mathrm{E}}\Big\{\Big(A_1 \epsilon_1(\varsigma_h, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)\Big(A_2 \epsilon_1(\varsigma_h, \varsigma_v) x(\varsigma_h - 1, \varsigma_v)\Big)^\mathsf{T}\Big\} \\
\beth_1 =\ & \hat{\mathrm{E}}\Big\{\Big(F_2 A_1 \tilde{x}_u^{\zeta_2, 1}(\varsigma_h, \varsigma_v - 1)\Big)\Big(E_2 A_2 \tilde{x}_u^{\zeta_2, 1}(\varsigma_h - 1, \varsigma_v) \\
& + F_2 A_2 \zeta_2(\varsigma_h, \varsigma_v - 1) x(\varsigma_h - 1, \varsigma_v) \\
& + E_2 A_1 \zeta_2(\varsigma_h - 1, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\beth_2 =\ & \hat{\mathrm{E}}\Big\{\Big(E_2 A_2 \tilde{x}_u^{\zeta_2, 1}(\varsigma_h - 1, \varsigma_v)\Big)\Big(F_2 A_2 \zeta_2(\varsigma_h, \varsigma_v - 1) \\
& \times x(\varsigma_h - 1, \varsigma_v) + E_1 A_1 \zeta_1(\varsigma_h - 1, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\beth_3 =\ & \hat{\mathrm{E}}\Big\{\Big(F_2 A_2 \zeta_2(\varsigma_h, \varsigma_v - 1) x(\varsigma_h - 1, \varsigma_v)\Big)\Big(E_2 A_1 \\
& \times \zeta_2(\varsigma_h - 1, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\beth_4 =\ & \hat{\mathrm{E}}\Big\{\Big(F_2 B_1 \zeta_2(\varsigma_h, \varsigma_v - 1) w(\varsigma_h, \varsigma_v - 1)\Big)\Big(E_2 B_1 \\
& \times \zeta_2(\varsigma_h - 1, \varsigma_v) w(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\beth_5 =\ & \hat{\mathrm{E}}\Big\{\Big(F_2 B_2 \zeta_2(\varsigma_h, \varsigma_v - 1) w(\varsigma_h - 1, \varsigma_v)\Big)\Big(E_2 B_2 \\
& \times \zeta_2(\varsigma_h - 1, \varsigma_v) w(\varsigma_h - 1, \varsigma_v)\Big)^\mathsf{T}\Big\} \\
\beth_6 =\ & \hat{\mathrm{E}}\Big\{\Big(A_1 \epsilon_2(\varsigma_h, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)\Big(A_2 \\
& \times \epsilon_2(\varsigma_h, \varsigma_v) x(\varsigma_h - 1, \varsigma_v)\Big)^\mathsf{T}\Big\}.
\end{aligned}
$$

Proof: Substituting (17c), (17e), and (18c) into (23) and (24), it is easy to obtain (39)-(43). The detailed proof is omitted here for brevity. ■

Lemma 6: Considering the 2-D systems (1)-(3) and the proposed estimator of the form (8)-(16), the recursive evolutions of the cross covariances $M_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)$ and $M_p^{\zeta_1,r}(\varsigma_h, \varsigma_v)$ are given as follows:

Step 1: For $(\varsigma_h, \varsigma_v) \in N_1$, under the initial conditions $M_p^{\zeta_1,1}(\varsigma_h, 0) = M_1^{\zeta_1}(\varsigma_h)$, $M_p^{\zeta_1,1}(0, \varsigma_v) = M_2^{\zeta_1}(\varsigma_v)$, $M_p^{\zeta_2,1}(\varsigma_h, 0) = M_1^{\zeta_2}(\varsigma_h)$, and $M_p^{\zeta_2,1}(0, \varsigma_v) = M_2^{\zeta_2}(\varsigma_v)$, one has

$$
\begin{aligned}
M_p^{\zeta_1,1}(\varsigma_h, \varsigma_v) =\ & A_1 M_u^{\zeta_1,1}(\varsigma_h, \varsigma_v - 1) A_1^\mathsf{T} F_1^\mathsf{T} \\
& + A_2 M_u^{\zeta_1,1}(\varsigma_h - 1, \varsigma_v) A_2^\mathsf{T} E_1^\mathsf{T} \\
& + \mathrm{sym}\{\daleth_1^1 + \daleth_2^1\}
\end{aligned} \tag{44}
$$

$$
\begin{aligned}
M_p^{\zeta_2,1}(\varsigma_h, \varsigma_v) =\ & A_1 M_u^{\zeta_2,1}(\varsigma_h, \varsigma_v - 1) A_1^\mathsf{T} F_2^\mathsf{T} \\
& + A_2 M_u^{\zeta_2,1}(\varsigma_h - 1, \varsigma_v) A_2^\mathsf{T} E_2^\mathsf{T} \\
& + \mathrm{sym}\{\daleth_1 + \daleth_2\}.
\end{aligned} \tag{45}
$$

Step 2: For $(\varsigma_h, \varsigma_v) \in N_2$, under the initiaf conditions $M_p^{\zeta_1,2}(\varsigma_h, \tau_v^1) = M_p^{\zeta_1,1}(\varsigma_h, \tau_v^1)$ and $M_p^{\zeta_1,2}(0, \varsigma_v) = M_2^{\zeta_1}(\varsigma_v)$, one has

$$
\begin{aligned}
M_p^{\zeta_1,2}(\varsigma_h, \varsigma_v) =\ & A_1 M_u^{\zeta_1,2}(\varsigma_h, \varsigma_v - 1) A_1^\mathsf{T} F_1^\mathsf{T} \\
& + A_2 M_u^{\zeta_1,2}(\varsigma_h - 1, \varsigma_v) A_2^\mathsf{T} E_1^\mathsf{T} \\
& + \mathrm{sym}\{\daleth_1^2 + \daleth_2^2\}.
\end{aligned} \tag{46}
$$

Step 3: For $(\varsigma_h, \varsigma_v) \in N_3$, under the initial conditions $M_p^{\zeta_1,3}(\varsigma_h, 0) = M_1^{\zeta_1}(\varsigma_h)$ and $M_p^{\zeta_1,3}(\tau_h^1, \varsigma_v) = M_p^{\zeta_1,2}(\tau_h^1, \varsigma_v)$, one has

$$
\begin{aligned}
M_p^{\zeta_1,3}(\varsigma_h, \varsigma_v) =\ & A_1 M_u^{\zeta_1,3}(\varsigma_h, \varsigma_v - 1) A_1^\mathsf{T} F_1^\mathsf{T} \\
& + A_2 M_u^{\zeta_1,3}(\varsigma_h - 1, \varsigma_v) A_2^\mathsf{T} E_1^\mathsf{T} \\
& + \mathrm{sym}\{\daleth_1^3 + \daleth_2^3\}.
\end{aligned} \tag{47}
$$

Step 4: For $(\varsigma_h, \varsigma_v) \in N_4$, under the initial conditions $M_p^{\zeta_1,4}(\tau_h^1, \varsigma_v) = M_p^{\zeta_1,2}(\tau_h^1, \varsigma_v)$ and $M_p^{\zeta_1,4}(\varsigma_h, \tau_v^1) = M_p^{\zeta_1,3}(\varsigma_h, \tau_v^1)$, one has

$$
\begin{aligned}
M_p^{\zeta_1,4}(\varsigma_h, \varsigma_v) =\ & A_1 M_u^{\zeta_1,4}(\varsigma_h, \varsigma_v - 1) A_1^\mathsf{T} F_1^\mathsf{T} \\
& + A_2 M_u^{\zeta_1,4}(\varsigma_h - 1, \varsigma_v) A_2^\mathsf{T} E_1^\mathsf{T} \\
& + \mathrm{sym}\{\daleth_1^4 + \daleth_2^4\}
\end{aligned} \tag{48}
$$

where

$$
\begin{aligned}
\daleth_1^r =\ & \hat{\mathrm{E}}\Big\{\Big(A_1 \tilde{x}_u^r(\varsigma_h, \varsigma_v - 1)\Big)\Big(E_1 A_2 \tilde{x}_u^{\zeta_1, r}(\varsigma_h - 1, \varsigma_v) \\
& + F_1 A_2 \zeta_1(\varsigma_h, \varsigma_v - 1) x(\varsigma_h - 1, \varsigma_v) \\
& + E_1 A_1 \zeta_1(\varsigma_h - 1, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\daleth_2^r =\ & \hat{\mathrm{E}}\Big\{\Big(A_2 \tilde{x}_u^r(\varsigma_h - 1, \varsigma_v)\Big)\Big(F_1 A_1 \tilde{x}_u^{\zeta_1, r}(\varsigma_h - 1, \varsigma_v) \\
& + F_1 A_2 \zeta_1(\varsigma_h, \varsigma_v - 1) x(\varsigma_h - 1, \varsigma_v) \\
& + E_1 A_1 \zeta_1(\varsigma_h - 1, \varsigma_v) x(\varsigma_h, \varsigma_v - 1)\Big)^\mathsf{T}\Big\} \\
\daleth_1 =\ & \hat{\mathrm{E}}\Big\{\Big(A_1 \tilde{x}_u^1(\varsigma_h, \varsigma_v - 1)\Big)\Big(E_2 A_2 \tilde{x}_u^{\zeta_2, 1}(\varsigma_h - 1, \varsigma_v) \\
& + F_2 A_2 \zeta_2(\varsigma_h, \varsigma_v - 1) x(\varsigma_h - 1, \varsigma_v)
\end{aligned}
$$

$$
\begin{aligned}
&\quad + E_2 A_1 \zeta_2(\varsigma_h-1,\varsigma_v) x(\varsigma_h,\varsigma_v-1)\Big)^{\mathsf{T}}\Big\} \\
\daleth_2 &= \hat{\mathrm{E}}\Big\{\Big(A_2 \tilde{x}_u^1(\varsigma_h-1,\varsigma_v)\Big)\Big(F_2 A_1 \tilde{x}_u^{\zeta_2,1}(\varsigma_h-1,\varsigma_v) \\
&\quad + F_2 A_2 \zeta_2(\varsigma_h,\varsigma_v-1) x(\varsigma_h-1,\varsigma_v) \\
&\quad + E_2 A_1 \zeta_2(\varsigma_h-1,\varsigma_v) x(\varsigma_h,\varsigma_v-1)\Big)^{\mathsf{T}}\Big\}.
\end{aligned}
$$

Proof: Inserting (17a), (17c), (17e), (18a), and (18c) into (29) and (30), it is not difficult to obtain (44)-(48). The detailed proof is omitted here for brevity. ■

Lemma 7: Consider the 2-D systems (1)-(3) and the proposed estimator of the form (8)-(16). Under the initial conditions $N_p^{\zeta_{12},1}(\varsigma_h,0)=N_1^{\zeta_{12}}(\varsigma_h)$ and $N_p^{\zeta_{12},1}(0,\varsigma_v)=N_2^{\zeta_{12}}(\varsigma_v)$ for any $(\varsigma_h,\varsigma_v)\in N_1$, the recursive evolution of the cross covariance $N_p^{\zeta_{12},1}(\varsigma_h,\varsigma_v)$ is given as follows:

$$
\begin{aligned}
N_p^{\zeta_{12},1}(\varsigma_h,\varsigma_v) &= F_1 A_1 N_u^{\zeta_{12},1}(\varsigma_h,\varsigma_v-1) A_1^{\mathsf{T}} F_2^{\mathsf{T}} \\
&\quad + E_1 A_2 N_u^{\zeta_{12},1}(\varsigma_h-1,\varsigma_v) A_2^{\mathsf{T}} E_2^{\mathsf{T}} \\
&\quad + \gimel_1 + \gimel_2 + \gimel_3 + \gimel_4 \qquad (49)
\end{aligned}
$$

where

$$
\begin{aligned}
\gimel_1 &= \hat{\mathrm{E}}\Big\{\Big(F_1 A_1 \tilde{x}_u^{\zeta_1,1}(\varsigma_h,\varsigma_v-1)\Big)\Big(E_2 A_2 \tilde{x}_u^{\zeta_2,1}(\varsigma_h-1,\varsigma_v) \\
&\quad + F_2 A_2 \zeta_2(\varsigma_h,\varsigma_v-1) x(\varsigma_h-1,\varsigma_v) \\
&\quad + E_2 A_1 \zeta_2(\varsigma_h-1,\varsigma_v) x(\varsigma_h,\varsigma_v-1)\Big)^{\mathsf{T}}\Big\} \\
\gimel_2 &= \hat{\mathrm{E}}\Big\{\Big(E_1 A_2 \tilde{x}_u^{\zeta_1,1}(\varsigma_h-1,\varsigma_v)\Big)\Big(F_2 A_1 \tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v-1) \\
&\quad + F_2 A_2 \zeta_2(\varsigma_h,\varsigma_v-1) x(\varsigma_h-1,\varsigma_v) \\
&\quad + E_2 A_1 \zeta_2(\varsigma_h-1,\varsigma_v) x(\varsigma_h,\varsigma_v-1)\Big)^{\mathsf{T}}\Big\} \\
\gimel_3 &= \hat{\mathrm{E}}\Big\{\Big(F_1 A_2 \zeta_1(\varsigma_h,\varsigma_v-1) x(\varsigma_h-1,\varsigma_v)\Big) \\
&\quad \times \Big(F_2 A_1 \tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v-1) + E_2 A_2 \tilde{x}_u^{\zeta_2,1}(\varsigma_h-1,\varsigma_v)\Big)^{\mathsf{T}}\Big\} \\
\gimel_4 &= \hat{\mathrm{E}}\Big\{\Big(E_1 A_1 \zeta_1(\varsigma_h-1,\varsigma_v) x(\varsigma_h,\varsigma_v-1)\Big) \\
&\quad \times \Big(F_2 A_1 \tilde{x}_u^{\zeta_2,1}(\varsigma_h,\varsigma_v-1)\Big)^{\mathsf{T}}\Big\}.
\end{aligned}
$$

Proof: Substituting (17c) and (17e) into (31), we can obtain (49). The detailed proof is omitted here for brevity. ■

D. Minimum-variance filter design

In the following theorem, the estimator gains are designed to minimize the estimation error covariances at each step.

Theorem 1: Consider the 2-D systems (1)-(3) and the proposed estimator of the form (8)-(16) for every $s\in[2,4]$. The estimator gains $K_1(\varsigma_h,\varsigma_v)$, $K_s(\varsigma_h,\varsigma_v)$, $K_1^{\zeta_1}(\varsigma_h,\varsigma_v)$, $K_1^{\zeta_2}(\varsigma_h,\varsigma_v)$, and $K_s^{\zeta_1}(\varsigma_h,\varsigma_v)$ are determined by

$$
\begin{aligned}
K_1(\varsigma_h,\varsigma_v) &= L_1(\varsigma_h,\varsigma_v)(\bar{R}_1^{\zeta_{12}}(\varsigma_h,\varsigma_v))^{-1} &(50)\\
K_s(\varsigma_h,\varsigma_v) &= L_s(\varsigma_h,\varsigma_v)(\bar{R}_s^{\zeta_1}(\varsigma_h,\varsigma_v))^{-1} &(51)\\
K_1^{\zeta_1}(\varsigma_h,\varsigma_v) &= L_1^{\zeta_1}(\varsigma_h,\varsigma_v)(\bar{R}_1^{\zeta_{12}}(\varsigma_h,\varsigma_v))^{-1} &(52)\\
K_1^{\zeta_2}(\varsigma_h,\varsigma_v) &= L_1^{\zeta_2}(\varsigma_h,\varsigma_v)(\bar{R}_1^{\zeta_{12}}(\varsigma_h,\varsigma_v))^{-1} &(53)\\
K_s^{\zeta_1}(\varsigma_h,\varsigma_v) &= L_s^{\zeta_1}(\varsigma_h,\varsigma_v)(\bar{R}_s^{\zeta_1}(\varsigma_h,\varsigma_v))^{-1} &(54)
\end{aligned}
$$

where

$$
\begin{aligned}
L_1(\varsigma_h,\varsigma_v) &= P_p^1(\varsigma_h,\varsigma_v) C_2^{\mathsf{T}}(\varsigma_h,\varsigma_v) + M_p^{\zeta_1,1}(\varsigma_h,\varsigma_v) \\
&\quad \times \check{C}_{21}^{\mathsf{T}}(\varsigma_h,\varsigma_v) + M_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)\check{C}_{22}^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
L_s(\varsigma_h,\varsigma_v) &= P_p^s(\varsigma_h,\varsigma_v) C_1^{\mathsf{T}}(\varsigma_h,\varsigma_v) + M_p^{\zeta_1,s}(\varsigma_h,\varsigma_v)\check{C}_{11}^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
L_1^{\zeta_1}(\varsigma_h,\varsigma_v) &= P_p^{\zeta_1,1}(\varsigma_h,\varsigma_v)\check{C}_{21}^{\mathsf{T}}(\varsigma_h,\varsigma_v) + (M_p^{\zeta_1,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}} \\
&\quad \times C_2^{\mathsf{T}}(\varsigma_h,\varsigma_v) + N_p^{\zeta_{12},1}(\varsigma_h,\varsigma_v)\check{C}_{22}^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
L_1^{\zeta_2}(\varsigma_h,\varsigma_v) &= P_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)\check{C}_{22}^{\mathsf{T}}(\varsigma_h,\varsigma_v) + (M_p^{\zeta_2,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}} \\
&\quad \times C_2^{\mathsf{T}}(\varsigma_h,\varsigma_v) + (N_p^{\zeta_{12},1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\check{C}_{21}^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
L_s^{\zeta_1}(\varsigma_h,\varsigma_v) &= P_p^{\zeta_1,s}(\varsigma_h,\varsigma_v)\check{C}_{11}^{\mathsf{T}}(\varsigma_h,\varsigma_v) + (M_p^{\zeta_1,s}(\varsigma_h,\varsigma_v))^{\mathsf{T}} \\
&\quad \times C_1^{\mathsf{T}}(\varsigma_h,\varsigma_v)
\end{aligned}
$$

$$
\begin{aligned}
&\bar{R}_1^{\zeta_{12}}(\varsigma_h,\varsigma_v) \\
&= C_2(\varsigma_h,\varsigma_v) P_p^1(\varsigma_h,\varsigma_v) C_2^{\mathsf{T}}(\varsigma_h,\varsigma_v) + C_2(\varsigma_h,\varsigma_v) M_p^{\zeta_1,1}(\varsigma_h,\varsigma_v) \\
&\quad \times \check{C}_{21}^{\mathsf{T}}(\varsigma_h,\varsigma_v) + C_2(\varsigma_h,\varsigma_v) M_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)\check{C}_{22}^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
&\quad + \check{C}_{21}(\varsigma_h,\varsigma_v)(M_p^{\zeta_1,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}} C_2^{\mathsf{T}}(\varsigma_h,\varsigma_v) + \check{C}_{21}(\varsigma_h,\varsigma_v) \\
&\quad \times P_p^{\zeta_1,1}(\varsigma_h,\varsigma_v)\check{C}_{21}^{\mathsf{T}}(\varsigma_h,\varsigma_v) + \check{C}_{21}(\varsigma_h,\varsigma_v) N_p^{\zeta_{12},1}(\varsigma_h,\varsigma_v) \\
&\quad \times \check{C}_{22}^{\mathsf{T}}(\varsigma_h,\varsigma_v) + \check{C}_{22}(\varsigma_h,\varsigma_v)(M_p^{\zeta_2,1}(\varsigma_h,\varsigma_v))^{\mathsf{T}} C_2^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
&\quad + \check{C}_{22}(\varsigma_h,\varsigma_v)(N_p^{\zeta_{12},1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}\check{C}_{21}^{\mathsf{T}}(\varsigma_h,\varsigma_v) + R_2(\varsigma_h,\varsigma_v) \\
&\quad + \check{C}_{22}(\varsigma_h,\varsigma_v) P_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)\check{C}_{22}^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
&\bar{R}_s^{\zeta_1}(\varsigma_h,\varsigma_v) \\
&= C_1(\varsigma_h,\varsigma_v) P_p^s(\varsigma_h,\varsigma_v) C_1^{\mathsf{T}}(\varsigma_h,\varsigma_v) + R_1(\varsigma_h,\varsigma_v) \\
&\quad + \check{C}_1(\varsigma_h,\varsigma_v) P_p^{\zeta_1,s}(\varsigma_h,\varsigma_v)\check{C}_1^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
&\quad + C_1(\varsigma_h,\varsigma_v) M_p^{\zeta_1,s}(\varsigma_h,\varsigma_v)\check{C}_1^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
&\quad + \check{C}_1(\varsigma_h,\varsigma_v)(M_p^{\zeta_1,s}(\varsigma_h,\varsigma_v))^{\mathsf{T}} C_1^{\mathsf{T}}(\varsigma_h,\varsigma_v).
\end{aligned}
$$

Accordingly, the minimal estimation error covariances are

$$
\begin{aligned}
P_u^1(\varsigma_h,\varsigma_v) &= P_p^1(\varsigma_h,\varsigma_v) - K_1(\varsigma_h,\varsigma_v)(L_1(\varsigma_h,\varsigma_v))^{\mathsf{T}} &(55)\\
P_u^s(\varsigma_h,\varsigma_v) &= P_p^s(\varsigma_h,\varsigma_v) - K_s(\varsigma_h,\varsigma_v)(L_s(\varsigma_h,\varsigma_v))^{\mathsf{T}} &(56)\\
P_u^{\zeta_1,1}(\varsigma_h,\varsigma_v) &= P_p^{\zeta_1,1}(\varsigma_h,\varsigma_v) - K_1^{\zeta_1}(\varsigma_h,\varsigma_v)(L_1^{\zeta_1}(\varsigma_h,\varsigma_v))^{\mathsf{T}} &(57)\\
P_u^{\zeta_2,1}(\varsigma_h,\varsigma_v) &= P_p^{\zeta_2,1}(\varsigma_h,\varsigma_v) - K_1^{\zeta_2}(\varsigma_h,\varsigma_v)(L_1^{\zeta_2}(\varsigma_h,\varsigma_v))^{\mathsf{T}} &(58)\\
P_u^{\zeta_1,s}(\varsigma_h,\varsigma_v) &= P_p^{\zeta_1,s}(\varsigma_h,\varsigma_v) - K_s^{\zeta_1}(\varsigma_h,\varsigma_v)(L_s^{\zeta_1}(\varsigma_h,\varsigma_v))^{\mathsf{T}}. &(59)
\end{aligned}
$$

Proof: Since the random variable $v_2(\varsigma_h,\varsigma_v)$ is uncorrelated with $\tilde{x}_p^1(\varsigma_h,\varsigma_v)$, $\tilde{x}_p^{\zeta_1,1}(\varsigma_h,\varsigma_v)$, and $\tilde{x}_p^{\zeta_2,1}(\varsigma_h,\varsigma_v)$, it is easy to get $P_u^1(\varsigma_h,\varsigma_v)$ from (17b). To determine the estimator gain $K_1(\varsigma_h,\varsigma_v)$ that minimizes $P_p^1(\varsigma_h,\varsigma_v)$, the method of completing the square is utilized as follows:

$$
\begin{aligned}
P_u^1(\varsigma_h,\varsigma_v) &= P_p^1(\varsigma_h,\varsigma_v) - K_1(\varsigma_h,\varsigma_v)(L_1(\varsigma_h,\varsigma_v))^{\mathsf{T}} + L_1(\varsigma_h,\varsigma_v) \\
&\quad \times K_1^{\mathsf{T}}(\varsigma_h,\varsigma_v) + K_1(\varsigma_h,\varsigma_v)\bar{R}_1^{\zeta_{12}}(\varsigma_h,\varsigma_v) K_1^{\mathsf{T}}(\varsigma_h,\varsigma_v) \\
&= P_p^1(\varsigma_h,\varsigma_v) + [K_1(\varsigma_h,\varsigma_v) - \bar{K}_1(\varsigma_h,\varsigma_v)] \\
&\quad \times \bar{R}_1^{\zeta_{12}}(\varsigma_h,\varsigma_v)[K_1(\varsigma_h,\varsigma_v) - \bar{K}_1(\varsigma_h,\varsigma_v)]^{\mathsf{T}} \\
&\quad - \bar{K}_1(\varsigma_h,\varsigma_v)\bar{R}_1^{\zeta_{12}}(\varsigma_h,\varsigma_v)\bar{K}_1^{\mathsf{T}}(\varsigma_h,\varsigma_v) \qquad (60)
\end{aligned}
$$

where $\bar{K}_1(\varsigma_h,\varsigma_v) = L_1(\varsigma_h,\varsigma_v)(\bar{R}_1^{\zeta_{12}}(\varsigma_h,\varsigma_v))^{-1}$. Clearly, $P_u^1(\varsigma_h,\varsigma_v)$ is minimized if and only if $K_1(\varsigma_h,\varsigma_v) =$

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$\bar{K}_1(\varsigma_h, \varsigma_v)$, which indicates that (50) is valid. Substituting (50) into (60) yields

$$\begin{aligned} P_u^1(\varsigma_h, \varsigma_v) &= P_p^1(\varsigma_h, \varsigma_v) - \bar{K}_1(\varsigma_h, \varsigma_v)\bar{R}_1^{\zeta_{12}}(\varsigma_h, \varsigma_v)\bar{K}_1^\top(\varsigma_h, \varsigma_v) \\ &= P_p^1(\varsigma_h, \varsigma_v) - K_1(\varsigma_h, \varsigma_v)(L_1^{\zeta_{12}}(\varsigma_h, \varsigma_v))^\top. \end{aligned}$$

Similarly, we can obtain (51)-(54) and (56)-(59). The proof is now complete ■

Theorem 2: Consider the 2-D systems (1)-(3) and the proposed estimator of the form (8)-(16) for every $s \in [2, 4]$. The recursive evolutions of the cross covariances $M_u^{\zeta_1,1}(\varsigma_h, \varsigma_v)$, $M_u^{\zeta_2,1}(\varsigma_h, \varsigma_v)$, and $M_u^{\zeta_1,s}(\varsigma_h, \varsigma_v)$ are given as follows:

$$\begin{aligned} M_u^{\zeta_1,1}(\varsigma_h, \varsigma_v) = &M_p^{\zeta_1,1}(\varsigma_h, \varsigma_v) - L_1(\varsigma_h, \varsigma_v)(K_1^{\zeta_1}(\varsigma_h, \varsigma_v))^\top \\ &- K_1(\varsigma_h, \varsigma_v)(L_1^{\zeta_1}(\varsigma_h, \varsigma_v))^\top \\ &+ K_1(\varsigma_h, \varsigma_v)\bar{R}_1^{\zeta_{12}}(\varsigma_h, \varsigma_v)(K_1^{\zeta_1}(\varsigma_h, \varsigma_v))^\top \end{aligned} \tag{61}$$

$$\begin{aligned} M_u^{\zeta_2,1}(\varsigma_h, \varsigma_v) = &M_p^{\zeta_2,1}(\varsigma_h, \varsigma_v) - L_1(\varsigma_h, \varsigma_v)(K_1^{\zeta_2}(\varsigma_h, \varsigma_v))^\top \\ &- K_1(\varsigma_h, \varsigma_v)(L_1^{\zeta_2}(\varsigma_h, \varsigma_v))^\top \\ &+ K_1(\varsigma_h, \varsigma_v)\bar{R}_1^{\zeta_{12}}(\varsigma_h, \varsigma_v)(K_1^{\zeta_2}(\varsigma_h, \varsigma_v))^\top \end{aligned} \tag{62}$$

$$\begin{aligned} M_u^{\zeta_1,s}(\varsigma_h, \varsigma_v) = &M_p^{\zeta_1,s}(\varsigma_h, \varsigma_v) - L_s^{\zeta_1}(\varsigma_h, \varsigma_v)(K_s^{\zeta_1}(\varsigma_h, \varsigma_v))^\top \\ &- K_s(\varsigma_h, \varsigma_v)(L_s^{\zeta_1}(\varsigma_h, \varsigma_v))^\top \\ &+ K_s(\varsigma_h, \varsigma_v)\bar{R}_s^{\zeta_1}(\varsigma_h, \varsigma_v)(K_s^{\zeta_1}(\varsigma_h, \varsigma_v))^\top. \end{aligned} \tag{63}$$

Proof: Since the random variable $v_2(\varsigma_h, \varsigma_v)$ is uncorrelated with $\tilde{x}_p^1(\varsigma_h, \varsigma_v)$, $\tilde{x}_p^{\zeta_1,1}(\varsigma_h, \varsigma_v)$, and $\tilde{x}_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)$, inserting (17b), (17d), and (17f) into (29) and (30), it is easy to obtain (61)-(63). The detailed proof is omitted here for brevity. ■

Theorem 3: Consider the 2-D systems (1)-(3) and the proposed estimator of the form (8)-(16). The recursive evolution of the cross covariance $N_u^{\zeta_{12},1}(\varsigma_h, \varsigma_v)$ is given as follows:

$$\begin{aligned} N_u^{\zeta_{12},1}(\varsigma_h, \varsigma_v) = &N_p^{\zeta_{12},1}(\varsigma_h, \varsigma_v) - L_1^{\zeta_1}(\varsigma_h, \varsigma_v)(K_1^{\zeta_2}(\varsigma_h, \varsigma_v))^\top \\ &- K_1^{\zeta_1}(\varsigma_h, \varsigma_v)(L_1^{\zeta_2}(\varsigma_h, \varsigma_v))^\top \\ &+ K_1^{\zeta_1}(\varsigma_h, \varsigma_v)\bar{R}_1^{\zeta_{12}}(\varsigma_h, \varsigma_v)(K_1^{\zeta_2}(\varsigma_h, \varsigma_v))^\top. \end{aligned} \tag{64}$$

Proof: Substituting (17d) and (17f) into (32), it is not difficult to obtain (64). The detailed proof is omitted here for brevity. ■

IV. Performance Analysis

In this section, the performance of the iterative coupled estimator is analyzed in terms of boundedness, and an online recursive algorithm is summarized for the implementation of the proposed iterative coupled estimator. Before presenting the main results, let us make the following assumption.

Assumption 2: For every $\varsigma_h, \varsigma_v \in [0, \Im]$ and $\iota \in [1, 2]$, there exist positive scalars a_ι, b_ι, and q such that

$$\begin{aligned} &A_\iota(\varsigma_h, \varsigma_v)A_\iota^\top(\varsigma_h, \varsigma_v) \le a_\iota I, \quad B_\iota(\varsigma_h, \varsigma_v)B_\iota^\top(\varsigma_h, \varsigma_v) \le b_\iota I, \\ &Q(\varsigma_h, \varsigma_v)Q^\top(\varsigma_h, \varsigma_v) \le qI. \end{aligned}$$

Remark 6: Note that the aforementioned assumption imposes constraints on the system parameters and noise covariance. Such an assumption is reasonable since the system energy is usually constrained in practical applications. A similar assumption for 1-D systems can be found in [21], [22]. Based on this assumption, we are able to derive an upper bound for the estimation error covariance.

Theorem 4: Under Assumption 2, the minimal estimation error covariance $P_u^r(\varsigma_h, \varsigma_v)$ for any $r \in [1, 4]$ satisfies the following conditions:

For $(\varsigma_h, \varsigma_v) \in N_1$,

$$\begin{aligned} P_u^1(\varsigma_h, \varsigma_v) \le &\sum_{s=1}^{\varsigma_h} \bar{a}_1\kappa(\varsigma_h - s, \varsigma_v - 1)P_u^1(s, 0) \\ &+ \sum_{t=1}^{\varsigma_v} \bar{a}_2\kappa(\varsigma_h - 1, \varsigma_v - t)P_u^1(0, t) \\ &+ \sum_{s=0}^{\varsigma_h-1}\sum_{t=0}^{\varsigma_v-1} \kappa(\varsigma_h - s - 1, \varsigma_v - t - 1)\mu_0 I \end{aligned}$$

$$\vdots$$

For $(\varsigma_h, \varsigma_v) \in N_4$,

$$\begin{aligned} P_u^4(\varsigma_h, \varsigma_v) \le &\sum_{s=\tau_h^1+1}^{\varsigma_h} \bar{a}_1\kappa(\varsigma_h - s, \varsigma_v - (\tau_v^1 + 1))P_u^4(s, \tau_v^1) \\ &+ \sum_{t=\tau_v^1+1}^{\varsigma_v} \bar{a}_2\kappa(\varsigma_h - (\tau_h^1 + 1), \varsigma_v - t)P_u^3(\tau_h^1, t) \\ &+ \sum_{s=\tau_h^1}^{\varsigma_h-1}\sum_{t=\tau_v^1}^{\varsigma_v-1} \kappa(\varsigma_h - s - (\tau_h^1 + 1) \\ &\qquad , \varsigma_v - t - (\tau_v^1 + 1))\mu_0 I \end{aligned}$$

where $\epsilon > 0$ is an arbitrary scalar, $\nu_1 = 1 + \epsilon$, $\nu_2 = 1 + \epsilon^{-1}$, $\bar{a}_1 = \nu_1 a_1^2$, $\bar{a}_2 = \nu_2 a_2^2$ and $\mu_0 = (b_1 + b_2)q$. $P_u^2(\varsigma_h, \tau_v^1) = P_u^1(\varsigma_h, \tau_v^1)(0 \le \varsigma_h \le \tau_h^1)$, $P_u^3(\tau_h^1, \varsigma_v) = P_u^1(\tau_h^1, \varsigma_v)(0 \le \varsigma_v \le \tau_v^1)$, $P_u^4(\varsigma_h, \tau_v^1) = P_u^3(\varsigma_h, \tau_v^1)(\tau_h^1 + 1 \le \varsigma_h \le \tau_h)$, and $P_u^4(\tau_h^1, \varsigma_v) = P_u^2(\tau_h^1, \varsigma_v)(\tau_v^1 + 1 \le \varsigma_v \le \tau_v)$. The term $\varsigma(\cdot, \cdot)$ with $\varsigma(0, 0) = \varsigma(0, \tau_v^1) = \varsigma(\tau_h^1, 0) = \varsigma(\tau_h^1, \tau_v^1) = 1$ is recursively determined by

$$\begin{aligned} \kappa(0, \varsigma_v) &= \bar{a}_1\kappa(0, \varsigma_v - 1), \\ \kappa(\varsigma_h, 0) &= \bar{a}_2\kappa(\varsigma_h - 1, 0), \\ \kappa(\kappa_h, \kappa_v) &= \bar{a}_1\kappa(\varsigma_h, \varsigma_v - 1) + \bar{a}_2\kappa(\varsigma_h - 1, \varsigma_v). \end{aligned}$$

Proof: The proof proceeds by mathematical induction, which is rather tedious and is therefore omitted here for brevity. ■

Remark 7: Up to now, we have established an upper bound for the estimation error covariance. From the recursive structure of the upper bound function, it can be seen that some important factors affecting the system complexity, such as the shift-varying system parameters, noise information and initial conditions, are explicitly reflected in such an upper bound.

The recursive algorithm of the proposed estimator in N_1 region is summarized in Algorithm 1.

Remark 8: By resorting to the observation reconstruction approach, the overall recursive estimation algorithm is divided into four parts, each of which corresponds to one area, and the other three parts of the algorithm are similar to Algorithm 1. Notably, it can be seen from Fig. 2 that the N_1 area provides

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Algorithm 1: The 2-D estimation algorithm in N_1 region

Set $\varsigma_h = 1$ and $\varsigma_v = 1$.
while $\varsigma_h \in [1, \tau_h^1]$ and $\varsigma_v \in [1, \tau_v^1]$ **do**
  Calculate predictions $\hat{x}_p^1(\varsigma_h, \varsigma_v)$, $\hat{x}_p^{\zeta_1,1}(\varsigma_h, \varsigma_v)$ and $\hat{x}_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)$;
  Calculate second-order moments $\tau_c(\varsigma_h, \varsigma_v)$, $X(\varsigma_h, \varsigma_v)$, $P_p^1(\varsigma_h, \varsigma_v)$, $P_p^{\zeta_1,1}(\varsigma_h, \varsigma_v)$, $P_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)$, $M_p^{\zeta_1,1}(\varsigma_h, \varsigma_v)$, $M_p^{\zeta_2,1}(\varsigma_h, \varsigma_v)$ and $N_p^{\zeta_{12},1}(\varsigma_h, \varsigma_v)$;
  Calculate gains $K_1^{\zeta_1}(\varsigma_h, \varsigma_v)$, $K_1^{\zeta_1}(\varsigma_h, \varsigma_v)$ and $K_1(\varsigma_h, \varsigma_v)$;
  Update state estimates $\hat{x}_u^1(\varsigma_h, \varsigma_v)$, $\hat{x}_u^{\zeta_1,1}(\varsigma_h, \varsigma_v)$ and $\hat{x}_u^{\zeta_2,1}(\varsigma_h, \varsigma_v)$;
  Calculate updated second-order moments $P_u^1(\varsigma_h, \varsigma_v)$, $P_u^{\zeta_1,1}(\varsigma_h, \varsigma_v)$, $P_u^{\zeta_2,1}(\varsigma_h, \varsigma_v)$, $M_u^{\zeta_1,1}(\varsigma_h, \varsigma_v)$, $M_u^{\zeta_2,1}(\varsigma_h, \varsigma_v)$ and $N_u^{\zeta_{12},1}(\varsigma_h, \varsigma_v)$;
  if $\varsigma_h \in [1, \tau_h^1]$ and $\varsigma_v = \tau_v^1$ **then**
    Set $\varsigma_h = \varsigma_h + 1$ and $\varsigma_v = 1$.
  else
    Set $\varsigma_v = \varsigma_v + 1$.
  end if
end while

recursive initial value for the N_2 and N_3 areas, and the N_2 and N_3 areas provide recursive initial value for the N_4 area, which means that the calculation order of each area is fixed. Analogously, it can be seen from Theorem 4 that the minimal estimation error covariance is convergent at each area.

V. Illustrative Examples

In this section, a numerical example is provided to demonstrate the effectiveness of the proposed estimation scheme.

Consider a 2-D system (1)-(3) defined on a finite horizon $[0, \Im]$ with $\Im = 80$ and $d = 25$. The system parameters are given by

$$A_1(\tau_h, \tau_v) = \begin{bmatrix} 0.1 + 0.01\cos(\tau_h) & 0.2 & 0 \\ 0 & 0.2 & 0.1 \\ 0 & 0.1 + 0.1e^{-\tau_h} & 0 \end{bmatrix}$$

$$A_2(\tau_h, \tau_v) = \begin{bmatrix} 0.1 & 0 & 0.3 + 0.08e^{-\tau_h} \\ 0.2 & 0.2 + 0.05\sin(\tau_v) & 0 \\ 0.2 & 0.4 & 0 \end{bmatrix}$$

$$B_1(\tau_h, \tau_v) = \begin{bmatrix} 0.5 & 0.6 + 0.01\sin(\tau_h) & 0.1 \end{bmatrix}^{\mathrm{T}}$$

$$B_2(\tau_h, \tau_v) = \begin{bmatrix} 0.2 & 0.3 & 0.2 + 0.01e^{-2\tau_h} \end{bmatrix}^{\mathrm{T}}$$

$$C_{(1)}(\tau_h, \tau_v) = \begin{bmatrix} 8 & 2.2 & 3 \end{bmatrix} \quad C_{(2)}(\tau_h, \tau_v) = \begin{bmatrix} 7 & 2 & 4 \end{bmatrix}$$

$$\check{C}_{(1)}(\tau_h, \tau_v) = \begin{bmatrix} 5.6 & 3 & 2.8 \end{bmatrix} \quad \check{C}_{(2)}(\tau_h, \tau_v) = \begin{bmatrix} 6.7 & 4 & 3 \end{bmatrix}$$

$$F_{(1)}(\tau_h, \tau_v) = 0.15, \ F_{(2)}(\tau_h, \tau_v) = 0.21$$

$$E_{(1)}(\tau_h, \tau_v) = 0.11, \ E_{(2)}(\tau_h, \tau_v) = 0.2.$$

The process and observation noises $w(\tau_h, \tau_v)$, $v_{(1)}(\tau_h, \tau_v)$ and $v_{(2)}(\tau_h, \tau_v)$ obey the zero-mean Gaussian distributions with covariance matrices $Q(\tau_h, \tau_v) = 1$, $R_{(1)}(\tau_h, \tau_v) = 0.81$ and $R_{(2)}(\tau_h, \tau_v) = 0.9$. The random variables $\epsilon_{(1)}(\tau_h, \tau_v)$ and $\epsilon_{(2)}(\tau_h, \tau_v)$ have variances $\sigma_{(1)}(\tau_h, \tau_v) = 0.1$ and $\sigma_{(2)}(\tau_h, \tau_v) = 0.15$.

For all $\tau_h, \tau_v \in [0, 80]$ and $c \in [1, 2]$, the initial conditions are given by $\tau_{(c),1}(\tau_h) = 0.3$, $\tau_{(c),2}(\tau_v) = 0.25$, $\hat{x}_1(\tau_h) = \hat{x}_2(\tau_v) = [\,0\;0\;0\,]^{\mathrm{T}}$, $\hat{x}_1^{\zeta_c}(\tau_h) = \hat{x}_2^{\zeta_c}(\tau_v) = [\,1\;1\;1\,]^{\mathrm{T}}$, $P_1(\tau_h) = P_2(\tau_v) = 0.6I$, $P_1^{\zeta_c}(\tau_h) = P_2^{\zeta_c}(\tau_v) = 0.5I$, $M_1^{\zeta_c}(\tau_h) = 0.2I$, $M_2^{\zeta_c}(\tau_v) = 0.1I$, $N_1^{\zeta_{12}}(\tau_h) = 0.2I$, and $N_2^{\zeta_{12}}(\tau_v) = 0.15I$.

Denote $\tilde{x}_u^{(s)}(\tau_h, \tau_v)$, $\hat{x}_u^{(s)}(\tau_h, \tau_v)$, and $x^{(s)}(\tau_h, \tau_v)$ as the s-th element of $\tilde{x}_u(\tau_h, \tau_v)$, $\hat{x}_u(\tau_h, \tau_v)$, and $x(\tau_h, \tau_v)$, respectively. The root mean squared error (RMSE) is introduced as follows:

$$x^{(s)}(\tau_h, \tau_v) \text{ RMSE} = \sqrt{(x^{(s)}(\tau_h, \tau_v) - \hat{x}_u^{(s)}(\tau_h, \tau_v))^2}.$$

Based on the established theoretical results, the system state can be estimated and the minimal estimation error covariance can be obtained at each step. Simulation results are shown in Figs. 3-5. The regions are partitioned by $N_1 = [0, 55] \times [0, 55]$, $N_2 = [0, 55] \times (55, 80]$, $N_3 = (55, 80] \times [0, 55]$, and $N_4 = (55, 80] \times (55, 80]$. In view of Theorem 4, the upper bound of estimation error covariance converges on N_1, N_2, N_3, and N_4. For simplicity, we only display the evolution trajectories of $x^{(1)}(\tau_h, \tau_v)$ RMSE in Fig. 3, and the cross sections of $\hat{x}_u^{(1)}(\tau_h, \tau_v)$ and $x^{(1)}(\tau_h, \tau_v)$ on $\tau_h = 20$ or $\tau_v = 20$ in Fig. 4. Moreover, Fig. 5 plots the trace evolution of the minimal estimation error covariance $P_u(\tau_h, \tau_v)$. The simulation results confirm the effectiveness of the proposed iterative coupled estimator.

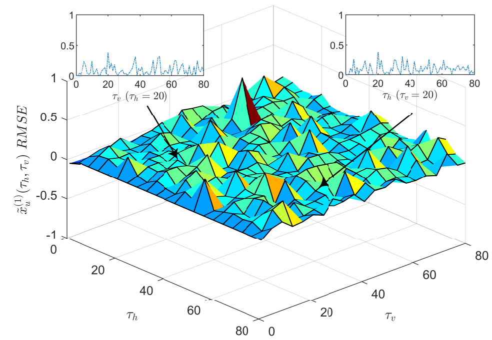

Fig. 3: Evolution trajectory of $x^{(1)}(\tau_h, \tau_v)$ RMSE.

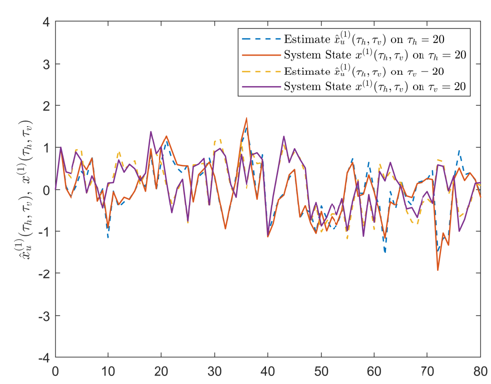

Fig. 4: Cross sections of $\hat{x}_u^{(1)}(\tau_h, \tau_v)$ and $x^{(1)}(\tau_h, \tau_v)$ on $\tau_h = 20$ or $\tau_v = 20$.

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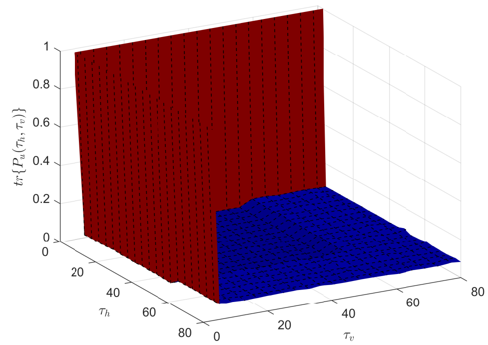

Fig. 5: Trace evolution of the minimal error covariance $P_u(\tau_h, \tau_v)$.

VI. Conclusion

In this paper, the state estimation problem has been investigated for a class of 2-D shift-varying systems subject to autocorrelated multiplicative noises and multi-step observation delays. An iterative coupled four-step estimator has been proposed based on the globally coupled correlation terms and the reconstructed observation sequence, and some sufficient conditions have been derived to guarantee the boundedness of the estimation error covariance. The proposed estimation algorithm has adopted a recursive form, which is capable of avoiding the increase of state dimension. Finally, a numerical example has been provided to demonstrate the feasibility of the proposed estimation algorithm. The limitation of this paper is that the delayed observations processed is limited to the dual-channel scenario. To handle the more general N-channel case, N globally coupled correlation terms must be defined, which inevitably complicates the filter design. Therefore, generalizing the proposed estimation framework to the multi-channel case $(N > 2)$ is considered a key direction for future research.

References

[1] A. Acharya and S. Meher, Iterative spatial domain 2-D signal decomposition for effectual image up-scaling, *Multimedia Tools and Applications*, vol. 80, no. 4, pp. 5577–5616, 2021.

[2] T. P. Azevedo-Perdicoúlis, G. Jank, and P. Lopes dos Santos, Modelling a gas pipeline as a repetitive process: Controllability, observability and stability, *Multidimensional Systems and Signal Processing*, vol. 26, no. 4, pp. 967–984, 2015.

[3] M. Bisiacco and M. E. Valcher, The general fault detection and isolation problem for 2D state-space models, *Systems and Control Letters*, vol. 55, no. 11, pp. 894–899, 2006.

[4] R. Caballero-Águila, M. P. Frías-Bustamante and A. Oya-Lechuga, Least-squares linear estimation for multirate uncertain systems subject to DoS attacks, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 2, art. no. 100014, Jun. 2025.

[5] Y. Chen, W. Wang, Z. Wang, C. Han and S. Du, Finite-horizon filtering for networked 2-D systems with asynchronous measurement delays under non-logarithmic sensor resolution, *IEEE Transactions on Signal and Information Processing over Networks*, vol. 11, pp. 805–820, 2025.

[6] W. Chen, J. Hu, Z. Wu and S. Ma, A survey on fault detection for networked systems under communication constraints, *Systems Science & Control Engineering*, vol. 13, no. 1, art. no. 2460434, 2025.

[7] R. Caballero-Águila and J. Linares-Pérez, Centralized fusion estimation in networked systems: addressing deception attacks and packet dropouts with a zero-order hold approach, *International Journal of Network Dynamics and Intelligence*, vol. 3, no. 4, art. no. 100021, Dec. 2024.

[8] F. Carravetta and A. Germani, Suboptimal solutions for the cubic sensor problem, in *Proceedings of the 36th IEEE Conference on Decision and Control*, San Diego, CA, USA, 1997, pp. 4460-4461.

[9] F. Carravetta, A. Germani, and M. Raimondi, Polynomial filtering for linear discrete time non-gaussian systems, *SIAM Journal on Control and Optimization*, vol. 34, no. 5, pp. 1666–1690, 1996.

[10] F. Carravetta, A. Germani, and M. K. Shuakayev, A new suboptimal approach to the filtering problem for bilinear stochastic differential systems, *SIAM Journal on Control and Optimization*, vol. 38, no. 4, pp. 1171–1203, 2000.

[11] H. Chen, Z. Wang, B. Shen, and J. Liang, Distributed recursive filtering over sensor networks with nonlogarithmic sensor resolution, *IEEE Transactions on Automatic Control*, vol. 67, no. 10, pp. 5408–5415, 2022.

[12] D. Dai, J. Li, Y. Song and F. Yang, Event-based recursive filtering for nonlinear bias-corrupted systems with amplify-and-forward relays, *Systems Science & Control Engineering*, vol. 12, no. 1, art. no. 2332419, 2024.

[13] D. Ding, Q.-L. Han, Y. Xiang, X. Ge, and X.-M. Zhang, A survey on security control and attack detection for industrial cyber-physical systems, *Neurocomputing*, vol. 275, pp. 1674–1683, 2018.

[14] H. Gao and T. Chen, H_∞ estimation for uncertain systems with limited communication capacity, *IEEE Transactions on Automatic Control*, vol. 52, no. 11, pp. 2070–2084, 2007.

[15] X. Ge, Q.-L. Han, X.-M. Zhang, L. Ding, and F. Yang, Distributed event-triggered estimation over sensor networks: A survey, *IEEE Transactions on Cybernetics*, vol. 50, no. 3, pp. 1306–1320, 2020.

[16] F. Han, G. Wei, D. Ding, and Y. Song, Local condition based consensus filtering with stochastic nonlinearities and multiple missing measurements, *IEEE Transactions on Automatic Control*, vol. 62, no. 9, pp. 4784–4790, 2017.

[17] L. V. Hien and H. Trinh, Stability analysis and control of two-dimensional fuzzy systems with directional time-varying delays, *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 3, pp. 1550–1564, 2018.

[18] A. Hmamed, R. Chaibi, and F. Tadeo, Stabilization of discrete-time 2-D T-S fuzzy system via a sum of squares (sos) approach, in *Proceedings of the 2016 5th International Conference on Systems and Control*, Marrakesh, Morocco, 2016, pp. 109–114.

[19] M. Imran, A. Ghafoor, and M. Imran, Stability preserving model reduction technique for 1-D and 2-D systems with error bounds, *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 3, pp. 1084–1088, 2022.

[20] T. Kaczorek, *Two-Dimensional Linear Systems*. Berlin: Springer-Verlag, 1985.

[21] W. Li, Z. Wang, D. W. C. Ho, and G. Wei, On boundedness of error covariances for Kalman consensus filtering problems, *IEEE Transactions on Automatic Control*, vol. 65, no. 6, pp. 2654–2661, 2020.

[22] W. Li, G. Wei, D. Ding, Y. Liu, and F. E. Alsaadi, A new look at boundedness of error covariance of Kalman filtering, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 2, pp. 309–314, 2018.

[23] J. Liang, F. Wang, Z. Wang, and X. Liu, Robust Kalman filtering for two-dimensional systems with multiplicative noises and measurement degradations: The finite-horizon case, *Automatica*, vol. 96, pp. 166–177, 2018.

[24] J. Liang, F. Wang, Z. Wang, and X. Liu, Minimum-variance recursive filtering for two-dimensional systems with degraded measurements: Boundedness and monotonicity, *IEEE Transactions on Automatic Control*, vol. 64, no. 10, pp. 4153–4166, 2019.

[25] H. Lin, J. Lam, and Z. Wang, Secure state estimation for systems under mixed cyber-attacks: Security and performance analysis, *Information Sciences*, vol. 546, pp. 943–960, 2021.

[26] B. Qu, D. Peng, Y. Shen, L. Zou and B. Shen, A survey on recent advances on dynamic state estimation for power systems, *International Journal of Systems Science*, vol. 55, no. 16, pp. 3305–3321, 2024.

[27] W. Liu, Optimal filtering for discrete-time linear systems with timecorrelated multiplicative measurement noises, *IEEE Transactions on Automatic Control*, vol. 61, no. 7, pp. 1972–1978, 2016.

[28] W. Liu and P. Shi, Convergence of optimal linear estimator with multiplicative and time-correlated additive measurement noises, *IEEE Transactions on Automatic Control*, vol. 64, no. 5, pp. 2190–2197, 2019.

[29] X. Lv, Y. Niu, J. H. Park and J. Song, Sliding mode control for 2D FMII systems: a bidirectional dynamic event-triggered strategy, *Automatica*, vol. 147, art. no. 110727, Jan. 2023.

[30] M. Olynik, M. Petovello, M. Cannon, and G. Lachapelle, The effect of GPS errors on relative single point positioning over time, *GPS Solutions*, vol. 6, no. 1-2, pp. 47–57, 2002.

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Citation information: DOI: 10.1109/JIOT.2025.3616849, IEEE Internet of Things Journal

[31] M. Petovello, K. O'Keefe, and G. Lachapelle, Consideration of time-correlated errors in a Kalman filter applicable to GNSS, *Journal of Geodesy*, vol. 83, pp. 51–56, 2009.

[32] S. Rathinasamy, H. R. Karimi, R. K, P. Selvaraj, and Y. Ren, Observer-based tracking control for switched stochastic systems based on a hybrid 2-D model, *International Journal of Robust and Nonlinear Control*, vol. 28, no. 2, pp. 478–491, 2018.

[33] W. Wang, L. Ma, Q. Rui and C. Gao, A survey on privacy-preserving control and filtering of networked control systems, *International Journal of Systems Science*, vol. 55, no. 11, pp. 2269–2288, 2024.

[34] B. Shen, Z. Wang, D. Wang, and H. Liu, Distributed state-saturated recursive filtering over sensor networks under round-robin protocol, *IEEE Transactions on Cybernetics*, vol. 50, no. 8, pp. 3605–3615, 2020.

[35] D. Stenger, M. Nitsch, and D. Abel, Joint constrained bayesian optimization of planning, guidance, control, and state estimation of an autonomous underwater vehicle, in *Proceedings of the 2022 European Control Conference*, London, United Kingdom, 2022, pp. 1982–1987.

[36] W. Wang, Y. Chen, J. Xu, and S. Gao, Finite-horizon estimation for 2-D systems with time-correlated multiplicative noises: A recursive iterative coupled estimator design, *International Journal of Robust and Nonlinear Control*, vol. 34, no. 12, pp. 8013–8032, Aug. 2024.

[37] Z. Sun and C. Han, Linear state estimation for multi-rate NCSs with multi-channel observation delays and unknown Markov packet losses, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 1, art. no. 100005, Mar. 2025.

[38] Y. Zou and E. Tian, Guaranteed cost intermittent control for discrete-time system: a data-driven method, *International Journal of Network Dynamics and Intelligence*, vol. 3, no. 3, art. no. 100015, Sept. 2024.

[39] F. Wang, J. Liang, J. Lam, J. Yang, and C. Zhao, Robust filtering for 2-D systems with uncertain-variance noises and weighted try-once-discard protocols, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 53, no. 5, pp. 2914–2924, 2023.

[40] F. Wang, Z. Wang, J. Liang, and X. Liu, Robust finite-horizon filtering for 2-D systems with randomly varying sensor delays, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 1, pp. 220–232, 2020.

[41] S. Wang, Z. Wang, H. Dong, and F. E. Alsaadi, Recursive state estimation for linear systems with lossy measurements under time-correlated multiplicative noises, *Journal of the Franklin Institute*, vol. 357, no. 3, pp. 1887–1908, 2020.

[42] W. Wang, C. Han, and X. Wang, Optimal H_2 filtering for measurement delay systems with multiplicative noise and sampled data, *Optimal Control Applications and Methods*, vol. 42, no. 4, pp. 987–1000, 2021.

[43] Y. Wang, C. Tang, M. Cai, J. Yin, S. Wang, L. Cheng, R. Wang, and M. Tan, Real-time underwater onboard vision sensing system for robotic gripping, *IEEE Transactions on Instrumentation and Measurement*, vol. 70, pp. 1–11, 2021.

[44] Y. Song, B. Zhang, C. Wen, D. Wang and G. Wei, Model predictive control for complicated dynamic systems: a survey, *International Journal of Systems Science*, vol. 56, no. 9, pp. 2168–2193, 2025.

[45] L. Wu, R. Yang, P. Shi, and X. Su, Stability analysis and stabilization of 2-D switched systems under arbitrary and restricted switchings, *Automatica*, vol. 59, pp. 206–215, 2015.

[46] L. Zhang, H. Gao, and O. Kaynak, Network-induced constraints in networked control systems–A survey, *IEEE Transactions on Industrial Informatics*, vol. 9, no. 1, pp. 403–416, 2013.

[47] W. Wang, X. Kan, D. Ding, H. Liu and X. Gao, Distributed correntropy Kalman filtering over sensor networks with FlexRay-based protocols, *International Journal of Systems Science*, vol. 56, no. 6, pp. 1347–1359, 2025.

[48] Y. Wang, C. Wen and X. Wu, Fault detection and isolation of floating wind turbine pitch system based on Kalman filter and multi-attention 1DCNN, *Systems Science & Control Engineering*, vol. 12, no. 1, art. no. 2362169, 2024.

[49] D. Zhao, D. Zhou, and Y. Wang, Sensor fault reconstruction for a class of 2-D nonlinear systems with application to fault compensation, *Multidimensional Systems and Signal Processing*, vol. 26, no. 4, pp. 1061–1080, 2015.

[50] K. Zhu, Z. Wang, Q. L. Han, and G. Wei, Distributed set-membership fusion filtering for nonlinear 2-D systems over sensor networks: An encoding-decoding scheme, *IEEE Transactions on Cybernetics*, vol. 53, no. 1, pp. 416–427, 2023.

[51] L. Zou, Z. Wang, and H. Gao, Set-membership filtering for timevarying systems with mixed time-delays under round-Robin and weighted try-once-discard protocols, *Automatica*, vol. 74, pp. 341–348, Dec. 2016.

[52] L. Zou, B. Song, J. Suo, N. Li and C. Wang, A survey on outlier-resistant state estimation and its applications, *Systems Science & Control Engineering*, vol. 13, no. 1, art. no. 2474471, 2025.

[53] Y. Zou, M. Sheng, N. Zhong, and S. Xu, A generalized Kalman filter for 2D discrete systems, *Circuits, Systems, and Signal Processing*, vol. 23, no. 5, pp. 351–364, 2004.

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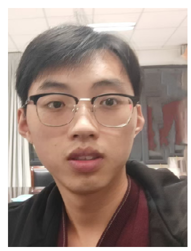

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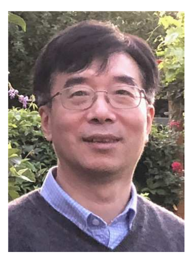

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