This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2025.3616848, IEEE Transactions on Systems Man and Cybernetics Systems

Recursive Quadratic Filter Design for Non-Gaussian Systems Under Random Access Protocol: A Zero-Order Hold Strategy

Shaoying Wang, Zidong Wang, and Hongli Dong

***Abstract*—This paper deals with the recursive quadratic filtering problem for a class of linear discrete-time systems with the random access protocol (RAP) and non-Gaussian noises (NGNs). In order to mitigate undesirable data collisions, the RAP scheduling, used in conjunction with the zero-order hold strategy (ZOHS), is exploited in the sensor-to-filter channel. This coordination of the transmission order of sensors is characterized by a set of independent and identically distributed random variables. The objective of this paper is to design a RAP-based quadratic filtering algorithm within the minimum-variance framework. The addressed system is first transformed into an enhanced system, which offers more information about the RAP and NGNs, by assembling the augmented states (including the original state and the latest measurement) and its second-order Kronecker power. With the assistance of two difference equations, an upper bound on the filtering error covariance is then established, and the gain parameter is subsequently designed by minimizing this upper bound. To address challenges from the RAP scheduling with ZOHS, a matrix decomposition technique is employed. The effectiveness of the proposed RAP-based quadratic filtering algorithm is ultimately confirmed by various simulation results.**

***Index Terms*—Quadratic filtering, random access protocol, non-Gaussian noises, stochastic system, zero-order hold strategy.**

Abbreviations and Notations

RAP	Random access protocol
NGNs	Non-Gaussian noises
ZOHS	Zero-order hold strategy
FEC	Filtering error covariance
$\mathbb{R}^{n_l}$	The n_l-dimensional Euclidean space
M^T	The transpose of matrix M
$\mathrm{vec}(M)$	The vectorization of matrix M
$\mathrm{st}(M)$	The operation that transfers $\mathrm{vec}(M)$ to M
$\mathrm{Sym}\{\star\}$	$\star + \star^T$
$\mathbb{P}\{\cdot\}$	The occurring probability of the event "$\cdot$"
$\mathbb{E}\{\cdot\}$	The expectation of random variable "$\cdot$"
$\otimes$	The Kronecker product
$\varphi_x^{(l)}$	The lth-order moment of x
$\tilde{S}_{n_l}(m \otimes n)$	$m \otimes n + n \otimes m$ with $m \in \mathbb{R}^{n_l}$ and $n \in \mathbb{R}^{n_l}$
$\delta(\cdot)$	The Kronecker delta function

This work was supported in part by the National Natural Science Foundation of China under Grants 62473235 and U21A2019, the Royal Society of the UK, and the Alexander von Humboldt Foundation of Germany. *(corresponding author: Hongli Dong)*

Shaoying Wang is with the College of Science, Shandong University of Aeronautics, Binzhou 256603, China. (Email: shaoying2004@163.com)

Zidong Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (E-mail: Zidong.Wang@brunel.ac.uk)

Hongli Dong is with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing 163318, China, and is also with the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Northeast Petroleum University, Daqing 163318, China. (E-mail: shiningdhl@vip.126.com)

I. Introduction

In recent years, the Kalman filter and its variants, including the extended Kalman filter and the unscented Kalman filter, have received significant research interest, which stems from their promising applications in diverse fields such as target tracking, manufacturing, navigation, and environmental monitoring [8]–[10], [12], [17], [27], [35], [37], [41], [43]. It is noteworthy that a common assumption made is that both process and measurement noises are Gaussian with known covariances. Unfortunately, such an assumption frequently proves to be unrealistic in real-world systems due to factors like severe maneuvering and the presence of measurement outliers [1], [3], [9], [13], [36], [39]. Based on the types of system noises and specific performance specifications, a plethora of filtering algorithms have been meticulously developed and implemented in engineering practices, and some notable examples are H_∞ filtering approaches [14], [21]–[23], [32], set-membership filtering schemes [25], [34], and particle filtering methods [27], [41].

Polynomial filtering, recognized as an effective mechanism to address non-Gaussian noises (NGNs), has been the focus of considerable attention due to its potential to enhance filtering accuracy. As a result, dedicated research efforts have enriched the literature with numerous significant findings in this domain [2], [6], [16], [17], [28], [37]. For instance, an optimal polynomial filtering issue has been explored in [6] for linear non-Gaussian systems affected by multiplicative state noise, extending the study to include the stationary behavior of such systems. In [16], a minimum-variance filtering challenge has been extended for stochastic singular systems tainted by NGNs. More recently, a novel polynomial filtering technique has been introduced in [37] for estimating the toolface of dynamic point-the-bit rotary steerable systems. However, it should be emphasized that deploying *high-degree* polynomial filtering algorithms in practical settings can be quite challenging due to their inherent complexity.

Recently, *quadratic* filtering algorithms have received heightened attention due to their ability to strike an optimal balance between computational complexity and estimation accuracy. Consequently, substantial research has been dedicated to the design and development of the quadratic filter, see e.g. [4], [5], [7], [30], [42]. In particular, the quadratic filtering problem has been addressed in [42] by emphasizing NGNs and developing a buffer-aided approach. Meanwhile, robust quadratic covariance-constrained filters have been formulated in [20] for linear and non-linear non-Gaussian systems. Furthermore, the feedback quadratic distributed filter has been introduced in [30] for multi-agent systems influenced by NGNs. Distinct from the traditional Kalman filter, the quadratic filter is renowned for its superior filtering accuracy (especially in scenarios dominated by NGNs), which is attributed to its ability to fully harness the information embedded in the high-order moments of the state/measurement vector and NGNs.

In an effort to alleviate network congestion and circumvent data collisions, recent research has shown a burgeoning interest in communication protocols. These protocols find their relevance in a myriad of applications such as security monitoring, load balancing, and the allocation and scheduling of computing and server resources [11], [38], [46], [47]. Noteworthy among these models are the random access protocol (RAP) [23], [31], the round-robin protocol (RRP) [19], [29], [44], and the try-once-discard protocol [24], [26], [33]. Specifically, the RAP ensures that only a single sensor node accesses the network at any given time, with the transmission order of these nodes being dictated by a series of independent and identically distributed variables. While there have been initial explorations into filtering challenges under RAP (e.g. [15], [18], [23], [31], [45], [48]), they have predominantly focused on areas like artificial neural networks, complex dynamical networks [18], and nonlinear cyber-physical systems [15]. Regrettably, there exists obvious gap in addressing the RAP-based filtering issues for linear non-Gaussian systems, and such a gap serves as a significant driving force behind our present research.

Given the rising prominence of communication protocols and the integration of the zero-order hold strategy (ZOHS), which notably counterbalances measurements that fail to access the network at a particular transmission moment, there has been a resurgence of interest in the associated filtering challenges [14], [22], [40], [45]. For instance, an energy-to-peak filter has been introduced in [45] for linear parameter-varying systems facing time delays and influenced by the RAP. In [40], the RRP-based filtering problem has been examined for genetic regulatory networks by taking into account time-varying delays, stochastic process noises, and circumscribed external disturbances. It is worth emphasizing that the state augmentation method, which elevates the system state to encompass the most recent measurement, emerges as a potent strategy to navigate the complexities presented by RAP and ZOHS. While some preliminary insights into the protocol-centric filtering challenge have been garnered, there remains a compelling need to intensify focus on the RAP-based quadratic filtering topic in conjunction with ZOHS.

Building on the aforementioned discussion, this paper sets out to investigate the recursive quadratic filtering problem for linear stochastic systems that integrate RAP, ZOHS, and NGNs. Throughout our research, we have identified several pressing challenges: 1) how can we effectively navigate the intrinsic complexities stemming from the combination of RAP and ZOHS when crafting the quadratic filter? 2) how can we determine the variance of the process/measurement noises in relation to the augmented systems in question? 3) how can a recursive quadratic filter be formulated by taking into account the influences of RAP, ZOHS, and NGNs? It's worth noting that, when juxtaposed with the traditional quadratic filtering algorithm (which is devoid of RAP and ZOHS influences), the RAP-centric quadratic filtering problem demands a much more intricate analysis, and such a challenge necessitates novel insights and approaches.

In response to the challenges outlined earlier, the principal contributions of this paper can be underscored as follows.

1) We decompose the coefficient matrices (associated with the estimation error dynamics), shaped by the interplay of RAP with ZOHS, into a constant matrix and a random parameter matrix, and such a step effectively separates the random variables dictating the transmission order of sensor nodes from their original matrices.
2) Drawing on the characteristics of the stochastic parameter matrices at hand, we undertake a comprehensive exploration of the statistical attributes of the augmented process/measurement noises.
3) We propose a novel recursive RAP-based quadratic filtering algorithm tailored for linear non-Gaussian systems and, subsequent to this design, we delve deeper into an analysis concerning the upper bound of the filtering error covariance (FEC).

The structure of this paper unfolds as detailed below. In Section II, we lay the groundwork by formulating the quadratic filtering issue for linear discrete-time systems integrated with RAP, ZOHS, and NGNs. Section III offers foundational lemmas alongside the design considerations for the quadratic filter, while Section IV showcases a simulation example, underscoring the efficacy of the devised quadratic filter. Concluding remarks and reflections are reserved for Section V.

II. Problem Formulation

Consider the following linear discrete-time non-Gaussian system:

$$\begin{cases} x_{s+1} = A_s x_s + B_s w_s \\ z_s = C_s x_s + D_s v_s \end{cases} \tag{1}$$

where $x_s \in \mathbb{R}^{n_x}$ is the system state with initial value x_0, $z_s \in \mathbb{R}^{n_z}$ is the measurement output, w_s and v_s denote the non-Gaussian process noise and measurement noise, respectively. A_s, B_s, C_s and D_s are known matrices with appropriate dimensions. Also, x_0, w_s and v_s are assumed to be zero-mean uncorrelated random sequences and their second-, third- and fourth-order moments are known.

In this study, the RAP is employed to mitigate data collisions. The core principle of RAP is that, at each transmission moment, only a single node is selected to send data through the communication network. Assume that the system's sensors are grouped into m sensor nodes based on their spatial placement. The measurement output z_s can then be expressed by $z_s = [z_{1,s}^T, z_{2,s}^T, \cdots, z_{m,s}^T]^T \in \mathbb{R}^{n_z}$, where $z_{i,s}$ $(i = 1, 2, \cdots, m)$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2025.3616848, IEEE Transactions on Systems Man and Cybernetics Systems

represents the measurement from the ith sensor node prior to transmission.

Let $\zeta_s \in \{1, 2, \cdots, m\}$ be the selected sensor node which gets access to the communication network at time s. Under the RAP scheduling, $\{\zeta_s\}_{s\geq 0}$ can be regarded as a set of mutually independent random variable sequences, and the occurrence probability of $\zeta_s = i$ is described by

$$\mathbb{P}\{\zeta_s = i\} = p_i \tag{2}$$

where $p_i > 0$ (with $\sum_{i=1}^{m} p_i = 1$) is the occurrence probability for the ith sensor node to be selected to transmit. Moreover, ζ_s is supposed to be independent of x_0, w_s and v_s.

Denoting the actually received measurement after being transmitted via the network by

$$\bar{z}_s = [\bar{z}_{1,s}^T, \bar{z}_{2,s}^T, \cdots, \bar{z}_{m,s}^T]^T \tag{3}$$

the updating rule of $\bar{z}_{i,s}$ $(i \in \{1, 2, \cdots, m\})$ under the RAP can be characterized by

$$\bar{z}_{i,s} = \begin{cases} z_{i,s}, & i = \zeta_s \\ \bar{z}_{i,s-1}, & \text{otherwise.} \end{cases} \tag{4}$$

Consequently, $\bar{z}_s$ can be reformulated in a concise manner as:

$$\bar{z}_s = \Phi_{\zeta_s} z_s + (I - \Phi_{\zeta_s})\bar{z}_{s-1} \tag{5}$$

where $\Phi_{\zeta_s} \triangleq \text{diag}\{\delta(\zeta_s - 1)I, \delta(\zeta_s - 2)I, \cdots, \delta(\zeta_s - m)I\}$.

Defining $X_{s+1} \triangleq [x_{s+1}^T, \bar{z}_s^T]^T$, the addressed system (1) can be rewritten as

$$\begin{cases} X_{s+1} = \tilde{A}_s X_s + W_s \\ \bar{z}_s = \tilde{C}_s X_s + V_s \end{cases} \tag{6}$$

where

$$\tilde{A}_s \triangleq \begin{bmatrix} A_s & 0 \\ \Phi_{\zeta_s} C_s & I - \Phi_{\zeta_s} \end{bmatrix}, W_s \triangleq \begin{bmatrix} B_s w_s \\ \Phi_{\zeta_s} D_s v_s \end{bmatrix},$$
$$\tilde{C}_s \triangleq \begin{bmatrix} \Phi_{\zeta_s} C_s & I - \Phi_{\zeta_s} \end{bmatrix}, V_s \triangleq \Phi_{\zeta_s} D_s v_s.$$

From (6), we know that

$$\begin{aligned} X_{s+1}^{[2]} =& \tilde{A}_s^{[2]} X_s^{[2]} + \varphi_{W_s}^{(2)} + \tilde{W}_s \\ \bar{z}_s^{[2]} =& \tilde{C}_s^{[2]} X_s^{[2]} + \varphi_{V_s}^{(2)} + \tilde{V}_s \end{aligned} \tag{7}$$

where

$$\begin{aligned} \tilde{W}_s \triangleq& \tilde{S}_{n_x+n_z}(\tilde{A}_s X_s \otimes W_s) + W_s^{[2]} - \varphi_{W_s}^{(2)}, \\ \tilde{V}_s \triangleq& \tilde{S}_{n_z}(\tilde{C}_s X_s \otimes V_s) + V_s^{[2]} - \varphi_{V_s}^{(2)}. \end{aligned} \tag{8}$$

Let us introduce the following augmented state and measurement vectors:

$$\vec{X}_s \triangleq \begin{bmatrix} X_s \\ X_s^{[2]} \end{bmatrix}, \quad \vec{Z}_s \triangleq \begin{bmatrix} \bar{z}_s \\ \bar{z}_s^{[2]} \end{bmatrix}.$$

Then, the resulting system (6) can be further transformed into

$$\begin{cases} \vec{X}_{s+1} = \vec{A}_s \vec{X}_s + \vec{g}_s + \vec{W}_s \\ \vec{Z}_s = \vec{C}_s \vec{X}_s + \vec{h}_s + \vec{V}_s \end{cases} \tag{9}$$

where

$$\vec{A}_s \triangleq \begin{bmatrix} \tilde{A}_s & 0 \\ 0 & \tilde{A}_s^{[2]} \end{bmatrix}, \vec{g}_s \triangleq \begin{bmatrix} 0 \\ \varphi_{W_s}^{(2)} \end{bmatrix}, \vec{W}_s \triangleq \begin{bmatrix} W_s \\ \tilde{W}_s \end{bmatrix},$$

$$\vec{C}_s \triangleq \begin{bmatrix} \tilde{C}_s & 0 \\ 0 & \tilde{C}_s^{[2]} \end{bmatrix}, \vec{h}_s \triangleq \begin{bmatrix} 0 \\ \varphi_{V_s}^{(2)} \end{bmatrix}, \vec{V}_s \triangleq \begin{bmatrix} V_s \\ \tilde{V}_s \end{bmatrix}. \tag{10}$$

Based on the available measurements $\vec{Z}_s$, for the augmented system (9), a *quadratic* filter is constructed as follows:

$$\begin{cases} \hat{\vec{X}}_{s+1|s} = \vec{A}_s \hat{\vec{X}}_{s|s} + \vec{g}_s \\ \hat{\vec{X}}_{s+1|s+1} = \hat{\vec{X}}_{s+1|s} + \Re_{s+1}(\vec{Z}_{s+1} - \vec{C}_{s+1}\hat{\vec{X}}_{s+1|s} - \vec{h}_{s+1}) \end{cases} \tag{11}$$

where $\hat{\vec{X}}_{s+1|s}$ and $\hat{\vec{X}}_{s+1|s+1}$ represent, respectively, the one-step predictor and the filter, and $\Re_{s+1}$ means the gain parameter to be designed. Moreover, the prediction error and the filtering error are defined as

$$\begin{aligned} \chi_{s+1|s} \triangleq& \vec{X}_{s+1} - \hat{\vec{X}}_{s+1|s} \\ \chi_{s+1|s+1} \triangleq& \vec{X}_{s+1} - \hat{\vec{X}}_{s+1|s+1} \end{aligned} \tag{12}$$

and the corresponding prediction and filtering error covariances are given by

$$\begin{aligned} \Im_{s+1|s} \triangleq& \mathbb{E}\{\chi_{s+1|s}\chi_{s+1|s}^T\} \\ \Im_{s+1|s+1} \triangleq& \mathbb{E}\{\chi_{s+1|s+1}\chi_{s+1|s+1}^T\}. \end{aligned} \tag{13}$$

The objective of this paper is to devise a recursive quadratic filter in the format of (11). Within the concurrent presence of NGNs, RAP, and ZOHS, the intent is to ensure that there exists an upper bound for the FEC denoted as $\Im_{s+1|s+1}$. Specifically, a positive definite matrix $\bar{\Im}_{s+1|s+1}$ is sought such that $\Im_{s+1|s+1} \leq \bar{\Im}_{s+1|s+1}$. Furthermore, the gain matrix $\Re_{s+1}$ is designed to minimize this acquired upper bound $\bar{\Im}_{s+1|s+1}$.

Remark 1: The issue of RAP-based filtering for linear *Gaussian* systems has been tackled in seminal works such as [15], [48]. Unfortunately, directly applying these established protocol-based filtering techniques to *non-Gaussian* systems is inadequate as the filtering performance could be largely compromised, and this underscores the importance of developing a novel RAP-based *quadratic* filtering method in this paper. The distinctive differences between our work and [15], [48] lie in the following aspects: 1) the focus of [15] is on the design of fusion filter for nonlinear cyber-physical systems with RAP and the denial-of-service attacks; 2) the main objective of [48] is to propose a protocol-based recursive filter for a class of linear Gaussian systems and discuss the boundedness of the associated FEC; and 3) the main contribution of this study is to design a RAP-based quadratic filter for linear discrete-time systems with NGNs.

Remark 2: Contrary to the traditional RAP scheduling model which lacks the ZOHS feature, the ZOHS is incorporated in the measurement model (4) with hope to make up for the data lost when sensor node i is not selected at the current moment. This incorporation, though beneficial, introduces a myriad of challenges when designing the quadratic filter. Furthermore, as depicted in (11), the devised recursive filter meticulously considers the ramifications stemming from RAP scheduling and NGNs. To be specific, the coefficient matrices Φ_{ζ_s}, $I - \Phi_{\zeta_s}$ and their second-order Kronecker powers in $\vec{A}_s$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2025.3616848, IEEE Transactions on Systems Man and Cybernetics Systems

signify the influence of the RAP combined with ZOHS. Concurrently, $\varphi^{(2)}_{W_s}$, $\varphi^{(2)}_{V_s}$ in $\vec{g}_s$ and $\vec{h}_s$ underscore the implications of NGNs on the quadratic filter.

III. Main Results

In this section, attention is dedicated to the design of the recursive RAP-based quadratic filtering algorithm. Preliminary lemmas provide the statistical properties of the augmented process and measurement noises. Subsequently, a sufficient condition is established to ensure the existence of an upper bound of the FEC, and such an upper bound is then minimized by the design of the appropriate gain matrix $\Re_{s+1}$.

A. Preliminaries

Before proceeding, some useful lemmas are given to facilitate the subsequent analysis.

Let us define

$$\begin{aligned}&\tilde{A}_{c,s} \triangleq \begin{bmatrix} A_s & 0 \\ 0 & I \end{bmatrix}, \tilde{B}_{c,s} \triangleq \begin{bmatrix} B_s & 0 \\ 0 & 0 \end{bmatrix},\\ &C_{c,s} \triangleq \begin{bmatrix} C_s & -I \end{bmatrix}, \tilde{C}_{c,s} \triangleq \begin{bmatrix} 0 & I \end{bmatrix},\\ &\Phi_i \triangleq \mathrm{diag}\{\underbrace{0,0,\cdots,0}_{i-1}, I, \underbrace{0,\cdots,0}_{m-i}\},\\ &\Psi_i \triangleq \begin{bmatrix} 0 \\ \Phi_i \end{bmatrix}, \tilde{D}_{c,s} \triangleq \begin{bmatrix} 0 & D_s \end{bmatrix},\\ &\Psi_{\zeta_s} \triangleq \sum_{i=1}^{m}\delta(\zeta_s - i)\Psi_i, \gamma_s \triangleq \begin{bmatrix} w_s \\ v_s \end{bmatrix}. \end{aligned} \tag{14}$$

Then, we have

$$\begin{aligned}\Phi_{\zeta_s} =&\sum_{i=1}^{m}\delta(\zeta_s - i)\Phi_i, W_s = (\tilde{B}_{c,s} + \Psi_{\zeta_s}\tilde{D}_{c,s})\gamma_s,\\ \tilde{A}_s =&\tilde{A}_{c,s} + \Psi_{\zeta_s}C_{c,s}, \tilde{C}_s = \Phi_{\zeta_s}C_{c,s} + \tilde{C}_{c,s},\\ \tilde{A}^{[2]}_s =&\tilde{A}^{[2]}_{c,s} + \tilde{A}_{c,s}\otimes(\Psi_{\zeta_s}C_{c,s}) + (\Psi_{\zeta_s}C_{c,s})\otimes\tilde{A}_{c,s}\\ &+\sum_{i=1}^{m}\delta(\zeta_s - i)\Psi^{[2]}_i C^{[2]}_{c,s},\\ \tilde{C}^{[2]}_s =&\Phi^{[2]}_{\zeta_s}C^{[2]}_{c,s} + \Phi_{\zeta_s}C_{c,s}\otimes\tilde{C}_{c,s}\\ &+\tilde{C}_{c,s}\otimes(\Phi_{\zeta_s}C_{c,s}) + \tilde{C}^{[2]}_{c,s},\\ W^{[2]}_s =&\Big[\tilde{B}^{[2]}_{c,s} + \tilde{B}_{c,s}\otimes(\Psi_{\zeta_s}\tilde{D}_{c,s}) + (\Psi_{\zeta_s}\tilde{D}_{c,s})\\ &\otimes\tilde{B}_{c,s} + \Psi^{[2]}_{\zeta_s}\tilde{D}^{[2]}_{c,s}\Big]\gamma^{[2]}_s. \end{aligned} \tag{15}$$

Lemma 1: The dynamic evolution of the state covariance $\Pi_{s+1} \triangleq \mathbb{E}\{X_{s+1}X^T_{s+1}\}$ for the system (6) obeys

$$\begin{aligned}\Pi_{s+1} =&\tilde{A}_{c,s}\Pi_s\tilde{A}^T_{c,s} + \sum_{i=1}^{m}p_i\Psi_iC_{c,s}\Pi_sC^T_{c,s}\Psi^T_i\\ &+\mathrm{Sym}\Big\{\tilde{A}_{c,s}\Pi_s(\sum_{i=1}^{m}p_i\Psi_iC_{c,s})^T\Big\} + \mathcal{Q}^{(11)}_{\vec{W}_s} \end{aligned} \tag{16}$$

where $\mathcal{Q}^{(11)}_{\vec{W}_s} \triangleq \mathbb{E}\{W_sW^T_s\}$ is given in (22), and Π_0 satisfies

$$\Pi_0 = \begin{bmatrix}\mathrm{st}(\varphi^{(2)}_{x_0}) & 0\\ 0 & 0\end{bmatrix}.$$

Proof: From the definition of Π_{s+1}, it follows immediately that

$$\begin{aligned}\Pi_{s+1} =&\mathbb{E}\{\tilde{A}_sX_sX^T_s\tilde{A}^T_s\} + \mathbb{E}\{W_sW^T_s\}\\ &+\mathbb{E}\{\tilde{A}_sX_sW^T_s\} + \mathbb{E}\{W_sX^T_s\tilde{A}^T_s\}. \end{aligned} \tag{17}$$

Utilizing $\tilde{A}_s$ in (15) leads to

$$\begin{aligned}&\mathbb{E}\{\tilde{A}_sX_sX^T_s\tilde{A}^T_s\}\\ =&\tilde{A}_{c,s}\Pi_s\tilde{A}^T_{c,s} + \mathbb{E}\{\tilde{A}_{c,s}X_sX^T_s(\Psi_{\zeta_s}C_{c,s})^T\}\\ &+\mathbb{E}\{\Psi_{\zeta_s}C_{c,s}X_sX^T_s\tilde{A}^T_{c,s}\}\\ &+\mathbb{E}\Big\{(\Psi_{\zeta_s}C_{c,s})X_sX^T_s(\Psi_{\zeta_s}C_{c,s})^T\Big\} \end{aligned} \tag{18}$$

which, together with the following property

$$\delta(\zeta_s - i)\delta(\zeta_s - j) = \begin{cases}\delta(\zeta_s - i), & i = j\\ 0, & i \neq j\end{cases} \tag{19}$$

and $\mathbb{E}\{\delta(\zeta_s - i)\} = p_i$, gives rise to the first three terms in the right-hand side of (16).

Noting that x_s, w_s and v_s are zero-mean uncorrelated random variables, we derive that

$$\mathbb{E}\{X_sW^T_s\} = \mathbb{E}\begin{bmatrix} x_sw^T_sB^T_s & x_sV^T_s\\ \bar{z}_{s-1}w^T_sB^T_s & \bar{z}_{s-1}V^T_s\end{bmatrix} = 0. \tag{20}$$

Then, by substituting (18)-(20) into (17), we arrive at (16) and the proof is complete. ■

Next, we are ready to derive $\mathcal{Q}_{\vec{W}_s} \triangleq \mathbb{E}\{\vec{W}_s\vec{W}^T_s\}$ and $\mathcal{Q}_{\vec{V}_s} \triangleq \mathbb{E}\{\vec{V}_s\vec{V}^T_s\}$.

Lemma 2: The variance of the augmented noise $\vec{W}_s$ (i.e., $\mathcal{Q}_{\vec{W}_s} \triangleq \mathbb{E}\{\vec{W}_s\vec{W}^T_s\}$) satisfies

$$\mathcal{Q}_{\vec{W}_s} = \begin{bmatrix}\mathcal{Q}^{(11)}_{\vec{W}_s} & \mathcal{Q}^{(12)}_{\vec{W}_s}\\ (\mathcal{Q}^{(12)}_{\vec{W}_s})^T & \mathcal{Q}^{(22)}_{\vec{W}_s}\end{bmatrix} \tag{21}$$

where

$$\begin{aligned}\mathcal{Q}^{(11)}_{\vec{W}_s} \triangleq& \begin{bmatrix} B_s\mathrm{st}(\varphi^{(2)}_{w_s})B^T_s & 0\\ 0 & \sum\limits_{i=1}^{m}p_i\Phi_iD_s\mathrm{st}(\varphi^{(2)}_{v_s})D^T_s\Phi^T_i\end{bmatrix},\\ \mathcal{Q}^{(12)}_{\vec{W}_s} \triangleq&\tilde{B}_{c,s}\mathrm{st}(\varphi^{(3)}_{\gamma_s})\Big\{\tilde{B}^{[2]}_{c,s} + \tilde{B}_{c,s}\otimes(\sum_{i=1}^{m}p_i\Psi_i\tilde{D}_{c,s})\\ &+(\sum_{i=1}^{m}p_i\Psi_i\tilde{D}_{c,s})\otimes\tilde{B}_{c,s} + \sum_{i=1}^{m}p_i\Psi^{[2]}_i\tilde{D}^{[2]}_{c,s}\Big\}^T\\ &+\sum_{i=1}^{m}p_i\Psi_i\tilde{D}_{c,s}\mathrm{st}(\varphi^{(3)}_{\gamma_s})\Big\{\tilde{B}^{[2]}_{c,s} + \Psi^{[2]}_i\tilde{D}^{[2]}_{c,s}\\ &+\tilde{B}_{c,s}\otimes(\Psi_i\tilde{D}_{c,s}) + (\Psi_i\tilde{D}_{c,s})\otimes\tilde{B}_{c,s}\Big\}^T,\\ \Lambda^{(1)}_{i,s} \triangleq&\tilde{A}_{c,s}\Pi_s\tilde{A}^T_{c,s} + \tilde{A}_{c,s}\Pi_sC^T_{c,s}\Psi^T_i\\ &+\Psi_iC_{c,s}\Pi_s\tilde{A}^T_{c,s} + \Psi_iC_{c,s}\Pi_sC^T_{c,s}\Psi^T_i,\\ \Lambda^{(2)}_{i,s} \triangleq&\tilde{B}_{c,s}\mathrm{st}(\varphi^{(2)}_{\gamma_s})\tilde{B}^T_{c,s} + \tilde{B}_{c,s}\mathrm{st}(\varphi^{(2)}_{\gamma_s})\tilde{D}^T_{c,s}\Psi^T_i\\ &+\Psi_i\tilde{D}_{c,s}\mathrm{st}(\varphi^{(2)}_{\gamma_s})\tilde{B}^T_{c,s} + \Psi_i\tilde{D}_{c,s}\mathrm{st}(\varphi^{(2)}_{\gamma_s})\tilde{D}^T_{c,s}\Psi^T_i,\\ \Lambda^{(3)}_{i,s} \triangleq&\tilde{B}^{[2]}_{c,s} + \tilde{B}_{c,s}\otimes(\Psi_i\tilde{D}_{c,s}) + (\Psi_i\tilde{D}_{c,s})\otimes\tilde{B}_{c,s} + \Psi^{[2]}_i\tilde{D}^{[2]}_{c,s},\\ \mathcal{Q}^{(22)}_{\vec{W}_s} \triangleq&\tilde{S}_{n_x+n_z}\Big(\sum_{i=1}^{m}p_i\Lambda^{(1)}_{i,s}\otimes\Lambda^{(2)}_{i,s}\Big)\tilde{S}^T_{n_x+n_z} + \sum_{i=1}^{m}p_i\Lambda^{(3)}_{i,s}\end{aligned}$$

$$\times \operatorname{st}(\varphi_{\gamma_s}^{(4)})(\Lambda_{i,s}^{(3)})^T - \varphi_{W_s}^{(2)}(\varphi_{W_s}^{(2)})^T. \tag{22}$$

Proof: According to the expression of W_s, we have

$$\begin{aligned}\mathbb{E}\{W_s W_s^T\} =&\mathbb{E}\{\tilde{B}_{c,s}\gamma_s\gamma_s^T\tilde{B}_{c,s}^T\} \\ &+ \mathbb{E}\{\tilde{B}_{c,s}\gamma_s\gamma_s^T(\Psi_{\zeta_s}\tilde{D}_{c,s})^T\} \\ &+ \mathbb{E}\{\Psi_{\zeta_s}\tilde{D}_{c,s}\gamma_s\gamma_s^T\tilde{B}_{c,s}^T\} \\ &+ \mathbb{E}\{\Psi_{\zeta_s}\tilde{D}_{c,s}\gamma_s\gamma_s^T(\Psi_{\zeta_s}\tilde{D}_{c,s})^T\}.\end{aligned} \tag{23}$$

Taking the statistical properties of w_s and v_s into account yields $\mathbb{E}\{w_s v_s^T\} = 0$. Therefore, (23) guarantees that $\mathcal{Q}_{\vec{W}_s}^{(11)}$ holds. Furthermore, it can be deduced that

$$\begin{aligned}\mathcal{Q}_{\vec{W}_s}^{(12)} =&\mathbb{E}\Big\{W_s\big[\tilde{S}_{n_x+n_z}(\tilde{A}_s X_s \otimes W_s) + W_s^{[2]} - \varphi_{W_s}^{(2)}\big]^T\Big\} \\ =&\mathbb{E}\{W_s(W_s^{[2]})^T\},\end{aligned} \tag{24}$$

in which we have utilized the facts that W_s is uncorrelated with X_s and $\mathbb{E}\{W_s\} = 0$.

Utilizing W_s and $W_s^{[2]}$ in (15), after tedious calculations, we obtain $\mathcal{Q}_{\vec{W}_s}^{(12)}$ in (22). Moreover, $\mathcal{Q}_{\vec{W}_{22,s}}$ can be computed as

$$\begin{aligned}\mathbb{E}\{\tilde{W}_s\tilde{W}_s^T\} =&\mathbb{E}\Big\{\big[\tilde{S}_{n_x+n_z}(\tilde{A}_s X_s \otimes W_s) + W_s^{[2]} - \varphi_{W_s}^{(2)}\big] \\ &\times \big[\tilde{S}_{n_x+n_z}(\tilde{A}_s X_s \otimes W_s) + W_s^{[2]} - \varphi_{W_s}^{(2)}\big]^T\Big\} \\ =&\tilde{S}_{n_x+n_z}\mathbb{E}\{(\tilde{A}_s X_s X_s^T\tilde{A}_s^T)\otimes(W_s W_s^T)\}\tilde{S}_{n_x+n_z}^T \\ &+ \mathbb{E}\{W_s^{[2]}(W_s^{[2]})^T\} - \varphi_{W_s}^{(2)}(\varphi_{W_s}^{(2)})^T.\end{aligned} \tag{25}$$

Concerning the first term in the right-hand side of (25), we have

$$\begin{aligned}&\mathbb{E}\Big\{(\tilde{A}_s X_s X_s^T\tilde{A}_s^T)\otimes(W_s W_s^T)\Big\} \\ =&\mathbb{E}\Big\{\big[\tilde{A}_{c,s}X_s X_s^T\tilde{A}_{c,s}^T + \tilde{A}_{c,s}X_s X_s^T C_{c,s}^T\Psi_{\zeta_s}^T \\ &+ \Psi_{\zeta_s}C_{c,s}X_s X_s^T\tilde{A}_{c,s}^T + \Psi_{\zeta_s}C_{c,s}X_s X_s^T C_{c,s}^T\Psi_{\zeta_s}^T\big] \\ &\otimes\big[\tilde{B}_{c,s}\gamma_s\gamma_s^T\tilde{B}_{c,s}^T + \tilde{B}_{c,s}\gamma_s\gamma_s^T\tilde{D}_{c,s}^T\Psi_{\zeta_s}^T \\ &+ \Psi_{\zeta_s}\tilde{D}_{c,s}\gamma_s\gamma_s^T\tilde{B}_{c,s}^T + \Psi_{\zeta_s}\tilde{D}_{c,s}\gamma_s\gamma_s^T\tilde{D}_{c,s}^T\Psi_{\zeta_s}^T\big]\Big\} \\ =&\sum_{i=1}^m p_i\Lambda_{i,s}^{(1)}\otimes\Lambda_{i,s}^{(2)}.\end{aligned} \tag{26}$$

Applying $W^{[2]}$ in (15) and $\Lambda_{i,s}^{(3)}$ defined in (22), we obtain

$$\mathbb{E}\{W_s^{[2]}(W_s^{[2]})^T\} = \sum_{i=1}^m p_i\Lambda_{i,s}^{(3)}\operatorname{st}(\varphi_{\gamma_s}^{(4)})(\Lambda_{i,s}^{(3)})^T, \tag{27}$$

and therefore

$$\begin{aligned}\varphi_{W_s}^{(2)} =&\Big[\tilde{B}_{c,s}^{[2]} + \sum_{i=1}^m p_i\tilde{B}_{c,s}\otimes(\Psi_i\tilde{D}_{c,s}) + \sum_{i=1}^m p_i(\Psi_i\tilde{D}_{c,s}) \\ &\otimes\tilde{B}_{c,s} + \sum_{i=1}^m p_i\Psi_i^{[2]}\tilde{D}_{c,s}^{[2]}\Big]\varphi_{\gamma_s}^{(2)}.\end{aligned} \tag{28}$$

Combining (25)-(28), we obtain the expression of $\mathcal{Q}_{\vec{W}_s}^{(22)}$, and this completes the proof. ■

Remark 3: In Lemma 2, the statistical properties of $\vec{W}_s$ are discussed. These properties appear to be influenced by the system coefficient matrices, state covariance, RAP, and high-order moments of the noises. A noteworthy challenge emerges from the correlations found among $\vec{W}_s$, w_s and v_s during the derivation. To navigate this challenge, W_s is first decomposed into $(\tilde{B}_{c,s} + \Psi_{\zeta_s}\tilde{D}_{c,s})\gamma_s$, and the augmented noise γ_s is introduced as its high-order moments can be more readily discerned. Moreover, given the concurrent existence of Ψ_{ζ_s} in both $\tilde{A}_s$ and W_s, the equation $\mathbb{E}\Big\{(\tilde{A}_s X_s X_s^T\tilde{A}_s^T)\otimes(W_s W_s^T)\Big\} \neq \mathbb{E}\{(\tilde{A}_s X_s X_s^T\tilde{A}_s^T)\}\otimes\mathbb{E}\{W_s W_s^T\}$ is yielded. A viable solution involved isolating Ψ_{ζ_s} directly from the original matrices $\tilde{A}_s$ and W_s. Subsequent to this, the properties of $\delta(\zeta_s - i)$ are leveraged, for instance, using $\delta(\zeta_s - i)\delta(\zeta_s - j) = 0 (i \neq j)$.

Lemma 3: The variance of the augmented noise $\vec{V}_s$ (i.e., $\mathcal{Q}_{\vec{V}_s} \triangleq \mathbb{E}\{\vec{V}_s\vec{V}_s^T\}$) satisfies

$$\mathcal{Q}_{\vec{V}_s} = \begin{bmatrix} \mathcal{Q}_{\vec{V}_s}^{(11)} & \mathcal{Q}_{\vec{V}_s}^{(12)} \\ (\mathcal{Q}_{\vec{V}_s}^{(12)})^T & \mathcal{Q}_{\vec{V}_s}^{(22)} \end{bmatrix} \tag{29}$$

where

$$\begin{aligned}\mathcal{Q}_{\vec{V}_s}^{(11)} \triangleq&\sum_{i=1}^m p_i\Phi_i D_s\operatorname{st}(\varphi_{v_s}^{(2)})D_s^T\Phi_i^T, \\ \mathcal{Q}_{\vec{V}_s}^{(12)} \triangleq&\sum_{i=1}^m p_i\Phi_i D_s\operatorname{st}(\varphi_{v_s}^{(3)})(D_s^{[2]})^T(\Phi_i^{[2]})^T, \\ \mathcal{Q}_{\vec{V}_s}^{(22)} \triangleq&\tilde{S}_{n_z}\Big\{\sum_{i=1}^m p_i[\Phi_i C_{c,s}\Pi_s C_{c,s}^T\Phi_i^T + \tilde{C}_{c,s}\Pi_s\tilde{C}_{c,s}^T \\ &+ \Phi_i C_{c,s}\Pi_s\tilde{C}_{c,s}^T + \tilde{C}_{c,s}\Pi_s C_{c,s}^T\Phi_i^T]\otimes(\Phi_i D_s\operatorname{st}(\varphi_{v_s}^{(2)}) \\ &\times D_s^T\Phi_i^T)\Big\}\tilde{S}_{n_z}^T + \sum_{i=1}^m p_i\Phi_i^{[2]}D_s^{[2]}\operatorname{st}(\varphi_{v_s}^{(4)})(D_s^{[2]})^T(\Phi_i^{[2]})^T \\ &- (\sum_{i=1}^m p_i\Phi_i^{[2]}D_s^{[2]}\varphi_{v_s}^{(2)})(\sum_{i=1}^m p_i\Phi_i^{[2]}D_s^{[2]}\varphi_{v_s}^{(2)})^T.\end{aligned} \tag{30}$$

Proof: From the expression of V_s, we see that

$$\begin{aligned}\mathbb{E}\{V_s V_s^T\} =&\mathbb{E}\{\Phi_{\zeta_s}D_s v_s v_s^T D_s^T\Phi_{\zeta_s}^T\} \\ =&\sum_{i=1}^m p_i\Phi_i D_s\operatorname{st}(\varphi_{v_s}^{(2)})D_s^T\Phi_i^T.\end{aligned} \tag{31}$$

Moreover, it is not difficult to verify that

$$\begin{aligned}&\mathbb{E}\{V_s(\tilde{V}_s^T)^T\} \\ =&\mathbb{E}\{V_s[\tilde{S}_{n_z}(\tilde{C}_s X_s\otimes V_s) + V_s^{[2]} - \varphi_{V_s}^{(2)}]^T\} \\ =&\mathbb{E}\{V_s(V_s^{[2]})^T\} = \mathbb{E}\{\Phi_{\zeta_s}D_s v_s(v_s^{[2]})^T(D_s^{[2]})^T(\Phi_{\zeta_s}^{[2]})^T\} \\ =&\sum_{i=1}^m p_i\Phi_i D_s\operatorname{st}(\varphi_{v_s}^{(3)})(D_s^{[2]})^T(\Phi_i^{[2]})^T.\end{aligned} \tag{32}$$

With regard to the last term $\mathbb{E}\{\tilde{V}_s\tilde{V}_s^T\}$, we have

$$\begin{aligned}\mathbb{E}\{\tilde{V}_s\tilde{V}_s^T\} =&\mathbb{E}\{\tilde{S}_{n_z}(\tilde{C}_s X_s X_s^T\tilde{C}_s^T)\otimes(V_s V_s^T)\tilde{S}_{n_z}^T\} \\ &+ \mathbb{E}\{V_s^{[2]}(V_s^{[2]})^T\} - \varphi_{V_s}^{(2)}(\varphi_{V_s}^{(2)})^T.\end{aligned} \tag{33}$$

Along the similar line of deriving (26), we obtain

$$\begin{aligned}&\mathbb{E}\{(\tilde{C}_s X_s X_s^T\tilde{C}_s^T)\otimes(V_s V_s^T)\} \\ =&\mathbb{E}\{\big[\Phi_{\zeta_s}C_{c,s}X_s X_s^T C_{c,s}^T(\Phi_{\zeta_s})^T + \tilde{C}_{c,s}X_s X_s^T\tilde{C}_{c,s}^T \\ &+ \Phi_{\zeta_s}C_{c,s}X_s X_s^T\tilde{C}_{c,s}^T + \tilde{C}_{c,s}X_s X_s^T C_{c,s}^T\Phi_{\zeta_s}^T\big]\end{aligned}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2025.3616848, IEEE Transactions on Systems Man and Cybernetics Systems

$$\begin{aligned}&\otimes(\Phi_{\zeta_s}D_s v_s v_s^T D_s^T\Phi_{\zeta_s}^T)\}\\ =&\sum_{i=1}^{m}p_i\Big[\Phi_i C_{c,s}\Pi_s C_{c,s}^T\Phi_i^T+\tilde{C}_{c,s}\Pi_s\tilde{C}_{c,s}^T\\ &+\Phi_i C_{c,s}\Pi_s\tilde{C}_{c,s}^T+\tilde{C}_{c,s}\Pi_s C_{c,s}^T\Phi_i^T\Big]\\ &\otimes(\Phi_i D_s\mathrm{st}(\varphi_{v_s}^{(2)})D_s^T\Phi_i^T).\end{aligned}\tag{34}$$

It follows readily from $V_s=\Phi_{\zeta_s}D_s v_s$ that

$$V_s^{[2]}=\Phi_{\zeta_s}^{[2]}D_s^{[2]}v_s^{[2]},\tag{35}$$

and $\varphi_{V_s}^{(2)}$ also satisfies

$$\varphi_{V_s}^{(2)}=\sum_{i=1}^{m}p_i\Phi_i^{[2]}D_s^{[2]}\varphi_{v_s}^{(2)}.\tag{36}$$

In addition, it is straightforward to see that

$$\begin{aligned}&\mathbb{E}\{V_s^{[2]}(V_s^{[2]})^T\}\\ =&\mathbb{E}\Big\{\sum_{i=1}^{m}\delta(\zeta_s-i)\Phi_i^{[2]}D_s^{[2]}v_s^{[2]}(v_s^{[2]})^T\\ &\times(D_s^{[2]})^T(\sum_{i=1}^{m}\delta(\zeta_s-i)\Phi_i^{[2]})^T\Big\}\\ =&\sum_{i=1}^{m}p_i\Phi_i^{[2]}D_s^{[2]}\mathrm{st}(\varphi_{v_s}^{(4)})(D_s^{[2]})^T(\Phi_i^{[2]})^T.\end{aligned}\tag{37}$$

Finally, substituting (34)-(37) into (33) results in $\mathcal{Q}_{\vec{V}_s}^{(22)}$ in (30), which completes the proof. ■

B. Filter Design

In this subsection, an upper bound on the FEC is to be investigated and, subsequently, the gain parameter $\Re_{s+1}$ will be designed by minimizing the established upper bound.

Lemma 4: The prediction error covariance $\Im_{s+1|s}$ satisfies

$$\begin{aligned}\Im_{s+1|s}=&\vec{A}_s^{(1)}\Im_{s|s}(\vec{A}_s^{(1)})^T+\vec{A}_s^{(1)}\Im_{s|s}(\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)})^T\\ &+\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)}\Im_{s|s}(\vec{A}_s^{(1)})^T+\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)}\Im_{s|s}(\vec{A}_{i,s}^{(2)})^T\\ &+\mathbb{E}\{\vec{A}_s\chi_{s|s}\vec{W}_s^T\}+\mathbb{E}\{\vec{W}_s\chi_{s|s}^T\vec{A}_s^T\}+\mathcal{Q}_{\vec{W}_s}\end{aligned}\tag{38}$$

where

$$\vec{A}_s^{(1)}\triangleq\begin{bmatrix}\tilde{A}_{c,s}&0\\0&\tilde{A}_{c,s}^{[2]}\end{bmatrix},\quad \vec{A}_{i,s}^{(2)}\triangleq\begin{bmatrix}\Psi_i C_{c,s}&0\\0&\vec{A}_{i,s}^{(22)}\end{bmatrix},\tag{39}$$

with

$$\vec{A}_{i,s}^{(22)}\triangleq\tilde{A}_{c,s}\otimes\Psi_i C_{c,s}+\Psi_i C_{c,s}\otimes\tilde{A}_{c,s}+\Psi_i^{[2]}C_{c,s}^{[2]}.$$

Proof: Subtracting $\hat{\vec{X}}_{s+1|s}$ from X_{s+1} yields

$$\chi_{s+1|s}=\vec{A}_s\chi_{s|s}+\vec{W}_s.\tag{40}$$

Then, it can be deduced that

$$\begin{aligned}\Im_{s+1|s}=&\mathbb{E}\{\vec{A}_s\chi_{s|s}\chi_{s|s}^T\vec{A}_s^T\}+\mathbb{E}\{\vec{W}_s\vec{W}_s^T\}\\ &+\mathbb{E}\{\vec{A}_s\chi_{s|s}\vec{W}_s^T\}+\mathbb{E}\{\vec{W}_s\chi_{s|s}^T\vec{A}_s^T\}.\end{aligned}\tag{41}$$

Obviously, $\vec{A}_s$ can be rewritten as follows:

$$\vec{A}_s=\vec{A}_s^{(1)}+\sum_{i=1}^{m}\delta(\zeta_s-i)\vec{A}_{i,s}^{(2)},\tag{42}$$

from which one infers that

$$\begin{aligned}&\mathbb{E}\{\vec{A}_s\chi_{s|s}\chi_{s|s}^T\vec{A}_s^T\}\\ =&\vec{A}_s^{(1)}\Im_{s|s}(\vec{A}_s^{(1)})^T+\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)}\Im_{s|s}(\vec{A}_s^{(1)})^T\\ &+\vec{A}_s^{(1)}\Im_{s|s}(\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)})^T+\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)}\Im_{s|s}(\vec{A}_{i,s}^{(2)})^T.\end{aligned}\tag{43}$$

Combining (41) with (43) results in (38), and the proof is complete. ■

Remark 4: It should be noted that a pivotal step in determining the prediction error covariance involves solving (43). However, the complexity of $\vec{A}_s$, brought about by matrices Φ_{ζ_s} and $I-\Phi_{\zeta_s}$, presents a significant challenge in deriving $\mathbb{E}\{\vec{A}_s\chi_{s|s}\chi_{s|s}^T\vec{A}_s^T\}$. With the application of the matrix decomposition technique, $\vec{A}_s$ can be simplified as (42), thereby addressing the identified challenge. Moreover, owing to the correlation between $\vec{W}_s$ and $\vec{V}_s$, $\mathbb{E}\{\vec{A}_s\chi_{s|s}\vec{W}_s^T\}$ is not a null matrix, and this allows for an efficient solution to find an upper bound of $\Im_{s+1|s}$ by focusing on $\mathbb{E}\{\vec{A}_s\chi_{s|s}\vec{W}_s^T\}+\mathbb{E}\{\vec{W}_s\chi_{s|s}^T\vec{A}_s^T\}$.

Lemma 5: The FEC $\Im_{s+1|s+1}$ satisfies

$$\begin{aligned}\Im_{s+1|s+1}=&\Im_{s+1|s}-\Re_{s+1}\Theta_{s+1}\Im_{s+1|s}-\Im_{s+1|s}\Theta_{s+1}^T\Re_{s+1}^T\\ &+\Re_{s+1}\Big\{\sum_{i=1}^{4}\aleph_{M_{s+1}^{(ii)}}+\mathrm{Sym}\{\sum_{i=1}^{3}\sum_{j>i}^{4}\aleph_{M_{s+1}^{(ij)}}\}\\ &+\mathcal{Q}_{\vec{V}_{s+1}}\Big\}\Re_{s+1}^T\end{aligned}\tag{44}$$

where

$$\Theta_{s+1}\triangleq\begin{bmatrix}\sum_{i=1}^{m}p_i\Phi_i C_{c,s+1}+\tilde{C}_{c,s+1}&0\\0&\vec{C}_{s+1}^{(22)}\end{bmatrix},$$

$$\vec{C}_{c,s+1}\triangleq\begin{bmatrix}C_{c,s+1}&0\\0&C_{c,s+1}^{[2]}\end{bmatrix},\hat{\Phi}_i\triangleq\begin{bmatrix}\Phi_i&0\\0&\Phi_i^{[2]}\end{bmatrix},$$

$$\aleph_{M_{s+1}^{(11)}}\triangleq\sum_{i=1}^{m}p_i\hat{\Phi}_i\vec{C}_{c,s+1}\Im_{s+1|s}\vec{C}_{c,s+1}^T\hat{\Phi}_i^T,$$

$$\aleph_{M_{s+1}^{(22)}}\triangleq\mathcal{M}_{s+1}^{(2)}\Im_{s+1|s}(\mathcal{M}_{s+1}^{(2)})^T,$$

$$\aleph_{M_{s+1}^{(33)}}\triangleq\begin{bmatrix}0&0\\0&\aleph_{M_{s+1}^{(33)}}^{(22)}\end{bmatrix},\aleph_{M_{s+1}^{(44)}}\triangleq\begin{bmatrix}0&0\\0&\aleph_{M_{s+1}^{(44)}}^{(22)}\end{bmatrix},$$

$$\aleph_{M_{s+1}^{(12)}}\triangleq\sum_{i=1}^{m}p_i\hat{\Phi}_i\vec{C}_{c,s+1}\Im_{s+1|s}(\mathcal{M}_{s+1}^{(2)})^T,$$

$$\aleph_{M_{s+1}^{(13)}}\triangleq\begin{bmatrix}0&\aleph_{M_{s+1}^{(13)}}^{(12)}\\0&\aleph_{M_{s+1}^{(13)}}^{(22)}\end{bmatrix},\aleph_{M_{s+1}^{(14)}}\triangleq\begin{bmatrix}0&\aleph_{M_{s+1}^{(14)}}^{(12)}\\0&\aleph_{M_{s+1}^{(14)}}^{(22)}\end{bmatrix},$$

$$\aleph_{M_{s+1}^{(23)}}\triangleq\mathcal{M}_{s+1}^{(2)}\Im_{s+1|s}\begin{bmatrix}0&0\\0&\sum_{i=1}^{m}p_i\Phi_i C_{c,s+1}\otimes\tilde{C}_{c,s+1}\end{bmatrix}^T,$$

$$\aleph_{M^{(24)}_{s+1}} \triangleq \mathcal{M}^{(2)}_{s+1}\Im_{s+1|s}\begin{bmatrix} 0 & 0 \\ 0 & \tilde{C}_{c,s+1}\otimes(\sum_{i=1}^{m}p_i\Phi_iC_{c,s+1}) \end{bmatrix}^T,$$

$$\aleph_{M^{(34)}_{s+1}} \triangleq \begin{bmatrix} 0 & 0 \\ 0 & \aleph^{(22)}_{M^{(34)}_{s+1}} \end{bmatrix}, H_1 \triangleq [0, I], H_2 \triangleq [I, 0], \tag{45}$$

with

$$\begin{aligned}\vec{C}^{(22)}_{s+1} \triangleq &\sum_{i=1}^{m}p_i\Phi_i^{[2]}C^{[2]}_{c,s+1} + \sum_{i=1}^{m}p_i\Phi_iC_{c,s+1}\otimes\tilde{C}_{c,s+1} \\ &+ \tilde{C}_{c,s+1}\otimes(\sum_{i=1}^{m}p_i\Phi_iC_{c,s+1}) + \tilde{C}^{[2]}_{c,s+1},\\
\aleph^{(22)}_{M^{(33)}_{s+1}} \triangleq &\sum_{i=1}^{m}p_i(\Phi_iC_{c,s+1}\otimes\tilde{C}_{c,s+1})H_1\Im_{s+1|s}H_1^T \\ &\times(\Phi_iC_{c,s+1}\otimes\tilde{C}_{c,s+1})^T,\\
\aleph^{(22)}_{M^{(44)}_{s+1}} \triangleq &\sum_{i=1}^{m}p_i(\tilde{C}_{c,s+1}\otimes\Phi_iC_{c,s+1})H_1\Im_{s+1|s}H_1^T \\ &\times(\tilde{C}_{c,s+1}\otimes\Phi_iC_{c,s+1})^T,\\
\aleph^{(12)}_{M^{(13)}_{s+1}} = &\sum_{i=1}^{m}p_i\Phi_iC_{c,s+1}H_2\Im_{s+1|s}H_1^T(\Phi_iC_{c,s+1}\otimes\tilde{C}_{c,s+1})^T,\\
\aleph^{(22)}_{M^{(13)}_{s+1}} = &\sum_{i=1}^{m}p_i\Phi_i^{[2]}C^{[2]}_{c,s+1}H_1\Im_{s+1|s}H_1^T(\Phi_iC_{c,s+1}\otimes\tilde{C}_{c,s+1})^T,\\
\aleph^{(12)}_{M^{(14)}_{s+1}} \triangleq &\sum_{i=1}^{m}p_i\Phi_iC_{c,s+1}H_2\Im_{s+1|s}H_1^T(\tilde{C}_{c,s+1}\otimes\Phi_iC_{c,s+1})^T,\\
\aleph^{(22)}_{M^{(14)}_{s+1}} \triangleq &\sum_{i=1}^{m}p_i\Phi_i^{[2]}C^{[2]}_{c,s+1}H_1\Im_{s+1|s}H_1^T(\tilde{C}_{c,s+1}\otimes\Phi_iC_{c,s+1})^T,\\
\aleph^{(22)}_{M^{(34)}_{s+1}} \triangleq &\sum_{i=1}^{m}p_i(\Phi_iC_{c,s+1}\otimes\tilde{C}_{c,s+1})H_1\Im_{s+1|s}H_1^T \\ &\times(\tilde{C}_{c,s+1}\otimes\Phi_iC_{c,s+1})^T.\end{aligned}\tag{46}$$

Proof: From $\hat{\vec{X}}_{s+1|s+1}$ and X_{s+1}, it is evident that

$$\begin{aligned}\chi_{s+1|s+1} =&\chi_{s+1|s} - \Re_{s+1}(\vec{C}_{s+1}\chi_{s+1|s} + \vec{V}_{s+1}) \\ =&(I - \Re_{s+1}\vec{C}_{s+1})\chi_{s+1|s} - \Re_{s+1}\vec{V}_{s+1}\end{aligned}\tag{47}$$

and, therefore, $\Im_{s+1|s+1}$ satisfies

$$\begin{aligned}\Im_{s+1|s+1} =&\mathbb{E}\{(I - \Re_{s+1}\vec{C}_{s+1})\chi_{s+1|s}\chi^T_{s+1|s}(I - \Re_{s+1}\vec{C}_{s+1})^T\} \\ &- \mathbb{E}\{(I - \Re_{s+1}\vec{C}_{s+1})\chi_{s+1|s}\vec{V}^T_{s+1}\Re^T_{s+1}\} \\ &- \mathbb{E}\{\Re_{s+1}\vec{V}_{s+1}\chi^T_{s+1|s}(I - \Re_{s+1}\vec{C}_{s+1})^T\} \\ &+ \Re_{s+1}\mathcal{Q}_{\vec{V}_{s+1}}\Re^T_{s+1}.\end{aligned}\tag{48}$$

It is easy to see that

$$\begin{aligned}&\mathbb{E}\{\chi_{s+1|s}\vec{V}^T_{s+1}\} \\ =&\mathbb{E}\{(\vec{A}_s\chi_{s|s}\vec{V}^T_{s+1}\} + \mathbb{E}\{\vec{W}_s\vec{V}^T_{s+1}\} = 0\end{aligned}\tag{49}$$

which indicates that $\Im_{s+1|s+1}$ can be simplified as

$$\begin{aligned}\Im_{s+1|s+1} =&\mathbb{E}\{(I - \Re_{s+1}\vec{C}_{s+1})\chi_{s+1|s}\chi^T_{s+1|s}(I - \Re_{s+1}\vec{C}_{s+1})^T\} \\ &+ \Re_{s+1}\mathcal{Q}_{\vec{V}_{s+1}}\Re^T_{s+1}.\end{aligned}\tag{50}$$

In order to facilitate subsequent discussion, let us define

$$\vec{C}_{s+1} = \sum_{i=1}^{4}\mathcal{M}^{(i)}_{s+1}\tag{51}$$

where

$$\begin{aligned}\mathcal{M}^{(1)}_{s+1} \triangleq &\begin{bmatrix} \Phi_{\zeta_{s+1}} & 0 \\ 0 & \Phi^{[2]}_{\zeta_{s+1}} \end{bmatrix}\vec{C}_{c,s+1},\\
\mathcal{M}^{(2)}_{s+1} \triangleq &\begin{bmatrix} \tilde{C}_{c,s+1} & 0 \\ 0 & \tilde{C}^{[2]}_{c,s+1} \end{bmatrix},\\
\mathcal{M}^{(3)}_{s+1} \triangleq &\begin{bmatrix} 0 & 0 \\ 0 & \Phi_{\zeta_{s+1}}C_{c,s+1}\otimes\tilde{C}_{c,s+1} \end{bmatrix},\\
\mathcal{M}^{(4)}_{s+1} \triangleq &\begin{bmatrix} 0 & 0 \\ 0 & \tilde{C}_{c,s+1}\otimes(\Phi_{\zeta_{s+1}}C_{c,s+1}) \end{bmatrix}.\end{aligned}\tag{52}$$

Then, it is straightforward to verify that

$$\begin{aligned}&\mathbb{E}\{\vec{C}_{s+1}\chi_{s+1|s}\chi^T_{s+1|s}\vec{C}^T_{s+1}\} \\ =&\mathbb{E}\{(\sum_{i=1}^{4}\mathcal{M}^{(i)}_{s+1})\chi_{s+1|s}\chi^T_{s+1|s}(\sum_{i=1}^{4}\mathcal{M}^{(i)}_{s+1})^T\} \\ =&\sum_{i=1}^{4}\mathbb{E}\{\mathcal{M}^{(i)}_{s+1}\chi_{s+1|s}\chi^T_{s+1|s}(\mathcal{M}^{(i)}_{s+1})^T\} \\ &+ \mathrm{Sym}\Big\{\mathbb{E}\{\sum_{i=1}^{3}\sum_{j>i}^{4}\mathcal{M}^{(i)}_{s+1}\chi_{s+1|s}\chi^T_{s+1|s}(\mathcal{M}^{(j)}_{s+1})^T\}\Big\}.\end{aligned}\tag{53}$$

Next, we move onto the computation of $\aleph_{M^{(ij)}_{s+1}}$. It follows from the definition of $\Phi_{\zeta_{s+1}}$ that

$$\Phi^{[2]}_{\zeta_{s+1}} = \sum_{i=1}^{m}\delta(\zeta_{s+1} - i)\Phi^{[2]}_i,\tag{54}$$

and thus $\mathcal{M}^{(1)}_{s+1}$ can be expressed as

$$\mathcal{M}^{(1)}_{s+1} = \sum_{i=1}^{m}\delta(\zeta_{s+1} - i)\hat{\Phi}_i\vec{C}_{c,s+1}.\tag{55}$$

Furthermore, one immediately has

$$\begin{aligned}\aleph_{M^{(11)}_{s+1}} \triangleq&\mathbb{E}\{\sum_{i=1}^{m}\delta(\zeta_{s+1} - i)\hat{\Phi}_i\vec{C}_{c,s+1}\chi_{s+1|s}\chi^T_{s+1|s} \\ &\times(\sum_{i=1}^{m}\delta(\zeta_{s+1} - i)\hat{\Phi}_i\vec{C}_{c,s+1})^T\} \\ =&\sum_{i=1}^{m}p_i\hat{\Phi}_i\vec{C}_{c,s+1}\Im_{s+1|s}\vec{C}^T_{c,s+1}\hat{\Phi}^T_i.\end{aligned}\tag{56}$$

Since $\mathcal{M}^{(2)}_{s+1}$ is a constant matrix, it is derived that

$$\begin{aligned}\aleph_{M^{(22)}_{s+1}} \triangleq&\mathbb{E}\{\mathcal{M}^{(2)}_{s+1}\chi_{s+1|s}\chi^T_{s+1|s}(\mathcal{M}^{(2)}_{s+1})^T\} \\ =&\mathcal{M}^{(2)}_{s+1}\Im_{s+1|s}(\mathcal{M}^{(2)}_{s+1})^T.\end{aligned}\tag{57}$$

For the $(2, 2)$-element of $\aleph_{M^{(33)}_{s+1}}$ (i.e., $\aleph^{(22)}_{M^{(33)}_{s+1}}$), we have

$$\begin{aligned}\aleph^{(22)}_{M^{(33)}_{s+1}} \triangleq&\mathbb{E}\{(\Phi_{\zeta_{s+1}}C_{c,s+1}\otimes\tilde{C}_{c,s+1})H_1\chi_{s+1|s}\chi^T_{s+1|s}H_1^T \\ &\times(\Phi_{\zeta_{s+1}}C_{c,s+1}\otimes\tilde{C}_{c,s+1})^T\}\end{aligned}$$

$$=\sum_{i=1}^{m} p_i(\Phi_i C_{c,s+1}\otimes\tilde{C}_{c,s+1})H_1\Im_{s+1|s}H_1^T \times(\Phi_i C_{c,s+1}\otimes\tilde{C}_{c,s+1})^T. \quad (58)$$

Along the similar line of deriving $\aleph^{(22)}_{M^{(33)}_{s+1}}$, we can obtain $\aleph^{(22)}_{M^{(44)}_{s+1}}$. Now, for the term $\aleph_{M^{(12)}_{s+1}}$, it is clear that

$$\begin{aligned}\aleph_{M^{(12)}_{s+1}} \triangleq&\mathbb{E}\Big\{\sum_{i=1}^{m}\delta(\zeta_{s+1}-i)\hat{\Phi}_i\vec{C}_{c,s+1}\chi_{s+1|s}\chi^T_{s+1|s}(\mathcal{M}^{(2)}_{s+1})^T\Big\}\\ =&\sum_{i=1}^{m}p_i\hat{\Phi}_i\vec{C}_{c,s+1}\Im_{s+1|s}(\mathcal{M}^{(2)}_{s+1})^T. \end{aligned}\quad (59)$$

For the terms $\aleph^{(12)}_{M^{(13)}_{s+1}}$ and $\aleph^{(22)}_{M^{(13)}_{s+1}}$, we know

$$\begin{aligned}\aleph^{(12)}_{M^{(13)}_{s+1}} \triangleq&\mathbb{E}\{(\Phi_{\zeta_{s+1}}C_{c,s+1})H_2\Im_{s+1|s}H_1^T\\ &\times(\Phi_{\zeta_{s+1}}C_{c,s+1}\otimes\tilde{C}_{c,s+1})^T\}\\ =&\sum_{i=1}^{m}p_i\Phi_i C_{c,s+1}H_2\Im_{s+1|s}H_1^T\\ &\times(\Phi_i C_{c,s+1}\otimes\tilde{C}^T_{c,s+1})^T \end{aligned}\quad (60)$$

and

$$\begin{aligned}\aleph^{(22)}_{M^{(13)}_{s+1}} \triangleq&\mathbb{E}\{\Phi^{[2]}_{\zeta_{s+1}}C^{[2]}_{c,s+1}H_1\Im_{s+1|s}H_1^T\\ &\times(\Phi_{\zeta_{s+1}}C_{c,s+1}\otimes\tilde{C}_{c,s+1})^T\}\\ =&\sum_{i=1}^{m}p_i\Phi_i^{[2]}C^{[2]}_{c,s+1}H_1\Im_{s+1|s}H_1^T\\ &\times(\Phi_i C_{c,s+1}\otimes\tilde{C}_{c,s+1})^T. \end{aligned}\quad (61)$$

Similarly, we can derive $\aleph^{(12)}_{M^{(14)}_{s+1}}$ and $\aleph^{(22)}_{M^{(14)}_{s+1}}$. Meanwhile, it is straightforward to show that

$$\begin{aligned}\aleph_{M^{(23)}_{s+1}} \triangleq&\mathbb{E}\{\mathcal{M}^{(2)}_{s+1}\chi_{s+1|s}\chi^T_{s+1|s}(\mathcal{M}^{(3)}_{s+1})^T\}\\ =&\mathcal{M}^{(2)}_{s+1}\Im_{s+1|s}\begin{bmatrix}0 & 0\\ 0 & \sum_{i=1}^{m}p_i\Phi_i C_{c,s+1}\otimes\tilde{C}_{c,s+1}\end{bmatrix}^T,\\ \aleph_{M^{(24)}_{s+1}} \triangleq&\mathbb{E}\{\mathcal{M}^{(2)}_{s+1}\chi_{s+1|s}\chi^T_{s+1|s}(\mathcal{M}^{(4)}_{s+1})^T\}\\ =&\mathcal{M}^{(2)}_{s+1}\Im_{s+1|s}\begin{bmatrix}0 & 0\\ 0 & \tilde{C}_{c,s+1}\otimes(\sum_{i=1}^{m}p_i\Phi_i C_{c,s+1})^T\end{bmatrix}^T. \end{aligned}\quad (62)$$

For the term $\aleph^{(22)}_{M^{(34)}_{s+1}}$, one has

$$\begin{aligned}\aleph^{(22)}_{M^{(34)}_{s+1}} \triangleq&\mathbb{E}\{(\Phi_{\zeta_{s+1}}C_{c,s+1}\otimes\tilde{C}_{c,s+1})H_1\Im_{s+1|s}H_1^T\\ &\times(\tilde{C}_{c,s+1}\otimes\Phi_{\zeta_{s+1}}C_{c,s+1})^T\}\\ =&\sum_{i=1}^{m}p_i(\Phi_i C_{c,s+1}\otimes\tilde{C}_{c,s+1})H_1\Im_{s+1|s}H_1^T\\ &\times(\tilde{C}_{c,s+1}\otimes\Phi_i C_{c,s+1})^T. \end{aligned}\quad (63)$$

Utilizing (51), after the tedious algebraic manipulation, we have $\mathbb{E}\{\vec{C}_{s+1}\}=\Theta_{s+1}$. Finally, substituting (56)-(63) and Θ_{s+1} into (50) leads to

$$\begin{aligned}\Im_{s+1|s+1}=&\Im_{s+1|s}-\Re_{s+1}\Theta_{s+1}\Im_{s+1|s}-\Im_{s+1|s}\Theta^T_{s+1}\Re^T_{s+1}\\ &+\Re_{s+1}\Big\{\sum_{i=1}^{4}\aleph_{M^{(ii)}_{s+1}}+\mathrm{Sym}\Big\{\sum_{i=1}^{3}\sum_{j>i}^{4}\aleph_{M^{(ij)}_{s+1}}\Big\}\Big\}\Re^T_{s+1}\\ &+\Re_{s+1}\mathcal{Q}_{\vec{V}_{s+1}}\Re^T_{s+1}, \end{aligned}\quad (64)$$

which completes this proof. ■

Remark 5: Up to this point, the FEC $\Im_{s+1|s+1}$ has been presented in Lemma 5 for the proposed quadratic filtering algorithm. Significant effort has been dedicated to addressing the core term $\mathbb{E}\{\vec{C}_{s+1}\chi_{s+1|s}\chi^T_{s+1|s}\vec{C}^T_{s+1}\}$ within $\Im_{s+1|s+1}$. In detail, the matrix $\vec{C}_{s+1}$ is partitioned into four terms $\mathcal{M}^{(i)}_{s+1}$ $(i=1,2,3,4)$ in an attempt to isolate $\Phi_{\zeta_{s+1}}$. Leveraging the properties of $\delta(\zeta_s-i)$, $\aleph_{M^{(ij)}_{s+1}}$ is subsequently derived, which effectively addresses the challenges posed by the RAP combined with ZOHS.

Theorem 1: Let $\tau_{1,s}>0$ be a given scalar. With the initial condition $\Im_{0|0}=\bar{\Im}_{0|0}>0$, if the following Riccati-like difference equations

$$\begin{aligned}\bar{\Im}_{s+1|s}=&(1+\tau_{1,s})\Big[\vec{A}^{(1)}_s\bar{\Im}_{s|s}(\vec{A}^{(1)}_s)^T+\sum_{i=1}^{m}p_i\vec{A}^{(2)}_{i,s}\bar{\Im}_{s|s}(\vec{A}^{(1)}_s)^T\\ &+\vec{A}^{(1)}_s\bar{\Im}_{s|s}(\sum_{i=1}^{m}p_i\vec{A}^{(2)}_{i,s})^T+\sum_{i=1}^{m}p_i\vec{A}^{(2)}_{i,s}\bar{\Im}_{s|s}(\vec{A}^{(2)}_{i,s})^T\Big]\\ &+(1+\tau^{-1}_{1,s})\mathcal{Q}_{\vec{W}_s} \end{aligned}\quad (65)$$

$$\begin{aligned}\bar{\Im}_{s+1|s+1}=&\bar{\Im}_{s+1|s}-\Re_{s+1}\Theta_{s+1}\bar{\Im}_{s+1|s}-\bar{\Im}_{s+1|s}\Theta^T_{s+1}\Re^T_{s+1}\\ &+\Re_{s+1}\Big\{\sum_{i=1}^{4}\bar{\aleph}_{M^{(ii)}_{s+1}}+\mathrm{Sym}\Big\{\sum_{i=1}^{3}\sum_{j>i}^{4}\bar{\aleph}_{M^{(ij)}_{s+1}}\Big\}\\ &+\mathcal{Q}_{\vec{V}_{s+1}}\Big\}\Re^T_{s+1} \end{aligned}\quad (66)$$

have positive-definite solutions $\bar{\Im}_{s+1|s}$ and $\bar{\Im}_{s+1|s+1}$, then the inequalities

$$\Im_{s+1|s}\le\bar{\Im}_{s+1|s},\quad \Im_{s+1|s+1}\le\bar{\Im}_{s+1|s+1} \quad (67)$$

hold for $s\ge0$, where $\bar{\aleph}_{M^{(ij)}_{s+1}}$ $(i,j=1,2,3,4)$ are derived from $\aleph_{M^{(ij)}_{s+1}}$ through replacing $\Im_{s+1|s}$ by $\bar{\Im}_{s+1|s}$.

Proof: This theorem will be proved by the mathematical induction method. Clearly, it is known from the initial condition that $\Im_{0|0}\le\bar{\Im}_{0|0}$. Assuming that $\Im_{s|s}\le\bar{\Im}_{s|s}$ holds, we need to prove $\Im_{s+1|s+1}\le\bar{\Im}_{s+1|s+1}$.

Using the following elementary inequality

$$\mathcal{G}\mathcal{H}^T+\mathcal{H}\mathcal{G}^T\le\tau\mathcal{G}\mathcal{G}^T+\tau^{-1}\mathcal{H}\mathcal{H}^T \quad (68)$$

where $\mathcal{G}$, $\mathcal{H}$ are known matrices and τ is a positive scalar, we have

$$\begin{aligned}&\mathbb{E}\{\vec{A}_s\chi_{s|s}\vec{W}^T_s\}+\mathbb{E}\{\vec{W}_s\chi^T_{s|s}\vec{A}^T_s\}\\ \le&\tau_{1,s}\mathbb{E}\{\vec{A}_s\chi_{s|s}\chi^T_{s|s}\vec{A}^T_s\}+\tau^{-1}_{1,s}\mathbb{E}\{\vec{W}_s\vec{W}^T_s\}. \end{aligned}\quad (69)$$

Based on (38) and (69), it is verified that

$$\Im_{s+1|s}$$

$$
\begin{aligned}
&\leq(1+\tau_{1,s})\mathbb{E}\{\vec{A}_s\chi_{s|s}\chi_{s|s}^T\vec{A}_s^T\}+(1+\tau_{1,s}^{-1})\mathbb{E}\{\vec{W}_s\vec{W}_s^T\}\\
&=(1+\tau_{1,s})\Big[\vec{A}_s^{(1)}\Im_{s|s}(\vec{A}_s^{(1)})^T+\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)}\Im_{s|s}(\vec{A}_s^{(1)})^T\\
&\quad+\vec{A}_s^{(1)}\Im_{s|s}(\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)})^T+\sum_{i=1}^{m}p_i\vec{A}_{i,s}^{(2)}\Im_{s|s}(\vec{A}_{i,s}^{(2)})^T\Big]\\
&\quad+(1+\tau_{1,s}^{-1})\mathcal{Q}_{\vec{W}_s}.
\end{aligned} \tag{70}
$$

Bearing the assumption $\Im_{s|s}\leq\bar{\Im}_{s|s}$ in mind, one has $\Im_{s+1|s}\leq\bar{\Im}_{s+1|s}$. On the other hand, the difference of $\bar{\Im}_{s+1|s+1}-\Im_{s+1|s+1}$ can be formulated as

$$
\begin{aligned}
&\bar{\Im}_{s+1|s+1}-\Im_{s+1|s+1}\\
=&\mathbb{E}\{(I-\Re_{s+1}\vec{C}_{s+1})(\bar{\Im}_{s+1|s}-\Im_{s+1|s})(I-\Re_{s+1}\vec{C}_{s+1})^T\}.
\end{aligned} \tag{71}
$$

From $\Im_{s+1|s}\leq\bar{\Im}_{s+1|s}$, it can be concluded that $\Im_{s+1|s+1}\leq\bar{\Im}_{s+1|s+1}$, which completes this proof. ∎

Remark 6: From (65)-(66), it can be observed that the FEC is significantly influenced by the RAP, ZOHS, and NGNs. Specifically, the presence of p_i and Φ_i in $\aleph_{M_{s+1}^{(ij)}}$ signifies the impact of the RAP. Meanwhile, the terms $\varphi_{w_s}^{(j)}$, $\varphi_{v_s}^{(j)}$ ($j=2,3,4$), found in $\mathcal{Q}_{\vec{W}_s}$ and $\mathcal{Q}_{\vec{V}_s}$, highlight the effect of the high-order moments of NGNs on $\bar{\Im}_{s+1|s+1}$.

In what follows, the optimal filter gain will be designed to minimize the aforementioned upper bound.

Theorem 2: The upper bound of the FEC $\bar{\Im}_{s+1|s+1}$ in (66) is minimized by the gain matrix as follows:

$$\Re_{s+1}=\bar{\Im}_{s+1|s}\Theta_{s+1}^T\Omega_{s+1}^{-1} \tag{72}$$

and the corresponding minimal upper bound is given by

$$\bar{\Im}_{s+1|s+1}=\bar{\Im}_{s+1|s}-\bar{\Im}_{s+1|s}\Theta_{s+1}^T\Omega_{s+1}^{-1}\Theta_{s+1}\bar{\Im}_{s+1|s} \tag{73}$$

where

$$\Omega_{s+1}\triangleq\sum_{i=1}^{4}\bar{\aleph}_{M_{s+1}^{(ii)}}+\mathrm{Sym}\{\sum_{i=1}^{3}\sum_{j>i}^{4}\bar{\aleph}_{M_{s+1}^{(ij)}}\}+\mathcal{Q}_{\vec{V}_{s+1}}. \tag{74}$$

Proof: Taking the partial derivation of $\mathrm{tr}(\bar{\Im}_{s+1|s+1})$ with respect to the parameters $\Re_{s+1}$, we have

$$\frac{\partial\mathrm{tr}(\bar{\Im}_{s+1|s+1})}{\partial\Re_{s+1}}=-2\bar{\Im}_{s+1|s}\Theta_{s+1}^T+2\Re_{s+1}\Omega_{s+1}. \tag{75}$$

Letting

$$\frac{\partial\mathrm{tr}(\bar{\Im}_{s+1|s+1})}{\partial\Re_{s+1}}=0,$$

we obtain

$$\Re_{s+1}=\bar{\Im}_{s+1|s}\Theta_{s+1}^T\Omega_{s+1}^{-1}, \tag{76}$$

which is (72), and the proof is complete. ∎

Remark 7: In Theorems 1-2, we develop a RAP-centric quadratic filtering algorithm grounded in the minimum-variance paradigm. To initiate the process, the underlying system is morphed into an enhanced version that reveals more about the interplay of RAP and NGNs, which is achieved by consolidating the augmented states (comprising the original state and the most recent measurement) and their second-order Kronecker powers. Subsequently, with the support of two distinct equations, an upper bound for the FEC is defined and, based on this upper bound, the gain parameter is meticulously devised. Given the intricacies introduced by integrating RAP scheduling with ZOHS, the matrix decomposition method proves indispensable in ensuring the practicality and efficiency of the proposed filtering algorithm.

Remark 8: So far, a novel quadratic filtering scheme has been introduced for linear discrete-time systems subjected to RAP, ZOHS and NGNs. When compared to existing literature on RAP, the filtering results presented in this study possess several distinctive features: 1) the filtering problem addressed is new since the combination of RAP scheduling with ZOHS has been explored for the first time in the context of linear non-Gaussian discrete-time systems; 2) a unique processing technique is developed to leverage matrix decomposition to separate the influence of RAP on the FEC; and 3) the proposed RAP-based quadratic filtering algorithm is innovative which seamlessly integrates information from RAP, ZOHS, and NGNs within a unified framework.

IV. An Illustrative Example

In this section, a simulation example is given to show the validity of the designed quadratic filtering scheme under the RAP.

Let us consider a linear non-Gaussian system with parameters given by

$$
\begin{aligned}
&A_s=\begin{bmatrix}0.35 & 0.4\sin(s)\\ 0.15 & 0.29+0.3\cos(s)\end{bmatrix},\quad B_s=\begin{bmatrix}0.1\\0.1\end{bmatrix},\\
&C_s=\begin{bmatrix}0.2 & 0.18\\ 0.5 & 0.3\end{bmatrix},\quad D_s=\begin{bmatrix}0.1\\0.1\end{bmatrix}.
\end{aligned}
$$

In this simulation, the sensor nodes are chosen as $m=2$, and the occurrence probabilities of the RAP scheduling are taken as $\mathrm{P}\{\zeta_s=1\}=0.6$ and $\mathrm{P}\{\zeta_s=2\}=0.4$. The initial state vector x_0 obeys the Gaussian distribution with $\mathbb{E}\{x_0\}=0$ and $\mathbb{E}\{x_0^2\}=0.01I_2$. The non-Gaussian noises w_s and v_s satisfy the following relationships

$$
\begin{aligned}
w_s=&-0.4\varphi_{w_s}+0.6(1-\varphi_{w_s})\\
v_s=&0.2\varphi_{v_s}-0.8(1-\varphi_{v_s})
\end{aligned}
$$

with the probability distribution of φ_{w_s} and φ_{v_s} being

$$
\begin{aligned}
\mathbb{P}\{\varphi_{w_s}=1\}=&0.6,\quad \mathbb{P}\{\varphi_{w_s}=0\}=0.4,\\
\mathbb{P}\{\varphi_{v_s}=1\}=&0.8,\quad \mathbb{P}\{\varphi_{v_s}=0\}=0.2.
\end{aligned}
$$

Also, we set $\tau_{1,s}=0.5$ and provide the 2nd, 3rd and 4th-order moments of w_s and v_s as follows:

$$
\begin{aligned}
\mathbb{E}\{w_s^2\}=&0.2400,\mathbb{E}\{w_s^3\}=0.0480,\mathbb{E}\{w_s^4\}=0.0672,\\
\mathbb{E}\{v_s^2\}=&0.1600,\mathbb{E}\{v_s^3\}=-0.0960,\mathbb{E}\{v_s^4\}=0.0832.
\end{aligned}
$$

Utilizing the quadratic filtering scheme presented in this paper, the simulation results are illustrated in Figs. 1-6. The trajectories of the system state $x_s=[x_{1,s},x_{2,s}]^T$ and their estimates are given in Figs. 1-2, from which we can observe that the designed RAP-based filter can effectively track the system states. Figs. 3-4 depict the curves of the upper bound of FEC and the mean square error (MSE). It is obvious to

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TSMC.2025.3616848, IEEE Transactions on Systems Man and Cybernetics Systems

see that the curves of upper bound are always higher than those of the MSE, which is consistent with our filtering result. Moreover, for the proposed quadratic filter and the linear filter that only uses $\bar{z}_s$, we plot comparison curves of their MSEs in Figs. 5-6, respectively. These comparison results provide additional evidence for the superiority of the RAP-based quadratic filtering method in enhancing filtering accuracy.

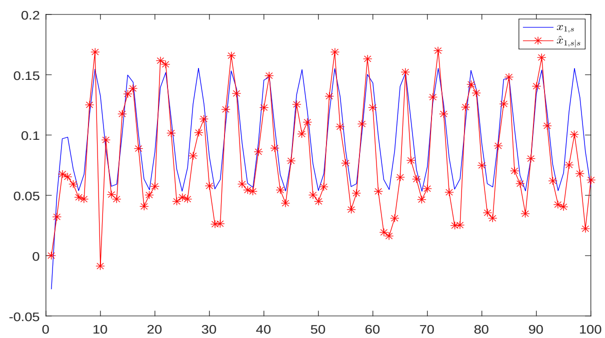


Fig. 1. $x_{1,s}$ and its estimation $\hat{x}_{1,s|s}$

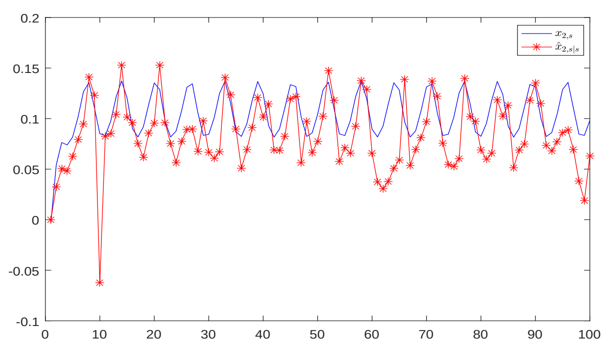


Fig. 2. $x_{2,s}$ and its estimation $\hat{x}_{2,s|s}$

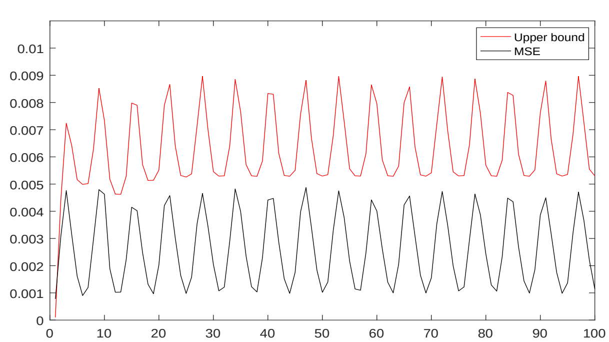


Fig. 3. The upper bound and MSE for $x_{1,s}$

V. Conclusions

In this paper, the RAP-based quadratic filtering algorithm has been developed for an array of linear non-Gaussian

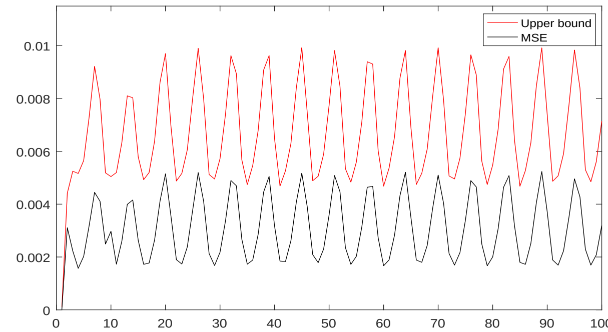


Fig. 4. The upper bound and MSE for $x_{2,s}$

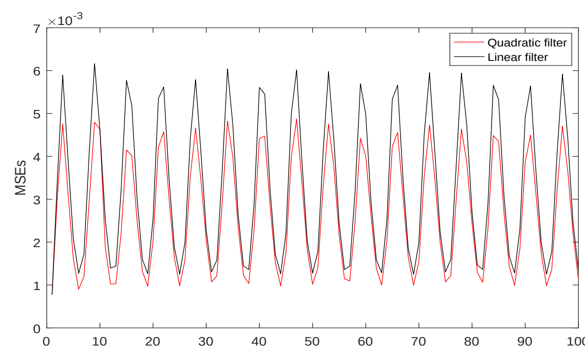


Fig. 5. Comparison of MSEs between quadratic filter and linear filter for $x_{1,s}$

systems, where the ZOHS has been introduced to offset the lost measurement. The associated enhanced system, involving more information about the RAP and NGNs, has been constructed in terms of the state/measurement vectors and their corresponding second-order Kronecker powers. On this basis, a recursive quadratic filter has been designed. Some sufficient conditions have been given to guarantee the existence of an upper bound on the FEC, and then the optimal gain matrix has been designed by minimizing such an upper bound. A matrix decomposition method has been given to address the issues caused by the RAP with ZOHS. Finally, the presented RAP-based quadratic filtering scheme has been confirmed by a numerical example.

References

[1] M. Afshari and A. Mahajan, Decentralized linear quadratic systems with major and minor agents and non-Gaussian noise, *IEEE Transactions on Automatic Control*, vol. 68, no. 8, pp. 889–896, 2023.

[2] V. Bhateja, M. Misra, S. Urooj and A. Lay-Ekuakille, A robust polynomial filtering framework for mammographic image enhancement from biomedical sensors, *IEEE Sensors Journal*, vol. 13, no. 11, pp. 4147–4156, 2013.

[3] R. Caballero-Águila, I. García-Garrido and J. Linares-Pérez, Quadratic estimation problem in discrete-time stochastic systems with random parameter matrices, *Applied Mathematics and Computation*, vol. 273, pp. 308–320, 2016.

[4] F. Cacace, F. Conte, A. Germani and G. Palombo, Feedback quadratic filtering, *Automatica*, vol. 82, pp. 158–164, 2017.

[5] F. Cacace, F. Conte, A. Germani and G. Palombo, Optimal linear and quadratic estimators for tracking from distance measurements, *Systems & Control Letters*, vol. 139, art. no. 104674, 2020.

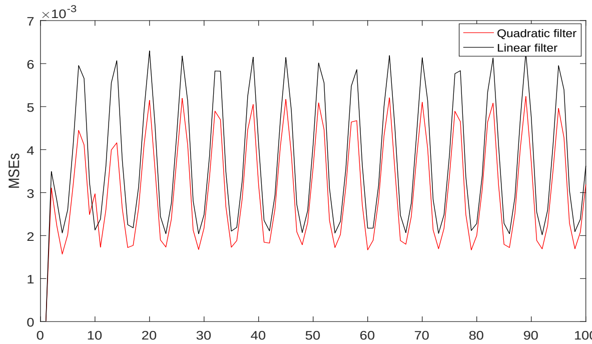


Fig. 6. Comparison of MSEs between quadratic filter and linear filter for $x_{2,s}$

[6] F. Carravetta, A. Germani and M. Raimondi, Polynomial filtering of discrete-time stochastic linear systems with multiplicative state noise, *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 1106–1126, 1997.

[7] F. Carravetta and G. Mavelli, Minimax quadratic filtering of uncertain linear stochastic systems with partial fourth-order information, *IEEE Transactions on Automatic Control*, vol. 44, no. 6, pp. 1287–1292, 1999.

[8] S. Chen, Q. Zhang, T. Zhang, L. Zhang, L. Peng and S. Wang, Robust state estimation with maximum correntropy rotating geometric unscented Kalman filter, *IEEE Transactions on Instrumentation and Measurement*, vol. 71, art. no. 2501714, 2022.

[9] D. Cilden-Guler, M. Raitoharju, R. Piche and C. Hajiyev, Nanosatellite attitude estimation using Kalman-type filters with non-Gaussian noise, *Aerospace Science and Technology*, vol. 92, pp. 66–76, 2019.

[10] D. Ciuonzo, A. Aubry and V. Carotenuto, Rician MIMO channel- and jamming-aware decision fusion, *IEEE Transactions on Signal Processing*, vol. 65, no. 15, pp. 3866–3880, 2017.

[11] Y. Dong, Y. Song and G. Wei, Efficient model-predictive control for nonlinear systems in interval type-2 T-S fuzzy form under round-robin protocol, *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 1, pp. 63–74, 2022.

[12] P. Duan, Z. Duan, Y. Lv and G. Chen, Distributed finite-horizon extended Kalman filtering for uncertain nonlinear systems, *IEEE Transactions on Cybernetics*, vol. 51, no. 2, pp. 512–520, 2021.

[13] X. Fan, G. Wang, J. Han and Y. Wang, A background-impulse Kalman filter with non-Gaussian measurement noises, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 53, no. 4, pp. 2434–2443, 2023.

[14] H. Fu, H. Dong, F. Han, Y. Shen and N. Hou, Outlier-resistant H_∞ filtering for a class of networked systems under round-robin protocol, *Neurocomputing*, vol. 403, pp. 133–142, 2020.

[15] H. Geng, Z. Wang, Y. Chen, X. Yi and Y. Cheng, Variance-constrained filtering fusion for nonlinear cyber-physical systems with the denial-of-service attacks and stochastic communication protocol, *IEEE/CAA Journal of Automatica Sinica*, vol. 9, no. 6, pp. 978–989, 2022.

[16] A. Germani, C. Manes and P. Palumbo, Polynomial filtering for stochastic non-Gaussian descriptor systems, *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 51, no. 8, pp. 1561–1576, 2004.

[17] B. Haus and P. Mercorelli, Polynomial augmented extended Kalman filter to estimate the state of charge of lithium-ion batteries, *IEEE Transactions on Vehicular Technology*, vol. 69, no. 2, pp. 1452–1463, 2019.

[18] N. Hou, H. Dong, Z. Wang and H. Liu, A partial-node-based approach to state estimation for complex networks with sensor saturations under random access protocol, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 32, no. 11, pp. 5167–5178, 2021.

[19] Z. Hu, J. Hu, H. Tan, J. Huang and Z. Cao, Distributed resilient fusion filtering for nonlinear systems with random sensor delay under round-robin protocol, *International Journal of Systems Science*, vol. 53, no. 13, pp. 2786–2799, 2022.

[20] E. Javanfar and M. Rahmani, Quadratic covariance-constrained filtering for linear and non-linear systems with non-Gaussian noises, *Optimal Control Applications and Methods*, vol. 44, pp. 2900–2915, 2023.

[21] J. Jiao, L. Trentelman and M. K. Camlibel, $\mathcal{H}_2$ and $\mathcal{H}_\infty$ suboptimal distributed filters design for linear systems, *IEEE Transactions on Automatic Control*, vol. 68, no. 4, pp. 2560–2567, 2023.

[22] Y. Jin, X. Ma, X. Meng and Y. Chen, Distributed fusion filtering for cyber-physical systems under round-robin protocol: A mixed H_2/H_∞ framework, *International Journal of Systems Science*, vol. 54, no. 8, pp. 1661–1675, 2023.

[23] D. Li, J. Liang, F. Wang and X. Ren, Observer-based H_∞ control of two-dimensional delayed networks under the random access protocol, *Neurocomputing*, vol. 401, pp. 353–363, 2020.

[24] M. Li, J. Liang and J. Qiu, Set-membership filtering for 2-D systems under uniform quantization and weighted try-once-discard protocol, *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 70, no. 9, pp. 3474–3478, 2023.

[25] X. Li, J. Song, N. Hou, D. Dai and F. Yang, Finite-horizon distributed set-membership filtering with dynamical bias and DoS attacks under binary encoding schemes, *Information Sciences*, vol. 641, art. no. 119084, 2023.

[26] X. Li, G. Wei, D. Ding and S. Liu, Recursive filtering for time-varying discrete sequential systems subject to deception attacks: Weighted try-once-discard protocol, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 6, pp. 3704–3713, 2022.

[27] Y. Li, C. Bian and H. Chen, Object tracking in satellite videos: Correlation particle filter tracking method with motion estimation by Kalman filter, *IEEE Transactions on Geoscience and Remote Sensing*, vol. 60, art. no. 5630112, 2022.

[28] Z. Li, S. Li, B. Liu, S. Y. Samson and P. Shi, A stochastic event-triggered robust cubature Kalman filtering approach to power system dynamic state estimation with non-Gaussian measurement noises, *IEEE Transactions on Control Systems Technology*, vol. 31, no. 2, pp. 889–896, 2023.

[29] J. Liu, E. Gong, L. Zha, X. Xie and E. Tian, Outlier-resistant recursive security filtering for multirate networked systems under fading measurements and round-robin protocol, *IEEE Transactions on Control of Network Systems*, vol. 10, no. 4, pp. 1962–1974, 2023.

[30] Q. Liu, S. Zheng, Z. Wang, N. D. Alotaibi and F. E. Alsaadi, Distributed estimation for multi-agent systems with relative measurements and quantized communication: A feedback quadratic framework, *International Journal of Robust and Nonlinear Control*, vol. 33, no. 5, pp. 3164–3184, 2023.

[31] S. Liu, X. Zhao, E. Tian and G. Wei, Distributed recursive filtering under random access protocols: A multirate strategy, *International Journal of Robust and Nonlinear Control*, vol. 32, no. 12, pp. 7132–7148, 2022.

[32] Y. Liu, W. Yang, J. Zhou and Y. Luo, H_∞ filter for discrete-time periodic piecewise systems with missing measurements, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 53, no. 9, pp. 5648–5657, 2023.

[33] X. Meng, Y. Chen, L. Ma and H. Liu, Protocol-based variance-constrained distributed secure filtering with measurement censoring, *International Journal of Systems Science*, vol. 53, no. 15, pp. 3322–3338, 2022.

[34] Z. Pan, S. Zhao, B. Huang and F. Liu, Confidence set-membership FIR filter for discrete time-variant systems, *Automatica*, vol. 157, art. no. 111231, 2023.

[35] K. Ryu and J. Back, Consensus optimization approach for distributed Kalman filtering: Performance recovery of centralized filtering, *Automatica*, vol. 149, art. no. 110843, 2023.

[36] B. Shen, X. Wang and L. Zou, Maximum correntropy Kalman filtering for non-Gaussian systems with state saturations and stochastic nonlinearities, *IEEE/CAA Journal of Automatica Sinica*, vol. 10, no. 5, pp. 1223–1233, 2023.

[37] L. Sheng, Y. Niu, W. Wang, M. Gao, Y. Geng and D. Zhou, Estimation of Toolface for dynamic point-the-bit rotary steerable systems via nonlinear polynomial filtering, *IEEE Transactions on Industrial Electronics*, vol. 69, no. 7, pp. 7192–7201, 2022.

[38] L. Sun, X. Tian, D. Ding and G. Wei, Consensus control of cooperation-competition multi-agent systems with round-robin protocols: A set-membership approach, *International Journal of Robust and Nonlinear Control*, vol. 32, pp. 5005–5022, 2022.

[39] B. Tian, Y. Wang and L. Guo, Disturbance observer-based minimum entropy control for a class of disturbed non-Gaussian stochastic systems, *IEEE Transactions on Cybernetics*, vol. 52, no. 6, pp. 4916–4925, 2022.

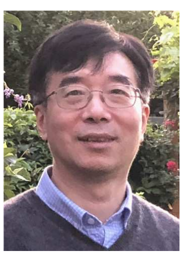
[40] X. Wan, Z. Wang, M. Wu and X. Liu, State estimation for discrete time-delayed genetic regulatory networks with stochastic noises under the round-robin protocols, *IEEE Transactions on Nanobioscience*, vol. 17, no. 2, pp. 145–154, 2018.

[41] B. Wang, J. Zhu, Z. Ma, Z. Deng and M. Fu, Improved particle filter-based matching method with gravity sample vector for underwater

Citation information: DOI: 10.1109/TSMC.2025.3616848, IEEE Transactions on Systems Man and Cybernetics Systems

gravity-aided navigation, *IEEE Transactions on Industrial Electronics*, vol. 68, no. 6, pp. 5206–5216, 2021.
[42] X. Wang, B. Shen and S. Gao, Quadratic filtering for non-Gaussian stochastic parameter systems with buffer-aided strategy, *Information Sciences*, vol. 644, art. no. 119304, 2023.
[43] Y. Wang, H.-J. Liu and H.-L. Tan, An overview of filtering for sampled-data systems under communication constraints, *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 3, art. no. 100011, 2023.
[44] M. Yao, G. Wei, D. Ding and W. Li, Output-feedback control for stochastic impulsive systems under round-robin protocol, *Automatica*, vol. 143, art. no. 110394, 2022.
[45] H. Yu, J. Hu, B. Song, H. Liu and X. Yi, Resilient energy-to-peak filtering for linear parameter-varying systems under random access protocol, *International Journal of Systems Science*, vol. 53, no. 11, pp. 2421–2436, 2022.
[46] Y. Zhang, L. Zou, B. Song, Z. Zhao and Y. Wang, Neural-network-based output feedback control for networked multirate systems: A bit rate allocation scheme, *Information Sciences*, vol. 637, art. no. 118952, 2023.
[47] Y. Zhang, L. Zou, Y. Wang and Y.-A. Wang, Estimator design for complex networks with encoding decoding mechanism and buffer-aided strategy: A partial-nodes accessible case, *ISA Transactions*, vol. 127, pp. 68–79, 2022.
[48] L. Zou, Z. Wang, Q.-L. Han and D. Zhou, Recursive filtering for time-varying systems with random access protocol, *IEEE Transactions on Automatic Control*, vol. 64, no. 2, pp. 720–727, 2019.

Shaoying Wang received her B.Sc. degree in mathematics in 2004 from Linyi University, Linyi, China, the M.Sc. degree in applied mathematics in 2006 from Wuhan University, Wuhan, China, and the Ph.D. degree in control theory and control engineering in 2016 from Huazhong University of Science and Technology, Wuhan, China. From 2023 to 2024, she was an academic visitor with the Department of Computer Science, Brunel University London, Uxbridge, UK, and was also a postdoctoral research fellow in the College of Electrical and Information Engineering, Northeast Petroleum University, Daqing, China. She is currently a professor in the College of Science, Shandong University of Aeronautics, Binzhou, China. Her research interests include networked control systems and state estimation.



Zidong Wang received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the UK. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published a number of papers in international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, the Editor-in-Chief for *Systems Science & Control Engineering*, and an Associate Editor for 12 international journals including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a Member of the Academia Europaea, a Member of the European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a Fellow of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

Hongli Dong received the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2012. From 2009 to 2010, she was a Research Assistant with the Department of Applied Mathematics, City University of Hong Kong, Hong Kong. From 2010 to 2011, she was a Research Assistant with the Department of Mechanical Engineering, The University of Hong Kong, Hong Kong. From 2011 to 2012, she was a Visiting Scholar with the Department of Information Systems and Computing, Brunel University London, London, U.K. From 2012 to 2014, she was an Alexander von Humboldt Research Fellow with the University of Duisburg–Essen, Duisburg, Germany. She is currently a Professor with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing, China. She is also the Director of the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Daqing. Her current research interests include robust control and networked control systems. She is a very active reviewer for many international journals.