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Distributed Polynomial Set-Membership Fusion Estimation for Target Tracking Systems Under a Binary Encoding Scheme

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***Abstract*—In this paper, a distributed polynomial set-membership fusion estimation approach is proposed for target tracking systems under a binary encoding scheme. Each distance measurement of the considered target is encoded via a binary encoding scheme to facilitate digital transmission. Based on the decoding signals, local estimators are designed by employing a polynomial set-membership estimation method. Furthermore, the effects of possible flipping bits occurring during the transmissions from the distance sensors to the local estimators are considered, and a detection scheme for flipping bits is presented by utilizing the obtained local estimation results. Subsequently, an optimal matrix-weighted distributed fusion estimator is developed in the F-radius sense of the zonotope restraining the global estimation error. Finally, simulation studies on a target tracking scenario are provided to demonstrate the effectiveness of the proposed approach.**

***Index Terms*—Target tracking, set-membership estimation, polynomial estimation, distributed fusion, binary encoding scheme.**

I. INTRODUCTION

Target tracking is referred to as the process of employing sensor measurement information to obtain satisfactory estimations for the states of the target based on the established target motion model. Over the past decades, such a technology has been applied in numerous areas including intelligent driving systems [15], vehicles [16], robotics [48], and security monitoring [1], among others. Accordingly, considerable research efforts have been devoted to target tracking, resulting in numerous significant advancements (see, e.g., [26], [30], [37], [39]–[42]).

An important issue in target tracking is the modeling of the unknown control inputs of the targets. In this regard, a commonly used method is to model the inputs of the targets as stochastic noises (e.g., Gaussian noises) with given statistics. In this case, the target states are estimated by employing methods that deal with stochastic noises [39], [40]. Since the control input of a target is always bounded due to physical limitations, bounded noises have been utilized in [2], [6], [18], [37], [38] to describe these unknown control inputs of the targets, and set-membership estimation (SME) that aims to recursively calculate the compact sets restraining the actual target states has been adopted. The SME-based approach relies solely on the bounds of the target control inputs, and thus is much easier to implement compared with those methods coping with stochastic noises. Nevertheless, relatively few studies have been devoted to SME-based target tracking, which motivates the further investigation of this subject.

In most studies concerning target tracking, the target states are estimated based on the nonlinear measurement outputs from the distance sensors and/or the angle sensors. It should be noted that, in these studies, the estimations for the target states are generally derived by employing the first- and the second-order partial derivatives of the concerned nonlinear function, while the information provided by the higher-order partial derivatives is usually neglected. The polynomial estimation technique is capable of utilizing the information of the higher-order partial derivatives, and thus the estimation accuracy of the target states can be potentially improved [25], [29], [32], [33]. As a result, it is considered a natural progression to apply the polynomial estimation technique to the problem of target tracking.

With the rapid development of network techniques, in an increasing number of target tracking problems, the target states are estimated by utilizing the measurement information transmitted through communication networks. Owing to the requirement of network-based transmissions, the measurement information is often transformed into a binary bit string by employing an encoding scheme [22], [36]. It should be noted that some bits in the binary bit string might be flipped due to the effects of channel noises [8], [23]. When the measurement information from one or more sensors is affected by flipping bits, the estimation performance for the target states might degrade significantly under the widely used centralized fusion estimation scheme because of its inherent characteristics [4], [12]. Considering this issue, together with the advantage of the distributed fusion estimation scheme, where the global estimation performance is no worse than the performance of any local estimation [10], [17], [27], it is considered practically

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meaningful to employ the distributed fusion scheme to handle the effects of the possible flipping bits.

Based on the above arguments, a distributed polynomial set-membership fusion estimation (DPSMFE) approach is proposed for target tracking under a binary encoding scheme. In this context, two key issues are addressed as follows:

1) how to design a polynomial SME (PSME) algorithm that leverages the higher-order partial derivatives of the distance measurements to generate the local estimates for the considered target with an established motion model based on the decoding signals?
2) how to mitigate the effects of the potential flipping bits that may exist in each decoding signal?

Correspondingly, the main contributions of this paper are summarized as follows.

1) A novel DPSMFE approach is proposed for target tracking under a binary encoding scheme. A PSME method is employed to fully utilize the information of the higher-order partial derivatives of the distance functions for the design of the local estimators, and a zonotope-based fusion method is adopted to process the outputs of the local estimators.
2) Zonotopes restraining the local estimation errors are recursively calculated, and their F-radii are minimized by designing the feasible local estimator parameters under the PSME framework. Furthermore, a new detection method is developed to detect possible flipping bits in the decoding signals based on the constructed zonotopes. Compared with the detection scheme in [8], which is designed for codewords derived from linear measurements, the proposed method can be applied to binary strings obtained from nonlinear measurements. Moreover, unlike the method in [8], which requires computing the intersection of two zonotopes, the proposed approach is easier to verify.
3) An optimal matrix-weighted distributed set-membership fusion scheme is presented in the sense of the F-radius of the zonotope enclosing the global estimation error, which is capable of suppressing the side effects of the measurement noises, the coding errors, as well as the possible flipping bits.

The rest of this paper is organized as follows. In Section II, several necessary concepts related to zonotopes and Kronecker powers are introduced. In Section III, the studied target tracking problem under the binary encoding scheme is formulated. In Section IV, the local estimation algorithm is designed by employing the PSME method, and a detection scheme for possible flipping bits is established; additionally, the optimal distributed fusion estimator with matrix weights is designed. Section V presents an illustrative example, and the conclusions are drawn in Section VI.

Notations: For a matrix $\mathcal{X}$, $\|\mathcal{X}\|_\infty$ denotes its infinity norm. $\mathrm{Tr}\{\mathcal{Z}\}$ denotes the trace of a square matrix $\mathcal{Z}$. I is an identity matrix of appropriate dimensions. $\mathrm{vec}_p\{\mathcal{Z}_\xi\}$ represents the matrix $\begin{bmatrix}\mathcal{Z}_1^T & \cdots & \mathcal{Z}_p^T\end{bmatrix}^T$. $\mathrm{diag}_p\{\mathcal{Z}_\xi\}$ represents the block-diagonal matrix $\mathrm{diag}\{\mathcal{Z}_1, \ldots, \mathcal{Z}_p\}$. For a matrix $\mathcal{X}$ with total p rows, $\mathrm{rs}\{\mathcal{X}\} \triangleq \mathrm{diag}_p\{\|\mathcal{X}_\xi\|_\infty\}$ with $\mathcal{X}_\xi$ being the ξ-th row of $\mathcal{X}$.

II. PRELIMINARIES

In this paper, zonotopes are employed to confine the state of the target. The definition is provided as follows.

Definition 1: [21] Given a center vector $\lambda \in \mathbb{R}^n$ and a generator matrix $\Lambda \in \mathbb{R}^{n\times m}$, an m-order zonotope $\langle\lambda, \Lambda\rangle \subset \mathbb{R}^n$ is defined as $\langle\lambda, \Lambda\rangle \triangleq \{\lambda + \Lambda c : \|c\|_\infty \leq 1\}$.

The F-radius of zonotopes is adopted to measure their sizes.

Definition 2: [9] For a zonotope $\langle\lambda, \Lambda\rangle$, its F-radius is defined as

$$\|\Lambda\|_F \triangleq \sqrt{\mathrm{Tr}\{\Lambda\Lambda^T\}}.$$

Remark 1: Different from [2], which employs the ellipsoidal SME for target tracking, this paper adopts the zonotopic SME for the following advantages of zonotopes [19]: 1) the zonotope is closed under the Minkowski sum and linear mapping operations (two operations frequently used in SME); and 2) the order reduction technique of zonotopes enables a trade-off between estimation accuracy and computational complexity. To quantify the size of a zonotope, the F-radius is adopted, as it allows the optimization to be reformulated as a quadratic minimization problem in terms of the estimator gain, thereby simplifying the design process compared to other measures, such as volume.

The Kronecker powers of matrices and the gradient operator (applied to vector-valued functions) are utilized in developing the desired DPSMFE algorithm. Their definitions are given as follows.

Definition 3: [3] For a matrix $\Lambda \in \mathbb{R}^{n\times m}$, its Kronecker power is defined as

$$\Lambda^{[0]} = 1,\ \Lambda^{[l]} = \Lambda^{[l-1]} \otimes \Lambda,\ l = 1, 2, \ldots$$

where "$\otimes$" represents the matrix Kronecker product.

Definition 4: [14] For a vector-valued function $h(x) : \mathbb{R}^n \mapsto \mathbb{R}^m$, the operation $\nabla_x^{[l]}\otimes$ applied to $h(\cdot)$ is defined as

$$\begin{aligned}&\nabla_x^{[0]} \otimes h = h,\\ &\nabla_x^{[l+1]} \otimes h = \nabla_x \otimes (\nabla_x^{[l]} \otimes h),\ l = 0, 1, 2, \ldots\end{aligned}$$

where $\nabla_x \triangleq \begin{bmatrix}\partial/\partial x^{(1)} & \partial/\partial x^{(2)} & \cdots & \partial/\partial x^{(n)}\end{bmatrix}$, and $x^{(1)}, x^{(2)}, \ldots, x^{(n)}$ are the components of vector x (i.e., $x = \begin{bmatrix}x^{(1)} & x^{(2)} & \cdots & x^{(n)}\end{bmatrix}^T$).

III. PROBLEM FORMULATION

A. System Model

Consider a target tracking problem in a two-dimensional (2-D) region where the motion model of the target is given by:

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\check{w}(k). \tag{1}$$

Here, $\mathbf{x}(k) = \begin{bmatrix}x(k) & y(k) & v_x(k) & v_y(k)\end{bmatrix}^T$ denotes the target state vector, with $x(k), v_x(k) \in \mathbb{R}$ representing the target position and velocity in the x-coordinate, and $y(k), v_y(k) \in \mathbb{R}$ representing the target position and velocity in the y-coordinate. $\check{w}(k) \in \mathbb{R}^2$ denotes a bounded signal modeling the control input of the target, and A and B are matrices expressed as

$$A=\begin{bmatrix}1&0&T&0\\0&1&0&T\\0&0&1&0\\0&0&0&1\end{bmatrix},B=\begin{bmatrix}\frac{T^2}{2}&0\\0&\frac{T^2}{2}\\T&0\\0&T\end{bmatrix}$$

with T being the sampling period.

The target is measured by N sensors with the measurement of the j-th sensor being

$$z_j(k)=g_j(\mathbf{x}(k))+v_j(k) \tag{2}$$

where, for $j\in\{1,2,\ldots,N\}$, $z_j(k)\in\mathbb{R}$ represents the distance measurement; $g_j(\cdot)$ denotes the distance function defined as

$$g_j(\mathbf{x}(k))\triangleq\sqrt{(x(k)-\underline{x}_j)^2+(y(k)-\underline{y}_j)^2} \tag{3}$$

with $(\underline{x}_j,\underline{y}_j)$ being the located position of the j-th sensor in the 2-D region; and $v_j(k)$ stands for the bounded measurement noise.

Assumption 1: The initial state $\mathbf{x}(0)$ of the target belongs to $\langle 0,X(0)\rangle$ where $X(0)\in\mathbb{R}^{4\times4}$ is a known positive diagonal matrix.

Assumption 2: The signal $\check{w}(k)$ resides in $\langle\hat{w},W\rangle$ where $\hat{w}\in\mathbb{R}^2$ is a known vector and $W=\operatorname{diag}\{w^{(1)},w^{(2)}\}\in\mathbb{R}^{2\times2}$ is a positive diagonal matrix. In addition, for $j\in\{1,2,\ldots,N\}$, the measurement noise $v_j(k)$ is confined to $[-V_j,V_j]$ where $V_j>0$ is a positive scalar.

Assumption 3: For $j\in\{1,2,\ldots,N\}$, the distance measurement of the j-th sensor (i.e., $z_j(k)$) takes values from the interval $[0,\bar{z}_j]$ where $\bar{z}_j>0$ is a known scalar.

Remark 2: The parameters $X(0)$, W, and V_j should be chosen to ensure that Assumptions 1 and 2 are satisfied, which is crucial for guaranteing the subsequent SME performance. A practical method to achieving this is to incorporate prior knowledge, such as the physical constraints of the target.

B. Transmission Model Under Binary Encoding Mechanism

To facilitate the digital transmission over communication networks, the distance measurement of the j-th sensor (i.e., $z_j(k)$) is transformed into a binary bit string before being sent to the j-th local estimator. To this end, a binary encoding mechanism is utilized, and the detailed encoding process is given as follows.

Assuming that $z_j(k)$ is encoded as a binary bit string of length L_j, the interval $[0,\bar{z}_j]$ is divided into $2^{L_j}-1$ segments by using 2^{L_j} uniformly spaced points. In this case, there exists a nonnegative integer $\gamma_j(k)$ such that

$$\gamma_j(k)\frac{\bar{z}_j}{2^{L_j}-1}\le z_j(k)\le(\gamma_j(k)+1)\frac{\bar{z}_j}{2^{L_j}-1}. \tag{4}$$

Based on (4), let $\mathcal{L}_j=\bar{z}_j/(2^{L_j}-1)$ and define

$$\mathbf{c}_j(k)\triangleq\begin{cases}\gamma_j(k), & \text{if } \gamma_j(k)\mathcal{L}_j\le z_j(k)\le\left(\gamma_j(k)+\frac{1}{2}\right)\mathcal{L}_j\\ \gamma_j(k)+1, & \text{if } \left(\gamma_j(k)+\frac{1}{2}\right)\mathcal{L}_j<z_j(k)\le(\gamma_j(k)+1)\mathcal{L}_j.\end{cases}$$

Then, the desired binary bit string is obtained by converting the decimal number $\mathbf{c}_j(k)$ to its binary equivalent. For convenience, such a binary number is denoted as $b_{j,L_j}(k)b_{j,L_j-1}(k)\cdots b_{j,1}(k)$ throughout the rest of this paper, where $b_{j,1}(k),b_{j,2}(k),\ldots,b_{j,L_j}(k)\in\{0,1\}$.

The binary bit string $b_{j,L_j}(k)b_{j,L_j-1}(k)\cdots b_{j,1}(k)$ is transmitted to the j-th local estimator via a communication network, where some bits might be flipped due to the effects of channel noises [8], [23]. Denote

$$\acute{b}_{j,t_j}(k)\triangleq\begin{cases}b_{j,t_j}(k), & \text{if } b_{j,t_j} \text{ is not flipped}\\ 1-b_{j,t_j}(k), & \text{otherwise}\end{cases} \tag{5}$$

for $t_j=1,2,\ldots,L_j$. Then, the received binary bit string by the j-th local estimator can be represented as $\acute{b}_{j,L_j}(k)\acute{b}_{j,L_j-1}(k)\cdots\acute{b}_{j,1}(k)$. Furthermore, the decoding signal, denoted as $\mathbf{d}_j(k)$, is obtained by

$$\mathbf{d}_j(k)=\mathcal{L}_j\left(2^0\acute{b}_{j,1}+2^1\acute{b}_{j,2}+\ldots+2^{L-1}\acute{b}_{j,L_j}\right). \tag{6}$$

Remark 3: To obtain the desired binary bit string, a commonly used uniform quantization mechanism is applied to encode the distance measurement $z_j(k)$. Compared with non-uniform quantization schemes (e.g., the one in [44]), the uniform approach is typically easier to implement. Moreover, the resulting quantization error is independent of the target's state [13], [34], [43], [45].

C. Problem Statement

The aim of this paper is threefold.

1) In the case $\mathbf{d}_j(k)=\mathcal{L}_j\mathbf{c}_j(k)$ (i.e., no flipping bit occurs), a local PSME algorithm is to be designed for the target in (1) such that the output zonotope of the algorithm can always enclose the actual system state.
2) Based on the designed local PSME algorithm, a detection approach is to be established to judge whether there exists one or more flipping bits in the binary bit string $\acute{b}_{j,L_j}\acute{b}_{j,L_j-1}\cdots\acute{b}_{j,1}$.
3) The information provided by the local estimators is to be fused in the fusion center.

IV. MAIN RESULTS

A. Some Preliminary Lemmas

Lemma 1: [19] For zonotopes $\langle\lambda_1,\Lambda_1\rangle,\langle\lambda_2,\Lambda_2\rangle\subset\mathbb{R}^n$ and a matrix $C\in\mathbb{R}^{m\times n}$, it holds that

$$\langle\lambda_1,\Lambda_1\rangle\oplus\langle\lambda_2,\Lambda_2\rangle=\left\langle\lambda_1+\lambda_2,\begin{bmatrix}\Lambda_1 & \Lambda_2\end{bmatrix}\right\rangle$$
$$C\odot\langle\lambda_1,\Lambda_1\rangle=\langle C\lambda_1,C\Lambda_1\rangle.$$

Lemma 2: Given a vector $\vec{\omega}\in\langle 0,\vec{\Omega}\rangle\subset\mathbb{R}^m$ with $\vec{\Omega}$ being a positive diagonal matrix and a positive integer l, there exist a vector $\omega(l)$ and a matrix $\Omega(l)$ such that $\vec{\omega}^{[l]}\in\langle\omega(l),\Omega(l)\rangle$.

Proof: In light of the boundedness of $\vec{\omega}$, $\vec{\omega}^{[l]}$ must be bounded. Hence, there must exist a vector $\omega(l)$ and a matrix $\Omega(l)$ such that $\vec{\omega}^{[l]}\in\langle\omega(l),\Omega(l)\rangle$. ■

Remark 4: There are various methods that can be employed to calculate the vector $\omega(l)$ and matrix $\Omega(l)$ in Lemma 2. For example, based on $\vec{\omega}\in\langle 0,\vec{\Omega}\rangle$ and Definition 1, it can be easily observed that $\vec{\omega}^{[l]}=(\vec{\Omega}\breve{\omega})^{[l]}=\vec{\Omega}^{[l]}\breve{\omega}^{[l]}$ for some $\breve{\omega}$

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with $\|\breve{\omega}\|_\infty \leq 1$. Thus, $\omega(l)$ and $\Omega(l)$ can be taken as $\omega(l) = 0_{m^l\times 1}$ and $\Omega(l) = \vec{\Omega}^{[l]}$. When l is even, some elements of $\vec{\omega}^{[l]}$ may not be less than 0. In such cases, $\omega(l)$ and $\Omega(l)$ can be appropriately selected such that the zonotope $\langle\omega(l), \Omega(l)\rangle$ becomes tighter.

Lemma 3: The motion model of the considered target in (1) can be rewritten as

$$\mathbf{x}(k+1) = f(\mathbf{x}(k)) + w(k) \tag{7}$$

where $f(\mathbf{x}(k)) = A\mathbf{x}(k) + B\hat{w}$ and $w(k) = B\underline{w}(k)$ with $\underline{w}(k) \in \langle 0_{2\times 1}, W\rangle$. Moreover, for a given positive integer l, there exist a vector $\mathbf{w}(l)$ and a matrix $\mathbf{W}(l)$ such that

$$w^{[l]}(k) \in \langle \mathbf{w}(l), \mathbf{W}(l)\rangle. \tag{8}$$

Proof: By Assumption 2 and Definition 1, the signal $\breve{w}(k)$ in (1) can be expressed as

$$\breve{w}(k) = \hat{w} + \underline{w}(k) \tag{9}$$

which, together with (1), yields (7).

Applying Lemma 2 to $\underline{w}(k) \in \langle 0_{2\times 1}, W\rangle$, it is seen that $\underline{w}^{[l]}(k)$ belongs to a zonotope. In accordance with $w^{[l]}(k) = B^{[l]}\underline{w}^{[l]}(k)$ and Lemma 1, the existence of the vector $\mathbf{w}(l)$ and the matrix $\mathbf{W}(l)$ ensuring (8) is guaranteed. ■

Remark 5: In Lemma 3, the target's dynamic equation is reformulated into the form given in (9). The primary purpose of this transformation is to facilitate the subsequent derivation of a polynomial nonlinear system using the Carleman linearization [14].

Lemma 4: In the case that $\acute{b}_{j,t_j} = b_{j,t_j}$ for all $t_j = 1, 2, \ldots, L_j$, the decoding signal $\mathbf{d}_j(k)$ in (6) can be expressed as

$$\mathbf{d}_j(k) = g_j(\mathbf{x}(k)) + \varsigma_j(k) \tag{10}$$

where $\varsigma_j(k)$ is an unknown signal satisfying $\varsigma_j(k) \in [-\bar{\varsigma}_j, \bar{\varsigma}_j]$ with $\bar{\varsigma}_j \triangleq V_j + \mathcal{L}_j/2$.

Proof: In the case that $\acute{b}_{j,t_j} \equiv b_{j,t_j}$ for $t_j = 1, 2, \ldots, L_j$, one can obtain $\mathbf{d}_j(k) = \mathcal{L}_j\mathbf{c}_j(k)$, which, together with the expression of $\mathbf{c}_j(k)$ (below (4)), yields

$$|\mathbf{d}_j(k) - z_j(k)| \leq \frac{1}{2}\mathcal{L}_j. \tag{11}$$

Based on (2), (11) and $v_j(k) \in [-V_j, V_j]$ (see Assumption 2), (10) can be readily obtained, which completes the proof. ■

Lemma 5: [5] For an integer $\ell \geq 0$ and vectors $a, b \in \mathbb{R}^n$, one has

$$(a+b)^{[\ell]} = \sum_{\upsilon=0}^{\ell} M(\ell, \upsilon, n)\left(a^{[\upsilon]} \otimes b^{[\ell-\upsilon]}\right)$$

where $M(\ell, \upsilon, n)$ $(\upsilon = 0, 1, \cdots, \ell)$ are suitably defined matrices.

Lemma 6: For a vector $\lambda \in \langle 0, \Lambda\rangle$, it holds that $\|\lambda\|_\infty \leq \|\Lambda\|_\infty$.

Proof: From Definition 1, there exists a vector $\vec{\lambda}$ with $\|\vec{\lambda}\|_\infty \leq 1$ such that $\lambda = \Lambda\vec{\lambda}$, from which it can be obtained that $\|\lambda\|_\infty = \|\Lambda\vec{\lambda}\|_\infty \leq \|\Lambda\|_\infty \cdot \|\vec{\lambda}\|_\infty \leq \|\Lambda\|_\infty$. ■

Lemma 7: For matrices $\Lambda_1 = (\lambda_{1,\underline{i}\underline{j}})_{m_1\times n_1}$ and $\Lambda_2 = (\lambda_{2,\underline{i}\underline{j}})_{m_2\times n_2}$, one has

$$\|\Lambda_1 \otimes \Lambda_2\|_\infty = \|\Lambda_1\|_\infty \cdot \|\Lambda_2\|_\infty.$$

Proof: According to the definition of the matrix Kronecker product, it follows that

$$\Lambda_1 \otimes \Lambda_2 = \begin{bmatrix} \lambda_{1,11}\Lambda_2 & \lambda_{1,12}\Lambda_2 & \cdots & \lambda_{1,1n_1}\Lambda_2 \\ \lambda_{1,21}\Lambda_2 & \lambda_{1,22}\Lambda_2 & \cdots & \lambda_{1,2n_1}\Lambda_2 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{1,m_11}\Lambda_2 & \lambda_{1,m_12}\Lambda_2 & \cdots & \lambda_{1,m_1n_1}\Lambda_2 \end{bmatrix}.$$

Then, in light of the definition of the matrix infinity norm, it can be obtained that

$$\begin{aligned}
&\|\Lambda_1 \otimes \Lambda_2\|_\infty \\
&= \max_{\underline{i}\in\{1,2,\ldots,m_1\}} \left\|\begin{bmatrix}\lambda_{1,\underline{i}1}\Lambda_2 & \ldots & \lambda_{1,\underline{i}n_1}\Lambda_2\end{bmatrix}\right\|_\infty \\
&= \max_{\underline{i}\in\{1,2,\ldots,m_1\}} \max_{\bar{i}\in\{1,2,\ldots,m_2\}} \sum_{\underline{j}=1}^{n_1} |\lambda_{1,\underline{i}\underline{j}}| \cdot \sum_{\bar{j}=1}^{n_2} |\lambda_{2,\bar{i}\bar{j}}| \\
&= \Big(\max_{\underline{i}\in\{1,2,\ldots,m_1\}} \sum_{\underline{j}=1}^{n_1} |\lambda_{1,\underline{i}\underline{j}}|\Big)\Big(\max_{\bar{i}\in\{1,2,\ldots,m_2\}} \sum_{\bar{j}=1}^{n_2} |\lambda_{2,\bar{i}\bar{j}}|\Big) \\
&= \|\Lambda_1\|_\infty \cdot \|\Lambda_2\|_\infty.
\end{aligned}$$

This completes the proof. ■

Lemma 8: [47] Suppose that a vector λ satisfies $\|\lambda\|_\infty \leq \bar{\lambda}$ with $\bar{\lambda}$ being a positive scalar. Then, one has $\lambda \in \langle 0, \bar{\lambda}I\rangle$.

B. Polynomial Nonlinear Systems and Local Estimators

To utilize the higher-order partial derivatives of the distance function $g_j(\cdot)$ $(j = 1, 2, \ldots, N)$ in the design of the local estimators, a polynomial nonlinear system is derived in this subsection using the Carleman linearization [14] for the case $\mathbf{d}_j(k) = \mathcal{L}_j\mathbf{c}_j(k)$, and the local estimators are constructed based on the derived nonlinear system. To this end, let a positive integer ν_j be given. Then, for $l \in \{0, 1, \ldots, \nu_j\}$, by using the Taylor polynomial around a given local estimate $\hat{\mathbf{x}}_j(k) \in \mathbb{R}^4$, it follows from Lemma 3 that

$$\mathbf{x}^{[l]}(k+1) = \sum_{p=0}^{\nu_j} \mathscr{F}(l, p, \hat{\mathbf{x}}_j(k), w(k))(\mathbf{x}(k) - \hat{\mathbf{x}}_j(k))^{[p]}. \tag{12}$$

Similarly, by using the Taylor polynomial around a given prediction $\grave{\mathbf{x}}_j(k) \in \mathbb{R}^4$, it can be obtained from Lemma 4 that

$$\begin{aligned}\mathbf{d}_j(k) = &\sum_{p=0}^{\nu_j} \mathscr{G}_j(p, \grave{\mathbf{x}}_j(k))(\mathbf{x}(k) - \grave{\mathbf{x}}_j(k))^{[p]} + \mathscr{O}_j(k) \\ &+ \varsigma_j(k)\end{aligned} \tag{13}$$

where

$$\begin{aligned}
\mathscr{F}_j(l, p, \hat{\mathbf{x}}_j(k), w(k)) &= \frac{1}{p!}\left(\nabla_{\mathbf{x}}^{[p]} \otimes (f + w)^{[l]}\right)\Big|_{\mathbf{x}(k)=\hat{\mathbf{x}}_j(k)} \\
\mathscr{G}_j(p, \grave{\mathbf{x}}_j(k)) &= \frac{1}{p!}\left(\nabla_{\mathbf{x}}^{[p]} \otimes g_j\right)\Big|_{\mathbf{x}=\grave{\mathbf{x}}_j(k)} \\
\mathscr{O}_j(k) &= \mathscr{R}_j(\breve{\mathbf{x}}_j(k))(\mathbf{x}(k) - \grave{\mathbf{x}}_j(k))^{[\nu_j+1]}
\end{aligned}$$

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$$\mathscr{R}_j(\check{\mathbf{x}}_j(k)) = \frac{1}{(\nu_j+1)!}\left(\nabla_{\mathbf{x}}^{[\nu_j+1]} \otimes g_j\right)\Big|_{\mathbf{x}=\check{\mathbf{x}}_j(k)}$$
$$\check{\mathbf{x}}_j(k) = \theta_j(k)\mathbf{x}(k) + (1-\theta_j(k))\grave{\mathbf{x}}_j(k)$$
$$\theta_j(k) \in [0,1].$$

Applying Lemma 5 to (12) and (13) gives

$$\begin{cases} \mathbf{x}^{[l]}(k+1) = \sum_{q=0}^{\nu_j} \mathcal{F}_j(l,q,k)\mathbf{x}^{[q]}(k) + \vec{w}_j(l,k) \\ \mathbf{d}_j(k) = \sum_{q=0}^{\nu_j} \mathcal{G}_j(q,k)\mathbf{x}^{[q]}(k) + \mathscr{O}_j(k) + \varsigma_j(k) \end{cases} \tag{14}$$

where

$$\begin{aligned} &\mathcal{F}_j(l,q,k) \\ &= \sum_{p=q}^{\nu_j}\sum_{s=0}^{l} \frac{1}{p!}M(l,s,4)\bigg(\Big(\big(\nabla_{\mathbf{x}}^{[p]} \otimes f^{[s]}\big)\Big|_{\mathbf{x}(k)=\hat{\mathbf{x}}_j(k)}\Big) \\ &\quad \otimes \mathbf{w}(l-s)\bigg)M(p,q,4)\left(I_{4^q} \otimes (-\hat{\mathbf{x}}_j(k))^{[p-q]}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} &\vec{w}_j(l,k) \\ &= \sum_{p=0}^{\nu_j}\sum_{s=0}^{l} \frac{1}{p!}M(l,s,4)\bigg\{\bigg(\Big(\big(\nabla_{\mathbf{x}}^{[p]} \otimes f^{[s]}\big)\Big|_{\mathbf{x}(k)=\hat{\mathbf{x}}_j(k)}\Big) \\ &\quad \times (\mathbf{x}(k) - \hat{\mathbf{x}}_j(k))^{[p]}\bigg) \otimes \left(w^{[l-s]} - \mathbf{w}(l-s)\right)\bigg\} \end{aligned} \tag{16}$$

$$\begin{aligned} &\mathcal{G}_j(q,k) \\ &= \sum_{p=q}^{\nu_j} \frac{1}{p!}\left(\left(\nabla_{\mathbf{x}}^{[p]} \otimes g_j\right)\Big|_{\mathbf{x}=\grave{\mathbf{x}}_j(k)}\right)M(p,q,4) \\ &\quad \times \left(I_{4^q} \otimes (-\grave{\mathbf{x}}_j(k))^{[p-q]}\right) \end{aligned} \tag{17}$$
$$\mathbf{w}(0) = 1.$$

Moreover, it follows from Assumption 1 and Lemma 2 that parameters $c(l,0)$ and $\mathbf{X}(l,0)$ exist such that

$$\mathbf{x}^{[l]}(0) \in \langle c(l,0), \mathbf{X}(l,0)\rangle. \tag{18}$$

Define

$$\tilde{\mathbf{x}}_j(k) \triangleq \begin{bmatrix} 1 & \mathbf{x}^T(k) & \cdots & \left(\mathbf{x}^{[\nu_j]}(k)\right)^T \end{bmatrix}^T.$$

Then, one can obtain from (14) that

$$\begin{cases} \tilde{\mathbf{x}}_j(k+1) = \tilde{\mathcal{F}}_j(k)\tilde{\mathbf{x}}_j(k) + \tilde{w}_j(k) \\ \mathbf{d}_j(k) = \tilde{\mathcal{G}}_j(k)\tilde{\mathbf{x}}_j(k) + \mathscr{O}_j(k) + \varsigma_j(k) \end{cases} \tag{19}$$

where

$$\tilde{w}_j(k) = \begin{bmatrix} \vec{w}_j^T(0,k) & \vec{w}_j^T(1,k) & \cdots & \vec{w}_j^T(\nu_j,k) \end{bmatrix}^T$$
$$\tilde{\mathcal{F}}_j(k) = \begin{bmatrix} \mathcal{F}_j(0,0,k) & \mathcal{F}_j(0,1,k) & \cdots & \mathcal{F}_j(0,\nu_j,k) \\ \mathcal{F}_j(1,0,k) & \mathcal{F}_j(1,1,k) & \cdots & \mathcal{F}_j(1,\nu_j,k) \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{F}_j(\nu_j,0,k) & \mathcal{F}_j(\nu_j,1,k) & \cdots & \mathcal{F}_j(\nu_j,\nu_j,k) \end{bmatrix} \tag{20}$$
$$\tilde{\mathcal{G}}_j(k) = \begin{bmatrix} \mathcal{G}_j(0,k) & \mathcal{G}_j(1,k) & \cdots & \mathcal{G}_j(\nu_j,k) \end{bmatrix}. \tag{21}$$

Moreover, one can derive from (18) that

$$\begin{aligned} \tilde{\mathbf{x}}_j(0) &\in \left\langle \begin{bmatrix} 1 & \mathrm{vec}_{\nu_j}^T\{c(\xi,0)\} \end{bmatrix}^T, \mathrm{diag}\{0, \mathrm{diag}_{\nu_j}\{\mathbf{X}(\xi,0)\}\} \right\rangle \\ &\triangleq \langle \tilde{c}_j(0), \tilde{\mathbf{X}}_j(0)\rangle. \end{aligned} \tag{22}$$

Remark 6: It can be observed that the information regarding the higher-order derivatives of the nonlinear function g_j is embedded in the matrix $\tilde{\mathcal{G}}_j(k)$. Such information can be utilized when the local estimator is designed based on (19).

Based on (19), the j-th local estimator is constructed as follows:

$$\begin{cases} \acute{\mathbf{x}}_j(k) = \tilde{\mathcal{F}}_j(k-1)\widehat{\mathbf{x}}_j(k-1) \\ \widehat{\mathbf{x}}_j(k) = \acute{\mathbf{x}}_j(k) + \mathcal{K}_j(k)\left(\mathbf{d}_j(k) - \tilde{\mathcal{G}}_j(k)\acute{\mathbf{x}}_j(k)\right) \\ \widehat{\mathbf{x}}_j(0) = \tilde{c}_j(0) \end{cases} \tag{23}$$

where $\acute{\mathbf{x}}_j(k)$ and $\widehat{\mathbf{x}}_j(k)$ denote the prediction and the local estimate of $\tilde{\mathbf{x}}_j(k)$, respectively, and $\mathcal{K}_j(k)$ is the gain matrix of the local estimator, which is in the following form:

$$\mathcal{K}_j(k) = \begin{bmatrix} 0 & K_j^T(k) \end{bmatrix}^T \tag{24}$$

with $K_j(k) \in \mathbb{R}^{(\sum_{q=1}^{\nu_j} 4^q)\times 1}$ being the parameter to be designed.

Remark 7: Since the first component of $\tilde{\mathbf{x}}_j(k)$ is 1, the estimation error of this component can be guaranteed to be zero by setting $\tilde{c}_j(0)$ as in (22) and the estimation gain given by (24).

C. Design of the Local Estimators

In this subsection, the local estimation algorithm will be developed.

Define the prediction error and the local estimation error of the j-th local estimator as $\acute{\mathbf{e}}_j(k) \triangleq \tilde{\mathbf{x}}_j(k) - \acute{\mathbf{x}}_j(k)$ and $\widehat{\mathbf{e}}_j(k) \triangleq \tilde{\mathbf{x}}_j(k) - \widehat{\mathbf{x}}_j(k)$, respectively. In the following theorem, the zonotopes restraining $\widehat{\mathbf{e}}_j(k)$ will be recursively calculated.

Theorem 1: Assume that

$$\widehat{\mathbf{e}}_j(k-1) \in \langle 0, \widehat{E}_j(k-1)\rangle \tag{25}$$

where $\widehat{E}_j(k-1)$ is a given matrix. Then, the following relations hold:

$$\acute{\mathbf{e}}_j(k) \in \langle 0, \acute{E}_j(k)\rangle \tag{26}$$
$$\widehat{\mathbf{e}}_j(k) \in \langle 0, \widehat{E}_j(k)\rangle \tag{27}$$

where

$$\acute{E}_j(k) \triangleq \left[\tilde{\mathcal{F}}_j(k-1)\widehat{E}_j(k-1) \quad \bar{\mathbf{W}}_j(k-1)\right] \tag{28}$$
$$\begin{aligned} \widehat{E}_j(k) \triangleq \big[&(I - \mathcal{K}_j(k)\tilde{\mathcal{G}}_j(k))\acute{E}_j(k) \quad -\bar{o}_j(k)\mathcal{K}_j(k) \\ &-\bar{\varsigma}_j\mathcal{K}_j(k)\big] \end{aligned} \tag{29}$$

with

$$\bar{\mathbf{W}}_j(k-1) \triangleq \mathrm{diag}\{0, \bar{\mathbf{w}}_j(1,k-1)I_4, \ldots, \bar{\mathbf{w}}_j(\nu_j,k-1)I_{4^{\nu_j}}\} \tag{30}$$

$$\bar{\mathbf{w}}_j(l,k-1) \triangleq \sum_{p=0}^{\nu_j}\sum_{s=0}^{l} \frac{1}{p!}\|M(l,s,4)\|_\infty \cdot \|\widehat{E}_{j,1}(k-1)\|_\infty^p$$

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$$\cdot \left\| \left(\nabla_{\mathbf{x}}^{[p]} \otimes f^{[s]} \right) \Big|_{\mathbf{x}(k-1)=\hat{\mathbf{x}}_j(k-1)} \right\|_{\infty} \cdot \|\mathbf{W}(l-s)\|_{\infty},\ l=1,2,\ldots,\nu_j \tag{31}$$

$$\mathbf{W}(0) \triangleq 0 \tag{32}$$

$$\widehat{E}_{j,1}(k-1) \triangleq \mathcal{I}_j \widehat{E}_j(k-1) \tag{33}$$

$$\hat{\mathbf{x}}_j(k-1) \triangleq \mathcal{I}_j \widehat{\mathbf{x}}_j(k-1) \tag{34}$$

$$\mathcal{I}_j \triangleq \begin{bmatrix} 0_{4\times 1} & I_4 & 0_{4\times 4^2} & \cdots & 0_{4\times 4^{\nu_j}} \end{bmatrix} \tag{35}$$

$$\bar{o}_j(k) \triangleq \frac{1}{(\nu_j+1)!} \bar{g}_j(k) \|\mathcal{I}_j \acute{E}_j(k)\|_{\infty}^{\nu_j+1} \tag{36}$$

$$\bar{g}_j(k) \triangleq \max_{\mathbf{x}\in\langle \dot{\mathbf{x}}_j(k), \mathcal{I}_j \acute{E}_j(k)\rangle} \left\| \nabla_{\mathbf{x}}^{[\nu_j+1]} \otimes g_j \right\|_{\infty} \tag{37}$$

$$\dot{\mathbf{x}}_j(k) = \mathcal{I}_j \acute{\mathbf{x}}_j(k). \tag{38}$$

Proof: First, it follows from (19) and (23) that

$$\acute{\mathbf{e}}_j(k) = \tilde{\mathcal{F}}_j(k-1)\widehat{\mathbf{e}}_j(k-1) + \tilde{w}_j(k-1) \tag{39}$$

$$\widehat{\mathbf{e}}_j(k) = (I - \mathcal{K}_j(k)\tilde{\mathcal{G}}_j(k))\acute{\mathbf{e}}_j(k) - \mathcal{K}_j(k)\mathscr{O}_j(k) - \mathcal{K}_j(k)\varsigma_j(k). \tag{40}$$

Next, let us prove (26) based on (39). For this purpose, we aim to show

$$\tilde{w}_j(k-1) \in \langle 0, \bar{\mathbf{W}}_j(k-1)\rangle. \tag{41}$$

According to Definition 1 and Lemma 3, one has

$$w^{[l-s]}(k-1) - \mathbf{w}(l-s) \in \langle 0, \mathbf{W}(l-s)\rangle$$

and $w^{[0]}(k-1) - \mathbf{w}(0) = 0$, which further imply

$$\left\| w^{[l-s]}(k-1) - \mathbf{w}(l-s) \right\|_{\infty} \le \|\mathbf{W}(l-s)\|_{\infty}. \tag{42}$$

From (34), (35) and the definitions of $\tilde{\mathbf{x}}_j(k)$ and $\widehat{\mathbf{e}}_j(k)$, it can be seen that

$$\mathbf{x}(k-1) - \hat{\mathbf{x}}_j(k-1) = \mathcal{I}_j \widehat{\mathbf{e}}_j(k-1)$$

which, together with (25), (33) and Lemma 1, leads to

$$\begin{aligned} \mathbf{x}(k-1) - \hat{\mathbf{x}}_j(k-1) &\in \mathcal{I} \odot \langle 0, \widehat{E}_j(k-1)\rangle \\ &= \langle 0, \widehat{E}_{j,1}(k-1)\rangle. \end{aligned} \tag{43}$$

Then, applying Lemma 6 to (43) yields

$$\|\mathbf{x}(k-1) - \hat{\mathbf{x}}_j(k-1)\|_{\infty} \le \|\widehat{E}_{j,1}(k-1)\|_{\infty}. \tag{44}$$

By virtue of Lemma 7, one has

$$\| \left(\mathbf{x}(k-1) - \hat{\mathbf{x}}_j(k-1)\right)^{[p]} \|_{\infty} = \|\mathbf{x}(k-1) - \hat{\mathbf{x}}_j(k-1)\|_{\infty}^{p}$$

which, together with (44), gives

$$\| \left(\mathbf{x}(k-1) - \hat{\mathbf{x}}_j(k-1)\right)^{[p]} \|_{\infty} \le \|\widehat{E}_{j,1}(k-1)\|_{\infty}^{p}. \tag{45}$$

With (42) and (45), applying the triangle inequality and Lemma 7 to (16), one can obtain

$$\|\vec{w}_j(l,k-1)\|_{\infty} \le \bar{\mathbf{w}}_j(l,k-1).$$

Applying Lemma 8 to the above inequality, one can see $\vec{w}_j(l,k-1) \in \langle 0, \bar{\mathbf{w}}_j(l,k-1)I_{4^l}\rangle$. Noticing the definition of $\tilde{w}_j(k)$ (below (19)), one can immediately derive (41) from Definition 1.

With (25), (39) and (41), it can be deduced that

$$\acute{\mathbf{e}}_j(k) \in (\tilde{\mathcal{F}}_j(k-1) \odot \langle 0, \widehat{E}_j(k-1)\rangle) \oplus \langle 0, \bar{\mathbf{W}}_j(k-1)\rangle$$

by which one can readily arrive at (26) by applying Lemma 1.

After proving (26), it remains to show that (27) is true. With (38), it follows from the expression of $\breve{\mathbf{x}}_j(k)$ (below (12)) that

$$\begin{aligned} \breve{\mathbf{x}}_j(k) &= \dot{\mathbf{x}}_j(k) + \theta_j(k)(\mathbf{x}(k) - \dot{\mathbf{x}}_j(k)) \\ &= \dot{\mathbf{x}}_j(k) + \theta_j(k)\mathcal{I}_j \acute{\mathbf{e}}_j(k). \end{aligned} \tag{46}$$

In view of (26) and (46), it is easy to see from Definition 1 that

$$\breve{\mathbf{x}}_j(k) \in \langle \dot{\mathbf{x}}_j(k), \mathcal{I}_j \acute{E}_j(k)\rangle$$

which, together with the expression of $\mathscr{R}_j(\breve{\mathbf{x}}_j(k))$, leads to

$$\|\mathscr{R}_j(\breve{\mathbf{x}}_j(k))\|_{\infty} \le ((\nu_j+1)!)^{-1}\bar{g}_j(k). \tag{47}$$

Similar to (44) and (45), one can obtain

$$\|(\mathbf{x}(k) - \dot{\mathbf{x}}_j(k))^{[\nu_j+1]}\|_{\infty} \le \|\mathcal{I}_j \acute{E}_j(k)\|_{\infty}^{\nu_j+1}. \tag{48}$$

According to (47), (48) and the expression of $\mathscr{O}_j(k)$, it can be verified that

$$\|\mathscr{O}_j(k)\|_{\infty} \le \bar{o}_j(k)$$

which, together with Lemma 8, implies

$$\mathscr{O}_j(k) \in [-\bar{o}_j(k), \bar{o}_j(k)]. \tag{49}$$

Based on (40), (49), (26) and the fact $\varsigma_j(k) \in [-\bar{\varsigma}_j, \bar{\varsigma}_j]$ (see Lemma 4), it is easy to prove (27) by using Lemma 1 again. ■

Remark 8: Theorem 1 establishes a recursive relationship between the zonotope that bounds the prediction error $\acute{\mathbf{e}}_j(k)$ and the zonotope that bounds the estimation error $\widehat{\mathbf{e}}_j(k)$. Note that deriving the zonotope enclosing $\widehat{\mathbf{e}}_j(k)$ requires solving the optimization problem described in (37). Various methods can be employed to obtain an relatively accurate solution to this problem such as evolutionary algorithms [7], [11], [24], [28], [31], [35].

In the next theorem, the parameter $K_j(k)$ that can optimize the F-radius of $\langle 0, \widehat{E}_j(k)\rangle$ will be given.

Theorem 2: Let the parameter $K_j(k)$ in (24) be designed as

$$K_j(k) = \mathscr{I}_j^T \acute{E}_j(k) \acute{E}_j^T(k) \tilde{\mathcal{G}}_j^T(k) \Upsilon_j^{-1}(k) \tag{50}$$

where

$$\mathscr{I}_j = \begin{bmatrix} 0_{(\sum_{q=1}^{\nu_j} 4^q)\times 1} & I_{\sum_{q=1}^{\nu_j} 4^q} \end{bmatrix}^T$$

$$\Upsilon_j(k) = \tilde{\mathcal{G}}_j(k)\acute{E}_j(k)\acute{E}_j^T(k)\tilde{\mathcal{G}}_j^T(k) + \bar{o}_j^2(k) + \bar{\varsigma}_j^2.$$

Then, the F-radius of the zonotope in (27) is minimized.

Proof: From Definition 2, it can be derived that the F-radius of the zonotope in (27) satisfies

$$\begin{aligned} &\|\widehat{E}_j(k)\|_F^2 \\ &= \mathrm{Tr}\Big\{ \left(K_j^T(k)\mathscr{I}_j^T - \Upsilon_j^{-1}(k)\tilde{\mathcal{G}}_j(k)\acute{E}_j(k)\acute{E}_j^T(k) \right)^T \Upsilon_j(k) \\ &\quad \times \left(K_j^T(k)\mathscr{I}_j^T - \Upsilon_j^{-1}(k)\tilde{\mathcal{G}}_j(k)\acute{E}_j(k)\acute{E}_j^T(k) \right) \\ &\quad - \acute{E}_j(k)\acute{E}_j^T(k)\tilde{\mathcal{G}}_j^T(k)\Upsilon_j^{-1}(k)\tilde{\mathcal{G}}_j(k)\acute{E}_j(k)\acute{E}_j^T(k) \end{aligned}$$

$$+\acute{E}_j(k)\acute{E}_j^T(k)\Big\}. \tag{51}$$

On the basis of (51) and $\mathscr{I}_j^T\mathscr{I}_j = I$, it is easy to see that the parameter $K_j(k)$ in (50) can minimize the F-radius of the zonotope $\langle 0, \widehat{E}_j(k)\rangle$, which completes the proof. ■

Remark 9: To calculate the parameter $K_j(k)$ in (50), it is necessary to ensure that $\Upsilon_j(k)$ is invertible. Based on the definition of $\bar{\varsigma}_j$ (given in Lemma 4) and the assumption $V_j > 0$ (see Assumption 2), it follows that $\bar{\varsigma}_j > 0$, which implies $\Upsilon_j(k) > 0$.

D. Detection Scheme for the Flipping Bits

In this subsection, a detection scheme will be proposed to judge whether there is a flipping bit in the binary bit string $\acute{b}_{j,L_j}(k)\acute{b}_{j,L_j-1}(k)\cdots\acute{b}_{j,1}(k)$. For this purpose, the zonotope restraining the decoding signal $\mathbf{d}_j(k)$ in the case $\mathbf{d}_j(k) = \mathbf{c}_j(k)$ will be calculated in the following theorem.

Theorem 3: If $\mathbf{d}_j(k) = \mathcal{L}_j\mathbf{c}_j(k)$, then the decoding signal $\mathbf{d}_j(k)$ satisfies

$$\begin{aligned}\mathbf{d}_j(k) \in \Big\langle \tilde{\mathcal{G}}_j(k)\acute{\mathbf{x}}_j(k), \big[\tilde{\mathcal{G}}_j(k)\acute{E}_j(k) \quad -\bar{o}_j(k) \quad -\bar{\varsigma}_j\big]\Big\rangle \\ \triangleq \mathscr{D}_j(k).\end{aligned} \tag{52}$$

Proof: From Theorem 1, it is known that (26) holds, which implies that

$$\tilde{\mathbf{x}}_j(k) = \acute{\mathbf{x}}_j(k) + \acute{\mathbf{e}}_j(k) \in \langle \acute{\mathbf{x}}_j(k), \acute{E}_j(k)\rangle. \tag{53}$$

With (49), (53) and $\varsigma_j(k) \in [-\bar{\varsigma}_j, \bar{\varsigma}_j]$, it follows from (19) that

$$\begin{aligned}\mathbf{d}_j(k) \in \tilde{\mathcal{G}}_j(k) \odot \langle \acute{\mathbf{x}}_j(k), \acute{E}_j(k)\rangle \oplus [-\bar{o}_j(k), \bar{o}_j(k)] \\ \oplus [-\bar{\varsigma}_j, \bar{\varsigma}_j] = \mathscr{D}_j(k)\end{aligned} \tag{54}$$

which ends the proof. ■

Corollary 1: If $\mathbf{d}_j(k) \notin \mathscr{D}_j(k)$, then there has one or more flipping bits in the binary bit string $\acute{b}_{j,L_j}\acute{b}_{j,L_j-1}\ldots\acute{b}_{j,1}$.

Proof: Using Theorem 3, the result of this corollary can be readily obtained. ■

As a summary of the above results, the local estimation algorithm is given as follows.

Remark 10: As in [47], the order reduction technique of zonotopes, which increases the F-radius of the zonotope in (27), is utilized in Algorithm 1 to guarantee that the number of columns of the matrix $\widehat{E}_j(k)$ remains bounded. Note that a larger M_j results in less frequent application of the order reduction, thereby reducing the computational complexity of the local estimation algorithm. Consequently, M_j should be selected to balance estimation accuracy and computational complexity.

Remark 11: It should be noted that the detection method proposed in Corollary 1 is based on a sufficient condition for the existence of flipping bits. This means that the zonotope $\langle 0, \widehat{E}_j(k)\rangle$, as computed by Algorithm 1, can enclose $\widehat{\mathbf{e}}_j(k)$ only when no flipping bit exists or when a flipping bit is successfully detected. Otherwise, $\langle 0, \widehat{E}_j(k)\rangle$ may fail to contain $\widehat{\mathbf{e}}_j(k)$. In such cases, the distributed fusion scheme to be discussed in the next subsection can mitigate the side effects caused by the flipping bits.

Algorithm 1: Local Estimation Algorithm

Input: Positive integer $k_{\max}$ and matrix $\tilde{\mathbf{X}}_j(0)$ in (22).
Output: $\widehat{\mathbf{x}}_j(k)$ and $\widehat{E}_j(k)$.
Initialization: Set $k = 0$, $\widehat{\mathbf{x}}_j(0) = \tilde{c}_j(0)$, and $\widehat{E}_j(0) = \tilde{\mathbf{X}}_j(0)$;
for $k = 1, 2, \ldots, k_{\max}$ **do**
    Compute $\widehat{E}_{j,1}(k-1)$ and $\hat{\mathbf{x}}_j(k-1)$ by (33)–(35) ;
    Compute $\tilde{\mathcal{F}}_j(k-1)$ by (15) and (20);
    Compute $\acute{\mathbf{x}}_j(k)$ by (23);
    Compute $\bar{\mathbf{W}}_j(k-1)$ by (30)–(35);
    Compute $\acute{E}_j(k)$ by (28);
    Compute $\tilde{\mathcal{G}}_j(k)$ by (17) and (21);
    Compute $\bar{o}_j(k)$ by (36) and (37);
    Compute $\mathscr{D}_j(k)$ by (52);
    if $\mathbf{d}_j(k) \in \mathscr{D}_j(k)$ **then**
        Compute $K_j(k)$ and $\mathcal{K}_j(k)$ by (50) and (24);
        Compute $\widehat{E}_j(k)$ by (29);
        Compute $\widehat{\mathbf{x}}_j(k)$ by (23);
    else
        $\widehat{\mathbf{x}}_j(k) = \acute{\mathbf{x}}_j(k)$, $\widehat{E}_j(k) = \acute{E}_j(k)$;
if *columns of* $\widehat{E}_j(k)$ *is larger than a pre-set positive integer* M_j **then**
    $\widehat{E}_j(k) = \mathrm{rs}\{\widehat{E}_j(k)\}$;

E. The Distributed Fusion Scheme

In this subsection, a distributed fusion scheme will be given to fuse the information calculated by the local estimators.

For the state $\mathbf{x}(k)$ of the target, its estimate calculated by the j-th local estimator is $\mathcal{I}_j\widehat{\mathbf{x}}_j(k)$. Let $\sum_{j=1}^{N}\Psi_j(k)\mathcal{I}_j\widehat{\mathbf{x}}_j(k)$ be the fused estimate, where $\Psi_j(k)$ $(j = 1, 2, \ldots, N)$, satisfying $\sum_{j=1}^{N}\Psi_j(k) = I$, are matrix weights to be determined. From Lemma 1 and Theorem 1, it can be easily obtained that $\mathbf{x}(k) - \sum_{j=1}^{N}\Psi_j(k)\mathcal{I}_j\widehat{\mathbf{x}}_j(k) \in \langle 0, \widehat{\mathcal{E}}(k)\rangle$ where $\widehat{\mathcal{E}}(k) = \Big[\Psi_1(k)\widehat{E}_{1,1}(k) \quad \cdots \quad \Psi_N(k)\widehat{E}_{N,1}(k)\Big]$.

To ensure that the global estimation performance is better than that of each local estimation, the weights $\Psi_j(k)$ $(j = 1, 2, \ldots, N)$ can be given by minimizing the F-radius of the zonotope $\langle 0, \widehat{\mathcal{E}}(k)\rangle$. According to [46], it is known that the solution to this optimization problem is

$$\Psi_j(k) = \left(\sum_{j=1}^{N}\Big(\widehat{E}_{j,1}(k)\widehat{E}_{j,1}^T(k)\Big)^{-1}\right)^{-1}\Big(\widehat{E}_{j,1}(k)\widehat{E}_{j,1}^T(k)\Big)^{-1}.$$

Remark 12: Compared with the existing target tracking methods based on SME, the proposed DPSMFE approach offers the following distinguishable advantages: 1) the information of the higher-order partial derivatives of the distance functions is utilized in the design of the local estimators, whereby the local estimation accuracy is expected to be higher than that achieved by using methods based on the first- and/or second-order partial derivatives, such as those in [2], [18], [37]; and 2) a distributed fusion scheme is adopted, which suppresses the side effects of the flipping bits that might not

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be detected by the detection scheme. Moreover, the proposed DPSMFE algorithm is readily extendable to more complex cases of target tracking, such as the target tracking problem in a three-dimensional (3-D) region or scenarios where the target is monitored by both distance and angle sensors with time-varying sampling periods.

V. ILLUSTRATIVE EXAMPLE

Consider a target in the form (1) with $T = 0.01$. The initial state $\mathbf{x}(0)$ of the target is chosen as $\mathbf{x}(0) = \begin{bmatrix}-0.1 & 0.1 & -0.1 & 0.1\end{bmatrix}^T$. The control input of the target is set to be $\breve{w}(k) = \begin{bmatrix}-0.3\cos(0.1k) & 0.2\sin(0.1k)\end{bmatrix}^T$. Three distance sensors are employed with their positions being $(\underline{x}_1, \underline{y}_1) = (5, 10)$, $(\underline{x}_2, \underline{y}_2) = (5, 5)$, and $(\underline{x}_3, \underline{y}_3) = (5, -5)$. The measurement noises are given by

$$(v_1(k), v_2(k), v_3(k)) = (0.1\cos(0.1k), 0.15\sin(0.1k), \\ 0.2\cos(0.2k)).$$

To ensure that Assumptions 1 and 2 hold, we select

$$X(0) = \mathrm{diag}\{0.1, 0.1, 0.1, 0.1\},\ \hat{w} = 0_{2\times 1}, \\ W = \mathrm{diag}\{0.3, 0.2\},\ V_1 = 0.1,\ V_2 = 0.15,\ V_3 = 0.2.$$

Moreover, parameters of the binary encoding scheme are chosen as $\bar{z}_1 = 12$, $\bar{z}_2 = 8$, $\bar{z}_3 = 8$ and $L_1 = L_2 = L_3 = 8$.

To show effectiveness of the proposed detection-based local estimation algorithm and the distributed fusion estimation method, the case that there are some flipping bits in the received codeword is considered. Fig. 1 plots the actual codewords $b_{j,8}(k)b_{j,7}(k)\cdots b_{j,1}(k)$ $(j = 1, 2, 3)$ and the codewords $\acute{b}_{j,8}(k)\acute{b}_{j,7}(k)\cdots\acute{b}_{j,1}(k)$ received by the local estimators (represented in decimal). It can be seen that there are flipping bits in the codeword $\acute{b}_{1,8}(k)\acute{b}_{1,7}(k)\cdots\acute{b}_{1,1}(k)$ at time instants 50 and 110, in the codeword $\acute{b}_{2,8}(k)\acute{b}_{2,7}(k)\cdots\acute{b}_{2,1}(k)$ at time instants 30, 100 and 170, and in the codeword $\acute{b}_{3,8}(k)\acute{b}_{3,7}(k)\cdots\acute{b}_{3,1}(k)$ at time instants 20, 50, 130 and 160. Employ the index

$$\tau_j(k) \triangleq \begin{cases} 1, & \text{if } \mathbf{d}_j(k) \notin \mathscr{D}_j(k) \\ 0, & \text{otherwise} \end{cases}$$

to describe whether the flipping bits are detected for $j = 1, 2, 3$. The values of $\tau_j(k)$ $(j = 1, 2, 3)$ are shown in Fig. 2. Obviously, all flipping cases are detected. Fig. 3, which is depicted under $\nu_1 = \nu_2 = \nu_3 = 3$, gives the information about the trajectories of the target position, its three local estimates and the fused estimate, which shows that the proposed DPSMFE method performs well.

To verify that usage of the higher-order partial derivatives of the distance function in (3) could improve the estimation accuracy, the F-radius of the fused zonotope (i.e., $\|\widehat{\mathcal{E}}(k)\|_F$) under different ν_j $(j = 1, 2, 3)$ is given in Fig. 4. It can be seen that, with the increase of parameters ν_j $(j = 1, 2, 3)$, the F-radius of the fused zonotope decreases.

Remark 13: As shown in Fig. 4, the reduction of $\|\widehat{\mathcal{E}}(k)\|_F$ slows down as ν_j $(j = 1, 2, 3)$ increases. This raises a natural question: is there an optimal upper bound for ν_j in practical applications, and how should ν_j be selected? This constitutes one of the key directions for our future research.

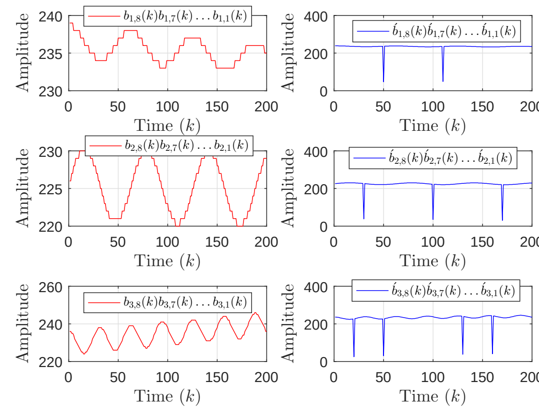

Fig. 1: The actual codewords $b_{j,L_j}(k)b_{j,L_j-1}(k)\cdots b_{j,1}(k)$ $(j = 1, 2, 3)$ and the codewords $\acute{b}_{j,L_j}(k)\acute{b}_{j,L_j-1}(k)\cdots\acute{b}_{j,1}(k)$ $(j = 1, 2, 3)$ received by the local estimators, which are represented in decimal.

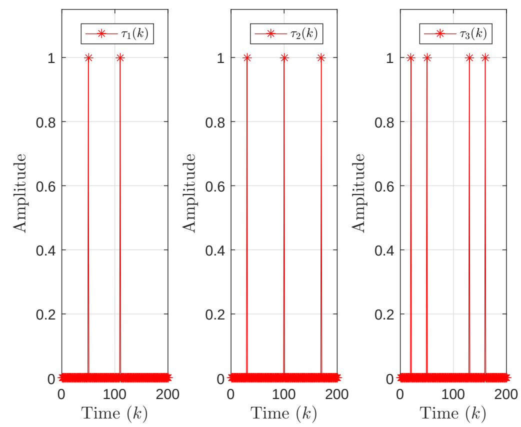

Fig. 2: The values of $\tau_j(k)$ $(j = 1, 2, 3)$.

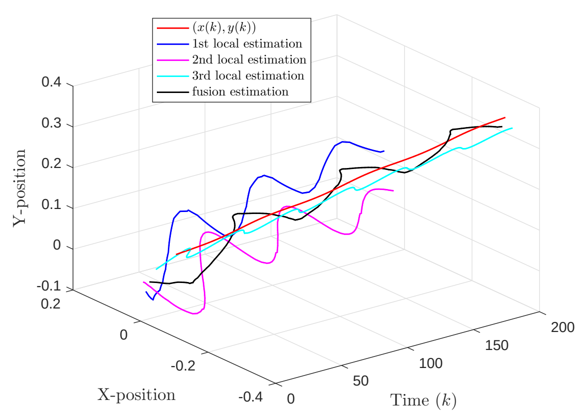

Fig. 3: The trajectories of the target position, its three local estimates and the fused estimate.

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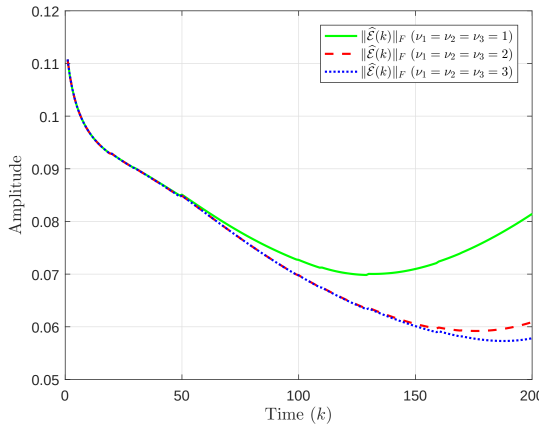

Fig. 4: $\|\widehat{\mathcal{E}}(k)\|_F$ under different ν_j $(j = 1, 2, 3)$.

VI. CONCLUSION

In this paper, a distributed polynomial set-membership fusion estimation (DPSMFE) approach has been proposed for target tracking under a binary encoding scheme. Multiple distance sensors have been deployed to monitor the target, with the measurements encoded and transmitted to local estimators. The influence of possible flipping bits has been addressed during transmissions. Local estimators have been designed by employing a polynomial set-membership estimation (PSME) method, which has incorporated higher-order partial derivatives of the distance function, thereby enhancing estimation accuracy compared to methods using only lower-order derivatives. A zonotope-based detection scheme has been developed to identify flipping bits in decoding signals. An optimal distributed fusion estimator, formulated in terms of the F-radius of the zonotope restraining the global estimation error, has been designed to suppress the side effects of undetected flipping bits. Simulation studies have been conducted to illustrate the effectiveness of the proposed approach. Future research will extend the DPSMFE approach to more complex target dynamics, including maneuvering targets in three-dimensional (3-D) environments and integrating measurements from heterogeneous sensors with varying sampling periods and communication delays.

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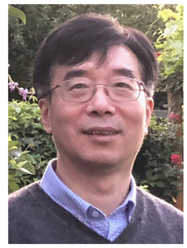

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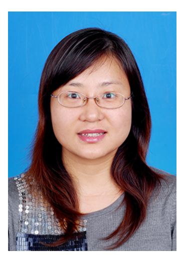

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