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Privacy-Preserving Distributed Energy Management for Battery Energy Storage Systems Over Time-Varying Networks

Wei Chen, Zidong Wang, Jimmy Chih-Hsien Peng, and Guo-Ping Liu

***Abstract*—This paper addresses the privacy-preserving energy management problem of battery energy storage systems (BESSs). An autonomous privacy-preserving distributed optimization (APPDO) scheme is developed over time-varying networks with the aim of regulating the power output of local BESS to fulfill the total load demand at the minimum economic cost under battery capacity constraints without privacy leakage. To this end, a linearly convergent distributed algorithm is proposed by combining the gradient descent algorithm with leaderless and leader-following consensus schemes. This algorithm is applicable to both islanded and grid-connected modes of BESSs. Furthermore, a novel privacy-preserving approach is constructed by injecting well-designed perturbation sequences into the data exchanged between neighboring nodes, making it effective against malicious eavesdroppers. Furthermore, a comprehensive analysis framework is established to evaluate the convergence, optimality, and privacy-preserving performance of the APPDO algorithm. Finally, numerical studies are conducted to demonstrate the effectiveness of the developed APPDO scheme.**

***Index Terms*—Distributed energy management, privacy preservation, linearly convergent distributed algorithm, correlated perturbations, battery energy storage systems.**

I. Introduction

In recent years, the push for green and sustainable development has significantly increased the penetration of distributed renewable generation in power grids. However, distributed renewable resources (e.g., solar and wind) are typically subject to characteristics such as uncertainty, randomness, and intermittency, which pose serious challenges to energy management in power systems [1]–[4]. To address the limitations of renewable resources, battery energy storage systems (BESSs) can be introduced to buffer short-term power imbalances, thereby enhancing power dispatch performance. BESSs have been widely applied in various renewable energy systems due to their crucial role in improving power supply quality, shaving peak demand, and providing backup power during emergencies (e.g., grid failures and electrical shortages [5]–[7]).

Energy management has become a cutting-edge topic in both industry and academia, driven by the widespread adoption of environmental, social, and governance (ESG) standards by many nations. Consequently, it is critical to ensure the efficient, safe, reliable, and economical operation of BESSs. With the increasing prevalence of energy storage technology, the number of power suppliers continues to grow. In this context, centralized energy management schemes struggle to manage system operations and data processing for a large number of participants dispersed over vast geographical areas [8]–[11]. As a promising alternative, the distributed approach has garnered significant attention, which leads to the development of several notable algorithms, including but not limited to, the distributed alternating direction method of multipliers [12], [13], the decentralized exact first-order algorithm [14], [15], and the distributed primal-dual algorithm [16], [17].

In distributed implementations, local agents need to frequently exchange data with their neighbors over networks to achieve a global goal [18]–[20]. However, in actual engineering, communication networks may be time-varying due to the plug-and-play characteristics of the grid and unexpected communication link failures. Several initial studies have explored distributed algorithms over time-varying networks [16], [17], [21]. For instance, a distributed scheme inspired by the dual averaging algorithm has been developed in [16] for use in time-varying topologies. Furthermore, an optimization algorithm has been developed for time-varying networks by adopting the push-(sub)gradient method [17]. Noted that the algorithms mentioned above require a decaying step size to achieve exact convergence, which degrades the convergence speed. Moreover, these algorithms are restricted to operating in the islanded mode. Therefore, it is imperative to develop an autonomous distributed algorithm with a rapid convergence rate that can function in both islanded and grid-connected modes, and such a necessity motivates our current study.

Within the landscape of the energy internet, BESSs are increasingly interconnected and reliant on digital technologies, resulting in a substantial increase in the volume of data generated and processed. While these data are valuable for optimizing operations and improving efficiency, they also pose new privacy and security challenges [22], [23]. Specifically, if sensitive data are compromised, they can be exploited to disrupt decision-making processes, interfere with system

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Wei Chen and Jimmy Chih-Hsien Peng are with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117583, Singapore (Emails: wei_chen@nus.edu.sg; jpeng@nus.edu.sg).

Zidong Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk).

Guo-Ping Liu is with School of Automation and Intelligent Manufacturing, Southern University of Science and Technology, Shenzhen 518055, China, (Email: liugp@sustech.edu.cn).

monitoring, and potentially threaten overall system stability [24], [25]. Hence, it is urgent to develop a privacy-preserving approach to prevent data breaches or theft that could lead to system disruptions, thereby ensuring the reliable and efficient operation of BESSs.

To preserve sensitive data, some privacy-preserving techniques have been reported in the literature [12], [13], [17], [26]. For instance, confidential communication has been achieved by utilizing the additive homomorphic property of the Paillier cryptosystem [12], [26]–[28]. However, cryptography-based approaches may require considerable computation and communication resources due to the inherent complexity of asymmetric cryptographic algorithms [29]. Additionally, a privacy-preserving distributed algorithm has been designed in [14] through state decomposition. This approach divides state information into two components: an external substate, which can communicate with neighboring nodes, and an internal substate, which interacts solely with the external substate and remains invisible to eavesdroppers. It should be pointed out that such a scheme may reduce convergence speed due to the altered connectivity of the communication topology.

Apart from cryptography-based and state-decomposition methods, noise-injected privacy preservation is also a mainstream approach. The primary concept involves injecting well-designed noise into sensitive information to confuse adversaries [30], [31]. However, it should be emphasized that existing noise-injected schemes may compromise the convergence accuracy of algorithms [13], [32] and reduce privacy performance against external eavesdroppers [17], [21]. Therefore, there is a need to explore an effective privacy-preserving technique that maintains both convergence accuracy and robust privacy performance, and this necessity forms the primary motivation for our present study.

Given the aforementioned discussions, we endeavor to develop an APPDO algorithm by combining the leaderless and leader-following consensus schemes with a correlated perturbation mechanism, aiming to achieve optimal energy management without exposing sensitive information. The main contributions of this work are articulated as follows.

1) A consensus-based distributed algorithm over time-varying topologies is developed with a linear convergence rate, suitable for both grid-connected and islanded modes of BESSs. Compared to existing works [5], [7], our algorithm covers both modes and enables smooth mode transitions. Furthermore, in comparison to [16], [17], [21], our algorithm achieves rapid convergence with a geometric rate;
2) A correlated perturbation scheme is incorporated into the corresponding distributed algorithm, demonstrating robustness against malicious eavesdroppers. Unlike differentially private approaches [13], [32] and homomorphic encryption schemes [26], [33], our scheme offers significant advantages in terms of precise convergence, minimal computational complexity, and uncompromised privacy-preserving performance;
3) By employing the small gain theorem, a sufficient condition is derived to verify that the developed APPDO algorithm can exponentially attain to the globally optimal solution in both modes. Unlike the eigenvalue analysis technique [34], [35], our technique is more general through addressing the case of time-varying topology.

Notation: $\|\cdot\|$ denotes the Euclidean norm. $\mathbf{1}_N$ ($\mathbf{0}_N$) is the N-dimensional column vector of ones (zeros). I_N is the N-dimensional identical matrix. $\mathrm{col}_N\{a_i\}$, $\mathrm{diag}_N\{a_i\}$, $[a_{ij}]_N$ are the N-dimensional column vector, diagonal matrix, and real matrix with elements being a_i, a_i, and a_{ij}, respectively.

II. Problem Formulation and Preliminaries

A. Network Model

In BESSs, the cyber layer can be described as a multi-agent system, where each node collects local power supply and demand information from a BESS, local loads, and local distributed energy resources (DERs), and provides an optimal nominal value to BESSs via local data exchange. The communication network of N agents can be represented by an undirected graph $\mathscr{G}(k) \triangleq \{\mathcal{V}, \mathcal{E}(k)\}$, where $\mathcal{V} \triangleq \{1, 2, \cdots, N\}$ and $\mathcal{E}(k) \triangleq \{(i,j)|i,j \in \mathcal{V}\}$ are, respectively, the vertex and edge sets at iteration k. The pair $(i,j) \in \mathcal{E}(k)$ indicates that agent j can receive information from agent i. Let $\mathcal{N}_i(k) \triangleq \{j|(j,i) \in \mathcal{E}(k), j \neq i\}$ denote the neighboring set of node i at iteration k. Denote the adjacency matrix of graph $\mathscr{G}(k)$ as $A_k \triangleq [a_{ij,k}]_N$, where the weight $a_{ij,k} > 0$ if $(j,i) \in \mathcal{E}(k)$, and $a_{ij,k} = 0$ otherwise. Define the Laplacian matrix as $L_k \triangleq C_k - A_k$, where $C_k \triangleq \mathrm{diag}_N\{c_{i,k}\}$ is the degree matrix with $c_{i,k} \triangleq \sum_{j \in \mathcal{N}_i(k)} a_{ij,k}$. Specially, the utility grid can be labeled by agent 0 (i.e., the leader), where $a_{i0} = a_{0i} = 1$ if agent i is connected to the utility grid via energy router (ER), and $a_{i0} = a_{0i} = 0$ otherwise.

Assumption 1: The sequence of undirected graph $\{\mathscr{G}(k)\}_{k \geq 0}$ is jointly connected. Specifically, there exists an integer $d > 0$ such that the graphs $\cup_{k=ld}^{(l+1)d-1} \mathscr{G}(k)$ are connected for $\forall l \geq 0$.

B. Optimization Problem

Consider a distributed energy management problem of BESSs over a finite time horizon T. First, we assume that the local demand $P_{d,i}(t)$, the DER generation power $P_{G,i}(t)$, and the BESS generation power $P_{B,i}(t)$ remain unchanged in each period $t \in \{1, 2, \ldots \mathcal{T}\}$.

Given the characteristics of BESSs, we have the following constraints. For BBSS i (i.e., agent i), the charging/discharging power is constrained by

$$\underline{P}_{B,i} \leq P_{B,i}(t) \leq \overline{P}_{B,i}, \ t \in \mathcal{T}, \ i \in \mathcal{V}. \tag{1}$$

To extend the operational lifespan of the energy storage devices, the state of energy (SoE) of BESS i is subjected to

$$\underline{SoE}_i \leq SoE_i(0) - \frac{\Delta t}{E_i} \sum_{l=1}^{t} \varrho_i^{\varepsilon(l)} P_{B,i}(l) \leq \overline{SoE}_i, \tag{2}$$

where $\varrho_i \in (0,1)$ is the charing/discharing efficiency, $\Delta t \triangleq T/\mathcal{T}$ is the length of each time interval, E_i is the maximum available energy, and $\varepsilon(l) \triangleq -\mathrm{sgn}(P_{B,i}(l))$.

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To achieve supply-demand balance, the power must satisfy

$$\sum_{i=1}^{N} P_{B,i}(t) + \tau(t) P_{UG}(t) = \sum_{i=1}^{N} P_{d,i}(t) - \sum_{i=1}^{N} P_{G,i}(t), \quad (3)$$

where $P_{UG}(t)$ is the power from the utility grid, and $\tau(t) \in \{0, 1\}$ is the mode converter. If $\tau = 1$, BESSs carry out in the islanded mode, and in the grid-connected mode otherwise. In this paper, $P_{d,i}(t)$ and $P_{G,i}(t)$ are known but uncontrollable, and $P_{G,i}(t)$ can be regarded as the negative load.

In BESSs, the main aim of energy management is to enhance the efficiency of electricity utilization and minimize the expenses associated with electrical supply. Generally speaking, the cost for BESS i primarily consists of the energy storage capacity deterioration cost and a fixed cost [7], [36], [37], which can be described by

$$C_i(P_{B,i}(t)) = b_i (P_{B,i}(t))^2 + a_i P_{B,i}(t) + c_i. \quad (4)$$

In the sequel, the power dispatch problem for multiple BESSs over the finite horizon can be formulated by

$$\begin{aligned} \underset{P_{B,i}(t)\ (i \in \mathcal{V},\ t \in \mathcal{T})}{\arg\min} \ & \sum_{t=1}^{\mathcal{T}} \sum_{i=1}^{N} C_i(P_{B,i}(t)) + \tau \lambda_0(t) P_{UG}(t) \\ \text{s.t.} \ & (1) - (3), \end{aligned} \quad (5)$$

where $\lambda_0(t)$ represents the electricity price from the utility grid.

Denote the marginal cost of BESS i as

$$\lambda_i(t) \triangleq \frac{\partial C_i(P_{B,i}(t))}{\partial P_{B,i}(t)} = 2 b_i P_{B,i}(t) + a_i. \quad (6)$$

Note that the optimization issue (5) is addressed by utilizing the classic Lagrange multiplier approach, with the expression of the optimal solution given by $\lambda_i(t) = \lambda^*(t)$. If $\tau = 1$, then $\lambda^*(t) = \lambda_0(t)$, where $\lambda^*(t)$ is related to the cost function of BESSs. More details can be found in [38].

C. Objective

Before outlining the main aims, we first provide the following definition of privacy in BESSs.

Definition 1: Consider a BESS of N agents, the privacy of BESS i is preserved, if malicious agent j cannot estimate/infer sensitive information pair $\{a_i, b_i\}$.

Consequently, the purpose of this study is twofold: 1) to design an autonomous distributed algorithm to solve the optimization problem (5); and 2) to construct a privacy-preserving mechanism that prevents sensitive data from being estimated by malicious agents.

Remark 1: Note that the sensitive information of BESSs plays a critical role in bidding strategies within the energy market, avoiding malicious competition, and ensuring the secure and dependable functioning of power grids. In this study, the malicious node can be regarded as a competitor in BESSs. The competitor could exploit the sensitive information from other BESS units to strategically adjust its own power output, thus gaining a dominant position in the electricity market and maximizing its own benefits. Hence, it is essential to develop schemes that enhance the preservation of power privacy data and prevent the exposure of sensitive information.

III. Distributed Algorithm Design and Its Performance Analysis

In this section, we first develop an APPDO algorithm designed for time-varying communication networks. Then, by leveraging the small gain theorem, we demonstrate that the developed algorithm achieves linear convergence in both islanded and grid-connected modes. Additionally, the constructed algorithm ensures a smooth transition between these two modes. Furthermore, a privacy analysis is conducted to verify the security of the developed algorithm against malicious agents.

A. APPDO Algorithm

To address the energy management problem in (5), we first focus on the single time-step energy management problem and then extend it to the multi-step case with inter-temporal constraints.

Inspired by the leaderless and leader-following consensus-based distributed scheme and the correlated perturbation mechanism, an APPDO algorithm is constructed as follows:

$$\begin{aligned} \lambda_i^{k+1}(t) &= \lambda_i^k(t) + \epsilon_1 \Big(\sum_{j \in \mathcal{N}_i(k)} w_{ij,k} (\hat{\lambda}_{j^i}^k(t) - \lambda_i^k(t)) \\ &\quad + \tau w_{i0} (\lambda_0(t) - \lambda_i^k(t)) + s_{1,i^i}^k(t) \Big) + \varsigma z_i^k(t) \\ P_{B,i}^k(t) &= \underset{P_{B,i}^k(t) \in [\underline{P}_{B,i},\ \overline{P}_{B,i}]}{\arg\min} \left\| P_{B,i}^k(t) - \frac{\lambda_i^k(t) - a_i}{2 b_i} \right\| \\ y_i^{k+1}(t) &= z_i^k(t) + \epsilon_2 \sum_{j \in \mathcal{N}_i(k)} l_{ij,k} (\hat{z}_{j^i}^k(t) - z_i^k(t)) \\ &\quad + \epsilon_2 s_{2,i^i}^k(t) - (P_{B,i}^{k+1}(t) - P_{B,i}^k(t)) \\ \delta P_{U,i}^{k+1}(t) &= \tau w_{0i} y_i^{k+1}(t) \\ P_{U,i}^{k+1}(t) &= \tau (P_{U,i}^k(t) + w_{i0} \delta P_{U,i}^{k+1}(t)) \\ z_i^{k+1}(t) &= y_i^{k+1}(t) + w_{i0} (P_{U,i}^k(t) - P_{U,i}^{k+1}(t)) \\ P_{UG}^k(t) &= \sum_{i=1}^{N} P_{U,i}^k(t), \end{aligned} \quad (7)$$

where

$$\hat{\lambda}_{j^i}^k(t) \triangleq \lambda_j^k(t) + s_{1,j^i}^k(t), \ \hat{z}_{j^i}^k(t) \triangleq z_j^k(t) + s_{2,j^i}^k(t) \quad (8)$$

are received information from agent j, with added random perturbations $s_{1,j^i}^k(t), s_{2,j^i}^k(t) \in [-\vartheta \sigma^k, \vartheta \sigma^k]$, where $\vartheta > 0$, $\sigma \in (0,\ 1)$, to preserve sensitive information. To eliminate the influence of these introduced perturbations on convergence accuracy, the perturbations $s_{1,i^i}^k(t)$ and $s_{2,i^i}^k(t)$ are subject to the following conditions:

$$s_{1,i^i}^k(t) \triangleq -\sum_{j \in \mathcal{N}_i} w_{ij,k} s_{1,i^j}^k(t), s_{2,i^i}^k(t) \triangleq -\sum_{j \in \mathcal{N}_i} l_{ij,k} s_{2,i^j}^k(t).$$

In (7), the variables $\lambda_i^k(t)$ and $P_{B,i}^k(t)$ are, respectively, the marginal cost and the output power of BESS i at iteration k. $\delta P_{U,i}^k(t)$ refers to the incremental power, $P_{U,i}^k(t)$ stands for the local accumulated power, and $P_{UG}^k(t)$ represents the total power that are exchanged with the utility grid at iteration k. $y_i^{k+1}(t)$ and $z_i^{k+1}(t)$ are the estimated power error before and after power replenishment by the utility grid. Specifically, if BESS i is linked with the utility grid, then the mismatch

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$y_i^{k+1}(t)$ is replenished by the utility grid, resulting in $z_i^{k+1}(t)$ being set as 0. Otherwise, $y_i^{k+1}(t) = z_i^{k+1}(t)$. $w_{ij,k}$, $l_{ij,k}$, w_{i0}, and w_{0i} are connected weights associated with the graph $\mathscr{G}(k)$, which are defined in II-A. $\varsigma > 0$ is the learning gain to be determined. $\epsilon_1, \epsilon_2 \in \left(0, \frac{1}{\varpi}\right)$ are the coupling scalars with ϖ being the maximum degree of the graph $\mathscr{G}(k), \forall k \geq 0$. Additionally, the initial state is set as follows:

$$\begin{cases} \lambda_i^0(t) = \frac{P_{B,i}^0(t) - a_i}{2b_i}, \quad \underline{P}_{B,i} < P_{B,i}^0(t) < \overline{P}_{B,i}, \\ z_i^0(t) = P_{d,i}(t) - P_{G,i}(t) - P_{B,i}^0(t), \\ P_{UG}^0(t) = 0. \end{cases} \tag{9}$$

The proposed APPDO algorithm is presented as follows.

Algorithm 1: APPDO Algorithm

► **Initialization**:
1. Initialize $P_{B,i}^0 \in (\underline{P}_{B,i}, \overline{P}_{B,i})$, $P_{UG}^0(t)$, $P_{G,i}(t)$, and $P_{d,i}(t)$, respectively, and calculate $\lambda_i^0(t)$ and $z_i^0(t)$ via (9) where $i \in \mathcal{V}$.

► **Loop**:
1. Agent i produces independent perturbations $s_{1,i^j}^k(t)$, $s_{2,i^j}^k(t) \in [-\vartheta\sigma^k, \vartheta\sigma^k]$ $(j \in \mathcal{N}_i(k))$, and then calculates $s_{1,i^i}^k(t)$, $s_{2,i^i}^k(t)$;
2. Agent i generates noise-injected signals $\hat{\lambda}_{i^j}^k(t)$ and $\hat{y}_{i^j}^k(t)$ via (8), and then sends them to its neighboring nodes as well as receives the information $\hat{\lambda}_{j^i}^k(t)$, and $\hat{y}_{j^i}^k(t)$, simultaneously;
3. Each agent i updates $\lambda_i^{k+1}(t)$ and $z_i^{k+1}(t)$ via (7);
4. If $|\lambda_i^{k+1}(t) - \lambda_i^k(t)| < o_1$, and $|z_i^{k+1}(t)| < o_2$ for $\forall i \in \mathcal{V}$, where o_1, o_2 are the error tolerance, break.

► **Output**:
1. $\lambda_i^k(t)$ and $P_{B,i}^k(t)$.

Remark 2: Our algorithm can be easily extended to the directed graph case. Let us define the in-neighborhood set of node i as $\mathcal{N}_i^{in} = \{j | (i,j) \in \mathcal{E}, i \neq j\}$ where node i can revive the information from node j. First, the undirected graph case can smoothly extend to the strongly connected and balanced directed graph due to the doubly stochastic weight matrix. For the case of the unbalanced directed graph, we can introduce a push-sum protocol [39], and construct the following distributed algorithm:

$$\begin{aligned} x_i^{k+1}(t) &= x_i^k(t) + \epsilon_1 \Big(\sum_{j \in \mathcal{N}_i^{in}(k)} w_{ij,k}(\hat{x}_{j^i}^k(t) - x_i^k(t)) \\ &\quad + \tau w_{i0}(x_0(t) - x_i^k(t)) + s_{1,i^i}^k(t) \Big) + \varsigma z_i^k(t) \\ v_i^{k+1}(t) &= v_i^k(t) + \epsilon_1 \Big(\sum_{j \in \mathcal{N}_i^{in}(k)} w_{ij,k}(v_{j^i}^k(t) - v_i^k(t)) \\ &\quad + \tau w_{i0}(v_0(t) - v_i^k(t)) \Big), \ v_i^0(t) = 1 \\ \lambda_i^{k+1}(t) &= \frac{x_i^{k+1}(t)}{v_i^{k+1}(t)}, \end{aligned}$$

and the iterative process of $P_{B,i}^k(t)$, $y_i^{k+1}(t)$, $\delta P_{U,i}^{k+1}(t)$, $P_{U,i}^{k+1}(t)$, $z_i^{k+1}(t)$, and $P_{UG}^k(t)$ is the same as that of (7).

For simplicity, denote $X \triangleq \mathrm{col}_N\{X_i\}$, $X_i = \lambda_i^k(t), z_i^k(t), P_{B,i}^k(t), y_i^k(t), P_{U,i}^k(t), \delta P_{U,i}^k(t)$, $S_{v,k}(t) \triangleq [s_{v,i^j}^k(t)]_N, v = 1,2$, $W_k \triangleq [w_{ij,k}]_N$, $L_k \triangleq [l_{ij,k}]_N$, $\Lambda \triangleq \mathrm{diag}_N\{w_{i0}\} \triangleq \mathrm{diag}_N\{w_{0i}\}$.

The compact form of (7) can be expressed as

$$\begin{aligned} \lambda^{k+1}(t) &= (I_N - \epsilon_1(W_k + \tau\Lambda))\lambda^k(t) \\ &\quad + \epsilon_1 \tau \lambda_0(t) \Lambda \mathbf{1}_N + \varsigma z^k(t) + \epsilon_1 S_{1,k}^T(t) \mathbf{1}_N \\ y^{k+1}(t) &= (I_N - \epsilon_2 L_k) z^k(t) + \epsilon_2 S_{2,k}^T(t) \mathbf{1}_N \\ &\quad - (P_B^{k+1}(t) - P_B^k(t)) \\ \delta P_U^{k+1}(t) &= \tau \Lambda y^{k+1}(t) \\ P_U^{k+1}(t) &= \tau(P_U^k(t) + \Lambda \delta P_U^{k+1}(t)) \\ z^{k+1}(t) &= y^{k+1}(t) + \Lambda(P_U^k(t) - P_U^{k+1}(t)) \\ P_{UG}^k(t) &= \mathbf{1}_N^T P_U^k(t). \end{aligned} \tag{10}$$

Next, we proceed to present the multi-step case of the APPDO algorithm. Denote

$$\begin{aligned} &\boldsymbol{X}_i \triangleq \mathrm{col}_{\mathcal{T}}\{X_i\}, X_i = \lambda_i^k(t), y_i^k(t), z_i^k(t), P_{B,i}^k(t), \delta P_{U,i}^k(t), \\ &P_{U,i}^k(t), \ \boldsymbol{P}_{UG}^k \triangleq \mathrm{col}_{\mathcal{T}}\{P_{UG}^k(t)\}, \ \boldsymbol{\lambda}_0 \triangleq \mathrm{col}_{\mathcal{T}N}\{\lambda_0(t)\}, \\ &\boldsymbol{X} \triangleq \mathrm{col}_N\{\boldsymbol{X}_i\}, \boldsymbol{X}_i = \boldsymbol{\lambda}_i^k, \boldsymbol{y}_i^k, \boldsymbol{z}_i^k, \boldsymbol{P}_{B,i}^k, \delta \boldsymbol{P}_{U,i}^k, \boldsymbol{P}_{U,i}^k. \end{aligned} \tag{11}$$

The multi-step case is an augmentation of (10) via (11), and corresponding constraints are reformulated as $\boldsymbol{P}_{B,i}^k \in \mathcal{X} \triangleq \{\text{constraints } (1)-(3), \forall i \in \mathcal{V}, t \in \{1, \ldots, \mathcal{T}\}\}$. That is

$$\begin{aligned} \boldsymbol{\lambda}^{k+1} &= \big((I_N - \epsilon_1(W_k + \tau\Lambda)) \otimes I_{\mathcal{T}}\big)\boldsymbol{\lambda}^k + \varsigma \boldsymbol{z}^k \\ &\quad + \epsilon_1 \tau (\Lambda \otimes I_{\mathcal{T}}) \boldsymbol{\lambda_0} + \epsilon_1 \boldsymbol{S}_{1,k}^T \mathbf{1}_{N\mathcal{T}} \\ \boldsymbol{y}^{k+1} &= \big((I_N - \epsilon_2 L_k) \otimes I_{\mathcal{T}}\big) \boldsymbol{z}^k + \epsilon_2 \boldsymbol{S}_{2,k}^T(t) \mathbf{1}_{N\mathcal{T}} \\ &\quad - (\boldsymbol{P}_B^{k+1} - \boldsymbol{P}_B^k) \\ \delta \boldsymbol{P}_U^{k+1} &= \tau (\Lambda \otimes I_{\mathcal{T}}) \boldsymbol{y}^{k+1} \\ \boldsymbol{P}_U^{k+1} &= \tau(\boldsymbol{P}_U^k + \Lambda \delta \boldsymbol{P}_U^{k+1}) \\ \boldsymbol{z}^{k+1} &= \boldsymbol{y}^{k+1} + \Lambda(\boldsymbol{P}_U^k - \boldsymbol{P}_U^{k+1}) \\ \boldsymbol{P}_{UG}^k &= (\mathbf{1}_N^T \otimes I_{\mathcal{T}}) \boldsymbol{P}_U^k, \end{aligned} \tag{12}$$

where

$$\boldsymbol{S}_{v,k} \triangleq [\boldsymbol{s}_{v,i^j}^k]_N, \boldsymbol{s}_{v,i^j}^k \triangleq \mathrm{diag}_{\mathcal{T}}\{s_{v,i^j}^k(t)\}, v = 1,2.$$

B. Supporting Lemmas

Assuming that $s = \{s^k\}_{k=0}^{+\infty}$ with $s^k \in \mathbb{R}^N$ is an infinite sequence, we define the following two norms:

$$\|s\|^{\gamma,K} \triangleq \max_{k \in \{0,\ldots,K\}} \gamma^{-k} \|s^k\|, \ \|s\|^{\gamma} \triangleq \sup_{k \geq 0} \gamma^{-k} \|s^k\| \tag{13}$$

with the parameter $\gamma \in (0,1)$. If $\|s\|^{\gamma,K}$ for $\forall K \geq 0$ (or $\|s\|^{\gamma}$) is bounded, then $\gamma^{-k}\|s^k\|$ is bounded and, furthermore, the sequence s^k approaches to $\mathbf{0}_N$ at least at a geometric rate of $\mathcal{O}(\gamma^k)$.

Lemma 1: (Small Gain Theorem [40], [41]): Consider sequences $s_1, s_2, \ldots, s_l$ such that the following cyclic inequalities $s_i \to s_{(i \bmod l)+1}$ are satisfied:

$$\|s_{(i \bmod l)+1}\|^{\gamma,K} \leq \kappa_i \|s_i\|^{\gamma,K} + \varphi_i, \ \forall K \geq 0, \tag{14}$$

where φ_i are constants. If $\kappa_1 \kappa_2 \ldots \kappa_l < 1$ with $\kappa_i > 0$, then

$$\|s_1\|^{\gamma} \leq \frac{\varphi_1 \kappa_2 \cdots \kappa_l + \varphi_2 \kappa_3 \cdots \kappa_l + \cdots + \kappa_l}{1 - \kappa_1 \kappa_2 \cdots \kappa_l}. \tag{15}$$

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Lemma 2: [16] Under Assumption 1, there exist an integer $d > 0$, a parameter $\eta \in (0,1)$, and a stochastic vector $\vartheta = \mathrm{col}_N\{\vartheta_i\}$ satisfying $\vartheta_i \geq 0$ and $\sum_{i=1}^N \vartheta_i = 1$ such that

$$\eta > \rho(\mathcal{L}_k^d - \vartheta \mathbf{1}^T),\ \forall k > 0, \tag{16}$$

where $\mathcal{L}_k^d \triangleq \mathcal{L}_k \mathcal{L}_{k-1} \cdots \mathcal{L}_{k-d+1}$ is the state transition matrix, $\mathcal{L}_k \triangleq I_N - \epsilon_2 L_k$ refers to the double stochastic matrix, and $\rho(\cdot)$ stands for the spectral radius.

C. Convergence Analysis under Islanded Mode

Note that $\tau = 0$ means that BESSs are operating in the islanded mode. To facilitate the analysis, we denote

$$\tilde{\lambda}^k(t) \triangleq \lambda^k(t) - \lambda^*(t)\mathbf{1}_N,\ \Delta P_B^k(t) \triangleq P_B^k(t) - P_B^{k-1}(t),$$
$$\check{z}^k(t) \triangleq \Xi z^k(t),\ \check{\lambda}^k(t) \triangleq \Xi\lambda^k(t),\ \Xi \triangleq I_N - 1/N\mathbf{1}_N\mathbf{1}_N^T.$$

By leveraging the small gain theorem, we are in a position to demonstrate the convergence of the proposed APPDO algorithm in light of the circle of arrows as follows:

$$\|\tilde{\lambda}(t)\|^\gamma \to \|\Delta P_B(t)\|^\gamma \to \|\check{z}(t)\|^\gamma \to \|\check{\lambda}(t)\|^\gamma \to \|\tilde{\lambda}(t)\|^\gamma.$$

Stage 1:) The arrow $\|\tilde{\lambda}^k(t)\|^\gamma \to \|\Delta P_B^k(t)\|^\gamma$

Lemma 3: If $\gamma \in (0,1)$, then one has $\|\Delta P_B(t)\|^\gamma \leq \kappa_1\|\check{\lambda}(t)\|^\gamma$ where $\kappa_1 \triangleq (1+1/\gamma)\hat{b}$ and $\hat{b} \triangleq \max\{1/2b_1, \ldots, 1/2b_N\}$.

Proof: We have from (7) that

$$\begin{aligned}\|\Delta P_B^{k+1}(t)\| &\leq \hat{b}\|\lambda^{k+1}(t) - \lambda^k(t)\| \\ &= \hat{b}\|\lambda^{k+1}(t) - \lambda^*(t)\mathbf{1}_N - \lambda^k(t) + \lambda^*(t)\mathbf{1}_N\| \\ &= \hat{b}\|\tilde{\lambda}^{k+1}(t) - \tilde{\lambda}^k(t)\| \\ &= \hat{b}\|\tilde{\lambda}^{k+1}(t)\| + \hat{b}\|\tilde{\lambda}^k(t)\|,\end{aligned} \tag{17}$$

and there exists a $\gamma \in (0,1)$ satisfying

$$\frac{1}{\gamma^{k+1}}\|\Delta P_B^{k+1}(t)\| \leq \frac{\hat{b}}{\gamma^{k+1}}\|\check{\lambda}^{k+1}(t)\| + \frac{\hat{b}}{\gamma^{k+1}}\|\check{\lambda}^k(t)\|. \tag{18}$$

Maximizing over $k = 0, \ldots, K-1$ on both sides of (18) yields

$$\begin{aligned}\|\Delta P_B(t)\|^{\gamma,K} \leq\ & \hat{b}\|\check{\lambda}(t)\|^{\gamma,K} + \frac{\hat{b}}{\gamma}\|\check{\lambda}(t)\|^{\gamma,K-1} \\ &\leq \hat{b}\|\check{\lambda}(t)\|^{\gamma,K} + \frac{\hat{b}}{\gamma}\|\check{\lambda}(t)\|^{\gamma,K} \\ &= \kappa_1\|\check{\lambda}(t)\|^{\gamma,K},\end{aligned} \tag{19}$$

which ends the proof. ■

Stage 2:) The arrow $\|\Delta P_B(t)\|^\gamma \to \|\check{z}(t)\|^\gamma$

Lemma 4: If $\gamma \in (\pi, 1)$ with $\pi = \max\{\sqrt[d]{\eta}, \sigma\}$, then one has $\|\check{z}(t)\|^\gamma \leq \kappa_2\|\Delta P_B(t)\|^\gamma + \varphi_2$ where $\kappa_2 \triangleq \frac{\gamma(\gamma^d - 1)}{(\gamma^d - \eta)(\gamma - 1)}$, $\varphi_2 \triangleq \frac{\gamma^d}{\gamma^d - \eta}\left(\sum_{s=0}^{d-1}\gamma^{-s}\|\check{z}^s(t)\| + 2\frac{\epsilon_2 N\vartheta}{\gamma^d}\frac{1-\sigma^d}{1-\sigma}\|\sigma\|^\gamma\right)$.

Proof: Based on the fact that $\Xi L_k = L_k\Xi = L_k$, we have from (10) that

$$\begin{aligned}\check{z}^{k+1}(t) &= \mathcal{L}_k\check{z}^k(t) + \epsilon_2\Xi S_{2,k}^T(t)\mathbf{1}_N - \Xi\Delta P_B^{k+1}(t) \\ &= \mathcal{L}_k^d\check{z}^{k-d+1}(t) + \epsilon_2\Xi\sum_{s=0}^{d-1}\mathcal{L}_k^s S_{2,k-s}^T(t)\mathbf{1}_N \\ &\quad - \Xi\sum_{s=0}^{d-1}\mathcal{L}_k^s\Delta P_B^{k+1-s}(t).\end{aligned} \tag{20}$$

For $\forall k \geq d-1$, we have

$$\begin{aligned}\|\check{z}^{k+1}(t)\| &\leq \eta\|\check{z}^{k-d+1}(t)\| + \epsilon_2\sum_{s=0}^{d-1}\|S_{2,k-s}^T(t)\mathbf{1}_N\| \\ &\quad + \sum_{s=0}^{d-1}\|\Delta P_B^{k+1-s}(t)\| \\ &\leq \eta\|\check{z}^{k-d+1}(t)\| + \sum_{s=0}^{d-1}\|\Delta P_B^{k+1-s}(t)\| \\ &\quad + 2\epsilon_2 N\vartheta\frac{1-\sigma^d}{1-\sigma}\sigma^{k-d+1},\end{aligned} \tag{21}$$

and further

$$\begin{aligned}\gamma^{-k-1}\|\check{z}^{k+1}(t)\| \leq\ & \frac{\eta}{\gamma^d}\gamma^{-k+d-1}\|\check{z}^{k-d+1}(t)\| \\ & + \sum_{s=0}^{d-1}\frac{1}{\gamma^s}\gamma^{-k-1+s}\|\Delta P_B^{k+1-s}(t)\| \\ & + 2\frac{\epsilon_2 N\vartheta}{\gamma^d}\frac{1-\sigma^d}{1-\sigma}\left(\frac{\sigma}{\gamma}\right)^{k-d+1}.\end{aligned} \tag{22}$$

For $\forall k = -1, \ldots, d-2$, it is easy to see

$$\gamma^{-k-1}\|\check{z}^{k+1}(t)\| \leq \gamma^{-k-1}\|\check{z}^{k+1}(t)\|. \tag{23}$$

Combining (22) with (23), it follows that

$$\begin{aligned}\|\check{z}(t)\|^{\gamma,K} &\leq \frac{\eta}{\gamma^d}\|\check{z}(t)\|^{\gamma,K-d} + \sum_{s=0}^{d-1}\frac{1}{\gamma^s}\|\Delta P_B(t)\|^{\gamma,K-s} \\ &\quad + \sum_{s=0}^{d-1}\gamma^{-s}\|\check{z}^s(t)\| + 2\frac{\epsilon_2 N\vartheta}{\gamma^d}\frac{1-\sigma^d}{1-\sigma}\|\sigma\|^{\gamma,K-d} \\ &\leq \frac{\eta}{\gamma^d}\|\check{z}(t)\|^{\gamma,K} + \sum_{s=0}^{d-1}\frac{1}{\gamma^s}\|\Delta P_B(t)\|^{\gamma,K} \\ &\quad + \sum_{s=0}^{d-1}\gamma^{-s}\|\check{z}^s(t)\| + 2\frac{\epsilon_2 N\vartheta}{\gamma^d}\frac{1-\sigma^d}{1-\sigma}\|\sigma\|^{\gamma,K} \\ &= \kappa_2\|\Delta P_B(t)\|^{\gamma,K} + \varphi_2,\end{aligned} \tag{24}$$

and the proof is now complete. ■

Stage 3:) The arrow $\|\check{z}(t)\|^\gamma \to \|\check{\lambda}(t)\|^\gamma$

Lemma 5: If $\gamma \in (\pi, 1)$ with $\pi = \max\{\sqrt[d]{\eta}, \sigma\}$, then one has $\|\check{\lambda}(t)\|^\gamma \leq \kappa_3\|\check{z}(t)\|^\gamma + \varphi_3$, where $\kappa_3 \triangleq \frac{\varsigma(\gamma^d - 1)}{(\gamma^d - \eta)(\gamma - 1)}$, $\varphi_3 \triangleq \frac{\gamma^d}{\gamma^d - \eta}\left(\sum_{s=0}^{d-1}\gamma^{-s}\|\check{\lambda}^s(t)\| + 2\frac{\epsilon_1 N\vartheta}{\gamma^d}\frac{1-\sigma^d}{1-\sigma}\|\sigma\|^\gamma\right)$.

Proof: The proof follows a similar approach as in Lemma 4, and is therefore not detailed here. ■

Stage 4:) The arrow $\|\check{\lambda}(t)\|^\gamma \to \|\tilde{\lambda}(t)\|^\gamma$

Denoting $\bar{z}^k(t) \triangleq \frac{1}{N}\mathbf{1}_N^T z^k(t)$ and $\bar{\lambda}^k(t) \triangleq \frac{1}{N}\mathbf{1}_N^T\lambda^k(t)$, it follows from (10) that

$$\begin{aligned}&\bar{z}^{k+1}(t) + \frac{1}{N}\sum_{i=1}^N(P_{B,i}^{k+1}(t) - P_{d,i}(t) + P_{G,i}(t)) \\ &= \bar{z}^0(t) + \frac{1}{N}\sum_{i=1}^N(P_{B,i}^0(t) - P_{d,i}(t) + P_{G,i}(t)) = 0,\end{aligned} \tag{25}$$

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and

$$\bar{\lambda}^{k+1}(t) = \bar{\lambda}^k(t) + \varsigma \bar{z}^k(t) = \bar{\lambda}^k(t) - \varsigma \frac{1}{N} \sum_{i=1}^{N} \left(P_{B,i}^k(t) - P_{d,i}(t) + P_{G,i}(t)\right).$$

Lemma 6: For $\theta_1 > 0, \theta_2 > 0$, if

$$\sqrt{1 - \frac{\varsigma \bar{b}\theta_1}{\theta_1 + 1}} \leq \gamma < 1,\ 0 < \varsigma \leq \frac{1}{(1+\theta_2)\bar{b}}, \tag{26}$$

then one has

$$\|\bar{\lambda}(t) - \lambda^*(t)\|^\gamma \leq \frac{1}{\gamma\sqrt{N}} \left(\sqrt{\frac{\hat{b}(1+\theta_2)}{\bar{b}\theta_2}} + \frac{\hat{b}}{\bar{b}}\theta_1\right) \sum_{i=1}^{N} \|\bar{\lambda}(t) - \lambda_i(t)\|^\gamma + 2\|\bar{\lambda}^0(t) - \lambda^*(t)\|, \tag{27}$$

where $\bar{b} \triangleq 1/N \sum_{i=1}^{N} \frac{1}{2b_i}$.

Proof: The proof can follows that of Lemma 8 in [40], and is therefore skipped here. ■

Lemma 7: For $\theta_1 > 0, \theta_2 > 0$, if (26) holds, then one has $\|\tilde{\lambda}(t)\|^\gamma \leq \kappa_4 \|\check{\lambda}(t)\|^\gamma + \varphi_4$, where $\kappa_4 \triangleq 1 + \frac{\sqrt{N}}{\gamma}\left(\sqrt{\frac{\hat{b}(1+\theta_2)}{\bar{b}\theta_2}} + \frac{\hat{b}}{\bar{b}}\theta_1\right)$, $\varphi_4 \triangleq 2\sqrt{N}\|\bar{\lambda}^0(t) - \lambda^*(t)\|$.

Proof: In light of

$$\sum_{i=1}^{N} \|\bar{\lambda}^k(t) - \lambda_i^k(t)\| \leq \sqrt{N \sum_{i=1}^{N} (\|\bar{\lambda}^k(t) - \lambda_i^k(t)\|)^2} = \sqrt{N}\|\check{\lambda}^k(t)\|, \tag{28}$$

it follows from (27) that

$$\|\bar{\lambda}(t) - \lambda^*(t)\|^\gamma = \frac{1}{\gamma}\left(\sqrt{\frac{\hat{b}(1+\theta_2)}{\bar{b}\theta_2}} + \frac{\hat{b}}{\bar{b}}\theta_1\right)\|\check{\lambda}\|^\gamma + 2\|\bar{\lambda}^0(t) - \lambda^*(t)\|. \tag{29}$$

Note that

$$\begin{aligned}\|\tilde{\lambda}(t)\|^\gamma &= \|\lambda(t) - \lambda^*(t)\mathbf{1}_N\|^\gamma \\ &= \|\lambda(t) - \bar{\lambda}(t)\mathbf{1}_N + \bar{\lambda}(t)\mathbf{1}_N - \lambda^*(t)\mathbf{1}_N\|^\gamma \\ &\leq \|\check{\lambda}(t)\|^\gamma + \sqrt{N}\|\bar{\lambda}(t) - \lambda^*(t)\|^\gamma \\ &\leq \kappa_4\|\tilde{\lambda}\|^\gamma + \varphi_4,\end{aligned} \tag{30}$$

which concludes the proof. ■

Theorem 1: Under Assumption 1, if $\tau = 0$, then there exists a small gain $\varsigma > 0$ such that the APPDO algorithm (10) over the time-varying graph $\mathscr{G}(k)$ achieves linear convergence towards the optimal solution to the problem (5).

Proof: To prove the exponential convergence of the developed APPDO algorithm, we need to ensure that $\|\tilde{\lambda}\|^\gamma$ is bounded. It follows from Lemma 1 that there must exist a small gain $\varsigma > 0$ such that $\kappa_1\kappa_2\kappa_3\kappa_4 < 1$ holds, where κ_1, κ_2, κ_3, and κ_4 are given in Lemmas 3-7, respectively. The proof is ended. ■

D. Convergence Analysis under Grid-Connected Mode

Note that $\tau = 1$ indicates that BESSs carry out in the grid-connected mode. To facilitate the analysis, we denote

$$e^k(t) \triangleq \lambda^k(t) - \lambda_0(t)\mathbf{1}_N.$$

The convergence analysis follows the sequence of stages represented by the following cycle of arrows:

$$\|e(t)\|^\gamma \to \|\Delta P_B(t)\|^\gamma \to \|z(t)\|^\gamma \to \|e(t)\|^\gamma.$$

Lemma 8: If $\gamma \in (\pi, 1)$ with $\pi = \max\{\sqrt[d]{\eta}, \sigma\}$, then one has $\|\Delta P_B(t)\|^\gamma \leq \kappa_1'\|e(t)\|^\gamma$, $\|z(t)\|^\gamma \leq \kappa_2'\|\Delta P_B(t)\|^\gamma + \varphi_2'$, $\|e(t)\|^\gamma \leq \kappa_3'\|z(t)\|^\gamma + \varphi_3'$, where $\kappa_1' \triangleq \kappa_1$, $\kappa_2' \triangleq \kappa_2$, $\kappa_3' \triangleq \kappa_3$, $\varphi_2' \triangleq \frac{\gamma^d}{\gamma^d - \eta}\left(\sum_{s=0}^{d-1}\gamma^{-s}\|z^s(t)\| + 2\frac{\epsilon_2 N\vartheta}{\gamma^d}\frac{1-\sigma^d}{1-\sigma}\|\sigma\|^\gamma\right)$, $\varphi_3' \triangleq \frac{\gamma^d}{\gamma^d - \eta}\left(\sum_{s=0}^{d-1}\gamma^{-s}\|e^s(t)\| + 2\frac{\epsilon_1 N\vartheta}{\gamma^d}\frac{1-\sigma^d}{1-\sigma}\|\sigma\|^\gamma\right)$.

Proof: The proof follows a similar approach as that of Lemma 3-5, and is therefore skipped here. ■

Theorem 2: Under Assumption 1, if $\tau = 1$, then there exists a small gain $\varsigma > 0$ such that the APPDO algorithm (10) over the time-varying graph $\mathscr{G}(k)$ linearly converge to the optimal solution to the problem (5).

Proof: The proof follows a similar approach as that of Theorem 1, and is therefore omitted here. ■

E. Smooth Transition Between Two Modes

Theorem 3: Under Assumption 1, the APPDO algorithm (10) can facilitate seamless transitions between two modes (i.e., islanded mode and grid-connected mode). Specifically, the estimated mismatch is always equal to the real power mismatch, i.e., $\sum_{i=1}^{N} z_i^k(t) = \sum_{i=1}^{N} P_{d,i}(t) - \sum_{i=1}^{N} P_{G,i}(t) - \sum_{i=1}^{N} P_{B,i}^k(t) - \tau P_{UG}^k(t),\ \forall\tau \in \{0,1\},\ \forall k \geq 0$.

Proof: Without loss of generality, we assume that BESSs transition from the islanded mode to the grid-connected mode at iteration $\mathcal{K}+1$.

When $k \in [0, \mathcal{K}]$, the APPDO algorithm (10) operates in the islanded mode, i.e., $\tau = 0$. It follows from (25) that

$$\sum_{i=1}^{N} z_i^k(t) = \sum_{i=1}^{N}\left(P_{d,i}(t) - P_{G,i}(t) - P_{B,i}^k(t)\right). \tag{31}$$

When $k = \mathcal{K}+1$ (i.e, $\tau = 1$), it follows from (10) that

$$z^{\mathcal{K}+1}(t) = \mathcal{L}_k z^{\mathcal{K}}(t) + \epsilon_2 S_{2,\mathcal{K}}^T(t)\mathbf{1}_N - (P_B^{\mathcal{K}+1}(t) - P_B^{\mathcal{K}}(t)) - \Lambda P_U^{\mathcal{K}+1}(t). \tag{32}$$

Due to $\mathbf{1}_N^T\mathcal{L}_k = \mathbf{1}_N^T$, $\mathbf{1}_N^T S_{2,\mathcal{K}}^T(t) = \mathbf{0}^T$, and $\sum_{i=1}^{N} z_i^k(t) = \mathbf{1}_N^T z^k(t)$, we have

$$\sum_{i=1}^{N} z_i^{\mathcal{K}+1}(t) = \sum_{i=1}^{N}\left(P_{d,i}(t) - P_{G,i}(t) - P_{B,i}^{\mathcal{K}+1}(t)\right) - P_{UG}^{\mathcal{K}+1}(t). \tag{33}$$

When $k > T+1$ (i.e., $\tau = 1$), recalling (10), we have

$$\begin{aligned}&\mathbf{1}_N^T\left(z^{k+1}(t) + P_B^{k+1}(t)\right) + P_{UG}^{k+1}(t) \\ &= \mathbf{1}_N^T\left(y^{k+1}(t) - \Lambda y^{k+1}(t) + P_B^{k+1}(t) + \Lambda P_U^k(t) + \Lambda y^{k+1}(t)\right) \\ &= \mathbf{1}_N^T\left(z^k(t) + P_B^k(t) + \Lambda P_U^k(t)\right)\end{aligned}$$

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$$= \mathbf{1}_N^T\big(z^{\mathcal{K}+1}(t) + P_B^{\mathcal{K}+1}(t)\big) + P_{UG}^{\mathcal{K}+1}(t), \tag{34}$$

which means that

$$\sum_{i=1}^{N} z_i^{k+1}(t) = \sum_{i=1}^{N} P_{d,i}(t) - \sum_{i=1}^{N} P_{G,i}(t) - \sum_{i=1}^{N} P_{B,i}^{k+1}(t) - P_{UG}^{k+1}(t), \quad k > T+1. \tag{35}$$

Next, we can follow a similar approach to prove that BESSs smoothly switch from the grid-connected mode back to the islanded mode, which ends the proof. ■

The analysis of the multi-step case is similar to that of the single-step case and is therefore skipped here.

F. Privacy Analysis

In this subsection, a privacy analysis is conducted to demonstrate the security of the proposed APPDO algorithm against malicious agents. The primary idea behind the privacy analysis is to assess whether malicious agents can infer sensitive information about their neighbors based on their observable information set.

Define $\mathcal{V}_U$ and $\mathcal{V}_M$ as the sets of the legitimate and malicious agents, respectively. Malicious agents can store all the data exchanged with their neighboring nodes. Consequently, the observable set for a malicious agent $j \in \mathcal{V}_M$ can be denoted by $\mathcal{I}_j(t) = \{\hat{\lambda}_{ij}^k, \hat{z}_{ij}^k, P_{B,i}^0(t), i \in \mathcal{N}_j(k), k \geq 0\}$.

Theorem 4: Suppose that Assumption 1 holds, the APPDO algorithm (10) over the time-varying graph $\mathscr{G}(k)$ can prevent sensitive information from being inferred by malicious agents.

Proof: For the case where all neighbors of the legitimate agent i are also legitimate (i.e., $j \notin \mathcal{N}_i(k) \cap \mathcal{V}_M, \forall k \geq 0$), malicious agents cannot estimate the sensitive information of agent i thanks to the unobservable information set.

In the case where $j \in \mathcal{N}_i(k) \cap \mathcal{V}_M, \forall k \geq 0$, we consider the worst scenario in which agent i is only connected to malicious agents. Given that malicious agents can collude with each other, we can, without loss of generality, consider that agent j is the only neighbor of agent i. Under this worst scenario, the malicious agent j can record all data that agent i shares with its neighbors, potentially allowing it to infer the privacy of node i. Consequently, it follows from (7)-(8) that

$$\lambda_i^{k+1}(t) = \lambda_i^k(t) + \epsilon_1 w_{ij,k}(\hat{\lambda}_{ji}^k(t) - \hat{\lambda}_{ij}^k(t)) + \varsigma z_i^k(t) \tag{36a}$$

$$z_i^{k+1}(t) = z_i^k(t) + \epsilon_2 l_{ij,k}(\hat{z}_{ji}^k(t) - \hat{z}_{ij}^k(t)) - (P_{B,i}^{k+1}(t) - P_{B,i}^k(t)). \tag{36b}$$

In the subsequent, by summing equation (36b) over k from 0 to k, we have

$$z_i^{k+1}(t) = z_i^0(t) + P_{B,i}^0(t) - P_{B,i}^{k+1}(t) + \epsilon_2 \sum_{s=0}^{k} l_{ij,s}(\hat{z}_{ji}^s(t) - \hat{z}_{ij}^s(t)). \tag{37}$$

As $k \to \infty$, we obtain

$$P_{B,i}^*(t) = z_i^0(t) + P_{B,i}^0(t) + \epsilon_2 \sum_{s=0}^{\infty} l_{ij,s}(\hat{z}_{ji}^s(t) - \hat{z}_{ij}^s(t)).$$

Note that $z_i^0(t) = \hat{z}_{ij}^0(t) - s_{2,ij}^0(t) \in [z_{ij}^0(t) - 0.5\vartheta, z_{ij}^0(t) + 0.5\vartheta]$ is unknown to agent j, and thus the optimal power $P_{B,i}^*(t)$ cannot be estimated exactly. Consequently, the malicious agent j cannot infer sensitive information $\{a_i, b_i\}$ via the established relationship (6), even though $\lambda_i^*(t)$ is observable. The proof is now ended. ■

Remark 3: In this work, a novel correlated perturbation-based privacy-preserving mechanism has been incorporated into the proposed distributed algorithm. Compared to existing privacy-preserving approaches, the distinct aspects of our results are as follows: 1) in contrast to the differential privacy method [13], [32], the proposed privacy-preserving mechanism has no influence on the convergence accuracy of the algorithm, thanks to the constructed correlated perturbation mechanism; 2) unlike the homomorphic encryption scheme [12], [26], [33], our scheme is easy to implement, requiring only simple addition and multiplication operations; and 3) in comparison with [17], [21], our algorithm achieves rapid convergence with a geometric rate without compromising performance.

Remark 4: So far, we have developed an APPDO algorithm over time-varying topologies to address the optimization problem (5) in both islanded and grid-connected modes. Compared to existing results, our APPDO algorithm offers three prominent features: 1) in comparison with [5], [7], the proposed distributed algorithm exhibits linear convergence under both modes over time-varying networks; 2) unlike existing privacy-preserving algorithms [12], [17], [21], [32], the proposed scheme demonstrates superior performance with regard to uncompromised convergence accuracy, strong privacy protection, and low computational complexity; and 3) unlike the eigenvalue analysis method [34], [35], [38], the adopted technique is capable of handling time-varying topologies.

IV. Case Study

In this section, case studies are given to verify the feasibility and validity of the developed APPDO algorithm. The test system includes 6 BESSs, each controlled by one of 6 agents, with the time-varying graphs $\mathscr{G}(k)$ transition between $\mathscr{G}_1$ and $\mathscr{G}_2$ at iteration k as described in Fig. 1, and the corresponding switching rule is given as follows:

$$\mathcal{G}(k) = \begin{cases} \mathcal{G}_1, & \text{if } k = 2s-1, \\ \mathcal{G}_2, & \text{if } k = 2s-2, \end{cases} \tag{38}$$

where s is a positive integer. The parameters of BESSs are given in Table I. The parameters are set as $\epsilon_1 = \epsilon_2 = 0.25$, $\varsigma = 0.01$, and $\sigma = 0.9$. The random variable ϑ follows a uniform distribution $\vartheta \sim U(0,1)$.

A. Case 1: Test on Two Operation Modes.

In this case, the total demand is set as $TL(t) \triangleq \sum_{i=1}^N P_{d,i}(t) - \sum_{i=1}^N P_{G,i}(t) = 190kW$, and the local supply $P_{B,i}^0(t)$ is set as $10kW$, $22kW$, $18kW$, $16kW$, $17kW$, and $16kW$, respectively, for each of the BESSs. Set $\lambda_0(t) = 2.7$. According to the classical Lagrangian method [38], we obtain $\lambda^*(t) = 4.12$ if $\tau = 0$, and $\lambda^*(t) = \lambda_0(t)$ if $\tau = 1$.

Here, we assume that BESSs change the operation mode at iterations $k = 300$ and $k = 1200$. The corresponding results

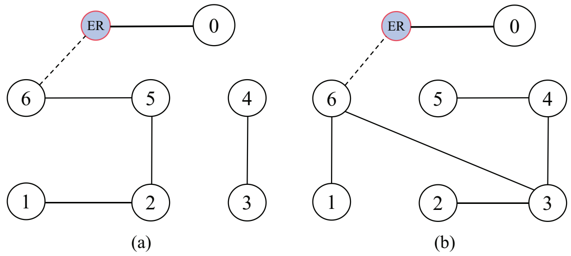

Fig. 1. Time-varying networks. (a) $\mathcal{G}_1$; (b) $\mathcal{G}_2$.

TABLE I
BESSs PARAMETERS [34]

BESS i	1	2	3	4	5	6
b_i	0.025	0.032	0.034	0.04	0.037	0.038
a_i	1.22	1.5	2.4	2.0	1.75	2.2
$\underline{P}_{B,i}$	-40	-50	-35	-45	-40	-30
$\overline{P}_{B,i}$	40	50	35	45	40	30
$\Delta\underline{P}_{B,i}$	-50	-80	-50	-50	-45	-60
$\Delta\overline{P}_{B,i}$	50	80	50	50	45	60
$\underline{SoE}_i$	0.1	0.2	0.2	0.15	0.15	0.1
$\overline{SoE}_i$	0.8	0.8	0.85	0.9	0.9	0.85
E_i	100	150	100	125	85	60
ϱ_i	0.85	0.8	0.85	0.80	0.90	0.90

are displayed in Fig. 2, which showcase the evolutions of the marginal cost $\lambda_i^k(t)$, the output power $P_{B,i}^k(t)$, $P_{UG}^k(t)$, the estimated mismatch $z_i^k(t)$, and total supply $P_S^k(t) = \sum_{i=1}^N P_{B,i}^k(t)+\tau P_{UG}^k(t)$. The results indicate that the marginal cost and output power attain to the optimal values of $\lambda^*(t)$ and $P_{B,i}^*(t)$, demonstrating the effectiveness of the developed algorithm. In addition, it is observed that all variables reach new optimal values when the operation mode changes, showing that the employed algorithm can smoothly transition between islanded and grid-connected modes.

B. Case 2: Test on Time-Varying Demand.

Assume that the total demand is initially selected as $190kW$, and then reduced to $170kW$ and $140kW$ at iterations $k = 500$ and $k = 1000$, respectively. In this study, assume that BESSs operate in the islanded mode. The results are displayed in Fig. 3, where the marginal cost $\lambda_i^k(t)$ and output power $P_{B,i}^k(t)$ converge to the respective optimal value, the estimated mismatch $z_i^k(t)$ approaches to 0, and the supply meets the demand under varying demand conditions. It is observed that the developed algorithm effectively achieves optimal power dispatch under time-varying demand.

C. Case 3: Test on Privacy Preservation.

In this case, BESS 4 is chosen to test the privacy property of the algorithm. Based on the results obtained in Theorem 4, the worst-case scenario is taken into account, where node 4's neighbors, nodes 5 and 6, are legitimate but malicious agents capable of receiving all data from node 4. Fig. 4 shows the estimate of $P_{B,4}^0(t)$ with and without the privacy-preserving scheme. The black line represents the actual evolution of the power-sensitive information, while the red line represents the value inferred by eavesdroppers. It is evident that there is a significant gap between the exact value and the inferred value, demonstrating that the developed privacy-preserving scheme effectively prevents sensitive data from being inferred.

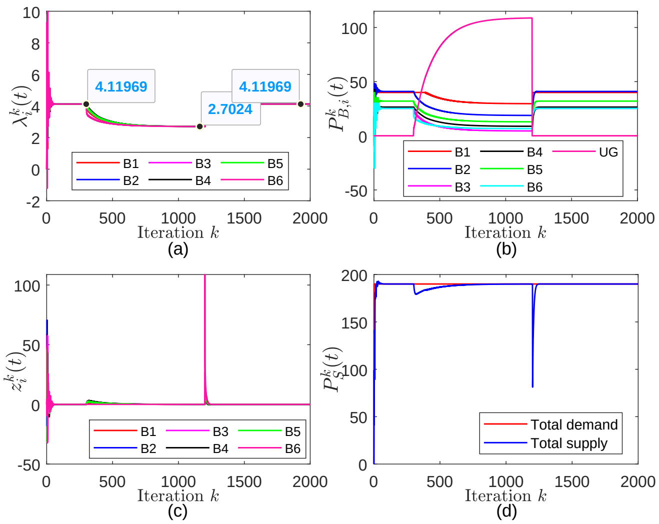

Fig. 2. Numerical results of the developed energy management algorithm. (a) Marginal cost $\lambda_i^k(t)$. (b) Output power $P_{B,i}^k(t)$ and $P_{UG}^k(t)$. (c) Estimated mismatch $z_i^k(t)$. (d) Power balance.

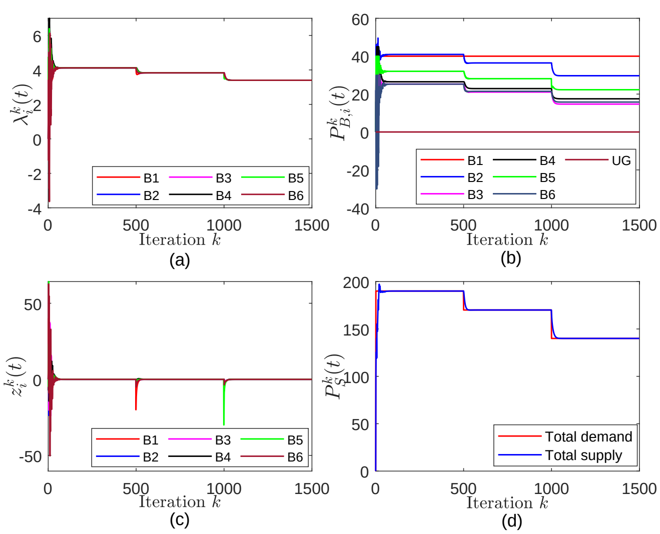

Fig. 3. Numerical results of the proposed algorithm under changing load demands. (a) Marginal cost $\lambda_i^k(t)$. (b) Output power $P_{B,i}^k(t)$, $P_{UG}^k(t)$. (c) Estimated mismatch $z_i^k(t)$. (d) Power balance.

D. Case 4: Comparison with Existing Distributed Algorithms

In this case study, a comparative analysis of convergence performance is conducted against the following four distributed algorithms in the islanded mode. 1) PDO: the proposed

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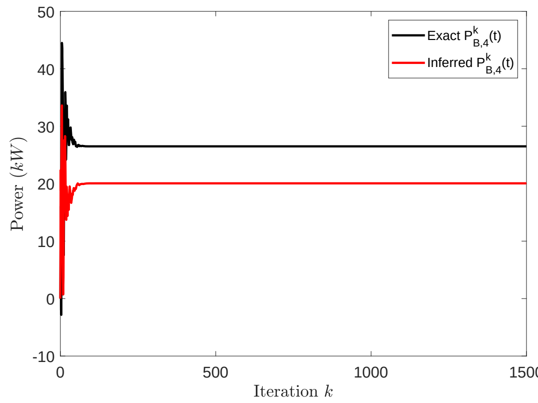

Fig. 4. Estimate of $P^0_{B,4}(t)$ with and without privacy preservation for BESS 4.

distributed optimization scheme; 2) CBDO: the consensus-based distributed optimization algorithm [38]; 3) PDDO: the primal and dual distributed optimization scheme [17]; and 4) DADO: the dual averaging distributed optimization [16]. Denote the total error between supply and demand at iteration k as $\Delta P_k = \sum_{i=1}^{6}(P_{B,i}(t) - P_{d,i}(t) + P_{G,i}(t))$. The evolutions of ΔP_k for four algorithms are plotted in Fig. 5. Note that the proposed algorithm have an advantage in fast convergence.

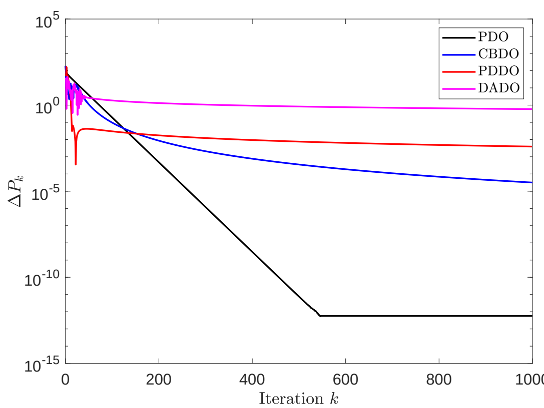

Fig. 5. Comparisons with different distributed algorithms.

E. Case 5: Utility Grid Power Supply with and without BESSs.

The total power demand in different periods is presented in Table II. Note that if the utility grid operates without BESSs, then this demand is fully supplied by the utility grid. However, when BESSs are introduced for each period, the supply from the utility grid changes. The utility grid power supply with and without BESSs is plotted in Fig. 6, where the values of red bar charts (i.e., the utility grid power supply with BESSs for different periods $P_{UG}(t), t = 1, 2, \ldots, 6$) are $212.9kW$, $219.1kW$, $229.5kW$, $223.9kW$, $227.0kW$, and $231.0kW$, respectively.

To further measure the dispersion of power supply of the utility grid, the sample variance is defined as $SV \triangleq \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ where $\bar{x}$ is the sample mean. We can obtain that the variance of power outputs from the utility grid with and without BESSs are 48.1 and 1180.6, respectively. It is noted that BESSs significantly contribute to smoothing the power output from the utility grid.

TABLE II
TOTAL POWER DEMAND IN EACH PERIOD.

t	1	2	3	4	5	6
Power(kW)	188.7	209.2	227.5	244.7	262.5	281.8

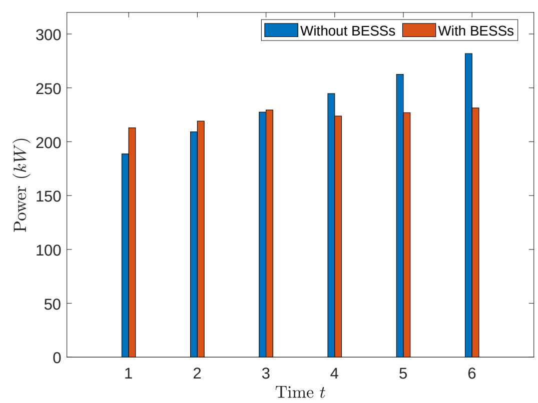

Fig. 6. Utility grid power supply with and without BESSs.

V. CONCLUSION

In this study, a novel APPDO algorithm has been developed to deal with the optimization issue while considering the physical constraints in BESSs. It has been demonstrated theoretically that the developed APPDO scheme can attain optimal energy management of BESSs in both modes at a linear convergence rate over time-varying networks. To protect sensitive information of BESSs, a correlated perturbation mechanism has been constructed to mask private values, verifying the resilience of the proposed privacy-preserving algorithm against malicious eavesdroppers. Finally, numerical results have validated the efficacy of the developed APPDO scheme. Future directions would be the extensions of other energy management problems for BESSs with the power flow and thermal constraints as well as transmission loss over time-varying directed graphs [26], [38], [40]; and the incorporation of the proposed scheme into practical experiments [42]–[48].

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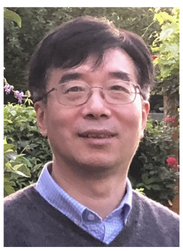
Wei Chen (Member, IEEE) received the Ph.D. degree in Control Science and Engineering from University of Shanghai for Science and Technology, Shanghai, China, in 2021.

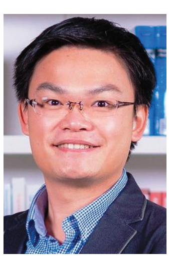
From 2019 to 2020, he was a visiting Ph.D. student with the Brunel University London, Uxbridge, U.K. From 2021 to 2022, he was a Research Associate with the City University of Hong Kong Shenzhen Research Institute, Shenzhen, China. From 2022 to 2024, he was a Post-Doctoral Research Fellow with the Southern University of Science and Technology, Shenzhen, China. He is currently a Research Fellow with the National University of Singapore, Singapore. His current research interests include distributed optimization/control, privacy preservation, energy management of smart grids. He is a very active reviewer for many international journals.



Zidong Wang (Fellow, IEEE) received the B.Sc. degree in mathematics from Suzhou University, Suzhou, China, in 1986, and the M.Sc. degree in applied mathematics and the Ph.D. degree in electrical engineering from the Nanjing University of Science and Technology, Nanjing, China, in 1990 and 1994, respectively.

He is currently a Professor in dynamical systems and computing with the Department of Computer Science, Brunel University London, Uxbridge, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany, and the U.K. He has published a number of articles in international journals. His research interests include dynamical systems, signal processing, bioinformatics, control theory and applications.

Prof. Wang is a Holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, and William Mong Visiting Research Fellowship of Hong Kong. He serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, *Neurocomputing*, and *Systems Science & Control Engineering*; and an Associate Editor for 12 international journals, including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a member of the Academia Europaea and European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a fellow of the Royal Statistical Society, and a member of program committee for many international conferences.



Jimmy Chih-Hsien Peng (Senior Member, IEEE) received the B.E. and Ph.D. degrees in electrical and computer engineering from the University of Auckland, Auckland, New Zealand, in 2008 and 2012, respectively. He is currently an Associate Professor with the Electrical and Computer Engineering, National University of Singapore, Singapore. Previously, he was with the Masdar Institute (now part of the Khalifa University), Abu Dhabi, UAE. In 2013, he was appointed a Visiting Scientist with the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA, USA, where he became a Visiting Assistant Professor in 2014.

His research interests include power system stability, grid resilience, cyber security, microgrids, and high-performance computing. Dr. Peng is currently a Member of the Electrical and Electronic Standards Committee, under the Singapore Standards Council. He is also an Associate Editor of the IEEE Transaction on Power Systems.

Guo-Ping Liu (Fellow, IEEE) received the Ph.D. degree in control engineering from the University of Manchester, Manchester, U.K., in 1992.

He is a professor with the Southern University of Science and Technology, Shenzhen, China. He has authored/co-authored over 400 journal papers and 10 books on control systems. His current research interests include the Internet of Things for renewable energy integration, networked control systems and advanced control of industrial systems. He is a Member of the Academy of Europe, a Fellow of IET and a Fellow of CAA.