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Privacy-Preserving Distributed Economic Dispatch of Microgrids over Directed Networks via State Decomposition: A Fast Consensus Algorithm

Wei Chen, Zidong Wang, Hongli Dong, Jingfeng Mao, and Guo-Ping Liu

***Abstract*—This paper is concerned with the privacy-preserving distributed economic dispatch problem of microgrids. The main goal of this work is to develop a privacy-preserving distributed optimization algorithm over directed networks, aiming to achieve supply-demand balance at the lowest economic cost under practical constraints while preventing the leakage of power sensitive information. For this purpose, a distributed optimization algorithm with a *constant* step size is proposed by combining the decentralized exact first-order algorithm with the push-sum protocol, which offers an advantage in terms of fast convergence. Additionally, to ensure privacy preservation, a state-decomposition approach is employed by randomly dividing the state into two parts, where only partial state information is transmitted. Moreover, the effectiveness of the privacy-preserving scheme against honest-but-curious nodes and external eavesdroppers is demonstrated through rigorous analysis. Finally, simulation studies demonstrate the validity and superiority of the developed privacy-preserving distributed algorithm.**

***Index Terms*—Microgrids, privacy preservation, economic dispatch, consensus-based optimization algorithm, push-sum protocol, state decomposition.**

I. Introduction

With the increasing electricity demand and emerging environmental issues, microgrids have been the subject of ever-increasing research interest in recent years owing to their flexibility, efficiency, scalability, and sustainability [1]–[6]. As one of the most fundamental optimization problems in microgrids, the economic dispatch (ED) has become an active research topic, with the primary goal of achieving supply-demand balance at the minimal economic cost under practical constraints. Due to the increasing system scale and the high integration of distributed energy resources (DERs), serious challenges are faced by the existing centralized ED schemes in system operation, communication, and calculation. In order to overcome these difficulties, the distributed implementation approach has gained favor in research, which is mainly attributed to the outstanding advantages in terms of reliability, autonomy, scalability, and decentralization [7]–[10].

Recently, several distributed optimization algorithms have been developed to solve the ED problem of microgrids, which can be broadly classified into distributed gradient- and ADMM-based methods [1], [11]–[14]. The former exhibits computationally simple and easy-to-implement features. However, most gradient-related algorithms require a diminishing step size to ensure exact convergence, resulting in a significant reduction in the convergence rate of the algorithm. In contrast, the distributed ADMM-based method can achieve fast convergence, albeit at the cost of consuming a certain amount of computing resources, requiring the resolution of a subproblem at each iteration. To combine the advantages of the above two types of algorithms, the decentralized exact first-order algorithm (EXTRA) has been introduced in [15], [16], which achieves fast and exact convergence by employing a fixed step size.

Note that the optimal ED achieved by the aforementioned distributed optimization algorithms is based on the assumption that the communication topology is undirected. In networked systems, data communication over directed graphs is more general and practical in consideration of physical and cost constraints. Particularly, in large-scale networks, directed communication exhibits remarkable advantages [17], [18]. However, designing a distributed optimization algorithm over directed networks poses difficulties due to asymmetric communication. Fortunately, the push-sum protocol, originally proposed in [19], has been recognized as a powerful technique for addressing the imbalance challenge stemming from directed communication. The main idea involves calculating the stationary distribution of the column-stochastic matrix corresponding to the directed graph and eliminating the imbalance by dividing it with the right eigenvector of the column-stochastic matrix. It is important to stress that most push-sum gradient-based results are constrained by the diminishing step size, which leads to slow convergence [20]–[22], and this serves as motivation for our current study.

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In the context of the energy internet, considerable attention has been given to the data privacy of power-sensitive information, which encompasses the value of the power market and the security of power systems [23]–[25]. So far, several privacy-preserving schemes have successfully been integrated into DED algorithms, including cryptography-based schemes [26]–[28] and noise-injected approaches [12], [20], [22], [29]–[32]. It should be noted that the privacy preservation achieved by the homomorphic encryption method proposed in [26]–[28] relies on the utilization of algebraic number theory. However, this approach may require a certain amount of calculation and storage resources due to its intrinsic computational complexity. In contrast to cryptography-based methods, noise-injected approaches mask sensitive information by introducing a sequence of stochastic noises. For example, the differential privacy scheme has been successfully applied to DED algorithms via adding independent stochastic noises [12], [31]. Nevertheless, the differential privacy algorithm compromises optimality to preserve sensitive information, resulting in a trade-off between privacy level and optimality. Additionally, in [30], a correlated noise sequence has been constructed to obfuscate the privacy value. However, as mentioned in [33], this approach may compromise privacy performance against external eavesdroppers.

To overcome the limitations of the aforementioned privacy-preserving algorithms, a state-decomposition-based privacy-preserving technique has been proposed in [34], where the state variable is randomly divided into two parts. The external substate can exchange information with neighboring nodes, while the internal substate only shares data with the external substate and remains completely unknown to other nodes. Furthermore, the initial values of these two substates are randomly generated but their sum is twice the initial value of the original state. This privacy-preserving method has demonstrated comprehensive merits in terms of exact convergence, low computational complexity, and uncompromised privacy performance. Its applicability has been extended to dynamic average consensus of multi-robot formation control [35], robust consensus of microgrid control [36], and resilient consensus of multi-agent systems under cyber-attacks [37]. It should be noted that the privacy-preserving ED problems via state decomposition have not yet received adequate investigation, possibly due to the structural complexity of the consensus-based DED algorithm, let alone the challenges posed by data communication over directed graphs.

In light of the foregoing discussions, the focus of this paper is on the privacy-preserving DED problem of microgrids. The following two aspects have been identified as substantial challenges: *1) how to develop a DED algorithm over directed networks with a fast convergence rate and a low computation burden? and 2) how to integrate a privacy-preserving scheme into a distributed algorithm with both well convergence and privacy-preserving performance?* To address the aforementioned difficulties, efforts are dedicated to developing a privacy-preserving push-sum EXTRA algorithm via state decomposition to achieve optimal ED and preserve power-sensitive information.

The primary contributions of this work can be summarized as follows.

1) A novel distributed algorithm is proposed to solve the DED problem over directed graphs by combining the EXTRA [15], [16] and the push-sum protocol [38]. Compared to most DED algorithms [20]–[22] with a diminishing step size, the designed distributed optimization algorithm achieves fast and exact convergence to the globally optimal solution with a constant step size.
2) For the first time, a state-decomposition privacy-preserving scheme is integrated into the DED algorithm under directed graphs. Furthermore, rigorous privacy analysis is presented to demonstrate that the developed privacy-preserving scheme is resilient against honest-but-curious nodes and external eavesdroppers. Unlike noise-injected and cryptographic-based privacy-preserving schemes [26], [30], [31], our approach offers prominent advantages in terms of exact convergence, low computational complexity, and well privacy-preserving performance.

The remaining sections of this paper are outlined as follows. Section II formulates the privacy-preserving DED issue of microgrids. Section III presents the design and convergence analysis of the privacy-preserving DED algorithm, while privacy analysis is provided in Section IV. Simulation results are presented in Section V to validate the obtained theoretical results. Finally, Section VI concludes this paper.

Notation. $W = [w_{ij}]_N$ denotes an N-dimensional matrix whose elements are w_{ij}. $\mathbf{1}_N$ ($\mathbf{0}_N$) represents the N-dimensional column vector of ones (zeros). I_N denotes N-dimensional identical matrix, and diag$\{\cdots\}$ represents a diagonal matrix. The symbol "$\backslash$" represents set subtraction. $\{a,b\}^+$ and $\{a,b\}^-$ stand for the larger and smaller value between a and b, respectively. $|a|$ is the absolute value of a.

II. Problem Formulation and Preliminaries

A. Network Model

The communication network of agents is described by a directed graph $\mathscr{G} = \{\mathcal{V}, \mathcal{E}\}$ with the node set $\mathcal{V} = \{1, 2, \cdots, N\}$ and the edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. The pair $(j, i) \in \mathcal{E}$ indicates that agent j can receive data from agent i. Particularly, the self-loop (i, i) is allowed, i.e., $(i, i) \in \mathcal{E}$. The agent j is denoted as the in-neighbor of agent i if the agent i can receive data from the agent j, and the set of in-neighbors is defined as $\mathcal{N}_i^{\text{in}} = \{j|(i, j) \in \mathcal{E}\}$. Similarly, the agent j is denoted as the out-neighbor of agent i if the agent i sends data to the agent j, and the set of out-neighbors is defined as $\mathcal{N}_i^{\text{out}} = \{j|(j, i) \in \mathcal{E}\}$.

Assumption 1: The directed graph $\mathscr{G}$ is strongly connected.

B. The ED Problem

In this paper, the ED problem of islanded microgrids is essentially framed as an optimization problem, with the main goal of power supply-demand balance being maintained at the least economic cost under practical physical constraints.

Consider an N-bus (agent) microgrid system, where the cost function of the agent i is described as the quadratic form [11], [16]:

$$F_i(P_i) = \frac{1}{2} a_i P_i^2 + b_i P_i + c_i, \quad i \in \mathcal{V} \tag{1}$$

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where $a_i > 0$, $b_i, c_i \geq 0$ are the cost function coefficients, $P_i > 0$ (or $P_i < 0$) indicates that the active power $|P_i|$ is injected into (or drawn from) the microgrid system for distributed generators (or loads). In addition, for a storage device, $P_i > 0$ (or $P_i < 0$) signifies discharging (or charging) operations, where the storage device acts as a generator (or a load).

In the energy management system, the optimization problem aims to minimize the total economic cost [11], [16], [39], which is described by the following formulation:

$$\begin{aligned} \underset{\{P_1,\ldots,P_N\}}{\arg\min} \quad & \sum_{i=1}^{N} F_i(P_i) \\ \text{s.t.} \quad & \sum_{i=1}^{N} P_i = 0, \quad \underline{P}_i \leq P_i \leq \overline{P}_i, \end{aligned} \tag{2}$$

where $\underline{P}_i$ and $\overline{P}_i$ are the lower and upper power limits of the agent i. Note that if the local load is fixed, then the physical constraints of loads are degraded into $\underline{P}_i = P_i = \overline{P}_i$, which is a special case of our results.

Denote the set $\Omega(P_i) = \{i | \underline{P}_i \leq P_i \leq \overline{P}_i\}$. For the optimization problem (2), the Lagrange function can be constructed as follows:

$$\begin{aligned} L(P_i, \lambda, \overline{\mu}_i, \underline{\mu}_i) = & \sum_{i=1}^{N} F_i(P_i) - \lambda \sum_{i=1}^{N} P_i + \sum_{i=1}^{N} \overline{\mu}_i (P_i - \overline{P}_i) \\ & + \sum_{i=1}^{N} \underline{\mu}_i (\underline{P}_i - P_i) \end{aligned} \tag{3}$$

where λ, $\overline{\mu}_i$, $\underline{\mu}_i$ $(i \in \mathcal{V})$ are the Lagrangian multipliers for each node. According to the Karush-Kuhn-Tucker (KKT) condition, if P_i^* $(i \in \mathcal{V})$ is the optimal solution of the optimization problem (2), then there exists the unique triple $(\lambda^*, \overline{\mu}_i^*, \underline{\mu}_i^*)$ satisfying

$$\begin{cases} \frac{\partial L(P_i^*, \lambda^*, \overline{\mu}_i^*, \underline{\mu}_i^*)}{\partial P_i} = 0, \\ \overline{\mu}_i^* \geq 0, \ \underline{\mu}_i^* \geq 0, \quad i \notin \Omega_i, \\ \overline{\mu}_i^* = 0, \ \underline{\mu}_i^* = 0, \quad i \in \Omega_i. \end{cases} \tag{4}$$

Furthermore, the global optimal solution P_i^* $(i \in \mathcal{V})$ is determined by

$$P_i^* = \left\{ \left\{ \frac{\lambda^* - b_i}{a_i}, \underline{P}_i \right\}^+, \overline{P}_i \right\}^-, \tag{5}$$

where λ^* can also be called as the optimal increment cost.

Based on the above observations, the optimization problem (2) can be transformed as the following restricted consensus-based problem whose aim is to develop a distributed optimization algorithm, where the local increment cost $\lambda_{i,k}$ $(i \in \mathcal{V})$ converges to λ^* such that

$$\lim_{k \to \infty} \sum_{i=1}^{N} P_{i,k} = 0 \tag{6}$$

with

$$P_{i,k} = \left\{ \left\{ \frac{\lambda_{i,k} - b_i}{a_i}, \underline{P}_i \right\}^+, \overline{P}_i \right\}^-. \tag{7}$$

C. The Objective

Before presenting the main goal of this paper, two types of adversaries are defined as follows: 1) internal honest-but-curious nodes who are agents following the distributed algorithm update and making attempts to estimate the sensitive data of their neighboring nodes based on the knowledge obtained, and 2) external eavesdroppers who can steal the shared data between neighboring agents by wiretapping all communication links and attempt to learn the sensitive information of each agent using the available information.

The primary objective of this paper is to develop a privacy-preserving distributed algorithm under directed graphs that can converge to the optimal solution to the optimization problem (2) while preserving the power sensitive information against the aforementioned two types of adversaries.

Remark 1: In this paper, the power sensitive information, including the generation power, load demand, and cost coefficients (i.e., $P_{i,k}, \{a_i, b_i\}$ $(i \in \mathcal{V})$), plays an essential role in maintaining market order and ensuring the safe and reliable operation of power grids. Strategic bidding in the energy market often relies on cost coefficients. If individual sensitive information of other power providers is exposed to competitors, a competitor may modify their operational cost to disrupt the electricity market. Additionally, load demand represents the electricity habits of individuals or the production status of companies. If the demand information is stolen, individuals or companies may face property damage or loss. Therefore, exploring a privacy-preserving technique to prevent the leakage of power-sensitive information is preferable. In this work, to ensure the existence of the optimal P_i^*, the cost coefficients can be selected such that there exists the unique triple $(\lambda^*, \overline{\mu}_i^*, \underline{\mu}_i^*)$ satisfying the KKT condition (4).

III. Distributed Algorithm Deign and Convergence Analysis

In this section, the optimization problem (2) is addressed by first developing a fast DED algorithm under directed communication graphs by combining the EXTRA [15] with the push-sum algorithm [19], [38]. Additionally, a privacy-preserving scheme is proposed through state decomposition, and the convergence and optimality of the employed privacy-preserving distributed scheme are then discussed.

A. Privacy-Preserving DED Algorithm via State Decomposition over Directed Graphs

Before proceeding further, the fast DED algorithm under undirected graphs is presented as follows [16]:

$$\begin{aligned} \lambda_{i,k+1} = & \lambda_{i,k} + \sum_{j=1}^{N} h_{ij} \lambda_{j,k} - \sum_{j=1}^{N} \tilde{h}_{ij} \lambda_{j,k-1} \\ & - \kappa \big(P_{i,k} - P_{i,k-1} \big) \\ P_{i,k} = & \left\{ \left\{ \frac{\lambda_{i,k} - b_i}{a_i}, \underline{P}_i \right\}^+, \overline{P}_i \right\}^- \end{aligned} \tag{8}$$

where $\lambda_{i,k}$ and $P_{i,k}$ are, respectively, the increment cost and the active power of node i, h_{ij} is the (i,j)-th element of the

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weight matrix $H = [h_{ij}]_N$ that is doubly stochastic, $\tilde{h}_{ij}$ is the (i,j)-th element of the weight matrix $\tilde{H} = [\tilde{h}_{ij}]_N$ that satisfies $\tilde{H} = \delta I_n + (1-\delta)H$ with $\delta \in (0, 0.5]$, and the step size $\kappa > 0$ is a small known scalar.

To solve the optimization problem (2) in a distributed manner under directed graphs, a novel DED algorithm can be constructed as follows by combining the distributed algorithm (8) with the push-sum protocol [19], [38]:

$$\begin{cases} \lambda_{i,k} = \dfrac{\phi_{i,k}}{x_{i,k}}, \\ \phi_{i,k+1} = \phi_{i,k} + \displaystyle\sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} w_{ij}\phi_{j,k} - \sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} \tilde{w}_{ij}\phi_{j,k-1} \\ \qquad - \kappa\big(P_{i,k} - P_{i,k-1}\big) \\ P_{i,k} = \Big\{\Big\{\dfrac{\lambda_{i,k} - b_i}{a_i}, \underline{P}_i\Big\}^+, \overline{P}_i\Big\}^- \\ x_{i,k+1} = \displaystyle\sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} w_{ij}x_{j,k}, \end{cases} \tag{9}$$

where $\phi_{i,k}$, $x_{i,k}$ are the auxiliary variables, $w_{ij} \in (0,1)$ if $i \in \mathcal{N}_j^{\text{out}} \cup \{i\}$ and $w_{ij} = 0$, otherwise. Meanwhile, the weights satisfy $\sum_{i=1}^N w_{ij} = 1$ $(\forall j \in \mathcal{V})$. Furthermore, the weight $\tilde{w}_{ij}$ can be constructed as

$$\tilde{w}_{ij} = \begin{cases} \delta + (1-\delta)w_{ij}, & i = j, \\ (1-\delta)w_{ij}, & i \neq j, \end{cases}$$

where $\delta \in (0, 0.5]$. In addition, setting $\phi_{i,-1} = P_{i,-1} = 0$, $P_{i,0} \in [\underline{P}_i,\ \overline{P}_i]$, and $x_{i,0} = 1$ $(i \in \mathcal{V})$, it follows from (9) that

$$\begin{aligned} &\lambda_{i,0} = a_i P_{i,0} + b_i,\ \phi_{i,0} = \lambda_{i,0}x_{i,0}, \\ &\phi_{i,1} = \sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} w_{ij}\phi_{j,0} - \kappa P_{i,0}, \\ &x_{i,1} = \sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} w_{ij}x_{j,0}. \end{aligned}$$

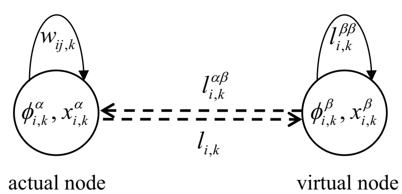

Fig. 1. State decomposition.

To ensure the prevention of power sensitive data leakage, a privacy-preserving algorithm is proposed using a state-decomposition approach. As illustrated in Fig. 1, the main idea involves randomly decomposing $\phi_{i,k}$, $x_{i,k}$ into two variables, namely, $\phi_{i,k}^{\alpha}, x_{i,k}^{\alpha}$ and $\phi_{i,k}^{\beta}, x_{i,k}^{\beta}$. The substate $\phi_{i,k}^{\alpha}, x_{i,k}^{\alpha}$ can be transmitted to neighboring nodes, while the substate $\phi_{i,k}^{\beta}, x_{i,k}^{\beta}$ only communicate with $\phi_{i,k}^{\alpha}, x_{i,k}^{\alpha}$. Intuitively, the substate $\phi_{i,k}^{\beta}, x_{i,k}^{\beta}$ remain invisible to the neighboring node of agent i, thereby achieving privacy preservation through the state-decomposition approach.

To be more specific, the proposed privacy-preserving DED algorithm can be summarized in Algorithm 1.

Algorithm 1 DED Algorithm via State Decomposition

▶ **Initialization**:

Step 1: Give the initial value $P_{i,0} \in [\underline{P}_i, \overline{P}_i]$, $x_{i,0} = 1$, $\delta \in (0, 0.5]$ and randomly generate $P_{i,0}^{\alpha} \in [\underline{P}_i, \overline{P}_i]$, $x_{i,0}^{\alpha} \in (0, 2)$;

Step 2: Calculate $P_{i,0}^{\beta} = 2P_{i,0} - P_{i,0}^{\alpha}$, $x_{i,0}^{\beta} = 2x_{i,0} - x_{i,0}^{\alpha}$, $\lambda_{i,0}^{c} = a_i P_{i,0}^{c} + b_i$, and $\phi_{i,0}^{c} = \lambda_{i,0}^{c} x_{i,0}^{c}$ $(c = \alpha, \beta)$.

▶ **Random weight construction**:

Step 1: Actual node i randomly generates weights $l_{i,k}$, $w_{ji,k} \in (0,1)$, $j \in \mathcal{N}_i^{\text{out}} \cup \{i\}$ such that $\sum_{j=1}^N w_{ji,k} + l_{i,k} = 1$, and $w_{ji,k} = 0$ if $j \notin \mathcal{N}_i^{\text{out}}$;

Step 2: Virtual node i randomly generates weights $l_{i,k}^{\alpha\beta}$, $l_{i,k}^{\beta\beta} \in (0,1)$ such that $l_{i,k}^{\alpha\beta} + l_{i,k}^{\beta\beta} = 1$.

▶ **Data exchange and update**:

Step 1: Agent i calculates $w_{ji,k}\phi_{i,k}^{\alpha}$, $w_{ji,k}x_{i,k}^{\alpha}$, $\tilde{w}_{ji,k}\phi_{i,k-1}^{\alpha}$, and transmits them to its out-neighbors $j \in \mathcal{N}_i^{\text{out}}$;

Step 2: After receiving the data from its in-neighbors $j \in \mathcal{N}_i^{\text{in}}$, agent i updates its substate as follows:

1) **When** $k = 0$:

$$\begin{aligned} \phi_{i,1}^{\alpha} &= \sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} w_{ij,0}\phi_{j,0}^{\alpha} + l_{i,0}^{\alpha\beta}\phi_{i,0}^{\beta} - \kappa P_{i,0}^{\alpha} \\ \phi_{i,1}^{\beta} &= l_{i,0}\phi_{i,0}^{\alpha} + l_{i,0}^{\beta\beta}\phi_{i,0}^{\beta} - \kappa P_{i,0}^{\beta} \\ P_{i,0}^{c} &= \Big\{\Big\{\frac{\lambda_{i,0}^{c} - b_i}{a_i}, \underline{P}_i\Big\}^+, \overline{P}_i\Big\}^-,\ c = \alpha, \beta \\ x_{i,1}^{\alpha} &= \sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} w_{ij,0}x_{j,0}^{\alpha} + l_{i,0}^{\alpha\beta}x_{i,0}^{\beta} \\ x_{i,1}^{\beta} &= l_{i,0}x_{i,0}^{\alpha} + l_{i,0}^{\beta\beta}x_{i,0}^{\beta}. \end{aligned} \tag{10}$$

2) **When** $k \geq 1$:

$$\begin{aligned} \lambda_{i,k}^{\alpha} &= \frac{\phi_{i,k}^{\alpha}}{x_{i,k}^{\alpha}},\ \lambda_{i,k}^{\beta} = \frac{\phi_{i,k}^{\beta}}{x_{i,k}^{\beta}} \\ \phi_{i,k+1}^{\alpha} &= \phi_{i,k}^{\alpha} + \sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} w_{ij,k}\phi_{j,k}^{\alpha} + l_{i,k}^{\alpha\beta}\phi_{i,k}^{\beta} \\ &\quad - \sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} \tilde{w}_{ij,k}\phi_{j,k-1}^{\alpha} - \tilde{l}_{i,k}^{\alpha\beta}\phi_{i,k-1}^{\beta} \\ &\quad - \kappa(P_{i,k}^{\alpha} - P_{i,k-1}^{\alpha}) \\ \phi_{i,k+1}^{\beta} &= \phi_{i,k}^{\beta} + l_{i,k}\phi_{i,k}^{\alpha} + l_{i,k}^{\beta\beta}\phi_{i,k}^{\beta} - \tilde{l}_{i,k}\phi_{i,k-1}^{\alpha} \\ &\quad - \tilde{l}_{i,k}^{\beta\beta}\phi_{i,k-1}^{\beta} - \kappa(P_{i,k}^{\beta} - P_{i,k-1}^{\beta}), \\ P_{i,k}^{c} &= \Big\{\Big\{\frac{\lambda_{i,k}^{c} - b_i}{a_i}, \underline{P}_i\Big\}^+, \overline{P}_i\Big\}^-,\ c = \alpha, \beta \\ x_{i,k+1}^{\alpha} &= \sum_{j \in \mathcal{N}_i^{\text{in}} \cup \{i\}} w_{ij,k}x_{j,k}^{\alpha} + l_{i,k}^{\alpha\beta}x_{i,k}^{\beta} \\ x_{i,k+1}^{\beta} &= l_{i,k}x_{i,k}^{\alpha} + l_{i,k}^{\beta\beta}x_{i,k}^{\beta}. \end{aligned} \tag{11}$$

Remark 2: It should be pointed out that the fixed size κ plays a vital role in convergence analysis of the distributed algorithm (9), whose upper bound $\bar{\kappa}$ usually depends on the underlying graph, the diagonally dominant weight matrix, and the cost function. For example, by applying the basic equality,

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the upper bound $\bar{\kappa} = 2\rho_{\min}(\tilde{\mathcal{W}}_k)\{a_i\}^-$ has been obtained in [15], [16], where $\rho_{\min}(\tilde{\mathcal{W}}_k)$ is the smallest eigenvalue of $\tilde{\mathcal{W}}_k$ for $\forall k \geq 0$, and $\{a_i\}^-$ is the smallest a_i $(i \in \mathcal{V})$. Note that the matrix $\tilde{\mathcal{W}}_k$ is diagonally dominant to ensure $\rho_{\min}(\tilde{\mathcal{W}}_k) > 0$. In this paper, the selection of $\kappa \in (0, \bar{\kappa})$ follows from the work [15], [16], [42].

B. Convergence Analysis

To facilitate later analysis, we denote the collection of the state variables as

$$\begin{aligned}
\lambda_k &= \left[\begin{array}{c} \lambda^\alpha_{1,k}, \cdots, \lambda^\alpha_{N,k}, \lambda^\beta_{1,k}, \cdots, \lambda^\beta_{N,k} \end{array}\right]^T, \\
\phi_k &= \left[\begin{array}{c} \phi^\alpha_{1,k}, \cdots, \phi^\alpha_{N,k}, \phi^\beta_{1,k}, \cdots, \phi^\beta_{N,k} \end{array}\right]^T, \\
x_k &= \left[\begin{array}{c} x^\alpha_{1,k}, \cdots, x^\alpha_{N,k}, x^\beta_{1,k}, \cdots, x^\beta_{N,k} \end{array}\right]^T, \\
P_k &= \left[\begin{array}{c} P^\alpha_{1,k}, \cdots, P^\alpha_{N,k}, P^\beta_{1,k}, \cdots, P^\beta_{N,k} \end{array}\right]^T,
\end{aligned}$$

the weight matrices as

$$\begin{aligned}
W_k &= [w_{ij,k}]_N,\ L_k = \mathrm{diag}\{l_{1,k}, \cdots, l_{N,k}\}, \\
L^{\alpha\beta}_k &= \mathrm{diag}\{l^{\alpha\beta}_{1,k}, \cdots, l^{\alpha\beta}_{N,k}\},\ L^{\beta\beta}_k = \mathrm{diag}\{l^{\beta\beta}_{1,k}, \cdots, l^{\beta\beta}_{N,k}\}, \\
\tilde{W}_k &= [\tilde{w}_{ij,k}]_N = \delta I_N + (1-\delta)W_k,\ \delta \in (0, 0.5], \\
\tilde{L}_k &= \mathrm{diag}\{\tilde{l}_{1,k}, \cdots, \tilde{l}_{N,k}\} = (1-\delta)L_k, \\
\tilde{L}^{\alpha\beta}_k &= \mathrm{diag}\{\tilde{l}^{\alpha\beta}_{1,k}, \cdots, \tilde{l}^{\alpha\beta}_{N,k}\} = (1-\delta)L^{\alpha\beta}_k, \\
\tilde{L}^{\beta\beta}_k &= \mathrm{diag}\{\tilde{l}^{\beta\beta}_{1,k}, \cdots, \tilde{l}^{\beta\beta}_{N,k}\} = (1-\delta)L^{\beta\beta}_k, \\
\mathcal{W}_k &= \left[\begin{array}{cc} W_k & L^{\alpha\beta}_k \\ L_k & L^{\beta\beta}_k \end{array}\right],\ \tilde{\mathcal{W}}_k = \left[\begin{array}{cc} \tilde{W}_k & \tilde{L}^{\alpha\beta}_k \\ \tilde{L}_k & \tilde{L}^{\beta\beta}_k \end{array}\right],
\end{aligned}$$

and the transition matrix $\Phi(t, s) = [\Phi_{ij}(t, s)]_{2N}$ as

$$\Phi(t, s) = \mathcal{W}_t \mathcal{W}_{t-1} \cdots \mathcal{W}_s,\ t > s \geq 0.$$

The compact form of the algorithm (11) can be written as follows:

$$\lambda_k = \mathcal{X}_k^{-1}\phi_k, \tag{12a}$$

$$\phi_{k+1} = \phi_k + \mathcal{W}_k\phi_k - \tilde{\mathcal{W}}_k\phi_{k-1} - \kappa(P_{k+1} - P_k), \tag{12b}$$

$$x_{k+1} = \mathcal{W}_k x_k, \tag{12c}$$

where matrices $\mathcal{W}_k$, $\tilde{\mathcal{W}}_k$ are column stochastic with the relationship $\tilde{\mathcal{W}}_k = \delta I_N + (1-\delta)\mathcal{W}_k$, and

$$\mathcal{X}_k = \mathrm{diag}\{x_k\}_{2N} = \mathrm{diag}\left\{\Phi(k-1, 0)x_0\right\}_{2N}. \tag{13}$$

It can be inferred from (13) that the diagonal matrix $\mathcal{X}_k > 0$ is positive definite and, consequently, is invertible at any time instant k. Subsequently, a lemma is provided to demonstrate the property of the transition matrix $\Phi(t, s)$.

Lemma 1: [40], [41] Under Assumption 1, there exist a scalar $\gamma \in (0, 1)$ and a stochastic vector $v = \left[\begin{array}{c} v_1, \cdots, v_i, \cdots, v_{2N} \end{array}\right]^T$ (i.e., $v_i > 0$ and $\mathbf{1}^T_{2N}v = 1$) such that

$$|\Phi_{ij}(t, s) - v_i| \leq K\gamma^{t-s},\ \forall i, j = 1, \cdots, 2N,\ t > s \tag{14}$$

with $K \geq 4$.

In light of Lemma 1, one has

$$\lim_{k\to\infty} \Phi(k, 0) = v\mathbf{1}^T_{2N}. \tag{15}$$

It follows from (12c) and (14) that

$$\begin{aligned}
\lim_{k\to\infty} x_k &= \lim_{k\to\infty} \Phi(k-1, 0)x_0 = 2Nv, \\
\mathcal{X}_\infty &= \lim_{k\to\infty} \mathcal{X}_k = 2N\mathrm{diag}\{v\}.
\end{aligned} \tag{16}$$

Furthermore, the variables λ_k and ϕ_k in (12b)-(12c) can converge linearly to their respective limits. The convergence analysis for these variables is similar to the proof provided in Theorems 1 and 2 of [42], and hence is omitted here.

In what follows, we are ready to prove that the algorithm (11) can achieve consensus and converge to the optimal value of the problem (2).

It follows from (12b) that

$$\phi^* = \phi^* + \mathcal{W}_\infty\phi^* - \tilde{\mathcal{W}}_\infty\phi^* - \kappa(P^* - P^*), \tag{17}$$

where $\phi^* = \lim_{k\to\infty}\phi_k$ and $P^* = \lim_{k\to\infty}P_k$. Furthermore, we have $(W_\infty - \tilde{W}_\infty)\phi^* = \mathbf{0}_{2N}$, which implies $\phi^* = kv,\ k \in \mathbb{R}\backslash\{0\}$.

In light of (12a), one has

$$\lim_{k\to\infty} \lambda_k = (\mathcal{X}^\infty)^{-1}\phi^* = k\mathbf{1}_{2N},\ k \in \mathbb{R}\backslash\{0\}, \tag{18}$$

which means that the algorithm (11) can achieve consensus.

The above analysis uncovers that the imbalance caused by asymmetric data exchange in directed graphs can be overcome by the proposed distributed algorithm (11). Specifically, both x_k and ϕ_k converge to the span of v, and the imbalance is eliminated by the element-wise division of x_k/ϕ_k, where the symbol "/" denotes the division between two vectors of the same dimension.

Next, summing up (12b) over k from 0 to ∞, we have

$$\phi^* = \mathcal{W}_\infty\phi^* - \kappa P^* - \sum_{k=0}^{\infty}(\tilde{\mathcal{W}}_k - \mathcal{W}_k)\phi_k. \tag{19}$$

Noting that the weight matrices $\tilde{\mathcal{W}}_k$ and $\mathcal{W}_k$ are column stochastic, one has $\mathbf{1}^T_{2N}\tilde{\mathcal{W}}_k = \mathbf{1}^T_{2N}\mathcal{W}_k = \mathbf{1}^T_{2N}$. Moreover, it follows from (19) that

$$\kappa\mathbf{1}^T_{2N}P^* = -\mathbf{1}^T_{2N}\sum_{k=0}^{\infty}(\tilde{\mathcal{W}}_k - \mathcal{W}_k)\phi_k = 0, \tag{20}$$

which satisfies the optimal condition in (7).

IV. Privacy Analysis

In this section, the proof will be presented to demonstrate that Algorithm 1 preserves the power sensitive information [20], [22] (i.e., $P_{i,k}$, $\{a_i, b_i\}$ $(i \in \mathcal{V})$) against external eavesdroppers and internal honest-but-curious nodes. First, the information set accessible to two types of eavesdroppers is defined, and then the update rule of the distributed algorithm is reconstructed in light of the obtained information set. Furthermore, the adversaries attempt to calculate the unknown but private variables via the established update rule and the known variable.

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A. Privacy Preservation Against External Eavesdroppers

Assuming that external eavesdroppers possess a certain level of knowledge regarding the algorithm and have the ability to wiretap all communication links, the information accessible to external eavesdroppers can be defined as follows:

$$\begin{aligned}\mathcal{D} = \{&\mathscr{G}, w_{ij,k}x^{\alpha}_{j,k}, w_{ij,k}\phi^{\alpha}_{j,k}, \tilde{w}_{ij,k}\phi^{\alpha}_{j,k-1}, \kappa \\ &|i \in \mathcal{V}, j \in \mathcal{N}^{\text{in}}_i, k = 0, 1, \cdots\}. \end{aligned} \tag{21}$$

Theorem 1: For a directed graph $\mathscr{G}$ under Assumption 1, the privacy of agent i $(i \in \mathcal{V})$ can be preserved against external eavesdroppers by using Algorithm 1.

Proof: We carry out privacy analysis from the perspective of external eavesdroppers. Denoting $z^x_{i,k} = x^{\alpha}_{i,k} + x^{\beta}_{i,k}$, it follows from (11) that

$$\begin{aligned} z^x_{i,k+1} &= \sum_{i\in\mathcal{N}^{\text{in}}_i} w_{ij,k}x^{\alpha}_{j,k} + (w_{ii,k} + l_{i,k})x^{\alpha}_{j,k} + x^{\beta}_{i,k} \\ &= z^x_{i,k} + \sum_{j\in\mathcal{N}^{\text{in}}_i} w_{ij,k}x^{\alpha}_{j,k} - \sum_{s\in\mathcal{N}^{\text{out}}_i} w_{si,k}x^{\alpha}_{i,k}. \end{aligned} \tag{22}$$

Then, summing up (22) over k from 0 to $k-1$, one has

$$z^x_{i,k} = z^x_{i,0} + \sum_{t=0}^{k-1}\left(\sum_{j\in\mathcal{N}^{\text{in}}_i} w_{ij,t}x^{\alpha}_{j,t} - \sum_{s\in\mathcal{N}^{\text{out}}_i} w_{si,t}x^{\alpha}_{i,t}\right) \tag{23}$$

where $k = 1, 2, \cdots$, $z^x_{i,0} = 2$. Note that $x_{i,k} = \frac{1}{2}z^x_{i,k}$ means that the actual state $x_{i,k}$ can be inferred by (23) at any time instant k, but the weight $w_{ij,k}$ $(i, j) \in \mathcal{E}$ is unknown to external eavesdroppers since $x^{\alpha}_{i,k}$ is unknown.

Similarly, in light of (11), we obtain

$$\begin{aligned} z^{\phi}_{i,k+1} &= \phi^{\alpha}_{i,k+1} + \phi^{\beta}_{i,k+1} \\ &= \phi^{\alpha}_{i,k} + \phi^{\beta}_{i,k} + \sum_{j\in\mathcal{N}^{\text{in}}_i} w_{ij,k}\phi^{\alpha}_{j,k} + (w_{ii,k} + l_{i,k})\phi^{\alpha}_{i,k} \\ &\quad + \phi^{\beta}_{i,k} - \phi^{\beta}_{i,k-1} - \sum_{j\in\mathcal{N}^{\text{in}}_i} \tilde{w}_{ij,k}\phi^{\alpha}_{j,k-1} \\ &\quad - (\tilde{w}_{ii,k} + \tilde{l}_{i,k})\phi^{\alpha}_{i,k-1} - \kappa(P_{i,k} - P_{i,k-1}) \\ &= 2z^{\phi}_{i,k} - z^{\phi}_{i,k-1} - \kappa(P_{i,k} - P_{i,k-1}) + \Phi_{i,k} - \tilde{\Phi}_{i,k} \end{aligned} \tag{24}$$

where

$$\begin{aligned} \Phi_{i,k} &= \sum_{j\in\mathcal{N}^{\text{in}}_i} w_{ij,k}\phi^{\alpha}_{j,k} - \sum_{s\in\mathcal{N}^{\text{out}}_i} w_{si,k}\phi^{\alpha}_{i,k}, \\ \tilde{\Phi}_{i,k} &= \sum_{j\in\mathcal{N}^{\text{in}}_i} \tilde{w}_{ij,k}\phi^{\alpha}_{j,k-1} - \sum_{s\in\mathcal{N}^{\text{out}}_i} \tilde{w}_{si,k}\phi^{\alpha}_{i,k-1}. \end{aligned}$$

Note that $\phi_{i,k} = \frac{1}{2}z^{\phi}_{i,k}$, and $\Phi_{i,k}$ and $\tilde{\Phi}_{i,k}$ can be calculated by external eavesdroppers.

In the following, defining $\eta_{i,k} = z^{\phi}_{i,k} - z^{\phi}_{i,k-1}$ $(k \geq 1)$, one has

$$\begin{aligned} \eta_{i,k+1} &= \eta_{i,k} - \kappa(P_{i,k} - P_{i,k-1}) + \Phi_{i,k} - \tilde{\Phi}_{i,k}, \\ &= -\kappa P_{i,k} + \sum_{t=0}^{k}\Phi_{i,t} - \sum_{t=1}^{k}\tilde{\Phi}_{i,t} \end{aligned} \tag{25}$$

where $\eta_{i,1} = -\kappa P_{i,0} + \Phi_{i,0}$. Furthermore, summing up (25) over k from 1 to k, we have

$$z^{\phi}_{i,k} = z^{\phi}_{i,0} - \kappa\sum_{t=0}^{k-1}P_{i,t} + \sum_{s=0}^{k-1}\sum_{t=0}^{s}\Phi_{i,t} - \sum_{s=1}^{k-1}\sum_{t=1}^{s}\tilde{\Phi}_{i,t}. \tag{26}$$

Since $z^{\phi}_{i,k}$ is unknown to external eavesdroppers at any time instant k, the sensitive information $P_{i,k}$ cannot be inferred, and the further privacy parameters $\{a_i, b_i\}$ can be preserved. The proof is now complete. ■

It is demonstrated by the above analysis that the employed state-decomposition-based privacy-preserving technique can effectively preserve the power sensitive information against external eavesdroppers. Subsequently, we will present the privacy leakage that occurs when the privacy-preserving scheme is not considered (i.e., when the distributed optimization algorithm (9) is executed).

Similar to the analysis in (22)-(23), $x_{i,k}$ can be inferred by external eavesdroppers through the following expression:

$$x_{i,k} = x_{i,0} + \sum_{t=0}^{k-1}\Big(\sum_{j\in\mathcal{N}^{\text{in}}_i} w_{ij,t}x_{j,t} - \sum_{s\in\mathcal{N}^{\text{out}}_i} w_{si,t}x_{i,t}\Big) \tag{27}$$

where $k = 1, 2, \cdots$, $x_{i,0} = 1$ $(i \in \mathcal{V})$. In this case, the weight w_{ij} $(j \in \mathcal{N}^{\text{in}}_i)$ is calculated by $w_{ij} = w_{ij}x_{j,k}/x_{j,k}$ and, furthermore, $\phi_{i,k}$ can be obtained via $w_{ij}\phi_{j,k}/w_{ij}$. Based on the relationships among $P_{i,k}, x_{i,k}, \lambda_{i,k}$, and $\phi_{i,k}$, external eavesdroppers can infer $P_{i,k}$ and $\{a_i, b_i\}$ via (9), which means that the sensitive information is leaked.

B. Privacy Preservation Against Honest-but-Curious Nodes

In this subsection, we proceed to show that Algorithm 1 is privacy-preserving against honest-but-curious nodes.

Denoting the set of all honest-but-curious nodes as $\mathcal{H} \subset \mathcal{V}$ and the information set accessible to the honest-but-curious node $h \in \mathcal{H}$ as

$$\begin{aligned} \mathcal{I}_h = \{&\phi^c_{h,k}, x^c_{h,k}, w_{dh,k}, w_{hj,k}\phi^{\alpha}_{j,k}, w_{dh,k}x^{\alpha}_{h,k}, \delta| \\ &d \in \mathcal{N}^{\text{out}}_h, j \in \mathcal{N}^{\text{in}}_h, c = \alpha, \beta, k = 0, 1, \cdots\}, \end{aligned} \tag{28}$$

one has

$$\mathcal{I}_{\mathcal{H}} = \{\mathcal{I}_h | h \in \mathcal{H}\}. \tag{29}$$

Theorem 2: For a directed graph $\mathscr{G}$ under Assumption 1, the privacy of agent i $(i \in \mathcal{V})$ can be preserved against the honest-but-curious nodes using Algorithm 1.

Proof: For the case of $(h, i) \notin \mathcal{E}$, the node h cannot infer the privacy of node i due to the unavailable information set.

In the case of $(h, i) \in \mathcal{E}$, the worst-case scenario is considered where node i has no legitimate neighbor. This is because, according to (22)-(23), honest-but-curious nodes can acquire all the data that node i can exchange with its neighbors, which facilitates to infer the sensitive information of node i.

For the purpose of analysis, the connection architecture is illustrated in Fig. 2. It should be noted that collusion among multiple honest-but-curious nodes is possible, making the considered connection architecture general.

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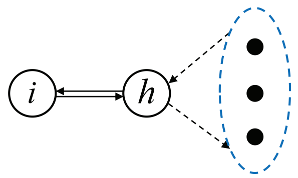

Fig. 2. Connection architecture

In light of the established results in (23) and (27), one has

$$\begin{aligned} z^x_{i,k+1} &= z^x_{i,0} + \sum_{t=0}^{k-1}(w_{ih,k}x^\alpha_{h,k} - w_{hi,k}x^\alpha_{i,k}) \\ z^\phi_{i,k} &= z^\phi_{i,0} - \kappa\sum_{t=0}^{k-1}P_{i,t} + \sum_{s=0}^{k-1}\sum_{t=0}^{s}\Phi^*_{i,t} - \sum_{s=1}^{k-1}\sum_{t=1}^{s}\tilde{\Phi}^*_{i,t}, \end{aligned} \tag{30}$$

where

$$\begin{aligned} \Phi^*_{i,k} &= w_{ih,k}\phi^\alpha_{h,k} - w_{hi,k}\phi^\alpha_{i,k}, \\ \tilde{\Phi}^*_{i,k} &= \tilde{w}_{ih,k}\phi^\alpha_{h,k-1} - \tilde{w}_{hi,k}\phi^\alpha_{i,k-1}. \end{aligned}$$

Due to the fact that $z^\phi_{i,0}$ is unknown to node h, the sensitive information $P_{i,k}$ cannot be inferred, and furthermore the privacy parameters $\{a_i, b_i\}$ can be preserved. ■

Remark 3: In this work, a state-decomposition-based privacy-preserving scheme has been successfully integrated into the DED algorithm. Compared with most privacy-preserving techniques, the adopted algorithm exhibits the following notable features. In comparison to the differential privacy method [12], [31], our scheme ensures the optimality of the ED problem due to its special construction decomposition of the distributed algorithm. In contrast to the homomorphic encryption scheme [26]–[28], the implementation of the proposed algorithm is simple, involving only basic addition and multiplication operations. Unlike the approach in [30], our method demonstrates robust privacy-preserving performance against external eavesdroppers.

Remark 4: Until now, a privacy-preserving DED algorithm has been developed to ensure supply-demand balance at the lowest economic cost without leaking sensitive information. In comparison to existing results, our paper presents the following distinctive merits: 1) the developed distributed optimization algorithm demonstrates the ability to achieve fast convergence over directed graphs, which is crucial for practical applications requiring real-time decision-making; and 2) the proposed privacy-preserving algorithm exhibits superior performance in terms of exact convergence, low computational complexity, and uncompromised privacy preservation. Overall, the proposed algorithm provides a practical and efficient solution for the privacy-preserving DED problem in islanded microgrids over directed graphs.

V. Simulation Study

In this section, a simulation example is presented to assess the validity of the developed algorithm. It is assumed that the microgrids consist of five generators (**G**), a storage device (**S**), and eight loads (**L**), with their communication links depicted in Fig. 3. Borrowed from [11], the parameters corresponding to each agent are listed in Table I. The step size κ is chosen as 0.0035, and δ is set to 0.1. The initial power $P_{i,0}$ $(i \in \mathcal{V})$ is selected as 32, 56, 50, 35, 42, -50, -30, -20, -14, -13, -76, -14, -56, and 10. In addition, $P^\alpha_{i,0}$, $P^\beta_{i,0}$, $x^\alpha_{i,0}$, $x^\beta_{i,0}$ $(i \in \mathcal{V})$ are generated in light of Algorithm 1.

TABLE I
Generation Parameters

Agent	Type	a_i	b_i	$\underline{P}_i$	$\overline{P}_i$
1	**G**	0.084	2	0	100
2	**G**	0.056	3	0	105
3	**G**	0.070	4	0	90
4	**L**	0.070	8	-80	-6
5	**L**	0.064	6.5	-30	-2
6	**G**	0.060	4	0	90
7	**S**	0.020	0	-50	50
8	**G**	0.080	2.5	0	80
9	**L**	0.060	8	-80	-10
10	**L**	0.076	7	-40	-5
11	**L**	0.070	7.5	-25	-2
12	**L**	0.080	8	-90	-8
13	**L**	0.070	7	-30	-2
14	**L**	0.084	8	-80	-10

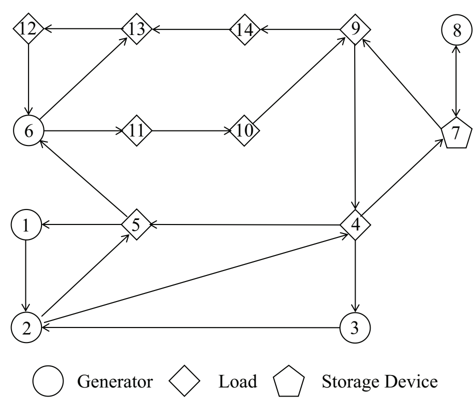

Fig. 3. Communication topology

The test results are depicted in Figs. 4-7. Fig. 4 illustrates the evolutions of the incremental cost $\lambda^\alpha_{i,k}$, the auxiliary variable $\phi^\alpha_{i,k}$, the local power $P^\alpha_{i,k}$, and total power demand, generation and their mismatch (i.e., $\tilde{P}_k \triangleq \sum_{i=1}^N P^\alpha_{i,k}$). Note that $\lambda^\alpha_{i,k}$ converges to $\lambda^* = 5.515$, which is identical to the optimal value obtained by the Lagrange multiplier algorithm. The local power $P^\alpha_{i,k}$ converges to the corresponding optimal value $P^*_{i,k}$, and the mismatch $\tilde{P}_k$ approaches to 0, which shows that the supply-demand balance is reached. Fig. 5 depicts the weighted information $w_{ij,k}\phi^\alpha_{j,k}$. Note that the received information are disordered to eavesdroppers due to the fact that the weight $w_{ij,k}$ is random and unknown. In other words, the eavesdroppers and honest-but-curious nodes cannot infer

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or estimate the privacy value $P_{i,k}$ and parameters a_i, b_i in light of the obtained information, which verifies the privacy property of the proposed privacy-preserving DED algorithm.

Next, to show the fast convergence of the proposed algorithm, the following four distributed algorithms are considered and compared: 1) PDED: the proposed DED algorithm; 2) CID; consensus+innovation distributed algorithm [11] under directed graphs; 3) DPG: the distributed push-gradient method [20]; and 4) PDDA: the push-based distributed dual averaging method [21]. The evolutions of the mismatch between supply and demand are presented in Fig. 6. It can be observed that the developed algorithm can rapidly converge to the globally optimal solution. In addition, we set δ as $0.05, 0.15, 0.25, 0.35, 0.45$ to test the convergence rate of the proposed distributed algorithm. In Fig. 7, we observe that a large δ leads to a faster convergence rate. The simulation study validates the accuracy and superiority of the obtained results.

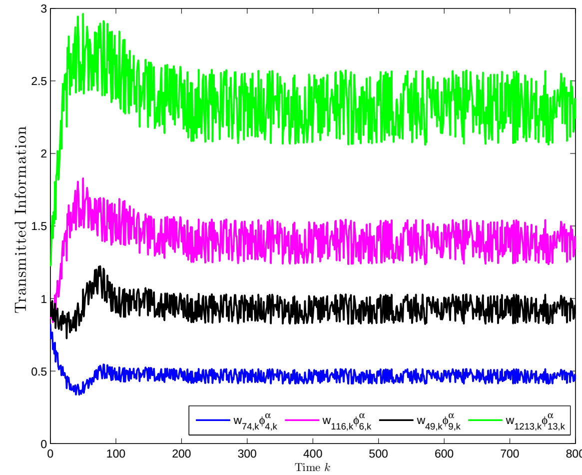

Fig. 5. Transmitted information.

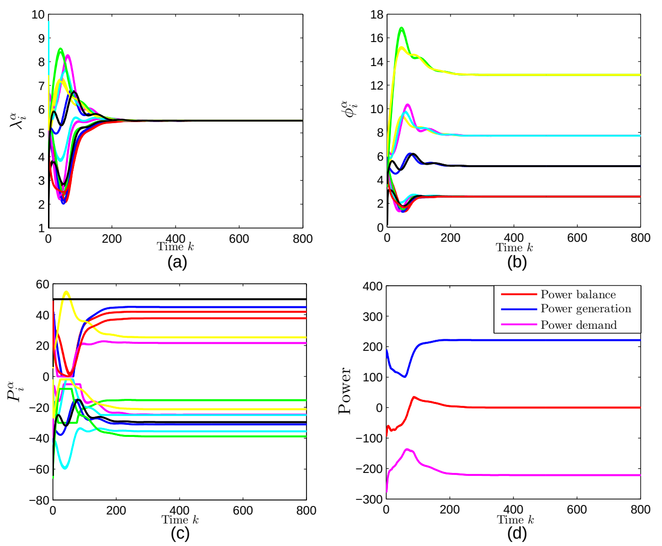

Fig. 4. Test results of the proposed algorithm. (a) Incremental cost $\lambda_{i,k}$. (b) Auxiliary variable $\phi_{i,k}$. (c) Local power $P_{i,k}$. (d) Power balance.

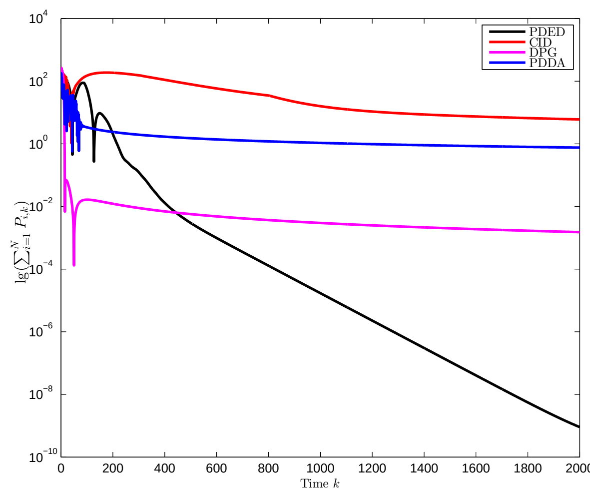

Fig. 6. The dynamic evolutions of $\lg(\sum_{i=0}^{N} P_{i,k})$ via different algorithms.

VI. Conclusion

In this paper, we have addressed the privacy-preserving DED problem of islanded microgrids over directed graphs. By combining the push-sum protocol with the EXTRA, it has been demonstrated that the proposed distributed optimization algorithm achieves fast convergence over directed graphs with low computational complexity. To prevent the leakage of power sensitive information, a privacy-preserving scheme has been integrated into the distributed optimization algorithm using the state-decomposition technique, which exhibits the privacy property against internal honest-but-curious agents and external eavesdroppers. Finally, a simulation case has been provided to validate the effectiveness of the developed algorithm. Future directions would be the extensions of other DED problems for grid-connected microgrids [1] with the power flow and thermal constraints via distributed game-based algorithms [43], [44] and distributed fixed-time convergent algorithms [45], [46], and privacy preservation in industrial applications using biometrics techniques [47], [48].

References

[1] W. Chen and T. Li, Distributed economic dispatch for energy internet based on multiagent consensus control, *IEEE Trans. Autom. Control*, vol. 66, no. 1, pp. 137–152, Jun. 2021.

[2] W. Chen, D. Ding, H. Dong, G. Wei, and X. Ge, Finite-horizon H_∞ bipartite consensus control of cooperation-competition multiagent systems with round-robin protocols, *IEEE Trans. Cybern.*, vol. 51, no. 7, pp. 3699–3709, Jul. 2021.

[3] Y. Jin, X. Ma, X. Meng and Y. Chen, Distributed fusion filtering for cyber-physical systems under Round-Robin protocol: a mixed H_2/H_∞ framework, *Int. J. Syst. Sci.*, vol. 54, no. 8, pp. 1661–1675, 2023.

[4] F. Yao, Y. Ding, S. Hong and S.-H. Yang, A survey on evolved LoRa-based communication technologies for emerging internet of things applications, *Int. J. Netw. Dyn. Intell.*, vol. 1, no. 1, pp. 4-19, Dec. 2022.

[5] Y. Yuan, X. Tang, W. Zhou, W. Pan, X. Li, H.-T. Zhang, H. Ding, and J. Goncalves, Data driven discovery of cyber physical systems, *Nature Commun.*, vol. 10, no. 1, pp. 1–9, 2019.

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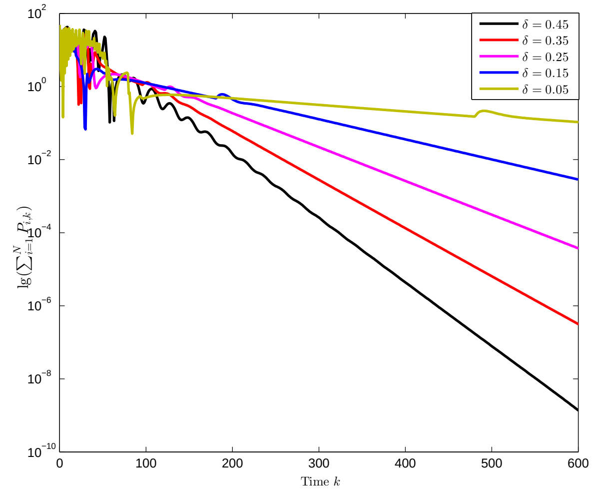

Fig. 7. The dynamic evolutions of $\lg(\sum_{i=0}^{N} P_{i,k})$ with different δ.

[6] Y. Yuan, G. Ma, C. Cheng, B. Zhou, H. Zhao, H.-T. Zhang, and H. Ding, A general end-to-end diagnosis framework for manufacturing systems, *Nat. Sci. Rev.*, vol. 7, no. 2, pp. 418–429, 2020.

[7] G. Bao, L. Ma, and X. Yi, Recent advances on cooperative control of heterogeneous multi-agent systems subject to constraints: A survey, *Syst. Sci. Control Eng.*, vol. 10, no. 1, pp. 539–551, 2022.

[8] P. Wen, X. Li, N. Hou, and S. Mu, Distributed recursive fault estimation with binary encoding schemes over sensor networks, *Syst. Sci. Control Eng.*, vol. 10, no. 1, pp. 417–427, 2022.

[9] F. Han, J. Liu, J. Li, J. Song, M. Wang, and Y. Zhang, Consensus control for multi-rate multi-agent systems with fading measurements: the dynamic event-triggered case, *Syst. Sci. Control Eng.*, vol. 11, no. 1, art. no. 2158959, 2023.

[10] Y.-A. Wang, B. Shen, L. Zou, and Q.-L. Han, A survey on recent advances in distributed filtering over sensor networks subject to communication constraints, *Int. J. Netw. Dyn. Intell.*, vol. 2, no. 2, art. no. 100007, Jun. 2023.

[11] G. Hug, S. Kar, and C. Wu, Consensus+innovations approach for distributed multiagent coordination in a microgrid, *IEEE Trans. Smart Grid*, vol. 6, no. 4, pp. 1893–1903, Jul. 2015.

[12] D. Zhao, C. Zhang, X. Cao, C. Peng, B. Sun, K. Li, and Y. Li, Differential privacy energy management for islanded microgrids with distributed consensus-based ADMM algorithm, *IEEE Trans. Control Syst. Tech.*, vol. 31, no. 3, pp. 1018–1031, May 2023.

[13] G. Chen and Q. Yang, An ADMM-based distributed algorithm for economic dispatch in islanded microgrids, *IEEE Trans. Ind. Informat.*, vol. 14, no. 9, pp. 3892–3903, Sept. 2018.

[14] X. Luo, Y. Zhong, Z. Wang and M. Li, An alternating-direction-method of multipliers-incorporated approach to symmetric non-negative latent factor analysis, *IEEE Tran. Neural Netw. Learn. Syst.*, vol. 34, no. 8, pp. 4826–4840, Aug. 2023.

[15] W. Shi, Q. Ling, G. Wu, and W. Yin, Extra: An exact first-order algorithm for decentralized consensus optimization, *SIAM J. Optim.*, vol. 25, no. 2, pp. 944–966, May 2015.

[16] Z. Tang, D. J. Hill, and T. Liu, A novel consensus-based economic dispatch for microgrids, *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 3920–3922, Jul. 2018.

[17] H. Dai, J. Jia, L. Yan, X. Fang, and W. Chen, Distributed fixed-time optimization in economic dispatch over directed networks, *IEEE Trans. Ind. Informat.*, vol. 17, no. 5, pp. 3011–3019, May 2021.

[18] X. Luo, H. Wu, Z. Wang, J. Wang, and D. Meng, A novel approach to large-scale dynamically weighted directed network representation, *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 44, no. 12, pp. 9756–9773, Dec. 2022.

[19] D. Kempe, A. Dobra, and J. Gehrke, Gossip-based computation of aggregate information, in *Proc. 44th Annu. IEEE Symp. Found. Comput. Sci.*, 2003, pp. 482–491.

[20] S. Mao, Y. Tang, Z. Dong, K. Meng, Z. Dong, and F. Qian, A Privacy preserving distributed optimization algorithm for economic dispatch over time-varying directed networks, *IEEE Trans. Ind. Informat.*, vol. 17, no. 3, pp. 1689–1701, Mar. 2021.

[21] Z. Wang, D. Wang, C. Wen, F. Guo, and W. Wang, Push-based distributed economic dispatch in smart grids over time-varying unbalanced directed graphs, *IEEE Trans. Smart Grid*, vol. 12, no. 4, pp. 3185–3199, Jul. 2021.

[22] D. Zhao, D. Liu, and L. Liu, Distributed privacy preserving algorithm for economic dispatch over time-varying communication, *IEEE Trans. Power Syst.*, doi: 10.1109/TPWRS.2023.3246998.

[23] Z.-H. Pang, L. Z. Fan, H. Guo, Y. Shi, R. Chai, J. Sun, and G. Liu, Security of networked control systems subject to deception attacks: a survey, *Int. J. Syst. Sci.*, vol. 53, no. 16, pp. 3577–3598, 2022.

[24] Z.-H. Pang, L.-Z. Fan, K. Liu, and G.-P. Liu, Detection of stealthy false data injection attacks against networked control systems via active data modification, *Inf. Sci.*, vol. 546, pp. 192–205, 2021.

[25] F. M. Shakiba, M. Shojaee, S. M. Azizi and M. Zhou, Real-time sensing and fault diagnosis for transmission lines, *Int. J. Netw. Dyn. Intell.*, vol. 1, no. 1, pp. 36-47, Dec. 2022.

[26] W. Chen, L. Liu, and G.-P. Liu, Privacy-preserving distributed economic dispatch of microgrids: A dynamic quantization based consensus scheme with homomorphic encryption, *IEEE Trans. Smart Grid*, vol. 14, no. 1, pp. 701–713, Jan. 2023.

[27] T. Wu, C. Zhao, and Y.-J. A. Zhang, Privacy-preserving distributed optimal power flow with partially homomorphic encryption, *IEEE Trans. Smart Grid*, vol. 12, no. 5, pp. 4506–4521, Sept. 2021.

[28] Y. Yan, Z. Chen, V. Varadharajan, M. J. Hossain, and G. E. Town, Distributed consensus-based economic dispatch in power grids using the Paillier cryptosystem, *IEEE Trans. Smart Grid*, vol. 12, no. 4, pp. 3493–3502, Jul. 2021.

[29] Z. Yang, Y. Liu, W. Zhang, F. E. Alsaadi, and K. H. Alharbi, Differentially private containment control for multi-agent systems, *Int. J. Syst. Sci.*, vol. 53, no. 13, pp. 2814–2831, 2022.

[30] C. Zhao, J. Chen, J. He, and P. Cheng, Privacy-preserving consensus-based energy management in smart grids, *IEEE Trans. Signal Process.*, vol. 66, no. 23, pp. 6162–6176, Dec. 2018.

[31] A. Wang, W. Liu, T. Dong, X. Liao, and T. Huang, DisEHPPC: enabling heterogeneous privacy-preserving consensus-based scheme for economic dispatch in smart grids, *IEEE Trans. Cybern.*, vol. 52, no. 6, pp. 5124–5135, Jun. 2022.

[32] W. Chen, Z. Wang, J. Hu, and G.-P. Liu, Differentially private average consensus with logarithmic dynamic encoding-decoding scheme, *IEEE Trans. Cybern.*, in press, doi: 10.1109/TCYB.2022.3233296.

[33] M. Ruan, H. Gao, and Y. Wang, Secure and privacy-preserving consensus, *IEEE Trans. Autom. Control*, vol. 64, no. 10, pp. 4035–4049, Oct. 2019.

[34] Y. Wang, Privacy-preserving average consensus via state decomposition, *IEEE Trans. Autom. Control*, vol. 54, no. 11, pp. 4711–4716, Nov. 2019.

[35] K. Zhang, Z. Li, Y. Wang, A. Louati, and J. Chen, Privacy-preserving dynamic average consensus via state decomposition: Case study on multi-robot formation control, *Automatica*, vol. 139, Feb. 2021, art. no. 110182.

[36] H. Tu, Y. Du, H. Yu, X. Lu, and S. Lukic, Privacy-preserving robust consensus for distributed microgrid control applications, *IEEE Trans. Indus. Electron.*, doi: 10.1109/TIE.2023.3274846.

[37] C. Ying, N. Zheng, Y. Wu, M. Xu, and W.-A. Zhang, Privacy-preserving adaptive resilient consensus for multi-agent systems under cyber attacks, *IEEE Trans Indus. Informat.*, doi: 10.1109/TII.2023.3280318.

[38] P. Rezaeinia, B. Gharesifard, T. Linder, and B. Touri, Push-sum on random graphs: Almost sure convergence and convergence rate, *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 1295–1302, Mar. 2020.

[39] M. H. Ullah, B. Babaiahgari, A. Alseyat, and J.-D. Park, A computationally efficient consensus-based multiagent distributed EMS for DC microgrids, *IEEE Trans. Indus. Electron.*, vol. 68, no. 6, pp. 5425–5435, Jun. 2021.

[40] A. Nedic and A. Olshevsky, Distributed optimization over time-varying directed graphs, *IEEE Trans. Autom. Control*, vol. 60, no. 3, pp. 601–615, Mar. 2015.

[41] X. Chen, L. Huang, K. Ding, S. Dey, and L. Shi, Privacy-preserving push-sum average consensus via state decomposition, *IEEE Trans. Autom. Control*, to be published, doi: 10.1109/TAC.2023.3256479.

[42] C. Xi and U. A. Khan, DEXTRA: A fast algorithm for optimization over directed graphs, *IEEE Trans. Autom. Control*, vol. 62, no. 10, pp. 4980–4993, Oct. 2017.

[43] M. Ye, Q.-L. Han, L. Ding, and S. Xu, Distributed nash equilibrium seeking in games with partial decision information: A survey, *Proc. IEEE*, vol. 111, no. 2, pp. 140–157, Feb. 2023.

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[44] X. Li, Q. Song, Y. Liu, and F. E. Alsaadi, Nash equilibrium and bang-bang property for the non-zero-sum differential game of multi-player uncertain systems with Hurwicz criterion, *Int. J. Syst. Sci.*, vol. 53, no. 10, pp. 2207–2218, Jul. 2022.

[45] B. Ning, Q.-L. Han, Z. Zuo, J. Jin, and J. Zheng, Collective behaviors of mobile robots beyond the nearest neighbor rules with switching topology, *IEEE Trans. Cybern.*, vol. 48, no. 5, pp. 1577–1590, May 2018.

[46] Y. Xu, Z. Yao, R. Lu, and B. K. Ghosh, A novel fixed-time protocol for first-order consensus tracking with disturbance rejection, *IEEE Trans. Autom. Control*, vol. 67, no. 11, pp. 6180–6186, Nov. 2022.

[47] Z. Gao, A. Castiglione, and M. Nappi, Guest Editorial: Biometrics in Industry 4.0: Open Challenges and Future Perspectives, *IEEE Trans. Ind. Informat.*, vol. 18, no. 12, pp. 9068-9071, Dec. 2022.

[48] S. Lu, Z. Gao, Q. Xu, C. Jiang, A. Zhang, and X. Wang, Class-imbalance privacy-preserving federated learning for decentralized fault diagnosis with biometric authentication, *IEEE Trans. Ind. Informat.*, vol. 18, no. 12, pp. 9101–9111, Dec. 2022.

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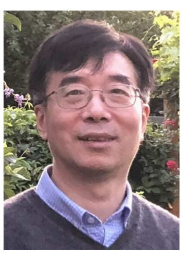

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