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RESEARCH ARTICLE



## Sovereign debt sustainability: some evidence for the US and Europe

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### ABSTRACT

This paper examines the sustainability of sovereign debt and its components by applying fractional integration methods to annual data starting in 1831 for the UK and the US, in 1862 for Italy and in 1881 for France and Germany, and ending in all cases in 2022. The results suggest in all cases I(d) behaviour of the differential between interest payments and primary deficits, but with different speeds of adjustment towards the long-run equilibrium. By contrast, since 1950 all estimates of the fractional differencing parameter are all within the same interval ( $0.5 < d < 1$ ), implying slower dynamic adjustment. This sustainability finding is confirmed by further tests based on the budget constraint as a whole, which, especially in the post-WWII period, suggest relatively low persistence.

### ARTICLE HISTORY

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### KEYWORDS

Fractional integration; mean reversion; persistence; debt sustainability

### JEL CLASSIFICATION

E52; C32

## 1. Introduction

Analysing the evolution of public debt is of paramount importance to establish whether or not it is on a sustainable path. Prior to the pandemic, the global debt-to-GDP ratio had been increasing for decades, with global public debt tripling since the mid-1970s to reach 92% of GDP by the end of 2022 (IMF, 2023a), and peaking at \$92 trillion in 2022 (UNCTAD, 2023). In advanced economies,<sup>1</sup> namely the Euro Area, Major Advanced Economies (G7), and some other countries of the ASEAN-5, the public debt to GDP ratio reached 112% in 2023 (IMF, 2023b).

The inflationary pressures of recent years (resulting in particular from the sharp increase in energy prices following the Russian invasion of Ukraine) have led to higher interest rates and cost of servicing the debt, adding to fiscal pressures and posing risks to financial stability (in addition to affecting growth and employment negatively). It is therefore essential to adopt fiscal measures to reduce global debt levels to more sustainable levels (IMF, 2024a). For a long time, debt dynamics had not been a major concern for governments, as interest rates were below the growth rate of the economy and therefore expanding fiscal deficits and debt stocks did not threaten solvency. Consequently, public debt has increased significantly in recent decades across both advanced and rising and middle-income economies,<sup>2</sup> with projections indicating it will reach 120% and 80% of GDP respectively by 2028 (Adrian et al., 2024). However, during the recent period of tight monetary policy and low growth, debt sustainability has again become a key issue, as real interest rates are no longer below growth rates.

In the US public debt is currently equivalent to 118.73% of GDP and it is estimated that the US government has 20 years to take corrective action, otherwise no amount of tax increases or spending cuts

<sup>1</sup>Details of the countries classified as Advanced economies are provided in Table A in the <sup>o</sup>, which follows the IMF classification, included. In general terms, they include the Euro Area, Major Advanced Economies (G7), and some others of the ASEAN-5.

<sup>2</sup>Details of the Emerging and Middle-Income economies are provided in Table B in the Appendix. These are countries still undergoing a process of industrialisation and globalisation, shifting from agricultural to industrial economies and being characterised by high GDP growth and increasing trade and foreign investment.

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will be able to prevent debt monetisation from producing significant inflation (IMF, 2024b). From 1950 to 1970 public debt in the US was relatively low thanks to high economic growth and a restrictive fiscal policy; however, in the eighties the increase in military spending drove it upwards, and debt did not stop growing until the nineties, when budget surpluses were achieved thanks to a combination of more disciplined fiscal policy and higher economic growth. Since then, US public debt as a percentage of GDP has started growing again, driven by large budget deficits. The Congressional Budget Office (CBO) projections<sup>3</sup> indicate that this figure is expected to grow over the next 30 years, driven by a persistent gap between spending and revenue driven by Social Security, health care, and especially interest costs. Specifically, in terms of GDP, spending is expected to rise from 23.1% in 2024 to 27.3% in 2054, while revenue is only expected to rise from 17.5% to 18.8% (CBO, 2024).

As for Europe, when the Economic and Monetary Union (EMU) was launched in 1992 with the signing of Maastricht Treaty, this imposed a public debt ceiling of 60% of GDP to ensure that all member states would converge towards a sustainable debt level. However, the 2007-08 global financial crisis led to a sharp increase of debt in those countries (84% in 2023 for EU members and 89% for the Eurozone (IMF, 2023c) either through expansionary fiscal policy or unconventional monetary policy measures such as the Quantitative Easing (QE) adopted by most central banks in response to the crisis.

One important issue in this context is the degree of persistence of sovereign debt, which might shed light on whether or not corrective policies are required to ensure sustainability. In a previous study, Martin-Valmayor et al. (2024) found that this is linked to the persistence of the primary deficit for the major European economies and the US. The current paper aims to extend their analysis by addressing the issue of sustainability along the lines of Trehan & Walsh (1991), namely by examining the stochastic properties of the budget constraint relationship, which is the differential between interest payments and primary deficits. In particular, the present study uses a more general econometric framework based on fractional integration (Baillie, 1996; Gil-Alana and Robinson, 1997) to measure the degree of persistence through the estimation of the differencing parameter  $d$ . It also examines the possible role of a wider set of variables affecting the evolution of debt by including bond yields, the nominal GDP growth rate and other financial transactions. Therefore, the key research question in this paper is whether or not sovereign debt and its specific components are sustainable in the long run in the main European economies and the US.

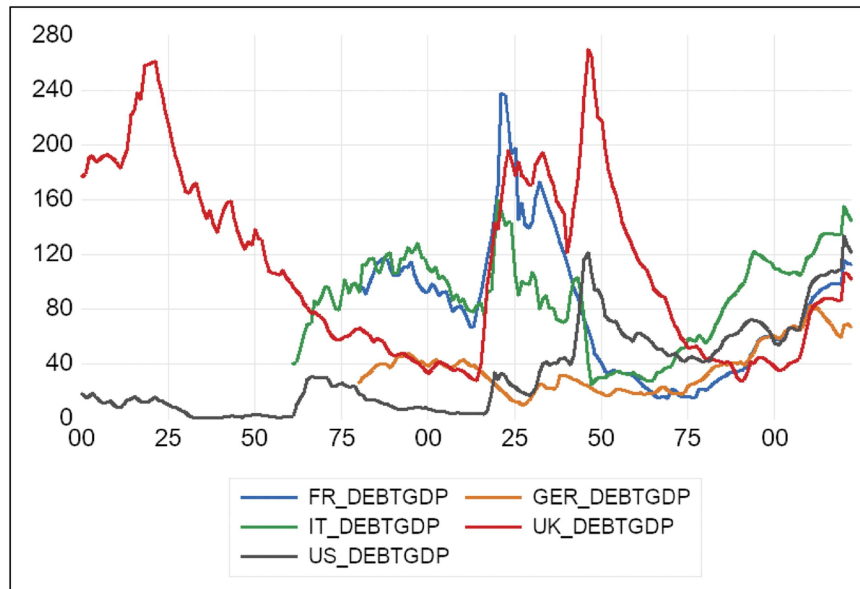
More specifically, as shown in Figure 1, long spans of data from the 19<sup>th</sup> century to the present are analysed for the US and the four main European economies, namely Germany, France, Italy and the UK. Figure 2 in the empirical section also provides plots of the debt components (i.e. the debt-to-GDP components) in the countries under investigation. It is noteworthy that there are some institutional differences between these countries which might have implications for the evolution of debt and its sustainability. Specifically, in all of them the government is responsible for fiscal policy; however, whilst the US and the UK are characterised by an independent monetary policy, the other countries in our sample belong to the eurozone. In the latter case, monetary policy decisions are made by the European Central Bank (ECB) and affect all the member countries. The ECB replaced central banks in the Euro zone members, which resulted in a loss of monetary independence for these countries.

The fractional integration framework used for the analysis is more general than standard models based on the dichotomy between stationary and non-stationary series since it allows the integration parameter to take any real value, including fractional ones, as opposed to integers only. As a result, a much wider range of dynamic processes can be considered, and valuable information can be obtained on the persistence and mean reversion properties of the series. Moreover, the integration parameter can be interpreted as a measure of the degree of persistence.

To sum up, the contribution of this paper is twofold. First, we intend to provide a more extensive evidence on whether or not public finances are on a sustainable path using long runs of data and a wide set of relevant variables for various countries. Second, we adopt an econometric approach that is more flexible

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<sup>3</sup>Note that the CBO's forecasts of economic variables are reliable and tend to exhibit small positive average errors in the short and medium term—specifically, for many variables, the average errors for two- and five-year forecasts are minimal and not statistically significant. However, the CBO does not compare forecasts beyond the five-year horizon (CBO, 2025). Various studies have indicated that errors can increase during periods of economic downturn or unexpected financial shifts. In particular, forecast errors made in 10- and 30-year projections published in reports by the CBO from 1996-2008 have, on average, overestimated the size of the cumulative budget balance by 3.9% of cumulative GDP across them (Inayatli, 2019)



**Figure 1.** Debt/GDP series for the countries under investigation. Note: The sample starts in 1800 for the UK and the US, 1860 for Italy and 1880 for Germany and France, and ends in 2022 in all cases. The data source is the Historical Public Debt Database (1800–2022) from the public IMF e-library (IMF, 2024b).

than the unit root testing methods generally carried out in previous studies on this topic. The rest of the paper is structured as follows: [Section 2](#) provides a brief review of the relevant literature; [Section 3](#) outlines the fractional integration approach used for the analysis; [Section 4](#) describes the data and discusses the empirical results; [Section 5](#) offers some concluding remarks.

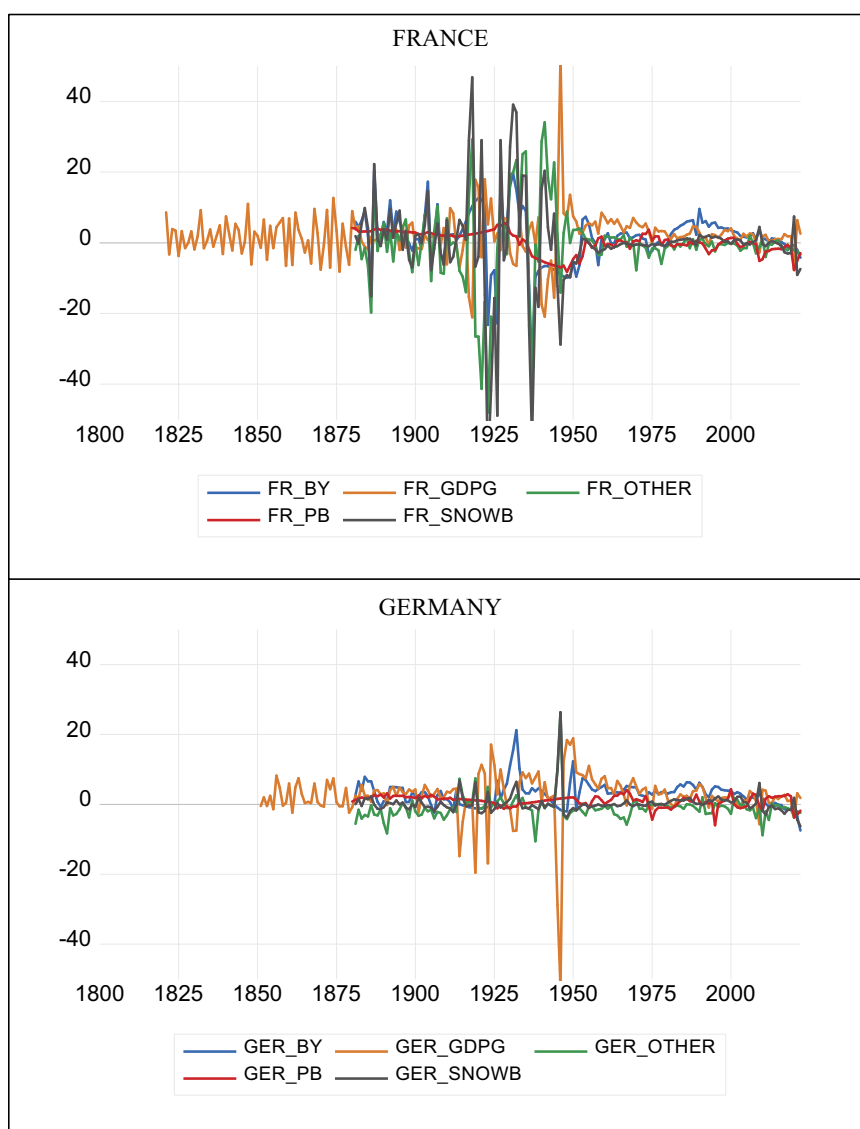
## 2. Literature review

The literature aiming to assess the persistent behaviour of sovereign debt is very extensive. Several studies have used unit root testing (Antonini et al., 2013; Brady & Magazzino, 2017; Camarero et al., 2013; Campos & Cysne, 2022 among others) finding evidence in all cases of unit roots, which implies that shocks have long-run effects. These papers used linear tests such as the Dickey & Fuller, 1979) or Phillips & Perron, 1988) ones, or non-linear and more efficient ones such as Kwiatkowski et al., 1992) or Elliot et al., 1996). Other studies which have tested for debt sustainability using unit root tests to establish whether or not mean reversion occurs include Caporale (1992), Feve and Henin (2000), Trehan & Walsh (1991), Uctum and Wickens (2000) and Chen and Wu (2018). To assess solvency, Caporale (1995) used a different method initially developed to detect speculative bubbles in financial markets and tested whether the government's budget is intertemporally balanced in a number of European countries; he reported that in some of them (Italy, Germany, Denmark and Greece) the government was not intertemporally solvent (see also Caporale, 1997, for another study focusing on the budget constraint to test for solvency). More recent papers have adopted a cointegration approach with or without structural breaks (Baharumshah et al., 2017, Bajo-Rubio et al., 2010), in some cases testing for cointegration between public expenditure and revenue (Camarero et al., 2013; Escario et al., 2012).

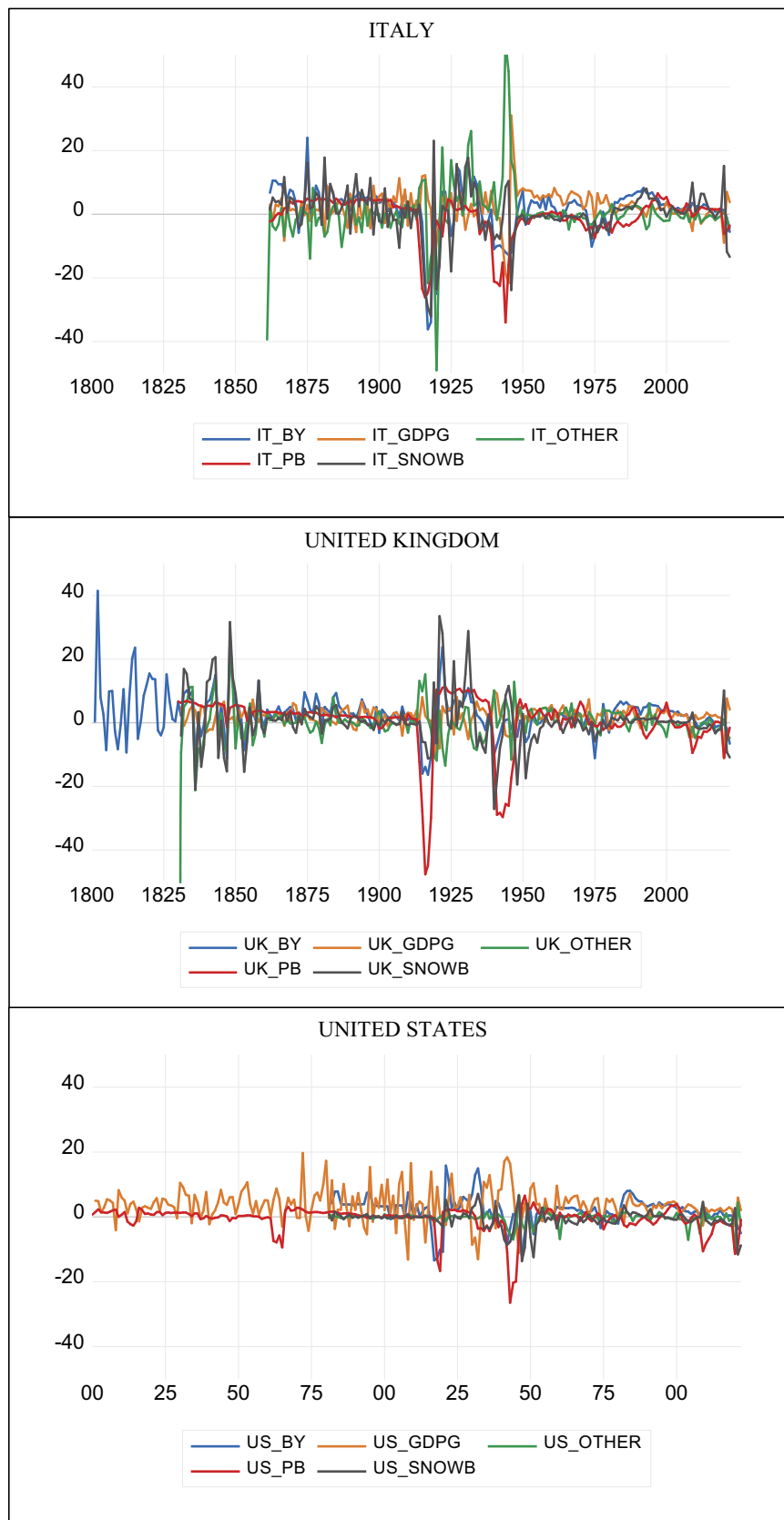
A useful paper on the issue of debt sustainability is due to Chalk and Hemming (2000), who reviewed early government solvency tests based on historical data. Previously, Hamilton & Flavin (1986) had suggested that a sufficient (but not necessary) condition for the stationarity of debt is that the primary balance be stationary. Subsequent paper such as Campbell & Shiller (1987), Hansen et al. (1987) and Trehan & Walsh (1988) introduced other sustainability conditions based on cointegration between net-of-interest expenditures, revenues, interest payments, and the outstanding stock of debt. Trehan & Walsh (1991) relaxed the requirement that expenditures and revenues be difference-stationary and provided sufficient conditions for sustainability. Specifically, if interest rates are constant, and debt as well as the

primary balance are integrated of order 1, a sufficient condition for sustainability is that the latter two variables should be cointegrated. Instead, if interest rates are not constant, as long as the expected real rate of interest is positive, debt sustainability holds if the deficit inclusive of interest payments is stationary.

Note that it has become apparent in recent decades that unit root tests do not provide reliable evidence. For instance, Diebold & Rudebush (1991) and Hassler & Wolters (1994) examined the properties of the Dickey-Fuller tests under fractionally integrated alternatives and showed that they have low power under this type of alternatives. Similarly, Lee & Schmidt (1996) examined the KPSS tests and found evidence of unbiasedness only against stationary long memory alternatives or  $0 < d < 0.5$ . In essence, imposing a dichotomy between  $I(0)$  and  $I(1)$  behaviour is very restrictive especially if the differencing parameter is in the range  $[0.5, 1)$ . In such cases, even though the series exhibit long memory and are non-stationary and mean-reverting, unit root tests may characterise them as  $I(1)$ . By contrast, a fractional integration approach will detect the presence of mean reversion and is therefore the most appropriate modelling framework. Furthermore, the estimated fractional differencing parameter can be interpreted as a measure of persistence.



**Figure 2.** Debt/GDP components in the countries under investigation. Note: This figure plots for each country the variables appearing in the budget constraint: bond yields—BY; GDP growth—GDPG; primary balance—PB; snowball contribution—SNOWB; and the other stock-flow adjustments - OTHER).



**Figure 2.** (Continued)

Finally, it should be noted that there is only a limited number of papers analysing debt persistence in the context of a fractional integration framework. Cuestas et al. (2014) examined government debt dynamics in the Eurozone following European economic integration (2000–2013). They found evidence of structural breaks around the 2007–2008 period in all countries except France and Germany, where persistence was very high ( $d > 1$ ). Evidence of mean reversion ( $d < 1$ ) was found in only five country subsamples, specifically Belgium and Spain prior to the break, and Finland, Greece and Portugal following the break. For other countries the results are more mixed Caporale et al. (2021) analysed the private debt-to-GDP ratio in 43 OECD countries from 1951 to 2020. Their findings also indicate high persistence, with the exception of Argentina ( $d = 0.39$ ). As for the remaining countries, only Turkey, Ireland, Brazil and Indonesia exhibit unit roots, while the other 27 countries display orders of integration significantly higher than 1. Finally, Martin-Valmayor et al. (2024) analysed debt persistence in the US and five major European economies (France, Germany, Italy, Spain and the UK) and found high degrees of persistence and clear evidence of no mean reversion ( $d \geq 1$ ) in the debt-to-GDP and debt per capita ratios. The findings also showed a correlation with the primary deficit, thereby supporting the hypothesis that debt persistence is attributable to the ongoing primary deficit. The purpose of the present paper is to provide a comprehensive analysis of this issue by analysing the persistence of the sustainability conditions and their associated components in a fractional integration framework.

### 3. Methodology

As already mentioned, the empirical analysis carried out in this paper is based on a fractional integration framework which allows the differencing parameter  $d$  to take any real value, including fractional ones. Within this context one can define a covariance or second-order stationary process  $\{x_t, t = 0, \pm 1, \dots\}$  with mean  $\mu$  as integrated of order 0 and denoted as  $I(0)$ , if the infinite sum of the autocovariances, defined as  $\gamma(u) = E[(x(t) - \mu)(x(t + u) - \mu)]$ , is finite, that is:

$$\sum_{j=-\infty}^{\infty} |\gamma(u)| < \infty. \quad (1)$$

These processes are said to exhibit short memory and include not only the white noise but also the stationary and invertible AutoRegressive Moving Average (ARMA) model, which is frequently employed for stationary series. By contrast, a process is said to exhibit long memory (so-named because of the relevance of observations in the distant past) if the infinite sum of its autocovariances is infinite:

$$\sum_{u=-\infty}^{\infty} \gamma_u = \infty. \quad (2)$$

There exist many models that satisfy the above condition, including those based on fractional integration or  $I(d)$  with  $d > 0$ . In such a case  $x_t$  is said to be integrated of order  $d$  and denoted by  $I(d)$  if it can be expressed as:

$$(1 - L)^d x_t = u_t, t = 1, 2, \dots, \quad (3)$$

where  $L$  is the lag operator and  $u_t$  is  $I(0)$  or short memory. Note that the polynomial in  $L$  on the left-hand side of (3) can be expanded in terms of its Binomial representation such that, for any real  $d$ ,

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \dots \quad (4)$$

and thus Equation (3) becomes:

$$x_t = dx_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \dots + u_t \quad (5)$$

According to Equation (5), if the differencing parameter  $d$  is an integer,  $x_t$  depends only on a finite number of previous observations; however, if it is a fractional value, the series will depend on its entire past history. Moreover, the higher  $d$  is, the higher will be the association between observations, and therefore this



parameter can be interpreted as a measure of the degree of persistence (dependence) in the series. In this context, if  $d$  is smaller than 0.5,  $x_t$  is covariance stationary; however,  $d \geq 0.5$  implies non-stationarity, although if  $d < 1$  the process is still mean-reverting, with shocks having transitory effects which decay to zero hyperbolically; in addition,  $x_t$  in (3) admits an infinite MA representation; finally, if  $d \geq 1$ , the process is explosive and shocks have permanent effects. These processes were proposed in Granger (1980), Granger and Joyeux 1980, Granger 1981, and Hosking 1981, and since then, they have been widely used in the analysis of economic and financial data (see, e.g., Abbritti et al., 2023; Gil-Alana and Robinson, 1997; Zheng et al., 2018; etc.). Gil-Alana and Hualde (2006) and Hualde and Nielsen (2022) provide extensive reviews on the issue of fractional integration.

In this paper we estimate the differencing parameter  $d$  using a simple version of the method developed by Robinson (1994) and widely used in empirical applications of fractional integration (Gil-Alana & Robinson, 1997), which is essentially a frequency domain version of the likelihood function. This approach has numerous advantages over others. First, it is based on testing the null hypothesis  $H_0: d = d_0$  in (3) for any real value  $d_0$ , including those outside the stationarity region ( $d_0 \geq 0.5$ ); second, it has a standard  $N(0,1)$  asymptotic distribution, and this behaviour holds whether or not deterministic terms are included in the model; third, it also allows for weak autocorrelation in the error term; fourth, it is the most efficient method in the Pitman sense against local departures from the null.

By Pitman (1937) efficiency we mean a measure of the effectiveness of the test when the alternative hypothesis is very close to the null hypothesis and indicates the test's ability to correctly reject the null hypothesis when the alternative is only slightly different. Since the starting date is not the same for all countries as a robustness check we have also re-estimated the models starting in 1881 in all cases. The results (not reported to save space but available upon request) are consistent with those discussed in the main text in terms of the order of integration of the series. The codes used for its implementation can be found in Gil-Alana (1998).

Our aim is to use this empirical framework to study the sustainability of the debt/GDP ratio. For this purpose, we start with the Trehan & Walsh (1991) general budget identity:

$$\Delta s_t = r_t s_{t-1} - pb_t, \quad (6)$$

where  $r_t$  is the real long-term government bond yield,  $s_t$  the real stock of outstanding debt, and  $pb_t$  the government primary balance-to-GDP ratio. Following Trehan & Walsh (1991), the stationarity of the inclusive-of-interest deficit would be sufficient to imply that intertemporal budget balance holds, as long as the expected real rate of interest is positive.

Let  $b_t$  denote the gross public debt-to-GDP ratio; since  $b_t = \frac{1+r_t}{1+g_t} b_{t-1} - pb_t$ , one can also perform the following stationarity test:

$$\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - pb_t, \quad (7)$$

where  $b_t$  is defined as above and  $g_t$  is the real GDP growth rate. One can also include an additional stock flow adjustment ( $sf_t$ ) variable concerning all financial transactions and other factors affecting the outstanding stock of debt but not the primary balance (ECB, 2011). Therefore, to test for debt sustainability (namely, for the stationarity of the growth or first-differenced debt series), one can use the following equation:

$$\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - pb_t + sf_t. \quad (8)$$

Note that the first term  $\frac{1+r_t}{1+g_t} b_{t-1}$  in Equation (8) is usually called the “snowball” effect or the interest rate growth rate differential; it shows that the debt ratio tends to rise (decline) if the GDP growth rate is lower (higher) than the interest rate paid on government debt (ECB, 2011), and consequently it captures the joint impact of interest payments and real GDP growth on the outstanding stock of debt relative to GDP. It is



therefore possible for the debt/GDP ratio to be stable even in the presence of an increasing primary deficit and/or other factors if this offset by the snowball effect.

If the other financial transactions term is zero ( $sf_t$ ), the debt ratio will stabilise when  $(r_t - g_t)b_{t-1} \approx pb_t$ . Thus, if  $r_t > g_t$  (which has often been the case in the past) a primary surplus is needed to prevent the debt burden from increasing and an ever-larger surplus is needed to reduce it (Bouabdallah et al., 2017). However, in recent times  $r_t < g_t$ , which has led to governments to being less concerned about debt increases since in such a scenario solvency is still achieved (Blanchard, 2019). In this situation ( $r_t < g_t$ ), a stable solution for (7) might be  $b_t = \frac{1+g_t}{r_t-g_t}pb_t$  (Willems & Zettelmeyer, 2022), implying that, for  $b_t$  to stabilise at a constant debt ratio, the primary balance must be in deficit and generating a “free lunch”. Whilst previous papers such as Martin-Valmayor et al. (2024) only analysed the primary balance-snowball relationship by carrying out Granger (1969) causality tests, the present one sheds more light on the issues of interest by applying fractional integration methods to assess debt sustainability.

#### 4. Empirical analysis

The data source for all series is the Historical Public Debt Database (1800-2022) from the public IMF e-library (IMF, 2024b). The frequency is annual, and the sample period starts in 1831 for the UK, in 1862 for Italy and in 1881 for France, Germany, and the US (these being the five countries being analysed), and it ends in all cases in 2022. The choice of the sample period is driven in each case by data availability since it is well known that long spans of data are ideally suited to the estimation of long-memory models.

The IMF series used for the analysis are gross public debt-to-GDP ratio at time  $t$ , as a percentage of GDP ( $b_t$ ); the real long-term government bond yield ( $r_t$ ); the real GDP growth rate ( $g_t$ ); and the government primary balance-to-GDP ratio ( $pb_t$ ). The snowball terms were calculated using the expression  $\frac{r_t - g_t}{1 + g_t}b_{t-1}$  appearing in Equation (7), while the stock-flow adjustment-to-GDP ratio was computed, from Equation (8), as  $sf_t = b_t - \frac{1+r_t}{1+g_t}b_{t-1} + pb_t$ . Figure 1 displays the debt/GDP ratio for the selected countries. Different trajectories can be observed in the earlier part of the sample, while similar patterns are noticeable in the following period, including the impact of the two world wars which led to a sharp increase of this ratio in all countries. Table 1 reports descriptive statistics for all the series. It can be seen that the snowball and other stock flow components are the most volatile, while bond yields and the debt/GDP ratios exhibit the lowest volatility. There is also evidence of positive or negative skewness in most cases, the distribution being symmetrical only in the case of GDP growth in the US and Italy, and of the debt/GDP ratio in the UK and Italy. Further, most series exhibit excess kurtosis ( $k > 3$ ), the only exceptions being the primary balance in France and the debt/GDP ratio in the UK, Italy and Germany.

Figure 2 displays for each country the evolution over time of each of the series entering the budget constraint. The evidence in this figure supplements that already provided by Figure 1 in the Introduction. It can be seen that war periods are characterised by high volatility and major stock-flow adjustments. This is consistent with the evidence provided by Martin-Valmayor et al. (2024) that the debt/GDP series exhibit structural breaks, especially during such war periods, such as the ones experienced between 1910 and 1950:

For each of the series we specify a fractional integration model that allows for deterministic terms such as a constant and/or a linear time trend. In particular, the estimated model is the following one:

$$y_t = \gamma + \delta t + x_t, (1 - L)^d x_t = u_t, \quad (9)$$

where  $y_t$  is the observed time series,  $\gamma$  and  $\delta$  are unknown parameters (specifically, the intercept and the coefficient on a linear time trend), and  $d$  is a real number corresponding to the order of integration of the series. Positive values imply that the series exhibits long memory, past observations having a greater

**Table 1.** Descriptive statistics for the terms of the budget constraint equation in each country under investigation.

FRANCE	Bond yield ( $r_t$ )	GDP growth ( $g_t$ )	Snowball $\frac{r_t - g_t}{1 + g_t} b_{t-1}$	Primary balance( $pb_t$ )	Other stock flow ( $sf_t$ )	DEBT/GDP ( $b_t$ )
Mean	1.46	2.14	-0.14	0.19	-0.41	79.97
Median	1.92	2.43	-0.17	0.53	-0.46	79.73
Maximum	20.32	52.05	46.81	5.67	34.09	237.04
Minimum	-34.91	-21.11	-63.76	-8.25	-48.03	14.39
Std.Dev.	7.55	6.59	13.59	3.07	11.38	48.93
Std.Dev/Mean	5.16	3.07	-97.30	16.44	-27.54	0.61
Skewness	-1.01	1.53	-0.91	-0.76	-0.73	0.76
Kurtosis	7.17	19.88	10.13	2.95	7.83	3.47
First obs.	1881	1821	1881	1880	1881	1880
No. of obs.	142	202	142	143	142	143
GERMANY	Bond yield ( $r_t$ )	GDP growth ( $g_t$ )	Snowball $\frac{r_t - g_t}{1 + g_t} b_{t-1}$	Primary balance( $pb_t$ )	Other stock flow ( $sf_t$ )	DEBT/GDP ( $b_t$ )
Mean	2.96	2.54	0.26	1.00	-1.02	36.43
Median	3.16	2.82	-0.11	1.26	-1.12	35.83
Maximum	21.23	18.90	26.34	4.34	26.10	81.99
Minimum	-7.51	-52.59	-6.28	-5.98	-10.57	9.73
Std.Dev.	3.43	6.88	2.95	1.54	3.38	17.74
Std.Dev/Mean	1.16	2.71	11.23	1.55	-3.30	0.49
Skewness	1.27	-3.69	5.26	-1.28	3.79	0.76
Kurtosis	9.01	28.93	45.16	6.13	32.12	2.72
First obs.	1881	1851	1881	1880	1881	1880
No. of obs.	142	172	142	143	142	143
ITALY	Bond yield ( $r_t$ )	GDP growth ( $g_t$ )	Snowball $\frac{r_t - g_t}{1 + g_t} b_{t-1}$	Primary balance( $pb_t$ )	Other stock flow ( $sf_t$ )	DEBT/GDP ( $b_t$ )
Mean	1.87	2.29	0.42	-0.92	0.43	87.87
Median	3.00	2.33	0.81	0.95	-0.37	91.76
Maximum	24.01	31.00	23.07	6.55	54.12	159.72
Minimum	-36.23	-21.71	-32.13	-34.05	-49.07	24.21
Std.Dev.	7.40	5.49	7.77	6.65	9.32	33.76
Std.Dev/Mean	3.95	2.40	18.69	-7.24	21.47	0.38
Skewness	-2.02	-0.22	-1.05	-2.42	0.69	-0.28
Kurtosis	11.00	10.57	7.04	9.81	18.38	2.24
First obs.	1861	1861	1861	1862	1861	1861
Observations	161	161	161	161	162	162
UK	Bond yield ( $r_t$ )	GDP growth ( $g_t$ )	Snowball $\frac{r_t - g_t}{1 + g_t} b_{t-1}$	Primary balance( $pb_t$ )	Other stock flow ( $sf_t$ )	DEBT/GDP ( $b_t$ )
Mean	2.84	1.98	-0.09	0.59	-0.61	111.61
Median	2.57	2.22	0.02	2.21	0.11	97.85
Maximum	41.45	10.01	12.90	11.39	22.07	269.80
Minimum	-16.41	-11.03	-13.02	-47.67	-172.30	27.27
Std.Dev.	6.56	3.04	2.42	8.57	13.43	65.84
Std.Dev/Mean	2.31	1.54	-25.48	14.50	-22.17	0.59
Skewness	1.06	-1.04	-0.85	-3.05	-10.87	0.50
Kurtosis	9.53	6.26	14.17	14.40	139.86	2.08
First obs.	1801	1831	1831	1830	1830	1800
No. of obs.	222	192	192	193	193	223
USA	Bond yield ( $r_t$ )	GDP growth ( $g_t$ )	Snowball $\frac{r_t - g_t}{1 + g_t} b_{t-1}$	Primary balance( $pb_t$ )	Other stock flow ( $sf_t$ )	DEBT/GDP ( $b_t$ )
Mean	2.12	3.65	-0.73	-0.58	-0.35	33.75
Median	2.71	3.49	-0.30	0.21	0.00	22.95
Maximum	15.84	19.73	7.08	6.56	6.74	133.50
Minimum	-13.38	-13.17	-13.63	-26.46	-10.63	0.00
Std.Dev.	4.34	4.91	2.98	3.88	1.89	32.28
Std.Dev/Mean	2.05	1.35	-4.05	-6.73	-5.38	0.96
Skewness	-0.74	0.11	-1.54	-3.37	-1.94	1.01
Kurtosis	6.40	4.96	8.25	18.06	12.38	3.21
First obs.	1,881	1,801	1,881	1,800	1,881	1,800
No. of obs.	142	222	142	223	142	223

Note: The data source for all series is the Historical Public Debt Database (1800-2022) from the public IMF e-library (IMF, 2024b). The frequency is annual, and the sample period starts in 1831 for the UK, in 1862 for Italy and in 1881 for France, Germany, and the US.

impact the greater the value of  $d$  is, with mean reversion occurring if this parameter is below 1. The error term  $u_t$  is assumed to follow the exponential spectral model of Bloomfield (1973) that approximates AutoRegressive (AR) structures in a non-parametric way. This model is non-parametric because it does not have an explicit specification and it is only described in terms of its spectral density function,

whoseloggd form is very close to the one produced by the log-spectrum of an autoregressive process. Moreover, it produces autocorrelations decaying exponentially but, unlike the AR model, it is stationary across all range of its values.

The estimates for the sustainability condition given by (8) are reported in Table 2. As istandard in the unit roots literature (Bhargava, 1986; Schmidt & Phillips, 1992), we consider three possible model specifications, namely with i) no regressors, ii) an intercept only, and iii) an intercept as well as a linear

**Table 2.** Estimates of the differencing parameter  $d$  for the whole sample from the selected specifications.

FRANCE [1881–2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Bond Yield ( $r$ )	$I(d)$	0.37 (0.25, 0.53)	–	–
GDP Growth ( $g$ )	$I(d)$	0.22 (0.11, 0.38)	6.031 (3.15)	–0.027 (–1.88)
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	$I(0)$	–0.02 (–0.21, 0.26)	0.989 (105.18)	–
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	$I(d)$	0.29 (0.10, 0.51)	–	–
Primary Balance	$I(d)$	0.68 (0.49, 0.93)	3.815 (3.31)	–0.046 (–1.78)
Others	$I(0)$	0.07 (–0.16, 0.39)	–	–
ITALY [1862–2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Bond Yield ( $r$ )	$I(d)$	0.55 (0.42, 0.71)	–	–
GDP Growth ( $g$ )	$I(d)$	0.14 (0.04, 0.29)	2.173 (2.67)	–
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	$I(0)$	0.09 (–0.06, 0.90)	1.001 (89.89)	–
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	$I(d)$	0.22 (0.13, 0.33)	–	–
Primary Balance	$I(1)$	0.67 (0.41, 1.00)	–	–
Others	$I(0)$	0.07 (–0.07, 0.27)	–	–
GERMANY [1881–2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Bond Yield	$I(d)$	0.65 (0.50, 0.85)	–	–
GDP Growth	$I(d)$	0.17 (0.01, 0.39)	–	–
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	$I(d < 0)$	–0.43 (–0.60, –0.17)	1.012 (90.42)	–
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	$I(0)$	0.13 (–0.03, 0.36)	–	–
Primary Balance	$I(0)$	0.16 (–0.05, 0.49)	1.852 (5.12)	–0.012 (–2.81)
Others	$I(0)$	0.06 (–0.10, 0.27)	–1.061 (–3.14)	–
UNITED KINGDOM [1831–2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Bond Yield ( $r$ )	$I(d)$	0.22 (0.11, 0.38)	6.031 (3.15)	–0.027 (–1.88)
GDP Growth ( $g$ )	$I(0)$	0.10 (–0.05, 0.31)	–	–
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	$I(d)$	0.25 (0.16, 0.35)	–0.000 (–0.29)	–0.000 (–0.11)
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	$I(d)$	0.26 (0.17, 0.34)	–0.025 (–0.72)	–
Primary Balance	$I(d)$	0.41 (0.11, 0.91)	5.232 (2.48)	–0.042 (–2.12)
Others	$I(d)$	0.26 (0.24, 0.28)	–	–
UNITED STATES [1881–2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Bond Yield	$I(d)$	0.60 (0.42, 0.86)	–	–
GDP Growth	$I(0)$	0.00 (–0.10, 0.15)	3.649 (11.709)	–
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	$I(0)$	–0.06 (–0.25, 0.23)	0.991 (214.56)	–
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	$I(d)$	0.22 (0.10, 0.39)	–	–
Primary Balance	$I(0)$	0.01 (–0.22, 0.34)	0.129 (2.25)	–0.017 (–2.82)
Others	$I(0)$	0.01 (–0.16, 0.25)	–0.350 (–2.32)	–

Note: The values in the first column refer to the estimates of  $d$  (and their 95% confidence bands) for the selected specification based on the statistical (in)significance of the deterministic terms with the minimum standard error estimation. The values in parenthesis in the last two columns refer to the  $t$ -values of the intercept and the linear time trend. Statistical evidence of stationary is found if all values in the confidence intervals are strictly smaller than 0.5.

time trend. The model displayed in Table 2 is the one selected on the basis of the statistical significance of the estimated coefficients, with the minimum standard error.<sup>4</sup>

When considering the results for the individual components of the budget constraint relationship (8), in the case of the snowball effect  $\left(\frac{r_t - g_t}{1 + g_t} b_{t-1}\right)$  we find I(d) stationary behaviour ( $0 < d < 0.5$ ) in all cases except in Germany where there is evidence of I(0) behaviour. Concerning the individual components of the snowball effect, all bond yield series ( $r_t$ ) exhibit similar I(d) behaviour, with larger values of  $d$  being estimated for the US, Italy and Germany ( $0.5 < d < 1$ ). We find similar patterns ( $0.5 < d < 1$ ) in the GDP growth series ( $g_t$ ), except for the US and the UK, where the stationarity I(0) hypothesis cannot be rejected. Finally, the differential between these two series, ( $r_t - g_t$ ), also appears to be I(0) stationary in all cases, except for Germany where anti-persistence ( $d < 0$ ) occurs, with the whole confidence interval being below 0.

Similarly, other financial components ( $sf_t$ ) are also characterised by I(0) stationarity, except in the case of the UK, for which there is evidence of I(d) stationary behaviour ( $0 < d < 0.5$ ). By contrast, in the case of the primary balance ( $pb_t$ ) different patterns emerge. Specifically, for Germany and the US we find evidence of I(0) behaviour with low persistence and mean reversion, while for the other countries the estimated values of  $d$  are much higher ( $0.5 < d < 1$ ), with  $d = 1$  being within the confidence interval in the case of Italy.

Given that the other stock-flow adjustments  $sf_t$  are stationary and  $\frac{g_t}{1 + g_t} \approx 0$  it follows that  $\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - pb_t + sf_t$  will be stationary if  $r_t b_{t-1} - pb_t$ , namely if the difference between interest payments and the primary balance, is I(0) stationary. As noted in Martin-Valmayor et al. (2024) the debt/GDP series often exhibit structural breaks, especially during war periods (France, 1901, 1922; Italy 1920; the UK, 1915, 1947; the US, 1914, 1948). As a check we have carried out the Bai and Perron (2003) tests for endogenous breaks using our data; the results (not reported for reasons of space) confirm the presence of breaks in Italy (1920), France (1922), the UK (1914, 1947) and the US (1913, 1947, 1982). whilst, as in Martin-Valmayor et al. (2024), we do not find structural breaks in Germany.

Therefore, we also re-estimated the model for both the full sample and a more recent period starting in 1950 (after World War II), during which no structural breaks appear to have occurred (except for the US in 1982). Table 3 reports the estimates of the differencing parameter  $d$  (and the 95% confidence interval) for the three series under investigation, i.e., bond yields, the primary balance and the debt sustainability condition (as in Trehan & Walsh, 1991) using the three specifications mentioned above.

In terms of the Trehan & Walsh (1991) sustainability condition, it can be seen that mean reversion ( $d < 1$ ) occurs in all cases (which would imply solvency), whilst I(0) behaviour is not found in any case. The full-sample results show I(d) stationary behaviour ( $0 < d < 0.5$ ) for France and Italy, with much higher values of  $d$  ( $0.5 < d < 1$ ) for the US and the UK, and especially for Germany (0.70). If the most recent period is analysed (with data starting in 1950), a more homogeneous I(d) behaviour is found, with  $0.5 < d < 1$  in all cases (see Figure 3), which implies mean reversion for the differential  $r_t b_{t-1} - pb_t$ .

To analyse the speed of adjustment we have also estimated the correlogram (Figure 4), whose confidence bands are given by  $\pm 1.96/\sqrt{n}$ . As can be seen from the left-hand side panel, in the case of the full sample there is one significant lag for Italy the US and France, two for the UK, and eight for Germany, which exhibits a slower decay. As for the shorter sample, there is one significant lag for the UK and the US, four for France, six for Italy and seven for Germany. This evidence suggests that the decay process is faster in the UK and the US, and slower in the other countries, especially in samples from 1950 onwards, which are characterised by longer memory. The faster transition from one state to another in the case of the Anglo-Saxon economies can be plausibly attributed to the fact that they feature fewer rigidities in terms of institutional and regulatory factors; consequently, policy measures aimed at ensuring debt sustainability have a more immediate impact than in the other countries.

<sup>4</sup>Since the starting date is not the same for all countries as a robustness check we have also re-estimated the models starting in 1881 in all cases. The results (not reported to save space but available upon request) are consistent with those discussed in the main text in terms of the order of integration of the series.

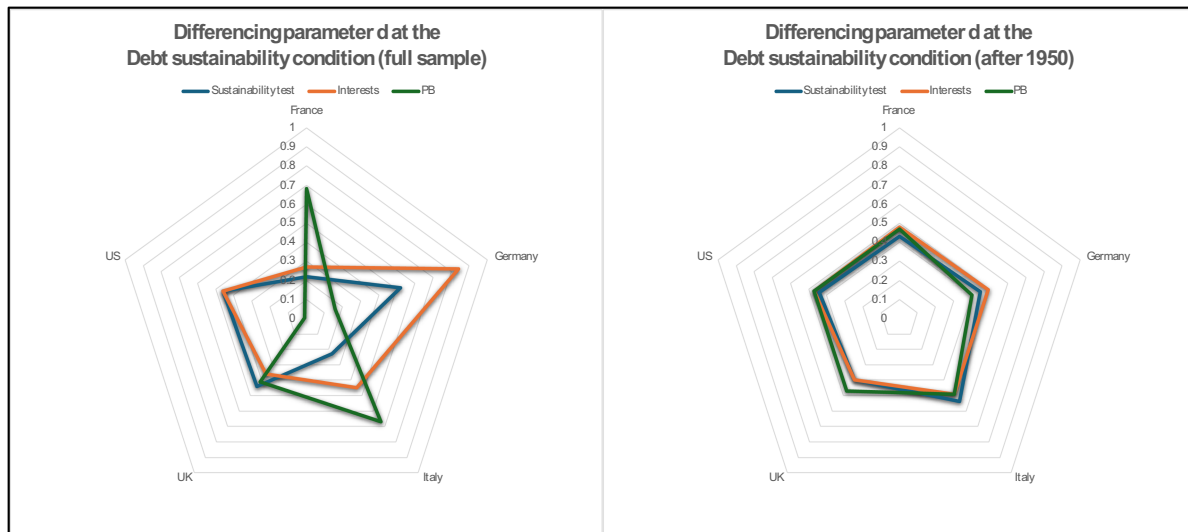
**Table 3.** Estimation of the differencing parameter  $d$  for the debt sustainability condition (Trehan & Walsh, 1991), from the start of the sample and from 1950.

Debt sustainability condition ( $r_t b_{t-1} - pb_t$ ) (Trehan and Walsh, 1991)–full sample				
FRANCE (1881–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	<b>0.22 (0.04, 0.43)</b>	0.22 (0.04, 0.43)	0.22 (0.03, 0.43)	I(d)
Primary Balance:	<b>0.27 (0.20, 0.34)</b>	0.27 (0.20, 0.34)	0.27 (0.19, 0.34)	I(d)
	0.67 (0.50, 0.90)	0.66 (0.48, 0.93)	<b>0.68 (0.49, 0.93)</b>	I(d)
GERMANY (1881–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	<b>0.52 (0.39, 0.70)</b>	0.51 (0.39, 0.70)	0.51 (0.38, 0.70)	I(d)
Primary Balance:	<b>0.84 (0.74, 0.93)</b>	0.84 (0.73, 0.92)	0.71 (0.81, 0.91)	I(d)
	0.24 (0.03, 0.50)	0.19 (0.03, 0.46)	<b>0.16 (-0.05, 0.49)</b>	I(0)
ITALY (1862–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	0.22 (0.11, 0.39)	<b>0.23 (0.11, 0.40)</b>	0.23 (0.11, 0.40)	I(d)
Primary Balance:	<b>0.45 (0.40, 0.50)</b>	0.45 (0.39, 0.50)	0.45 (0.38, 0.51)	I(d)
	<b>0.67 (0.41, 1.00)</b>	0.66 (0.41, 0.99)	0.66 (0.41, 0.99)	I(1)
UK (1831–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	<b>0.44 (0.37, 0.51)</b>	0.44 (0.37, 0.51)	0.44 (0.36, 0.51)	I(d)
Primary Balance:	<b>0.36 (0.30, 0.42)</b>	0.34 (0.28, 0.41)	0.32 (0.26, 0.39)	I(d)
	0.44 (0.15, 0.92)	0.43 (0.15, 0.91)	<b>0.41 (0.11, 0.91)</b>	I(d)
US (1881–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	<b>0.46 (0.40, 0.52)</b>	0.46 (0.39, 0.54)	0.46 (0.38, 0.53)	I(d)
Primary Balance:	<b>0.46 (0.40, 0.52)</b>	0.46 (0.38, 0.53)	0.46 (0.38, 0.53)	I(d)
	0.01 (-0.20, 0.35)	0.02 (-0.22, 0.33)	<b>0.01 (-0.22, 0.34)</b>	I(0)
Debt sustainability condition ( $r_t b_{t-1} - pb_t$ ) (Trehan and Walsh, 1991) – after 1950				
FRANCE (1950–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	<b>0.43 (0.29, 0.69)</b>	0.47 (0.31, 0.76)	0.46 (0.26, 0.76)	I(d)
Primary Balance:	<b>0.48 (0.41, 0.56)</b>	0.48 (0.41, 0.56)	0.48 (0.40, 0.56)	I(d)
	<b>0.47 (0.38, 0.57)</b>	0.47 (0.33, 0.60)	0.46 (0.33, 0.60)	I(d)
GERMANY (1950–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	<b>0.45 (0.29, 0.71)</b>	0.45 (0.29, 0.71)	0.44 (0.30, 0.72)	I(d)
Primary Balance:	0.49 (0.43, 0.54)	0.49 (0.43, 0.54)	<b>0.49 (0.44, 0.54)</b>	I(d)
	<b>0.40 (0.21, 0.59)</b>	0.39 (0.19, 0.59)	0.38 (0.18, 0.58)	I(d)
ITALY (1950–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	0.51 (0.37, 0.71)	<b>0.54 (0.39, 0.81)</b>	0.55 (0.40, 0.82)	I(d)
Primary Balance:	<b>0.49 (0.44, 0.54)</b>	0.49 (0.44, 0.54)	0.49 (0.44, 0.54)	I(d)
	<b>0.49 (0.44, 0.53)</b>	0.49 (0.44, 0.54)	0.49 (0.44, 0.54)	I(d)
UK (1950–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	<b>0.40 (0.30, 0.50)</b>	0.40 (0.25, 0.55)	0.34 (0.13, 0.55)	I(d)
Primary Balance:	<b>0.40 (0.30, 0.50)</b>	0.40 (0.26, 0.54)	0.40 (0.26, 0.54)	I(d)
	<b>0.47 (0.32, 0.62)</b>	0.47 (0.32, 0.62)	0.44 (0.26, 0.62)	I(d)
US (1950–2022)	No terms	An intercept	An intercept and a linear time trend	
Interests:	<b>0.44 (0.31, 0.57)</b>	0.43 (0.29, 0.56)	0.38 (0.15, 0.61)	I(d)
Primary Balance:	0.47 (0.38, 0.56)	0.47 (0.38, 0.56)	<b>0.47 (0.38, 0.56)</b>	I(d)
	<b>0.47 (0.38, 0.57)</b>	0.47 (0.36, 0.58)	0.45 (0.31, 0.58)	I(d)

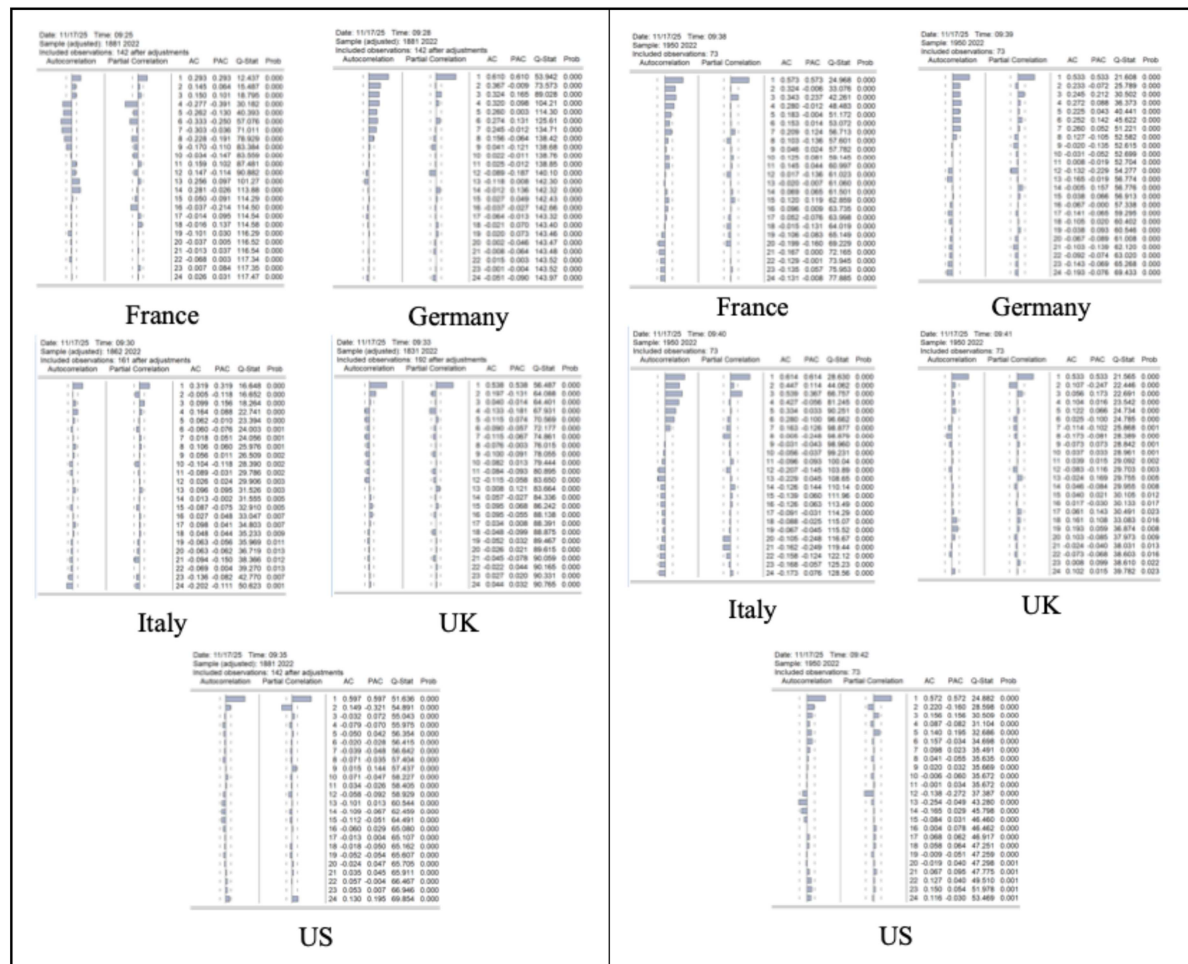
Note: The values in bold refer to the estimates of  $d$  (and their 95% confidence bands) for the selected specification based on the statistical (in) significance of the deterministic terms. Statistical evidence of stationary is found if all values in the confidence intervals are strictly smaller than 0.5.

It is also noteworthy that the primary balance has worsened in all countries in recent times. Table 4 shows the mean value of the budget constraint relationship and their components in both periods. After 1950, Germany is the only country which kept a significant primary surplus, whilst over time the UK moved from a surplus to a balanced budget. The other countries (France, Italy and the US) are characterised by persistent deficits.

Even though the Trehan & Walsh (1991) sustainability test indicates that the difference between interest payments and primary surplus is an I(d) series with slow mean reversion, the effects of those persistent deficits (given the budget constraint relationship  $\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - pb_t$ ) would require an average GDP



**Figure 3.** Estimation of the differencing parameter  $d$  for the debt sustainability condition (Trehan & Walsh, 1991), from the start of the sample and from 1950. Note: The figures concern the differential parameter results shown in Table 3.



**Figure 4.** Trehan and Walsh sustainability condition correlogram comparison: full sample (left), and short sample after 1950 (right). Note: The significance level is based on the standard errors, that is  $\pm \frac{1.96}{\sqrt{n}}$ . Therefore, for the full sample (left-hand side panel) it is 0.164 for France and Germany (143 observations); 0.154 for Italy (161 observations); 0.141 for the UK (192 observations). For the sample starting in 1950 (right-hand sidepanel) it is 0.229 for all countries (73 observations-).



**Table 4.** Average values of the sustainability tests in the whole data series and after 1950.

	FRANCE			GERMANY			ITALY		
	Interests (rB) (1)	Primary Bal. (2)	Sustain. Test (1-2)	Interests (rB) (1)	Primary Bal. (2)	Sustain. Test (1-2)	Interests (rB) (1)	Primary Bal. (2)	Sustain. Test (1-2)
All Time series:									
Mean	0.943	0.187	0.784	0.878	0.996	-0.119	1.672	-0.918	2.590
Std Dev.	10.747	3.073	10.351	1.121	1.540	1.908	6.888	6.646	5.840
Since 1950:									
Mean	0.628	-0.773	1.400	0.948	0.685	0.263	1.835	-0.572	2.407
Std Dev.	1.483	2.021	2.201	1.210	1.891	2.192	2.945	3.006	2.856
	UNITED KINGDOM			UNITED STATES					
	Interests (rB) (1)	Primary Bal. (2)	Sustain. Test (1-2)	Interests (rB) (1)	Primary Bal. (2)	Sustain. Test (1-2)			
All Time series:									
Mean	0.553	0.591	-0.040	0.606	-0.576	1.720			
Std Dev.	1.743	8.569	8.391	2.156	3.875	4.811			
Since 1950:									
Mean	0.955	0.078	0.877	1.094	-0.886	1.980			
Std Dev.	2.214	3.407	3.944	1.722	2.985	3.294			

**Table 5.** Selected estimates of the differencing parameter  $d$  for the debt budget constraint relationship

$$\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - p b_t$$

Budget constraint relationship $\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - p b_t$ condition. Full sample				
Series		No terms	An intercept	An intercept and a linear time trend
France (1881–2022)	I(d)	0.26 (0.19, 0.32)	—	—
Germany (1881–2022)	I(d)	0.24 (0.18, 0.30)	—	—
Italy (1862–2022)	I(d)	0.17 (0.11, 0.22)	—	—
UK (1831–2022)	I(d)	0.42 (0.34, 0.51)	-1.92 (0.17)	0.011 (0.183)
US (1881–2022)	I(d)	0.39 (0.32, 0.47)	—	—
Budget constraint relationship $\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - p b_t$ condition. After 1950				
Series		No terms	An intercept	An intercept and a linear time trend
France (1950–2022)	I(d)	0.20 (0.06, 0.34)	—	—
Germany (1950–2022)	I(d)	0.35 (0.13, 0.57)	—	—
Italy (1950–2022)	I(d)	0.20 (0.07, 0.33)	—	—
UK (1950–2022)	I(d)	0.33 (0.20, 0.47)	—	—
US (1950–2022)	I(d)	0.41 (0.33, 0.48)	—	—

Note: The values in parenthesis in the last two columns refer to the t-values of the intercept and the linear time trend. Statistical evidence of stationarity is found if all values in the confidence intervals are strictly smaller than 0.5.

growth rate ( $g_t$ ) of 1.4% in France, 1.98% in the US and 2.59% in Italy to balance the debt/GDP ratio on a sustainable path in the long term. This evidence of mean reversion implies that debt is on a sustainable path in all cases, possibly as a result of the positive effects of a relatively high GDP growth rate.

Some studies argue that the persistence of the primary deficit is a consequence of the way governments tend to respond to external shocks (Antonini et al., 2013): they will typically adopt expansionary fiscal policies which will generate deficits, and such policies will not be subsequently reversed, since this would require unpopular measures that could affect the outcome of future elections; thus governments do not make symmetrical corrections to generate primary surpluses (Beqiraj et al., 2018), especially if there is no pressure from bond holders or international organisations (Martin-Valmayor et al., 2024). This raises questions about debt sustainability, especially in the presence of relatively high yields as in the current economic environment, regardless of the results of the Trehan & Walsh (1991), with Japan being a special case, since it has a very large debt/GDP ratio (2.5) combined with a negative interest rate policy. As pointed out by Willems and Zettelmeyer (2022), the degree of credibility of the central bank is crucial in this context. Whenever government borrowing costs are below the growth rate of the economy ( $r < g$ ) debt sustainability does not require future primary surpluses and there is an apparent “free lunch”.



To obtain additional evidence on the possible role played by the GDP growth in terms of debt sustainability, we carry out additional tests based on the Trehan & Walsh (1991)  $(r_t B_{t-1} - PB_t)$  condition but incorporating the growth component. Specifically, we examine the degree of persistence of the budget constraint relationship,  $\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - pb_t$ , without including  $sf_t$ , which is assumed to be an  $I(0)$  series. These results are displayed in Table 5; in general, the estimates of the differencing parameter  $d$  are now lower, more in line with the stationary behaviour required by Trehan & Walsh (1991). In particular, for the full sample we find  $I(d)$  stationary behaviour ( $0 < d < 0.5$ ) in all countries except the UK (0.34, 0.51) and similarly, for the period after 1950 we find a  $I(d)$  stationary behaviour ( $0 < d < 0.5$ ) in all countries except for Germany (0.13, 0.57). Therefore, this evidence of lower values of  $d$  in the whole budget constraint relationship  $\frac{r_t - g_t}{1 + g_t} b_{t-1} - pb_t$ , implies that the debt is on a sustainable path in all cases, but possibly as a result of the positive effects of a relatively high GDP growth rate.

## 5. Conclusions

This paper analyses the persistence and mean reversion properties of sovereign debt and its individual components appearing in the budget constraint equation for the US and the largest European economies. Specifically, fractional integration methods are applied to long runs of annual data starting in 1831 for the UK and the US, in 1862 for Italy and in 1881 for France and Germany, and ending in all cases in 2022. The chosen approach is more general and flexible than the standard one based on the classical  $I(0)$  versus  $I(1)$  dichotomy, and thus it encompasses a wide range of stochastic processes, the unit root case being one of them.

This paper builds on the seminal work by Trehan & Walsh (1991) to evaluate the debt sustainability of the US and the main European economies. For this purpose, we analyse stationarity of the general budget identity  $(r_t B_{t-1} - PB_t)$  and their specific components. We then refine our analysis by incorporating the growth component. This allows us to examine the degree of persistence in the budget constraint relationship  $\Delta b_t = \frac{r_t - g_t}{1 + g_t} b_{t-1} - pb_t$  along with their specific components. This is under the assumption that the stock flow adjustment is stationary  $I(0)$ , which was found in France, Germany, Italy and the US. From a methodological perspective, we make a two-fold contribution to the existing literature. First, as already stated, we employ a fractional integration approach, which is more general than the unit root testing carried out in previous studies. Second, in contrast to the post-WWII focus of Trehan & Walsh (1991) and Hamilton & Flavin (1986), we examine long time spans from the 19th century to the present for the US and the main European economies.

The empirical results for the solvency test based on the differential between interest payments and primary deficits  $(r_t b - pb_t)$  provide evidence of an  $I(d)$  pattern with mean reversion ( $d < 1$ ) for all countries, which implies solvency. However, mean reversion occurs, with some values being above 0.5, namely the US ( $0.40 < d < 0.52$ ), the UK ( $0.37 < d < 0.51$ ), and especially Germany ( $0.30 < d < 0.70$ ) where some values are in the non-stationary range and shocks decay hyperbolically to zero. With regard to the post-WWII results, it is clear that there is more homogeneous  $I(d)$  behaviour, with values of  $d$  in all cases being within the range ( $0.5 < d < 1$ ). This still implies mean-reverting behaviour, but at a slower pace compared to the Trehan & Walsh (1991) finding of stationarity of the interest payments and primary deficit differential on the basis of unit root tests. These authors characterised those series as  $I(0)$  when in fact they are  $I(d)$  with  $d$ -values in the interval between (0.5, 1).

This sustainability result was confirmed by further persistence tests based on the budget constraint but incorporating the growth component  $\left( \frac{r_t - g_t}{1 + g_t} b_{t-1} - pb_t \right)$ , although the estimates of  $d$  are lower, which implies slower adjustment towards the long-run equilibrium. Nevertheless, debt appears to be on a sustainable path in all cases, which confirms the important role of a relatively high GDP growth rate in terms of solvency. However, we have found evidence that after 1950, the primary balance has worsened in all countries. Therefore, in order to balance the debt/GDP ratio on a sustainable path in the long term, the

impact of these persistent deficits would require a relatively high GDP growth rate. As Antonini et al. (2013) observe, the reason for these deficits lies in the governments' responses to external shocks, if expansionary policies are not reversed due to the unpopularity of the required measures.

This fact may give rise to questions regarding the issue of debt sustainability, particularly in the context of relatively high yields. Therefore, as Willems and Zettelmeyer (2022) have highlighted, it is particularly important that central banks have the credibility to sustain confidence, especially during periods of slow GDP growth and high yields. However, empirical evidence suggests that all countries under analysis show  $I(d)$  behaviour in the budget constraint relationship. This implies that the debt is on a sustainable path in all cases, possibly as a result of the positive effects of a relatively high GDP growth rate.

Note that the results based on Robinson (1994) tests reported in this paper appear to be robust to the use of alternative  $I(d)$  approaches such as semiparametric log periodogram methods (Geweke & Porter-Hudak, 1983; Robinson, 1995a) or local Whittle approaches (Robinson, 1995b; Shimotsu & Phillips, 2006) - although the results obtained with these methods are highly sensitive to the bandwidth parameters, they are qualitatively the same in terms of the  $I(0)$ -stationary  $I(d)$ -nonstationary  $I(d)$ -unit roots classification.

It should also be noted that our analysis has some limitations, which could be addressed by future research. Notably, the presence of structural breaks may indicate possible non-linearities affecting the current estimations of  $d$  with linear structures. Therefore, to capture them, a non-linear model with Chebyshev polynomials in time might be appropriate (see Bierens (1997), Hamming (1973), Smyth (1998), and Tomasevic et al. (2009) for details). In this context, the estimation of  $d$  might be carried out by means of a likelihood function in the frequency domain, using the Lagrange multiplier approach developed by Cuestas and Gil-Alana (2016) for non-linear structures.

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## Author contributions

All authors contributed to the study conception, analysis and design. Material preparation and data collection were performed by prof. Martin-Valmayor, prof. Carmona and prof. Gil-Alana. The first draft of the manuscript was written by prof. prof. Martin-Valmayor and prof. Carmona and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript. Funding and project management were respectively secured and conducted by prof. Martin-Valmayor and prof. Gil-Alana.





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No potential conflict of interest was reported by the author(s).

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## Data availability statement

The data that support the findings of this study are available upon reasonable request.  
Clinical trial number: not applicable. Study is no referred to a clinical trial.

## Compliance with ethical standards

All authors acknowledges that accepted principles of ethical and professional conduct have been followed, with no potential conflicts of interest (financial or non-financial). All authors inform that this research has not involved human participants or animals.

## Consent to participate

Authors acknowledge that there were not individual participants in the study, and therefore no Informed consent is needed.

## Consent to publish

Authors affirm that there is not human research participants in the study and therefore there is no need of informed consent for publication of the images in Figure 1.

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## Appendix

**Table A1.** IMF definition of advanced economies.

Andorra	Hong Kong SAR	Norway
Australia	Iceland	Portugal
Austria	Ireland	Puerto Rico
Belgium	Israel	San Marino
Canada	Italy	Singapore
Croatia	Japan	Slovak Republic
Cyprus	Korea	Slovenia
Czech Republic	Latvia	Spain
Denmark	Lithuania	Sweden
Estonia	Luxembourg	Switzerland
Finland	Macao SAR	Taiwan Province of China
France	Malta	United Kingdom
Germany	The Netherlands	United States
Greece	New Zealand	

**Table B1.** IMF definition of emerging and developing economies.

Afghanistan	Georgia	Paraguay
Albania	Ghana	Peru
Algeria	Grenada	Philippines
Angola	Guatemala	Poland
Antigua and Barbuda	Guinea	Qatar
Argentina	Guinea-Bissau	Romania
Armenia	Guyana	Russia
Aruba	Haiti	Rwanda
Azerbaijan	Honduras	Samoa
The Bahamas	Hungary	São Tomé and Príncipe
Bahrain	India	Saudi Arabia
Bangladesh	Indonesia	Senegal
Barbados	Iran	Serbia
Belarus	Iraq	Seychelles
Belize	Jamaica	Sierra Leone
Benin	Jordan	Solomon Islands
Bhutan	Kazakhstan	Somalia
Bolivia	Kenya	South Africa
Bosnia and Herzegovina	Kiribati	South Sudan
Botswana	Kosovo	Sri Lanka
Brazil	Kuwait	St. Kitts and Nevis
Brunei Darussalam	Kyrgyz Republic	St. Lucia
Bulgaria	Lao P.D.R.	St. Vincent and the Grenadines
Burkina Faso	Lebanon	Sudan
Burundi	Lesotho	Suriname
Cabo Verde	Liberia	Syria
Cambodia	Libya	Tajikistan
Cameroon	Madagascar	Tanzania
Central African Republic	Malawi	Thailand
Chad	Malaysia	Timor-Leste
Chile	Maldives	Togo
China	Mali	Tonga
Colombia	Marshall Islands	Trinidad and Tobago
Comoros	Mauritania	Tunisia
Democratic Republic of the Congo	Mauritius	Türkiye
Republic of Congo	Mexico	Turkmenistan
Costa Rica	Micronesia	Tuvalu
Côte d'Ivoire	Moldova	Uganda
Djibouti	Mongolia	Ukraine
Dominica	Montenegro	United Arab Emirates
Dominican Republic	Morocco	Uruguay
Ecuador	Mozambique	Uzbekistan
Egypt	Myanmar	Vanuatu
El Salvador	Namibia	Venezuela
Equatorial Guinea	Nauru	Vietnam
Eritrea	Nepal	West Bank and Gaza
Eswatini	Nicaragua	Yemen
Ethiopia	Niger	Zambia
Fiji	Nigeria	Zimbabwe
Gabon	North Macedonia	
The Gambia	Oman	
	Pakistan	
	Palau	
	Panama	
	Papua New Guinea	