Chapter 9
Simultaneous State and Fault Estimation over Bandwidth-Constrained Networks: A Relay-Aided Binary Encoding Strategy

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Abstract This paper is concerned with the joint state and fault estimation problem for a class of discrete-time systems over a bandwidth-constrained network subject to actuator and sensor faults. The signal attenuation is resisted by locating the amplify-and-forward relay between the sensor and the estimator. In the sensor-to-relay channel, a binary encoding mechanism is employed to encode the measurement signal into a series of binary numbers. The random bit error, governed by a series of Bernoulli distributed random variables, is considered due to long-distance transmission and channel noise. The objective of the problem addressed in this paper is to design a joint state and fault estimator by augmenting the system state and the sensor fault into a descriptor system model. Sufficient conditions are established to ensure that the joint estimation error is exponentially bounded in the mean-square sense. The desired joint estimator gain is parameterized based on the solution of certain matrix inequalities. Finally, a numerical simulation is provided to validate the effectiveness of the proposed joint estimator design method.

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9.1 Introduction

In the past few decades, there has been a great leap in the flourishing of the networked control system (NCS) based on the strengths of its less wiring, flexible configuration, easy implementation and installation, and friendly maintenance. The NCS has found broad applications in electric power systems, intelligent homes, unmanned aerial vehicles, and other domains [26, 31, 35, 38]. With the growth of the network scale, network structure, and interactions, the demand for the safety and reliability of the network (which serves as the most fundamental requirement for the normal operation of the system) has become an increasingly urgent concern, attracting significant research attention. To ensure the network's safety and reliability, considerable efforts have been dedicated to the issue of fault diagnosis, leading to fruitful research achievements in a variety of industrial systems (see [12, 13, 17, 18, 24, 34, 44]).

In general, fault diagnosis can be divided into three steps: fault detection, fault isolation, and fault estimation. The fault detection step utilizes the residual signal to determine whether the system is faulty and identifies the time instant when the fault occurs (see [9, 40]). Depending on the specific location, faults can be categorized as actuator faults or sensor faults. In the second step, the fault isolation technique is employed to identify the faulty component of the system and determine the corresponding fault (see [14, 45]). Fault estimation, as a crucial step in the fault diagnosis process, aims to estimate the fault magnitude and determine the type and intensity of the fault (see [12, 13, 25, 42]). Consequently, significant efforts have been devoted to addressing the fault estimation issue, leading to the development of various estimation methods (see [7, 17, 20, 28, 30, 39]).

It is widely recognized that the transmission capacity of the network channel is often limited due to factors such as significant path loss and technical/cost constraints [2, 5, 6, 10]. In many cases, low-cost sensors are used to measure the output, and the data is transmitted to a remote receiver through a wireless link [15, 19, 23]. In such scenarios, relays have been extensively deployed to forward the sensor signals to the filter/controller. The primary objectives of using relays are to extend the signal transmission distance and ensure the quality of long-distance signal transmission. In practical engineering, various types of relays have been proposed, including amplify-and-forward relays, full-duplex relays, and filter-and-forward relays (see [1, 16, 21, 33]). For instance, in [36], full-duplex relay networks have been employed to investigate the recursive filtering problem for nonlinear time-varying systems with self-interferences.

It is worth noting that the filtering and control problem associated with amplify-and-forward (AF) relays has recently garnered attention in NCSs. Various relay selection and power allocation methods have been investigated, including the average transmission power approach and the minimum total transmission power approach. For instance, in [37], the average transmission power approach has been employed to study the robust recursive filtering problem for a class of uncertain stochastic time-varying systems with AF relays. In [29], the particle filtering problem has been addressed in sensor networks with AF relays. In [11], an optimal relay selection scheme has been developed to maximize the effective signal-to-interference and noise ratio, and optimal power allocation and the selection of duplex mode have also been obtained.

Due to the rapid progress of network communication technologies, digital communication has gained significant popularity due to its low cost, high reliability, and strong anti-interference capability. Among various digital communication schemes, the binary encoding scheme has emerged as one of the most effective approaches, attracting substantial research interest. It is important to emphasize that the limited communication resources and bandwidth have a significant impact on the accuracy of binary encoding, which can result in information loss. Therefore, designing an appropriate binary encoding scheme that strikes a balance between limited network bandwidth and encoding accuracy has become a topic of special research attention, see, e.g., [32, 41, 43].

In most of the existing literature regarding the binary encoding scheme, a common assumption is that signal transmission is error-free in terms of bits. However, this assumption is often violated due to factors such as channel noise and long-distance transmission, leading to random bit errors. It is important to note that such errors can have a detrimental effect on communication quality, thereby degrading system performance. Therefore, it is both practically significant and theoretically important to establish a model for random bit errors and explore their impact on system performance. In recent research, initial efforts have been made to investigate control and filtering problems under the presence of random bit errors in binary encoding (see [3, 22]). For instance, in [27], the moving-horizon estimation problem has been addressed for linear dynamic networks using a noisy channel and binary encoding. In [8], a binary encoding scheme has been proposed to handle the consensusability problem for multi-agent systems over a resource-constrained network.

Building upon the previous discussions, this paper focuses on the remote estimation problem for a specific class of discrete-time systems using a bandwidth-limited network. The system model takes into account both actuator faults and sensor faults, which are commonly encountered in practical engineering scenarios. A joint estimation scheme is proposed to simultaneously estimate the system state and faults. To facilitate long-distance transmission, an AF relay is positioned between the sensor and the remote estimation. The sensor-to-relay channel employs a binary encoding mechanism, considering the presence of random bit errors resulting from channel noise. Additionally, the paper establishes sufficient conditions to ensure the exponential boundedness of the estimation error in the mean-square sense. The desired gain of the joint estimator is parameterized accordingly.

The paper makes several key contributions, which can be highlighted as follows:

1. The paper presents a comprehensive system model that incorporates sensor faults, actuator faults, AF relays, and the binary encoding mechanism with random bit errors. This model captures various important aspects of the considered system.
2. A novel descriptor system model is proposed to address the joint estimator design problem in the presence of both actuator faults and sensor faults. This model

provides a framework for integrating fault estimation with the system state estimation.
3. The binary encoding mechanism is included in the sensor-to-relay channel, which has not been explored previously. Random bit errors resulting from channel noise are explicitly modeled using a series of random variables.
4. sufficient conditions are established to guarantee the exponential mean-square boundedness of the joint estimation error. These conditions provide insights into the stability and performance of the proposed estimator.
5. The expected estimator gain is parameterized by solving specific matrix inequalities. This parameterization allows for a flexible and adjustable design of the joint estimator.

The remaining sections of this paper are organized as follows. Section 9.2 presents the system model, where a descriptor system model is introduced to address the joint estimation problem. The mathematical model of the AF relays is established, and the binary encoding mechanism is proposed, incorporating the random bit error governed by a series of Bernoulli-distributed random variables. Section 9.3 presents sufficient conditions that ensure the mean-square exponential boundedness of the estimation error. Furthermore, an analytical expression for the joint estimator gain is derived. Section 9.4 presents a numerical simulation to validate the effectiveness of the proposed joint estimator design scheme. The simulation results provide insights into the performance of the proposed approach. Finally, Sect. 9.5 concludes the paper, summarizing the contributions and highlighting the key findings.

Notations. The symbol $\mathbb{R}^n$ represents the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real-valued matrices. For a matrix M, M^T and M^{-1} denote its transpose and inverse, respectively. The notation $Y_1 \geq Y_2$ ($Y_1 > Y_2$) indicates that Y_1 and Y_2 are symmetric matrices, and $X_1 - X_2$ is non-negative definite (positive-definite). The asterisk symbol $*$ represents a term induced by symmetry. The notation diag$\{\cdots\}$ signifies a block-diagonal matrix.

9.2 Problem Formulation

Consider the following discrete-time system:

$$\begin{cases} x_{k+1} = Ax_k + F_a f_{a,k} + D_1 w_k \\ z_k = Cx_k + F_s f_{s,k} + D_2 v_k \end{cases} \tag{9.1}$$

where $x_k \in \mathbb{R}^{n_x}$ and $z_k \in \mathbb{R}^{n_z}$ stand for the system state and the measurement output, respectively; $f_a \in \mathbb{R}^{n_{fa}}$ indicates the actuator fault and $f_s \in \mathbb{R}^{n_{fs}}$ represents the sensor fault; $w_k \in l_2([0,\ N]; \mathbb{R}^{n_w})$ and $v_k \in l_2([0,\ N]; \mathbb{R}^{n_v})$ are the process noise and the measurement noise, respectively; A, C, D_1, D_2, F_a, F_s, and M are known constant matrices with appropriate dimensions.

The measurement signal z_k is transmitted from the sensor to the relay terminal through a sensor-to-relay channel with a specific transmission power. Consequently, we can express this relationship as follows:

$$h_k = \sqrt{q_1} C_1 z_k + \phi_{1,k} \tag{9.2}$$

where h_k is the signal received by the relay, $\phi_{1,k}$ is the channel noise from the sensor to the relay and q_1 is the transmission power. $\phi_{1,k}$ is a Gaussian distributed random vector with zero mean and variance matrix $R_1 > 0$.

Since the signal transmission from the sensor to the relay is implemented via a binary symmetric channel, the encoding function $\mathcal{F}(\cdot)$ is defined as follows:

$$\mathcal{F}(h_k) = \{s_{1,k}, s_{2,k}, \ldots, s_{L,k}\} \tag{9.3}$$

where $\{s_{1,k}, s_{2,k}, \ldots, s_{L,k}\}$ is the coding sequence with length L and $s_{i,k} \in \{0, 1\}$ ($i = 1, 2, \ldots, L$) is the ith codeword of the coding sequence. In the subsequent discussion, we will outline a specific procedure for generating a coding sequence for an original data h_k.

For a given interval $\mathcal{H} \triangleq [-\bar{h}, \bar{h}]$ in which the signal h_k is located, $\mathcal{H}$ is uniformly divided into $2^L - 1$ segments. Thus, the left endpoint of the ith segment, which is denoted by ξ_i, can be determined as

$$\xi_i = -\bar{h} + \frac{2\bar{h}}{2^L - 1}(i - 1), \quad i = 1, 2, \ldots, 2^L - 1.$$

In addition, we denote the right endpoint of interval $\mathcal{H}$ as ξ_{2^L} and it is easy to see that $\delta \triangleq \xi_{i+1} - \xi_i = \frac{2\bar{h}}{2^L-1}$.

Due to the limitation of communication bandwidth, the transmission through the channel can only accommodate signals with a finite number of bits. Without loss of generality, let's assume that L bits are allowed for transmission per unit time. Consequently, to meet the transmission requirement, the raw data h_k, which possesses arbitrary accuracy, needs to be truncated. In this paper, a stochastic truncation technique is employed to map the lossless signal $h_k \in [\xi_i, \xi_{i+1})$ to a truncated signal n_k according to the following rule:

$$\begin{cases} \text{Prob}\{n_k = \xi_i\} = 1 - \alpha_k \\ \text{Prob}\{n_k = \xi_{i+1}\} = \alpha_k \end{cases} \tag{9.4}$$

where

$$\alpha_k \triangleq \frac{h_k - \xi_i}{\xi_{i+1} - \xi_i} \in [0, 1).$$

Moreover, n_k can be rewritten as a linear combination of the first L terms of the series $\{2^{i-1}\delta\}$, i.e.,

$$n_k = -\bar{h} + \sum_{i=1}^{L} s_{i,k} 2^{i-1} \delta$$

and $s_{i,k} \in \{0, 1\}$ is the ith codeword of the coding sequence $\{s_{1,k}, s_{2,k}, \ldots, s_{L,k}\}$, which is denoted as $\mathcal{S}_k$ for convenience hereafter.

In the process of transmitting the coding sequence $\mathcal{S}_k$ through a memoryless binary symmetric channel, the presence of probabilistic bit flips is taken into consideration to better simulate the effects of a noisy channel environment. Consequently, the received code sequence can be described as follows:

$$\tilde{\mathcal{S}}_k = \{\tilde{s}_{1,k}, \tilde{s}_{2,k}, \ldots, \tilde{s}_{L,k}\} \tag{9.5}$$

where

$$\tilde{s}_{i,k} \triangleq (1 - \lambda_{i,k}) s_{i,k} + \lambda_{i,k}(1 - s_{i,k}).$$

Here, $\lambda_{i,k}$ is a Bernoulli distributed random variable with the following distribution:

$$\text{Prob}\{\lambda_{i,k} = 1\} = \bar{\lambda}, \quad \text{Prob}\{\lambda_{i,k} = 0\} = 1 - \bar{\lambda} \tag{9.6}$$

where $\bar{\lambda} \in [0, 1]$ and $\lambda_{i,k} = 1$ indicates that the ith bit $s_{i,k}$ flips while $\lambda_{i,k} = 0$ implies that there is no bit flip of $s_{i,k}$. After receiving the sequence $\tilde{\mathcal{S}}_k$, the original signal h_k is decoded by

$$\bar{n}_k = -\bar{h} + \sum_{i=1}^{L} \tilde{s}_{i,k} 2^{i-1} \delta \tag{9.7}$$

where $\bar{n}_k$ is the decoded value of h_k. It is easy to see that if there is no bit flips occurring, one has $\bar{n}_k = n_k$.

Define $e_{1,k} \triangleq h_k - n_k$ as the truncation error of the signal h_k. Recalling the expression of α_k as $h_k = \xi_i + \alpha_k \delta$, it is not difficult to see from (9.4) that

$$\begin{cases} \text{Prob}\{e_{1,k} = \alpha_k \delta\} = 1 - \alpha_k \\ \text{Prob}\{e_{1,k} = (\alpha_k - 1)\delta\} = \alpha_k. \end{cases} \tag{9.8}$$

The following two lemmas are given to show some statistical properties about the truncation error $e_{1,k}$.

Lemma 9.1 ([27]) *The mean and the variance of the truncation error $e_{1,k}$ satisfy $\mathbb{E}\{e_{1,k}\} = 0$ and*

$$\mathbb{E}\{e_{1,k}^2\} \leq \frac{\delta^2}{4}.$$

Lemma 9.2 ([27]) *For a memoryless binary symmetric channel with a crossover probability* $\bar{\lambda}$*, assume that the signal* n_k *is transmitted via such a channel. Then, the mean and the variance of the decoded signal* $\bar{n}_k$ *are, respectively, given by*

$$\mathbb{E}\{\bar{n}_k\} = (1 - 2\bar{\lambda})n_k \tag{9.9}$$

and

$$\mathbb{VAR}\{\bar{n}_k\} = \bar{\lambda}(1 - \bar{\lambda})\frac{4\bar{h}^2(2^{2L} - 1)}{3(2^L - 1)^2} \tag{9.10}$$

where the expectation is taken with respect to the random variables $\lambda_{i,k}$*.*

The relay receives the decoded signal $\bar{n}_k$, which is affected by the bit flips and causes a distortion between the original signal n_k and $\bar{n}_k$. To maintain the unbiasedness of this distortion, the signal

$$\hat{n}_k \triangleq \frac{1}{1 - 2\bar{\lambda}}\bar{n}_k$$

is used instead of directly utilizing $\bar{n}_k$. Subsequently, $\hat{n}_k$ is amplified and forwarded to the state estimator. Consequently, the actual signal entering the estimator can be described as follows:

$$\hat{z}_k = \epsilon\sqrt{q_2}C_2\hat{n}_k + \phi_{2,k} \tag{9.11}$$

where $\epsilon > 0$ is the amplification factor, q_2 is the transmission power of the relay, and $\phi_{2,k}$ is the zero-mean channel noise from the relay to the estimator with the variance R_2. $C_2 = \text{diag}\{c_{21}, c_{22}, \ldots, c_{2n_z}\}$ with c_{2i} $(1, 2, \ldots, n_z)$ being the channel coefficient of the ith channel.

Denoting $e_{2,k} \triangleq n_k - \hat{n}_k$ and recalling $e_{1,k} \triangleq h_k - n_k$, the decoded signal $\hat{n}_k$ can be represented by $\hat{n}_k = h_k - e_{1,k} - e_{2,k}$, which gives

$$\begin{aligned}\hat{z}_k &= \epsilon\sqrt{q_2}C_2(h_k - e_{1,k} - e_{2,k}) + \phi_{2,k}\\ &= \epsilon\sqrt{q_2}C_2\sqrt{q_1}C_1 z_k + \epsilon\sqrt{q_2}C_2\phi_{1,k}\\ &\quad - \epsilon\sqrt{q_2}C_2 e_{1,k} - \epsilon\sqrt{q_2}C_2 e_{2,k} + \phi_{2,k}\\ &= M z_k + N\zeta_k\end{aligned} \tag{9.12}$$

where

$$\begin{aligned}M &\triangleq \epsilon\sqrt{q_1 q_2}C_1 C_2\\ N &\triangleq \left[\epsilon\sqrt{q_2}C_2 \;\; -\epsilon\sqrt{q_2}C_2 \;\; -\epsilon\sqrt{q_2}C_2 \;\; I\right]\\ \zeta_k &\triangleq \left[\phi_{1,k}^T \;\; e_{1,k}^T \;\; e_{2,k}^T \;\; \phi_{2,k}^T\right]^T.\end{aligned}$$

Lemma 9.3 *The mean and the variance of* $e_{2,k}$ *are, respectively, 0 and*

$$\frac{1}{(1-2\bar{\lambda})^2}\mathbb{VAR}\{\bar{n}_k\}.$$

Proof It is straightforward to see from the definition of $e_{2,k}$ and Lemma 9.2 that $\mathbb{E}\{e_{2,k}\}=0$. Thus, the variance of $e_{2,k}$ is calculated by

$$\begin{aligned}
&\mathbb{E}\{e_{2,k}^2\}\\
=&\mathbb{E}\{(n_k-\hat{n}_k)^2\}\\
=&\mathbb{E}\left\{\left(n_k-\frac{1}{1-2\bar{\lambda}}\bar{n}_k\right)^2\right\}\\
=&\mathbb{E}\left\{n_k^2-\frac{2}{1-2\bar{\lambda}}n_k\bar{n}_k+\frac{1}{(1-2\bar{\lambda})^2}\bar{n}_k^2\right\}\\
=&n_k^2-2n_k^2+\frac{1}{(1-2\bar{\lambda})^2}\mathbb{E}\left\{\bar{n}_k^2\right\}
\end{aligned}$$

where

$$\mathbb{E}\{\bar{n}_k^2\}=\mathbb{VAR}\{\bar{n}_k\}+(\mathbb{E}\{\bar{n}_k\})^2=\mathbb{VAR}\{\bar{n}_k\}+(1-2\bar{\lambda})^2n_k^2$$

and therefore

$$\mathbb{E}\{e_{2,k}^2\}=\frac{1}{(1-2\bar{\lambda})^2}\mathbb{VAR}\{\bar{n}_k\}$$

which ends the proof of this lemma. □

To facilitate further analysis, the following assumptions are provided which lay the groundwork for subsequent discussions.

Assumption 9.1 The matrices F_a and F_s are of full-column rank.

Assumption 9.2 For the actuator fault signal $f_{a,k}$, the difference between two successive time instants is bounded, namely, $|f_{a,k+1}-f_{a,k}|\leq\bar{f}_a$ for $\forall k\geq 0$ where $\bar{f}_a$ is a known positive value.

By combining the state vector x_k and the sensor fault signal $f_{s,k}$ into a compacted vector $x_{f,k}\triangleq[x_k^T\ \ f_{s,k}^T]^T$ and considering the original system dynamics (9.1), we obtain the augmented system dynamics as follows:

$$\begin{cases}Ex_{f,k+1}=AEx_{f,k}+F_af_{a,k}+D_1w_k\\ \qquad\ z_k=C_fx_f+D_2v_k\end{cases}\tag{9.13}$$

where $E\triangleq[I\ \ 0]$ and $C_f\triangleq[C\ \ F_s]$.

Lemma 9.4 *There exist matrices $G \in \mathbb{R}^{(n_x+n_{fs})\times n_x}$ and $H \in \mathbb{R}^{(n_x+n_{fs})\times n_y}$ such that the following holds:*

$$\begin{bmatrix} G & H \end{bmatrix}\begin{bmatrix} E \\ C_f \end{bmatrix} = I_{n_x+n_{fs}}. \tag{9.14}$$

Proof Based on Assumption 9.1, the lemma can be directly derived and the detailed proof is thus omitted here. □

For any matrix G with an appropriate dimension, it follows from the first equation of (9.13) that

$$GEx_{f,k+1} = GAEx_{f,k} + GF_{a,k}f_{a,k} + GD_1w_k. \tag{9.15}$$

Similarly, for any matrix H with an appropriate dimension, it is not difficult to obtain from the second equation of (9.13) that

$$Hz_{k+1} = HC_{f,k+1}x_{f,k+1} + HD_2v_{k+1}. \tag{9.16}$$

With the help of Lemma 9.4 and the combination of (9.15)–(9.16), one obtains

$$\begin{cases} x_{f,k+1} = GAEx_{f,k} + Hz_{k+1} + GF_a f_{a,k} \\ \qquad\qquad - HD_2v_{k+1} + GD_1w_k \\ z_k = C_f x_{f,k} + D_2v_k. \end{cases} \tag{9.17}$$

Noting that $\hat{z}_k$ represents the actual measurement output that is used in the estimator, we proceed to develop a joint state estimator. The objective of this estimator is to simultaneously estimate both the fault signals and the system dynamics. The formulation of the joint state estimator is as follows:

$$\begin{cases} \hat{x}_{f,k+1} = GAE\hat{x}_{f,k} + HM^{-1}\hat{z}_{k+1} + GF_a\hat{f}_{a,k} \\ \qquad\qquad + K_a(\hat{z}_k - MC_f\hat{x}_{f,k}) \\ \hat{f}_{a,k+1} = \hat{f}_{a,k} + K_b(\hat{z}_k - MC_f\hat{x}_{f,k}) \end{cases} \tag{9.18}$$

where $\hat{x}_{f,k} \in \mathbb{R}^{n_x+n_{fs}}$ is an estimate of $x_{f,k}$, $\hat{f}_{a,k} \in \mathbb{R}^{n_{fa}}$ is an estimate of $f_{a,k}$ and $\hat{z}_k \in \mathbb{R}^{n_z}$ is an estimate of z_k; $\hat{z}_{k+1}$ stands for the received measurement signal by the estimator at $k+1$; and K_a and K_b are the estimator gains to be designed.

Denote $e_{f,k} \triangleq x_{f,k} - \hat{x}_{f,k}$ and $\tilde{f}_{a,k} \triangleq f_{a,k} - \hat{f}_{a,k}$ as the augment state estimation errors and the actuator fault estimation error $f_{a,k}$, respectively. It follows from (9.17) and (9.18) that the error dynamics of both $e_{f,k}$ and $\tilde{f}_{a,k}$ are given as follows:

$$\begin{cases} e_{f,k+1} = A_1 e_{f,k} + GF_a \tilde{f}_{a,k} - HD_2 v_{k+1} - HM^{-1}N\zeta_{k+1} \\ \qquad\qquad - K_a MD_2 v_k - K_a N\zeta_k + GD_1 w_k \\ \tilde{f}_{a,k+1} = \tilde{f}_{a,k} - K_b MC_f e_{f,k} - K_b MD_2 v_k - K_b N\zeta_k \\ \qquad\qquad + \Delta f_{a,k} \end{cases} \tag{9.19}$$

where $\Delta f_{a,k} \triangleq f_{a,k+1} - f_{a,k}$ and $A_1 \triangleq GAE - K_a MC_f$.

By augmenting the estimation errors $e_{f,k}$ and $\tilde{f}_{a,k}$ as $e_k \triangleq [e_{f,k}^T \ \tilde{f}_{a,k}^T]^T$, a further compact form of the estimation error dynamics is constructed with the following form:

$$e_{k+1} = \mathcal{A}e_k + \mathcal{D}\eta_k + \mathcal{E}\theta_k \tag{9.20}$$

where

$$\begin{aligned} \mathcal{A} &\triangleq \begin{bmatrix} A_1 & GF_a \\ -K_b MC_f & I \end{bmatrix} \\ \mathcal{D} &\triangleq \begin{bmatrix} -HD_2 & -K_a MD_2 & GD_1 & 0 \\ 0 & -K_b MD_2 & 0 & I \end{bmatrix} \\ \mathcal{E} &\triangleq \begin{bmatrix} -HM^{-1}N & -K_a N \\ 0 & -K_b N \end{bmatrix} \\ \eta_k &\triangleq \begin{bmatrix} v_{k+1}^T & v_k^T & w_k^T & \Delta f_{a,k}^T \end{bmatrix}^T \\ \theta_k &\triangleq \begin{bmatrix} \zeta_{k+1}^T & \zeta_k^T \end{bmatrix}^T . \end{aligned}$$

Definition 9.1 The estimation error dynamics characterized by (9.20) is said to be exponentially bounded in the mean-square sense if there exist scalars $\mu_1 \in (0,\ 1)$ and $\mu_2 > 0$ such that

$$\mathbb{E}\{\|e_k\|^2\} \le \mu_1^k \mathbb{E}\{\|e_0\|^2\} + \mu_2. \tag{9.21}$$

The goal of our current investigation is to design a joint state-and-fault estimator of structure (9.18) for system (9.17). The primary objective is to ensure that the estimation error is exponentially bounded in the mean-square sense under the relay-aided binary encoding scheme.

9.3 Main Results

In this section, we will commence by addressing the boundedness analysis problem for the estimation error system (9.20). We will establish a sufficient condition to ensure the exponential ultimate boundedness of the error system (9.20). Subsequently, we will delve into the problem of designing the estimator parameters. We will provide an analytical expression for the estimator parameters, which will facilitate the practical implementation of the estimation algorithm in engineering applications.

9.3.1 Stability Analysis

Theorem 9.1 *Let the scalar $\varrho \in (0, 1)$ and the estimator parameters K_a, K_b be given. Consider the error dynamics (9.20) under the signal relay scheme (9.2) and (9.11) as well as the binary coding mechanism (9.3)–(9.7). The estimation error dynamics (9.20) is exponentially ultimately bounded in the mean-square sense if there exist a positive definite matrix $\mathcal{P}$ and a positive scalar $\sigma > 0$ such that*

$$\Xi \triangleq \Xi_0 + \Xi_1 < 0 \tag{9.22}$$

where

$$\begin{aligned}
&\Xi_0 \triangleq \begin{bmatrix} \mathcal{A}^T\mathcal{P}\mathcal{A} - (1-\varrho)\mathcal{P} & \mathcal{A}^T\mathcal{P}\mathcal{D} \\ * & \mathcal{D}^T\mathcal{P}\mathcal{D} - \sigma I \end{bmatrix} \\
&\Xi_1 \triangleq \mathrm{diag}\{0, -\sigma I\}, \quad \rho \triangleq \lambda_{\max}\{\mathcal{E}\mathcal{P}\mathcal{E}\}^T \\
&\bar{\theta} \triangleq 2\left(R_1 + R_2 + \frac{\delta}{4}\right) + \frac{2}{(1-2p)^2}\mathbb{VAR}\{\bar{n}_k\} \\
&\bar{\eta} \triangleq 2\bar{w} + \bar{v} + \bar{f}_a, \quad \beta \triangleq \rho\bar{\theta} + \sigma\bar{\eta}.
\end{aligned}$$

Moreover, an ultimate upper bound of the estimation error is given by

$$\frac{\beta}{\lambda_{\min}\{\mathcal{P}\}}.$$

Proof Choose a Lyapunov function as $V_k = e_k^T\mathcal{P}e_k$. Calculating the difference between V_{k+1} and $(1-\varrho)V_k$ along the trajectory of the estimation dynamics (9.20) and taking its mathematical expectation yield

$$\begin{aligned}
&\mathbb{E}\{V_{k+1} - (1-\varrho)V_k\} \\
=&\mathbb{E}\{e_{k+1}^T\mathcal{P}e_{k+1} - (1-\varrho)e_k^T\mathcal{P}e_k\} \\
=&\mathbb{E}\{(\mathcal{A}e_k + \mathcal{D}\eta_k + \mathcal{E}\theta_k)^T\mathcal{P}(\mathcal{A}e_k + \mathcal{D}\eta_k + \mathcal{E}\theta_k)\} \\
&- (1-\varrho)e_k^T\mathcal{P}e_k \\
=&\mathbb{E}\{(\mathcal{A}e_k + \mathcal{D}\eta_k)^T\mathcal{P}(\mathcal{A}e_k + \mathcal{D}\eta_k) + \theta_k^T\mathcal{E}^T\mathcal{P}\mathcal{E}\theta_k\} \\
&- (1-\varrho)e_k^T\mathcal{P}e_k \\
\leq&\mathbb{E}\{(\mathcal{A}e_k + \mathcal{D}\eta_k)^T\mathcal{P}(\mathcal{A}e_k + \mathcal{D}\eta_k)\} + \rho\mathbb{E}\{\theta_k^T\theta_k\} \\
&- (1-\varrho)e_k^T\mathcal{P}e_k
\end{aligned} \tag{9.23}$$

where the third equality holds because of the facts that (1) the mathematical expectation of θ_k is zero; and (2) it is independent of e_k and η_k. Next, let us tackle the term $\mathbb{E}\{\theta_k^T\theta_k\}$, which can be further calculated as

$$\begin{aligned}\mathbb{E}\{\theta_k^T\theta_k\} =&\mathbb{E}\{\zeta_{k+1}^T\zeta_{k+1}+\zeta_k^T\zeta_k\}\\ =&2\mathbb{E}\{\zeta_k^T\zeta_k\}=2\sum_{i=1}^{2}\mathbb{E}\{\phi_{i,k}^2+e_{i,k}^2\}\\ \leq&2\left(R_1+R_2+\frac{\delta}{4}\right)+2\mathbb{E}\{e_{2,k}^2\}.\end{aligned} \tag{9.24}$$

It is inferred from Lemma 9.3 that $\mathbb{E}\{\theta_k^T\theta_k\}\leq\bar{\theta}$. Moreover, taking into account the constraint $\eta_k^T\eta_k\leq\bar{\eta}$ and the condition (9.22) in Theorem 9.1, one has

$$\begin{aligned}\mathbb{E}\{V_{k+1}\} \leq&(1-\varrho)\mathbb{E}\{V_k\}+\beta\\ \leq&(1-\varrho)\left[(1-\varrho)\mathbb{E}\{V_{k-1}\}+\beta\right]+\beta\\ &\vdots\\ \leq&(1-\varrho)^{k+1}\mathbb{E}\{V_0\}+\frac{\beta(1-(1-\varrho)^{k+1})}{\varrho},\end{aligned} \tag{9.25}$$

which further implies

$$\lambda_{\min}\{\mathcal{P}\}\mathbb{E}\{\|e_k\|^2\}\leq\mathbb{E}\{V_k\}\leq(1-\varrho)^k\lambda_{\max}\{\mathcal{P}\}\mathbb{E}\{\|e_0\|^2\}+\frac{\beta}{\varrho}.$$

Consequently, one directly obtains

$$\lim_{k\to\infty}\mathbb{E}\{\|e_k\|^2\}\leq\frac{\beta}{\lambda_{\min}\{\mathcal{P}\}}, \tag{9.26}$$

which ends the proof of this theorem. □

9.3.2 *Estimator Parameter Design*

Having derived a sufficient condition for the mean-square boundedness of the estimation error dynamics (9.20), we are now able to provide an explicit form of the estimator parameters by utilizing the solutions to certain matrix inequalities.

Theorem 9.2 *Consider the signal relay scheme* (9.2) *and* (9.11) *as well as the binary coding mechanism* (9.3)–(9.7)*. For a given scalar* $\varrho \in (0, 1)$*, the error dynamics* (9.20) *is exponentially ultimately bounded in the mean-square sense if there exist a positive definite matrix* $\mathcal{P}$*, a matrix* $\bar{\mathcal{K}}$ *with appropriate dimensions and a positive scalar* $\sigma > 0$ *such that the following matrix inequality*

$$\Pi \triangleq \begin{bmatrix} -(1-\varrho)\mathcal{P} & 0 & \bar{\mathcal{A}}^T \\ * & -\sigma I & \bar{\mathcal{D}}^T \\ * & * & -\mathcal{P} \end{bmatrix} < 0 \tag{9.27}$$

holds, where

$$\begin{aligned} &\bar{\mathcal{A}} \triangleq \mathcal{P}\mathcal{G} - \bar{\mathcal{K}}\mathcal{M}_1, \bar{\mathcal{D}} \triangleq \mathcal{P}\mathcal{H} - \bar{\mathcal{K}}\mathcal{M}_2 \\ &\mathcal{G} \triangleq \begin{bmatrix} GAE & GF_a \\ 0 & -I \end{bmatrix}, \mathcal{H} \triangleq \begin{bmatrix} -HD_2 & 0 & GD_1 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \\ &\mathcal{M}_1 \triangleq \begin{bmatrix} MC_f & 0 \end{bmatrix}, \mathcal{M}_2 \triangleq \begin{bmatrix} 0 & MD_2 & 0 & 0 \end{bmatrix} \end{aligned}$$

and other parameters have been defined in Theorem 9.1. In addition, an ultimate upper bound of the estimation error in the mean-square sense is calculated as

$$\frac{\beta}{\lambda_{\min}\{\mathcal{P}\}}$$

with the estimator parameters K_a *and* K_b *determined by*

$$[K_a^T \ \ K_b^T]^T = \mathcal{P}^{-1}\bar{\mathcal{K}}. \tag{9.28}$$

Proof Noting that $\bar{\mathcal{K}} = \mathcal{P}[K_a^T \ \ K_b^T]^T$ and using the Schur Complement Lemma [4], the inequality (9.22) is ensured by the inequality (9.27), and the rest of the proof is easily accessible from that of Theorem 9.1. □

Remark 9.1 In this study, the problem of simultaneous estimation of system state and fault signals is thoroughly investigated for discrete-time systems subject to both the binary coding scheme and the signal relay mechanism. The system model and communication model considered in this study are comprehensive, encompassing sensor faults, actuator faults, binary coding scheme, and signal relay strategy. To facilitate the synthesis and design of the joint estimator, the original system is transformed into a descriptor system model by augmenting the sensor fault and system state. Additionally, crucial information such as noise intensity, coding truncation error, and statistical information of code flipping are fully incorporated into the upper bound of the estimation error, making the joint estimator synthesis and design more effective.

Remark 9.2 A systematic investigation is launched on the joint estimation problem for a specific class of linear discrete-time systems by considering both sensor and actuator faults. The study incorporates the binary coding scheme and signal relay strategy in the data communication module. The major novelties of this research can be identified in the following aspects: (1) this study represents one of the first attempts to tackle the joint estimation problem in the presence of both the binary coding scheme and signal relay strategy; (2) a descriptor system model is developed to provide a viable solution for simultaneous state and fault estimation; and (3) a comprehensive theoretical framework is constructed to handle the various influences on the estimation performance, including the effects of sensor/actuator faults, binary coding scheme, and amplify-and-forward relay strategy.

9.4 An Illustrative Example

A numerical simulation is given in this section to further verify the validity of the proposed joint state and fault estimation scheme. For this purpose, we consider a maneuvering target tracking system whose dynamics is governed by (9.1) with the relevant parameters provided as follows:

$$
\begin{aligned}
A &= \begin{bmatrix} 0.6 & h \\ 0.2 & 0.5 \end{bmatrix}, \; F_a = \begin{bmatrix} 0.5 & 0.9 \end{bmatrix}^T \\
D_1 &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T, \; C = \begin{bmatrix} 5 & 5 \end{bmatrix} \\
F_s &= 30, \; D_2 = 0.5
\end{aligned}
$$

where $h = 0.004$ indicates the sampling period of the system state $x_k = [x_{1k} \;\; x_{2k}]^T$ with x_{1k} and x_{2k} being the position and velocity of the target, respectively.

In the data communication aspect, the channel parameters for the sensor-to-relay and relay-to-estimator channels are set as $C_1 = 0.8$ and $C_2 = 0.005$, respectively. Additionally, the amplification factor is chosen as $\epsilon = 1$. The transmission powers for both the sensor-to-relay channel and relay-to-estimator channel are specified as $q_1 = q_2 = 1.5$. The variances of the noises for both channels are given as $R_1 = R_2 = 0.5$.

The initial conditions are given as $x_0 = [1.5 \;\; 1.25]^T$, $f_{a,0} = 0$, $f_{s,0} = 0$, $\hat{x}_{f,0} = [0.01 \;\; 0.01 \;\; 0.02]^T$ and $\hat{f}_{a,0} = 0.5$. The noises of the system dynamics and the sensor measurement are selected as $v_k = 0.1 \sin(k)$ and $v_k = 0.1 \cos(k)$. For the binary coding scheme, the length of the coding sequence is chosen as $L = 8$. The interval parameter $\bar{h} = 10$ and flipping probability p is set as $p = 0.01$.

The sensor fault and the actuator fault are described with the following form

$$f_{s,k} = \begin{cases} 0.5, & 30 \leq k \leq 100 \\ 0, & \text{otherwise} \end{cases}$$

$$f_{a,k} = \begin{cases} 0.5 + 0.01 * \sin(0.1 * k), & 5 \leq k \leq 100 \\ 0, & \text{otherwise.} \end{cases}$$

Based on the given parameters, we solve linear matrix inequality (9.27) and obtain

$$\mathcal{P} = \begin{bmatrix} 14.5093 & -3.0544 & 1.2718 & -0.0010 \\ -3.0544 & 11.8759 & 1.7107 & -0.0011 \\ 1.2718 & 1.7107 & 18.5869 & 0.0004 \\ -0.0010 & -0.0011 & 0.0004 & 19.0874 \end{bmatrix}.$$

Moreover, the estimator parameters K_a and K_b are calculated as follows:

$$K_a = [5.5680\ 5.3464\ -1.8187]^T,\ K_b = -0.2462.$$

The simulation results are shown in Figs. 9.1, 9.2, 9.3, and 9.4. Among them, Figs. 9.1 and 9.2 depict the first and second state trajectories of the actual system states and their estimates. Figure 9.3 displays the sensor fault and its estimate and Fig. 9.4 illustrates the curves of the actuator fault and its estimate. The simulation results align with our expectations and provide further validation of the proposed joint estimation scheme.

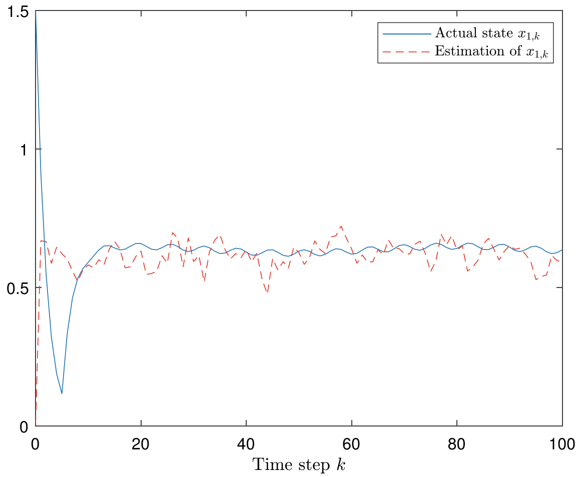

Fig. 9.1 The state trajectory $x_{1,k}$ and its estimation

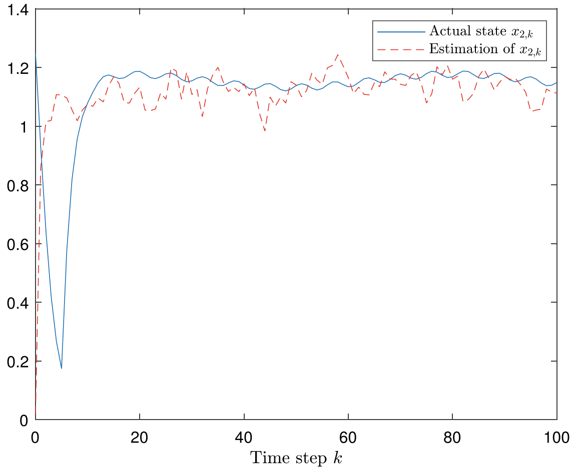

Fig. 9.2 The state trajectory $x_{2,k}$ and its estimation

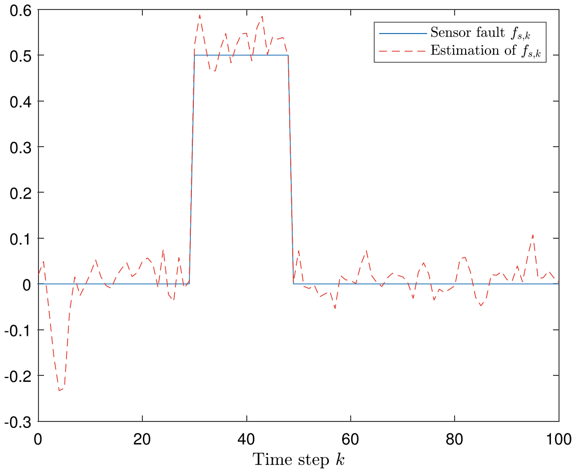

Fig. 9.3 The trajectory of the sensor fault $f_{s,k}$ and its estimation

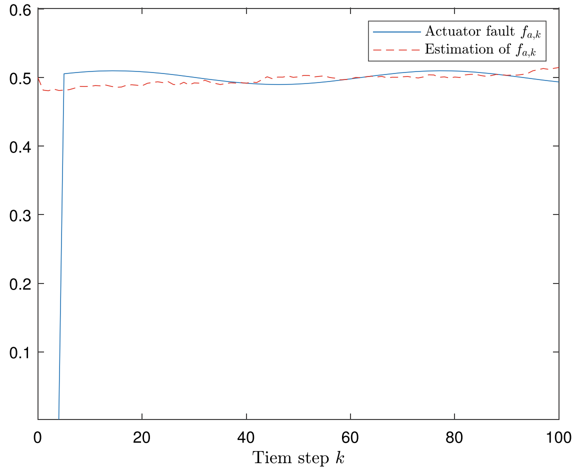

Fig. 9.4 The trajectory of the actuator fault $f_{a,k}$ and its estimation

9.5 Conclusion

This paper has addressed the problem of simultaneous estimation of both the state and sensor/actuator fault for a specific class of discrete-time linear systems. The binary coding scheme has been considered to facilitate digital signal transmission, and an amplify-and-forward relay strategy has been employed to mitigate signal attenuation from the sensor to the estimator. To tackle the joint estimation problem in a unified framework, the original system has been transformed into a descriptor system model, where the system state and sensor fault are augmented together. Sufficient conditions have been provided to guarantee the ultimate boundedness of the estimation error dynamics in the mean-square sense, along with a specific upper bound. A numerical example has been presented to demonstrate the effectiveness of the proposed joint estimation algorithm.

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