This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI: 10.1109/TCYB.2025.3626067, IEEE Transactions on Cybernetics

Dynamic Event-Driven State Estimation for Complex Networks via Partial Nodes' Sampled Outputs: An Encoding-Decoding Scheme

Yurong Liu, Zidong Wang, Luyang Yu, and Wenbing Zhang

***Abstract*—In this paper, the encoding-decoding-based state estimation problem is investigated for a class of continuous-time nonlinear complex networks subject to communication bandwidth constraints. Based on the sampled outputs from a subset of network nodes, a novel dynamic event-driven encoding mechanism is integrated into the design of state estimator, where a time-varying auxiliary parameter is utilized to modulate the triggering condition in a dynamical fashion, enabling the event detector to decide whether the data packet should be released at the periodic sampling instants. Specifically, when the dynamic triggering condition is satisfied, the data is first encoded into a codeword and subsequently transmitted to the estimator through a digital communication channel. The Zeno behavior can be naturally prevented owing to the periodic feature of the proposed event detector. By leveraging the Lyapunov theory and the matrix inequality techniques, sufficient conditions are established to ensure the exponential stability of the estimation error system. In addition, a convex optimization approach is employed to design the estimator gain with the goal of maximizing the allowable bound of the sampling intervals. Finally, an illustrative example and a practical example involving a three-area power system are provided to showcase the effectiveness of the proposed state estimation method.**

***Index Terms*—Complex networks, partial nodes, state estimation, sampled data, dynamic event-driven mechanism, encoding-decoding scheme.**

I. INTRODUCTION

Complex networks (CNs) are large-scale systems that consist of a substantial number of interconnected dynamic units, organized through a variety of topological structures. In mathematics, a network is typically characterized by a graph, where nodes represent the individual entities or agents within the network, and edges signify the connections or interactions between these nodes. By exchanging local information with neighboring nodes, these interconnected nodes are able to collaboratively accomplish a diverse range of tasks, such as control, optimization, and other engineering-oriented objectives. Owing primarily to their inherent flexibility and adaptability, CNs have become a powerful tool for modeling and analyzing various real-world systems which include, but are not limited to, sensor networks, social networks, and power grids [1]–[9]. Over the past several decades, significant research attention has been devoted to investigating CNs from a variety of perspectives, such as stability, robustness, synchronization, consensus, pinning control, cyber attacks, and output-feedback control under various scheduling protocols [10]–[17].

As is well known, gaining insight into the exact states of CNs is critically important for understanding and analyzing their dynamic behaviors, such as synchronization, consensus, and flocking. Undeniably, it may sometimes be feasible to directly measure the states of a CN by resorting to certain appropriate sensors. Frequently, however, it may be either impossible or simply impractical to obtain measurements for all states. Instead, in most cases, what can be available are system outputs or measurements. As a matter of fact, it is of paramount importance to take full advantage of the available measurement information and accordingly design effective state estimation algorithms to infer the unmeasurable system states. Up to now, considerable research interest has been focused on the state estimation problems for various types of CNs, leading to a wealth of valuable results in the literature, see e.g., [18]–[21] and the references therein.

It should be emphasized that, in the majority of existing results on state estimation for CNs, there is an implicit assumption that the measurement outputs from all the network nodes are accessible. Even though this assumption may be valid for low-dimensional systems or small-scale networks, it becomes a bit too harsh and unreasonable for large-scale CNs with a plethora of network nodes due to various factors, such as the limited availability of measuring resources, and the hardware constraints of physical sensors. Additionally, malicious denial-of-service attacks or jamming attacks may block the transmission channel, resulting in measurements from sensors potentially failing to reach their destination in a timely manner. In such a context, a more reasonable assumption is that only the measurement outputs from a small subset of network nodes are available, which underscores the necessity of developing feasible strategies that can estimate the states of interest based on the measurement information from a fraction of network nodes. This concept is commonly referred to as the partial-nodes-based (PNB) state estimation problem, which was first proposed and addressed in [22]–[24]. Nevertheless, the corresponding problem for nonlinear CNs has not received adequate research attention, despite its clear practical significance, and this constitutes the first motivation

This work was supported in part by National Natural Science Foundation of China under Grants 62173292, 62376242, 62103359 and 62506326, Postdoctoral Fellowship Program of CPSF of China under Grant GZC20232210, and Key Laboratory of Yangzhou of China under Grant YZ2024245. *(Corresponding author: Luyang Yu.)*

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of launching our current investigation.

The past several decades have witnessed the popularity of networked systems due mainly to the rapid advancement of the networked communication technology. In modern industrial systems, the data transmissions across the sensor-to-estimator and controller-to-actuator channels are usually facilitated by the digital communication networks [25], [26]. Compared with the traditional analog communication, the digital counterpart has recently gained widespread attention from academia and industry owing to its distinctive advantages in terms of high reliability, enhanced maintainability, low susceptibility to interference, and reduced power consumption. Nevertheless, the finite channel capacity of a digital network would inevitably limit the total amount of data that can be simultaneously and reliably transmitted via such a network. To tackle the network bandwidth constraints, the encoding-decoding technologies, which aim to compress the data before transmission, have emerged as both effective and appealing solutions. So far, a large volume of research has been conducted on the encoding-decoding-based analysis and synthesis problems, see, e.g., [27]–[33].

In the context of digital channels with limited bandwidth, the traditional encoding-decoding-based control or state estimation methods usually rely on a periodic transmission mechanism, where the data is only processed at the discrete sampling instants. Notably, such a transmission mechanism would largely simplify the design and analysis issues of controllers or state estimators. Nevertheless, from the resource utilization perspective, the periodic strategy might be inefficient since the data is transmitted at a fixed rate, regardless of the difference between two successive data, thereby leading to a waste of scarce resources. In this sense, a preferred scheme is to transmit the data packet only when certain predefined triggering conditions are fulfilled [34]–[41], which is known as the event-driven transmission (EDT) strategy.

With the purpose of further lowering the data transmission frequency, a dynamic event-driven transmission (DEDT) mechanism has recently been proposed in [42] by introducing a time-varying adjustable parameter, which is generated via an auxiliary system closely related to the target states or measurements. Since then, a great deal of research attention has been focused on designing novel and effective DEDT strategies, and some elegant research results have appeared in the literature regarding various control and state estimation tasks [43]–[48]. In the implementation of the aforementioned EDT strategies, additional hardware should be utilized to continuously monitor the system states/measurements and accordingly detect whether the current value exceeds the predefined triggering threshold. Clearly, the continuous monitoring would lead to excessive energy consumption and increase the complexity of practical implementation. On the other hand, to guarantee the feasibility of EDT strategies, the Zeno behavior must be eliminated by ensuring that the minimum time interval between any two consecutive events is strictly greater than zero.

Recently, the sampled-data-based EDT mechanisms have been put forward to overcome the above-identified shortcomings. The underlying idea behind the sampled-data-based EDT strategy is that the system states or measurements are periodically sampled, and the decision of whether to transmit the sampled data is based on a prescribed event-triggering condition. Apparently, under the sampled-data-based EDT strategy, the extra hardware is no longer required and the undesired Zeno behavior can be naturally excluded since the minimum inter-event time is always equal to or greater than the sampling period. Therefore, the sampled-data-based EDT strategy has gained prominence in the relevant research, which gives rise to a number of interesting results in the literature, see, e.g., [49]–[52]. Particularly, the sampled-data-based DEDT strategy, integrating the traditional DEDT mechanism with the sampled-data-based technique, has shown potential to become a major research focus [39]. Nevertheless, to the best of our knowledge, the encoding-decoding-based state estimation for nonlinear CNs under such a strategy is still an open problem, not to mention the case where the measurement outputs are only available for a fraction of network nodes. The second motivation of this paper is to shorten such a gap.

In response to the above discussions, in this paper, we are committed to investigating the encoding-decoding-based PNB state estimation problem for a class of continuous-time nonlinear CNs with communication bandwidth constraints. This appears to be a non-trivial task for the following challenges. 1) How to construct an effective state estimator to trace the states of the target network based on the outputs only from a proportion of nodes? 2) How to conduct the analysis on dynamical behavior of the estimation error system when the dynamic event-driven encoding mechanism is integrated into the design of state estimator? 3) How to establish some easy-to-check criteria to ensure the estimator performance? The main contributions of this paper can be highlighted as follows.

1) The encoding-decoding-based PNB state estimation problem is new for the continuous-time nonlinear CNs under communication bandwidth constraints. Specifically, the encoding-decoding strategy is utilized to enhance the efficiency and confidentiality of the data transmission over a digital communication channel, and the state estimates are generated by only employing the sampled measurement outputs from a fraction of network nodes.
2) By incorporating the discrete-time event detectors, a novel sampled-data-based DEDT strategy is proposed to reduce the consumption of limited network resources. Compared with the existing results [43], [44], [53], the proposed DEDT strategy determines whether the current data should be transmitted at the periodic sampling instants, avoiding the continuous monitoring of measurements and guaranteeing the natural elimination of Zeno phenomenon.
3) Based on the Lyapunov stability theory and the matrix inequality techniques, sufficient conditions are established to ensure that the estimation error system is exponentially stable. In addition, a convex optimization approach is utilized to design the estimator gain, aiming to maximize the allowable bound of the sampling intervals.

Notations: $\mathbb{R}^p$ and $\mathbb{R}^{p\times q}$ stand for, respectively, the p-dimensional Euclidean space and the set of $p\times q$ real matrices. $\varPhi^T$ and $\varPhi^{-1}$ represent, respectively, the transpose and inverse of matrix $\varPhi$. $\lambda_M(\varPi)$ and $\lambda_m(\varPi)$ are, respectively, the maximum and minimum eigenvalues of a symmetric matrix $\varPi\in\mathbb{R}^{p\times p}$. For $z\in\mathbb{R}^p$, its norm, denoted as $\|z\|$, is defined as the square root of z^Tz. For a matrix $\varPsi$, its norm, denoted as $\|\varPsi\|$, is defined as the square root of the largest eigenvalue of the matrix $\varPsi^T\varPsi$. The notation $\varPsi>\varPhi$ ($\varPsi\geq\varPhi$) implies that $\lambda_m(\varPsi-\varPhi)>0$ ($\lambda_m(\varPsi-\varPhi)\geq 0$). $\mathbb{I}_p^q\triangleq\{p,p+1,\dots,q\}$, where p and q are integers satisfying $q>p\geq 0$.

II. Problem Formulation

Consider a nonlinear CN consisting of N non-identical nodes with the following form:

$$\dot{s}_p(t)=C_ps_p(t)+g(s_p(t))+\sum_{q\in\mathbb{I}_1^N}w_{pq}U(s_q(t)-s_p(t)), \tag{1}$$

where $s_p(t)\in\mathbb{R}^n$ stands for the state vector of the p-th node; $g:\mathbb{R}^n\to\mathbb{R}^n$ is a known nonlinear vector-valued function; $C_p\in\mathbb{R}^{n\times n}$ is a known constant matrix; $W\triangleq(w_{pq})_{N\times N}\in\mathbb{R}^{N\times N}$ denotes the weighted adjacency matrix, where $w_{pq}>0$ if node p can receive information from node q, and $w_{pq}=0$ otherwise; and U is a known matrix representing the inner coupling matrix of the network.

Without loss of generality, we assume that only the measurement information of the first N_0 nodes is available, that is,

$$z_p(t)=E_ps_p(t),\ p\in\mathbb{I}_1^{N_0}, \tag{2}$$

where $z_p(t)\in\mathbb{R}^m$ denotes the measurement output of the p-th node and $E_p\in\mathbb{R}^{m\times n}$ is a known constant matrix.

In a networked environment, the communication channel is inevitably constrained by the limited bandwidth resources. To this end, we are going to put forward a novel dynamic event-driven encoding scheme to improve the efficiency of data transmission, which is focused on reducing the transmission frequency via the sampled-data-based DEDT mechanism and compressing the signal through encoding technique. More specifically, the measurement signal is firstly sampled at a constant period $d>0$ with the sampling instants denoted by $\{t_r\}_{r=0}^{+\infty}$, where $t_0=0$ and $t_{r+1}=t_r+d$. Then, the event generator is responsible for determining whether the current sampled data should be transmitted or not. If the triggering condition is satisfied, the signal is encoded into a codeword by the encoders and the codeword is subsequently transmitted to the decoders through a communication network. Finally, the decoders receive the codeword and the codeword is decoded for the purpose of the state estimation of nonlinear CNs. For the p-th node ($p\in\mathbb{I}_1^{N_0}$), the triggered sequence is denoted by $t_0=T_0^p<T_1^p<T_2^p<\cdots<T_k^p<\cdots$, where the employed event generator will be specified later.

Based on the EDT mechanism, we are now going to present the event-driven encoding and decoding scheme. The event-driven encoding algorithm for the p-th node is described as follows:

$$\begin{cases}\dot{\eta}_p(t)=C_p\eta_p(t)+\sum\limits_{q\in\mathbb{I}_1^N}w_{pq}U(\eta_q(t)-\eta_p(t))\\ \qquad\qquad+g\left(\eta_p(t)\right)+h\left(t_r\right)L_pf_p\left(T_k^p\right),\\ \qquad\qquad\qquad t\in[t_r,t_{r+1}),p\in\mathbb{I}_1^{N_0},\\ f_p\left(T_k^p\right)=\Phi\left(\dfrac{z_p(T_k^p)-E_p\eta_p(T_k^p)}{h\left(T_k^p\right)}\right),\ p\in\mathbb{I}_1^{N_0},\\ \dot{\eta}_p(t)=C_p\eta_p(t)+g\left(\eta_p(t)\right)\\ \qquad\qquad+\sum\limits_{q\in\mathbb{I}_1^N}w_{pq}U(\eta_q(t)-\eta_p(t)),\ p\in\mathbb{I}_{N_0+1}^N,\\ \eta_p\left(t_0\right)=\mathbf{0}_n,\ p\in\mathbb{I}_1^N,\end{cases} \tag{3}$$

where $t_r\in[T_k^p,T_{k+1}^p)$, $\eta_p(t)\in\mathbb{R}^n$ is the encoder's internal variable, $f_p(t)$ denotes the codeword to be transmitted at time instant t, and L_p is the estimator gain matrix. $h(t)=h_0e^{-\rho t}$, where $h_0>0$ is a given parameter and $\rho>0$ is a design parameter to be determined later.

In (3), $\Phi(\cdot)$ represents a finite-level vector quantizer. Suppose that $x\in\mathbb{R}^m$ is the signal to be quantized, then one has $\Phi(x)\triangleq(\phi(x_1),\phi(x_2),\dots,\phi(x_m))^T$, where $\phi(\cdot)$ is a $(2M+1)$-level uniform quantizer given by

$$\phi(x_l)=\begin{cases}\hbar\Delta, & (\hbar-\frac{1}{2})\Delta\leq x_l<(\hbar+\frac{1}{2})\Delta,\\ & \hbar=0,1,2,\dots,M,\\ M\Delta, & x_l\geq(M+\frac{1}{2})\Delta,\\ -\phi(-x_l), & x_l<-\frac{1}{2}\Delta,\end{cases} \tag{4}$$

where $l=1,2,\dots,m$, $\Delta>0$ and $M\in\mathbb{N}^+$ denote the given quantization parameters. Obviously, if $|x_l|\leq(M+1/2)\Delta$, one has $|\phi(x_l)-x_l|\leq\Delta/2$.

The corresponding decoding (data recovery) algorithm for the p-th node is given by

$$\begin{cases}\dot{\hat{s}}_p(t)=C_p\hat{s}_p(t)+\sum\limits_{q\in\mathbb{I}_1^N}w_{pq}U(\hat{s}_q(t)-\hat{s}_p(t))\\ \qquad\qquad+g\left(\hat{s}_p(t)\right)+h\left(t_r\right)L_pf_p\left(T_k^p\right),\\ \qquad\qquad\qquad t\in[t_r,t_{r+1}),\ p\in\mathbb{I}_1^{N_0},\\ \dot{\hat{s}}_p(t)=C_p\hat{s}_p(t)+g\left(\hat{s}_p(t)\right)\\ \qquad\qquad+\sum\limits_{q\in\mathbb{I}_1^N}w_{pq}U(\hat{s}_q(t)-\hat{s}_p(t)),\ p\in\mathbb{I}_{N_0+1}^N,\\ \hat{s}_p\left(t_0\right)=\mathbf{0}_n,\ p\in\mathbb{I}_1^N,\end{cases} \tag{5}$$

where $\hat{s}_p(t)$ is the estimate of $s_p(t)$. It is not difficult to verify that $\hat{s}_p(t)=\eta_p(t)$ for all $t\geq 0$.

Based on the above formulations, the state estimator for the nonlinear CN (1) is constructed as follows:

$$\dot{\hat{s}}_p(t)=C_p\hat{s}_p(t)+g(\hat{s}_p(t))+\sum_{q\in\mathbb{I}_1^N}w_{pq}U(\hat{s}_q(t)-\hat{s}_p(t))+h\left(t_r\right)L_pf_p\left(T_k^p\right),\ t\in[t_r,t_{r+1}),\ p\in\mathbb{I}_1^{N_0}, \tag{6a}$$

$$\dot{\hat{s}}_p(t)=C_p\hat{s}_p(t)+g(\hat{s}_p(t))+\sum_{q\in\mathbb{I}_1^N}w_{pq}U(\hat{s}_q(t)-\hat{s}_p(t)),\quad p\in\mathbb{I}_{N_0+1}^N. \tag{6b}$$

Now, let us move on to design the event generator and accordingly regulate the process of information transmission. Specifically, the triggering instants for the p-th node are iteratively determined by

$$T^p_{k+1} = \min\Big\{t_r > T^p_k \,|\, \sigma_p\|y_p(t_r)\|^2 - \gamma_p\|\zeta_p(t_r)\|^2 > \varphi_p\phi_p(t_r)\Big\},\ p\in\mathbb{I}_1^{N_0}, \quad (7)$$

where φ_p is a given positive constant, $\zeta_p(t) = z_p(t) - E_p\eta_p(t)$, $y_p(t) = \zeta_p(t) - h(t)\frac{\zeta_p(T^p_k)}{h(T^p_k)}$, and $\phi_p(t)$ is an auxiliary variable generated by

$$\dot{\phi}_p(t) = -\alpha_p\phi_p(t) - \beta_p(\sigma_p\|y_p(t_r)\|^2 - \gamma_p\|\zeta_p(t_r)\|^2),\quad t\in[t_r, t_{r+1}), \quad (8)$$

where $\phi_p(0) \triangleq \phi^0_p$, ϕ^0_p, α_p, β_p, and φ_p are given positive constants, σ_p and γ_p are positive parameters to be designed.

Remark 1: Notably, the proposed sampled-data-based DEDT strategy in (7) and (8) presents three significant advantages over conventional event-driven methods [23], [35], [41]: 1) For the p-th node ($p\in\mathbb{I}_1^{N_0}$), the sampled-data-based DEDT strategy only requires transmission of discrete-time data packets $z_p(t_r)$ at sampling instants, unlike conventional methods that rely on the continuous measurement information $z_p(t)$. 2) Whether or not the data need to be transmitted is only examined at the discrete sampling times, thereby removing the requirements of extra hardware to continuously monitor and compute. 3) The operation of data transmission only occurs at the sampling instants, implying that the Zeno behavior is naturally excluded.

Let $\varepsilon_p(t) = \hat{s}_p(t) - s_p(t)$ be the estimation error and $B = (b_{pq})_{N\times N}$ be the Laplacian matrix of the CN (1) with $b_{pq} = -w_{pq}$ (if $p\neq q$) and $b_{pp} = -\sum_{q\neq p} w_{pq}$. Then, the estimation error $\varepsilon_p(t)$ satisfies the following relationships

$$\dot{\varepsilon}_p(t) = C_p\varepsilon_p(t) + \bar{g}(\varepsilon_p(t)) - \sum_{q\in\mathbb{I}_1^N} b_{pq}U\varepsilon_q(t) + h(t_r)L_pf_p\left(T^p_k\right),\ t\in[t_r,t_{r+1}),\ p\in\mathbb{I}_1^{N_0}, \quad (9a)$$

$$\dot{\varepsilon}_p(t) = C_p\varepsilon_p(t) + \bar{g}(\varepsilon_p(t)) - \sum_{q\in\mathbb{I}_1^N} b_{pq}U\varepsilon_q(t),\quad p\in\mathbb{I}_{N_0+1}^N, \quad (9b)$$

where $\bar{g}(\varepsilon_p(t)) = g(\hat{s}_p(t)) - g(s_p(t))$. For the nonlinear CN (1) and the state estimator (6), let us denote $\varepsilon(t) = (\varepsilon_1^T(t), \varepsilon_2^T(t), \ldots, \varepsilon_N^T(t))^T$.

Definition 1: The encoding-decoding-based state estimator (6) under the sampled-data-based DEDT mechanism (7) and (8) is termed as an exponential state estimator for the CN (1) if there exist two positive numbers $\mathfrak{M}$ and $\mathfrak{N}$ such that

$$\|\varepsilon(t)\| \le \mathfrak{M}e^{-\mathfrak{N}t},\ t\ge 0. \quad (10)$$

In this paper, we are interested in dealing with the PNB state estimation problem for nonlinear CN (1) under the novel sampled-data-based DEDT scheme (7) and (8), whose schematic structure is depicted in Fig. 1. More specifically, we aim to derive certain sufficient conditions to ensure that the encoding-decoding-based estimator (6) is an exponential state estimator for the considered CN. Moreover, by solving a convex optimization problem, the estimator gain matrix will be designed to maximize the allowable bound of the sampling intervals.

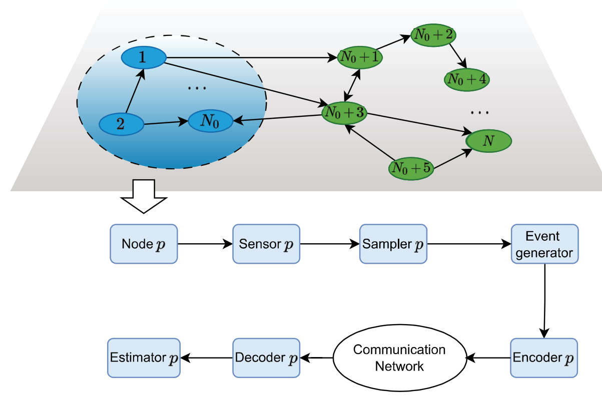

Fig. 1. Schematic of PNB state estimation problem for CNs with communication bandwidth constraints under the sampled-data-based DEDT scheme.

III. Main Results

In this section, the encoding-decoding-based state estimator will be designed for the nonlinear CN (1) under the partial nodes' measurement outputs and the sampled-data-based DEDT strategy (7) and (8). To begin with, let us first introduce the following lemma and assumptions.

Lemma 1: For the given positive scalars ϕ^0_p, β_p, φ_p, and α_p, if the sampling interval satisfies $d \le -\frac{1}{\alpha_p}\ln\frac{\beta_p\varphi_p}{\alpha_p+\beta_p\varphi_p}$, then the auxiliary variable $\phi_p(t)$ ($p\in\mathbb{I}_1^{N_0}$) is non-negative at all time instants under the sampled-data-based DEDT strategy (7) and (8).

Proof: For any $t\ge 0$, there exists a non-negative integer r such that $t\in[t_r, t_{r+1})$. Based on the sampled-data-based DEDT mechanism (7), one has

$$(\sigma_p\|y_p(t_r)\|^2 - \gamma_p\|\zeta_p(t_r)\|^2) \le \varphi_p\phi_p(t_r). \quad (11)$$

Bearing in mind the fact that

$$\dot{\phi}_p(t) = -\alpha_p\phi_p(t) - \beta_p(\sigma_p\|y_p(t_r)\|^2 - \gamma_p\|\zeta_p(t_r)\|^2),\quad t\in[t_r,t_{r+1}), \quad (12)$$

we have

$$\dot{\phi}_p(t) \ge -\alpha_p\phi_p(t) - \beta_p\varphi_p\phi_p(t_r),\ t\in[t_r,t_{r+1}). \quad (13)$$

After some simple computations, it is clear to see that

$$\begin{aligned}\phi_p(t) &\ge \left(\left(1+\frac{\beta_p\varphi_p}{\alpha_p}\right)e^{-\alpha_p(t-t_r)} - \frac{\beta_p\varphi_p}{\alpha_p}\right)\phi_p(t_r)\\ &\ge \left(\left(1+\frac{\beta_p\varphi_p}{\alpha_p}\right)e^{-\alpha_p d} - \frac{\beta_p\varphi_p}{\alpha_p}\right)\phi_p(t_r),\\ &\qquad t\in[t_r,t_{r+1}). \end{aligned} \quad (14)$$

Let $\pi_p = \left(1+\frac{\beta_p\varphi_p}{\alpha_p}\right)e^{-\alpha_p d} - \frac{\beta_p\varphi_p}{\alpha_p}$. Noting that $d \le -\frac{1}{\alpha_p}\ln\frac{\beta_p\varphi_p}{\alpha_p+\beta_p\varphi_p}$, one has $\pi_p\ge 0$. Then, we can arrive at

$$\phi_p(t) \ge \pi_p\phi_p(t_r) \ge \pi_p^2\phi_p(t_{r-1})$$

$$\geq \cdots \geq \pi_p^{r+1}\phi_p(0) \geq 0,\ t \geq 0, \tag{15}$$

which completes the proof. ∎

Remark 2: Letting $\mathfrak{f}_p(\alpha_p, \beta_p, \varphi_p) = -\frac{1}{\alpha_p}\ln\frac{\beta_p\varphi_p}{\alpha_p+\beta_p\varphi_p}$, it is not difficult to see that $\mathfrak{f}_p(\alpha_p, \beta_p, \varphi_p) > 0$ holds for arbitrary positive numbers α_p, β_p, and φ_p. In other words, there must exist some $d > 0$ satisfying $d \leq \mathfrak{f}_p(\alpha_p, \beta_p, \varphi_p)$ for any given positive scalars α_p, β_p, and φ_p. Moreover, for the given $\alpha_p > 0$ and $\varphi_p > 0$, we have $\lim_{\beta_p\to 0^+}\mathfrak{f}_p(\alpha_p, \beta_p, \varphi_p) = +\infty$. Therefore, when implementing the proposed sampled-data-based DEDT strategy, a smaller positive number β_p would lead to a larger sampling period d.

Assumption 1: The nonlinear vector-valued function $g(\cdot)$ is continuous and satisfies the following relationship

$$[g(\theta_1) - g(\theta_2) - Z_1(\theta_1 - \theta_2)]^T \times [g(\theta_1) - g(\theta_2) - Z_2(\theta_1 - \theta_2)] \leq 0, \tag{16}$$

for arbitrary θ_1 and $\theta_2 \in \mathbb{R}^n$, where Z_1 and Z_2 are known constant matrices.

Assumption 2: There exists a known scalar $\chi > 0$ such that $\|s_p(0)\| \leq \chi$, $p \in \mathbb{I}_1^N$.

For notational convenience, let us denote

$$\begin{aligned}
\varepsilon(t) &= (\varepsilon_1^T(t), \varepsilon_2^T(t), \ldots, \varepsilon_N^T(t))^T,\\
\bar{\varepsilon}(t) &= (\varepsilon_1^T(t), \varepsilon_2^T(t), \ldots, \varepsilon_{N_0}^T(t))^T,\\
C &= \mathrm{diag}\{C_1, C_2, \ldots, C_N\},\\
\bar{L} &= \mathrm{diag}\{L_1, L_2, \ldots, L_{N_0}\},\\
\bar{E} &= \mathrm{diag}\{E_1, E_2, \ldots, E_{N_0}\},\\
L &= \begin{bmatrix}\bar{L}\\ 0\end{bmatrix}, E = \begin{bmatrix}\bar{E} & 0\end{bmatrix}.
\end{aligned}$$

The following theorem provides a sufficient condition to guarantee the existence of an exponential state estimator for the CN (1) with communication bandwidth constraints.

Theorem 1: Let the estimator gain matrix L and the scalars $\alpha_p \geq \theta_1 > 0$, $\beta_p > 0$ and $\varphi_p > 0$ ($p \in \mathbb{I}_1^{N_0}$) be given. Under Assumptions 1 and 2, the encoding-decoding-based estimator (6) with the triggering mechanism (7) and (8) is an exponential state estimator for the CN (1) if there exists a block diagonal matrix $P = \mathrm{diag}\{P_1, P_2, \ldots, P_N\} > 0$, a diagonal matrix $\Psi > 0$, and positive scalars δ_1, δ_2, δ_3, ϖ_1, ϖ_2, σ_p and γ_p ($p \in \mathbb{I}_1^{N_0}$) such that the following inequalities hold

$$\varpi_1 - \delta_2 > 0, \tag{17}$$

$$\sigma_p\beta_p - \varpi_1 \geq 0, \tag{18}$$

$$\gamma_p\beta_p - \varpi_2 \leq 0, \tag{19}$$

$$\theta_1 - \varpi_2\lambda_M(P^{-1})\lambda_M(E^TE) > 0, \tag{20}$$

$$\Pi \triangleq \begin{bmatrix}\Pi_0 & P + \Psi\otimes\check{Z}_2\\ * & -\Psi\otimes I\end{bmatrix} < 0, \tag{21}$$

and the sampling interval satisfies

$$d < \min\{d_1, d_2, d_3\}, \tag{22}$$

with the following relationships

$$M = \max\{M_1, M_2\}, \tag{23}$$

$$\varrho > 2\rho > 0, \tag{24}$$

where $\Pi_0 = P(C - B\otimes U - LE) + (C - B\otimes U - LE)^TP + \theta_1P + \delta_1^{-1}PLEE^TL^TP + (\delta_2^{-1} + \delta_3^{-1})PLL^TP - \Psi\otimes\check{Z}_1$, $\check{Z}_1 = (Z_1^TZ_2 + Z_2^TZ_1)/2$, $\check{Z}_2 = (Z_1^T + Z_2^T)/2$, $d_1 = \sqrt{\frac{\theta_1-\theta_2}{2\delta_1\mu^2\lambda_M(P^{-1})}}$, $d_2 = \min_{p\in\mathbb{I}_1^{N_0}}\left\{-\frac{1}{\alpha_p}\ln\frac{\beta_p\varphi_p}{\alpha_p+\beta_p\varphi_p}\right\}$, $d_3 = \frac{\sqrt{\varpi_1-\delta_2}}{2\|L\|}$, $M_1 = \left\lceil\max_{p\in\mathbb{I}_1^{N_0}}\left\{\frac{\|E_p\|\chi}{h_0\Delta}\right\}\right\rceil$, $M_2 = \left\lceil\max_{p\in\mathbb{I}_1^{N_0}}\left\{\frac{\|E_p\|}{h_0\Delta}\left(\sqrt{\frac{\vartheta}{\lambda_m(P)}} + \frac{\sqrt{\theta_5}}{\sqrt{\lambda_m(P)}(\sqrt{\mathfrak{n}}-\sqrt{\mathfrak{m}})}\right)\right\}\right\rceil$, $\mathfrak{m} = e^{-\varrho d}$, $\mathfrak{n} = e^{-2\rho d}$, $\vartheta = \lambda_M(P)N\chi^2 + \sum_{p\in\mathbb{I}_1^{N_0}}\phi_p^0$, $\mu = \|C - B\otimes U\| + \kappa + \|L\bar{E}\|$, $\theta_2 = \varpi_2\lambda_M(P^{-1})\lambda_M(E^TE)$, $\theta_3 = 2\delta_1d^2\mu^2\lambda_M(P^{-1})$, $\theta_4 = (\delta_3 + 4\delta_1d^2\|L\|^2)h_0^2\frac{mN_0\Delta^2}{4}$, $\theta_5 = \frac{\theta_4}{\theta_1-\theta_2-\theta_3}$, $\kappa = (\|Z_1 + Z_2\| + \|Z_1 - Z_2\|)/2$, and ϱ is the positive solution to the following equation

$$\theta_1 - (\theta_2 + \theta_3)e^{\varrho d} - \varrho = 0. \tag{25}$$

Proof: It follows from (9) that

$$\begin{aligned}
h(t_r)f_p(T_k^p) &= h(t_r)f_p(T_k^p) - h(t_r)\frac{\zeta_p(T_k^p)}{h(T_k^p)}\\
&\quad + h(t_r)\frac{\zeta_p(T_k^p)}{h(T_k^p)} - \zeta_p(t_r) + \zeta_p(t_r)\\
&= h(t_r)\xi_p(T_k^p) - E_p\varepsilon_p(t_r) - y_p(t_r),
\end{aligned} \tag{26}$$

where $\xi_p(t) = f_p(t) - \frac{\zeta_p(t)}{h(t)}$.

With a slight abuse of notation, let us denote $\xi_p(T_k^p)$ by $\xi_p(\bar{T})$. Then, based on (9) and (26), the estimation error $\varepsilon_p(t)$ satisfies the following dynamics

$$\begin{aligned}
\dot{\varepsilon}_p(t) &= C_p\varepsilon_p(t) + \bar{g}(\varepsilon_p(t)) - \sum_{q\in\mathbb{I}_1^N} b_{pq}U\varepsilon_q(t)\\
&\quad + h(t_r)L_p\xi_p(\bar{T}) - L_pE_p\varepsilon_p(t_r) - L_py_p(t_r),\\
&\qquad t \in [t_r, t_{r+1}),\ p \in \mathbb{I}_1^{N_0},
\end{aligned} \tag{27a}$$

$$\dot{\varepsilon}_p(t) = C_p\varepsilon_p(t) + \bar{g}(\varepsilon_p(t)) - \sum_{q\in\mathbb{I}_1^N} b_{pq}U\varepsilon_q(t),\quad p \in \mathbb{I}_{N_0+1}^N, \tag{27b}$$

which can be rewritten in a compact form

$$\begin{aligned}
\dot{\varepsilon}(t) =& (C - B\otimes U)\varepsilon(t) + G(\varepsilon(t)) - LY(t_r)\\
&+ h(t_r)L\xi(\bar{T}) - L\bar{E}\bar{\varepsilon}(t_r),\ t \in [t_r, t_{r+1}),
\end{aligned} \tag{28}$$

where $G(\varepsilon(t)) = (\bar{g}(\varepsilon_1^T(t)), \bar{g}(\varepsilon_2^T(t)), \ldots, \bar{g}(\varepsilon_N^T(t)))^T$, $Y(t) = (y_1^T(t), y_2^T(t), \ldots, y_{N_0}^T(t))^T$, and $\xi(t) = (\xi_1^T(t), \xi_2^T(t), \ldots, \xi_{N_0}^T(t))^T$.

Letting $\Upsilon_p(t) = \varepsilon_p(t) - \varepsilon_p(t_r)$ and $\Upsilon(t) = (\Upsilon_1^T(t), \Upsilon_2^T(t), \ldots, \Upsilon_{N_0}^T(t))^T$, $t \in [t_r, t_{r+1})$, the error dynamics (28) can be further rewritten by

$$\begin{aligned}
\dot{\varepsilon}(t) &= (C - B\otimes U)\varepsilon(t) + G(\varepsilon(t)) - LY(t_r)\\
&\quad + h(t_r)L\xi(\bar{T}) - L\bar{E}\bar{\varepsilon}(t_r) + L\bar{E}\bar{\varepsilon}(t) - L\bar{E}\bar{\varepsilon}(t)\\
&= (C - B\otimes U - LE)\varepsilon(t) + G(\varepsilon(t))\\
&\quad + L\bar{E}\Upsilon(t) - LY(t_r) + h(t_r)L\xi(\bar{T}),\ t \in [t_r, t_{r+1}).
\end{aligned} \tag{29}$$

Now, we are in a position to claim that the vector quantizer $\Phi(\cdot)$ will never be saturated. In fact, according to Assumption 2 and (23), one has

$$\frac{\|E_p\varepsilon_p(0)\|_\infty}{h(0)} \leq \frac{\|E_p\varepsilon_p(0)\|}{h(0)} \leq \frac{\|E_p\|\|\varepsilon_p(0)\|}{h(0)}$$

$$\leq \left(M+\frac{1}{2}\right)\Delta,\ p\in\mathbb{I}_1^{N_0}, \tag{30}$$

which implies that when $r=0$, the quantizer is unsaturated.

Next, let us consider the worst situation where the event occurs at each sampling instant t_r. Suppose that when $r=1,2,\ldots,l$, the quantizer is unsaturated, namely,

$$\sup_{r\in\mathbb{I}_0^l}\frac{\|E_p\varepsilon_p(t_r)\|_\infty}{h(t_r)}\leq\left(M+\frac{1}{2}\right)\Delta,\ p\in\mathbb{I}_1^{N_0}. \tag{31}$$

In what follows, we shall show that when r = $l+1$, the quantizer is unsaturated.

Consider the following Lyapunov function candidate

$$V(t)=V_1(t)+V_2(t), \tag{32}$$

where

$$V_1(t)=\varepsilon^T(t)P\varepsilon(t), \tag{33}$$

$$V_2(t)=\sum_{p\in\mathbb{I}_1^{N_0}}\phi_p(t). \tag{34}$$

Taking the time derivative of $V(t)$ along the error system (28) yields

$$\dot V(t)=\dot V_1(t)+\dot V_2(t),\ t\in[t_l,t_{l+1}), \tag{35}$$

where

$$\dot V_1(t)=2\varepsilon^T(t)P\Big[(C-B\otimes U-LE)\varepsilon(t)+G(\varepsilon(t)) + L\bar E\Upsilon(t)-LY(t_l)+h(t_l)L\xi(\bar T)\Big] \tag{36}$$

and

$$\begin{aligned}\dot V_2(t)=&\sum_{p\in\mathbb{I}_1^{N_0}}\Big(-\alpha_p\phi_p(t)-\beta_p(\sigma_p\|y_p(t_l)\|^2 - \gamma_p\|\zeta_p(t_l)\|^2)\Big)\\ \leq&-\theta_1V_2(t)-\varpi_1\|Y(t_l)\|^2+\varpi_2\|E\varepsilon(t_l)\|^2.\end{aligned} \tag{37}$$

It is clear that

$$2\varepsilon^T(t)PL\bar E\Upsilon(t)\leq\delta_1\Upsilon^T(t)\Upsilon(t) +\delta_1^{-1}\varepsilon^T(t)PL\bar E\bar E^TL^TP\varepsilon(t), \tag{38}$$

$$-2\varepsilon^T(t)PLY(t_l)\leq\delta_2Y^T(t_l)Y(t_l) +\delta_2^{-1}\varepsilon^T(t)PLL^TP\varepsilon(t), \tag{39}$$

$$2h(t_l)\varepsilon^T(t)PL\xi(\bar T)\leq\delta_3^{-1}\varepsilon^T(t)PLL^TP\varepsilon(t) +\delta_3h^2(t_l)\xi^T(\bar T)\xi(\bar T). \tag{40}$$

In virtue of Assumption 1, one can conclude that

$$\sum_{p\in\mathbb{I}_1^N}\psi_p\Big(\varepsilon_p^T(t)\check Z_1\varepsilon_p(t)-2\varepsilon_p^T(t)\check Z_2\bar g(\varepsilon_p(t)) +\bar g^T(\varepsilon_p(t))\bar g(\varepsilon_p(t))\Big)\leq 0, \tag{41}$$

which implies that

$$\varepsilon^T(t)(\Psi\otimes\check Z_1)\varepsilon(t)-2\varepsilon^T(t)(\Psi\otimes\check Z_2)G(\varepsilon(t)) +G^T(\varepsilon(t))(\Psi\otimes I)G(\varepsilon(t))\leq 0. \tag{42}$$

Consequently, it can be derived from (35)-(42) that

$$\begin{aligned}\dot V(t)\leq&\ 2\varepsilon^T(t)P(C-B\otimes U-LE)\varepsilon(t)+2\varepsilon^T(t)PG(\varepsilon(t))\\ &+\delta_1^{-1}\varepsilon^T(t)PL\bar E\bar E^TL^TP\varepsilon(t)+\delta_1\Upsilon^T(t)\Upsilon(t)\\ &+\delta_2^{-1}\varepsilon^T(t)PLL^TP\varepsilon(t)+\delta_2Y^T(t_l)Y(t_l)\\ &+\delta_3^{-1}\varepsilon^T(t)PLL^TP\varepsilon(t)+\delta_3h^2(t_l)\xi^T(\bar T)\xi(\bar T)\\ &-\varepsilon^T(t)(\Psi\otimes\check Z_1)\varepsilon(t)+2\varepsilon^T(t)(\Psi\otimes\check Z_2)G(\varepsilon(t))\\ &-G^T(\varepsilon(t))(\Psi\otimes I)G(\varepsilon(t))\\ &-\theta_1V_2(t)-\varpi_1\|Y(t_l)\|^2+\varpi_2\|E\varepsilon(t_l)\|^2\\ =&\ \nu^T(t)\Pi\nu(t)-\theta_1V(t)+\delta_1\Upsilon^T(t)\Upsilon(t)\\ &+\delta_3h^2(t_l)\xi^T(\bar T)\xi(\bar T)+\varpi_2\varepsilon^T(t_l)E^TE\varepsilon(t_l)\\ &-(\varpi_1-\delta_2)Y^T(t_l)Y(t_l)\\ \leq&\ -\theta_1V(t)+\varpi_2\lambda_M(P^{-1})\lambda_M(E^TE)V(t_l)\\ &+\delta_1\Upsilon^T(t)\Upsilon(t)+\delta_3h^2(t_l)\xi^T(\bar T)\xi(\bar T)\\ &-(\varpi_1-\delta_2)Y^T(t_l)Y(t_l),\end{aligned} \tag{43}$$

where $\nu(t)=(\varepsilon^T(t),G^T(\varepsilon(t)))^T$.

Next, we are going to give an estimate for $\Upsilon(t)$. Note that $\Upsilon(t_l)=0$ and

$$\begin{aligned}\dot\Upsilon(t)=&\dot{\bar\varepsilon}(t)=\mathcal{I}\dot\varepsilon(t)\\ =&\mathcal{I}\Big((C-B\otimes U)\varepsilon(t)+G(\varepsilon(t))-LY(t_l)\\ &+h(t_l)L\xi(\bar T)-L\bar E\bar\varepsilon(t_l)\Big),\ t\in[t_l,t_{l+1}),\end{aligned} \tag{44}$$

where $\mathcal{I}=\begin{bmatrix}\bar{\mathcal{I}} & 0\end{bmatrix}$ with $\bar{\mathcal{I}}=\mathrm{diag}\{\underbrace{I,I,\ldots,I}_{N_0}\}$. Then, we can arrive at

$$\begin{aligned}\Upsilon(t)=&\int_{t_l}^t\mathcal{I}\Big((C-B\otimes U)\varepsilon(\tau)+G(\varepsilon(\tau))-LY(t_l)\\ &+h(t_l)L\xi(\bar T)-L\bar E\bar\varepsilon(t_l)\Big)d\tau\end{aligned} \tag{45}$$

and

$$\begin{aligned}\|\Upsilon(t)\|\leq&\int_{t_l}^t\Big(\|C-B\otimes U\|\|\varepsilon(\tau)\|+h(t_l)\|L\|\|\xi(\bar T)\|\\ &+\|G(\varepsilon(\tau))\|+\|L\|\|Y(t_l)\|+\|L\bar E\|\|\bar\varepsilon(t_l)\|\Big)d\tau\\ \leq&\int_{t_l}^t\Big((\|C-B\otimes U\|+\kappa)\|\varepsilon(\tau)\|+\|L\|\|Y(t_l)\|\\ &+h(t_l)\|L\|\|\xi(\bar T)\|+\|L\bar E\|\|\varepsilon(t_l)\|\Big)d\tau\\ \leq&\ d\Big(\mu\max_{\tau\in[t_l,t)}\|\varepsilon(\tau)\|\\ &+\|L\|\big(\|Y(t_l)\|+h(t_l)\|\xi(\bar T)\|\big)\Big).\end{aligned} \tag{46}$$

It is not difficult to verify that

$$\begin{aligned}\Upsilon^T(t)\Upsilon(t)\leq&\ 2d^2\mu^2\max_{\tau\in[t_l,t)}\|\varepsilon(\tau)\|^2\\ &+2d^2\|L\|^2\big(\|Y(t_l)\|+h(t_l)\|\xi(\bar T)\|\big)^2\\ \leq&\ 2d^2\mu^2\lambda_M(P^{-1})\max_{\tau\in[t_l,t)}V(\tau)\\ &+4d^2\|L\|^2\big(\|Y(t_l)\|^2+h^2(t_l)\|\xi(\bar T)\|^2\big).\end{aligned} \tag{47}$$

Substituting (47) into (43) leads to

$$\begin{aligned}\dot V(t)\leq&-\theta_1V(t)+\theta_2V(t_l)\\ &+\delta_3h^2(t_l)\xi^T(\bar T)\xi(\bar T)-(\varpi_1-\delta_2)Y^T(t_l)Y(t_l)\end{aligned}$$

$$
\begin{aligned}
&+\delta_1\Bigg(2d^2\mu^2\lambda_M(P^{-1})\max_{\tau\in[t_l,t)}V(\tau)\\
&+4d^2\|L\|^2\big(\|Y(t_l)\|^2+h^2(t_l)\|\xi(t_l)\|^2\big)\Bigg)\\
\le&-\theta_1V(t)+\theta_2V(t_l)\\
&+2\delta_1d^2\mu^2\lambda_M(P^{-1})\max_{\tau\in[t_l,t)}V(\tau)\\
&+(\delta_3+4\delta_1d^2\|L\|^2)h_0^2e^{-2\rho t_l}\frac{mN_0\Delta^2}{4}\\
=&-\theta_1V(t)+\theta_2V(t_l)+\theta_3\max_{\tau\in[t_l,t)}V(\tau)+\theta_4e^{-2\rho t_l}.
\end{aligned}
\tag{48}
$$

Now, we assert that if $\theta_1>\theta_2+\theta_3$ (which is satisfied according to (20) and (22)), then it can be obtained from (48) that

$$
V(t)\le V(t_l)e^{-\varrho(t-t_l)}+\theta_5e^{-2\rho t_l},\ t\in[t_l,t_{l+1}). \tag{49}
$$

In order to prove (49), it suffices to show that for any $\mathfrak{h}>0$, the following inequality holds

$$
V(t)<(V(t_l)+\mathfrak{h})e^{-\varrho(t-t_l)}+\theta_5e^{-2\rho t_l},\ t\in[t_l,t_{l+1}). \tag{50}
$$

Suppose that (50) is not true, then there must exist some $t\in(t_l,t_{l+1})$ such that $V(t)\ge(V(t_l)+\mathfrak{h})e^{-\varrho(t-t_l)}+\theta_5e^{-2\rho t_l}$. Denote

$$
t^*=\inf\Big\{t\in(t_l,t_{l+1})\mid V(t)=(V(t_l)+\mathfrak{h})e^{-\varrho(t-t_l)}+\theta_5e^{-2\rho t_l}\Big\}. \tag{51}
$$

Then, it follows from (51) that

$$
\begin{aligned}
V(t)&<(V(t_l)+\mathfrak{h})e^{-\varrho(t-t_l)}+\theta_5e^{-2\rho t_l},\ t\in[t_l,t^*), && (52)\\
V(t^*)&=(V(t_l)+\mathfrak{h})e^{-\varrho(t^*-t_l)}+\theta_5e^{-2\rho t_l}, && (53)\\
\dot V(t^*)&\ge-\varrho(V(t_l)+\mathfrak{h})e^{-\varrho(t^*-t_l)}, && (54)
\end{aligned}
$$

which, together with (48), lead to

$$
\begin{aligned}
\dot V(t^*)\le&-\theta_1V(t^*)+\theta_2V(t_l)+\theta_3\max_{\tau\in[t_l,t^*)}V(\tau)+\theta_4e^{-2\rho t_l}\\
<&-\theta_1(V(t_l)+\mathfrak{h})e^{-\varrho(t^*-t_l)}+(\theta_2+\theta_3)(V(t_l)+\mathfrak{h})\\
\le&\big((\theta_2+\theta_3)e^{\varrho d}-\theta_1\big)(V(t_l)+\mathfrak{h})e^{-\varrho(t^*-t_l)}\\
=&-\varrho(V(t_l)+\mathfrak{h})e^{-\varrho(t^*-t_l)},
\end{aligned}
\tag{55}
$$

where the last equality holds because ϱ is the positive solution to the equation $\theta_1-(\theta_2+\theta_3)e^{\varrho d}-\varrho=0$. Clearly, this indicates a contradiction with (54), thereby affirming the validity of (50). Letting $\mathfrak{h}\to0^+$, it can be verified that (49) is true.

According to (49), one has

$$
\begin{aligned}
V(t_{l+1})&\le V(t_l)\mathfrak{m}+\theta_5\mathfrak{n}^l\\
&\le V(t_{l-1})\mathfrak{m}^2+\theta_5\mathfrak{m}\mathfrak{n}^{l-1}+\theta_5\mathfrak{n}^l\\
&\le V(t_{l-2})\mathfrak{m}^3+\theta_5\mathfrak{m}^2\mathfrak{n}^{l-2}+\theta_5\mathfrak{m}\mathfrak{n}^{l-1}+\theta_5\mathfrak{n}^l\\
&\le\cdots\\
&\le V(0)\mathfrak{m}^{l+1}+\theta_5\sum_{p=0}^{l}\mathfrak{m}^p\mathfrak{n}^{l-p}.
\end{aligned}
\tag{56}
$$

Then, we can verify that

$$
\begin{aligned}
&\frac{\|z_p(t_{l+1})-E_p\eta_p(t_{l+1})\|_\infty}{h(t_{l+1})}\\
\le&\frac{1}{h(t_{l+1})}\|E_p\|\|\varepsilon_p(t_{l+1})\|\\
\le&\frac{\|E_p\|}{h_0}\left(\sqrt{\frac{V(0)}{\lambda_m(P)}}\frac{\mathfrak{m}^{\frac{l+1}{2}}}{\mathfrak{n}^{\frac{l+1}{2}}}+\frac{\sqrt{\theta_5}}{\sqrt{\lambda_m(P)}}\sum_{p=0}^{l}\frac{\mathfrak{m}^{\frac{p}{2}}\mathfrak{n}^{\frac{l-p}{2}}}{\mathfrak{n}^{\frac{l+1}{2}}}\right)\\
\le&\frac{\|E_p\|}{h_0}\left(\sqrt{\frac{V(0)}{\lambda_m(P)}}+\frac{\sqrt{\theta_5}}{\sqrt{\lambda_m(P)}(\sqrt{\mathfrak{n}}-\sqrt{\mathfrak{m}})}\right)\\
\le&\left(M+\frac{1}{2}\right)\Delta,
\end{aligned}
\tag{57}
$$

where the penultimate inequality is obtained by $\mathfrak{n}>\mathfrak{m}$. This means that when $\mathrm{r}=l+1$, the quantizer is unsaturated.

For any $t\ge0$, there exists a non-negative integer r such that $t\in[t_r,t_{r+1})$. Then, it follows readily from (49) and (56) that

$$
\begin{aligned}
\frac{V(t)}{h^2(t_r)}&\le\frac{V(0)\mathfrak{m}^r+\theta_5\sum_{p=0}^{r}\mathfrak{m}^p\mathfrak{n}^{r-p}}{h_0^2\mathfrak{n}^r}\\
&\le\frac{V(0)}{h_0^2}+\frac{\theta_5\mathfrak{n}}{h_0^2(\mathfrak{n}-\mathfrak{m})},
\end{aligned}
\tag{58}
$$

which implies that

$$
\begin{aligned}
V(t)&\le\left(V(0)+\frac{\theta_5\mathfrak{n}}{\mathfrak{n}-\mathfrak{m}}\right)e^{-2\rho t_r}\\
&\le\left(V(0)+\frac{\theta_5\mathfrak{n}}{\mathfrak{n}-\mathfrak{m}}\right)e^{2\rho d}e^{-2\rho t},
\end{aligned}
\tag{59}
$$

namely,

$$
\|\varepsilon(t)\|\le\sqrt{\frac{1}{\lambda_m(P)}\left(V(0)+\frac{\theta_5\mathfrak{n}}{\mathfrak{n}-\mathfrak{m}}\right)}e^{\rho d}e^{-\rho t},\ t\ge0. \tag{60}
$$

Obviously, according to Definition 1, the encoding-decoding-based state estimator (6) under the sampled-data-based DEDT mechanism (7) and (8) is an exponential state estimator for the CN (1), which completes the proof. ■

So far, we have analyzed the convergence property of the estimation error system under the given estimator gain. Now, the focus will be shifted towards the design issue of the estimator.

Theorem 2: Let the positive scalars $\alpha_p\ge\theta_1$, β_p, and φ_p $(p\in\mathbb{I}_1^{N_0})$ be given. Under Assumptions 1 and 2, the encoding-decoding-based estimator (6) with the triggering mechanism (7) and (8) is an exponential state estimator for the nonlinear CN (1) if there exists a block diagonal matrix $P=\mathrm{diag}\{P_1,P_2,\ldots,P_N\}>0$, a diagonal matrix $\Psi>0$, matrices X_p $(p\in\mathbb{I}_1^{N_0})$, and positive scalars δ_1, δ_2, δ_3 and ϖ_2 such that the following inequalities hold

$$
\theta_1P-\varpi_2\lambda_M(E^TE)I>0, \tag{61}
$$

$$
\bar\Pi\triangleq\begin{bmatrix}\bar\Pi_0&\bar\Pi_1\\ *&\bar\Pi_2\end{bmatrix}<0, \tag{62}
$$

and the sampling interval satisfies

$$d < \min\{\bar{d}_1, d_2\}, \tag{63}$$

with the following relationships

$$\varpi_1 \geq \delta_2 + 4d^2\|P^{-1}X\|^2,\ \sigma_p \geq \frac{\varpi_1}{\beta_p},$$
$$\gamma_p \leq \frac{\varpi_2}{\beta_p},\ M = \max\{M_1, \bar{M}_2\},\ \bar{\varrho} > 2\rho > 0,$$

where $\bar{\Pi}_0 = P(C - B \otimes U) - XE + (C - B \otimes U)^T P - E^T X^T + \theta_1 P - \Psi \otimes \check{Z}_1$, $\bar{\Pi}_1 = \begin{bmatrix} P + \Psi \otimes \check{Z}_2 & XE & X & X \end{bmatrix}$, $\bar{\Pi}_2 = \mathrm{diag}\{-\Psi \otimes I, -\delta_1 I, -\delta_2 I, -\delta_3 I\}$, $\bar{d}_1 = \sqrt{\frac{\theta_1 - \theta_2}{2\delta_1 \bar{\mu}^2 \lambda_M(P^{-1})}}$, $\bar{M}_2 = \left\lceil \max_{p \in \mathbb{I}_1^{N_0}} \left\{ \frac{\|E_p\|}{h_0 \Delta} \left(\sqrt{\frac{\vartheta}{\lambda_m(P)}} + \frac{\sqrt{\bar{\theta}_5}}{\sqrt{\lambda_m(P)}(\sqrt{\mathfrak{n}} - \sqrt{\bar{\mathfrak{m}}})} \right) \right\} \right\rceil$, $\bar{\theta}_3 = 2\delta_1 d^2 \bar{\mu}^2 \lambda_M(P^{-1})$, $\bar{\theta}_4 = (\delta_3 + 4\delta_1 d^2 \|P^{-1}X\|^2) h_0^2 \frac{m N_0 \Delta^2}{4}$, $\bar{\theta}_5 = \frac{\bar{\theta}_4}{\theta_1 - \theta_2 - \bar{\theta}_3}$, $\bar{\mathfrak{m}} = e^{-\bar{\varrho} d}$, $\bar{\mu} = \|C - B \otimes U\| + \kappa + \|P^{-1} X \bar{E}\|$, and $X = \begin{bmatrix} \bar{X}^T & 0 \end{bmatrix}^T$ with $\bar{X} = \mathrm{diag}\{X_1, X_2, \ldots, X_{N_0}\}$, $\bar{\varrho}$ is the positive solution to the following equation

$$\theta_1 - (\theta_2 + \bar{\theta}_3) e^{\bar{\varrho} d} - \bar{\varrho} = 0,$$

and the remaining parameters are defined as before. Accordingly, the estimator gain L_p can be designed as follows:

$$L_p = P_p^{-1} X_p. \tag{64}$$

Proof: The proof is omitted here for brevity, as it can be directly obtained from Theorem 1. ■

Remark 3: As pointed out in Remark 2, for the given positive constants α_p and φ_p, we can pick a smaller positive scalar β_p such that d_2 in (63) becomes sufficiently large. Consequently, the restriction with respect to the sampling interval can be relaxed as $d < \bar{d}_1$.

Remark 4: The design issue of the estimator has been addressed in Theorem 2, where the estimator gain is determined by solving a set of matrix inequalities. It is clear that, under the given estimator gain and a smaller positive scalar β_p, the maximal-allowable bound of the sampling interval can be explicitly estimated by solving the algebraic inequality (63). Notably, a larger sampling interval is beneficial for reducing both computational resource consumption and communication frequency. Therefore, it makes sense to design the estimator gain by resorting to the convex optimization approach, thereby maximizing the allowable bound of the sampling interval.

Theorem 3: Let the positive scalars $\mathfrak{p}$, $\alpha_p \geq \theta_1 > \bar{\theta}_2$, δ_1, β_p, and φ_p $(p \in \mathbb{I}_1^{N_0})$ be given. Under Assumptions 1 and 2, the encoding-decoding-based estimator (6) with the triggering mechanism (7) and (8) is an exponential state estimator for the nonlinear CN (1) if an optimal problem with the following linear objective function

$$\min_{\substack{P>0,\Psi>0,X,\\ \delta_2>0,\delta_3>0,\mu_2>0,\varpi_2>0}} \quad \mathfrak{z} > 0 \tag{65}$$

and linear matrix inequality (LMI) constraints

$$\text{LMIs (62)} \tag{65a}$$
$$P - \mathfrak{p}I > 0 \tag{65b}$$
$$\varpi_2 \lambda_M(E^T E) - \bar{\theta}_2 P < 0 \tag{65c}$$
$$\begin{bmatrix} -\mathfrak{p}^2 \mu_2 I & X\bar{E} \\ * & -\mu_2 I \end{bmatrix} < 0 \tag{65d}$$
$$\begin{bmatrix} P & (\mu_1 + \mu_2) I \\ * & \mathfrak{z} I \end{bmatrix} > 0 \tag{65e}$$

has a set of solutions $P > 0$, $\Psi > 0$, X, $\delta_2 > 0$, $\delta_3 > 0$, $\mu_2 > 0$, and $\varpi_2 > 0$. Meanwhile, the sampling period satisfies

$$d < \min\left\{ \sqrt{\frac{\theta_1 - \bar{\theta}_2}{2\delta_1 \mathfrak{z}}}, d_2 \right\} \tag{66}$$

where $\mu_1 = \|C - B \otimes U\| + \kappa$, and the remaining parameters are defined as before. Accordingly, the estimator gain L_p can be designed as follows:

$$L_p = P_p^{-1} X_p. \tag{67}$$

Proof: Applying the Schur complement lemma to (65e) yields

$$P - \mathfrak{z}^{-1} (\mu_1 + \mu_2)^2 I > 0, \tag{68}$$

namely,

$$\mathfrak{z} P - (\mu_1 + \mu_2)^2 I > 0, \tag{69}$$

which implies that

$$\mathfrak{z} > (\mu_1 + \mu_2)^2 \lambda_M(P^{-1}). \tag{70}$$

Similarly, using the Schur complement lemma again, we can verify that (65d) holds if and only if the following inequality holds

$$-\mathfrak{p}^2 \mu_2 I + \mu_2^{-1} X \bar{E} \bar{E}^T X^T < 0. \tag{71}$$

Noting that $P - \mathfrak{p} I > 0$, it can be obtained from (71) that

$$-PP + \mu_2^{-2} X \bar{E} \bar{E}^T X^T < 0. \tag{72}$$

Utilizing the Schur complement lemma again, we have

$$\begin{bmatrix} -\mu_2^2 I & \bar{E}^T X^T \\ * & -PP \end{bmatrix} < 0 \tag{73}$$

or

$$-\mu_2^2 I + \bar{E}^T X^T P^{-1} P^{-1} X \bar{E} < 0, \tag{74}$$

which means that $\|P^{-1} X \bar{E}\| < \mu_2$. Recalling (65c) and (70), one has

$$\sqrt{\frac{\theta_1 - \bar{\theta}_2}{2\delta_1 \mathfrak{z}}} < \sqrt{\frac{\theta_1 - \theta_2}{2\delta_1 \lambda_M(P^{-1}) \bar{\mu}^2}}. \tag{75}$$

Clearly, the inequality (70) is valid. We can conclude that all the conditions stated in Theorem 2 are satisfied. The proof is now complete. ■

Remark 5: Theorem 3 has provided a novel convex optimization approach to design the desired estimator gains with the purpose of maximizing the allowable bound of the sampling period. It is important to note that several inequalities have been introduced in Theorem 3, and this might give rise to a certain level of conservativeness in our theoretical results. When determining the estimator gains, one can first solve the optimal problem (65) and then design the estimator gain matrices L_p by using (67). Subsequently, one can obtain the allowable bound of the sampling period d through (63). In light of Theorems 1 and 3, we propose the Algorithm 1 to compute the estimator gains L_p and the allowable bound of the sampling period d.

Algorithm 1 Computational algorithm for L_p and d

Step 1: Given the positive constants θ_1, $\mathfrak{p}$, $\alpha_p \geq \theta_1 > \bar{\theta}_2$, δ_1, β_p, and φ_p.
Step 2: Search for feasible solution to the optimal problem (65).
Step 3: Determine estimator gain matrices L_p via (67).
Step 4: Calculate the allowable bound of the sampling period d according to (22).

In what follows, we are going to consider the sampled-data-based EDT strategy in the design of encoding-decoding-based estimator, which is a special case of the proposed sampled-data-based DEDT mechanism (7) with $\varphi_p = 0$. Specifically, the triggering instants are determined by the following condition:

$$T^p_{k+1} = \min\Big\{t_r > T^p_k \,|\, \sigma_p\|y_p(t_r)\|^2 - \gamma_p\|\zeta_p(t_r)\|^2 \geq 0\Big\}, \quad p \in \mathbb{I}_1^{N_0}, \tag{76}$$

where $T^p_0 = t_0$, $\sigma_p > 0$ is a given constant, and $\gamma_p > 0$ is a design parameter to be determined.

The following theorem presents the design scheme of the estimator parameters under the sampled-data-based EDT strategy (76).

Theorem 4: Let the positive scalars θ_1 and σ_p $(p \in \mathbb{I}_1^{N_0})$ be given. Under Assumptions 1 and 2, the encoding-decoding-based estimator (6) with the triggering mechanism (76) is an exponential state estimator for the nonlinear CN (1) if there exists a block diagonal matrix $P = \text{diag}\{P_1, P_2, \dots, P_N\} > 0$, a diagonal matrix $\Psi > 0$, matrices X_p $(p \in \mathbb{I}_1^{N_0})$, and positive scalars δ_1, δ_2 and δ_3 such that the inequalities in (62) hold, and the sampling interval satisfies

$$d < \sqrt{\frac{\theta_1}{2\delta_1\bar{\mu}^2\lambda_M(P^{-1})}} \tag{77}$$

with the following relationships

$$\gamma_p < \frac{\tilde{\sigma}(\theta_1 - 2\delta_1 d^2\bar{\mu}^2\lambda_M(P^{-1}))}{\delta_2 + d^2\|P^{-1}X\|^2}, \tag{78}$$

$$M = \max\{M_1, \tilde{M}_2\}, \;\; \tilde{\varrho} > 2\rho > 0, \tag{79}$$

where $\tilde{\vartheta} = \lambda_M(P)N\chi^2$, $\tilde{\mathfrak{m}} = e^{-\tilde{\varrho}d}$, $\tilde{\theta}_5 = \frac{\bar{\theta}_4}{\theta_1 - \tilde{\theta}_2 - \bar{\theta}_3}$, $\tilde{M}_2 = \left\lceil \max_{p\in\mathbb{I}_1^{N_0}}\left\{\frac{\|E_p\|}{h_0}\left(\sqrt{\frac{\tilde{\vartheta}}{\lambda_m(P)}} + \frac{\sqrt{\tilde{\theta}_5}}{\sqrt{\lambda_m(P)}(\sqrt{\mathfrak{n}} - \sqrt{\tilde{\mathfrak{m}}})}\right)\right\}\right\rceil$, $\tilde{\theta}_2 = (\delta_2 + d^2\|L\|^2)\frac{\tilde{\gamma}}{\tilde{\sigma}}\lambda_M(E^TE)\lambda_M(P^{-1})$, $\tilde{\sigma} = \min_{p\in\mathbb{I}_1^{N_0}}\{\sigma_p\}$, $\tilde{\gamma} = \max_{p\in\mathbb{I}_1^{N_0}}\{\gamma_p\}$, $\tilde{\varrho}$ is the positive solution to the following equation

$$\theta_1 - (\tilde{\theta}_2 + \bar{\theta}_3)e^{\tilde{\varrho}d} - \tilde{\varrho} = 0, \tag{80}$$

and the remaining parameters are defined as before. Accordingly, the estimator gain L_p can be designed as follows:

$$L_p = P_p^{-1}X_p. \tag{81}$$

Proof: The proof is omitted here for brevity, as it follows immediately from the conclusions of Theorems 1 and 2. ■

Remark 6: So far, we have tackled the encoding-decoding-based state estimation problem for continuous-time nonlinear complex networks subject to communication bandwidth constraints, where the proposed method relies on sampled outputs from a subset of network nodes. A DEDT strategy has been utilized that integrates sampled-data mechanisms to determine whether data should be transmitted at periodic sampling instants, thereby reducing resource consumption and avoiding Zeno behavior. Sufficient conditions have been established to ensure the exponential stability of the estimation error dynamics, and a convex optimization technique has been developed to design the estimator gain in order to maximize the allowable sampling interval, thereby enhancing communication and computational efficiency. In the next section, we will employ a numerical example to demonstrate the effectiveness of the proposed method in reducing network communication burdens and maintaining estimation error convergence, even for an unstable complex network with limited measurements.

Remark 7: The key novelties of this article compared with existing literature are summarized as follows. 1) Unlike most existing studies that typically require full output measurements from all nodes, this paper focuses on state estimation using outputs from only a fraction of nodes. This is more realistic for large-scale networks where the data collection from all the nodes is impractical. 2) An encoding-decoding-based approach is developed to enhance transmission efficiency and data confidentiality under bandwidth limitations. 3) A sampled-data-based DEDT mechanism is employed to operate at periodic sampling intervals, combining sampled-data strategies with event-driven techniques, thereby eliminating the need for continuous monitoring of measurements, reducing data transmission frequency, and naturally preventing Zeno behavior. 4) Through a convex optimization approach, the paper explicitly designs the estimator gain to maximize the allowable sampling interval, significantly improving resource management compared to traditional methods.

IV. Simulation results

In this section, an illustrative example and a practical example are presented to demonstrate the effectiveness of the obtained theoretical results.

Example 1: Consider a nonlinear CN (1) with six non-identical nodes and assume that the measurement outputs of the first four nodes are accessible, namely, $N_0 = 4$. The relevant parameters are given as follows:

$$C_1 = C_2 = \begin{bmatrix} 0.25 & -0.5 \\ 0.5 & -0.05 \end{bmatrix}, \; C_3 = C_4 = \begin{bmatrix} 0.3 & -0.5 \\ 0.5 & -0.25 \end{bmatrix},$$

$$C_5 = C_6 = \begin{bmatrix} 0.25 & -0.5 \\ 0.5 & -0.4 \end{bmatrix}, \; U = 0.2I, \; E_p = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$

$$B=\begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\-1&0&0&0&0&1\end{bmatrix},$$
$$g(s)=0.1\begin{bmatrix}-0.8s_1+0.2(|s_1+8|-|s_1-8|)+0.4s_2\\0.8s_2+\tanh(-0.6s_2)\end{bmatrix}$$
with $s=\begin{bmatrix}s_1 & s_2\end{bmatrix}^T$.

It is straightforward to confirm that the nonlinear function g satisfies Assumption 1 with the following parameters:

$$Z_1=0.1\begin{bmatrix}-0.8&0.4\\0&0.8\end{bmatrix}\text{ and }Z_2=0.1\begin{bmatrix}-0.4&0.4\\0&0.2\end{bmatrix}.$$

Choosing $\theta_1=0.1$, $\bar{\theta}_2=0.025$, $\delta_1=0.008$, and $\mathfrak{p}=0.01$, we can solve the minimization problem (65) using YALMIP and acquire a set of feasible solutions. The corresponding estimator gains are obtained as follows:

$$L_1=\begin{bmatrix}3.9257\\2.4109\end{bmatrix},\ L_2=\begin{bmatrix}3.8429\\2.3330\end{bmatrix},$$
$$L_3=\begin{bmatrix}3.9703\\2.6493\end{bmatrix},\ L_4=\begin{bmatrix}3.9233\\2.8493\end{bmatrix},$$

and the sampling period is $d=0.1081$.

According to Theorem 3, it can be concluded that the encoding-decoding-based estimator (6) is an exponential state estimator for the nonlinear CN (1). The simulation results further confirm the correctness of the theoretical findings. Specifically, Fig. 2 illustrates the state evolutions of six nodes with the initial states $s_p(0)=\begin{bmatrix}-p & 7-p\end{bmatrix}^T$, $p\in\mathbb{I}_1^6$. Letting $d=0.1$, $\Delta=0.1$, $\alpha_p=0.1$, $\phi_p^0=10$, $\gamma_p=0.05$, $\sigma_p=26$, $\beta_p=0.2$ ($p\in\mathbb{I}_1^4$), $\chi=6.1$, $h_0=2$. After some calculations, one can get $\bar{\varrho}=0.113$, $M_1=5$, $\bar{M}_2=36524$. In such case, we select $M=36524$ and $\rho=0.05$. Then, the evolution of the estimation error is shown in Fig. 3, which is exactly consistent with the theoretical results. It should be pointed out that the considered CN with the given parameters is unstable. Nevertheless, the estimation errors still converge to zero. Figs. 4 and 5 display, respectively, the triggering time sequences of the first four nodes and the trajectories of the transmitted codewords.

In addition, we perform a comparative simulation to highlight the superiority of the proposed sampled-data-based DEDT strategy. Fig. 6 depicts the triggering time sequences for the first four nodes under the sampled-data-based EDT strategy (76). A comparison of triggered numbers between the sampled-data-based DEDT strategy and sampled-data-based EDT are listed in Table I. It is evident from Figs. 4 and 6 and Table I that the proposed sampled-data-based DEDT strategy is more effective than the static one in reducing the network communication burden.

Example 2 (A practical example): Consider a three-area power system [40], [54], which can be described by CN (1)

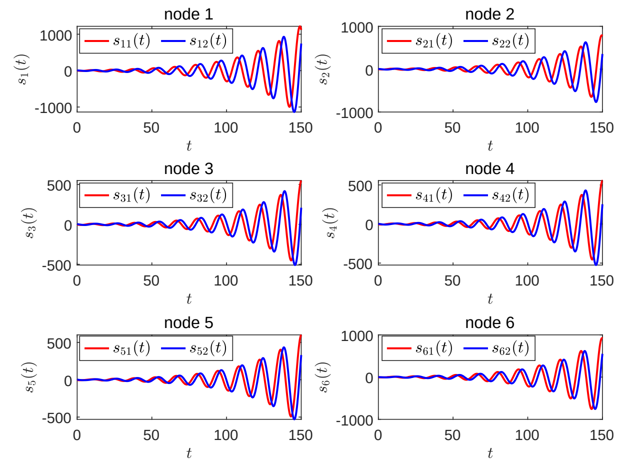

Fig. 2. State evolution of each node in the nonlinear CN.

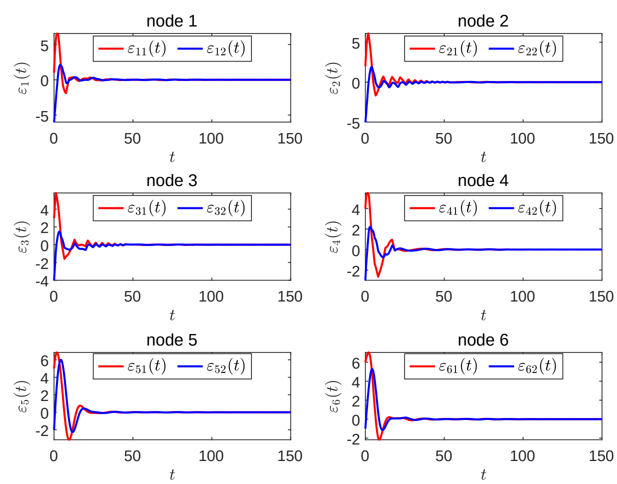

Fig. 3. State estimation error of each node.

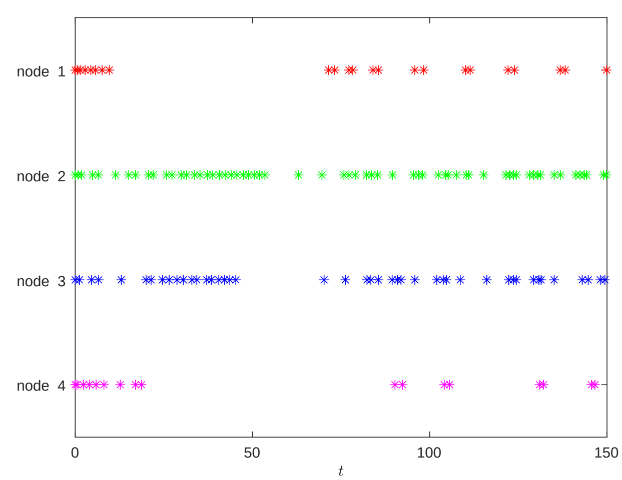

Fig. 4. Triggering time sequences of the first four nodes under the sampled-data-based DEDT strategy.

TABLE I
TRIGGERED NUMBERS FOR FIRST FOUR NODES UNDER SAMPLED-DATA-BASED EDT STRATEGY VERSUS SAMPLED-DATA-BASED DEDT STRATEGY.

Time intervals	node 1		node 2		node 3		node 4	
	S-EDT	S-DEDT	S-EDT	S-DEDT	S-EDT	S-DEDT	S-EDT	S-DEDT
$[0, 30)$	20	8	17	13	17	10	22	9
$[30, 60)$	22	0	19	14	21	9	22	0
$[60, 90)$	24	6	21	9	22	6	22	0
$[90, 120)$	22	4	21	10	19	8	20	4
$[0, 120)$	88	18	78	46	79	33	86	13

S-EDT and S-DEDT represent, respectively, sampled-data-based EDT strategy and sampled-data-based DEDT strategy.

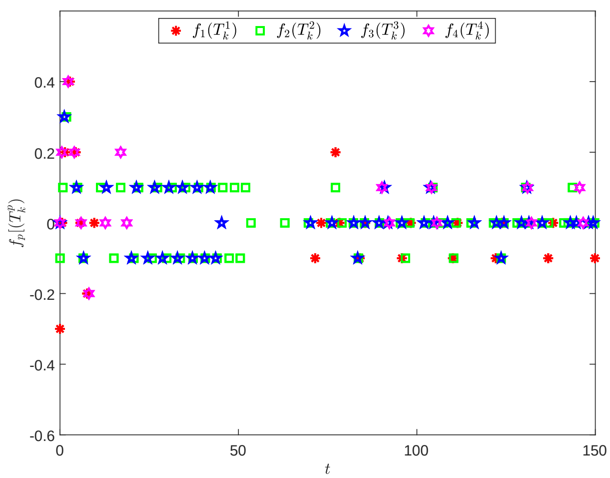

Fig. 5. Trajectories of the transmitted codewords.

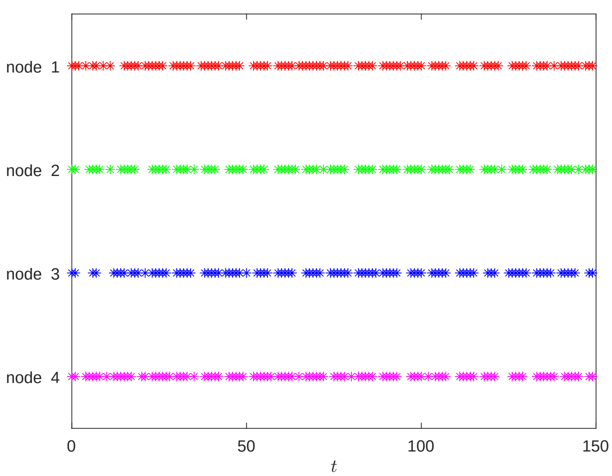

Fig. 6. Triggering time sequences of the first four nodes under the sampled-data-based EDT strategy.

with the following parameters:

$$C_p = \begin{bmatrix} -\frac{D_p}{2H_p} & \frac{1}{2H_p} & 0 & -\frac{1}{2H_p} \\ 0 & -\frac{1}{T_{\text{ch}_p}} & \frac{1}{T_{\text{ch}_p}} & 0 \\ -\frac{1}{R_p T_{\text{g}_p}} & 0 & -\frac{1}{T_{\text{g}_p}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$U = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \; w_{pq} = 2\pi T_{pq},$$

where the practical meanings of the relevant parameters are detailed in [54]. Here, both the load generation balance point and the load deviation are assumed to be zero.

The relevant parameters for the simulation are listed as follows: $D_1 = D_2 = 1.5$, $H_1 = H_2 = 5$, $T_{\text{ch}_1} = T_{\text{ch}_2} = 0.3$, $R_1 = R_2 = 0.1$, $T_{\text{g}_1} = T_{\text{g}_2} = 1$, $D_3 = 1.8$, $H_3 = 6$, $T_{\text{ch}_3} = 0.2$, $R_3 = 0.3$, $T_{\text{g}_3} = 1.2$, $T_{12} = T_{13} = T_{21} = T_{23} = T_{31} = T_{32} = 0.25$. Suppose that the outputs of first two nodes can be available with $E_1 = E_2 = \begin{bmatrix} 0.1 & 2 & 0 & 0 \end{bmatrix}$.

Given $\theta_1 = 0.11$, $\bar{\theta}_2 = 0.01$, $\delta_1 = 24.08$, and $\mathfrak{p} = 0.39$, we can solve the minimization problem (65) using YALMIP and acquire a set of feasible solutions. According to Theorem 3, the encoding-decoding-based estimator (6) with the triggering mechanism (7) and (8) is an exponential state estimator for the CN (1), and estimator gain matrices can be computed as

$$L_1 = \begin{bmatrix} -0.0120 \\ 0.1389 \\ 0.1362 \\ 0.1181 \end{bmatrix}, \; L_2 = \begin{bmatrix} -0.0141 \\ 0.1686 \\ 0.1641 \\ 0.1404 \end{bmatrix},$$

and the sampling period is $d = 0.0201$.

Let us pick $d = 0.02$, $\Delta = 0.2$, $\alpha_p = 0.11$, $\phi_p^0 = 10$, $\gamma_p = 2$, $\sigma_p = 31$, $\beta_p = 1$ ($p \in \mathbb{I}_1^2$), $h_0 = 10$. Then, the simulation figures with the above parameters are shown in Figs. 7–9, which considerably coincide with the theoretical results. Fig. 7 depicts the fact that the corresponding estimation error system is exponentially stable. Fig. 8 illustrates the triggering time sequences of the first two nodes, while Fig. 9 displays the trajectories of the transmitted codewords.

V. CONCLUSIONS

In this paper, the PNB state estimation issue has been investigated for a class of continuous-time nonlinear CNs subject to the communication bandwidth constraints. To cater for the engineering reality, only the measurement outputs from a fraction of network nodes have been utilized to conduct the state estimation task. For efficient resource management, a novel sampled-data-based DEDT mechanism combined with the encoding-decoding technique has been employed in the process of data transmission. Specifically, the sampled-data-based DEDT mechanism has been introduced to regulate the

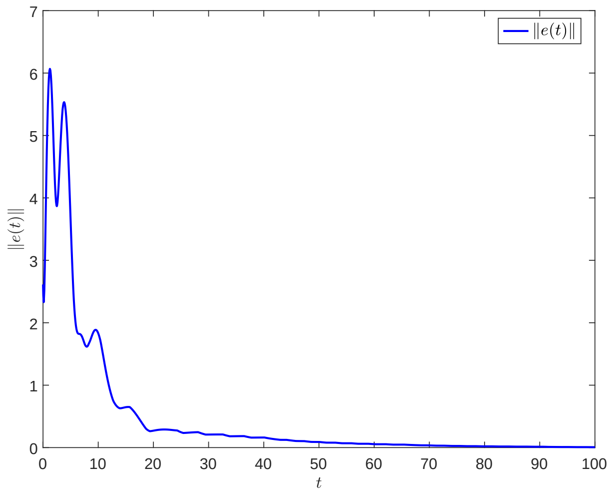

Fig. 7. State estimation error of CN.

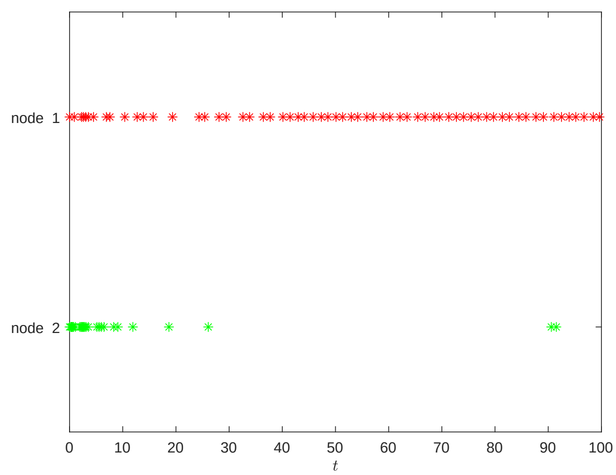

Fig. 8. Triggering time sequences of the first two nodes under the sampled-data-based EDT strategy.

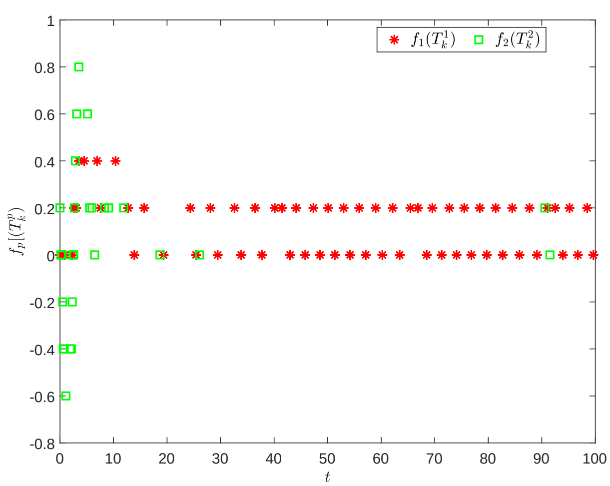

Fig. 9. Trajectories of the transmitted codewords.

transmission of measurement signals by determining when to transmit them. Under the encoding-decoding scheme, the measurement signals have been encoded as codewords and sent to the estimator via a digital communication channel. Some sufficient criteria have been established to ensure that the estimation error system is exponentially stable. Moreover, a new convex optimization approach has been employed to design the estimator gain by maximizing the allowable bound of the sampling intervals. Finally, a demonstrative example and a real-world case involving a three-area power grid have been provided to illustrate the effectiveness and correctness of the obtained main results. One potential direction for future research is to extend the current results to settings with more intricate communication constraints, such as limited transmission range and restricted bandwidth [55], [56].

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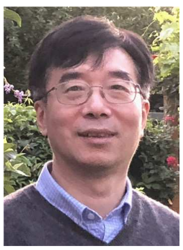

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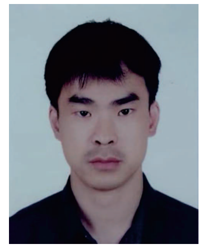

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