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H_∞ Fuzzy Control for A Class of Cyber-Physical Systems Under Frequency-Duration-Constrained Replay Attacks

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***Abstract*—In this article, the H_∞ fuzzy control problem is investigated for a class of nonlinear systems subject to replay attacks with frequency-duration constraints. Owing to the vulnerability of the open shared communication network, the information transmitted from the sensor to the controller may be exposed to replay attackers. A novel yet comprehensive replay attack model is constructed to characterize the repeated replay behavior of the adversary. On the basis of the constructed model, a fuzzy controller is designed to guarantee asymptotic stability and the desired H_∞ performance. By employing Lyapunov stability theory and the orthogonal decomposition technique, sufficient conditions are derived to ensure the existence of the desired controller parameter. Finally, simulation results are presented to verify the effectiveness and correctness of the proposed fuzzy controller for T-S fuzzy systems under replay attacks.**

***Index Terms*—T-S Fuzzy Systems, cyber-physical systems, H_∞ control, cyber attacks, replay attacks, frequency-duration constraints.**

I. Introduction

Cyber-physical systems (CPSs) are a class of large-scale systems in which physical layers and computation processes are integrated by network layers. Such integration brings the higher computing efficiency and wider applications. Meanwhile, the advantages (e.g., low installation costs, flexible structure, and remote control) have been inherited by CPSs since network technologies are adopted. Thus, CPSs have been widely applied in various fields such as smart grids [1], [2], unmanned aerial vehicles [3], [4], and industrial automation systems [5]–[7].

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Although the advantages of communication networks are inherited by CPSs, the application of network technologies has inevitably given rise to induced phenomena such as packet dropouts [8]–[10], time delays [11], [12], and fading channels [13], [14]. Moreover, transmission signals have been rendered vulnerable to cyber-attacks owing to the open shared communication network in CPSs. It has been demonstrated in several studies that system performance can be disrupted by malicious attackers [15]–[17]. Hence, it is of significant importance that a control law be designed such that the desired system performance is maintained while accounting for the effects of potential cyber-attacks. For example, secure control problems have been investigated for CPSs modeled by Markov jump systems under cyber-attacks in [18], [19]. A composite H_∞ control scheme and a novel resilient hybrid learning scheme are designed to guarantee the desired system performance, respectively.

According to the security requirements of transmission data, cyber-attacks have been classified into availability attacks (e.g., Denial-of-Service (DoS) attacks) [20]–[22] and integrity attacks (e.g., false-data injection (FDI) attacks and replay attacks) [23]–[26]. From the perspective of attackers, the DoS attack strategy can be launched by blocking data transmissions, the FDI attack strategy can be executed by injecting false data packets into the original data, and the replay attack strategy can be carried out by replacing real-time data with recorded historical data. Owing to their high stealthiness and ease of implementation, replay attacks have been regarded as one of the most threatening cyber-attacks for CPSs [27], [28].

During the execution of replay attacks, a sequence of historical data is recorded, and the collected data is replayed to replace the real-time data. In comparison with FDI attacks, replay attacks are easier to carry out since no prior system information is required by the adversary. Over the past decade, secure filtering and control problems under replay attacks have been investigated, and several models have been proposed to illustrate the principles of replay attacks. For instance, in [29], a model of replay attacks has been established in which the measurement output has been represented as a model with constant time delays. In [30], replay attacks have been modeled as the measurement output with bounded time-varying delays. It is worth noting that most of the existing studies have assumed that the collected data can only be replayed once. Clearly, this assumption is overly restrictive, since adversaries may repeatedly replay the collected data. Therefore, a novel model needs to be developed to characterize the behavior of repeated replay attacks, and the secure control problem under such attacks should be investigated.

The Takagi-Sugeno (T-S) system has been proposed to approximate nonlinear functions by employing a set of linear subsystems interconnected through time-varying membership functions. It has been demonstrated in the relevant literature that any smooth nonlinear function can be approximated to any desired accuracy by means of T-S fuzzy technology [31]–[33]. In recent years, the security control problem of networked T-S fuzzy systems has received considerable attention owing to the prevalence of cyber-attacks in nonlinear CPSs [34], [35]. For instance, the secure control problem of T-S fuzzy systems has been investigated in [36], where DoS and FDI attacks have been taken into account. Nevertheless, the impact of replay attacks on the secure control problem has not yet been addressed for T-S fuzzy systems, which serves as the motivation of the present study.

Based on the above discussions, the H_∞ fuzzy control problem is investigated for networked T-S fuzzy systems under replay attacks. The main challenges of this problem can be summarized as follows: 1) how to design a reasonable model of replay attacks that can capture the behavior of repeated replay? and 2) how to analyze the dynamical behavior of the system state subject to replay attacks? Correspondingly, the primary contributions of this article can be highlighted as follows:

1) a novel mathematical model of replay attacks is developed to characterize the repeated replay behavior;
2) sufficient conditions are derived to guarantee the exponential stability and the H_∞ performance of the closed-loop system under replay attacks; and
3) the relations between the frequency/duration of replay attacks and the H_∞ disturbance attenuation level are thoroughly established.

The remainder of this article is organized as follows. In Section II, the fuzzy model under replay attacks and the fuzzy controller are introduced. In Section III, several sufficient conditions are proposed to ensure the stability and H_∞ performance of the closed-loop system under replay attacks, and the desired controller gains are derived. A simulation example is provided in Section IV to demonstrate the feasibility of the proposed fuzzy control scheme. Finally, Section V concludes this article.

Notations: In this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $l_2[0,+\infty)$ denotes the space of square summable sequences. For a matrix A, its transposition is denoted as A^T. B^{-1} and $\lambda_{\max}(B)$ $(\lambda_{\min}(B))$ denotes the inverse and the maximum (minimum) eigenvalue of the square matrix B, respectively. $C^\perp$ represents the orthogonal basis for the null space of the full column rank matrix C. A diagonal-block matrix D is denoted by $D = \mathrm{diag}\{D_{11}, D_{22}, \ldots, D_{nn}\}$. In a symmetric matrix, the symmetric parts are denoted as an asterisk "$*$".

II. Problem Formulation and Preliminaries

A. System Model

Consider a class of nonlinear systems described by the following T-S fuzzy system:

System Rule i: IF $\rho_1(k)$ is W_{i1}, and $\rho_2(k)$ is W_{i2}, and $\cdots$, and $\rho_s(k)$ is W_{is}, THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) + E_i \omega(k) \\ \quad y(k) = Cx(k) + F\omega(k) \\ \quad z(k) = G_i x(k), \quad i \in \mathbb{I} \triangleq \{1, 2, \cdots, r\} \end{cases} \tag{1}$$

where r is the number of fuzzy rules; $W_{i1}, \cdots, W_{is}$ are fuzzy sets; $\rho_i(k)$ $(i = 1, 2, \cdots, s)$ are the measurable variables; $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^m$, $z(k) \in \mathbb{R}^h$, and $u(k) \in \mathbb{R}^o$ denote, respectively, the system state, the measurement output, the control output, and the control input; $\omega(k) \in (\mathbb{R}^{n_\omega},\ l_2[0,+\infty))$ is the energy-bounded external noise (including process and measurement noises); A_i, B_i, E_i, C, F, and G_i are real constant matrices of appropriate dimensions.

By using the standard fuzzy inference technique, the fuzzy system (1) can be described by

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} \varphi_i(\rho(k))(A_i x(k) + B_i u(k) + E_i \omega(k)) \\ \quad y(k) = Cx(k) + F\omega(k) \\ \quad z(k) = \sum_{i=1}^{r} \varphi_i(\rho(k)) G_i x(k) \end{cases} \tag{2}$$

where $\rho(k) \triangleq \begin{bmatrix} \rho_1(k) & \rho_2(k) & \cdots & \rho_s(k) \end{bmatrix}^T \in \mathbb{R}^s$ is the premise variable vector, and

$$\varphi_i(\rho(k)) \triangleq \frac{\prod_{\iota=1}^{s} W_{i\iota}(\rho_\iota(k))}{\sum_{i=1}^{r} \prod_{\iota=1}^{s} W_{i\iota}(\rho_\iota(k))}$$

is the normalized membership function with the membership grade $0 \le W_{i\iota}(\rho_\iota(k)) \le 1$ of $\rho_\iota(k)$ in $W_{i\iota}$. For $\forall k \ge 0$, the normalized membership function satisfies $\varphi_i(\rho(k)) \ge 0$ and $\sum_{i=1}^{r} \varphi_i(\rho(k)) = 1$.

B. Replay Attack

An original replay attack model has been constructed in [37], where an assumption has been made that the attacker is able to record and erase the real sensor measurements. When a replay attack occurs, the actual measured output $y(k)$ is replaced by the recorded historical measured output $y(k-\tau)$. It is worth noting that τ is a positive integer denoting the number of recorded data packets.

In practice, owing to the existence of certain defensive measures and technical limitations, attackers cannot record the real sensor measurements at will. Consequently, the recorded data packets may be replayed repeatedly, and the number of packets may be time-varying in practical engineering. For example, adversaries have been shown capable of launching multiple consecutive replay attacks in [38], [39]. A novel model has been proposed in [26] to describe the recording of a varying number of data packets. In reality, replay attackers can determine not only the length of the recorded data packets but also the number of times they are replayed. Therefore, a novel model of replay attacks will be proposed in this work to characterize these features.

Based on the above analysis, it is reasonable to assume that the attacker can 1) record and erase the sensor measurements;

and 2) inject the recorded data into the system. The process of replay attacks is divided into the following phases:

1) In the attack sleeping time $[k_t,\ k_t+\Delta_t-1]$, the attacker records a series of the measurement output $\{y(k_t+\Delta_t-\tau_t), y(k_t+\Delta_t-\tau_t+1), \ldots, y(k_t+\Delta_t-1\}$, where Δ_t and τ_t $(t=1,2,\ldots)$ denote the sleeping length and the number of recording data packets for the tth replay attack. k_t is the beginning of sleeping time before the tth attack;
2) In the attack time $[k_t+\Delta_t,\ k_{t+1}-1]$, the measurement output is first erased by the attacker. Then, the recorded measurement output is repeatedly injected into the channel in sequence. During this process, the attacker replays the recorded measurement outputs $\{y(k_t+\Delta_t-\tau_t), y(k_t+\Delta_t-\tau_t+1), \ldots, y(k_t+\Delta_t-1\}$ periodically for certain times. Without loss of generality, we assume that the recorded measurement outputs are replayed for d_t times with d_t being a positive integer. In this situation, it is obvious that $k_{t+1}=k_t+\Delta_t+d_t\tau_t$.

For simplicity, the following symbols are defined to indicate the specific time instants:

$$\begin{aligned} k_{t(1)} &\triangleq k_t+\Delta_t, \quad k_{t(2)} \triangleq k_t+\Delta_t+\tau_t, \\ k_{t(3)} &\triangleq k_t+\Delta_t+2\tau_t, \cdots, \\ k_{t(d_t)} &\triangleq k_t+\Delta_t+(d_t-1)\tau_t. \end{aligned}$$

To illustrate the timing of replay attacks, Fig. 1 is presented, and it is clear that the number of recorded data packets τ_t and the replay times d_t are determined by the attacker in each attack. Note that, in practical applications, a replay attacker is only able to record a limited number of sensor measurements and to launch a finite number of attacks. Therefore, it is assumed that $\tau(k) \le \bar{\tau}$ and $d(k) \le \bar{d}$.

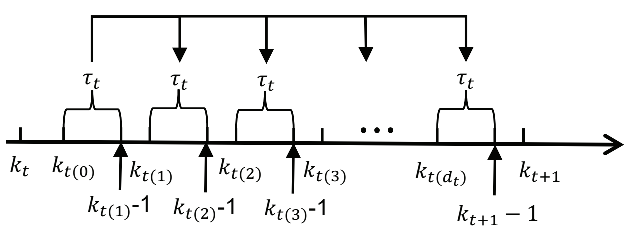

Fig. 1: The timing diagram of replay attacks.

According to the above discussions, a novel replay attack model is established as:

$$\tilde{y}(k) = (1-\alpha(k))y(k) + \alpha(k)y(k-d(k)\tau(k)) \tag{3}$$

where

$$\alpha(k) \triangleq \begin{cases} 0, & k \in [k_t,\ k_t+\Delta_t-1] \\ 1, & k \in [k_t+\Delta_t,\ k_{t+1}-1] \end{cases},$$

$$d(k) \triangleq \begin{cases} 1, & k \in [k_t+\Delta_t,\ k_t+\Delta_t+\tau_t-1] \\ 2, & k \in [k_t+\Delta_t+\tau_t,\ k_t+\Delta_t+2\tau_t-1] \\ \vdots & \qquad \vdots \\ d_t, & k \in [k_t+\Delta_t+(d_t-1)\tau_t,\ k_{t+1}-1] \end{cases},$$

and $\tau(k) \triangleq \tau_t$ when $k \in [k_t,\ k_{t+1}-1]$.

Remark 1: In the replay attack model (3), the variable $\alpha(k)$ is introduced to indicate whether replay attacks occur at time instant k. It is easy to see that the controller receives the actual signal $y(k)$ when $\alpha(k)=0$. When $\alpha(k)=1$, the real-time signals are replaced by the recorded measurement output $y(k-d(k)\tau(k))$. Note that, in each attack phase, the replay attacker can decide both the number of recorded data packets and the replay times. However, this does not imply that an infinite number of sensor measurements can be recorded or replayed, since storage and energy resources are limited. Hence, it is assumed that the number of recorded sensor measurements and replay times are bounded. As shown in Fig. 1, the designed attack model (3) is more general and can reduce to the replay attack models in [40], [26] by setting $d(k)=1$.

Remark 2: As introduced in the proposed replay attack strategy, the measurement output is recorded and replayed. The stealthiness of replay attacks is realized since the replay measurement output is recorded in the absence of an attack. In addition, the existence of time-delay caused by replay attacks may bring about performance degradation and even system instability.

For the convenience of subsequent controller design, the following assumptions on the frequency and duration of replay attacks are adopted.

Assumption 1: (Replay Attack Frequency) Within the time interval $[k_g, k_h]$ for any $0 \le k_g < k_h$, there exist scalars $\sigma > 0$ and $\theta > 1$ such that the number of replay attacks $H(k_g, k_h)$ satisfies the following inequality:

$$H(k_g, k_h) \le \sigma + \frac{k_h - k_g}{\theta}. \tag{4}$$

Assumption 2: (Replay Attack Duration) Within the time interval $[k_g, k_h]$ for any $0 \le k_g < k_h$, there exist scalars $\alpha > 0$ and $\beta > 1$ such that the length of total time interval of replay attacks $M(k_g, k_h)$ satisfies the following inequality:

$$M(k_g, k_h) \le \alpha + \frac{k_h - k_g}{\beta}. \tag{5}$$

Remark 3: The frequency and duration constraints have been widely employed to describe general DoS attacks in [41], where such constraints are less restrictive compared with strict assumptions on statistical behaviors. Furthermore, these assumptions are motivated by practical considerations: in real-world scenarios, continuously injecting attack signals without constraints on frequency or duration would significantly increase the likelihood of detection. Moreover, sustaining such persistent attacks would demand considerable energy resources, which is often undesirable from the attacker's perspective. Therefore, a large body of work addressing secure estimation and control problems under DoS attacks with frequency and duration constraints has appeared in the literature; see [42]–[44]. Motivated by this literature, frequency and duration constraints are adopted here to describe the behaviors of replay attacks.

Under the effects of the above replay attack, the control input is constructed as:

Controller Rule j: IF $\varrho_1(k)$ is M_{i1}, and $\varrho_2(k)$ is M_{i2}, and $\cdots$, and $\varrho_e(k)$ is M_{ie}, THEN

$$u(k) = K_j \tilde{y}(k) \tag{6}$$

where $\varrho(k) \triangleq \begin{bmatrix} \varrho_1(k) & \varrho_2(k) & \cdots & \varrho_e(k) \end{bmatrix}^T$ is the premise variable vector of the controller; $M_{i1}, M_{i2}, \ldots, M_{ie}$ are fuzzy sets; K_j is the controller gain to be designed.

The controller (6) can be rewritten in the following compact form:

$$u(k) = \sum_{j=1}^{f} \psi_j(\varrho(k)) K_j \tilde{y}(k) \tag{7}$$

where $\psi_j(\varrho(k))$ is the normalized membership function and calculated by

$$\psi_j(\varrho(k)) \triangleq \frac{\prod_{\iota=1}^{s} W_{j\iota}(\varrho_\iota(k))}{\sum_{j=1}^{f} \prod_{\iota=1}^{s} W_{j\iota}(\varrho_\iota(k))}.$$

Considering the replay attack model (3) and the structure of controller (7), the system (2) can be expressed as:

$$\begin{aligned} x(k+1) = &\sum_{i=1}^{r} \varphi_i(\rho(k)) \sum_{j=1}^{f} \psi_j(\varrho(k)) \\ &\times \Big((A_i + (1-\alpha(k)) B_i K_j C) x(k) \\ &+ \alpha(k) B_i K_j C x(k - d(k)\tau(k)) \\ &+ (E_i + (1-\alpha(k)) B_i K_j F) \omega(k) \\ &+ \alpha(k) B_i K_j F \omega(k - d(k)\tau(k)) \Big). \end{aligned} \tag{8}$$

Remark 4: In most literature on PID control for nonlinear systems, it is commonly necessary to impose assumptions on the nonlinear functions, such as satisfying the Lipschitz condition or being sector-bounded. Whereas, the fuzzy feedback control strategy we employ only requires the nonlinear system to be smooth, thereby leading to broader applicability of our results. In contrast to other model-based control methods, such as state-feedback control, the proposed output-feedback approach does not require the states to be strictly measurable, which further reduces implementation constraints. More importantly, under reasonable replay attack assumptions, the designed fuzzy controller exhibits strong robustness against both external disturbances and replay attacks, effectively maintaining closed-loop stability and performance even when the system is subject to repeated signal injection.

The objective of this paper is to design a controller that ensures the closed-loop system (8) subject to replay attacks satisfies the following two requirements:

R1) With $\omega(k) = 0$, the system (8) is asymptotically stable;

R2) For all nonzero $\omega(k) \in l_2[0, +\infty)$, under the zero initial condition, the controlled output $z(k)$ satisfies the following H_∞ performance:

$$\sum_{k=0}^{\infty} z^T(k) z(k) \le \lambda^2 \sum_{k=0}^{\infty} \omega^T(k) \omega(k) \tag{9}$$

where $\lambda > 0$ is a given constant.

III. Main Results

In this section, the stability and H_∞ performance of the closed-loop system under frequency-duration-constrained replay attacks are analyzed, and the desired controller gains are computed.

The following theorem provides a sufficient criterion to ensure both stability and H_∞ performance of the closed-loop system (8).

Theorem 1: Let the controller gains K_j $(j = 1, 2, \ldots, f)$, and scalars $\lambda_1 > \lambda_0 > 0$, $\gamma_1 > 1$, and $0 < \gamma_0 < 1$ be given. The closed-loop system (8) is asymptotically stable and satisfies the H_∞ performance in (9) if there exist a positive-definite matrix P and a positive scalar μ such that

$$\bar{\Upsilon}_{ij}^T P \bar{\Upsilon}_{ij} + \bar{\Upsilon}_i < 0 \tag{10}$$

$$\bar{\Psi}_{ij}^T P \bar{\Psi}_{ij} + \bar{\Psi}_{i\breve{i}} < 0 \tag{11}$$

$$\bar{\Psi}_{ij}^T P \bar{\Psi}_{ij} + \bar{\Phi}_{i\breve{i}} < 0 \tag{12}$$

$$(1-\gamma_0)^{-\bar{\tau}} < \mu \tag{13}$$

$$\begin{aligned} &\theta^{-1} \ln(1 + \bar{d}\mu) + \beta^{-1} \ln \gamma_1 \\ &+ (1-\beta^{-1}) \ln(1-\gamma_0) < 0 \end{aligned} \tag{14}$$

where

$$\begin{aligned} \bar{\Upsilon}_{ij} &\triangleq \begin{bmatrix} A_i + B_i K_j C & E_i + B_i K_j F \end{bmatrix}, \\ \bar{\Upsilon}_i &\triangleq \mathrm{diag}\{G_i^T G_i - (1-\gamma_0)P, -\lambda_0^2 I\}, \\ \bar{\Psi}_{ij} &\triangleq \begin{bmatrix} A_i & B_i K_j C & E_i & B_i K_j F \end{bmatrix}, \\ \bar{\Psi}_{i\breve{i}} &\triangleq \mathrm{diag}\{G_i^T G_i - \gamma_1 P, -(\gamma_1 + \gamma_0 - 1)P, \\ &\qquad - \lambda_0^2 I, (\lambda_0^2 - \lambda_1^2) I\}, \\ \bar{\Phi}_{i\breve{i}} &\triangleq \mathrm{diag}\{G_i^T G_i - \gamma_1 P, -\gamma_1 P, -\lambda_0^2 I, -\lambda_1^2 I\}. \end{aligned}$$

Proof: To analyze the stability of system (8), the following Lyapunov function is constructed:

$$\begin{aligned} V(k) \triangleq\ & x^T(k) P x(k) + \alpha(k) x^T(k - d(k)\tau(k)) P \\ & \times x(k - d(k)\tau(k)). \end{aligned} \tag{15}$$

On the basis of (8) and (15), the proof is divided into the following steps.

Step 1: Under condition (10), prove

$$V(k+1) - (1-\gamma_0) V(k) < 0, \text{ when } k \in [k_t, k_{t(1)} - 1]. \tag{16}$$

When $k \in [k_t, k_{t(1)} - 1]$, we calculate that

$$\begin{aligned} & V(k+1) - V(k) + \gamma_0 V(k) \\ &= \sum_{i=1}^{r} \sum_{j=1}^{f} \sum_{\breve{i}=1}^{r} \sum_{\breve{j}=1}^{f} \varphi_i(\rho(k)) \psi_j(\varrho(k)) \varphi_{\breve{i}}(\rho(k)) \psi_{\breve{j}}(\varrho(k)) \\ & \times \big((A_i + B_i K_j C) x(k) + (E_i + B_i K_j F) \omega(k) \big)^T P \\ & \times \big((A_{\breve{i}} + B_{\breve{i}} K_{\breve{j}} C) x(k) + (E_{\breve{i}} + B_{\breve{i}} K_{\breve{j}} F) \omega(k) \big) \\ & \quad - (1-\gamma_0) x^T(k) P x(k) \\ & \le \sum_{i=1}^{r} \sum_{j=1}^{f} \varphi_i(\rho(k)) \psi_j(\varrho(k)) \big((A_i + B_i K_j C) x(k) \\ & \quad + (E_i + B_i K_j F) \omega(k) \big)^T P \big((A_i + B_i K_j C) x(k) \\ & \quad + (E_i + B_i K_j F) \omega(k) \big) - (1-\gamma_0) x^T(k) P x(k) \end{aligned}$$

$$
\begin{aligned}
=&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k))\\
&\times\begin{bmatrix}x(k)\\ \omega(k)\end{bmatrix}^T(\bar{\Upsilon}_{ij}^TP\bar{\Upsilon}_{ij}-(1-\gamma_0)\bar{P})\begin{bmatrix}x(k)\\ \omega(k)\end{bmatrix}
\end{aligned}
$$

where $\bar{P}\triangleq\mathrm{diag}\{P,0\}$.

Letting $\omega(k)=0$ and considering (10), we can see that (16) holds.

Step 2: Under condition (11), prove that the following inequality

$$
V(k+1)-\gamma_1V(k)<0 \tag{17}
$$

holds when $k\in[k_{t(l)},k_{t(l+1)})$.

For $k\in[k_{t(l)},k_{t(l+1)})$, it follows from (15) that

$$
\begin{aligned}
&V(k+1)-\gamma_1V(k)\\
=&x^T(k+1)Px(k+1)-\gamma_1x^T(k)Px(k)\\
&+x^T(k+1-l\tau_t)Px(k+1-l\tau_t)\\
&-\gamma_1x^T(k-l\tau_t)Px(k-l\tau_t)\\
=&\sum_{i=1}^{r}\sum_{j=1}^{f}\sum_{\breve{i}=1}^{r}\sum_{\breve{j}=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k))\varphi_{\breve{i}}(\rho(k))\psi_{\breve{j}}(\varrho(k))\\
&\times\big(A_ix(k)+B_iK_jCx(k-l\tau_t)+E_i\omega(k)\\
&+B_iK_jF\omega(k-l\tau_t)\big)^TP\big(A_{\breve{i}}x(k)+E_{\breve{i}}\omega(k)\\
&+B_{\breve{i}}K_{\breve{j}}Cx(k-l\tau_t)+B_{\breve{i}}K_{\breve{j}}F\omega(k-l\tau_t)\big)\\
&+\sum_{i=1}^{r}\sum_{j=1}^{f}\sum_{\breve{i}=1}^{r}\sum_{\breve{j}=1}^{f}\varphi_i(\rho(k-l\tau_t))\psi_j(\varrho(k-l\tau_t))\\
&\times\varphi_{\breve{i}}(\rho(k-l\tau_t))\psi_{\breve{j}}(\varrho(k-l\tau_t))\\
&\times\big((A_i+B_iK_jC)x(k-l\tau_t)+(E_i+B_iK_jF)\\
&\times\omega(k-l\tau_t)\big)^TP\big((A_{\breve{i}}+B_{\breve{i}}K_{\breve{j}}C)x(k-l\tau_t)\\
&+(E_{\breve{i}}+B_{\breve{i}}K_{\breve{j}}F)\omega(k-l\tau_t)\big)\\
&-\gamma_1x^T(k)Px(k)-\gamma_1x^T(k-l\tau_t)Px(k-l\tau_t)\\
\leq&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k))\\
&\times\big(A_ix(k)+B_iK_jCx(k-l\tau_t)+E_i\omega(k)\\
&+B_iK_jF\omega(k-l\tau_t)\big)^TP\big(A_ix(k)+E_i\omega(k)\\
&+B_iK_jCx(k-l\tau_t)+B_iK_jF\omega(k-l\tau_t)\big)\\
&+\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k-l\tau_t))\psi_j(\varrho(k-l\tau_t))\\
&+\big((A_i+B_iK_jC)x(k-l\tau_t)+(E_i+B_iK_jF)\\
&\times\omega(k-l\tau_t)\big)^TP\big((A_i+B_iK_jC)x(k-l\tau_t)\\
&+(E_i+B_iK_jF)\omega(k-l\tau_t)\big)-\gamma_1x^T(k)Px(k)\\
&-\gamma_1x^T(k-l\tau_t)Px(k-l\tau_t)\\
=&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k))\\
&\times\begin{bmatrix}\bar{x}(k)\\ \bar{\omega}(k)\end{bmatrix}^T(\bar{\Psi}_{ij}^TP\bar{\Psi}_{ij}-\gamma_1\tilde{P}+(1-\gamma_0)\vec{P})\begin{bmatrix}\bar{x}(k)\\ \bar{\omega}(k)\end{bmatrix}\\
&+\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k-l\tau_t))\psi_j(\varrho(k-l\tau_t))\\
&\times\begin{bmatrix}x(k-l\tau_t)\\ \omega(k-l\tau_t)\end{bmatrix}^T(\bar{\Upsilon}_{ij}^TP\bar{\Upsilon}_{ij}-(1-\gamma_0)\bar{P})\\
&\times\begin{bmatrix}x(k-l\tau_t)\\ \omega(k-l\tau_t)\end{bmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
&\bar{x}(k)\triangleq\begin{bmatrix}x(k)\\ x(k-l\tau_t)\end{bmatrix},\quad\bar{\omega}(k)\triangleq\begin{bmatrix}\omega(k)\\ \omega(k-l\tau_t)\end{bmatrix},\\
&\tilde{P}\triangleq\mathrm{diag}\{P,P,0,0\},\quad\vec{P}\triangleq\mathrm{diag}\{0,P,0,0\}.
\end{aligned}
$$

According to the definition of time series, it is easy to see that $k-l\tau_t\in[k_t,k_{t(1)}-1]$ when $k\in[k_{t(l)},k_{t(l+1)})$. Letting $\bar{\omega}(k)=0$, and considering (10), (11), and (16), we obtain

$$
\begin{aligned}
&V(k+1)-\gamma_1V(k)\\
<&V(k-l\tau_t+1)-(1-\gamma_0)V(k-l\tau_t)\\
<&0.
\end{aligned}
$$

Step 3: Under condition (12), prove

$$
V(k_{t(l)})-\gamma_1V(k_{t(l)}-1)-V(k_{t(1)}-\tau_t)<0. \tag{18}
$$

Recalling the definition of $V(k)$, we have

$$
\begin{aligned}
V(k_{t(l)})=&x^T(k_{t(l)})Px(k_{t(l)})\\
&+x^T(k_{t(1)}-\tau_t)Px(k_{t(1)}-\tau_t),\\
V(k_{t(l)}-1)=&x^T(k_{t(l)}-1)Px(k_{t(l)}-1)\\
&+x^T(k_{t(1)}-1)Px(k_{t(1)}-1).
\end{aligned}
$$

Furthermore, the system state suffered from replay attacks at $k_{t(l)}$ can be represented as:

$$
\begin{aligned}
x(k_{t(l)})=&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k_{t(l)}-1))\psi_j(\varrho(k_{t(l)}-1))\\
&\times\Big(A_ix(k_{t(l)}-1)+B_iK_jCx(k_{t(1)}-1)\\
&+E_i\omega(k_{t(l)}-1)+B_iK_jF\omega(k_{t(1)}-1)\Big).
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
&V(k_{t(l)})-\gamma_1V(k_{t(l)}-1)-V(k_{t(1)}-\tau_t)\\
=&\sum_{i=1}^{r}\sum_{j=1}^{f}\sum_{\breve{i}=1}^{r}\sum_{\breve{j}=1}^{f}\varphi_i(\rho(k_{t(l)}-1))\psi_j(\varrho(k_{t(l)}-1))\\
&\times\varphi_{\breve{i}}(\rho(k_{t(l)}-1))\psi_{\breve{j}}(\varrho(k_{t(l)}-1))\big(A_ix(k_{t(l)}-1)\\
&+B_iK_jCx(k_{t(1)}-1)+E_i\omega(k_{t(l)}-1)\\
&+B_iK_jF\omega(k_{t(1)}-1)\big)^TP\big(A_{\breve{i}}x(k_{t(l)}-1)\\
&+B_{\breve{i}}K_{\breve{j}}Cx(k_{t(1)}-1)+E_{\breve{i}}\omega(k_{t(l)}-1)\\
&+B_{\breve{i}}K_{\breve{j}}F\omega(k_{t(1)}-1)\big)-\gamma_1\big(x^T(k_{t(l)}-1)Px(k_{t(l)}-1)\\
&+x^T(k_{t(1)}-1)Px(k_{t(1)}-1)\big)\\
\leq&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k_{t(l)}-1))\psi_j(\varrho(k_{t(l)}-1))\\
&\times\big(A_ix(k_{t(l)}-1)+B_iK_jCx(k_{t(1)}-1)+E_i\omega(k_{t(l)}-1)
\end{aligned}
$$

$$
\begin{aligned}
&+ B_iK_jF\omega(k_{t(1)}-1)\big)^TP\big(A_ix(k_{t(l)}-1)+B_iK_jC\\
&\times x(k_{t(1)}-1)+E_i\omega(k_{t(l)}-1)+B_iK_jF\omega(k_{t(1)}-1)\big)\\
&-\gamma_1\big(x^T(k_{t(l)}-1)Px(k_{t(l)}-1)\\
&+x^T(k_{t(1)}-1)Px(k_{t(1)}-1)\big)\\
=&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k_{t(l)}-1))\psi_j(\varrho(k_{t(l)}-1))\\
&\times\begin{bmatrix}\bar{x}(k_{t(l)}-1)\\ \bar{\omega}(k_{t(l)}-1)\end{bmatrix}^T(\bar{\Psi}_{ij}^TP\bar{\Psi}_{ij}-\gamma_1\tilde{P})\begin{bmatrix}\bar{x}(k_{t(l)}-1)\\ \bar{\omega}(k_{t(l)}-1)\end{bmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{x}(k_{t(l)}-1)&\triangleq\begin{bmatrix}x^T(k_{t(l)}-1) & x^T(k_{t(1)}-1)\end{bmatrix}^T,\\
\bar{\omega}(k_{t(l)}-1)&\triangleq\begin{bmatrix}\omega^T(k_{t(l)}-1) & \omega^T(k_{t(1)}-1)\end{bmatrix}^T.
\end{aligned}
$$

Similarly, letting $\bar{\omega}(k_{t(l)}-1)=0$, it is easy to obtain that

$$
\begin{aligned}
&V(k_{t(l)})-\gamma_1V(k_{t(l)}-1)-V(k_{t(1)}-\tau_t)\\
\le&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k_{t(l)}-1))\psi_j(\varrho(k_{t(l)}-1))\\
&\times\bar{x}^T(k_{t(l)}-1)(\Phi_{ij}^TP\Phi_{ij}-\gamma_1\bar{P})\bar{x}(k_{t(l)}-1)
\end{aligned}
$$

where $\Phi_{ij}\triangleq\begin{bmatrix}A_i & B_iK_jC\end{bmatrix}$ which, together with (11), leads to (18).

Step 4: Under condition (10), prove

$$
V(k_{t(1)})-(1-\gamma_0)V(k_{t(1)}-1)-V(k_{t(1)}-\tau_t)<0. \quad (19)
$$

With (15) and the state dynamics of system (8) at $k_{t(1)}-1$

$$
\begin{aligned}
x(k_{t(1)})=&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k_{t(1)}-1))\psi_j(\varrho(k_{t(1)}-1))\\
&\times\big((A_i+B_iK_jC)x(k_{t(1)}-1)\\
&+(E_i+B_iK_jF)\omega(k_{t(1)}-1)\big),
\end{aligned}
$$

we can see that

$$
\begin{aligned}
&V(k_{t(1)})-(1-\gamma_0)V(k_{t(1)}-1)-V(k_{t(1)}-\tau_t)\\
\le&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k_{t(1)}-1))\psi_j(\varrho(k_{t(1)}-1))\\
&\times\begin{bmatrix}x(k_{t(1)}-1)\\ \omega(k_{t(1)}-1)\end{bmatrix}^T(\bar{\Upsilon}_{ij}^TP\bar{\Upsilon}_{ij}-(1-\gamma_0)\bar{P})\begin{bmatrix}x(k_{t(1)}-1)\\ \omega(k_{t(1)}-1)\end{bmatrix}.
\end{aligned}
$$

Letting $\omega(k_{t(1)}-1)=0$, we have

$$
\begin{aligned}
&V(k_{t(1)})-(1-\gamma_0)V(k_{t(1)}-1)-V(k_{t(1)}-\tau_t)\\
\le&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k_{t(1)}-1))\psi_j(\varrho(k_{t(1)}-1))x^T(k_t^1-1)\\
&\times\big((A_i+B_iK_jC)^TP(A_i+B_iK_jC)\\
&-(1-\gamma_0)P\big)x(k_t^1-1).
\end{aligned}
$$

Noticing (10), we know that the inequality (19) holds.

Step 5: Under condition (12), prove

$$
V(k_t)-\gamma_1V(k_t-1)<0. \quad (20)
$$

Similar to the above analysis process, according to the Lyapunov functions $V(k_t)$, $V(k_t-1)$ and the dynamics of system state $x(k_t)$, one obtains

$$
\begin{aligned}
&V(k_t)-\gamma_1V(k_t-1)\\
=&x^T(k_t)Px(k_t)-\gamma_1\Big(x^T(k_t-1)Px(k_t-1)\\
&+x^T(k_{t-1(1)}-1)Px(k_{t-1(1)}-1)\Big)\\
\le&\sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k_t-1))\psi_j(\varrho(k_t-1))\Big(A_ix(k_t-1)\\
&+B_iK_jCx(k_{t-1(1)}-1)+E_i\omega(k_t-1)\\
&+B_iK_jF\omega(k_{t-1(1)}-1)\Big)^TP\Big(A_ix(k_t-1)\\
&+B_iK_jCx(k_{t-1(1)}-1)+E_i\omega(k_t-1)\\
&+B_iK_jF\omega(k_{t-1(1)}-1)\Big)-\gamma_1\Big(x^T(k_t-1)\\
&\times Px(k_t-1)+x^T(k_{t-1(1)}-1)Px(k_{t-1(1)}-1)\Big)\\
=&\begin{bmatrix}\bar{x}(k_t-1)\\ \bar{\omega}(k_t-1)\end{bmatrix}^T(\bar{\Psi}_{ij}^TP\bar{\Psi}_{ij}-\gamma_1\tilde{P})\begin{bmatrix}\bar{x}(k_t-1)\\ \bar{\omega}(k_t-1)\end{bmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{x}(k_t-1)&\triangleq\begin{bmatrix}x^T(k_t-1) & x^T(k_{t-1(1)}-1)\end{bmatrix}^T,\\
\bar{\omega}(k_t-1)&\triangleq\begin{bmatrix}\omega^T(k_t-1) & \omega^T(k_{t-1(1)}-1)\end{bmatrix}^T.
\end{aligned}
$$

Letting $\bar{\omega}(k_t-1)=0$, the inequality (20) can be concluded from the condition (12).

Step 6: Based on the inequalities proved in the above five steps and under conditions (13) and (14), prove that the closed-loop system (8) is asymptotically stable.

It follows from (17) that

$$
V(k)<\gamma_1V(k-1)<\cdots<\gamma_1^{k-k_{t(l)}}V(k_{t(l)}). \quad (21)
$$

The following inequality is inferred from (16):

$$
\begin{aligned}
V(k_{t(1)}-1)<&(1-\gamma_0)V(k_{t(1)}-2)\\
&<\ldots<(1-\gamma_0)^{\Delta_t-1}V(k_t).
\end{aligned} \quad (22)
$$

Without loss of generality, we assume $k\in[k_{t(l)},k_{t(l+1)})$. Combining (18), (19), and (20) with (21), (22), we obtain that:

$$
\begin{aligned}
V(k)<&\gamma_1^{k-k_t^l}H(l,\tau_t,\Delta_t)V(k_t)\\
<&\gamma_1^{k-k_t^l}H(l,\tau_t,\Delta_t)H(d_{t-1},\tau_{t-1},\Delta_{t-1})V(k_{t-1})\\
<&\ldots\\
<&\gamma_1^{k-k_t^l}H(l,\tau_t,\Delta_t)\prod_{p=1}^{t-1}H(d_p,\tau_p,\Delta_p)V(k_1)
\end{aligned}
$$

where

$$
\begin{aligned}
H(l,\tau_t,\Delta_t)\triangleq&\gamma_1^{(l-1)\tau_t}(1-\gamma_0)^{\Delta_t}\\
&+(1-\gamma_0)^{\Delta_t-\tau_t}\sum_{q=0}^{l-1}\gamma_1^{q\tau_t},\\
H(d_p,\tau_p,\Delta_p)\triangleq&\gamma_1^{d_p\tau_p}(1-\gamma_0)^{\Delta_p}\\
&+(1-\gamma_0)^{\Delta_p-\tau_p}\sum_{q=1}^{d_p}\gamma_1^{q\tau_p}.
\end{aligned}
$$

Under condition (13) and taking $\gamma_1 > 1$, $\tau_t \leq \bar{\tau}$, and $d_t \leq \bar{d}$ into account, it is easy to see that

$$\mu > (1-\gamma_0)^{-\bar{\tau}} > (1-\gamma_0)^{-\tau_p},$$
$$\sum_{q=1}^{d_p} \gamma_1^{q\tau_p} < d_p \gamma_1^{d_p\tau_p} < \bar{d}\gamma_1^{d_p\tau_p},$$

and then

$$H(l,\tau_t,\Delta_t) \leq (1+l\mu)\gamma_1^{(l-1)\tau_t}(1-\gamma_0)^{\Delta_t},$$
$$H(d_p,\tau_p,\Delta_p) \leq (1+\bar{d}\mu)\gamma_1^{d_p\tau_p}(1-\gamma_0)^{\Delta_p}.$$

Thus, we have

$$\begin{aligned} V(k) <& (1+l\mu)(1+\bar{d}\mu)^{t-1}\gamma_1^{k-k_t^l+(l-1)\tau_t+\sum_{p=1}^{t-1} d_p\tau_p} \\ &\times (1-\gamma_0)^{\sum_{p=1}^{t}\Delta_p} V(k_1) \\ <& (1+\bar{d}\mu)^t \gamma_1^{k-k_t^l+(l-1)\tau_t+\sum_{p=1}^{t-1} d_p\tau_p} \\ &\times (1-\gamma_0)^{\sum_{p=1}^{t}\Delta_p} V(k_1). \end{aligned} \tag{23}$$

In light of Assumption 1 and Assumption 2, we have

$$\begin{aligned} t =& H(k,k_1) \leq \sigma + \frac{k-k_1}{\theta}, \\ M(k,k_1) =& k - k_t^l + (l-1)\tau_t + \sum_{p=1}^{t-1} d_p\tau_p \\ \leq& \alpha + \frac{k-k_1}{\beta}, \\ \sum_{p=1}^{t}\Delta_p =& k - k_1 - M(k,k_1) \\ \geq& -\alpha + (1-\frac{1}{\beta})(k-k_1). \end{aligned} \tag{24}$$

Substituting (24) into (23) yields

$$\begin{aligned} V(k) <& (1+\bar{d}\mu)^{H(k_1,k)}\gamma_1^{M(k_1,k)}(1-\gamma_0)^{k-k_1-M(k_1,k)}V(k_1) \\ <& (1+\bar{d}\mu)^{\sigma+\frac{k-k_1}{\theta}}(\frac{\gamma_1}{1-\gamma_0})^{\alpha+\frac{k-k_1}{\beta}}(1-\gamma_0)^{k-k_1}V(k_1). \end{aligned}$$

When $k_1 = 0$, we have

$$V(k) < e^{\kappa}e^{\varsigma k}V(0)$$

where

$$\begin{aligned} \kappa \triangleq& \sigma\ln(1+\bar{d}\mu) + \alpha\ln\gamma_1 - \alpha\ln(1-\gamma_0), \\ \varsigma \triangleq& \theta^{-1}\ln(1+\bar{d}\mu) + \beta^{-1}\ln\gamma_1 + (1-\beta^{-1})\ln(1-\gamma_0). \end{aligned}$$

Furthermore, it is worth noting that

$$\begin{aligned} \underline{\eta}x^T(k)x(k) \leq& x^T(k)Px(k), \\ x^T(0)Px(0) \leq& \bar{\eta}x^T(0)x(0) \end{aligned}$$

where $\underline{\eta} \triangleq \lambda_{\min}\{P\}$ and $\bar{\eta} \triangleq \lambda_{\max}\{P\}$. Finally, it follows that

$$\|x(k)\|^2 < \frac{\bar{\eta}}{\underline{\eta}}e^{\kappa}e^{\varsigma k}\|\tilde{x}(0)\|^2.$$

In term of the condition (14), one has $\varsigma < 0$, which implies $0 < e^{\varsigma} < 1$, and we have $x(k) \to 0$ as $k \to \infty$. Thus, the closed-loop system is asymptotically stable.

Next, we will proceed with the H_∞ performance analysis of system (8). To this end, define the following function as:

$$\begin{aligned} J(k) \triangleq& z^T(k)z(k) - \lambda_0^2\omega^T(k)\omega(k) \\ &- \alpha(k)\lambda_1^2\omega^T(k-d(k)\tau(k))\omega(k-d(k)\tau(k)). \end{aligned}$$

When $k \in [k_t, k_{t(1)}-1]$, one has

$$\begin{aligned} &V(k+1) - (1-\gamma_0)V(k) + J(k) \\ =& \sum_{i=1}^{r}\sum_{j=1}^{f}\sum_{\breve{i}=1}^{r}\sum_{\breve{j}=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k))\varphi_{\breve{i}}(\rho(k))\psi_{\breve{j}}(\varrho(k)) \\ &\times \big((A_i+B_iK_jC)x(k) + (E_i+B_iK_jF)\omega(k)\big)^T P \\ &\times \big((A_{\breve{i}}+B_{\breve{i}}K_{\breve{j}}C)x(k) + (E_{\breve{i}}+B_{\breve{i}}K_{\breve{j}}F)\omega(k)\big) \\ &- (1-\gamma_0)x^T(k)Px(k) + z^T(k)z(k) - \lambda_0^2\omega^T(k)\omega(k) \\ \leq& \sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k))\Big(\big((A_i+B_iK_jC)x(k) \\ &+ (E_i+B_iK_jF)\omega(k)\big)^T P\big((A_{\breve{i}}+B_{\breve{i}}K_{\breve{j}}C)x(k) \\ &+ (E_{\breve{i}}+B_{\breve{i}}K_{\breve{j}}F)\omega(k)\big) + x^T(k)G_i^TG_ix(k)\Big) \\ &- (1-\gamma_0)x^T(k)Px(k) - \lambda_0^2\omega^T(k)\omega(k) \\ =& \begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix}^T (\bar{\Upsilon}_{ij}^T P\bar{\Upsilon}_{ij} + \bar{\Upsilon}_i)\begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix}. \end{aligned}$$

It can be concluded from (10) that

$$V(k+1) - (1-\gamma_0)V(k) + J(k) < 0, \tag{25}$$

For $k \in [k_{t(l)}, k_{t(l+1)})$, we have

$$\begin{aligned} &V(k+1) - \gamma_1V(k) + J(k) \\ =& \sum_{i=1}^{r}\sum_{j=1}^{f}\sum_{\breve{i}=1}^{r}\sum_{\breve{j}=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k))\varphi_{\breve{i}}(\rho(k))\psi_{\breve{j}}(\varrho(k)) \\ &\times \big(A_ix(k) + B_iK_jCx(k-l\tau_t) + E_i\omega(k) \\ &+ B_iK_jF\omega(k-l\tau_t)\big)^T P\big(A_{\breve{i}}x(k) + E_{\breve{i}}\omega(k) \\ &+ B_{\breve{i}}K_{\breve{j}}Cx(k-l\tau_t) + B_{\breve{i}}K_{\breve{j}}F\omega(k-l\tau_t)\big) \\ &- \gamma_1x^T(k)Px(k) - \gamma_1x^T(k-l\tau_t)Px(k-l\tau_t) \\ &+ z^T(k)z(k) - \lambda_0^2\omega^T(k)\omega(k) + x^T(k-l\tau_t+1) \\ &\times Px(k-l\tau_t+1) - \lambda_1^2\omega^T(k-l\tau_t)\omega(k-l\tau_t) \\ \leq& \sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k)) \\ &\times \Big(\big(A_ix(k) + B_iK_jCx(k-l\tau_t) + E_i\omega(k) \\ &+ B_iK_jF\omega(k-l\tau_t)\big)^T P\big(A_ix(k) + E_i\omega(k) \\ &+ B_iK_jCx(k-l\tau_t) + B_iK_jF\omega(k-l\tau_t)\big) \\ &+ x^T(k)G_i^TG_ix(k)\Big) - \gamma_1x^T(k)Px(k) \\ &- \gamma_1x^T(k-l\tau_t)Px(k-l\tau_t) - \lambda_0^2\omega^T(k)\omega(k) \\ &- \lambda_1^2\omega^T(k-l\tau_t)\omega(k-l\tau_t) + x^T(k-l\tau_t+1) \\ &\times Px(k-l\tau_t+1). \end{aligned}$$

According to the inequality (25), one has

$$V(k+1) - \gamma_1V(k) + J(k)$$

$$
\begin{aligned}
&< \sum_{i=1}^{r}\sum_{j=1}^{f}\varphi_i(\rho(k))\psi_j(\varrho(k))\\
&\quad\times\Big(\big(A_ix(k)+B_iK_jCx(k-l\tau_t)+E_i\omega(k)\\
&\quad+B_iK_jF\omega(k-l\tau_t)\big)^TP\big(A_ix(k)+E_i\omega(k)\\
&\quad+B_iK_jCx(k-l\tau_t)+B_iK_jF\omega(k-l\tau_t)\big)\\
&\quad+x^T(k)G_i^TG_ix(k)\Big)-\gamma_1x^T(k)Px(k)\\
&\quad-(\gamma_1+\gamma_0-1)x^T(k-l\tau_t)Px(k-l\tau_t)\\
&\quad-\lambda_0^2\omega^T(k)\omega(k)-(\lambda_1^2-\lambda_0^2)\omega^T(k-l\tau_t)\omega(k-l\tau_t).
\end{aligned}
$$

Based on the condition (11), it is deduced that

$$V(k+1)-\gamma_1V(k)+J(k)<0.$$

Similarly, the following inequalities can be inferred from (11) and (12), respectively

$$
\begin{aligned}
&V(k_{t(l)})-\gamma_1V(k_{t(l)}-1)-V(k_{t(1)}-\tau_t)\\
&\qquad+J(k_{t(l)}-1)<0,\\
&V(k_{t(1)})-(1-\gamma_0)V(k_{t(1)}-1)-V(k_{t(1)}-\tau_t)\\
&\qquad+J(k_{t(1)}-1)<0,\\
&V(k_t)-\gamma_1V(k_t-1)+J(k_t-1)<0.
\end{aligned}
$$

Thus, we conclude that:

$$
\begin{aligned}
V(k)&<\gamma_1V(k-1)-J(k-1)\\
&<\gamma_1^2V(k-2)-\gamma_1J(k-2)-J(k-1)\\
&<\dots\\
&<\gamma_1^{k-k_{t(l)}}H(l,\tau_t,\Delta_t)\prod_{p=1}^{t-1}H(d_p,\tau_p,\Delta_p)V(k_1)\\
&\quad-\gamma_1^{k-k_{t(l)}}H(l,\tau_t,\Delta_t)\sum_{\bar p=2}^{t-1}\prod_{p=\bar p}^{t-1}H(d_p,\tau_p,\Delta_p)\\
&\quad\times\Big(\bar H(d_p,\tau_p,\Delta_p)\sum_{s=k_{p-1}}^{k_{p-1(1)}-1-\tau_{p-1}}(1-\gamma_0)^{k_{p-1(1)}-\tau_{p-1}}\\
&\quad\times(1-\gamma_0)^{-s-1}J(s)+\gamma_1^{d_{p-1}\tau_{p-1}}\\
&\quad\times\sum_{s=k_{p-1(1)}-\tau_{p-1}}^{k_{p-1(1)}-1}(1-\gamma_0)^{k_{p-1(1)}-s-1}J(s)\\
&\quad+\sum_{s=k_{p-1(1)}}^{k_p-1}\gamma_1^{k_p-s-1}J(s)\Big)-\gamma_1^{k-k_{t(l)}}H(l,\tau_t,\Delta_t)\\
&\quad\times\Big(\sum_{s=k_{t-1}}^{k_{t-1(1)}-1-\tau_{t-1}}(1-\gamma_0)^{k_{t-1(1)}-s-1-\tau_{t-1}}J(s)\\
&\quad+\gamma_1^{d_{t-1}\tau_{t-1}}\sum_{s=k_{t-1(1)}-\tau_{t-1}}^{k_{t-1(1)}-1}(1-\gamma_0)^{k_{t-1(1)}-s-1}J(s)\\
&\quad+\sum_{s=k_{t-1(1)}}^{k_t-1}\gamma_1^{k_t-s-1}J(s)\Big)-\gamma_1^{k-k_{t(l)}}\bar H(l,\tau_t,\Delta_t)\\
&\quad\times\sum_{s=k_t}^{k_{t(1)}-1-\tau_t}(1-\gamma_0)^{k_{t(0)}-s-1}J(s)\\
&\quad-\gamma_1^{k-k_{t(1)}}\sum_{s=k_{t(1)}-\tau_t}^{k_{t(1)}-1}(1-\gamma_0)^{k_{t(1)}-s-1}J(s)\\
&\quad-\sum_{s=k_{t(1)}}^{k-1}\gamma_1^{k-s-1}J(s)
\end{aligned}
$$

where

$$
\begin{aligned}
\bar H(d_p,\tau_p,\Delta_p)&\triangleq H(d_p,\tau_p,\Delta_p)(1-\gamma_0)^{\tau_p-\Delta_p},\\
\bar H(l,\tau_t,\Delta_t)&\triangleq H(l,\tau_t,\Delta_t)(1-\gamma_0)^{\tau_t-\Delta_t}.
\end{aligned}
$$

Under the zero initial condition $V(0)=0$ and considering the fact $V(k)\geq 0$, one has

$$
\begin{aligned}
&\gamma_1{}^{k-k_{t(l)}}H(l,\tau_t,\Delta_t)\sum_{\bar p=2}^{t-1}\prod_{p=\bar p}^{t-1}H(d_p,\tau_p,\Delta_p)\\
&\quad\times\Big(\bar H(d_p,\tau_p,\Delta_p)\sum_{s=k_{p-1}}^{k_{p-1(1)}-1-\tau_{p-1}}(1-\gamma_0)^{k_{p-1(1)}-\tau_{p-1}}\\
&\quad\times(1-\gamma_0)^{-s-1}J(s)+\gamma_1^{d_{p-1}\tau_{p-1}}\\
&\quad\times\sum_{s=k_{p-1(1)}-\tau_{p-1}}^{k_{p-1(1)}-1}(1-\gamma_0)^{k_{p-1(1)}-s-1}J(s)\\
&\quad+\sum_{s=k_{p-1(1)}}^{k_p-1}\gamma_1^{k_p-s-1}J(s)\Big)+\gamma_1^{k-k_{t(l)}}H(l,\tau_t,\Delta_t)\\
&\quad\times\Big(\sum_{s=k_{t-1}}^{k_{t-1(1)}-1-\tau_{t-1}}(1-\gamma_0)^{k_{t-1(1)}-s-1-\tau_{t-1}}J(s)\\
&\quad+\gamma_1^{d_{t-1}\tau_{t-1}}\sum_{s=k_{t-1(1)}-\tau_{t-1}}^{k_{t-1(1)}-1}(1-\gamma_0)^{k_{t-1(1)}-s-1}J(s)\\
&\quad+\sum_{s=k_{t-1(1)}}^{k_t-1}\gamma_1^{k_t-s-1}J(s)\Big)+\gamma_1^{k-k_{t(l)}}\bar H(l,\tau_t,\Delta_t)\\
&\quad\times\sum_{s=k_t}^{k_{t(1)}-1-\tau_t}(1-\gamma_0)^{k_{t(0)}-s-1}J(s)\\
&\quad+\gamma_1^{k-k_{t(1)}}\sum_{s=k_{t(1)}-\tau_t}^{k_{t(1)}-1}(1-\gamma_0)^{k_{t(1)}-s-1}J(s)\\
&\quad+\sum_{s=k_{t(1)}}^{k-1}\gamma_1^{k-s-1}J(s)<0.
\end{aligned}
$$

From the definitions of $\bar H(d_p,\tau_p,\Delta_p)$ and $\bar H(l,\tau_t,\Delta_t)$, it is easy to see that:

$$
\begin{aligned}
\bar H(d_p,\tau_p,\Delta_p)&>\gamma_1^{d_p\tau_p}(1-\gamma_0)^{\tau_p},\\
\bar H(l,\tau_t,\Delta_t)&>\gamma_1^{(l-1)\tau_t}(1-\gamma_0)^{\tau_t}
\end{aligned}
$$

and

$$
\begin{aligned}
&\sum_{s=0}^{k-1}\gamma_1^{M(s,k-1)}(1-\gamma_0)^{k-s-1-M(s,k-1)}z^T(s)z(s)\\
&\quad<\sum_{s=0}^{k-1}(1+\bar d\mu)^{H(s,k-1)}\gamma_1^{M(s,k-1)}
\end{aligned}
$$

$$\times (1-\gamma_0)^{k-s-1-M(s,k-1)}\Big(\lambda_0^2\omega^T(s)\omega(s) + \alpha(s)\lambda_1^2\omega^T(s-d(s)\tau(s))\omega(s-d(s)\tau(s))\Big).$$

Recalling Assumption 1 and Assumption 2, we have

$$\begin{aligned}&\sum_{s=0}^{k-1}(1-\gamma_0)^{k-s-1}\big(z^T(s)z(s)\\ &<\sum_{s=0}^{k-1}(1+\bar{d}\mu)^{\sigma+\frac{k-s-1}{\theta}}(\frac{\gamma_1}{1-\gamma_0})^{\alpha+\frac{k-s-1}{\beta}}\\ &\times(1-\gamma_0)^{k-s-1}\big(\lambda_0^2\omega^T(s)\omega(s)\\ &+\alpha(s)\lambda_1^2\omega^T(s-d(s)\tau(s))\omega(s-d(s)\tau(s))\big).\end{aligned}$$

Taking summation on both sides of the above inequality from $k=1$ to $k=\infty$ results in

$$\begin{aligned}&\sum_{k=1}^{\infty}\sum_{s=0}^{k-1}(1-\gamma_0)^{k-s-1}z^T(s)z(s)\\ &<\sum_{k=1}^{\infty}\sum_{s=0}^{k-1}(1+\bar{d}\mu)^{\sigma+\frac{k-s-1}{\theta}}(\frac{\gamma_1}{1-\gamma_0})^{\alpha+\frac{k-s-1}{\beta}}\\ &\times(1-\gamma_0)^{k-s-1}\big(\lambda_0^2\omega^T(s)\omega(s)\\ &+\alpha(s)\lambda_1^2\omega^T(s-d(s)\tau)\omega(s-d(s)\tau)\big),\end{aligned}$$

which can be rewritten as:

$$\begin{aligned}&\sum_{s=0}^{\infty}z^T(s)z(s)\sum_{k=s+1}^{\infty}(1-\gamma_0)^{k-s-1}\\ &<\sum_{s=0}^{\infty}\big(\lambda_0^2\omega^T(s)\omega(s)+\alpha(s)\lambda_1^2\omega^T(s-d(s)\tau(s))\\ &\times\omega(s-d(s)\tau(s))\big)\\ &\times\sum_{k=s+1}^{\infty}(1+\bar{d}\mu)^{\frac{k-s-1}{\theta}}(\frac{\gamma_1}{1-\gamma_0})^{\frac{k-s-1}{\beta}}(1-\gamma_0)^{k-s-1}.\end{aligned}$$

Owing to $\lambda_1>\lambda_0>0$, it is easy to see that

$$\begin{aligned}&\sum_{s=0}^{\infty}\big(\lambda_0^2\omega^T(s)\omega(s)\\ &+\alpha(s)\lambda_1^2\omega^T(s-d(s)\tau(s))\omega(s-d(s)\tau(s))\big)\\ &\le 2\lambda_1^2\sum_{q=0}^{\infty}\omega^T(s)\omega(s).\end{aligned}$$

Considering the condition (14), we have

$$\sum_{k=0}^{\infty}z^T(k)z(k)<\sum_{k=0}^{\infty}\bar{\lambda}^2\omega^T(k)\omega(k)$$

where

$$\bar{\lambda}\triangleq\sqrt{\frac{2(1+\bar{d}\mu)^{\sigma}\gamma_0(1-\gamma_0)^{-\alpha}\gamma_1^{\alpha}}{1-(1+\bar{d}\mu)^{\frac{1}{\theta}}(1-\gamma_0)^{1-\frac{1}{\beta}}\gamma_1^{\frac{1}{\beta}}}}\lambda_1.$$

The proof is complete now. ■

In Theorem 1, sufficient conditions have been established, with given controller gains, to ensure the asymptotic stability and the H_∞ performance of system (8). Based on Theorem 1, the desired fuzzy controller gains are computed in Theorem 2.

Theorem 2: Let scalars $\mu>1$, $\gamma_1>1$, $0<\gamma_0<1$, and $\lambda>0$ be given. The closed-loop system in (8) is asymptotically stable and satisfies the H_∞ performance if there exist a positive-definite matrix $\check{P}>0$, and matrices K_j, $\bar{R}_j$, Q_{11}, Q_{21}, and Q_{22} satisfying (13), (14), and the following inequalities:

$$\begin{bmatrix}\check{\Psi}^{11} & *\\ \check{\Psi}_{ij}^{21} & \check{\Psi}^{22}\end{bmatrix}<0 \tag{26}$$

$$\begin{bmatrix}\check{\Phi}^{11} & *\\ \check{\Psi}_{ij}^{21} & \check{\Psi}^{22}\end{bmatrix}<0 \tag{27}$$

$$\begin{bmatrix}\check{\Upsilon}^{11} & *\\ \check{\Upsilon}^{21} & \check{\Upsilon}^{22}\end{bmatrix}<0 \tag{28}$$

where

$$\begin{aligned}\check{\Psi}_{11}\triangleq&-\mathrm{diag}\{\gamma_1(\bar{Q}^T+\bar{Q}-\check{P}),\\ &(\gamma_1+\gamma_0-1)(\bar{Q}^T+\bar{Q}-\check{P}),\\ &\lambda_0^2I,(\lambda_1^2-\lambda_0^2)I\},\\ \check{\Psi}_{ij}^{21}\triangleq&\begin{bmatrix}A_i\bar{Q} & B_i\bar{R}_j & E_i & B_iK_jF\\ G_i\bar{Q} & 0 & 0 & 0\end{bmatrix},\\ \check{\Psi}_{22}\triangleq&-\mathrm{diag}\{\check{P},I\},\quad \bar{Q}\triangleq T_CQ,\\ \check{\Phi}_{11}\triangleq&-\mathrm{diag}\{\gamma_1(\bar{Q}^T+\bar{Q}-\check{P}),\gamma_1(\bar{Q}^T+\bar{Q}-\check{P}),\\ &-\lambda_0^2I,-\lambda_1^2I\},\\ \check{\Upsilon}^{11}\triangleq&\mathrm{diag}\{(\gamma_0-1)(\bar{Q}^T+\bar{Q}-\check{P}),-\lambda^2I\},\\ \check{\Upsilon}_{ij}^{21}\triangleq&\begin{bmatrix}A_i\bar{Q}+B_i\bar{R}_j & E_i+B_iK_jF\\ G_i\bar{Q} & 0\end{bmatrix},\\ Q\triangleq&\begin{bmatrix}Q_{11} & 0\\ Q_{21} & Q_{22}\end{bmatrix},\quad T_C\triangleq\begin{bmatrix}C^T(CC^T)^{-1} & C^{\perp}\end{bmatrix}.\end{aligned}$$

Proof: By a simple matrix transformation, the inequality in (11) can be rewritten as:

$$\tilde{\Psi}_{ij}^T\tilde{\Psi}^{(1)}\tilde{\Psi}_{ij}+\tilde{\Psi}^{(2)}<0 \tag{29}$$

where

$$\begin{aligned}\tilde{\Psi}_{ij}\triangleq&\begin{bmatrix}A_i & B_iK_jC & E_i & B_iK_jF\\ G_i & 0 & 0 & 0\end{bmatrix},\\ \tilde{\Psi}^{(1)}\triangleq&\mathrm{diag}\{P,I\},\\ \tilde{\Psi}^{(2)}\triangleq&-\mathrm{diag}\{\gamma_1P,(\gamma_1+\gamma_0-1)P,\lambda_0^2I,(\lambda_1^2-\lambda_0^2)I\}.\end{aligned}$$

By applying the Schur complement lemma, it follows that (29) holds if and only if

$$\begin{bmatrix}\tilde{\Psi}^{(2)} & *\\ \tilde{\Psi}_{ij} & -(\tilde{\Psi}^{(1)})^{-1}\end{bmatrix}<0. \tag{30}$$

Next, pre- and post-multiplying the matrix in (30) by $\mathrm{diag}\{\bar{Q}^T,\bar{Q}^T,I,I,I,I\}$ and its transpose, the following inequality is obtained:

$$\begin{bmatrix}\check{\Psi}^{(2)} & *\\ \check{\Psi}_{ij} & -(\tilde{\Psi}^{(1)})^{-1}\end{bmatrix}<0 \tag{31}$$

where

$$\begin{aligned}\check{\Psi}^{(2)}\triangleq&-\mathrm{diag}\{\gamma_1\bar{Q}^TP\bar{Q},(\gamma_1+\gamma_0-1)\bar{Q}^TP\bar{Q},\\ &\lambda_0^2I,(\lambda_1^2-\lambda_0^2)I\},\\ \check{\Psi}_{ij}\triangleq&\begin{bmatrix}A_i\bar{Q} & B_iK_jC\bar{Q} & E_i & B_iK_jF\\ G_i\bar{Q} & 0 & 0 & 0\end{bmatrix}.\end{aligned}$$

Since $P > 0$, it follows that

$$\begin{aligned}&\bar{Q}^T + \bar{Q} - \bar{Q}^T P \bar{Q} - P^{-1} \\ &= -(\bar{Q} - P^{-1})^T P (\bar{Q} - P^{-1}) \leq 0\end{aligned}$$

which implies $\bar{Q}^T P \bar{Q} \geq \bar{Q}^T + \bar{Q} - P^{-1}$. Therefore, it can be concluded that (31) is satisfied if (26) holds. By following a similar reasoning, the inequalities in (27) and (28) can also be derived. The proof is thus complete. ■

Remark 5: So far, the fuzzy controller design problem for T-S fuzzy systems under replay attacks has been investigated. In particular, a more general model has been proposed to describe the behavior of replay attacks. Under the given assumptions on the frequency and duration of replay attacks, sufficient conditions have been established to guarantee the existence of the desired controller gains.

Remark 6: In comparison with the existing literature, the distinguishing novelties of this article are emphasized as follows.

1) This work represents the first attempt to investigate the fuzzy control problem for T-S fuzzy systems under replay attacks subject to frequency and duration constraints. While previous studies have primarily considered replay attacks in terms of single occurrences or simplified assumptions, the present study provides a rigorous framework that explicitly incorporates both the frequency of attacks and the duration of their impact, thereby offering a more realistic and practical treatment of the problem.
2) A novel replay attack model is developed to characterize the behavior of repeated replays. Unlike earlier models reported in [29], [40], [45], which typically restricted the number of replay times or assumed a fixed delay structure, the proposed model allows both the number of recorded data packets and the number of replays to be determined by the adversary. As a result, the model is more general and capable of capturing a wider range of attack scenarios that may occur in practice.
3) The proposed replay attack model offers greater generality by capturing the process of repeatedly replaying the same recorded data. While this generalization better reflects practical attack scenarios, it also introduces significant challenges in theoretical analysis. Unlike existing studies where only the switching between the normal and attacked modes needs to be considered, our model further requires analyzing the switching between different replay stages within the attack mode itself, which substantially increases the complexity of deriving system performance.
4) A fuzzy controller is designed to effectively cope with the measurement delays induced by replay attacks while ensuring the desired system performance. The controller design is derived under a set of sufficient conditions formulated using Lyapunov stability theory, which guarantee asymptotic stability as well as the required H_∞ performance. This contribution provides a systematic approach to achieving robust control for T-S fuzzy systems in the presence of replay attacks.

IV. Numerical Example

In this section, an illustrative example is given to verify the effectiveness of the fuzzy controller for the plant under replay attacks.

The nonlinear system is described by the following T-S fuzzy model with two rules:

$$\begin{cases} x(k+1) = \sum_{i=1}^{2} \varphi_i(\rho(k))(A_i x(k) + B_i u(k) + E_i \omega(k)) \\ y(k) = C x(k) + F\omega(k) \\ z(k) = \sum_{i=1}^{2} \varphi_i(\rho(k)) G_i x(k) \end{cases}$$

where

$$\begin{aligned} &A_1 = \begin{bmatrix} 1.02 & 0.3 \\ 0.2 & 0.32 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.94 & 0.2 \\ 0.3 & 0.2 \end{bmatrix}, \\ &B_1 = \begin{bmatrix} 1.1 \\ 0.8 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.2 \\ 0.6 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ &E_1 = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ &C = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}, \quad F = 0.1, \\ &\varphi_1(\rho(k)) = \sin^2(k), \quad \varphi_2(\rho(k)) = \cos^2(k). \end{aligned}$$

In the simulation, the noise is set as $\omega(k) = (0.8\sin(k))/k$ and the prescribed disturbance attenuation level is set to be $\lambda_0 = 1.2$ and $\lambda_1 = 1.8$. The parameters of the replay attack are taken as $\bar{\tau} = 4$, $\bar{d} = 5$, $\alpha = 0.1$, $\beta = 50$, $\sigma = 0.1$, and $\theta = 40$.

By solving matrix inequalities (13) and (14) in Theorem 1 and (26)-(28) in Theorem 2, the controller gains are calculated by

$$K_1 = -0.39558, \quad K_2 = -0.39694.$$

Simulation results are shown in Figs. 2–6. The state evolution of the open-loop system is plotted in Fig. 2. Fig. 3 shows the difference between the normal measurement output $y(k)$ and the manipulated measurement output $\tilde{y}(k)$, from which it is seen that the recorded measurement output is replayed in sequence when $k \in [6, 14]$, $k \in [31, 45]$, $k \in [56, 63]$, and $k \in [71, 90]$. Fig. 4 plots the state evolution under replay attacks, and the control output is plotted in Fig. 5. The time instants of replay attacks are depicted in Fig. 6. Fig. 7 plots the state evolution under replay attacks and possible packet loss, where time delays caused by replay attacks are bounded and the probability of packet loss is 20%. It can be seen that the designed H_∞ fuzzy controller keeps robustness against network latency caused by replay attacks and packet loss.

V. Conclusions

In this article, the H_∞ fuzzy control problem has been investigated for T-S fuzzy systems subject to replay attacks. In practice, adversaries may repeatedly launch replay attacks by exploiting historical data. To capture this feature, a novel mathematical model has been proposed to describe the repetitive replay behavior. On the basis of Lyapunov stability theory, sufficient conditions have been derived to guarantee

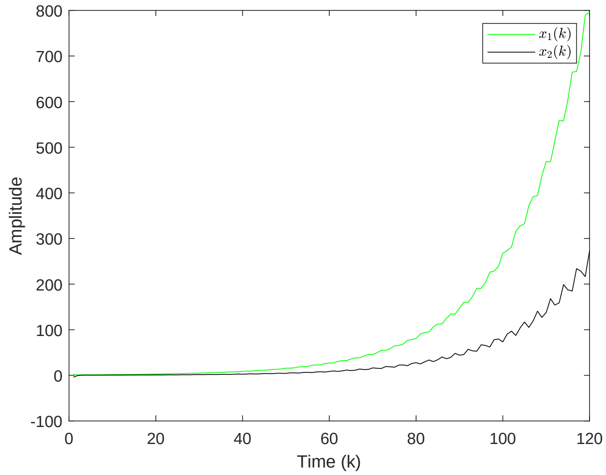

Fig. 2: Evolution of the system state $x(k)$ without control.

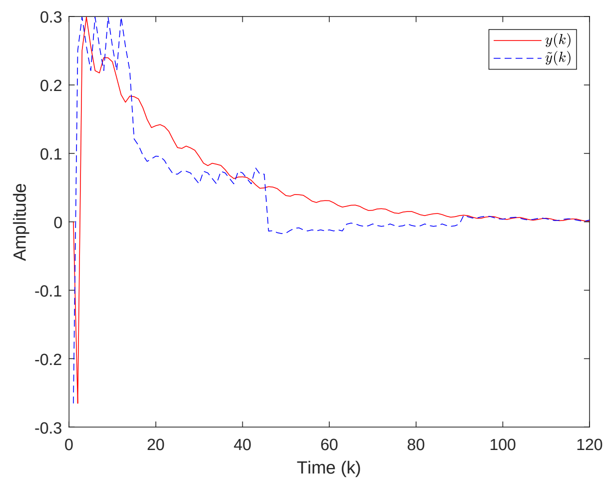

Fig. 3: The unmanipulated measurement output $y(k)$ and the attacked measurement output $\tilde{y}(k)$.

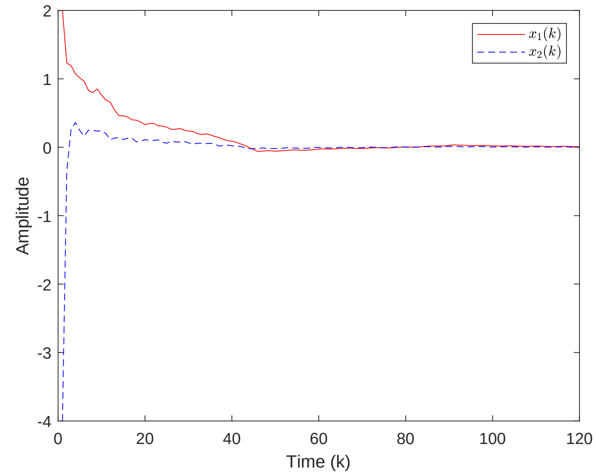

Fig. 4: State evolution $x(k)$ of the closed-loop system.

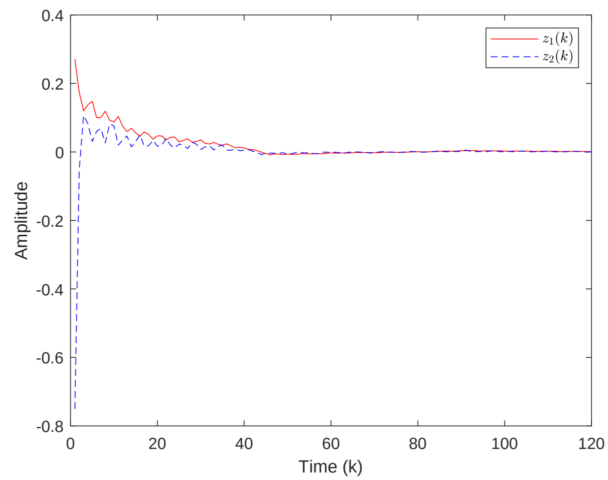

Fig. 5: Evolution of the control output $z(k)$.

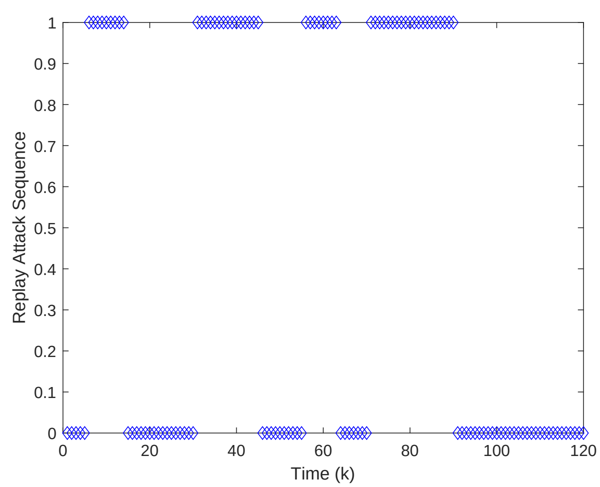

Fig. 6: Time instants of replay attacks.

both the stability and the H_∞ performance of the closed-loop system. By incorporating assumptions on the frequency, duration, record length, and replay count of replay attacks, the disturbance attenuation level has been established. Subsequently, the fuzzy controller gains have been designed with the aid of the orthogonal decomposition technique. Finally, a simulation example has been presented to demonstrate the effectiveness of the proposed fuzzy controller against replay attacks.

Future research directions include the development of active control strategies for T-S fuzzy systems under replay attacks and the extension of the secure control problem to distributed systems such as multi-agent systems [46], [47], sensor networks [48], [49], and complex networks [50].

REFERENCES

[1] W. Chen and G.-P. Liu, Privacy-preserving consensus-based distributed economic dispatch of smart grids via state decomposition, *IEEE/CAA Journal of Automatica Sinica*, vol. 11, no. 5, pp. 1250-1261, 2024.

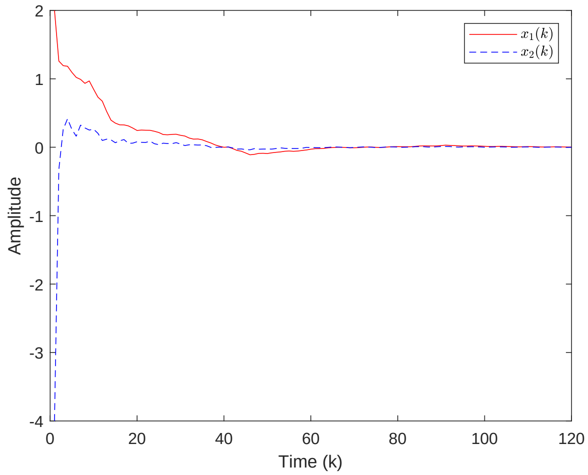

Fig. 7: State evolution $x(k)$ under replay attacks and packet loss.

[2] B. Qu, D. Peng, Y. Shen, L. Zou, and B. Shen, A survey on recent advances on dynamic state estimation for power systems, *International Journal of Systems Science*, vol. 55, no. 16, pp. 3305–3321, 2024.

[3] Y. Wang, C. Shen, J. Huang, and H. Chen, Model-free adaptive control for unmanned surface vessels: a literature review, *Systems Science & Control Engineering*, vol. 12, no. 1, 2024, art. no. 2316170.

[4] B. Xin, Y. Kang, and W. Liu, Tracking-oriented formation control for unmanned aerial vehicle swarm through output feedback under jointly-connected topologies, *IEEE Transactions on Vehicular Technology*, vol. 73, no. 1, pp. 438-449, 2024.

[5] Y. Song, B. Zhang, C. Wen, D. Wang, and G. Wei, Model predictive control for complicated dynamic systems: a survey, *International Journal of Systems Science*, vol. 56, no. 9, pp. 2168–2193, 2025.

[6] W. Wang, L. Ma, Q. Rui, and C. Gao, A survey on privacy-preserving control and filtering of networked control systems, *International Journal of Systems Science*, vol. 55, no. 11, pp. 2269–2288, 2024.

[7] C. Zhou, S. Huang, N. Xiong, S.-H. Yang, H. Li, and Y. Qin, Design and analysis of multimodel-based anomaly intrusion detection systems in industrial process automation, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45, no. 10, pp. 1345-1360, 2015.

[8] R. Caballero-Águila and J. Linares-Pérez, Centralized fusion estimation in networked systems: addressing deception attacks and packet dropouts with a zero-order hold approach, *International Journal of Network Dynamics and Intelligence*, vol. 3, no. 4, Dec. 2024, art. no. 100021.

[9] Y. Ishido, K. Takaba, and D. E. Quevedo, Stability analysis of networked control systems subject to packet-dropouts and finite-level quantization, *Systems & Control Letters*, vol. 60, no. 5, pp. 325-332, 2011.

[10] Y. Jin, X. Ma, X. Meng, and Y. Chen, Distributed fusion filtering for cyber-physical systems under Round-Robin protocol: a mixed H_2/H_∞ framework. *International Journal of Systems Science*, vol. 54, no. 8, pp. 1661-1675, 2023.

[11] T. Seitz, R. K. Yedavalli, and A. Behbahani, Robust stability bounds for multi-delay networked control systems, *International Journal of Control*, vol. 91, no. 4, pp. 755-769, 2017.

[12] W. Zhang, T. Wang, and S. Tong, Event-triggered control for networked switched fuzzy time-delay systems with saturated inputs, *International Journal of Fuzzy Systems*, vol. 21, pp. 1455-1466, 2019.

[13] L. Su and G. Chesi, Robust stability analysis and synthesis for uncertain discrete-time networked control systems over fading channels, *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 1966-1971, 2017.

[14] C. Xu, H. Dong, Y. Shen, and J. Li, Dynamic event-triggered fault estimation for complex networks with time-correlated fading channels, *Information Sciences*, vol. 644, 2023, art. no. 119288.

[15] R. Caballero-Águila, M. P. Frías-Bustamante, and A. Oya-Lechuga, Least-squares linear estimation for multirate uncertain systems subject to DoS attacks, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 2, Jun. 2025, art. no. 100014.

[16] T. Sui, Y. Mo, D. Marelli, X. Sun, and M. Fu, The vulnerability of cyber-physical system under stealthy attacks, *IEEE Transactions on Automatic Control*, vol. 66, no. 2, pp. 637-650, 2021.

[17] D. Sun, I. Hwang, and J. Goppert, A stealthy man-in-the-middle attack strategy for switched systems, *International Journal of Systems Science*, vol. 55, no. 6, pp. 1206-1223, 2024.

[18] J. Wang, D. Wang, H. Yan and H. Shen, Composite antidisturbance H_∞ control for hidden Markov jump systems with multi-sensor against replay attacks, *IEEE Transactions on Automatic Control*, vol. 69, no. 3, pp. 1760-1766, Mar. 2024.

[19] H. Shen, Y. Wang, J. Wu, J. H. Park and J. Wang, Secure control for Markov jump cyber-physical systems subject to malicious attacks: a resilient hybrid learning scheme, *IEEE Transactions on Cybernetics*, vol. 54, no. 11, pp. 7068-7079, Nov. 2024.

[20] R. Caballero-Águila, J. Hu, and J. Linares-Pérez, Filtering and smoothing estimation algorithms from uncertain nonlinear observations with time-correlated additive noise and random deception attacks, *International Journal of Systems Science*, vol. 55, no. 10, pp. 2023–2035, 2024.

[21] A. Hasan, I. Kuncara, A. Widyotriatmo, O. Osen, and R. T. Bye, Secure state estimation and control for autonomous ships under cyber-attacks, *Systems Science & Control Engineering*, vol. 13, no. 1, 2025, art. no. 2518964.

[22] Y. Wang, Z. Wang, L. Zou, Y. Chen, and D. Yue, Distributed proportional-integral fuzzy state estimation over sensor networks under energy-constrained denial-of-service attacks, *IEEE Transactions on Cybernetics*, vol. 54, no. 1, pp. 641-654, 2024.

[23] L. Ma, H. Zhang, G. Wang, C. Yang, and L. Zhou, Security coordination control for the belt conveyor systems with false data injection attacks, *International Journal of Network Dynamics and Intelligence*, vol. 4, no. 1, Mar. 2025, art. no. 100001.

[24] H. Song, D. Ding, H. Dong, and X. Yi, Distributed filtering based on Cauchy-kernel-based maximum correntropy subject to randomly occurring cyber-attacks, *Automatica*, vol. 135, 2022, art. no. 110004.

[25] Y. Sun, Y. Ju, D. Ding, and H. Liu, Distributed H_∞ filtering of replay attacks over sensor networks, *ISA Transactions*, vol. 141, pp. 113-120, Oct. 2023.

[26] M. Xie, D. Ding, X. Ge, Q.-L. Han, H. Dong, and Y. Song, Distributed platooning control of automated vehicles subject to replay attacks based on proportional integral observers, *IEEE/CAA Journal of Automatica Sinica*, vol. 11, no. 9, pp. 1954-1966, 2024.

[27] D. Pan, D. Ding, X. Ge, Q.-L. Han, and X.-M. Zhang, Edge-based event-triggered platooning control of heterogeneous connected automated vehicles under replay attacks, *IEEE Transactions on Vehicular Technology*, vol. 73, no. 11, pp. 17443–17456, Nov. 2024.

[28] D. Zhao, B. Yang, Y. Li, and H. Zhang, Replay attack detection for cyber-physical control systems: a dynamical delay estimation method, *IEEE Transactions on Industrial Electronics*, in press, doi: 10.1109/TIE.2024.3406859.

[29] J. Huang, W. Yang, D. W. C. Ho, F. Li, and Y. Tang, Security analysis of distributed consensus filtering under replay attacks, *IEEE Transactions on Cybernetics*, vol. 54, no. 6, pp. 3526-3539, 2024.

[30] Q. Liu, Y. Long, T. Li, J. H. Park, and C. L. P. Chen, Fault detection for unmanned marine vehicles under replay attack, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 5, pp. 1716-1728, 2023.

[31] Y. Wang, Z. Wang, L. Zou, and H. Dong, On H_∞ fuzzy proportional-integral observer design under amplify-and-forward relays and multirate measurements, *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 4, pp. 1873-1885, 2024.

[32] D. Xu, E. Tian, and X. Xie, Constant-power and maximum efficiency tracking control of fuzzy IPT systems with communication delays, *IEEE Transactions on Automation Science and Engineering*, vol. 21, no. 4, pp. 7539–7549, Oct. 2024.

[33] S. Zhu, Y. Huang, E. Tian, Y. Luo, X. Xie, and H. Chen, Event-based H_∞ tracking control for the T-S fuzzy-based SP-IPT system with coil misalignment, *IEEE Transactions on Automation Science and Engineering*, vol. 21, no. 4, pp. 6056–6067, Oct. 2024.

[34] Y. Tan, Y. Yuan, X. Xie, E. Tian, and J. Liu, Observer-based event-triggered control for interval type-2 fuzzy networked system with network attacks, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 8, pp. 2788–2798, Aug. 2023.

[35] X. Wang, E. Tian, W. X. Zheng, and X. Xie, Important-data-based DoS attack mechanism and resilient H_∞ filter design for networked T-S fuzzy systems, *IEEE Transactions on Cybernetics*, vol. 54, no. 5, pp. 3352–3362, May. 2024.

[36] J. Liu, T. Yin, J. Cao, D. Yue, and H. R. Karimi, Security control for T-S fuzzy systems with adaptive event-triggered mechanism and multiple cyber-attacks, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 10, pp. 6544-6554, 2021.

[37] Y. Mo and B. Sinopoli, Secure control against replay attacks, *In Proceedings of the 47th annual allerton conference on communication, control, and computing*, pp. 911-918, 2009.
[38] B. Chen, D. W. C. Ho, G. Hu, and L. Yu, Secure fusion estimation for bandwidth constrained cyber-physical systems under replay attacks, *IEEE Transactions on Cybernetics*, vol. 48, no. 6, pp. 1862-1876, 2018.
[39] T. Li, Z. Wang, L. Zou, B. Chen, and L. Yu, A dynamic encryption-decryption scheme for replay attack detection in cyber-physical systems, *Automatica*, vol. 151, 2023, art. no. 110926.
[40] J. Huang, L. Zhao, and Q.-G. Wang, Adaptive control of a class of strict feedback nonlinear systems under replay attacks, *ISA Transactions*, vol. 107, pp. 134-142, 2020.
[41] M. Wakaiki, A. Cetinkaya, and H. Ishii, Stabilization of networked control systems under DoS attacks and output quantization, *IEEE Transactions on Automatic Control*, vol. 65, no. 8, pp. 3560-3575, 2020.
[42] S. Feng and P. Tesi, Resilient control under denial-of-service: robust design, *Automatica*, vol. 79, pp. 42-51, 2017.
[43] P. Zeng, F. Deng, H. Zhang, and X. Gao, Event-based H_∞ control for discrete-time fuzzy Markov jump systems subject to DoS attacks, *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 6, pp. 1853-1863, 2022.
[44] Y. Zhang, Z.-G. Wu, P. Shi, T. Huang, and P. Chakrabarti, Quantization-based event-triggered consensus of multiagent systems against aperiodic DoS attacks, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 53, no. 6, pp. 3774-3783, 2023.
[45] L. Su, D. Ye, and X. Zhao, Static output feedback secure control for cyber-physical systems based on multisensor scheme against replay attacks, *International Journal of Robust and Nonlinear Control*, vol. 30, no. 18, pp. 8313-8326, 2020.
[46] L. Chen, Y. Li, and S. Tong, Neural network adaptive consensus control for nonlinear multi-agent systems encountered sensor attacks, *International Journal of Systems Science*, vol. 54, no. 12, pp. 2536-2550, 2023.
[47] D. Yao, C. Dou, D. Yue, N. Zhao, and T. Zhang, Adaptive neural network consensus tracking control for uncertain multi-agent systems with predefined accuracy, *Nonlinear Dynamics*, vol. 101, pp. 2249-2262, 2020.
[48] R. Zhao, Z. Zuo, Y. Wang, and W. Zhang, Active control strategy for switched systems against asynchronous DoS attacks, *Automatica*, Vol. 148, 2023, art. no. 110765.
[49] C. Zheng, H. Fu, J. Bai, X. Meng, and Y. Chen, FlexRay protocol based distributed nonfragile dissipative filtering of state-saturated switched stochastic systems, *International Journal of Systems Science*, vol. 55, no. 4, pp. 714-727, 2023.
[50] Y.-W. Wang, H. O. Wang, J.-W. Xiao, and Z.-H. Guan, Synchronization of complex dynamical networks under recoverable attacks, *Automatica*, Vol. 46, no. 1, pp. 197-203, 2010.

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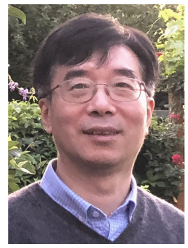

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