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Ultimately Bounded State Estimation for Discrete-Time Singularly Perturbed Complex Networks Under Bit Rate Constraints

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***Abstract*—In this paper, the ultimately bounded state estimation problem is investigated for discrete-time singularly perturbed complex networks (SPCNs) under bit rate constraints. The measurement outputs of the considered SPCNs are transmitted to the remote estimator through a bandwidth-limited wireless network where a coding-decoding mechanism is employed. A state estimator is developed by fully considering the effects of the constrained bit rate to achieve the desired estimation performance index. The boundedness of the estimation error dynamics is analyzed with the help of the constructed Lyapunov-like functional candidate. Finally, a simulation example is used to verify the correctness of the obtained theoretical results.**

***Index Terms*—Singularly perturbed systems, constrained bit rate, complex networks, discrete-time systems, ultimate boundedness.**

I. Introduction

Complex networks (CNs) have well be recognized to be an ideal abstraction of representing many practical scenarios (e.g. social networks, transportation networks, power grids and biological networks) and have therefore attracted widespread attention from research communities [1]–[3]. CNs exhibit a high degree of complexity since a large number of network nodes are coupled with each other. In real-world applications, it is difficult to accurately obtain the internal states of the network nodes due probably to measurement errors, sensor noises, and modeling errors. To obtain reliable internal state information of CNs, the state estimation strategy becomes a prerequisite in some dynamics analysis tasks such as control and optimization [4], [5].

Singularly perturbed complex networks (SPCNs), as a particular type of CNs exposing rich dynamics and structures, are characterized by two or more time scales that are reflected by the so-called singularly perturbed parameter (SPP) [6]. Different from the singularly perturbed system, the modeling and analysis for SPCNs is much more complicated because of the different weights of connections and interactions between the network nodes. Due to the wide applications in practical fields such as biology, power systems, transport networks and social networks, the investigation of SPCNs has drawn intense attention in recent years [7]–[11]. Nonetheless, there are still some open issues that require further studies, and an example is the dynamics analysis problem for SPCNs when implementing data transmission through digital communication networks with limited bandwidth.

Bandwidth limitation may lead to a series of issues during wireless transmission, such as signal fadings and packet dropouts [12], [13]. The bit rate, which indicates the data size that can be transmitted over digital communication networks at each unit of time, is a critical factor in depicting the communication bandwidth. In practice, the bit rate is inevitably restricted due to the limited bandwidth resources [14], [15]. In this case, it is of great significance to allocate the bit rate for each network node to avoid data collision.

Recently, the filtering problems under constrained bit have received initial research interest. For instance, in [16], the distributed filtering of wireless sensor networks with limited bit rate has been concerned, where a bit rate allocation protocol has been proposed. The cluster synchronization control of complex networks has been explored in [17] with data transmissions affected by the constrained bit rate. Note that most existing results only focus on the standard CNs models with limited bit rate. Regarding the SPCNs, the bit-rate-constrained filtering problem has not received adequate attention despite its practical significance.

Motivated by the above discussions, we study the ultimately bounded state estimation problem for SPCNs under the constrained bit rate in this paper. The main contributions of this paper are highlighted as follows: 1) the constrained bit rate is considered for the first time in the discrete-time SPCNs, and a coding-decoding model is constructed based on the fast and slow states; and 2) a state estimator is designed that can guarantee the ultimate boundedness of the estimation error dynamics.

This paper is divided into five parts. Section II provides the description of SPCNs, the coding-decoding process, and the estimator. Section III presents the main results concerning boundedness analysis and estimator design. Section IV provides a numerical simulation, followed by Section V, which

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draws the conclusion.

II. Problem Formulation

A. Singularly Perturbed Complex Networks

Consider a class of SPCNs of the following form:

$$\begin{cases} x_i(k+1) = A_\varepsilon x_i(k) + \tilde{J}_\varepsilon(x_i(k)) \\ \qquad\qquad + \sum_{j=1}^{N} \omega_{ij}\Gamma_\varepsilon x_j(k) + H_i\vartheta(k) \\ y_i(k) = C_i x_i(k) + M_i\vartheta(k) \end{cases} \tag{1}$$

with

$$\begin{aligned} x_i(k) &\triangleq \begin{bmatrix} x_{if}(k) \\ x_{is}(k) \end{bmatrix},\ y_i(k) \triangleq \begin{bmatrix} y_{if}(k) \\ y_{is}(k) \end{bmatrix}, \\ A_\varepsilon &\triangleq \begin{bmatrix} A_1 & A_2 \\ \varepsilon A_3 & \varepsilon A_4 \end{bmatrix},\ \Gamma_\varepsilon \triangleq \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ \varepsilon\Gamma_3 & \varepsilon\Gamma_4 \end{bmatrix}, \\ \tilde{J}_\varepsilon(x_i(k)) &\triangleq \begin{bmatrix} \tilde{g}(x_{if}(k)) + Dx_{is}(k) \\ \varepsilon\big(\tilde{h}(x_{is}(k)) + Fx_{is}(k)\big) \end{bmatrix}, \\ C_i &\triangleq \mathrm{diag}\{C_{if}, C_{is}\}, \\ H_i &\triangleq \begin{bmatrix} H_{i1} \\ H_{i2} \end{bmatrix},\ M_i \triangleq \begin{bmatrix} M_{i1} \\ M_{i2} \end{bmatrix} \end{aligned}$$

where $i \in \Psi \triangleq \{1, 2, \ldots, N\}$ denotes the node number, $x_i(k) \in \mathbb{R}^n$ represents the state of SPCNs that contains the fast state $x_{if}(k) \in \mathbb{R}^{n_f}$ and the slow state $x_{is}(k) \in \mathbb{R}^{n_s}$ with $n_f + n_s = n$, $y_i(k) \in \mathbb{R}^v$ denotes the measurement output consisting of $y_{if}(k) \in \mathbb{R}^{v_f}$ and $y_{is}(k) \in \mathbb{R}^{v_s}$ with $v_f + v_s = v$, $\vartheta(k) \in \mathbb{R}^\varsigma$ is the external disturbance satisfying $\|\vartheta(k)\| \le \vartheta_0$, and ε is a small positive constant controlling the separation of the fast and slow time scales.

In (1), A_ε is a parameter matrix with $A_1 \in \mathbb{R}^{n_f \times n_f}$, $A_2 \in \mathbb{R}^{n_f \times n_s}$, $A_3 \in \mathbb{R}^{n_s \times n_f}$ and $A_4 \in \mathbb{R}^{n_s \times n_s}$. The inner coupling matrix Γ_ε represents the relationship among components of a node, where $\Gamma_1 \in \mathbb{R}^{n_f \times n_f}$, $\Gamma_2 \in \mathbb{R}^{n_f \times n_s}$, $\Gamma_3 \in \mathbb{R}^{n_s \times n_f}$, $\Gamma_4 \in \mathbb{R}^{n_s \times n_s}$. The coupled configuration matrix $W = \{\omega_{ij}\} \in \mathbb{R}^{n \times n}$ satisfies $\sum_{j=1}^{N} \omega_{ij} = 0$ with $i \in \Psi$, which indicates that the node i can receive information form the node j if $\omega_{ij} > 0$, otherwise $\omega_{ij} = 0$. $C_{if} \in \mathbb{R}^{v_f \times n_f}$, $C_{is} \in \mathbb{R}^{v_s \times n_s}$, $D \in \mathbb{R}^{v_f \times n_s}$, $F \in \mathbb{R}^{v_s \times n_s}$, $H_{i1} \in \mathbb{R}^{n_f \times \varsigma}$, $H_{i2} \in \mathbb{R}^{n_s \times \varsigma}$, $M_{i1} \in \mathbb{R}^{n_f \times \varsigma}$ and $M_{i2} \in \mathbb{R}^{n_s \times \varsigma}$ are constant matrices.

The nonlinear functions $\tilde{g}(\cdot) \in \mathbb{R}^{n_f}$ and $\tilde{h}(\cdot) \in \mathbb{R}^{n_s}$ satisfy

$$\begin{aligned} &\big(\tilde{g}(a) - \tilde{g}(b) - \tilde{\psi}_1(a-b)\big)^T\big(\tilde{g}(a) - \tilde{g}(b) - \tilde{\psi}_2(a-b)\big) \le 0, \\ &\big(\tilde{h}(a) - \tilde{h}(b) - \tilde{\chi}_1(a-b)\big)^T\big(\tilde{h}(a) - \tilde{h}(b) - \tilde{\chi}_2(a-b)\big) \le 0 \end{aligned} \tag{2}$$

where $a, b \in \mathbb{R}^{n_f}$, $\tilde{\psi}_\imath \in \mathbb{R}^{n_f \times n_f}$ and $\tilde{\chi}_\imath \in \mathbb{R}^{n_s \times n_s}$ ($\imath \in \{1, 2\}$).

B. Coding-Decoding Process Under Constrained Bit Rate

In practice, the appropriate bit rate for each node can be allocated to effectively avoid data collisions during transmission in wireless networks due to the limited bandwidth of digital communication networks. The bit rates allocated to node i are denoted by B_i (a positive integer) and, in this case, the following condition holds [16]:

$$B_t \ge \sum_{i=1}^{N} B_i \tag{3}$$

where B_t are the total bit rates of the entire wireless network.

To facilitate the transmission of sensor data via a bandwidth-constrained channel, compression of data needs to be achieved, which can be done by a uniform quantizer. Given a scalar $\sigma_i > 0$, the quantization region of the ith sensor node is represented by

$$\digamma_{\sigma_i} \triangleq \big\{y_i(k) : |y_i^j(k)| \le \sigma_i, j \in \{1, 2, \ldots, v\}\big\} \tag{4}$$

where $y_i^j(k)$ represents the jth element of the measurement $y_i(k)$.

After choosing a quantization grade q_i for the sensor node i, the quantization region $\digamma_{\sigma_i}$ can be uniformly divided into some sub-hyperrectangles. Let $(d_i^1, \ldots, d_i^v)$ be the corresponding quantization region for the jth element of the sensor node i. Then, the ranges of sub-hyperrectangles are denoted by

$$\begin{cases} \mathcal{Q}_1^{ij}(\sigma_i) : -\sigma_i \le y_i^j(k) < -\sigma_i + \dfrac{2\sigma_i}{q_i} \\ \mathcal{Q}_2^{ij}(\sigma_i) : -\sigma_i + \dfrac{2\sigma_i}{q_i} \le y_i^j(k) < -\sigma_i + \dfrac{4\sigma_i}{q_i} \\ \qquad \vdots \\ \mathcal{Q}_{q_i}^{ij}(\sigma_i) : \sigma_i - \dfrac{2\sigma_i}{q_i} \le y_i^j(k) \le \sigma_i. \end{cases} \tag{5}$$

The maximum quantization level q_i^m of sensor node i is limited by allocated bits, i.e.,

$$q_i^m = \lfloor \sqrt[v]{2^{B_i}} \rfloor \tag{6}$$

where symbol $\lfloor\cdot\rfloor$ stands for rounding down.

When the sensor measurements are scattered in the quantization region defined in (5), the central value of sub-hyperrectangle is used to represent all the data falling in the region, which is denoted by

$$\hbar_{b_i}^i(d_i^1, \ldots, d_i^v) \triangleq [z_i^1, \ldots, z_i^v]^T \tag{7}$$

where $\hbar_{b_i}^i(\cdot)$ is a quantization function and $z_i^j \triangleq -\sigma_i + \frac{(2d_i^j - 1)\sigma_i}{q_i^m}$, $j \in \{1, 2, \ldots, v\}$. According to the above description, the quantization error of the measurement is derived as follows:

$$\|y_i(k) - \hbar_{\sigma_i}^i(d_i^1, \ldots, d_i^v)\|_2 \le \frac{\sqrt{v}\sigma_i}{q_i^m}. \tag{8}$$

According to the different time scales of SPCNs , the codeword generated by the coder is written as $\mathcal{Y}_i(k) \triangleq \big(\mathcal{Y}_{if}(k), \mathcal{Y}_{is}(k)\big)$ with $\mathcal{Y}_{if}(k) = (d_{if}^1, \ldots, d_{if}^{v_f})$ and $\mathcal{Y}_{is}(k) = (d_{is}^{v_f+1}, \ldots, d_{is}^{v})$.

The codeword is transmitted over the wireless digital network and is decoded in the following form:

$$\vec{y}_i(k) \triangleq \begin{bmatrix} \vec{y}_{if}(k) \\ \vec{y}_{is}(k) \end{bmatrix} = \begin{bmatrix} \hbar_{\sigma_i}^i(\mathcal{Y}_{if}(k)) \\ \hbar_{\sigma_i}^i(\mathcal{Y}_{is}(k)) \end{bmatrix} \tag{9}$$

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where the decoding error is defined as

$$\delta_i(k) \triangleq y_i(k) - \vec{y}_i(k). \tag{10}$$

C. State Estimator

Based on the output $\vec{y}_i(k)$ of the decoder, we construct the following state estimator for SPCNs (1):

$$\begin{aligned}\hat{x}_i(k+1) =& A_\varepsilon \hat{x}_i(k) + \tilde{J}_\varepsilon(\hat{x}_i(k)) + \sum_{j=1}^{N} \omega_{ij}\Gamma_\varepsilon \hat{x}_j(k) \\ &+ L_i\big(\vec{y}_i(k) - C_i\hat{x}_i(k)\big)\end{aligned} \tag{11}$$

where $\hat{x}_i(k) \triangleq \begin{bmatrix}\hat{x}_{if}^T(k) & \hat{x}_{is}^T(k)\end{bmatrix}^T$ represents the state estimate for node i, and $L_i \triangleq \mathrm{diag}\{L_{if}, L_{is}\}$ with $L_{if} \in \mathbb{R}^{n_f \times v_f}$ and $L_{is} \in \mathbb{R}^{n_s \times v_s}$ denoting the estimator gain to be designed.

Defining the state estimation error of the ith node as $e_i(k) \triangleq x_i(k) - \hat{x}_i(k) \triangleq \begin{bmatrix}e_{if}^T(k) & e_{is}^T(k)\end{bmatrix}^T$, we have

$$\begin{aligned}e_i(k+1) =& A_\varepsilon e_i(k) + J_\varepsilon(e_i(k)) + \sum_{j=1}^{N} \omega_{ij}\Gamma_\varepsilon e_j(k) \\ &- L_i\big(y_i(k) - \delta_i(k) - C_i\hat{x}_i(k)\big) + H_i\vartheta(k)\end{aligned} \tag{12}$$

where

$$\begin{aligned}e_{if}(k) &\triangleq x_{if}(k) - \hat{x}_{if}(k), \\ e_{is}(k) &\triangleq x_{is}(k) - \hat{x}_{is}(k), \\ J_\varepsilon(e_i(k)) &\triangleq \begin{bmatrix} g(e_{if}(k)) + De_{is}(k) \\ \varepsilon\big(h(e_{is}(k)) + Fe_{is}(k)\big)\end{bmatrix}, \\ g(e_{if}(k)) &\triangleq \tilde{g}(x_{if}(k)) - \tilde{g}(\hat{x}_{if}(k)), \\ h(e_{is}(k)) &\triangleq \tilde{h}(x_{is}(k)) - \tilde{h}(\hat{x}_{is}(k)).\end{aligned}$$

By denoting $e(k) \triangleq \begin{bmatrix}e_1^T(k) & e_2^T(k) & \dots & e_N^T(k)\end{bmatrix}^T$, the error dynamics is rewritten as the following compact form:

$$\begin{aligned}e(k+1) =& (I_N \otimes A_\varepsilon + I_N \otimes \mathcal{F}_\varepsilon)e(k) \\ &+ \varphi_\varepsilon(e(k)) + \mathcal{H}\nu(k) + (W \otimes \Gamma_\varepsilon)e(k) \\ &- \mathfrak{L}\mathcal{C}e(k) + \mathfrak{L}\phi(k) - \mathfrak{L}\mathcal{M}\nu(k)\end{aligned} \tag{13}$$

where

$$\begin{aligned}\mathcal{F}_\varepsilon &\triangleq \begin{bmatrix}0 & D \\ 0 & \varepsilon F\end{bmatrix}, \ \nu(k) \triangleq \mathrm{col_N}\{\vartheta(\mathrm{k})\}, \\ \varphi_\varepsilon(e(k)) &\triangleq \begin{bmatrix}\varphi_{\varepsilon,1}^T(e_1(k)) & \dots & \varphi_{\varepsilon,N}^T(e_N(k))\end{bmatrix}^T, \\ \varphi_{\varepsilon,i}(e_i(k)) &\triangleq \begin{bmatrix}g^T(e_{if}(k)) & \varepsilon h^T(e_{is}(k)\end{bmatrix}^T, \\ \mathfrak{L} &\triangleq \mathrm{diag}\{L_1, L_2, \dots, L_N\}, \\ \mathcal{C} &\triangleq \mathrm{diag}\{C_1, C_2, \dots, C_N\}, \\ \mathcal{H} &\triangleq \mathrm{diag}\{H_1, H_2, \dots, H_N\}, \\ \mathcal{M} &\triangleq \mathrm{diag}\{M_1, M_2, \dots, M_N\}, \\ \phi(k) &\triangleq \begin{bmatrix}\delta_1^T(k) & \delta_2^T(k) & \dots & \delta_N^T(k)\end{bmatrix}^T.\end{aligned}$$

Let a row-switching elementary matrix be $R \triangleq \prod_i^N R_i$ with $R_i \in \mathbb{R}^{nN \times nN}$ $(i \in \Psi)$. According to the properties of row-switching elementary transformation, one has $R_i = R_i^{-1}$. Defining $Re(k) \triangleq \tilde{e}(k)$, pre-multiplying (13) by the elementary matrix R gives

$$\begin{aligned}\tilde{e}(k+1) =& \Lambda_\varepsilon \tilde{e}(k) + \tilde{\varphi}_\varepsilon(e_f(k)) \\ &+ \tilde{\mathfrak{L}}\tilde{\phi}(k) + (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k)\end{aligned} \tag{14}$$

where

$$\begin{aligned}\Lambda_\varepsilon &\triangleq \begin{bmatrix}\Lambda^{11} & \Lambda^{12} \\ \Lambda_\varepsilon^{21} & \Lambda_\varepsilon^{22}\end{bmatrix}, \ \tilde{\mathfrak{L}} \triangleq \begin{bmatrix}\mathfrak{L}_f & 0 \\ 0 & \mathfrak{L}_s\end{bmatrix}, \\ \Lambda^{11} &\triangleq I_N \otimes A_1 + W \otimes \Gamma_1 - \mathfrak{L}_f\mathcal{C}_f, \\ \Lambda^{12} &\triangleq I_N \otimes A_2 + I_N \otimes D + W \otimes \Gamma_2, \\ \Lambda_\varepsilon^{21} &\triangleq \varepsilon(I_N \otimes A_3 + W \otimes \Gamma_3), \\ \Lambda_\varepsilon^{22} &\triangleq \varepsilon(I_N \otimes A_4 + I_N \otimes F + W \otimes \Gamma_4) - \mathfrak{L}_s\mathcal{C}_s, \\ \mathfrak{L}_f &\triangleq \mathrm{diag}\{L_{1f}, L_{2f}, \dots, L_{Nf}\}, \\ \mathfrak{L}_s &\triangleq \mathrm{diag}\{L_{1s}, L_{2s}, \dots, L_{Ns}\}, \\ \mathcal{C}_f &\triangleq \mathrm{diag}\{C_{1f}, C_{2f}, \dots, C_{Nf}\}, \\ \mathcal{C}_s &\triangleq \mathrm{diag}\{\mathrm{C_{1s}}, \mathrm{C_{2s}}, \dots, \mathrm{C_{Ns}}\}, \\ \tilde{\mathcal{H}} &\triangleq \begin{bmatrix}\mathcal{H}_1^T & \mathcal{H}_2^T\end{bmatrix}^T, \ \tilde{\mathcal{M}} \triangleq \begin{bmatrix}\mathcal{M}_1^T & \mathcal{M}_2^T\end{bmatrix}^T, \\ \mathcal{H}_\imath &\triangleq \mathrm{diag}\{H_{1\imath}, H_{2\imath}, \dots, H_{N\imath}\}, \\ \mathcal{M}_\imath &\triangleq \mathrm{diag}\{M_{1\imath}, M_{2\imath}, \dots, M_{N\imath}\}, \ \imath \in \{1, 2\}, \\ \tilde{\varphi}_\varepsilon(e(k)) &\triangleq \begin{bmatrix}\breve{g}^T(e_f(k)) & \varepsilon\breve{h}^T(e_s(k))\end{bmatrix}^T, \\ \breve{g}(e_f(k)) &\triangleq \begin{bmatrix}g^T(e_{1f}(k)) & \dots & g^T(e_{Nf}(k))\end{bmatrix}^T, \\ \breve{h}(e_s(k)) &\triangleq \begin{bmatrix}h^T(e_{1s}(k)) & \dots & h^T(e_{Ns}(k))\end{bmatrix}^T, \\ \tilde{\phi}(k) &\triangleq \begin{bmatrix}\tilde{\delta}_f^T(k) & \tilde{\delta}_s^T(k)\end{bmatrix}^T, \\ \tilde{\delta}_f(k) &\triangleq \begin{bmatrix}y_{1f}^T(k) - \vec{y}_{1f}^T(k) & \dots & y_{Nf}^T(k) - \vec{y}_{Nf}^T(k)\end{bmatrix}^T, \\ \tilde{\delta}_s(k) &\triangleq \begin{bmatrix}y_{1s}^T(k) - \vec{y}_{1s}^T(k) & \dots & y_{Ns}^T(k) - \vec{y}_{Ns}^T(k)\end{bmatrix}^T.\end{aligned}$$

Lemma 1: [10] Let $\varepsilon \in (0, \acute{\varepsilon}]$ be a positive scalar, and Ω_1 and Ω_2 be symmetric matrices with compatible dimensions. Then, the following inequality

$$\Omega_1 + \varepsilon\Omega_2 < 0 \tag{15}$$

holds if and only if $\Omega_1 \leq 0$ and $\Omega_1 + \acute{\varepsilon}\Omega_2 < 0$.

Definition 1: The estimation error dynamics (14) of SPCNs is said to be uniformly exponentially bounded if there exist constants $0 < \iota < 1$, $\gamma > 0$ and $\alpha > 0$ such that the following inequality holds:

$$\|\tilde{e}(k)\|^2 \leq \iota^k\gamma + \alpha \tag{16}$$

where α is an asymptotic upper bound of the error $\|\tilde{e}(k)\|^2$.

III. Main Results

In this section, a sufficient condition is obtained to guarantee the ultimate boundedness of the estimation error dynamics.

Theorem 1: Let the scalars $0 < \beta < 1$, $\varepsilon > 0$, the positive integers B_i $(i \in \Psi)$ and the estimator gains be given. If there exist positive scalars $\epsilon_1, \epsilon_2, \tau_1, \tau_2$ and positive definite matrix P_ε such that the following inequality holds:

$$\begin{bmatrix}\Pi_{11} & 0 & \Pi_{13} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & -P_\varepsilon^{-1}\end{bmatrix} < 0 \tag{17}$$

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where

$$\Pi_{11} \triangleq \begin{bmatrix} (\beta-1)P_\varepsilon - \mathfrak{J}_{11} & \mathfrak{J}_{12} & \mathfrak{J}_{13} \\ * & -\epsilon_1 I_{n_f N} & 0 \\ * & * & -\epsilon_2 I_{n_s N} \end{bmatrix},$$

$$\Pi_{22} \triangleq \begin{bmatrix} -\tau_1 I_{vN} & 0 \\ * & -\tau_2 I_{\varsigma N} \end{bmatrix},$$

$$\Pi_{13} \triangleq \begin{bmatrix} \Lambda_\varepsilon & Z_f & \varepsilon Z_s \end{bmatrix}^T, \ \Pi_{23} \triangleq \begin{bmatrix} \tilde{\mathfrak{L}} & \tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}} \end{bmatrix}^T,$$

$$\mathfrak{J}_{11} \triangleq \epsilon_1 Z_f \psi_1^T \psi_2 Z_f^T + \epsilon_2 Z_s \chi_1^T \chi_2 Z_s^T,$$

$$\mathfrak{J}_{12} \triangleq \epsilon_1 Z_f \frac{\psi_1^T + \psi_2^T}{2}, \ \mathfrak{J}_{13} \triangleq \epsilon_2 Z_s \frac{\chi_1^T + \chi_2^T}{2},$$

$$\psi_\imath \triangleq I_N \otimes \tilde{\psi}_\imath, \ \chi_\imath \triangleq I_N \otimes \tilde{\chi}_\imath, \ \imath \in \{1,2\},$$

$$Z_f \triangleq \begin{bmatrix} I_{n_f N} \\ 0_{n_s N \times n_f N} \end{bmatrix}, \ Z_s \triangleq \begin{bmatrix} 0_{n_f N \times n_s N} \\ I_{n_s N} \end{bmatrix},$$

then the estimation error dynamics (14) is ultimately bounded.

Proof: Choose the Lyapunov-like functional candidate as

$$V(k) = \tilde{e}^T(k) P_\varepsilon \tilde{e}(k). \tag{18}$$

Define the difference of $V(k)$ as $\Delta V(k) \triangleq V(k+1) - V(k)$. Combining (14) and (18), one has

$$\begin{aligned}\Delta V(k) =& \tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon \Lambda_\varepsilon \tilde{e}(k) + \tilde{\varphi}_\varepsilon^T(e(k)) P_\varepsilon \tilde{\varphi}_\varepsilon(e(k)) \\ &+ \tilde{\phi}^T(k)\tilde{\mathfrak{L}}^T P_\varepsilon \tilde{\mathfrak{L}}\tilde{\phi}(k) + 2\tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon \tilde{\varphi}_\varepsilon(e(k)) \\ &+ \nu^T(k)(\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k) \\ &+ 2\tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon \tilde{\mathfrak{L}}\tilde{\phi}(k) + 2\tilde{\varphi}_\varepsilon^T(e(k)) P_\varepsilon \tilde{\mathfrak{L}}\tilde{\phi}(k) \\ &- \tilde{e}^T(k) P_\varepsilon \tilde{e}(k) + 2\tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k) \\ &+ 2\tilde{\varphi}_\varepsilon^T(e(k)) P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k) \\ &+ 2\tilde{\phi}^T(k)\tilde{\mathfrak{L}}^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k). \end{aligned} \tag{19}$$

Letting $\tilde{\varphi}_\varepsilon(e(k)) \triangleq Z_f \breve{g}(e_f(k)) + \varepsilon Z_s \breve{h}(e_s(k))$, (19) can be rewritten as

$$\begin{aligned}\Delta V(k) =& \tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon \Lambda_\varepsilon \tilde{e}(k) + \breve{g}^T(e_f(k)) Z_f^T P_\varepsilon Z_f \breve{g}(e_f(k)) \\ &+ \varepsilon^2 \breve{h}^T(e_s(k)) Z_s^T P_\varepsilon Z_s \breve{h}(e_s(k)) + 2\varepsilon \breve{g}^T(e_f(k)) \\ &\times Z_f^T P_\varepsilon Z_s \breve{h}(e_s(k)) + 2\tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon Z_f \breve{g}(e_f(k)) \\ &+ 2\varepsilon \tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon Z_s \breve{h}(e_s(k)) + \tilde{\phi}_1^T(k)\tilde{\mathfrak{L}}^T P_\varepsilon \tilde{\mathfrak{L}} \\ &\times \tilde{\phi}_1(k) + \tilde{\phi}_2^T(k)\tilde{\mathfrak{L}}^T P_\varepsilon \tilde{\mathfrak{L}}\tilde{\phi}_2(k) + 2\tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon \\ &\times \tilde{\mathfrak{L}}\tilde{\phi}_1(k) + 2\varepsilon \breve{h}^T(e_s(k)) Z_s^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k) \\ &+ 2\breve{g}^T(e_f(k)) Z_f^T P_\varepsilon \tilde{\mathfrak{L}}\tilde{\phi}_1(k) + 2\varepsilon \breve{h}^T(e_s(k)) Z_s^T \\ &\times P_\varepsilon \tilde{\mathfrak{L}}\tilde{\phi}_1(k) + \nu^T(k)(\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}}) \\ &\times \nu(k) + 2\tilde{e}^T(k)\Lambda_\varepsilon^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k) \\ &+ 2\tilde{\phi}_1^T(k)\tilde{\mathfrak{L}}^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k) - \tilde{e}^T(k) P_\varepsilon \tilde{e}(k) \\ &+ 2\breve{g}^T(e_f(k)) Z_f^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})\nu(k). \end{aligned} \tag{20}$$

It follows from the nonlinearity conditions in (2) that

$$\begin{aligned} &\epsilon_1 \big(\breve{g}(e_f(k)) - \psi_1 e_f(k)\big)^T \big(\breve{g}(e_f(k)) - \psi_2 e_f(k)\big) \le 0, \\ &\epsilon_2 \big(\breve{h}(e_s(k)) - \chi_1 e_s(k)\big)^T \big(\breve{h}(e_s(k)) - \chi_2 e_s(k)\big) \le 0. \end{aligned} \tag{21}$$

Substituting $e_f(k) = Z_f^T \tilde{e}(k)$ and $e_s(k) = Z_s^T \tilde{e}(k)$ into (21), we have

$$\begin{bmatrix} \tilde{e}(k) \\ \breve{g}(e_f(k)) \\ \breve{h}(e_s(k)) \end{bmatrix}^T \begin{bmatrix} \mathfrak{J}_{11} & -\mathfrak{J}_{12} & -\mathfrak{J}_{13} \\ * & \epsilon_1 I_{n_f N} & 0 \\ * & * & \epsilon_2 I_{n_s N} \end{bmatrix} \begin{bmatrix} \tilde{e}(k) \\ \breve{g}(e_f(k)) \\ \breve{h}(e_s(k)) \end{bmatrix} \le 0. \tag{22}$$

Define $\xi(k) \triangleq \begin{bmatrix} \tilde{e}^T(k) \ \breve{g}^T(e_f(k)) \ \breve{h}^T(e_s(k)) \ \tilde{\phi}^T(k) \ \nu^T(k) \end{bmatrix}^T$. Combining the difference function (20) and nonlinear constraint (22), we derive that

$$\begin{aligned}\Delta V(k) \le& \xi^T(k)\tilde{\Pi}\xi(k) + \tau_1 \tilde{\phi}^T(k)\tilde{\phi}(k) \\ &+ \tau_2 \nu^T(k)\nu(k) - \beta V(k) \end{aligned} \tag{23}$$

where

$$\tilde{\Pi} \triangleq \begin{bmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13} & \Lambda_\varepsilon^T P_\varepsilon \tilde{\mathfrak{L}} & \tilde{\Pi}_{15} \\ * & \tilde{\Pi}_{22} & \varepsilon Z_f^T P_\varepsilon Z_s & Z_f^T P_\varepsilon \tilde{\mathfrak{L}} & \tilde{\Pi}_{25} \\ * & * & \tilde{\Pi}_{33} & \varepsilon Z_s^T P_\varepsilon \tilde{\mathfrak{L}} & \tilde{\Pi}_{35} \\ * & * & * & \tilde{\Pi}_{44} & \tilde{\Pi}_{45} \\ * & * & * & * & \tilde{\Pi}_{55} \end{bmatrix},$$

$$\tilde{\Pi}_{11} \triangleq \Lambda_\varepsilon^T P_\varepsilon \Lambda_\varepsilon - (1-\beta)P_\varepsilon - \mathfrak{J}_{11}, \ \tilde{\Pi}_{12} \triangleq \Lambda_\varepsilon^T P_\varepsilon Z_f + \mathfrak{J}_{12},$$

$$\tilde{\Pi}_{13} \triangleq \varepsilon \Lambda_\varepsilon^T P_\varepsilon Z_s + \mathfrak{J}_{13}, \ \tilde{\Pi}_{15} \triangleq \Lambda_\varepsilon^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}}),$$

$$\tilde{\Pi}_{22} \triangleq Z_f^T P_\varepsilon Z_f - \epsilon_1 I_{n_f N}, \ \tilde{\Pi}_{25} \triangleq Z_f^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}}),$$

$$\tilde{\Pi}_{33} \triangleq \varepsilon^2 Z_s^T P_\varepsilon Z_s - \epsilon_2 I_{n_s N}, \ \tilde{\Pi}_{35} \triangleq \varepsilon Z_s^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}}),$$

$$\tilde{\Pi}_{44} \triangleq \tilde{\mathfrak{L}}^T P_\varepsilon \tilde{\mathfrak{L}} - \tau_1 I_{vN}, \ \tilde{\Pi}_{45} \triangleq \tilde{\mathfrak{L}}^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}}),$$

$$\tilde{\Pi}_{55} \triangleq (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}})^T P_\varepsilon (\tilde{\mathcal{H}} - \tilde{\mathfrak{L}}\tilde{\mathcal{M}}) - \tau_2 I_{\varsigma N}.$$

Applying Schur Complement Lemma to (17) in Theorem 1, we have $\tilde{\Pi} < 0$, which implies

$$\Delta V(k) < -\beta V(k) + \tau_1 \tilde{\phi}^T(k)\tilde{\phi}(k) + \tau_2 \nu^T(k)\nu(k). \tag{24}$$

From the quantization error (8), it follows that

$$\|\tilde{\phi}(k)\| \le \sqrt{\sum_{i=1}^{N} v\sigma_i^2/(q_i^m)^2}. \tag{25}$$

For the sake of simplicity, we denote $\tilde{\pi} \triangleq \tau_1 \|\tilde{\phi}(k)\|^2 + \tau_2 \|\nu(k)\|^2$. Then, the inequality (24) is rewritten as

$$\begin{aligned} V(k) &< (1-\beta)V(k-1) + \tilde{\pi} \\ &< (1-\beta)^2 V(k-2) + (1 + (1-\beta))\tilde{\pi} \\ &< \cdots \\ &< (1-\beta)^k V(0) + \tilde{\pi} \sum_{\jmath=0}^{k-1} (1-\beta)^\jmath, \end{aligned} \tag{26}$$

which further indicates

$$\|\tilde{e}(k)\|^2 < \frac{(1-\beta)^k V(0)}{\lambda_{\min}(P_\varepsilon)} + \frac{\tilde{\pi} \sum_{\jmath=0}^{k-1} (1-\beta)^\jmath}{\lambda_{\min}(P_\varepsilon)}. \tag{27}$$

Recalling Definition 1, the error $\|\tilde{e}(k)\|^2$ is uniformly exponentially bounded. According to the Taylor expansion formula, we have that

$$\begin{aligned} \|\tilde{e}(k)\|^2 &< \frac{\tilde{\pi}}{\beta \lambda_{\min}(P_\varepsilon)} \\ &= \frac{\tau_1 \sum_{i=1}^{N} v\sigma_i^2/(q_i^m)^2 + \tau_2 N \vartheta_0^2}{\beta \lambda_{\min}(P_\varepsilon)} \triangleq \tilde{o}, \end{aligned} \tag{28}$$

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which ends the proof. ■

Based on Theorem 1 and Lemma 1, the following theorem is presented to obtain the desired estimator gains.

Theorem 2: Let scalars $0 < \beta < 1$, $\acute{\varepsilon} > 0$ and positive integers B_i $(i \in \Psi)$ be given. The error dynamics is ultimately bounded if there exist positive scalars $\epsilon_1, \epsilon_2, \tau_1, \tau_2$, matrices $\hat{P}$, $\breve{P}$, $\mathcal{K}$, and non-singular matrix Q such that the following inequalities hold:

$$\begin{bmatrix} \breve{\Pi}_{11} & 0 & \breve{\Pi}_{13} \\ * & \Pi_{22} & \breve{\Pi}_{23} \\ * & * & \mathfrak{Q} \end{bmatrix} \leq 0, \tag{29}$$

$$\begin{bmatrix} \hat{\Pi}_{11} & 0 & \hat{\Pi}_{13} \\ * & \Pi_{22} & \hat{\Pi}_{23} \\ * & * & \hat{\mathfrak{Q}} \end{bmatrix} < 0, \tag{30}$$

$$\breve{P} \geq 0, \quad \breve{P} + \acute{\varepsilon}\hat{P} > 0 \tag{31}$$

where

$$\breve{\Pi}_{11} \triangleq \begin{bmatrix} (\beta-1)\breve{P} - \mathfrak{J}_{11} & \mathfrak{J}_{12} & \mathfrak{J}_{13} \\ * & -\epsilon_1 I_{n_f N} & 0 \\ * & * & -\epsilon_2 I_{n_s N} \end{bmatrix},$$

$$\hat{\Pi}_{11} \triangleq \begin{bmatrix} (\beta-1)(\breve{P} + \acute{\varepsilon}\hat{P}) - \mathfrak{J}_{11} & \mathfrak{J}_{12} & \mathfrak{J}_{13} \\ * & -\epsilon_1 I_{n_f N} & 0 \\ * & * & -\epsilon_2 I_{n_s N} \end{bmatrix},$$

$$\breve{\Pi}_{13} \triangleq \begin{bmatrix} \Upsilon_0^T & Q^T Z_f & 0 \end{bmatrix}^T, \ \breve{\Pi}_{23} \triangleq \begin{bmatrix} \mathcal{K}^T & \tilde{\mathcal{H}} - \mathcal{K}^T \tilde{\mathcal{M}} \end{bmatrix}^T,$$

$$\hat{\Pi}_{13} \triangleq \begin{bmatrix} \Upsilon_0^T + \acute{\varepsilon}\Upsilon_1^T & Q^T Z_f & \acute{\varepsilon} Q^T Z_s \end{bmatrix}^T,$$

$$\mathfrak{Q} \triangleq \breve{P} - Q - Q^T, \ \hat{\mathfrak{Q}} \triangleq (\breve{P} + \acute{\varepsilon}\hat{P}) - Q - Q^T,$$

$$\Upsilon_0 \triangleq \begin{bmatrix} \Omega_{11} & 0 \\ (\Lambda^{12})^T & -\mathcal{C}_s^T \mathcal{K}_s \end{bmatrix},$$

$$\Upsilon_1 \triangleq \begin{bmatrix} 0 & (I_N \otimes A_3 + W \otimes \Gamma_3)^T \\ 0 & \Omega_{22} \end{bmatrix},$$

$$\Omega_{11} \triangleq (I_N \otimes A_1 + W \otimes \Gamma_1)^T Q_f - \mathcal{C}_f^T \mathcal{K}_f,$$

$$\Omega_{22} \triangleq (I_N \otimes A_4 + I_N \otimes F + W \otimes \Gamma_4)^T Q_s,$$

$$Q \triangleq \mathrm{diag}\{Q_f, Q_s\}, \ \mathcal{K} \triangleq \mathrm{diag}\{\mathcal{K}_f, \mathcal{K}_s\},$$

$$Q_f \triangleq \mathrm{diag}\{Q_{1f}, Q_{2f}, \ldots, Q_{Nf}\},$$

$$Q_s \triangleq \mathrm{diag}\{Q_{1s}, Q_{2s}, \ldots, Q_{Ns}\},$$

$$\mathcal{K}_f \triangleq \mathrm{diag}\{\mathcal{K}_{1f}\mathcal{K}_{2f}, \ldots, \mathcal{K}_{Nf}\},$$

$$\mathcal{K}_s \triangleq \mathrm{diag}\{\mathcal{K}_{1s}, \mathcal{K}_{2s}, \ldots, \mathcal{K}_{Ns}\}.$$

The gains of the estimator are calculated by $L_{if} = Q_{if}^{-T}\mathcal{K}_{if}^T$, and $L_{is} = Q_{is}^{-T}\mathcal{K}_{is}^T$.

Proof: From (29), (30) and Lemma 1, one has

$$\begin{bmatrix} \tilde{\Xi}_{11} & 0 & \tilde{\Xi}_{13} \\ * & \Pi_{22} & \breve{\Pi}_{23} \\ * & * & \mathfrak{Q} + \varepsilon\hat{P} \end{bmatrix} < 0 \tag{32}$$

where

$$\tilde{\Xi}_{11} \triangleq \begin{bmatrix} (\beta-1)(\breve{P} + \varepsilon\hat{P}) - \mathfrak{J}_{11} & \mathfrak{J}_{12} & \mathfrak{J}_{13} \\ * & -\epsilon_1 I_{n_f N} & 0 \\ * & * & -\epsilon_2 I_{n_s N} \end{bmatrix},$$

$$\tilde{\Xi}_{13} \triangleq \begin{bmatrix} \Upsilon_0^T + \varepsilon\Upsilon_1^T & Q^T Z_f & \varepsilon Q^T Z_s \end{bmatrix}^T.$$

Define $P_\varepsilon \triangleq \breve{P} + \varepsilon\hat{P}$, $\varepsilon \in (0, \acute{\varepsilon}]$. It follows from (31) and Lemma 1 that

$$\breve{P} + \varepsilon\hat{P} > 0. \tag{33}$$

Note that

$$(P_\varepsilon - Q)P_\varepsilon^{-1}(P_\varepsilon - Q)^T \geq 0 \tag{34}$$

implies

$$P_\varepsilon - Q - Q^T \geq -QP_\varepsilon^{-1}Q^T. \tag{35}$$

Define $\mathcal{K} \triangleq \tilde{\mathfrak{L}}^T Q$ and $\mathbb{I} \triangleq \{I_{nN}, I_{n_f N}, I_{n_s N}, I_{vN}\}$. By applying (35) into (32), then pre-multiplying it with $\mathbb{Q} \triangleq \mathrm{diag}\{\mathbb{I}, I_{vN}, Q^{-1}, Q^{-1}\}$ and post-multiplying it with $\mathbb{Q}^T$, we can conclude that the inequality (32) holds under the condition (17). The proof is now complete. ■

IV. Numerical Simulation

Consider an SPCN with the following parameters:

$$A_1 = \begin{bmatrix} 0.6 & 0.2 \\ 0.3 & 0.54 \end{bmatrix}, \ \Gamma_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.31 & 0.15 \end{bmatrix}^T, \ A_3 = \begin{bmatrix} 0.31 & 0.13 \end{bmatrix}, \ A_4 = 0.5,$$

$$\Gamma_2 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}^T, \ \Gamma_3 = \begin{bmatrix} 0.35 & 0.16 \end{bmatrix}, \Gamma_4 = 0.4,$$

$$C_1 = \begin{bmatrix} 0.8 & 0.1 & 0 \\ 0.2 & 0.9 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.9 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}, \ W = \begin{bmatrix} -0.3 & 0.1 & 0.2 \\ 0.1 & -0.3 & 0.2 \\ 0.1 & 0.2 & -0.3 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.15 & 0.16 & 0.2 \end{bmatrix}^T, \ H_2 = \begin{bmatrix} 0.25 & 0.36 & 0.3 \end{bmatrix}^T,$$

$$H_3 = \begin{bmatrix} 0.3 & 0.1 & 0.1 \end{bmatrix}^T, \ M_1 = \begin{bmatrix} 0.2 & 0.2 & 0.6 \end{bmatrix}^T,$$

$$M_2 = \begin{bmatrix} 0.5 & 0.2 & 0.9 \end{bmatrix}^T, \ M_3 = \begin{bmatrix} 0.4 & 0.1 & 0.3 \end{bmatrix}^T,$$

$$D = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}^T, \ F = 2.$$

Let the external disturbance be $\vartheta(k) = 1.1\cos(k)$, which implies $\vartheta_0 = 1.1$. The nonlinear functions are chosen as

$$\begin{cases} \tilde{g}(x_{if}(k)) = 0.24 \begin{bmatrix} \tanh(0.2x_{if}^1(k)) \\ \tanh(0.2x_{if}^2(k)) \end{bmatrix} \\ \tilde{h}(x_{is}(k)) = 0.6\big(|x_{is}(k)+1| - |x_{is}(k)-1|\big) \end{cases} \tag{36}$$

where $x_{if}^1(k)$ and $x_{if}^2(k)$ represent the first and the second component of the state $x_{if}(k)$, respectively.

Let the attenuation coefficient be $\beta = 0.86$ and the SPP be $\varepsilon \in (0, 0.1]$. Assume that the total bits available to the SPCNs are $B_t = 64$, and each node is allocated with $B_1 = B_2 = B_3 = 21$ bits. The upper bounds of the uniform quantizer are given as $\sigma_1 = 8$, $\sigma_2 = 7$ and $\sigma_3 = 6$.

The initial values of the state are set to be

$$x_1(k) = \begin{bmatrix} 0.4 & 0.2 & 0.1 \end{bmatrix}^T, \ x_2(k) = \begin{bmatrix} 0.2 & 0.2 & 0.3 \end{bmatrix}^T,$$

$$x_3(k) = \begin{bmatrix} 0.4 & 0.1 & 0.2 \end{bmatrix}^T, \ \hat{x}_i(k) = \mathrm{col}_3\{0\}, i \in \Psi.$$

In this example, we choose $\varepsilon = 0.09$. By using the estimator gains obtained from Theorem 2, the system states and their estimates are depicted in Fig. 1, where $x_{ij}(k)$ denotes the

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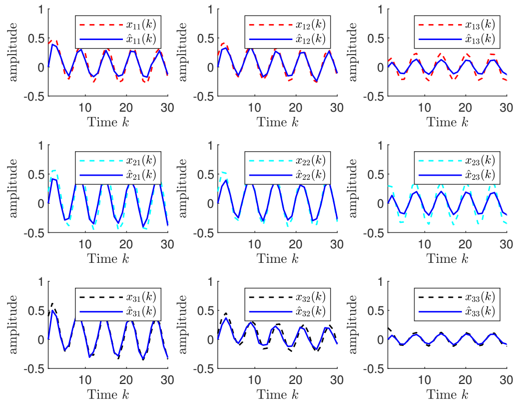

Fig. 1. State trajectory and its estimation.

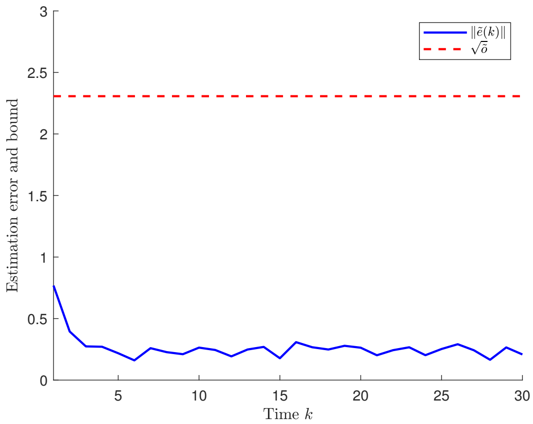

Fig. 2. Error dynamics and the bound.

jth component of the node i. The error norm $\|\tilde{e}(k)\|$ and the estimation error bound $\sqrt{\bar{o}} = 2.3073$ are plotted in Fig. 2, which verifies that the estimation error is indeed ultimately bounded.

V. Conclusion

In this paper, we have studied the ultimately bounded state estimation problem for PSCNs with bit rate constraints. A state estimator has been designed based on the structure of PSCNs and the feature of the encoding-decoding mechanism. A sufficient condition has been obtained to ensure the uniform boundedness of the estimation error dynamics. The desired estimator gains have been obtained using the linear matrix inequality technique. Finally, a simulation example has been provided to verify the estimation performance of the proposed estimator.

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