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Distributed Fuzzy Proportional-Integral State Estimation Over Sensor Networks With Pull-Type Gossip Protocols and Fading Data

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***Abstract*—This paper addresses the problem of distributed state estimation for smooth nonlinear systems over sensor networks by means of a generalized fuzzy proportional-integral observer (PIO). A sensor network is employed to collect system measurements, with a pull-type gossip protocol governing the intermittent data exchange among neighboring nodes. Under the gossip protocol, each sensor node randomly selects one neighbor to request data, facilitating distributed information updating. Furthermore, considering challenges such as long-distance communication and complex environmental conditions, signal transmission is subject to amplitude fading. To accommodate the characteristics of the gossip protocol, a generalized fuzzy PIO with a flexible structure is developed. Sufficient conditions are derived to guarantee the H_∞ estimation performance of the proposed observer. Based on established conditions, the parameters of both the gossip protocol and the fuzzy PIO are co-designed via a particle-swarm-optimization-based iterative algorithm, with emphasis on enhancing observer robustness. Finally, an engineering-oriented simulation example is presented to illustrate the effectiveness of the proposed methodology.**

***Index Terms*—Fuzzy proportional-integral observers (PIOs), gossip protocols, sensor networks, Takagi-Sugeno (T-S) fuzzy models, channel fading.**

I. Introduction

Sensor networks (SNs) have long been a hot research topic due to their wide applications in environmental monitoring, the Industrial Internet of Things, smart homes, and beyond. Typically composed of multiple low-cost sensors, SNs form interconnected nodes that communicate based on predefined topological structures, enabling the accomplishment of complex tasks [1]. A distinctive advantage of SNs lies in their ability to collect information about complex targets in a more flexible and efficient manner compared to traditional measurement approaches. Consequently, SN-based control, monitoring, and fault diagnosis have attracted significant research interest over the past decades, resulting in a substantial body of literature dedicated to various popular methodologies [2]–[6].

Acquiring system state information is beneficial for subsequent control and decision-making tasks. In recent years, the problem of state estimation based on SNs has attracted increasing attention. Benefiting from the distributed nature of sensor nodes, each node can be equipped with a local estimator, enabling distributed estimation by leveraging the network topology. To date, a number of effective distributed estimation methods have been developed, including Kalman-like filtering, set-membership filtering, fuzzy filtering, among others [7]–[13]. For example, a novel distributed network size estimation method with a specific stopping criterion has been proposed in [14]. In [15], an average-consensus protocol that achieves fast convergence has been designed using multi-hop average-consensus among multi-hop nodes. Experimental results have demonstrated that the proposed approach offers low complexity, high accuracy, and fast convergence. In [16], the distributed state estimation problem has been addressed using a novel finite-time approach, which has effectively handled the challenges induced by protocols and fading.

Among various estimation algorithms, the state estimation approach based on the Takagi-Sugeno (T-S) fuzzy model has garnered significant attention due to its remarkable performance in handling uncertainty and nonlinearity. The T-S fuzzy model approximates the dynamic behavior of complex systems using a set of linear subsystems coupled with nonlinear membership functions, effectively integrating the analytical tools of linear system theory with the powerful descriptive capability of fuzzy logic for nonlinear systems. The "local linearization, global nonlinearity" modeling framework greatly facilitates the design of estimators [17]–[25]. On the other hand, inspired by the classical proportional-integral-derivative control philosophy, researchers have begun to explore the integration of proportional-integral observer (PIO) techniques with fuzzy systems. Compared to conventional methods, the PIO-based estimation strategy offers greater design flexibility. By incorporating historical estimation error information, the robustness of the estimation algorithm is enhanced [26]–[29].

The management of inter-node communication poses a fundamental challenge in SNs. Specifically, practical SNs are large-scale yet rely on bandwidth-constrained long-distance links. Unstructured information exchange in such long-distance settings causes adverse effects like channel conges-

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tion, which severely degrades system performance [30]. In light of the inherent constraints, communication protocols are indispensable for regulating data flow. The gossip protocol has emerged as a notably effective solution, combining ease of implementation with a proven ability to facilitate the global dissemination of local information, thereby accounting for its extensive use in practical scenarios [31].

The gossip protocol mimics the spread of information in social networks by randomly selecting individuals within a group for information exchange, thereby enabling rapid dissemination of local information [32]. Under such a mechanism, at each sampling instant, a sensor node randomly selects one of its neighbors to either request or transmit information, unlike traditional methods where all neighbors are contacted simultaneously. The gossip-based approach significantly conserves communication resources and helps alleviate network congestion. Furthermore, by appropriately designing protocol parameters, nodes with different types of information can be assigned varying levels of priority, thereby optimizing the information transmission process and improving information utilization efficiency [23], [24], [33]–[38]. Common variants of the gossip protocol include push-pull, push-only, and pull-only types. Among these, the pull-based gossip protocol has attracted particular attention from researchers due to its characteristic of actively requesting information.

In SNs, another issue that warrants attention is channel fadings, which arises from multiple factors, including path loss, shadowing, and multipath effects. When channel fading occurs, the amplitude or phase of the transmitted signal deteriorates to some extent, leading to degraded signal quality [39], [40]. If not properly addressed, channel fadings can compromise the overall performance of the SN. To date, a substantial number of works on channel fadings have been reported. For example, the fading-affected control problem has been addressed in [41] using a stochastic framework. The effects of fading data have been dealt with for a kind of SNs in [42], where a distributed method has been employed.

A review of the literature on gossip-based distributed filtering reveals that, the majority of existing results focus on linear systems with Gaussian noises [21]–[24], [43], [44], primarily addressing the design of distributed Kalman filters. These methods, unfortunately, are not directly applicable to general nonlinear systems with non-Gaussian noise distributions. Furthermore, in the reported studies, the gossip mechanism is often employed as a generic consensus tool, with insufficient attention paid to the optimization of its parameters such as selection probabilities. To the best of the authors' knowledge, the problem of designing distributed fuzzy PIO schemes for general nonlinear systems under fading data constraints and a pull-type gossip protocol remains unaddressed, despite its relevance in practical engineering applications. This gap in the literature motivates the present study.

Based on the preceding discussions, this paper investigates the H_∞ state estimation problem via a distributed fuzzy PIO for systems operating over gossip-protocol-assisted SNs. The principal challenges addressed in this work are threefold: 1) How to characterize the behavior of the pull-type gossip protocol within a distributed fuzzy estimation framework? 2) How to handle the coupling terms introduced by multiple stochastic factors, including random request mechanisms of the gossip protocol and fading measurements? and 3) How to co-design the observer gains and the protocol parameters with respect to the H_∞ performance index?

In response to these challenges, the main contributions of this paper are summarized as follows: 1) A comprehensive model is developed to describe the dynamics of the pull-type gossip protocol in fading channels, where key protocol parameters are captured by a set of designable probability distributions; 2) A novel generalized fuzzy PIO is constructed that incorporates the structural features of the gossip protocol. Unlike existing PIO designs, the proposed observer utilizes limited information from neighboring nodes, thereby reducing communication resource consumption; and 3) By employing a specific matrix inequality transformation technique, the H_∞ performance of the estimation error system is rigorously analyzed. Furthermore, a particle swarm optimization (PSO)-based algorithm is developed to jointly design the observer gains and protocol parameters by minimizing the disturbance attenuation level.

The remainder of this paper is organized as follows. Section II presents the system model, along with descriptions of system complexities including the gossip protocol, fading data channels, and the proposed fuzzy PIO, followed by a statement of the design objectives. Section III provides the main results concerning performance analysis and parameter design based on the H_∞ framework. One engineering-motivated simulation example is provided in Section IV to verify the effectiveness of the proposed methodology. Finally, concluding remarks are summarized in Section V.

II. Problem Statement and Preliminaries

A. T-S Fuzzy-Model-Based Nonlinear Systems

Consider a type of smooth nonlinear systems that can be represented by the following discrete-time T-S fuzzy models:

$$\begin{cases} x(k+1) = \sum_{\tau=1}^{r} s_\tau(\alpha(k))\big(A_\tau x(k) + E_\tau \omega(k)\big) \\ p(k) = \sum_{\tau=1}^{r} s_\tau(\alpha(k)) G_\tau x(k) \end{cases} \tag{1}$$

where $x(k)$ with dimension n_x is the internal state vector; $\omega(k)$ with dimension n_ω is the energy-bounded noise; $p(k)$ with dimension n_p is the signal to be estimated; A_τ, E_τ, G_τ are known matrices; r denotes the number of fuzzy rules; $\alpha(k)$ is the premise variable of the fuzzy model; $s_\tau(\alpha(k))$ ($\tau \in \{1, 2, \cdots, r\} \triangleq \mathbb{H}$) are membership functions satisfying the following properties [45]:

$$\sum_{\tau=1}^{r} s_\tau(\alpha(k)) = 1, \quad s_\tau(\alpha(k)) \geq 0, \quad \tau = 1, 2, \cdots, r.$$

B. SN-Based Measurements

An SN with N nodes is utilized to monitor system (1). A directed graph $\mathbb{G} \triangleq \{\mathbb{V}, \mathbb{E}, \mathbb{L}\}$ is used to represent the topology of the SN, where $\mathbb{V} \triangleq \{1, 2, \ldots, N\}$ is the set of nodes; $\mathbb{E} \subset \mathbb{V} \times \mathbb{V}$ is the set of edges; $\mathbb{L} \triangleq [l_{i,j}]_{N\times N}$ is adjacency matrix with $l_{i,j} \in \{0,1\}$. If $l_{i,j} = 1$ ($\forall i, j \in \mathbb{V}$), it means that $(i,j) \in \mathbb{E}$ and node i can receive the information from node j. If $l_{i,j} = 0$, it indicates that $(i,j) \notin \mathbb{E}$ and there is no communication between node i and node j. For node i, the set of its neighboring nodes is denoted by $\mathbb{N}_i \triangleq \{j \in \mathbb{V}, (i,j) \in \mathbb{E}\}$.

The measurement output of sensor node i is given by

$$y_i(k) = C_i x(k) + D_i v_i(k) \tag{2}$$

where C_i, D_i are known matrices; $v_i(k)$ with dimension n_{v_i} is the energy-bounded noise; and the dimension of $y_i(k)$ is n_y.

C. Pull-Type Gossip Protocols

In SNs, local estimators can share their estimates with neighboring nodes according to the network topology, thereby achieving information consensus and improving estimation accuracy. Due to the large-scale nature of SNs and the constraints of shared communication resources, unrestricted information exchange under limited resources may significantly increase the communication burden. To address this issue, communication protocols are commonly employed to schedule data transmissions efficiently. Among the various gossip protocols (such as pull-type, push-type, and push-pull-type), the pull-type scheme is recognized for its straightforward implementation, as it operates by actively requesting data from neighboring nodes. In this paper, the pull-type gossip protocol is adopted to facilitate data transmissions among sensor nodes.

Under the pull-type gossip protocol, each node randomly selects a neighbor from its neighboring set for data request according to a predefined probability distribution. The assignment of these probabilities critically influences how frequently a node communicates with neighbors that hold significant information. Hence, their design requires careful consideration. In general, maintaining persistent communication with nodes that exhibit high estimation performance can enhance the overall estimation accuracy of the network. By employing the pull-type gossip protocol, not only is the consumption of communication resources reduced, but information exchange across the entire network is also effectively promoted.

Now, we adopt a simple yet effective model to characterize the behavior of the pull-type gossip protocol. Let $\beta_i(k)$ denote the neighbor node selected by node i at time instant k. Then, the following relation holds:

$$\beta_i(k) \in \mathbb{N}_i,\ \forall i \in \mathbb{V},\ \forall k \in \mathbb{N}^+. \tag{3}$$

The following assumption is made regarding the gossip protocol.

Assumption 1: Each sensor node has at least one neighbor. For any $k \in \mathbb{N}^+$, $\beta_i(k)$ is independent and identically distributed (i.i.d.). For any $i \neq j$, $\beta_i(k)$ and $\beta_j(k)$ are mutually independent.

Assumption 1 facilitates the design and implementation of the pull-type gossip protocol. Based on this assumption, the probability distribution of stochastic variable $\beta_i(k)$ is given as follows:

$$\text{Prob}\{\beta_i(k) = j\} = \epsilon_{i,j},\ \forall i, j \in \mathbb{V},\ i \neq j \tag{4}$$

where $\epsilon_{i,j}$ is the parameter to be designed and satisfies

$$\epsilon_{i,j} \in [0,1], \tag{5}$$

$$\sum_{j=1}^{N} \epsilon_{i,j} = 1, \tag{6}$$

$$\epsilon_{i,j} = 0,\ \forall j \notin \mathbb{N}_i. \tag{7}$$

Remark 1: It is worth emphasizing that the utilized model, while being a simplification of real-world interactions, effectively captures the core stochastic and distributed nature of the pull-type gossip protocol. This abstraction not only facilitates theoretical investigations into estimation performance but also offers valuable insights for the design of adaptive probability schemes aimed at improving collaborative estimation.

The pull-based gossip protocol requires no centralized preconfiguration. Before estimation begins, each node performs a one-time local neighbor discovery (e.g., via hello messages) to obtain its neighbor set. No global topology knowledge, synchronization, or leader election is needed. At each sampling instant, every node independently and uniformly selects one neighbor from its set to communicate with.

D. PIO Using Fading Data

The innovation term plays a critical role in correcting state estimates through appropriately designed feedback gains. In the SN, the innovation from each node is allowed to be transmitted over communication links. Taking into account fading effects, the actual information received by node i from a neighboring node j ($j \in \mathbb{N}_i$) is modeled as

$$\bar{y}_j(k) = \Upsilon(k)\big(y_j(k) - C_j \hat{x}_j(k)\big) \tag{8}$$

where $\hat{x}_j(k)$ is the local estimate of $x(k)$ by node j; $\Upsilon(k) \in \mathbb{R}^{n_y}$ is a stochastic matrix featuring the fading level of the received data, which has the following form:

$$\Upsilon(k) \triangleq \text{diag}\{\upsilon_1(k), \upsilon_2(k), \cdots, \upsilon_{n_y}(k)\}$$

with $\upsilon_d(k)$ ($d = 1, 2, \cdots, n_y$) being stochastic variables for representing the channel coefficients. For $\forall k \in \mathbb{N}^+$, $\upsilon_d(k)$ ($d = 1, 2, \cdots, n_y$) are i.i.d., and mutually independent with $\beta_i(k)$ ($\forall i \in \mathbb{V}$). In addition, the stochastic variables have the following statistical properties:

$$\begin{aligned}\bar{\upsilon}_d \triangleq& \mathcal{E}\{\upsilon_d(k)\},\\ \tilde{\upsilon}_{d,f} \triangleq& \mathcal{E}\{(\upsilon_d(k) - \bar{\upsilon}_d)(\upsilon_f(k) - \bar{\upsilon}_f)\},\end{aligned}$$

where $\bar{\upsilon}_d > 0$; $\tilde{\upsilon}_{d,f} > 0$ are known scalars for $d, f = 1, 2, \cdots, n_y$; and $\mathcal{E}\{\cdot\}$ denotes the mathematical expectation operator. We assume that $\Upsilon(k)$ is independent of $\beta_i(k)$ ($\forall i \in \mathbb{V}$).

To facilitate the following estimator construction and analysis, some auxiliary matrices are defined as follows:

$$\bar{\Upsilon} \triangleq \text{diag}\{\bar{\upsilon}_1, \bar{\upsilon}_2, \cdots, \bar{\upsilon}_{n_y}\},$$

$$\Omega \triangleq \mathrm{diag}\{\tilde{\upsilon}_{1,1}, \tilde{\upsilon}_{2,2}, \cdots, \tilde{\upsilon}_{n_y,n_y}\},$$
$$\bar{\Omega} \triangleq [\tilde{\upsilon}_{d,f}]_{d,f=1,2,\cdots,n_y}.$$

It is easy to see that $\bar{\Upsilon} > 0$, $\Omega > 0$, and $\bar{\Omega} \geq 0$.

Remark 2: As discussed above, the SN employs a shared communication infrastructure comprising multiple individual channels. The adopted fading model is general, as evidenced by the time-varying nature of $\Upsilon(k)$, which captures the effects of dynamically changing environments. Furthermore, correlations among different channels are explicitly modeled via the covariance structure. The formulation is capable of representing several classical fading models widely used in communication and control fields, such as the simplified Rayleigh fading model [42].

Based on the pull-type gossip protocol and the fading data, we adopt the following revised PIO for sensor node i:

$$\begin{cases} \hat{x}_i(k+1) = \sum_{\sigma=1}^{r} s_\sigma(\hat{\alpha}(k))\Big(A_\sigma \hat{x}_i(k) + M_{i,\sigma}\theta_i(k) \\ \qquad + K_{i,\sigma}\big(y_i(k) - C_i\hat{x}_i(k)\big) + H_{i,\sigma}\bar{y}_{\beta_i(k)}(k)\Big) \\ \theta_i(k+1) = L_i\theta_i(k) + T_i\big(y_i(k) - C_i\hat{x}_i(k)\big) \\ \hat{p}_i(k) = \sum_{\sigma=1}^{r} s_\sigma(\hat{\alpha}(k))G_\sigma\hat{x}_i(k) \end{cases} \tag{9}$$

where for node i, $\hat{x}_i(k)$ is the estimate of $x(k)$; $\theta_i(k)$ is the integral (accumulative) term; $\hat{p}_i(k)$ is the estimate of $p(k)$; $\hat{\alpha}(k)$ is the premise variable of the observer; matrices $K_{i,\sigma}$, $M_{i,\sigma}$, $H_{i,\sigma}$, L_i and T_i are observer gains that need to be designed.

By introducing the following Kronecker function:

$$\delta(i,j) \triangleq \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

and considering the relation:

$$\sum_{j\in\mathbb{N}_i} \delta(\beta_i(k), j) = \sum_{j=1}^{N} \delta(\beta_i(k), j),$$

the state equation in observer (9) is rewritten by

$$\begin{aligned} \hat{x}_i(k+1) = \sum_{\sigma=1}^{r} s_\sigma(\hat{\alpha}(k))\Big(&A_\sigma\hat{x}_i(k) + M_{i,\sigma}\theta_i(k) \\ &+ K_{i,\sigma}\big(y_i(k) - C_i\hat{x}_i(k)\big) \\ &+ H_{i,\sigma}\sum_{j=1}^{N}\delta(\beta_i(k), j)\bar{y}_j(k)\Big). \end{aligned} \tag{10}$$

Remark 3: The pull-based gossip protocol adopted in this paper reduces the probability of packet collisions by limiting concurrent transmissions, but it does not guarantee collision-free media access without centralization. Any remaining collisions are naturally incorporated into the fading model, where a fading factor of zero represents a collision-induced packet loss. Therefore, the proposed estimator is inherently robust to such events, and no explicit centralized coordination is required.

E. Estimation Error Systems

Define the estimation error for node i as $e_i(k) = x(k) - \hat{x}_i(k)$ and $\tilde{p}_i(k) = p(k) - \hat{p}_i(k)$. Then, from (1) and (10), the estimation error dynamics is derived as follows:

$$\begin{cases} e_i(k+1) = \sum_{\tau=1}^{r}\sum_{\sigma=1}^{r} s_\tau(\alpha(k))s_\sigma(\hat{\alpha}(k))\Big(\big(A_\tau - A_\sigma\big)x(k) \\ \qquad + \big(A_\sigma - K_{i,\sigma}C_i\big)e_i(k) - M_{i,\sigma}\theta_i(k) \\ \qquad - H_{i,\sigma}\sum_{j=1}^{N}\delta(\beta_i(k), j)\big(\vec{\Upsilon}(k) + \bar{\Upsilon}\big)C_j e_j(k) \\ \qquad - H_{i,\sigma}\sum_{j=1}^{N}\delta(\beta_i(k), j)\big(\vec{\Upsilon}(k) + \bar{\Upsilon}\big)D_j v_j(k) \\ \qquad + E_\tau\omega(k) - K_{i,\sigma}D_i v_i(k)\Big) \\ \theta_i(k+1) = L_i\theta_i(k) + T_iC_ie_i(k) + T_iD_iv_i(k) \\ \tilde{p}_i(k) = \sum_{\tau=1}^{r}\sum_{\sigma=1}^{r} s_\tau(\alpha(k))s_\sigma(\hat{\alpha}(k))\big(G_\sigma e_i(k) \\ \qquad + \bar{G}_{\tau,\sigma}x(k)\big) \end{cases} \tag{11}$$

where

$$\begin{aligned} \bar{G}_{\tau,\sigma} &\triangleq G_\tau - G_\sigma, \\ \vec{\Upsilon}(k) &\triangleq \Upsilon(k) - \bar{\Upsilon} \\ &= \mathrm{diag}\{\upsilon_1(k) - \bar{\upsilon}_1, \upsilon_2(k) - \bar{\upsilon}_2, \cdots, \upsilon_{n_y}(k) - \bar{\upsilon}_{n_y}\}. \end{aligned}$$

By defining the following augmentation vectors:

$$e(k) \triangleq \begin{bmatrix} e_1(k) \\ e_2(k) \\ \vdots \\ e_N(k) \end{bmatrix}, \quad \theta(k) \triangleq \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ \vdots \\ \theta_N(k) \end{bmatrix}, \quad \mu(k) \triangleq \begin{bmatrix} e(k) \\ \theta(k) \\ x(k) \end{bmatrix},$$
$$\bar{\omega}(k) \triangleq \begin{bmatrix} \omega^T(k) & v_1^T(k) & v_2^T(k) & \cdots & v_N^T(k) \end{bmatrix}^T,$$
$$\tilde{p}(k) \triangleq \begin{bmatrix} \tilde{p}_1^T(k) & \tilde{p}_2^T(k) & \cdots & \tilde{p}_N^T(k) \end{bmatrix},$$

the overall estimation error system is obtained as follows:

$$\begin{aligned} \mu(k+1) = \sum_{\tau=1}^{r}\sum_{\sigma=1}^{r} s_\tau(\alpha(k))s_\sigma(\hat{\alpha}(k))\bigg(&\Big(\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon}) \\ &\times \bar{\delta}(k)\bar{C}\bar{I}^T + \tilde{A}_{\tau,\sigma} + \bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\bar{\delta}(k) \\ &\times \bar{C}\bar{I}^T\Big)\mu(k) + \Big(\tilde{E}_{\tau,\sigma} + \bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{D} \\ &+ \bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\Big)\bar{\omega}(k)\bigg) \end{aligned} \tag{12}$$

and

$$\tilde{p}(k) = \sum_{\tau=1}^{r}\sum_{\sigma=1}^{r} s_\tau(\alpha(k))s_\sigma(\hat{\alpha}(k))\tilde{G}_{\tau,\sigma}\mu(k) \tag{13}$$

where

$$\tilde{A}_{\tau,\sigma} \triangleq \begin{bmatrix} \bar{A}_\sigma & \bar{M}_\sigma & \vec{A}_{\tau,\sigma} \\ \vec{C} & \bar{L} & 0 \\ 0 & 0 & A_\tau \end{bmatrix}, \quad \tilde{E}_{\tau,\sigma} \triangleq \begin{bmatrix} \bar{E}_\tau & \vec{E}_\sigma \\ 0 & \bar{D} \\ E_\tau & 0 \end{bmatrix},$$
$$\bar{A}_\sigma \triangleq \mathrm{diag}\big\{A_\sigma - K_{1,\sigma}C_1, \cdots, A_\sigma - K_{N,\sigma}C_N\big\},$$

$$
\begin{aligned}
\bar{M}_\sigma &\triangleq \mathrm{diag}\big\{-M_{1,\sigma}, -M_{2,\sigma}, \cdots, -M_{N,\sigma}\big\},\\
\bar{H}_\sigma &\triangleq \mathrm{diag}\big\{-H_{1,\sigma}, -H_{2,\sigma}, \cdots, -H_{N,\sigma}\big\},\\
\vec{A}_{\tau,\sigma} &\triangleq \mathbf{1}_N \otimes (A_\tau - A_\sigma), \quad \bar{C} \triangleq \mathrm{diag}\{C_1, C_2, \cdots, C_N\},\\
\vec{C} &\triangleq \mathrm{diag}\{T_1 C_1, T_2 C_2, \cdots, T_N C_N\}, \quad \bar{E}_\tau \triangleq \mathbf{1}_N \otimes E_\tau,\\
\bar{L} &\triangleq \mathrm{diag}\{L_1, L_2, \cdots, L_N\}, \quad \bar{\delta}(k) \triangleq \tilde{\delta}(k) \otimes I_{n_y},\\
\bar{I}^T &\triangleq \begin{bmatrix} I_{Nn_x} & 0_{Nn_x \times Nn_y} & 0_{Nn_x \times n_x} \end{bmatrix},\\
\tilde{\delta}(k) &\triangleq \begin{bmatrix} \delta(\beta_1(k),1) & \delta(\beta_1(k),2) & \cdots & \delta(\beta_1(k),N)\\ \delta(\beta_2(k),1) & \delta(\beta_2(k),2) & \cdots & \delta(\beta_2(k),N)\\ \vdots & \vdots & \ddots & \vdots \\ \delta(\beta_N(k),1) & \delta(\beta_N(k),2) & \cdots & \delta(\beta_N(k),N) \end{bmatrix},\\
\vec{E}_\sigma &\triangleq \mathrm{diag}\big\{-K_{1,\sigma}D_1, -K_{2,\sigma}D_2, \cdots, -K_{N,\sigma}D_N\big\},\\
\bar{D} &\triangleq \mathrm{diag}\{T_1 D_1, T_2 D_2, \cdots, T_N D_N\},\\
\tilde{G}_{\tau,\sigma} &\triangleq \begin{bmatrix} \tilde{G}^{1,1}_\sigma & 0 & \tilde{G}^{1,3}_{\tau,\sigma} \end{bmatrix}, \quad \tilde{G}^{1,3}_{\tau,\sigma} \triangleq \mathbf{1}_N \otimes (G_\tau - G_\sigma),\\
\tilde{G}^{1,1}_\sigma &\triangleq \mathrm{diag}\underbrace{\{G_\sigma, G_\sigma, \cdots, G_\sigma\}}_{N}.
\end{aligned}
$$

The objective of this paper is to design observer's gains and protocol's parameters that simultaneously satisfy the following requirements:

1) in the absence of external noises, the augmented error system (12) is asymptotically stable in the mean-square sense; and

2) under zero-initial conditions and in the presence of any non-zero energy-bounded noises, the following H_∞ performance criterion is guaranteed:

$$
\mathcal{E}\left\{\sum_{k=0}^{\infty} \tilde{p}^T(k)\tilde{p}(k)\right\} < \gamma^2 \sum_{k=0}^{\infty} \bar{\omega}^T(k)\bar{\omega}(k) \tag{14}
$$

where $\gamma > 0$ is a scalar.

Remark 4: Inequality (14) implies that, in the mean-square sense, the ratio of the energy of the estimation error to the energy of the external disturbance is bounded by γ. In other words, under the combined influence of external noises, signal attenuation, and the pull-based gossip protocol, the proposed fuzzy estimator guarantees that the energy of the estimation error remains smaller than a specified multiple of the energy of the external disturbance, which ensures a desirable level of robustness and disturbance attenuation capability.

III. Main Results

A. Performance Analysis and Observer Design

In the following, the performance analysis of the proposed PIO is detailed in Theorem 1.

Theorem 1: Consider system (1) and distributed fuzzy PIO (8) with given scalars γ, $\epsilon_{i,j}$, and matrices $M_{i,\sigma}$, $H_{i,\sigma}$, $K_{i,\sigma}$, T_i, L_i ($i, j \in \mathbb{V}$, $\sigma \in \mathbb{H}$). The resultant augmentation error system (12) is asymptotically stable in the mean-square sense and satisfies the prescribed H_∞ performance index (14) if, for τ, $\sigma \in \mathbb{H}$, $i \in \mathbb{V}$, there exist symmetric matrices $P_i > 0$, $Q_i > 0$, $R > 0$, $\check{A}_{\tau,\sigma} > 0$, $\check{E}_{\tau,\sigma} > 0$ and scalars $\alpha_\sigma > 0$, $\varpi_\sigma > 0$ such that the following inequalities hold:

$$
\mathcal{A}^T_{\tau,\sigma}\tilde{P}\mathcal{A}_{\tau,\sigma} + \mathcal{G}_{\tau,\sigma} < 0 \tag{15}
$$

$$
\tilde{A}^T_{\tau,\sigma}\tilde{P}\tilde{A}_{\tau,\sigma} - \check{A}_{\tau,\sigma} < 0 \tag{16}
$$

$$
\tilde{E}^T_{\tau,\sigma}\tilde{P}\tilde{E}_{\tau,\sigma} - \check{E}_{\tau,\sigma} < 0 \tag{17}
$$

$$
\bar{H}^T_\sigma\bar{P}\bar{H}_\sigma - \varpi_\sigma I_{n_y} < 0 \tag{18}
$$

$$
(I_N \otimes \bar{\Upsilon})^T\bar{H}^T_\sigma\bar{P}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon}) < \alpha_\sigma I_{n_y} \tag{19}
$$

where

$$
\begin{aligned}
\mathcal{A}_{\tau,\sigma} &\triangleq \begin{bmatrix} \tilde{A}_{\tau,\sigma} & \tilde{E}_{\tau,\sigma} \end{bmatrix}, \quad \bar{P} \triangleq \mathrm{diag}\{P_1, P_2, \cdots, P_N\},\\
\tilde{P} &\triangleq \mathrm{diag}\big\{\bar{P}, \bar{Q}, R\big\}, \quad \bar{Q} \triangleq \mathrm{diag}\{Q_1, Q_2, \cdots, Q_N\},\\
\mathcal{G}_{\tau,\sigma} &\triangleq \mathrm{diag}\{\tilde{G}^T_{\tau,\sigma}\tilde{G}_{\tau,\sigma} + \grave{A}_{\tau,\sigma} - \tilde{P}, \grave{E}_{\tau,\sigma} - \gamma^2 I\},\\
\grave{A}_{\tau,\sigma} &\triangleq 2\alpha_\sigma\bar{I}\bar{C}^T\bar{\Pi}\bar{C}\bar{I}^T + 2\bar{I}\bar{C}^T\bar{\Pi}(\varpi_\sigma I_N \otimes \Omega)\bar{C}\bar{I}^T\\
&\quad + 2\check{A}_{\tau,\sigma} + 2\alpha_\sigma\bar{I}\bar{C}^T\tilde{\Pi}^T\tilde{\Pi}\bar{C}\bar{I}^T,\\
\grave{E}_{\tau,\sigma} &\triangleq 2\alpha_\sigma\bar{D}^T\bar{\Pi}\bar{D} + 2\bar{D}^T\bar{\Pi}(\varpi_\sigma I_N \otimes \Omega)\bar{D}\\
&\quad + 2\check{E}_{\tau,\sigma} + 2\alpha_\sigma\bar{D}^T\tilde{\Pi}^T\tilde{\Pi}\bar{D},\\
\bar{\Pi} &\triangleq \mathrm{diag}\{\bar{\Pi}_1, \bar{\Pi}_2, \cdots, \bar{\Pi}_N\}, \quad \bar{\Pi}_i \triangleq \sum_{j=1}^{N}\epsilon_{j,i}I_{n_y},\\
\Pi &\triangleq \begin{bmatrix} \epsilon_{1,1} & \epsilon_{1,2} & \cdots & \epsilon_{1,N}\\ \epsilon_{2,1} & \epsilon_{2,2} & \cdots & \epsilon_{2,N}\\ \vdots & \vdots & \ddots & \vdots\\ \epsilon_{N,1} & \epsilon_{N,2} & \cdots & \epsilon_{N,N} \end{bmatrix}, \quad \tilde{\Pi} \triangleq \Pi \otimes I_{n_y}.
\end{aligned}
$$

Proof: Construct a Lyapunov candidate as follows:

$$
V(k) = \mu^T(k)\tilde{P}\mu(k). \tag{20}
$$

For notational convenience, the following is defined by omitting the variable $\alpha(k)$ and $\hat{\alpha}(k)$.

$$
\sum_{\tau=1}^{r}\sum_{\sigma=1}^{r} s_\tau s_\sigma \triangleq \sum_{\tau=1}^{r}\sum_{\sigma=1}^{r} s_\tau(\alpha(k))s_\sigma(\hat{\alpha}(k)).
$$

Then, calculate the difference of $V(k)$ in terms of conditional mathematical expectation, one has from (12) that

$$
\begin{aligned}
&\mathcal{E}\{V(k+1) - V(k)|\mu(k)\}\\
\le\,&\mathcal{E}\Bigg\{\sum_{\tau=1}^{r}\sum_{\sigma=1}^{r} s_\tau s_\sigma\Big(\mu^T(k)\tilde{A}^T_{\tau,\sigma}\tilde{P}\tilde{A}_{\tau,\sigma}\mu(k) + \mu^T(k)\big(\bar{I}\bar{H}_\sigma\\
&\times (I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\mu(k)\\
&+ \mu^T(k)\big(\bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\\
&\times \bar{\delta}(k)\bar{C}\bar{I}^T\mu(k) + \bar{\omega}^T(k)\tilde{E}^T_{\tau,\sigma}\tilde{P}\tilde{E}_{\tau,\sigma}\bar{\omega}(k) + \bar{\omega}^T(k)\Big(\bar{I}\bar{H}_\sigma\\
&\times (I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{D}\Big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{D}\bar{\omega}(k) + \bar{\omega}^T(k)\\
&\times \Big(\bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\\
&\times \bar{\omega}(k) + 2\mu^T(k)\tilde{A}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\mu(k)\\
&+ 2\mu^T(k)\tilde{A}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\mu(k) + 2\mu^T(k)\\
&\times \tilde{A}^T_{\tau,\sigma}\tilde{P}\tilde{E}_{\tau,\sigma}\bar{\omega}(k) + 2\mu^T(k)\tilde{A}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{D}\\
&\times \bar{\omega}(k) + 2\mu^T(k)\tilde{A}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\bar{\omega}(k)\\
&+ 2\mu^T(k)\big(\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N \otimes \vec{\Upsilon}(k))\\
&\times \bar{\delta}(k)\bar{C}\bar{I}^T\Big)\mu(k) + 2\mu^T(k)\big(\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\\
&\times \tilde{E}_{\tau,\sigma}\bar{\omega}(k) + 2\mu^T(k)\big(\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma\\
&\times (I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{D}\bar{\omega}(k) + 2\mu^T(k)\big(\bar{I}\bar{H}_\sigma(I_N \otimes \bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T
\end{aligned}
$$

$$
\begin{aligned}
&\times \tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\bar{\varpi}(k)+2\mu^T(k)\big(\bar{I}\bar{H}_\sigma\\
&\times(I_N\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\tilde{E}_{\tau,\sigma}\bar{\varpi}(k)+2\mu^T(k)\big(\bar{I}\bar{H}_\sigma\\
&+2\mu^T(k)\big(\bar{I}\bar{H}_\sigma(I_N\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\vec{\Upsilon}(k))\\
&\times\bar{\delta}(k)\bar{D}\bar{\varpi}(k)(I_N\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\\
&\times\bar{D}\bar{\varpi}(k)+2\bar{\varpi}^T(k)\tilde{E}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\bar{\varpi}(k)\\
&+2\bar{\varpi}^T(k)\tilde{E}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\bar{\varpi}(k)+2\bar{\varpi}^T(k)\\
&\times\Big(\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\Big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\bar{\varpi}(k)\\
&-\mu^T(k)\tilde{P}\mu(k)\Big)|\mu(k)\Big\}. \qquad (21)
\end{aligned}
$$

There are some complex coupling terms in (21) induced by the fading channel and the gossip protocol, which will be dealt with respectively.

Based on the structure of $\bar{I}_1$ and $\tilde{P}$, one derives that

$$\bar{I}_1^T\tilde{P}\bar{I}_1=\bar{P}.$$

Then, one has from (19) that

$$
\begin{aligned}
&\mathcal{E}\left\{\big(\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\right\}\\
&\quad=\mathcal{E}\left\{\bar{I}\bar{C}^T\bar{\delta}^T(k)(I_N\otimes\bar{\Upsilon})^T\bar{H}_\sigma^T\bar{P}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\right\}\\
&\quad<\mathcal{E}\left\{\bar{I}\bar{C}^T\bar{\delta}^T(k)\alpha_\sigma I_{n_y}\bar{\delta}(k)\bar{C}\bar{I}^T\right\}. \qquad (22)
\end{aligned}
$$

Using the definition of $\bar{\delta}(k)$, it is calculated that

$$\mathcal{E}\{\bar{\delta}^T(k)\bar{\delta}(k)\}=\bar{\Pi},\quad \mathcal{E}\{\bar{\delta}(k)\}=\tilde{\Pi}, \qquad (23)$$

which implies

$$
\begin{aligned}
&\mathcal{E}\left\{\big(\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\right\}\\
&\quad<\alpha_\sigma\bar{I}\bar{C}^T\bar{\Pi}\bar{C}\bar{I}^T. \qquad (24)
\end{aligned}
$$

One deduces from (18) that

$$
\begin{aligned}
&\mathcal{E}\left\{\big(\bar{I}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\right\}\\
&<\mathcal{E}\left\{\bar{I}\bar{C}^T\bar{\delta}^T(k)(I\otimes\vec{\Upsilon}(k))^T\varpi_\sigma I_{n_y}(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\right\}\\
&=\bar{I}\bar{C}^T\bar{\Pi}(\varpi_\sigma I_N\otimes\Omega)\bar{C}\bar{I}^T. \qquad (25)
\end{aligned}
$$

Similarly, one infers that

$$
\begin{aligned}
&\mathcal{E}\left\{\big(\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\right\}\\
&\quad=\mathcal{E}\left\{\bar{D}^T\bar{\delta}^T(k)(I_N\otimes\bar{\Upsilon})^T\bar{H}_\sigma^T\bar{P}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\right\}\\
&\quad<\mathcal{E}\left\{\bar{D}^T\bar{\delta}^T(k)\alpha_\sigma I_{n_y}\bar{\delta}(k)\bar{D}\right\}\\
&\quad=\alpha_\sigma\bar{D}^T\bar{\Pi}\bar{D}, \qquad (26)
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathcal{E}\left\{\big(\bar{I}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\right\}\\
&\quad=\mathcal{E}\left\{\bar{D}^T\bar{\delta}^T(k)(I\otimes\vec{\Upsilon}(k))^T\bar{H}_\sigma^T\bar{P}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\right\}\\
&\quad<\mathcal{E}\left\{\bar{D}^T\bar{\delta}^T(k)(I\otimes\vec{\Upsilon}(k))^T\varpi_\sigma I_{n_y}(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\right\}\\
&\quad=\bar{D}^T\bar{\Pi}(\varpi_\sigma I_N\otimes\Omega)\bar{D}. \qquad (27)
\end{aligned}
$$

By using the basic inequality techniques, one has that

$$
\begin{aligned}
&\mathcal{E}\left\{2\tilde{A}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\right\}\\
&\quad\le\mathcal{E}\Big\{\tilde{A}^T_{\tau,\sigma}\tilde{P}\tilde{A}_{\tau,\sigma}+\bar{I}\bar{C}^T\bar{\delta}^T(k)(I\otimes\vec{\Upsilon}(k))^T\bar{H}_\sigma^T\bar{P}\\
&\qquad\times\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\Big\}\\
&\quad<\tilde{A}^T_{\tau,\sigma}\tilde{P}\tilde{A}_{\tau,\sigma}+\bar{I}\bar{C}^T\bar{\Pi}(\varpi_\sigma I_N\otimes\Omega)\bar{C}\bar{I}^T,\\
&\mathcal{E}\left\{2\tilde{A}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\right\}\\
&\quad\le\mathcal{E}\Big\{\tilde{A}^T_{\tau,\sigma}\tilde{P}\tilde{A}_{\tau,\sigma}+\bar{D}^T\bar{\delta}^T(k)(I\otimes\bar{\Upsilon})^T\bar{H}_\sigma^T\bar{P}\\
&\qquad\times\bar{H}_\sigma(I\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\Big\}\\
&\quad<\tilde{A}^T_{\tau,\sigma}\tilde{P}\tilde{A}_{\tau,\sigma}+\alpha_\sigma\bar{D}^T\bar{\Pi}\bar{D},\\
&\mathcal{E}\left\{2\big(\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\tilde{E}_{\tau,\sigma}\right\}\\
&\quad\le\mathcal{E}\Big\{\big(\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\\
&\qquad+\tilde{E}^T_{\tau,\sigma}\tilde{P}\tilde{E}_{\tau,\sigma}\Big\}\\
&\quad<\tilde{E}^T_{\tau,\sigma}\tilde{P}\tilde{E}_{\tau,\sigma}+\bar{I}\bar{C}^T\alpha_\sigma\bar{\Pi}\bar{C}\bar{I}^T,\\
&\mathcal{E}\left\{2\big(\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\right\}\\
&\quad<\mathcal{E}\Big\{\bar{I}\bar{C}^T\bar{\delta}^T(k)(I_N\otimes\bar{\Upsilon})^T\bar{H}_\sigma^T\bar{P}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{C}\bar{I}^T\\
&\qquad+\bar{D}^T\bar{\delta}^T(k)(I_N\otimes\bar{\Upsilon})^T\bar{H}_\sigma^T\bar{P}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\Big\}\\
&\quad<\bar{I}\bar{C}^T\alpha_\sigma\bar{\Pi}\bar{C}\bar{I}^T+\alpha_\sigma\bar{D}^T\bar{\Pi}\bar{D},\\
&\mathcal{E}\Big\{2\big(\bar{I}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\big)^T\tilde{P}\bar{I}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\Big\}\\
&\quad\le\mathcal{E}\left\{\bar{I}\bar{C}^T\bar{\delta}^T(k)(I\otimes\vec{\Upsilon}(k))^T\bar{H}_\sigma^T\bar{P}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{C}\bar{I}^T\right\}\\
&\qquad+\mathcal{E}\left\{\bar{D}^T\bar{\delta}^T(k)(I\otimes\vec{\Upsilon}(k))^T\bar{H}_\sigma^T\bar{P}\bar{H}_\sigma(I\otimes\vec{\Upsilon}(k))\bar{\delta}(k)\bar{D}\right\}\\
&\quad\le\bar{I}\bar{C}^T\bar{\Pi}(\varpi_\sigma I_N\otimes\Omega)\bar{C}\bar{I}^T+\bar{D}^T\bar{\Pi}(\varpi_\sigma I_N\otimes\Omega)\bar{D},\\
&\mathcal{E}\Big\{2\tilde{E}^T_{\tau,\sigma}\tilde{P}\bar{I}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\Big\}\\
&\quad\le\mathcal{E}\Big\{\tilde{E}^T_{\tau,\sigma}\tilde{P}\tilde{E}_{\tau,\sigma}+\bar{D}^T\bar{\delta}^T(k)(I_N\otimes\bar{\Upsilon})^T\bar{H}_\sigma^T\\
&\qquad\times\tilde{P}\bar{H}_\sigma(I_N\otimes\bar{\Upsilon})\bar{\delta}(k)\bar{D}\\
&\quad<\tilde{E}^T_{\tau,\sigma}\tilde{P}\tilde{E}_{\tau,\sigma}+\alpha_\sigma\bar{D}^T\bar{\Pi}\bar{D}. \qquad (28)
\end{aligned}
$$

Conditions (16) and (17) imply that

$$\tilde{A}^T_{\tau,\sigma}\tilde{P}\tilde{A}_{\tau,\sigma}<\check{A}_{\tau,\sigma},\quad \tilde{E}^T_{\tau,\sigma}\tilde{P}\tilde{E}_{\tau,\sigma}<\check{E}_{\tau,\sigma} \qquad (29)$$

By taking into account (21)–(29), we have

$$
\begin{aligned}
&\mathcal{E}\{V(k+1)-V(k)|\mu(k)\}\\
&=\sum_{\tau=1}^{r}\sum_{\sigma=1}^{r}s_\tau s_\sigma\mathcal{E}\Big\{\big(\tilde{A}_{\tau,\sigma}\mu(k)+\tilde{E}_{\tau,\sigma}\bar{\varpi}(k)\big)^T\tilde{P}\big(\tilde{A}_{\tau,\sigma}\mu(k)\\
&\quad+\tilde{E}_{\tau,\sigma}\bar{\varpi}(k)\big)+\mu^T(k)\grave{A}_{\tau,\sigma}\mu(k)+\bar{\varpi}^T(k)\grave{E}_{\tau,\sigma}\bar{\varpi}(k)\\
&\quad-\mu^T(k)\tilde{P}\mu(k)|\mu(k)\Big\}. \qquad (30)
\end{aligned}
$$

For stability analysis of the estimation error system, we first let $\bar{\varpi}(k)\equiv 0$. By following a similar line with the proof in [26], condition (16) ensures $\mathcal{E}\{V(k+1)-V(k)|\mu(k)\}<0$ in the absence of noises. Then, by taking the mathematical expectation, one finally has

$$\mathcal{E}\{V(k+1)-V(k)\}<0. \qquad (31)$$

Thus, one deduces that the estimation error system (12) is asymptotically stable in the mean-square sense.

To check the H_∞ performance for any non-zero energy-bounded noises, we define the following functions for any integer $k_{\max} > 0$ under the zero initial condition $V(0) = 0$:

$$g(k_{\max}) \triangleq \sum_{k=0}^{k_{\max}} \mathcal{E}\Big\{\tilde{p}^T(k)\tilde{p}(k) - \gamma^2\bar{\omega}^T(k)\bar{\omega}(k)|\mu(k)\Big\}. \quad (32)$$

One infers that

$$\begin{aligned} g(k_{\max}) &= \sum_{k=0}^{k_{\max}} \mathcal{E}\Big\{\tilde{p}^T(k)\tilde{p}(k) - \gamma^2\bar{\omega}^T(k)\bar{\omega}(k) + V(k+1) \\ &\quad - V(k)|\mu(k)\Big\} - \mathcal{E}\{V(k_{\max}+1)|\mu(k)\} \\ &\le \sum_{k=0}^{k_{\max}} \mathcal{E}\Big\{\tilde{p}^T(k)\tilde{p}(k) - \gamma^2\bar{\omega}^T(k)\bar{\omega}(k) + V(k+1) \\ &\quad - V(k)|\mu(k)\Big\} \\ &\le \sum_{k=0}^{k_{\max}} \sum_{\tau=1}^{r}\sum_{\sigma=1}^{r} s_\tau s_\sigma \mathcal{E}\Big\{\bar{\mu}^T(k)\big(\mathcal{A}_{\tau,\sigma}^T\tilde{P}\mathcal{A}_{\tau,\sigma} + \mathcal{G}_{\tau,\sigma}\big) \\ &\quad \times \bar{\mu}(k)|\mu(k)\Big\} \end{aligned} \quad (33)$$

where $\bar{\mu}(k) \triangleq \begin{bmatrix}\mu^T(k) & \bar{\omega}^T(k)\end{bmatrix}^T$.

It is obtained from condition (16) that $g(k_{\max}) < 0$. By letting $k_{\max} \to +\infty$ and taking the mathematical expectation, one has

$$\sum_{k=0}^{\infty} \mathcal{E}\Big\{\tilde{p}^T(k)\tilde{p}(k) - \gamma^2\bar{\omega}^T(k)\bar{\omega}(k)\Big\} < 0. \quad (34)$$

The proof is complete. ■

Based on the conclusions of Theorem 1, Theorem 2 presents the methods for calculating the observer gains.

Theorem 2: Consider system (1) and distributed fuzzy PIO (8) with given scalars γ and $\epsilon_{i,j}$ $(i, j \in \mathbb{V})$. The resultant augmentation error system (12) is asymptotically stable in the mean-square sense and satisfies the prescribed H_∞ performance index (14) if, for $\tau,\ \sigma \in \mathbb{H}$, $i \in \mathbb{V}$, there exist symmetric matrices $P_i > 0$, $Q_i > 0$, $R > 0$, $\check{A}_{\tau,\sigma} > 0$, $\check{E}_{\tau,\sigma} > 0$, matrices $\bar{K}_{i,\sigma}$, $\bar{M}_{i,\sigma}$, $\bar{H}_{i,\sigma}$, $\bar{T}_i$, $\bar{L}_i$ and scalars $\alpha_i > 0$, $\varpi_\sigma > 0$, such that the following inequalities hold:

$$\Xi_{\tau,\sigma} < 0 \quad (35)$$
$$\Lambda_{\tau,\sigma} < 0 \quad (36)$$
$$\Psi_{\tau,\sigma} < 0 \quad (37)$$
$$\Gamma_\sigma < 0 \quad (38)$$
$$\Phi_\sigma < 0 \quad (39)$$

where

$$\Phi_\sigma \triangleq \begin{bmatrix} -\alpha_\sigma I_{n_y} & * \\ \vec{H}_\sigma(I_N \otimes \Upsilon) & -\bar{P}\end{bmatrix},$$

$$\bar{\Xi}_{\tau,\sigma} \triangleq \begin{bmatrix} \Xi_\sigma^{(1,1)} & \Xi_\sigma^{(1,2)} & \Xi_{\tau,\sigma}^{(1,3)} \\ \Xi^{(2,1)} & \Xi^{(2,2)} & 0 \\ 0 & 0 & RA_\tau \end{bmatrix},$$

$$\Xi_{\tau,\sigma} \triangleq \begin{bmatrix} \tilde{G}_{\tau,\sigma}^T\tilde{G}_{\tau,\sigma} + \grave{A}_{\tau,\sigma} - \tilde{P} & * & * \\ 0 & \grave{E}_{\tau,\sigma} - \gamma^2 I & * \\ \bar{\Xi}_{\tau,\sigma} & \vec{\Xi}_\tau & -\tilde{P}\end{bmatrix},$$

$$\Xi_{\tau,\sigma}^{(1,3)} \triangleq \begin{bmatrix} P_1(A_\tau - A_\sigma) \\ P_2(A_\tau - A_\sigma) \\ \vdots \\ P_N(A_\tau - A_\sigma)\end{bmatrix}, \quad \vec{\Xi}_\tau^{(1,1)} \triangleq \begin{bmatrix} P_1E_\tau \\ P_2E_\tau \\ \vdots \\ P_NE_\tau\end{bmatrix},$$

$$\Lambda_{\tau,\sigma} \triangleq \begin{bmatrix} -\check{A}_{\tau,\sigma} & * \\ \bar{\Xi}_{\tau,\sigma} & -\tilde{P}\end{bmatrix}, \quad \vec{\Xi}_{\tau,\sigma} \triangleq \begin{bmatrix} \vec{\Xi}_\tau^{(1,1)} & \vec{\Xi}_\sigma^{(1,2)} \\ 0 & \vec{\Xi}^{(2,2)} \\ RE_\tau & 0\end{bmatrix},$$

$$\Psi_{\tau,\sigma} \triangleq \begin{bmatrix} -\check{E}_{\tau,\sigma} & * \\ \vec{\Xi}_{\tau,\sigma} & -\tilde{P}\end{bmatrix}, \quad \Gamma_\sigma \triangleq \begin{bmatrix} -\varpi_\sigma I_{n_y} & * \\ \vec{H}_\sigma & -\bar{P}\end{bmatrix},$$

$$\begin{aligned} \Xi_\sigma^{(1,2)} &\triangleq \mathrm{diag}\{-\bar{M}_{1,\sigma}, -\bar{M}_{2,\sigma}, \cdots, -\bar{M}_{N,\sigma}\}, \\ \Xi^{(2,1)} &\triangleq \mathrm{diag}\{\bar{T}_1C_1, \bar{T}_2C_2, \cdots, \bar{T}_NC_N\}, \\ \Xi^{(2,2)} &\triangleq \mathrm{diag}\{\bar{L}_1, \bar{L}_2, \cdots, \bar{L}_N\}, \\ \vec{\Xi}^{(2,2)} &\triangleq \mathrm{diag}\{\bar{T}_1D_1, \bar{T}_2D_2, \cdots, \bar{T}_ND_N\}, \\ \vec{H}_\sigma &\triangleq \mathrm{diag}\{-\bar{H}_{1,\sigma}, -\bar{H}_{2,\sigma}, \cdots, -\bar{H}_{N,\sigma}\}, \\ \vec{\Xi}_\sigma^{(1,2)} &\triangleq \mathrm{diag}\{-\bar{K}_{1,\sigma}D_1, -\bar{K}_{2,\sigma}D_2, \cdots, -\bar{K}_{N,\sigma}D_N\}, \\ \Xi_\sigma^{(1,1)} &\triangleq \mathrm{diag}\{P_1A_\sigma - \bar{K}_{1,\sigma}C_1, \cdots, P_NA_\sigma - \bar{K}_{N,\sigma}C_N\}. \end{aligned}$$

The remaining parameters are defined in Theorem 1. Provided that the above inequalities are solvable, the observer gains can be directly determined as follows:

$$\begin{aligned} K_{i,\sigma} &= P_i^{-1}\bar{K}_{i,\sigma}, \quad M_{i,\sigma} = P_i^{-1}\bar{M}_{i,\sigma}, \quad T_i = Q_i^{-1}\bar{T}_i, \\ L_i &= Q_i^{-1}\bar{L}_i, \quad H_{i,\sigma} = P_i^{-1}\bar{H}_{i,\sigma}. \end{aligned} \quad (40)$$

Proof: For the matrix in (15), the following can be calculated:

$$\begin{aligned} &\mathcal{A}_{\tau,\sigma}^T\tilde{P}\mathcal{A}_{\tau,\sigma} + \mathcal{G}_{\tau,\sigma} \\ &\quad = \mathcal{A}_{\tau,\sigma}^T\tilde{P}\tilde{P}^{-1}\tilde{P}\mathcal{A}_{\tau,\sigma} + \mathcal{G}_{\tau,\sigma} \end{aligned} \quad (41)$$

By using the Schur Complement Lemma, $\mathcal{A}_{\tau,\sigma}^T\tilde{P}\mathcal{A}_{\tau,\sigma} + \mathcal{G}_{\tau,\sigma} < 0$ holds if and only if $\Xi_{\tau,\sigma} < 0$ holds, with the relation $\bar{K}_{i,\sigma} = P_iK_{i,\sigma}$, $\bar{M}_{i,\sigma} = P_iM_{i,\sigma}$, $\bar{H}_{i,\sigma} = P_iH_{i,\sigma}$, $\bar{T}_i = Q_iT_i$ and $\bar{L}_i = Q_iL_i$. Thus, condition (15) in Theorem 1 is ensured by condition (35) in Theorem 2.

By analogous reasoning, conditions (16)–(19) are guaranteed by (36)–(39), respectively, thus concluding the proof. ■

B. Co-design of Gossip Protocol and PIO

Theorem 2 formulates a methodology for computing the gains of the proposed PIO given predefined protocol parameters and a disturbance attenuation level γ. It is noted that a smaller γ corresponds to enhanced robustness of the observer. This subsection proceeds to establish a co-design framework aimed at minimizing γ through the simultaneous optimization of both observer gains and protocol parameters.

Corollary 1: Consider system (1) and fuzzy PIO (8). The resultant augmentation error system (12) is asymptotically stable in the mean-square sense and satisfies the prescribed H_∞ performance index (14) if, for $\tau,\ \sigma \in \mathbb{H}$, $i \in \mathbb{V}$, there exist symmetric matrices $P_i > 0$, $Q_i > 0$, $R > 0$, $\check{A}_{\tau,\sigma} > 0$, $\check{E}_{\tau,\sigma} > 0$, matrices $\bar{K}_{i,\sigma}$, $\bar{M}_{i,\sigma}$, $\bar{H}_{i,\sigma}$, $\bar{T}_i$, $\bar{L}_i$, and scalars $\alpha_\sigma > 0$, $\varpi_\sigma > 0$, $\epsilon_{i,j} \ge 0$, $\gamma > 0$, such that (5)–(7), (35)–(39) holds. Furthermore, the parameter γ can be minimized by solving the following optimization problem:

$$\min_{\mathcal{P}} \gamma \quad (42)$$

subject to (5)–(7) and (35)–(39), where

$$\mathcal{P} \triangleq \{P_i, Q_i, R, \check{A}_{\tau,\sigma}, \check{E}_{\tau,\sigma}, \bar{K}_{i,\sigma}, \bar{M}_{i,\sigma}, \bar{H}_{i,\sigma}, \bar{T}_i, \bar{L}_i, \alpha_\sigma, \varpi_\sigma, \epsilon_{i,j}\}.$$

If optimization problem (42) is solvable, then the parameter matrix of the pull-type gossip protocol Π is directly designed following obtained $\epsilon_{i,j}$, and observer gains are calculated in terms of (40).

Proof: The proof is straightforward from Theorem 2, which is omitted here. ■

Corollary 1 formulates an optimization problem aimed at minimizing the H_∞ performance index γ, subject to a set of equality and matrix inequality constraints. The resulting problem, denoted as (42), is inherently non-convex due to the presence of multiple bilinear terms among the decision variables in constraint (35), which places it outside the scope of conventional linear matrix inequality solvers.

To address this challenge, we resort to evolutionary algorithms, which are well-suited for complex, non-convex optimization. Specifically, the particle swarm optimization (PSO) algorithm is employed to solve minimization problem (42), owing to its recognized advantages in fast convergence and straightforward implementation.

The fundamental principle of PSO is to simulate the intelligent search behavior of a particle swarm within a solution space, with the objective of discovering a global optimum. Within this framework, each particle represents a candidate solution, defined by its spatial position, while the velocity vector of each particle serves as the driving mechanism for the population's evolution. This study focuses on employing PSO to explore the optimal design configurations for protocol parameter Π. Throughout this optimization process, the performance of each design candidate, represented by a particle's position, is quantitatively evaluated using Theorems 2 and Corollary 1. The outcome of this evaluation directly guides the entire particle swarm toward regions of the design space characterized by superior performance.

This subsection outlines the key parameters of the PSO algorithm, including the swarm size and evolution equations, and subsequently presents a systematic procedure for solving optimization problem (42).

For node i, its neighboring set is assumed to be

$$\mathbb{N}_i \triangleq \{j_1^{(i)}, j_2^{(i)}, \cdots, j_{\kappa_i}^{(i)}\}. \tag{43}$$

Notation (43) means that 1) node i can request data from node $j_1^{(i)}$, $j_2^{(i)}$, $\cdots$, $j_{\kappa_i}^{(i)}$; and 2) the parameters of the gossip protocol to be designed for node i are $\epsilon_{i,j_i^{(i)}}$, $\epsilon_{i,j_2^{(i)}}$, $\cdots$, $\epsilon_{j_{\kappa_i}^{(i)}}$. Correspondingly, the dimension of each particle is set as

$$l = \sum_{i=1}^{N} (\kappa_i - 1).$$

Remark 5: To efficiently handle the equality constraint (6) within the PSO framework, the parameters $\epsilon_{i,j_i^{(i)}}$, $\epsilon_{i,j_2^{(i)}}$, $\cdots$, $\epsilon_{j_{\kappa_i-1}^{(i)}}$ are treated as the design variables for node i. The remaining parameter is then determined uniquely by

$$\epsilon_{i,j_{\kappa_i}^{(i)}} = 1 - \bar{\epsilon}_i,$$

where

$$\bar{\epsilon}_i \triangleq \sum_{\varsigma=1}^{\kappa_i - 1} \epsilon_{i,j_\varsigma^{(i)}} \in (0, 1).$$

In addition, to satisfy constraint (7), all parameters $\epsilon_{i,j}$ corresponding to non-neighboring nodes of i are explicitly set to zero.

The position vector of a particle is defined as follows:

$$\mathbf{P} \triangleq \begin{bmatrix} \mathbf{p}_1^T & \mathbf{p}_2^T & \cdots & \mathbf{p}_N^T \end{bmatrix}^T \tag{44}$$

where

$$\mathbf{p}_i \triangleq \begin{bmatrix} \epsilon_{i,j_1^{(i)}} & \epsilon_{i,j_2^{(i)}} & \cdots & \epsilon_{i,j_{\kappa_i-1}^{(i)}} \end{bmatrix}^T.$$

Assume that the total of S particles are used, where the position and velocity of the s-th ($s \in \{1, 2, \cdots, S\}$) particle at the t-th ($t \in \mathbb{N}$) updating are denoted, respectively, by $\mathbf{P}_s(t)$ and $\mathbf{V}_s(t)$. Then, the updating process is defined as follows:

$$\begin{cases} \mathbf{V}_s(t+1) = \iota \mathbf{V}_s(t) + b_1 o_1 \big(\bar{\mathbf{P}}_L^{(s)}(t) - \mathbf{P}_s(t)\big) \\ \qquad\qquad\quad + b_2 o_2 \big(\bar{\mathbf{P}}_G(t) - \mathbf{P}_s(t)\big) \\ \mathbf{P}_s(t+1) = \mathbf{P}_s(t) + \mathbf{V}_s(t+1) \end{cases} \tag{45}$$

where ι, b_1, b_2 are given scalars; o_1, o_2 are two stochastic numbers taking values in $(0, 1)$; $\bar{\mathbf{P}}_L^{(s)}(t)$ is the historical best position found by the i-th particle up to the updating time t; and $\bar{\mathbf{P}}_G(t)$ is the global best position of the entire particle swarm up to the updating time t.

The PSO-based co-design method is concluded in Algorithm 1.

Algorithm 1: PSO-based PIO Design Under Fading Data

Step 1. Set $t = 0$ and give the maximum number of iterations $t_{\max}$. Generate S particles with given initial position $\mathbf{P}_s(0)$ such that (5)–(7) hold and (35)–(39) are feasible. Denote the obtained γ as $\gamma(\mathbf{P}_s(0))$. Let $\bar{\mathbf{P}}_L^{(s)}(0) = \mathbf{P}_s(0)$ and $\bar{\mathbf{P}}_G(0) = 0.5I$. Set an acceptable performance threshold $\gamma_{\text{acc}} > 0$ and a convergence tolerance $\varepsilon > 0$.

Step 2. Update the position and velocity according to (45). Obtain $\mathbf{P}_s(t+1)$ and $\mathbf{V}_s(t+1)$.

Step 3. Use $\mathbf{P}_s(t+1)$ to solve (35)–(39). If they are solvable, then go to *Step 4*. Otherwise, let $\mathbf{P}_s(t+1) = \mathbf{P}_s(t)$, $\mathbf{V}_s(t+1) = \mathbf{V}_s(t)$.

Step 4. Let $t = t + 1$. For the s-th particle with known parameters $\mathbf{P}_s(t)$, solve the minimization problem (42). Denote the obtained γ as $\gamma(\mathbf{P}_s(t))$. Update $\bar{\mathbf{P}}_L^{(s)}(t)$ and $\bar{\mathbf{P}}_G(t)$ as follows:
$\bar{\mathbf{P}}_L^{(s)}(t) = \arg\min_{\mathbf{P}_s(\bar{t}),\ 0 \le \bar{t} \le t} \gamma(\mathbf{P}_s(\bar{t}))$,
$\bar{\mathbf{P}}_G(t) \triangleq \arg\min_{\mathbf{P}_s(\bar{t}), 0 \le \bar{t} \le t, 1 \le s \le S} \gamma(\mathbf{P}_s(\bar{t}))$.

Step 5. If $\gamma(\bar{\mathbf{P}}_G(t)) \le \gamma_{\text{acc}}$, then go to *Step 7*.
If $\|\bar{\mathbf{P}}_G(t) - \bar{\mathbf{P}}_G(t-1)\| \le \varepsilon$, then go to *Step 7*.
Otherwise, go to *Step 6*.

Step 6. If $t < t_{\max}$, then go to *Step 2*. Otherwise, Update $\bar{\mathbf{P}}_L^{(s)}(t)$ and $\bar{\mathbf{P}}_G(t)$, and go to *Step 7*.

Step 7. Use $\bar{\mathbf{P}}_G(t_{\max})$ to solve the minimization problem (42). Then, obtain the parameters of the gossip protocol and PIO in terms of $\bar{\mathbf{P}}_G(t_{\max})$ and (40).

Remark 6: This paper presents a methodology for the design of a distributed fuzzy PIO over pull-based gossip-assisted SNs. A transmission model is established by concurrently accounting for the stochastic characteristics of both the gossip protocol and channel fading effects. Utilizing the constrained information available, a modified fuzzy PIO structure is proposed. Sufficient conditions for the design of observer gains are derived by employing inequality techniques and ensuring the H_∞ performance. Moreover, a co-design framework integrating the gossip protocol with the PIO is addressed through a PSO-based iterative algorithm. The T-S fuzzy modeling approach is adopted to handle the inherent nonlinearity of the system. Classical linear H_∞ filtering does not directly apply to nonlinear systems, while the T-S fuzzy framework provides a systematic solution.

Remark 7: The principal merits of the proposed method are threefold. First, the introduced protocol-based PIO features a flexible structure offering increased degrees of freedom, and is able to degenerate into the conventional proportional-type observer under specific configurations [46]. Second, in contrast to certain existing studies [47], [48] where scalar consensus gains are adopted to simplify the analysis, the proposed approach accommodates matrix-form gains, thereby contributing to a reduction in design conservatism. Third, unlike works that employ fixed parameters for the gossip protocol [43], [49], this paper develops an optimization algorithm to systematically design the protocol parameters with respect to a specified performance index.

IV. Simulation Example

A. System Configuration

In this section, we employ an engineering-motivated simulation example to verify the effectiveness of the proposed method. An SN with the pull-type gossip protocol is used to monitor the steering process of a controlled model car, and the schematic diagram is shown in Fig. 1.

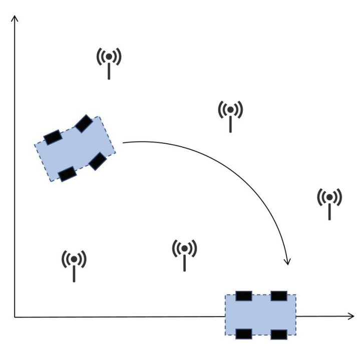

Fig. 1: SN-based state estimation of a model car

The dynamics of the model car subject to disturbances is described as follows [50]:

$$\begin{cases} \phi(k+1) = \phi(k) + \dfrac{d_1 d_2}{d_3} u(k) + 0.01\omega(k) \\ \xi(k+1) = d_1 d_2 \sin(\phi(k)) + \xi(k) + 0.02\omega(k) \end{cases} \tag{46}$$

where $\phi(k)$ is the vehicle's angle relative to the horizontal axis, as shown in Fig. 1; $\xi(k)$ denotes the vertical distance from the rear of the vehicle to the horizontal axis; $u(k)$ is the predefined steering angle; d_1 indicates the vehicle speed; d_2 is the sampling time interval; and d_3 corresponds to the vehicle length.

At the operating points 0° and $\pm 180^\circ$ with model parameters $d_1 = 1$ m/s, $d_2 = 1$ s, and $d_3 = 2.8$ m, the nonlinear system (46) is represented by a two-rule fuzzy model as follows:

$$x(k+1) = \sum_{\tau=1}^{2} s_\tau(x_1(k))\big(A_\tau x(k) + B_\tau u(k) + E_\tau \omega(k)\big)$$

where

$$x(k) \triangleq \begin{bmatrix} x^{(1)}(k) \\ x^{(2)}(k) \end{bmatrix} \triangleq \begin{bmatrix} \phi(k) \\ \xi(k) \end{bmatrix}, \; G_1 = G_2 = \begin{bmatrix} 0.5 & 0.6 \end{bmatrix},$$
$$A_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\ 0.003180 & 1 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0.357143 \\ 0 \end{bmatrix}, \; E_1 = E_2 = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}.$$

The membership functions of the fuzzy model is depicted in Fig. 2

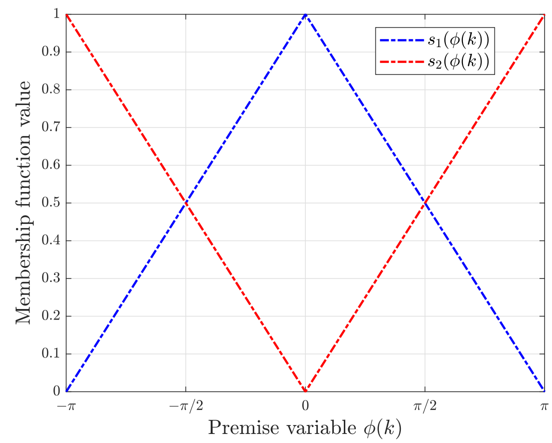

Fig. 2: Membership functions of the model car

In this paper, triangular membership functions are adopted for the T-S fuzzy model. The reasons are threefold. First, triangular functions are computationally efficient due to their linear nature, which is particularly important in our iterative co-design framework. Second, although different from Gaussian functions in mathematical form, triangular functions are fully effective in characterizing the nonlinear behavior of $\sin(\phi(k))$ over the bounded operating range $\phi(k) \in [-\pi, \pi]$, as validated by the satisfactory estimation performance in our simulations. Third, triangular membership functions have been widely used in the T-S fuzzy modeling literature for their simplicity and adequate approximation capability.

It is assumed that an SN with five nodes is implemented to achieve the estimation, whose possible communication topology is displayed in Fig. 3.

Under the pull-type gossip protocol, only one neighbor can be chosen for each node at each sampling instant. The initial values of the matrix Π featuring the protocol behavior are given as follows:

$$\Pi = \begin{bmatrix} 0 & 0.2 & 0.3 & 0 & 0.5 \\ 0.3 & 0 & 0.3 & 0 & 0.4 \\ 0.5 & 0.3 & 0 & 0.2 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.3 & 0.2 & 0 & 0.5 & 0 \end{bmatrix}.$$

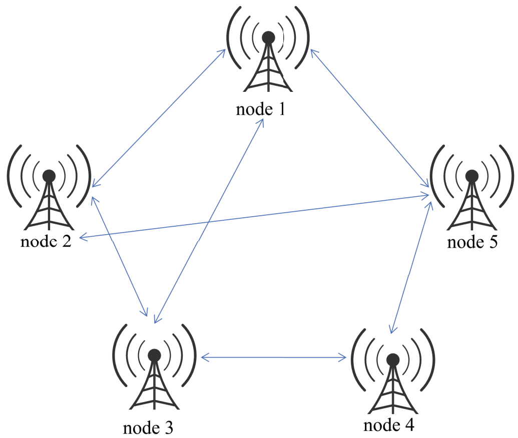

Fig. 3: Communication topology of SNs

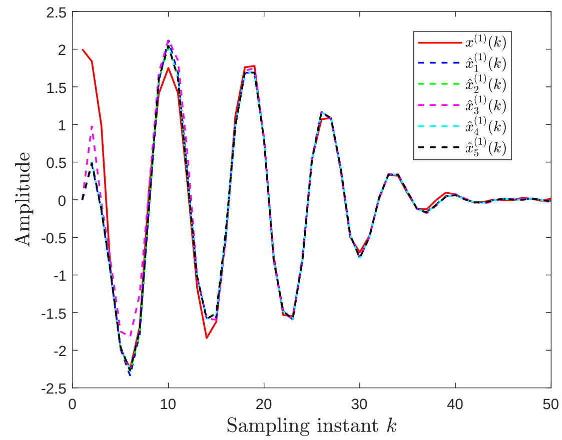

Fig. 4: Estimation of the first element of $x(k)$ in five nodes

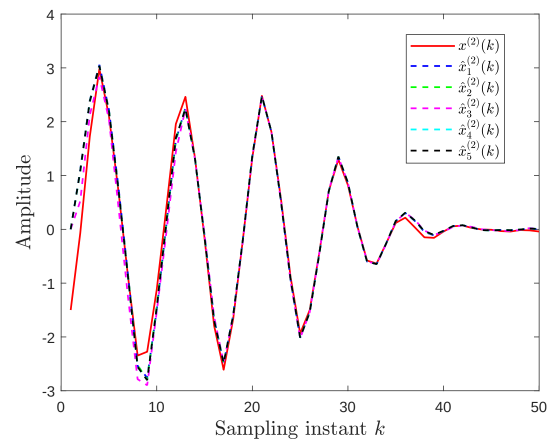

Fig. 5: Estimation of the second element of $x(k)$ in five nodes

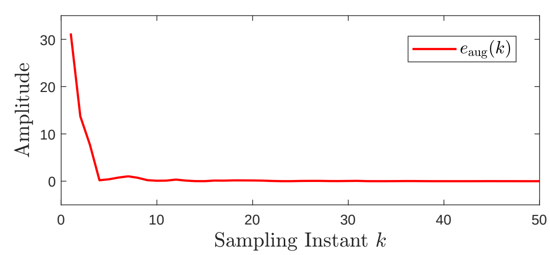

Fig. 6: The evolution of the average estimation error

The parameters of sensors are given as follows:

$$C_1 = \begin{bmatrix} -1 & 0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} -0.6 & 1 \end{bmatrix},\ C_3 = \begin{bmatrix} -0.05 & 0.05 \end{bmatrix},$$
$$C_4 = \begin{bmatrix} -0.4 & 0.5 \end{bmatrix},\ C_5 = \begin{bmatrix} -0.3 & 0.5 \end{bmatrix},\ D_1 = 0.1,$$
$$D_2 = 0.2,\ D_3 = 0.3,\ D_4 = 0.2,\ D_5 = 0.1.$$

The external noises are simulated according to:

$$\omega(k) = 10e^{-0.05k}\breve{\omega}(k)$$

where $\breve{\omega}(k)$ is zero-mean Gaussian white noise. The signal is energy-bounded due to the exponential decay term, which is used to check the performance of the observer.

The measurement noises are assumed to be

$$v_1(k) = \frac{1.3\sin(k)}{k},\ v_2(k) = \frac{1.4\sin(k)}{k},\ v_3(k) = \frac{1.2\cos(k)}{k},$$
$$v_4(k) = \frac{1.1\cos(k)}{k},\ v_5(k) = \frac{1.3\cos(k)}{k}.$$

In this example, the measurement output of each sensor is scalar. Correspondingly, the parameter of the fading channel modeled in (8) is $\Upsilon(k) = \upsilon_1(k)$ with $\bar{\upsilon}_1 = 0.8$ and $\tilde{\upsilon}_{1,1} = 0.01$.

B. Simulation Results and Explanations

Set the simulation length as $k_{\max} = 50$. With the given parameters, conducting Algorithm 1 for $t_{\max} = 20$, the constructed optimization problem is solved, and the protocol parameter after optimization is given as follows:

$$\Pi = \begin{bmatrix} 0 & 0.1856 & 0.2916 & 0 & 0.5228 \\ 0.1916 & 0 & 0.3916 & 0 & 0.4168 \\ 0.2748 & 0.4325 & 0 & 0.2927 & 0 \\ 0 & 0 & 0.4748 & 0 & 0.5252 \\ 0.2916 & 0.2000 & 0 & 0.5084 & 0 \end{bmatrix}.$$

By running distributed PIO (9) with the obtained observer gains, the simulation results are plotted in Fig. 4–7.

Figs. 4–5 present the state estimation results of five nodes for the original states $x^{(1)}(k)$ and $x^{(2)}(k)$. It can be observed that the proposed PIO is capable of rapidly tracking the system states, even under relatively large initial condition discrepancies. Fig. 6 illustrates the augmentation estimation error across the five nodes, defined as

$$e_{\text{aug}}(k) \triangleq \sum_{i=1}^{5} e_i^T(k) e_i(k).$$

This norm-based variable $e_{\text{aug}}(k)$ reflects the overall estimation performance in the presence of energy-bounded disturbances. As shown, during the initial interval $k \in [0, 4]$, $e_{\text{aug}}(k)$ exhibits a relatively large magnitude, followed by a rapid convergence process, which is attributed to the significant disparity between the initial conditions of the system and the observer. After $k = 10$, owing to the corrective feedback provided by both the proportional and integral terms, the

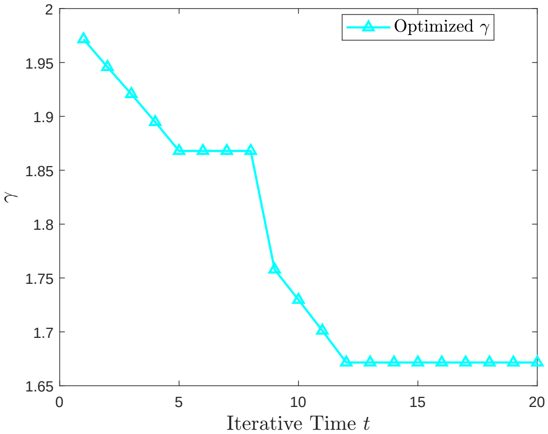

Fig. 7: The optimization of γ under the PSO algorithm

estimation error converges to a small neighborhood of the origin. Fig. 7 depicts the optimization process of parameter γ via a prescribed particle swarm search. The resulting curve can be broadly divided into two phases: a decreasing phase (e.g., $t \in [0, 5]$, $t \in [9, 12]$) and a stable phase (e.g., $t \in [6, 8]$, $t \in [13, 20]$). This behavior ensures that the designed gossip protocol does not degrade estimation performance, thereby validating the effectiveness of the proposed tuning strategy.

C. Comparisons With the Existing Results

To show the advantages of the proposed fuzzy PIO-based method, a series of comparative analyses is conducted in this section under various system configurations. In order to quantify the estimation error over a specified time interval, the following metric is defined:

$$\bar{e}_{\text{sum}} \triangleq \sum_{k=0}^{k_{\max}} \sum_{i=1}^{N} e_i^T(k) e_i(k).$$

A comparison of the cumulative estimation errors under different noises $\omega(k)$ is presented in Table I, evaluating the proposed method against the proportional-type observer (PTO) [51], the consensus-based observer (CBO) [47] and the traditional PIO (TPIO) without PSO-based optimization [27].

TABLE I: Cumulative Estimation Error Under Different Noises

Noise $\omega(k)$	$10e^{-0.05k}\tilde{\omega}(k)$	$\frac{2\cos(k)}{k}$	$12e^{-0.01k}$
$\bar{e}_{\text{sum}}$ (fuzzy PIO)	69.42	61.12	58.32
$\bar{e}_{\text{sum}}$ (PTO)	80.21	69.11	65.51
$\bar{e}_{\text{sum}}$ (CBO)	72.01	65.21	66.13
$\bar{e}_{\text{sum}}$ (TPIO)	90.32	65.25	62.13

From Table I, it can be observed that in the simulation setting, the proposed PIO method achieves the smallest $\bar{e}_{\text{sum}}$ in all three cases, indicating its superior estimation performance over the PTO, CBO and TPIO. This improvement can be attributed to the additional design degrees of freedom introduced in the proposed approach, which contribute to optimizing the disturbance attenuation level γ.

In this example, the third sensor node is configured as a "weak" sensor with the parameter $C_3 = \begin{bmatrix} -0.05 & 0.05 \end{bmatrix}$, meaning that the measurement information from this node is highly limited. To illustrate the effectiveness of the SNs-based estimation method assisted by the pull-based gossip protocol, we compare the cumulative estimation error of node 3 under two scenarios: the traditional measurement setup without inter-node communication, and the SN-based framework incorporating the gossip protocol.

TABLE II: Cumulative Estimation Error of Node 3 Under Different Fading Levels

Fading levels $(1 - \bar{\upsilon}_1)$	0.2	0.3	0.5
$\tilde{e}_{\text{sum}}$ (with SNs)	9.4511	13.2255	17.0633
$\tilde{e}_{\text{sum}}$ (without SNs)	12.0205	14.2512	18.1215

In Table II, the cumulative estimation error of node 3 is defined as

$$\tilde{e}_{\text{sum}} \triangleq \sum_{k=0}^{k_{\max}} e_3^T(k) e_3(k).$$

A smaller value of $\tilde{e}_{\text{sum}}$ indicates better estimation performance. As can be seen from the table, the estimation performance deteriorates as the level of channel fading increases, which is attributed to the poorer transmission quality caused by higher fading levels. On the other hand, under three different fading scenarios, the estimation errors obtained using the SN-based measurement method are consistently smaller than those from the traditional measurement method for sensor node 3. This also reflects the effectiveness of inter-node communication in improving estimation performance.

Fig. 8 shows the feasibility status of the linear matrix inequality (LMI) conditions for different values of γ. It can be observed that for $\gamma \leq 1.6$, the LMIs are infeasible and no feasible solution exists. The threshold reflects the inherent limitation imposed by the system's nonlinearity and network-induced effects. The PSO-based optimization successfully reduces γ from 2.0 to approximately 1.65, which is very close to the theoretical lower bound, demonstrating the effectiveness of the proposed method.

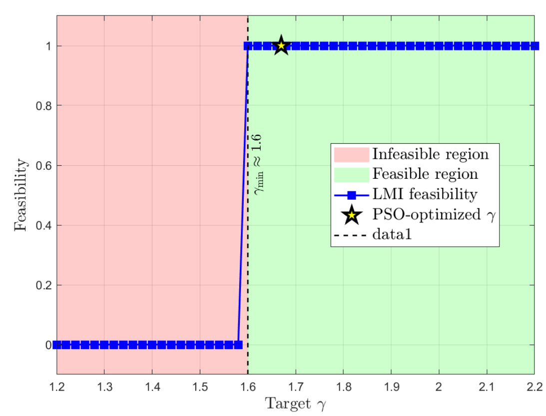

Fig. 8: Sensitivity of LMI feasibility to γ

Remark 8: The proposed co-design method is hybrid in nature, combining LMI-based feasibility checking with PSO-based parameter optimization. Consequently, the achieved H_∞ performance index γ may depend on the chosen PSO parameter settings (e.g., population size, number of iterations, acceleration coefficients). While a fully analytical closed-form solution is difficult to obtain for the considered complex network environment, the hybrid approach consistently yields a smaller γ compared to non-optimized or heuristic parameter selections, as demonstrated in our simulations.

Remark 9: The proposed PSO-based co-design method does not theoretically guarantee global optimality due to the non-convex nature of the problem. Nevertheless, it consistently yields a high-quality solution with improved H_∞ performance compared to baseline methods, as demonstrated in the simulations.

V. Conclusion

In this paper, the distributed fuzzy PIO design problem has been investigated for a class of nonlinear systems implemented over SNs. A pull-based gossip protocol has been adopted to schedule data transmissions among sensor nodes. A concise and efficient model has been introduced to characterize the mechanism by which nodes request data from their neighbors. For state estimation, a generalized PIO has been developed, accounting for the combined influence of the communication protocol and channel fading effects. Within the H_∞ framework and with the aid of specialized inequality techniques, sufficient conditions have been derived to ensure the robustness of the proposed observer. Moreover, a PSO-based algorithm has been developed to facilitate the co-design of the communication protocol and the observers. Finally, an application-oriented example has been provided to demonstrate the effectiveness of the proposed design method.

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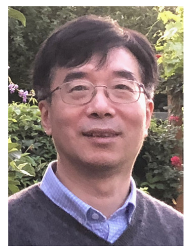

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