

Robust Model Predictive Control for Polytopic Uncertain Systems With Energy Harvesting Sensors Under Round-Robin Protocol

Hongbin Cai, Zidong Wang, Yan Song, and Ping Li

Abstract—This paper addresses the robust model predictive control problem for a class of networked control systems with polytopic uncertainties and hard constraints, where the controller design is complicated by the joint presence of an energy harvesting sensor in the forward channel and the round-robin protocol in the backward channel. In such a setting, stochastic transmission behavior caused by random energy availability, together with fixed communication scheduling and immeasurable states, makes it difficult to guarantee recursive feasibility of the online optimization and mean-square stability of the closed-loop system. To capture these features, the mathematical expectation of a quadratic function depending on both the sensor energy level and the transmission order over an infinite horizon is constructed to formulate the optimization problem. In order to cope with the terminal constraint set and the immeasurability of system states, an auxiliary optimization problem with guaranteed solvability is developed by employing inequality analysis and slack-matrix techniques, through which a sub-optimal solution is obtained. Furthermore, sufficient conditions are derived to ensure the recursive feasibility of the proposed algorithm and the mean-square stability of the resulting closed-loop system with and without hard constraints. Finally, a simulation example is provided to demonstrate the effectiveness of the proposed method.

Index Terms—Robust model predictive control; networked control systems; energy harvesting sensor; round-robin protocol; recursive feasibility; mean-square stability.

I. INTRODUCTION

Model predictive control (MPC) is recognized as a model-based optimal control strategy. Since its inception in the 1970s, MPC has attracted considerable attention in both academia and engineering [6], [29], [35], [36], due primarily to its strong control performance based on optimization and its effective capability of handling hard constraints in practical applications. The working principle of MPC lies in recursively solving an online optimization problem, which is constructed based

on the latest measurement, to determine the current control input [4], [30], [38]. In such recursive online optimization, feasibility has always been a challenging issue, particularly in the presence of model uncertainties. To address this difficulty, substantial efforts have been made, and the so-called robust MPC (RMPC) has been proposed. In RMPC, a “min-max” optimization problem is formulated with explicit consideration of model uncertainties, and a set of solvable inequality conditions is provided to ensure feasibility, robustness, and stability. Consequently, RMPC has received focused attention, and a wide range of results has been reported in the literature, see, e.g. [7], [10], [18], [20].

In recent years, with the widespread adoption of wireless network communication in daily life, networked control systems (NCSs) have attracted considerable attention [1], [8], [14], [15], [17], [25], [43], [44]. As a result, robust model predictive control (RMPC) for NCSs has become a prominent research topic within the MPC community. A major challenge in this area lies in addressing network-induced phenomena caused by the limited bandwidth of networks, such as time delays and packet dropouts, which may compromise the feasibility of the algorithm and degrade system stability. To date, extensive investigations have been conducted, and a substantial body of promising results has been reported [11], [27], [37], [40], [47], [48]. It is noteworthy that most existing RMPC results for NCSs, including the aforementioned works, focus on achieving desirable system performance by developing MPC strategies with enhanced resilience and robustness, rather than modifying the network transmission mechanism. However, an alternative and effective approach to ensuring the stability of NCSs is to enhance transmission reliability through sufficient energy supply and efficient scheduling, and to accordingly design a protocol-related control strategy. This consideration forms the primary motivation of the present paper.

To reduce packet loss during transmission, particularly over long-distance networks, an essential requirement is to ensure that sufficient energy is available in the sensor. On the other hand, as energy is inevitably consumed during data transmission, depletion may occur if a conventional sensor with limited energy capacity is adopted. To prevent energy exhaustion and enable successful transmissions, a new type of sensor with energy-harvesting capability has been proposed [2], [3], [13], [31], [45]. This energy harvesting sensor (EHS) is equipped with both an energy harvester and a storage module. The former collects energy from the external environment, such as through solar panels or windmills, while the latter (typically

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a rechargeable storage device) is used to retain the harvested energy. EHS technology has recently been increasingly applied to NCSs, giving rise to a number of emerging research directions [5], [21], [22], [26], [41], [42]. To the best of the authors' knowledge, the control problem for NCSs equipped with EHS remains in its early stages; in particular, the MPC problem has received very limited attention. This is largely because the transmission behavior introduced by EHS may compromise the recursive feasibility of the optimization process in MPC.

Another effective approach to improving network transmission reliability is to employ a carefully designed scheduling scheme to regulate the transmission order. Among the commonly used communication protocols, the so-called round-robin (RR) protocol has found wide application due to its simple implementation and favorable reliability. Under the RR protocol, each node is granted equitable access to the shared communication channel according to a fixed circular order [9], [12], [16], [23], [24]. When addressing the MPC problem for NCSs governed by the RR protocol, a significant challenge arises from the fixed scheduling. In particular, the variation in data transmission order may adversely affect the recursive feasibility of the algorithm. To overcome this issue, special efforts have been made by several researchers [39], [46]. However, most of these contributions are based on state-feedback control and are not well-suited to the RMPC problem for NCSs under the RR protocol with immeasurable states. Moreover, the investigation of the RMPC problem for NCSs incorporating both RR protocol and EHS remains at the periphery, which is because, in addition to transmission variations, the randomness introduced by the EHS further complicates the guarantee of recursive feasibility. TABLE I depicts the contributions and limitations of published papers of the RMPC problem for NCSs.

TABLE I: The contributions and limitations of published papers of the RMPC problem for NCSs

Study Content	[34]	[37]	[39]	[40]	[46]	[48]
EHS	×	×	×	×	×	×
RR protocol	×	×	✓	×	✓	×
Uncertainty	×	×	✓	✓	✓	✓
Output feedback	×	×	✓	✓	×	✓
Constraints	✓	✓	✓	✓	✓	✓

The present paper is motivated by the need to improve transmission reliability in NCSs from the sensing and communication mechanism itself, rather than solely enhancing robustness at the controller-design level. In the considered setting, the joint presence of the EHS and the RR protocol introduces stochastic transmission behavior and fixed scheduling effects, which, together with polytopic uncertainty, hard constraints, and immeasurable states, render the RMPC design significantly more involved. Compared with the existing RMPC literature, the main distinction of this paper lies in the unified treatment of polytopic uncertainty, hard constraints, stochastic transmission behavior caused by the EHS, and fixed communication scheduling induced by the RR protocol within an output-feedback framework. Such a combination has made the recursive-feasibility and mean-square-stability analysis considerably more involved than in the existing related

studies. For this reason, a new expectation-based performance formulation and a tractable auxiliary optimization framework are developed in this paper. To achieve this objective, the following three substantial difficulties must be addressed:

- 1) How to guarantee the recursive feasibility of the optimization problems over a moving horizon under the presence of the EHS and RR protocol?
- 2) How to effectively handle the non-convex obstacles arising from RMPC particularly for systems with EHS and immeasurable states?
- 3) How to ensure the mean-square stability of the system when transmitted information may be lost due to the randomness in energy absorption by the EHS?

With respect to the above identified difficulties, the primary contributions of this paper are highlighted as follows.

- 1) To the best of the authors' knowledge, one of the first few attempts is made to address an RMPC problem for NCSs with polytopic uncertainties and hard constraints, subject to both an EHS and RR scheduling.
- 2) To accurately capture the randomness of the EHS, the scheduling characteristics of the RR protocol, and the polytopic structure of parameter uncertainties, the mathematical expectation of a quadratic function (dependent on the sensor's energy level and the transmission order over the infinite time horizon) is constructed to formulate the optimization problem, and this contributes significantly to ensuring recursive feasibility.
- 3) To overcome the challenges posed by the terminal constraint set and the immeasurability of system states, the inequality analysis technique and slack matrices are utilized, and an auxiliary optimization problem with guaranteed solvability is proposed to obtain a sub-optimal solution, which supports the mean-square stability of the underlying system.

The scope of this paper is the RMPC design for a class of polytopic uncertain NCSs with hard state/input constraints, where stochastic sensing behavior caused by the EHS and scheduled communication induced by the RR protocol must be simultaneously taken into account. The potential of the proposed framework lies in its applicability to resource-constrained networked control environments, especially those in which transmission reliability, limited sensing energy, and communication scheduling must be jointly addressed. Moreover, the developed analysis and design ideas may provide a useful basis for future extensions to more complex settings, such as multisensor energy-harvesting systems and broader wireless industrial control applications.

The structure of this paper is organized as follows. Section II presents the problem formulation, including the modeling of the EHS and the RR protocol. In Section III, a detailed analysis is conducted on the stability of the proposed RMPC, and the gain parameters are determined by solving constrained optimization problems. Section IV provides one simulation example to verify the effectiveness and advantages of the proposed RMPC approach. Finally, the conclusions of this research are summarized in Section V. Before proceeding, the notations used throughout the paper are listed in TABLE II.

TABLE II: Notations

Symbols	Descriptions
P^{-1}	the inverse of P
P^T	the transpose of P
$\text{diag}(\dots)$	diagonal matrix
\mathbb{R}^n	n dimensional Euclidean space
0 and I	zero matrix and identity matrix
$\mathbb{R}^{n \times m}$	the set of all $n \times m$ real matrices
*	the symmetric term in a symmetric matrix
$P > 0$	symmetric and positive definite matrix
$\ x\ $	Euclidean norm of a vector x
\mathcal{E}	mathematical expectation
$\text{Prob}\{\star\}$	the occurrence probability of the event " \star "

II. PROBLEM FORMULATION

A. System description

Consider a class of linear discrete-time systems described as

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = C(k)x(k), \end{cases} \quad (1)$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$ and $y(k) \in \mathbb{R}^{n_y}$ denote the system state, control input, and system output, respectively. The matrices $A(k)$, $B(k)$, $C(k)$ are known and of compatible dimensions, satisfying

$$[A(k)|B(k)|C(k)] \in \Omega,$$

where the set $\Omega \triangleq \text{Co}\{[A^1|B^1|C^1], \dots, [A^L|B^L|C^L]\}$ with "Co $\{\cdot\}$ " denoting a convex hull. In this sense, there exists a group of non-negative parameters $\nu_l(k) \in [0, 1]$, $l = 1, \dots, L$ such that $\sum_{l=1}^L \nu_l(k) = 1$ and

$$[A(k)|B(k)|C(k)] = \sum_{l=1}^L \nu_l(k)[A^l|B^l|C^l]. \quad (2)$$

To better reflect practical requirements, the following hard constraints are imposed in the controller design:

$$\begin{cases} \max_e |[u(k)]_e| \leq \bar{u}, e \in \{1, \dots, n_u\} \\ \max_f |[x(k)]_f| \leq \bar{x}, f \in \{1, \dots, n_x\} \end{cases} \quad (3)$$

where $\bar{u} > 0$ and $\bar{x} > 0$ are known scalars determined from practice, and $[\star]_{\bar{i}}$ ($\bar{i} \in \{e, f\}$) represents the \bar{i} th element of the vector " \star ".

Since a remote controller is considered in this paper, energy consumption in the sensor occurs during the data transmission from the sensor to the controller. As illustrated in Fig. 1, the sensor is equipped with an energy storage module and is capable of harvesting energy from the surrounding environment, such as through solar panels. If sufficient energy is available, the transmission can be carried out normally; otherwise, communication is disrupted, and the sensor data is regarded as missing. Furthermore, to improve the reliability of transmitting the control signal from the controller to the actuator, a widely used and effective communication protocol, namely, the RR protocol, is adopted to coordinate the transmission order. The operating principles of both the EHS and the RR protocol will be detailed in the subsequent section.

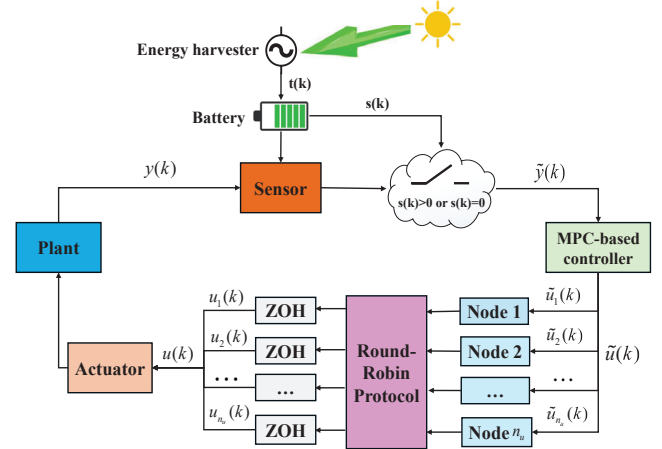


Fig. 1: The sketch of the MPC-based system with an EHS and RR protocol.

B. Energy harvesting sensor

As mentioned above, a new type of sensor with energy harvesting capability is employed to support signal propagation. Letting Ξ denote the maximum number of energy units that can be stored in the sensor, the energy level of the energy harvesting sensor at time instant k , denoted as $s(k)$, satisfies $s(k) \in \mathbb{S} \triangleq \{0, 1, 2, \dots, \Xi\}$. Meanwhile, the amount of energy harvested by the sensor at time instant k is represented by $t(k)$, which is modeled as an independent and identically distributed stochastic process with a probability distribution given by

$$\text{Prob}(t(k) = \psi) = p_\psi, \psi = 0, 1, 2, \dots \quad (4)$$

where p_ψ satisfies $0 \leq p_\psi \leq 1$ and $\sum_{\psi=0}^{\infty} p_\psi = 1$.

Without loss of generality, the following assumption regarding energy consumption and storage is introduced for subsequent analysis.

Assumption 1: At each time instant k , the sensor data can be transmitted to the remote controller if the stored energy is nonzero. One unit of energy is consumed if and only if such a transmission is successfully completed. In addition, any energy exceeding the sensor's storage capacity is arbitrarily discarded.

Under Assumption 1, the dynamics of the energy level $s(k+1)$ can be formulated as

$$s(k+1) = \min \{s(k) + t(k) - \mathbf{1}_{\{s(k)>0\}}, \Xi\}, \quad (5)$$

where $s(0) \in \mathbb{S}$ is the initial energy level, and $\mathbf{1}_{\{s(k)>0\}}$ denotes the indicator function, which equals 1 if the condition " $s(k) > 0$ " holds, and 0 otherwise.

Based on (5), the data received by the controller can be described by

$$\tilde{y}(k) = \mathbf{1}_{\{s(k)>0\}} y(k). \quad (6)$$

Remark 1: It is worth mentioning that, at time instant k , the energy collected by the harvester is random, and it is possible that $t(k) = 0$. Meanwhile, the sensor consumes one unit of energy to transmit data whenever $s(k) > 0$. According to the above discussion, there exist time instants at which $s(k) = 0$ may occur. In such cases, the rechargeable device is deemed

to lack sufficient energy to power the sensor, resulting in measurement loss due to energy insufficiency.

Since the energy level $s(k)$ is time-varying, the value of $\mathbf{1}_{\{s(k)>0\}}$ is unknown to the controller. Therefore, the expectation $\mathcal{E}\{\mathbf{1}_{\{s(k)>0\}}\} = \mu(k)$ is introduced to represent the expected transmission status for the remote sensor. It is straightforward to obtain

$$\begin{aligned} \mathcal{E}\{\mathbf{1}_{\{s(k)>0\}} - \mu(k)\} &= 0, \\ \mathcal{E}\{\mathbf{1}_{\{s(k)>0\}} - \mu(k)\}^2 &= \mu(k)(1 - \mu(k)) \triangleq \zeta^2(k). \end{aligned} \quad (7)$$

C. Round-Robin Protocol

As illustrated in Fig. 1, after the implementation of the MPC-based control strategy, the controller outputs are transmitted to the actuator via a long-distance and shared communication network. To mitigate data collisions and ensure the desired system performance, a widely used communication protocol, namely, the RR protocol, is employed to manage the transmission sequence. According to the RR protocol, at each transmission instant, only one node that holds the token is granted access to the network and allowed to transmit data, while the other nodes retain their most recent control values through the use of a zero-order holder (ZOH).

For the control input $u(k) \in \mathbb{R}^{n_u}$, let $q(k) \in \mathbb{Q} \triangleq \{1, 2, \dots, n_u\}$ denote the active controller node at time instant k under the RR protocol, which is determined by

$$q(k) \triangleq \text{mod}(k - 1, n_u) + 1. \quad (8)$$

Define $\tilde{u}(k) \triangleq [\tilde{u}_1^T(k) \ \tilde{u}_2^T(k) \ \dots \ \tilde{u}_{n_u}^T(k)]^T$ and $u(k) \triangleq [u_1^T(k) \ u_2^T(k) \ \dots \ u_{n_u}^T(k)]^T$, where $\tilde{u}_e \in \mathbb{R}$ and $u_e \in \mathbb{R}$ represent the pre-transmission and post-transmission control signals at time k under the RR protocol, respectively. Then, the j th component of $u(k)$, i.e., $u_j(k)$, is expressed as

$$u_j(k) = \begin{cases} \tilde{u}_j(k), & j = q(k) \\ u_j(k-1), & \text{otherwise.} \end{cases} \quad (9)$$

Next, the updated matrix is defined as $\Upsilon_{q(k)} \triangleq \text{diag}\{\delta(q(k)-1), \delta(q(k)-2), \dots, \delta(q(k)-n_u)\}$ where $\delta(\cdot) \in \{0, 1\}$ denotes the Kronecker delta function. Based on (9), the signal received by the actuator can then be expressed as

$$u(k) = \Upsilon_{q(k)} \tilde{u}(k) + \tilde{\Upsilon}_{q(k)} u(k-1), \quad (10)$$

where $\tilde{\Upsilon}_{q(k)} \triangleq I - \Upsilon_{q(k)}$.

Remark 2: Although transmission-order management can often be coordinated at higher communication layers in general networked systems, in the present study the RR protocol is introduced as a control-relevant scheduling mechanism rather than a purely communication-level detail. In the considered resource-constrained NCS, the transmission order determines the actual actuator update sequence and therefore directly affects the closed-loop control behavior. For this reason, its effect has been explicitly incorporated into the RMPC formulation and analysis.

D. Control law and optimization problem

1) *Control law:* To address the issue of immeasurable states, a dynamic output feedback controller within the MPC framework is adopted, which is computed as follows:

$$\begin{cases} \hat{x}(k+i+1|k) = \check{A}_{s(k+i|k), q(k+i|k)}(k) \hat{x}(k+i|k) \\ \quad + \check{B}_{s(k+i|k), q(k+i|k)}(k) \check{y}(k+i|k) \\ \tilde{u}(k+i|k) = \check{K}_{s(k+i|k), q(k+i|k)}(k) \hat{x}(k+i|k) \end{cases} \quad (11)$$

where $\star(k+i|k)$, for $i \in \mathcal{Y} \triangleq \{0, 1, 2, \dots\}$, denotes the predicted value at time $k+i$ based on available data at time k , with $\star(k|k) = \star(k)$. The matrices $\check{A}_{s(\cdot), q(\cdot)}(k)$, $\check{B}_{s(\cdot), q(\cdot)}(k)$ and $\check{K}_{s(\cdot), q(\cdot)}(k)$ are decision variables to be designed.

For notational simplicity in the subsequent analysis, the following shorthand is introduced: $q(k+i|k) \triangleq r$, $q(k+i+1|k) \triangleq g$, $s(k+i|k) \triangleq m$, $s(k+i+1|k) \triangleq n$, $\check{A}_{s(\cdot), q(\cdot)}(k) \triangleq \check{A}_{s(\cdot), q(\cdot)}$, $\check{B}_{s(\cdot), q(\cdot)}(k) \triangleq \check{B}_{s(\cdot), q(\cdot)}$, and $\check{K}_{s(\cdot), q(\cdot)}(k) \triangleq \check{K}_{s(\cdot), q(\cdot)}$.

By substituting (11) into (1) and defining $\xi(k+i|k) \triangleq [x^T(k+i|k) \ \hat{x}^T(k+i|k) \ u^T(k+i-1|k)]^T$, the following compact system representation is obtained:

$$\xi(k+i+1|k) = \bar{A}_{m,r}(k+i) \xi(k+i|k), \quad (12)$$

where

$$\bar{A}_{m,r}(k+i) \triangleq \begin{bmatrix} A_{k,i} & B_{k,i} \Upsilon_r \check{K}_{m,r} & B_{k,i} \tilde{\Upsilon}_r \\ \check{B}_{m,r} \mathbf{1}_{\{m>0\}} C_{k,i} & \check{A}_{m,r} & 0 \\ 0 & \Upsilon_r \check{K}_{m,r} & \tilde{\Upsilon}_r \end{bmatrix}$$

with $A_{k,i} \triangleq A(k+i)$, $B_{k,i} \triangleq B(k+i)$, $C_{k,i} \triangleq C(k+i)$. Furthermore, (12) can be rewritten by

$$\begin{aligned} \xi(k+i+1|k) &= \bar{A}_{m,r}(k+i) \xi(k+i|k) + \bar{B}_{m,r}(k+i) \\ &\quad \times (\mathbf{1}_{\{m>0\}} - \bar{\mu}_m) \xi(k+i|k), \end{aligned} \quad (13)$$

where

$$\bar{A}_{m,r}(k+i) \triangleq \begin{bmatrix} A_{k,i} & B_{k,i} \Upsilon_r \check{K}_{m,r} & B_{k,i} \tilde{\Upsilon}_r \\ \check{B}_{m,r} \bar{\mu}_m C_{k,i} & \check{A}_{m,r} & 0 \\ 0 & \Upsilon_r \check{K}_{m,r} & \tilde{\Upsilon}_r \end{bmatrix}$$

and

$$\bar{B}_{m,r}(k+i) \triangleq \begin{bmatrix} 0 & 0 & 0 \\ \check{B}_{m,r} C_{k,i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $A_{k,i}$, $B_{k,i}$ and $C_{k,i}$ are defined as before.

From (7), it is observed that $\mu(k+i|k)$ depends explicitly on $s(k+i|k)$. Therefore, it is denoted as $\mu(k+i|k) = \bar{\mu}_m$. In addition, the initial value at $k=0$ is denoted as $\xi(0) \triangleq [x^T(0) \ \hat{x}^T(0) \ u^T(-1)]^T$.

2) *A min-max optimization problem:* To design the controller for the parameter-uncertain system (12) with an EHS and the RR protocol within the MPC framework, a ‘‘min-max’’ optimization problem over an infinite time horizon is formulated based on the mathematical expectation:

$$\min_{\check{A}_{m,r}, \check{B}_{m,r}, \check{K}_{m,r}} \max_{\{A_{k,i}, B_{k,i}, C_{k,i}\} \in \Omega} J_\infty(k), \quad (14)$$

where $J_\infty(k) \triangleq \mathcal{E}\{\sum_{i=0}^{\infty} [\xi^T(k+i|k) Q \xi(k+i|k) + \tilde{u}^T(k+i|k) R \tilde{u}(k+i|k)]\}$, $Q = \text{diag}\{Q_1, Q_2, Q_3\}$ and R are known symmetric and positive-definite weighting matrices.

In accordance with (14), the following online optimization problem, denoted as OP1, is proposed for controller synthesis:

$$\text{OP1: } \begin{cases} \min_{\tilde{A}_{m,r}, \tilde{B}_{m,r}, \tilde{K}_{m,r}} \max_{\{A_{k,i}, B_{k,i}, C_{k,i}\} \in \Omega} J_\infty(k) \\ \text{s.t. } \begin{cases} (12), (3), \text{ and,} \\ \xi(k+i|k) \in \Psi_k, i \in \mathcal{Y} \end{cases} \end{cases} \quad (15)$$

It is worth noting that Ψ_k in (15) denotes the time-varying terminal constraint set (TCS) [34], defined as

$$\Psi_k \triangleq \{\xi(k+i|k) | \xi^T(k+i|k)P_{m,r}\xi(k+i|k) \leq 3\lambda\}, \quad (16)$$

where $P_{m,r} > 0$ and $\lambda > 0$ are both time-varying, with the dependence on k omitted for simplicity. Their detailed construction and associated analysis will be presented in the next section.

Remark 3: The OP1 formulates an online optimization problem for controller design, which corresponds to a one-mode control strategy. That is, OP1 must be solved at every time step over a moving prediction horizon. Although a sequence of predicted control inputs $u(k), u(k+1|k), \dots$ is generated at each time instant, only the first input $u(k)$ is applied to the system. This one-mode MPC approach may result in a relatively small initial feasible region, yet it ensures desirable performance, including system stability and convergence rate (particularly in complex scenarios involving both the EHS and the RR protocol). Furthermore, to address the randomness introduced by the EHS and RR protocol, the objective function is constructed based on the mathematical expectation. In this manner, the mean-square stability of the closed-loop system will be investigated in the subsequent analysis.

E. Preliminaries

Definition 1: [34] (Positive control invariant set) For system (1) under the output feedback control law (11), the set Ψ_k is said to be a positive control invariant set if, for any $\xi(k) \in \Psi_k$, there exist admissible control inputs such that $\xi(k+i|k) \in \Psi_k, i \in \mathcal{Y}$.

Definition 2: [28] System (1) under the output feedback control law (11) is said to be mean square (MS) stable if

$$\lim_{k \rightarrow \infty} \mathcal{E}_{\xi(0)} \{\|\xi(k)\|^2\} = 0$$

holds for any initially feasible condition $\xi(0)$.

In this paper, the objective is to design a series of controllers within the framework of RMPC such that the augmented system (12), subject to an EHS and the RR protocol, is MS stable. To accomplish this goal, the following two requirements must be satisfied:

- 1) $\mathcal{R}1$: Due to the potential insolvability of OP1 caused by the TCS condition (16) and the cost function $J_\infty(k)$ (14), an auxiliary optimization problem must be formulated to compute the controller gains $\tilde{A}_{m,r}, \tilde{B}_{m,r}, \tilde{K}_{m,r}$.
- 2) $\mathcal{R}2$: Sufficient conditions must be derived to guarantee the MS stability of the augmented system (12) under the influence of both the EHS and the RR protocol.

III. MAIN RESULT

To make the methodological development easier to follow, the proposed research procedure is briefly summarized here. i) First, the original RMPC problem is formulated for the considered NCS under the simultaneous effects of the EHS and the RR protocol. Since the original optimization problem is generally difficult to solve directly, a tractable auxiliary problem is then constructed by deriving an upper bound on the performance index. ii) Based on this formulation, sufficient conditions are established to guarantee recursive feasibility and mean-square stability. iii) The analysis is then extended to the constrained case by explicitly incorporating the hard state and input constraints. iv) Finally, the complete online implementation procedure is summarized in Algorithm 1.

A. Unconstrained RMPC with an EHS and RR protocol

This subsection focuses on formulating an auxiliary optimization problem to replace the unconstrained version of OP1 (i.e., OP1 without the hard constraints in (3)) to ensure solvability. To this end, the terminal constraint set condition (16) and the infinite-horizon cost function $J_\infty(k)$ must be addressed separately.

1) *Terminal constraint set:* According to [34], the set Ψ_k qualifies as a TCS for the augmented system (12) in the mean-square sense if the following two conditions are satisfied.

Condition C1: Define a quadratic function as

$$V(\xi(k+i|k)) \triangleq \xi^T(k+i|k)P_{m,r}\xi(k+i|k), \quad (17)$$

where $P_{m,r} = \text{diag}\{P_{m1,r1}, P_{m2,r2}, P_{m3,r3}\}$ consists of positive-definite and symmetric matrices to be designed. Then, the mathematical expectation of the difference in $V(\xi(k+i|k))$ on i must satisfy

$$\begin{aligned} \mathcal{E}\{\Delta V(\xi(k+i|k))\} &= \mathcal{E}\{V(\xi(k+i+1|k))\} - V(\xi(k+i|k)) \\ &\leq -\xi^T(k+i|k)Q\xi(k+i|k) - \tilde{u}^T(k+i|k)R\tilde{u}(k+i|k). \end{aligned} \quad (18)$$

Condition C2: The set Ψ_k defined in (16) is a positive control invariant set.

Next, conditions C1 and C2 for the TCS are addressed in turn.

i) *C1 of TCS:* Before proceeding, an essential lemma is introduced to describe the expectation of measurement transmission under the EHS. This result follows directly from the approach used in [32].

Lemma 1: [32] For the energy level $s(k)$ given by (5), the recursion of the probability distribution $\sigma(k) = [\text{Prob}(s(k) = 0) \text{Prob}(s(k) = 1) \dots \text{Prob}(s(k) = \Xi)]^T$ is formulated by

$$\begin{cases} \sigma(k+1) = b + \Pi\sigma(k), \\ \sigma(0) = \underbrace{[0 \dots 0]}_{s(0)} \mathbf{1} \underbrace{[0 \dots 0]}_{\Xi-s(0)} \end{cases} \quad (19)$$

where $b = \underbrace{[0 \ \cdots \ 0]^\top}_{\Xi} 1^\top$ and

$$\Pi = \begin{bmatrix} p_0 & p_0 & 0 & \cdots & 0 \\ p_1 & p_1 & p_0 & \cdots & 0 \\ p_2 & p_2 & p_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{\Xi-1} & p_{\Xi-1} & p_{\Xi-2} & \cdots & p_0 \\ -\sum_{\psi=0}^{\Xi-1} p_\psi & -\sum_{\psi=0}^{\Xi-1} p_\psi & -\sum_{\psi=0}^{\Xi-2} p_\psi & \cdots & -p_0 \end{bmatrix}.$$

Then, from the recursion of $\sigma(k)$, the expectation of the measurement transmission can be obtained by

$$\mu(k) = [0 \ \underbrace{1 \ \cdots \ 1}_{\Xi}] \sigma(k). \quad (20)$$

For the subsequent analysis, a matrix $G \triangleq [(C^l)^\top(C^l(C^l)^\top)^{-1} \ (C^l)^\perp]$ is defined, where $(C^l)^\perp$ denotes the orthogonal basis of the null space for $C(k)$. Then, the following slack matrices are introduced by

$$W = \begin{bmatrix} W_{11} & 0 \\ 0 & W_{22} \end{bmatrix}, \quad (21)$$

where W_{11} and W_{22} are a diagonal matrix and an arbitrary matrix with appropriate dimensions, respectively.

Now, to guarantee *Condition C1*, the following lemma is given.

Lemma 2: Let matrices $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R > 0$ be given. If there are symmetric positive-definite matrices $X_{m1,r1}$, $X_{m2,r2}$, $X_{m3,r3}$, $X_{n1,g1}$, $X_{n2,g2}$, $X_{n3,g3}$, matrices W , $\mathcal{A}_{m,r}$, $\mathcal{B}_{m,r}$, $\mathcal{F}_{m,r}$, and a scalar $\lambda > 0$, for any $\{r, g\} \in \mathbb{Q} \times \mathbb{Q}$, any $\{m, n\} \in \mathbb{S} \times \mathbb{S}$, and $l = 1, \dots, L$ such that

$$\begin{bmatrix} \hat{X}_{m,r} & * & * & * & * \\ \hat{A}_{m,r}^l & X_{n,g} & * & * & * \\ \bar{\zeta}_m \hat{B}_{m,r}^l & 0 & X_{n,g} & * & * \\ \hat{Q} & 0 & 0 & \lambda I & * \\ \hat{Y} & 0 & 0 & 0 & \lambda I \end{bmatrix} \geq 0, \quad (22)$$

where $\hat{X}_{m,r} = \text{diag}\{\Delta_{11}, X_{m2,r2}, \Delta_{33}\}$, $X_{n,g} = \text{diag}\{X_{n1,g1}, X_{n2,g2}, X_{n3,g3}\}$, $\hat{Q} = \text{diag}\{Q_1^{\frac{1}{2}}GW, Q_2^{\frac{1}{2}}X_{m2,r2}, Q_3^{\frac{1}{2}}W_{11}\}$, $\Delta_{11} = (GW)^\top + (GW) - X_{m1,r1}$, $\Delta_{33} = W_{11}^\top + W_{11} - X_{m3,r3}$, $\hat{Y} = [0 \ R^{\frac{1}{2}}\mathcal{F}_{m,r} \ 0]$, $\hat{\mathcal{B}}_{m,r} = [\mathcal{B}_{m,r} \ 0]$, $\bar{\zeta}_m = \zeta(k+i|k)$, $\mathcal{F}_{m,r} = K_{m,r}X_{m2,r2}$, $\mathcal{A}_{m,r} = \hat{A}_{m,r}X_{m2,r2}$, $\mathcal{B}_{m,r} = \hat{B}_{m,r}W_{11}$, and

$$\hat{A}_{m,r}^l = \begin{bmatrix} A^lGW & B^l\Upsilon_r\mathcal{F}_{m,r} & B^l\tilde{\Upsilon}_rW_{11} \\ \hat{B}_{m,r}^l\bar{\mu}_m & \mathcal{A}_{m,r} & 0 \\ 0 & \Upsilon_r\mathcal{F}_{m,r} & \tilde{\Upsilon}_rW_{11} \end{bmatrix},$$

$$\hat{B}_{m,r}^l = \begin{bmatrix} 0 & 0 & 0 \\ \hat{B}_{m,r}^l & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

then *Condition C1* holds.

Proof: Based on the quadratic function (17), taking the mathematical expectation of the difference in $V(\xi(k+i|k))$ along the system (13) yields

$$\begin{aligned} & \mathcal{E}\{\Delta V(\xi(k+i|k))\} \\ &= \mathcal{E}\{[\bar{A}_{m,r}(k+i)\xi(k+i|k) + \bar{B}_{m,r}(k+i) \\ & \times (\mathbf{1}_{\{m>0\}} - \bar{\mu}_m)\xi(k+i|k)]^\top P_{n,g}[\bar{A}_{m,r}(k+i) \\ & \times \xi(k+i|k) + \bar{B}_{m,r}(k+i)(\mathbf{1}_{\{m>0\}} - \bar{\mu}_m) \\ & \times \xi(k+i|k)]\} - \xi^\top(k+i|k)P_{m,r}\xi(k+i|k). \end{aligned} \quad (23)$$

Substituting (7) into (23) gives

$$\begin{aligned} & \mathcal{E}\{\Delta V(\xi(k+i|k))\} \\ &= \xi^\top(k+i|k)\{\bar{A}_{m,r}^\top(k+i)P_{n,g}\bar{A}_{m,r}(k+i) + \bar{\zeta}_m^2 \\ & \times \bar{B}_{m,r}^\top(k+i)P_{n,g}\bar{B}_{m,r}(k+i) - P_{m,r}\}\xi(k+i|k). \end{aligned} \quad (24)$$

Using a matrix analysis technique, the following inequalities are obtained:

$$\begin{aligned} (GW)^\top X_{m1,r1}^{-1}GW &\geq (GW)^\top + GW - X_{m1,r1}, \\ W_{11}^\top X_{m3,r3}^{-1}W_{11} &\geq W_{11}^\top + W_{11} - X_{m3,r3}. \end{aligned} \quad (25)$$

Based on (25), it can be inferred from (22) that

$$\begin{bmatrix} \tilde{X}_{m,r} & * & * & * & * \\ \tilde{A}_{m,r}^l & X_{n,g} & * & * & * \\ \bar{\zeta}_m \tilde{B}_{m,r}^l & 0 & X_{n,g} & * & * \\ \tilde{Q} & 0 & 0 & \lambda I & * \\ \tilde{Y} & 0 & 0 & 0 & \lambda I \end{bmatrix} \geq 0, \quad (26)$$

where

$$\begin{aligned} \tilde{A}_{m,r}^l &= \begin{bmatrix} A^lGW & B^l\Upsilon_r\mathcal{F}_{m,r} & B^l\tilde{\Upsilon}_rW_{11} \\ \tilde{B}_{m,r}^l\bar{\mu}_m C^lGW & \mathcal{A}_{m,r} & 0 \\ 0 & \Upsilon_r\mathcal{F}_{m,r} & \tilde{\Upsilon}_rW_{11} \end{bmatrix}, \\ \tilde{B}_{m,r}^l &= \begin{bmatrix} 0 & 0 & 0 \\ \tilde{B}_{m,r}^l C^lGW & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \tilde{X}_{m,r} &= \begin{bmatrix} (GW)^\top X_{m1,r1}^{-1}GW & 0 & 0 \\ 0 & X_{m2,r2} & 0 \\ 0 & 0 & W_{11}^\top X_{m3,r3}^{-1}W_{11} \end{bmatrix} \end{aligned}$$

By pre- and post-multiplying (26) with $\text{diag}\{(GW)^{-1}, I, W_{11}^{-1}, \underbrace{I, \dots, I}_{10}\}$ and its transpose, the

following is obtained:

$$\begin{bmatrix} \vec{X}_{m,r} & * & * & * & * \\ \vec{A}_{m,r}^l & X_{n,g} & * & * & * \\ \bar{\zeta}_m \vec{B}_{m,r}^l & 0 & X_{n,g} & * & * \\ \vec{Q} & 0 & 0 & \lambda I & * \\ \vec{Y} & 0 & 0 & 0 & \lambda I \end{bmatrix} \geq 0, \quad (27)$$

where $\vec{Q} = \text{diag}\{Q_1^{\frac{1}{2}}, Q_2^{\frac{1}{2}}X_{m2,r2}, Q_3^{\frac{1}{2}}\}$, $\vec{X}_{m,r} = \text{diag}\{X_{m1,r1}^{-1}, X_{m2,r2}, X_{m3,r3}^{-1}\}$, and

$$\begin{aligned} \vec{A}_{m,r}^l &= \begin{bmatrix} A^l & B^l\Upsilon_r\mathcal{F}_{m,r} & B^l\tilde{\Upsilon}_r \\ \tilde{B}_{m,r}^l\bar{\mu}_m C^l & \mathcal{A}_{m,r} & 0 \\ 0 & \Upsilon_r\mathcal{F}_{m,r} & \tilde{\Upsilon}_r \end{bmatrix}, \\ \vec{B}_{m,r}^l &= \begin{bmatrix} 0 & 0 & 0 \\ \tilde{B}_{m,r}^l C^l & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

By further pre- and post-multiplying (27) with $\text{diag}\{I, X_{m2,r2}^{-1}, \underbrace{I, \dots, I}_{11}\}$ and its transpose, we have

$$\begin{bmatrix} X_{m,r}^{-1} & * & * & * & * \\ \bar{A}_{m,r}^l & X_{n,g} & * & * & * \\ \bar{\zeta}_m \bar{B}_{m,r}^l & 0 & X_{n,g} & * & * \\ \bar{Q} & 0 & 0 & \lambda I & * \\ \bar{Y} & 0 & 0 & 0 & \lambda I \end{bmatrix} \geq 0, \quad (28)$$

where $X_{m,r}^{-1} = \text{diag}\{X_{m1,r1}^{-1}, X_{m2,r2}^{-1}, X_{m3,r3}^{-1}\}$, $\bar{Q} = \text{diag}\{Q_1^{\frac{1}{2}}, Q_2^{\frac{1}{2}}, Q_3^{\frac{1}{2}}\}$, $\bar{Y} = [0 \ R^{\frac{1}{2}} \tilde{K}_{m,r} \ 0]$ with $\tilde{K}_{m,r}$ denoted in (51), and

$$\bar{A}_{m,r}^l = \begin{bmatrix} A^l & B^l \Upsilon_r \tilde{K}_{m,r} & B^l \tilde{\Upsilon}_r \\ \bar{B}_{m,r} \bar{\mu}_m C^l & \bar{A}_{m,r} & 0 \\ 0 & \Upsilon_r \tilde{K}_{m,r} & \tilde{\Upsilon}_r \end{bmatrix}.$$

Due to the polytopic uncertainty (2), the condition in (28) is equivalent to:

$$\begin{bmatrix} X_{m,r}^{-1} & * & * & * & * \\ \bar{A}_{m,r}(k+i) & X_{n,g} & * & * & * \\ \bar{\zeta}_m \bar{B}_{m,r}(k+i) & 0 & X_{n,g} & * & * \\ \bar{Q} & 0 & 0 & \lambda I & * \\ \bar{Y} & 0 & 0 & 0 & \lambda I \end{bmatrix} \geq 0. \quad (29)$$

Applying the Schur complement, (29) holds if and only if:

$$\begin{aligned} & \bar{A}_{m,r}^T(k+i) X_{n,g}^{-1} \bar{A}_{m,r}(k+i) + \bar{\zeta}_m^2 \bar{B}_{m,r}^T(k+i) X_{n,g}^{-1} \\ & \times \bar{B}_{m,r}(k+i) - X_{m,r}^{-1} + \lambda^{-1} \bar{Q} + \lambda^{-1} \tilde{K}^T R \tilde{K} \leq 0, \end{aligned} \quad (30)$$

where $\tilde{K} = [0 \ \tilde{K}_{m,r}^T \ 0]^T$.

By multiplying both sides of (30) by $\lambda > 0$ and letting $\lambda X_{m,r}^{-1} \triangleq P_{m,r}$, $\lambda X_{n,g}^{-1} \triangleq P_{n,g}$, it is obtained that

$$\begin{aligned} & \bar{A}_{m,r}^T(k+i) P_{n,g} \bar{A}_{m,r}(k+i) + \bar{\zeta}_m^2 \bar{B}_{m,r}^T(k+i) P_{n,g} \\ & \times \bar{B}_{m,r}(k+i) - P_{m,r} + \bar{Q} + \tilde{K}^T R \tilde{K} \leq 0. \end{aligned} \quad (31)$$

Keeping (24) in mind, pre- and post-multiplying (31) with $\xi(k+i|k) > 0$ and its transpose, (18) can be obtained, and this completes the proof. \blacksquare

Remark 4: Note that C is a known system matrix and $\Upsilon_{q(k)}$ is a known scheduling matrix determined by the RR protocol at time instant k ; hence, when we take mathematical expectation on (17), the expectation is taken only in (18) with respect to the stochastic transmission behavior induced by the energy harvesting process, while C and $\Upsilon_{q(k)}$ are treated as given matrices.

ii) C2 of TCS: Now, to satisfy the C2 of the terminal constraint set (TCS), the following conditions need to be provided.

Based on Definition 1, Ψ_k is a positive control invariant set if the following two requirements are satisfied.

- R_{c1} : For the time instant k , there is $\xi(k) \in \Psi_k$, i.e.,

$$\xi^T(k) P_{m,r} \xi(k) \leq 3\lambda. \quad (32)$$

- R_{c2} : For the predicted states at future instants, one has $\xi(k+i|k) \in \Psi_k$, $\forall i \in \mathcal{Y}$ in the MS sense.

Using the Schur complement and the definition $\lambda X_{m,r}^{-1} \triangleq P_{m,r}$, condition (32) is equivalent to the following linear matrix inequality:

$$\begin{bmatrix} 3 & * & * & * \\ x(k) & X_{m1,r1} & * & * \\ \hat{x}(k) & 0 & X_{m2,r2} & * \\ u(k-1) & 0 & 0 & X_{m3,r3} \end{bmatrix} \geq 0. \quad (33)$$

Thus, requirement R_{c1} is guaranteed if (33) is satisfied. Furthermore, given that R_{c1} holds, it follows directly from (18) that

$$\begin{aligned} & \mathcal{E}\{\xi^T(k+1|k) P_{n,g} \xi(k+1|k)\} \\ & \leq \xi^T(k) P_{m,r} \xi(k) \leq 3\lambda, \end{aligned} \quad (34)$$

which implies $V(\xi(k+1|k)) \leq V(\xi(k)) \leq 3\lambda$. Repeating this process over the prediction horizon yields: $3\lambda \geq V(\xi(k)) \geq V(\xi(k+1|k)) \geq V(\xi(k+2|k)) \geq \dots$. Therefore, requirement R_{c2} is also satisfied.

According to the above discussion, *Condition C2* of the TCS is ensured by conditions (22) and (33). However, due to the immeasurability of the system state $x(k)$, (33) cannot be directly verified, which in turn leads to the potential insolvability of OP1. To address this issue, a necessary assumption regarding the initial system state is introduced as follows.

Assumption 2: The initial state of system (1) satisfies the condition:

$$x(0) \in \{x(k) | x^T(k) \Lambda^{-1} x(k) \leq 1\}, \quad (35)$$

where $\Lambda > 0$ is a pre-specified matrix determined by practical experience.

Lemma 3: Given (22) and Assumption 2, condition (33) is guaranteed if, for $\forall \{r, g\} \in \mathbb{Q} \times \mathbb{Q}$, $\forall \{m, n\} \in \mathbb{S} \times \mathbb{S}$, and $l = 1, \dots, L$, there exist symmetric and positive-definite matrices $X_{m1,r1}$, $X_{m2,r2}$, $X_{m3,r3}$, $X_{n1,g1}$, $X_{n2,g2}$, $X_{n3,g3}$ and $N = \text{diag}\{N_1, N_2, N_3\}$ such that the following conditions are satisfied:

$$\begin{bmatrix} 2 & * & * \\ \hat{x}(k) & X_{m2,r2} & * \\ u(k-1) & 0 & X_{m3,r3} \end{bmatrix} \geq 0, \quad (36)$$

$$\begin{bmatrix} \hat{X}_{m,r} & * & * \\ \hat{A}_{m,r}^l & N & * \\ \bar{\zeta}_m \hat{B}_{m,r}^l & 0 & N \end{bmatrix} \geq 0, \quad (37)$$

$$X_{n1,g1} \geq N_1, X_{n2,g2} \geq N_2, X_{n3,g3} \geq N_3, X_{m1,r1} \geq \Lambda, \quad (38)$$

$$\begin{bmatrix} \hat{X}_{m,r} & * & * & * & * \\ [0 \ I \ 0] \hat{A}_{m,r}^l & 2N_2 & * & * & * \\ [0 \ I \ 0] \bar{\zeta}_m \hat{B}_{m,r}^l & 0 & 2N_2 & * & * \\ [0 \ 0 \ I] \hat{A}_{m,r}^l & 0 & 0 & 2N_3 & * \\ [0 \ 0 \ I] \bar{\zeta}_m \hat{B}_{m,r}^l & 0 & 0 & 0 & 2N_3 \end{bmatrix} \geq 0. \quad (39)$$

Proof: At the current time instant $k=0, i=0$, Assumption 2 and inequality (38) imply that $x^T(0) X_{m1,r1}^{-1} x(0) \leq 1$. Then, using (36) and the Schur complement, condition (33) is satisfied.

At the next prediction step $i = 1$, substituting the matrix inequalities in (25) into (37) yields

$$\begin{bmatrix} \tilde{X}_{m,r} & * & * \\ \tilde{A}_{m,r}^l & N & * \\ \tilde{\zeta}_m \tilde{B}_{m,r}^l & 0 & N \end{bmatrix} \geq 0. \quad (40)$$

Pre- and post-multiplying (40) with $\text{diag}\{(GW)^{-1}, I, W_{11}^{-1}, I, \dots, I\}$ and its transpose results in

$$\begin{bmatrix} \vec{X}_{m,r} & * & * \\ \vec{A}_{m,r}^l & N & * \\ \vec{\zeta}_m \vec{B}_{m,r}^l & 0 & N \end{bmatrix} \geq 0. \quad (41)$$

Then, further pre- and post-multiplying (41) with $\text{diag}\{I, X_{m2,r2}^{-1}, I, \dots, I\}$ and its transpose yields

$$\begin{bmatrix} X_{m,r}^{-1} & * & * \\ \vec{A}_{m,r}^l & N & * \\ \vec{\zeta}_m \vec{B}_{m,r}^l & 0 & N \end{bmatrix} \geq 0. \quad (42)$$

Considering the polytopic uncertainty (2), inequality (42) is equivalent to

$$\begin{bmatrix} X_{m,r}^{-1} & * & * \\ \vec{A}_{m,r}(1) & N & * \\ \vec{\zeta}_m \vec{B}_{m,r}(1) & 0 & N \end{bmatrix} \geq 0. \quad (43)$$

Using the Schur complement, (43) holds if and only if

$$\vec{A}_{m,r}^T(1)N^{-1}\vec{A}_{m,r}(1) + \vec{\zeta}_m^2 \vec{B}_{m,r}^T(1)N^{-1}\vec{B}_{m,r}(1) \leq X_{m,r}^{-1} \quad (44)$$

Pre- and post-multiplying (44) with $\xi^T(0)$ and its transpose, and considering (13), it follows that $\mathcal{E}\{\|\xi(1|0)\|_{N^{-1}}^2\} \leq \|\xi(0)\|_{X_{m,r}^{-1}}^2 \leq 3$. Furthermore, from (38), it is obvious that $X_{n1,g1} \geq N_1, X_{n2,g2} \geq N_2, X_{n3,g3} \geq N_3$ imply that $X_{n,g} \geq N$. Thus, $\mathcal{E}\{\|\xi(1|0)\|_{X_{n,g}^{-1}}^2\} \leq \mathcal{E}\{\|\xi(1|0)\|_{N^{-1}}^2\} \leq 3$ can be obtained.

It remains to verify (36) at $i = 1$. From (25) and (39), the following inequality holds:

$$\begin{bmatrix} \tilde{X}_{m,r} & * & * & * & * \\ [0 \ I \ 0] \tilde{A}_{m,r}^l & 2N_2 & * & * & * \\ [0 \ I \ 0] \tilde{\zeta}_m \tilde{B}_{m,r}^l & 0 & 2N_2 & * & * \\ [0 \ 0 \ I] \tilde{A}_{m,r}^l & 0 & 0 & 2N_3 & * \\ [0 \ 0 \ I] \tilde{\zeta}_m \tilde{B}_{m,r}^l & 0 & 0 & 0 & 2N_3 \end{bmatrix} \geq 0. \quad (45)$$

Pre- and post-multiplying (45) with $\text{diag}\{(GW)^{-1}, I, W_{11}^{-1}, I, \dots, I\}$ and its transpose yields

$$\begin{bmatrix} \vec{X}_{m,r} & * & * & * & * \\ [0 \ I \ 0] \vec{A}_{m,r}^l & 2N_2 & * & * & * \\ [0 \ I \ 0] \vec{\zeta}_m \vec{B}_{m,r}^l & 0 & 2N_2 & * & * \\ [0 \ 0 \ I] \vec{A}_{m,r}^l & 0 & 0 & 2N_3 & * \\ [0 \ 0 \ I] \vec{\zeta}_m \vec{B}_{m,r}^l & 0 & 0 & 0 & 2N_3 \end{bmatrix} \geq 0. \quad (46)$$

Pre- and post-multiplying (46) with $\text{diag}\{I, X_{m2,r2}^{-1}, I, \dots, I\}$ and its transpose, it is got

that

$$\begin{bmatrix} X_{m,r}^{-1} & * & * & * & * \\ [0 \ I \ 0] \vec{A}_{m,r}^l & 2N_2 & * & * & * \\ [0 \ I \ 0] \vec{\zeta}_m \vec{B}_{m,r}^l & 0 & 2N_2 & * & * \\ [0 \ 0 \ I] \vec{A}_{m,r}^l & 0 & 0 & 2N_3 & * \\ [0 \ 0 \ I] \vec{\zeta}_m \vec{B}_{m,r}^l & 0 & 0 & 0 & 2N_3 \end{bmatrix} \geq 0. \quad (47)$$

By using the Schur Complement technique, (47) holds if and only if

$$\begin{aligned} & \{[0 \ I \ 0] \vec{A}_{m,r}^l\}^T N_2^{-1} \{[0 \ I \ 0] \vec{A}_{m,r}^l\} \\ & + \{[0 \ 0 \ I] \vec{A}_{m,r}^l\}^T N_3^{-1} \{[0 \ 0 \ I] \vec{A}_{m,r}^l\} \\ & + \vec{\zeta}_m^2 \{[0 \ I \ 0] \vec{B}_{m,r}^l\}^T N_2^{-1} \{[0 \ I \ 0] \vec{B}_{m,r}^l\} \\ & + \vec{\zeta}_m^2 \{[0 \ 0 \ I] \vec{B}_{m,r}^l\}^T N_3^{-1} \{[0 \ 0 \ I] \vec{B}_{m,r}^l\} \leq 2X_{m,r}^{-1}. \end{aligned} \quad (48)$$

Keeping the polytopic uncertain property (2) in mind, pre- and post-multiplying (48) with $\xi^T(0)$ and its transpose yields

$$\begin{aligned} & \xi^T(0) \{ \phi_{11}^T(1) N_2^{-1} \phi_{11}(1) + \phi_{21}^T(1) N_3^{-1} \phi_{21}(1) \\ & + \vec{\zeta}_m^2 \phi_{12}^T(1) N_2^{-1} \phi_{12}(1) + \vec{\zeta}_m^2 \phi_{22}^T(1) N_3^{-1} \phi_{22}(1) \} \xi(0) \\ & \leq 2\xi^T(0) X_{m,r}^{-1} \xi(0), \end{aligned} \quad (49)$$

where $\phi_{11}(1) = [0 \ I \ 0] \vec{A}_{m,r}(1)$, $\phi_{12}(1) = [0 \ I \ 0] \vec{B}_{m,r}(1)$, $\phi_{21}(1) = [0 \ 0 \ I] \vec{A}_{m,r}(1)$, $\phi_{22}(1) = [0 \ 0 \ I] \vec{B}_{m,r}(1)$. Then, based on (13), it is concluded that $\mathcal{E}\{\|\hat{x}(1|0)\|_{N_2^{-1}}^2 + \|u(0) - 1\|_{N_3^{-1}}^2\} \leq 2$. Furthermore, by virtue of (38), it can be achieved $\mathcal{E}\{\|\hat{x}(1|0)\|_{X_{n2,g2}^{-1}}^2\} \leq \mathcal{E}\{\|\hat{x}(1|0)\|_{N_2^{-1}}^2\}$ and $\mathcal{E}\{\|u(0) - 1\|_{X_{n3,g3}^{-1}}^2\} \leq \mathcal{E}\{\|u(0) - 1\|_{N_3^{-1}}^2\}$. Therefore, $\mathcal{E}\{\|\hat{x}(1|0)\|_{X_{n2,g2}^{-1}}^2 + \|u(0) - 1\|_{X_{n3,g3}^{-1}}^2\} \leq 2$ can be ensured, which implies that (36) is guaranteed at the predicted instant $i = 1$.

By now, the guarantee of condition (33) at the predicted instant $i = 1$ has been established. The above reasoning can be recursively extended to all future prediction steps $i = 2, i = 3, \dots$. By following the same procedure, it can be concluded that if conditions (36)-(39) are satisfied at any given time instant k , then condition (36) remains satisfied for all subsequent prediction steps $k+i$ ($i > 0$), and this completes the proof. ■

2) *An auxiliary optimization problem:* To address the polytopic uncertainty (2) in the underlying system (1), the objective function (14) explicitly incorporates parameter uncertainties. While this formulation enhances robustness, it also renders problem OP1 generally unsolvable, as noted in [34]. Following the approach in [18], to construct an auxiliary optimization problem, an upper bound on the robust performance objective need to be obtained. This auxiliary problem, which is solvable, serves as a sub-optimization of problem OP1.

Considering the nature of the infinite-horizon minimization problem, the objective function $J_\infty(k)$ in (14) must be finite, which implies that $\xi(\infty|k) = 0$, and hence $V(\xi(\infty|k)) = 0$. Summing up both sides of (18) from $i = 0$ to $i = \infty$ yields

$$J_\infty(k) \leq V(\xi(k)) \leq 3\lambda, \quad (50)$$

which implies that 3λ serves as an upper bound for $J_\infty(k)$.

Based on the obtained upper bound and the preceding results, the following auxiliary optimization problem OP2 can be given:

$$\begin{aligned} \text{OP2:} \quad & \min && 3\lambda, \\ & X_{m1,r1}, X_{m2,r2}, X_{m3,r3}, X_{n1,g1}, \\ & X_{n2,g2}, X_{n3,g3}, N_1, N_2, N_3, \\ & \{r,g\} \in \mathbb{Q} \times \mathbb{Q}, \{m,n\} \in \mathbb{S} \times \mathbb{S}, l=1, \dots, L \\ & \mathcal{A}_{m,r}, \mathcal{B}_{m,r}, \mathcal{F}_{m,r} \\ \text{s.t.} \quad & (22), (36), (37), (38), (39). \end{aligned}$$

Solving OP2 yields the output feedback controller gains as

$$\begin{cases} \check{A}_{m,r} = \mathcal{A}_{m,r} X_{m2,r2}^{-1} \\ \check{B}_{m,r} = \mathcal{B}_{m,r} W_{11}^{-1} \\ \check{K}_{m,r} = \mathcal{F}_{m,r} X_{m2,r2}^{-1} \end{cases} \quad (51)$$

Remark 5: Due to the difficulty of solving OP1 directly, we instead develop the auxiliary problem OP2. The solution of OP2 is sub-optimal, since the objective 3λ is merely an upper bound of the original cost $J_\infty(k)$. However, this sub-optimality does not compromise the stability ensured by OP1. The constraints in OP2 (i.e., (22), (36), (37), (38), and (39)) are sufficient to satisfy the TCS conditions required in OP1. Therefore, the controller designed via OP2 guarantees the stability of the system, provided that OP1 admits a stabilizing solution. A detailed stability analysis will be presented in the next section.

B. Feasibility and stability

Now, it is necessary to present a sufficient condition that guarantees both the feasibility of the proposed optimization problem and the mean-square (MS) stability of the resulting closed-loop system.

Theorem 1: Consider the system (1) with an EHS and RR protocol under the output feedback controller (11). At time instant k , if a feasible solution to problem OP2 exists, then feasibility will also hold for any future time instant $k+i$, $i > 1$, and the closed-loop system governed by (1) and (11) is MS stable.

Proof: Following the standard approach as in [18], the proof proceeds in two parts: (1) recursive feasibility of the proposed MPC strategy, and (2) MS stability of the closed-loop system.

(1) *Recursive feasibility:* Among the constraints in OP2, condition (36) is the only one that explicitly depends on the current system state $\hat{x}(k)$ and input $u(k-1)$. Therefore, given the initial feasibility of OP2 at time k , it suffices to prove that (36) remains feasible at all future time instants $k+i$, $i \in \mathcal{Y}$.

From the inequality (22) and the assumed feasibility at k , we have

$$\xi^\top(k+1|k)P_{n,g}\xi(k+1|k) \leq \xi^\top(k)P_{m,r}\xi(k) \leq 3\lambda \quad (52)$$

Due to the system evolution governed by (12), for some $[A(k)|B(k)|C(k)] \in \Omega$, it holds that

$$\xi(k+1|k) = \vec{A}_{m,r}\xi(k) = \xi(k+1). \quad (53)$$

Substituting (53) into (52) yields

$$\xi^\top(k+1)P_{n,g}\xi(k+1) \leq 3\lambda, \quad (54)$$

which confirms the feasibility of (36) at $k+1$.

By iterating this process for $k+2, k+3, \dots$, it is concluded that OP2 remains feasible at all future time instants, thereby establishing recursive feasibility.

(2) *MS stability:* Consider the quadratic Lyapunov function

$$\bar{V}(\xi(k)) = \xi^\top(k)P_{m,r}\xi(k). \quad (55)$$

where $P_{m,r}$ is obtained by solving OP2 at time k . Let $P_{m,r}^*$ and $P_{n,g}^*$ denote the optimal solutions at instants k and $k+1$, respectively.

From the optimality of $P_{n,g}^*$, it follows that

$$\xi^\top(k+1)P_{n,g}^*\xi(k+1) \leq \xi^\top(k+1)P_{n,g}\xi(k+1), \quad (56)$$

where $P_{n,g}$ represents any feasible solution at time $k+1$. In addition, inequality (22) implies

$$\mathcal{E}\{\xi^\top(k+1|k)P_{n,g}\xi(k+1|k)\} \leq \xi^\top(k)P_{m,r}^*\xi(k). \quad (57)$$

Combining (53), (56), and (57) leads to

$$\mathcal{E}\{\xi^\top(k+1)P_{n,g}^*\xi(k+1)\} \leq \xi^\top(k)P_{m,r}^*\xi(k), \quad (58)$$

which indicates that $\bar{V}(\xi(k))$ is non-increasing in the MS sense. Therefore, $\xi(k) \rightarrow 0$ in the MS sense as $k \rightarrow \infty$, which completes the proof. ■

Remark 6: The recursive-feasibility difficulty considered here should not be understood as a failure of the RR protocol itself, but rather as the need to guarantee feasibility of the moving-horizon optimization at every future sampling instant despite the time-varying scheduling index under RR and the stochastic transmission behavior induced by the EHS.

Remark 7: It should be emphasized that the proposed controller is not only obtained from a tractable auxiliary optimization problem, but is also supported by explicit theoretical guarantees. In particular, recursive feasibility is established for all future time instants once initial feasibility is achieved, and mean-square stability of the closed-loop system is ensured through a non-increasing quadratic Lyapunov function. Therefore, the combined effects of the EHS, the RR protocol, and the min-max formulation do not lead to uncontrolled divergence under the derived conditions.

C. Constrained RMPC with an EHS and RR protocol

In practical industrial applications, system operation is often subject to hard constraints due to safety and performance requirements. For instance, in a distillation column, process parameters such as pressure and temperature must remain within specified bounds. Consequently, it is essential to incorporate these hard constraints, as described in (3), into the controller design. Before formulating the corresponding optimization problem, to guarantee satisfaction of the constraints, a lemma that provides solvability conditions is first presented.

Lemma 4: For the RMPC problem with state and input hard constraints (3), let vectors \bar{u} and \bar{x} be given. If there exist symmetric and positive-definite matrices $U, Z, \mathcal{F}_{m,r}, X_{m1,r1}, X_{m2,r2}, X_{m3,r3}$ such that the following inequalities hold:

$$\begin{bmatrix} \frac{1}{2}U_{ee} & * & * \\ \mathcal{F}_{m,r}^\top \Upsilon_r^\top & X_{m2,r2} & * \\ X_{m3,r3} \tilde{\Upsilon}_r^\top & 0 & X_{m3,r3} \end{bmatrix} \geq 0, \quad (59)$$

$$U_{ee} \leq \bar{u}^2, e \in \{1, \dots, n_u\},$$

$$\begin{bmatrix} Z_{ff} & * \\ X_{m1,r1} & X_{m1,r1} \end{bmatrix} \geq 0, Z_{ff} \leq \bar{x}^2, f \in \{1, \dots, n_x\}, \quad (60)$$

where $[\star]_{ee}$ and $([\star]_{ff})$ is the e th and f th diagonal element of the “ \star ”, respectively, then the hard constraint (3) is satisfied.

Proof: The proof follows a similar approach as in [18] and is thus omitted for brevity. ■

Based on Lemma 4, the optimization problem for the system under hard constraints is formulated as follows.

$$\begin{aligned} \text{OP3:} \quad & \min && 3\gamma, \\ & X_{m1,r1}, X_{m2,r2}, X_{m3,r3}, X_{n1,g1}, \\ & X_{n2,g2}, X_{n3,g3}, N_1, N_2, N_3, U, Z, \\ & \{r,g\} \in \mathbb{Q} \times \mathbb{Q}, \{m,n\} \in \mathbb{S} \times \mathbb{S}, l=1, \dots, L \\ & \mathcal{A}_{m,r}, \mathcal{B}_{m,r}, \mathcal{F}_{m,r} \\ \text{s.t.} \quad & (22), (36), (37), (38), (39), (59), (60). \end{aligned}$$

Theorem 2: Consider the system (1) operating under the EHS and RR protocol with the output feedback controller (11) subject to the state and input constraints (3). If a feasible solution to OP3 exists at time instant k , then feasibility is ensured for all future instants $k+i$, $i > 1$, and the closed-loop system is MS stable.

Proof: The proof follows similar reasoning to Theorem 1 and is omitted for conciseness. ■

D. Algorithm of RMPC with an EHS and RR protocol subject to hard constraints

Next, the algorithm of the proposed RMPC with an EHS and RR protocol subject to hard constraints will be given.

Algorithm 1 RMPC procedure with EHS and RR protocol

- 1: **Initialization:**
 - 2: Select the initial energy level $s(0)$ and initial state values $\xi(0) \triangleq [x^T(0) \hat{x}^T(0) u^T(-1)]^T$, weighting matrices $Q = \text{diag}\{Q_1, Q_2, Q_3\}$ and R , the number of communication nodes h , the maximum energy capacity of the sensor Ξ and other appropriate parameters Λ , $x(0) \in \{x(k) | x^T(0)\Lambda^{-1}x(0) \leq 1\}$ such that problem OP3 is feasible.
 - 3: **for** $k = 1, 2, \dots$ **do**
 - 4: Using Lemma 1, compute the probability distribution of the energy level $s(k)$ and the corresponding expectation $\mu(k)$;
 - 5: Calculate the measured output $\tilde{y}(k)$;
 - 6: Solve the optimization problem OP3 to determine controller gains $\check{A}_{m,r}, \check{B}_{m,r}, \check{K}_{m,r}$;
 - 7: Calculate $\tilde{u}(k)$ based on (11);
 - 8: Calculate $u(k)$ based on (10);
 - 9: Apply $u(k)$ to the system. Set $k = k + 1$ and go to Step 4.
 - 10: **end for**
-

IV. SIMULATION EXAMPLES

In this section, a numerical example is presented to demonstrate the effectiveness of the proposed RMPC strategy for system (1) with an EHS and RR protocol. A second-order

system with parametric uncertainty, corresponding to (1), is considered and described as follows:

$$A_1 = \begin{bmatrix} -1.2 & 0.018 \\ 0.8779 & 0.382 \end{bmatrix}, A_2 = \begin{bmatrix} -0.85 & -0.0218 \\ 0.4171 & 0.2861 \end{bmatrix}, \\ B_1 = B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The initial conditions are selected as $x(0) = [0.0001 \ 0.001]^T$, $\hat{x}(0) = [10 \ 10]^T$, $u(-1) = [0.009 \ 0.074]^T$. Other parameters are chosen as $\bar{u} = 5$, $\bar{x} = 5$, $Q_1 = Q_2 = Q_3 = \text{diag}\{1, 1\}$, $R = \text{diag}\{1, 1\}$, $\Lambda = \text{diag}\{0.001, 0.001\}$. The number of communication nodes is $h = 2$.

The energy-harvesting sensor is initialized with energy level $s(0) = 1$ and a maximum capacity $\Xi = 2$. At each time instant k , the harvested energy $t(k)$ follows a Poisson distribution $\text{Prob}(t(k) = \psi) = \frac{\nu^\psi \exp(-\nu)}{\psi!}$ with the rate $\nu = 1$. Based on (19), the energy level distribution $\sigma(k)$ and expected transmission probability $\mu(k)$ are recursively computed as shown in Table III. To demonstrate the effectiveness of the proposed method, [19] was used for comparison.

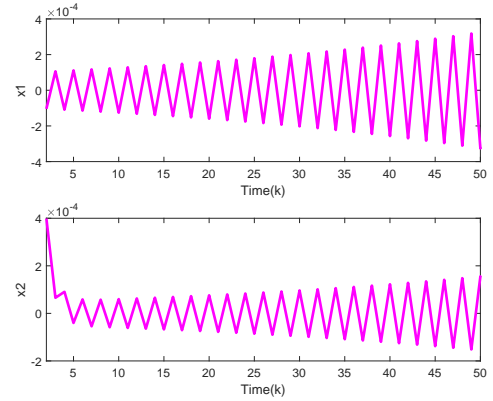


Fig. 2: The state response of open-loop system.

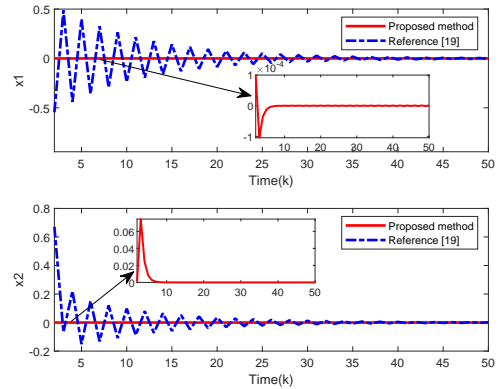


Fig. 3: The state response of the underlying system.

The simulation results are presented in Fig. 2, Fig. 3 and Fig. 4, under the RR protocol illustrated in Fig. 5 and the EHS mechanism shown in Fig. 6. Fig. 2 shows the state response

TABLE III: The probability distribution of the sensor energy level and the expectation of the measurement transmission.

k	0	1	2	3	...
$\sigma(k)$	$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.3679 \\ 0.3679 \\ 0.2642 \end{bmatrix}$	$\begin{bmatrix} 0.2707 \\ 0.3679 \\ 0.3614 \end{bmatrix}$	$\begin{bmatrix} 0.2349 \\ 0.3679 \\ 0.3972 \end{bmatrix}$...
$\mu(k)$	1	0.6321	0.7293	0.7651	...

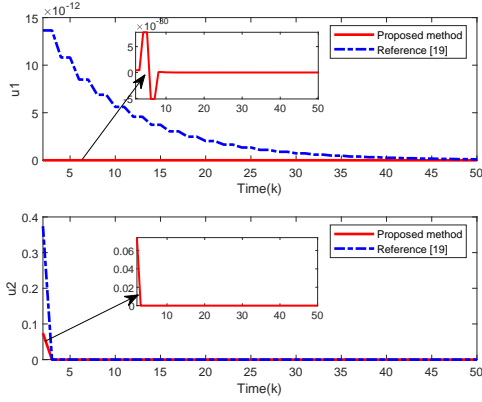


Fig. 4: The control input of the underlying system.

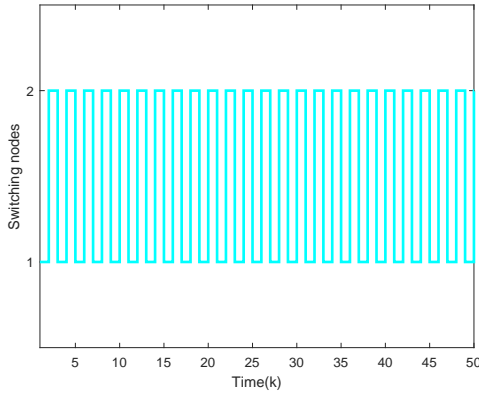


Fig. 5: Switching nodes.

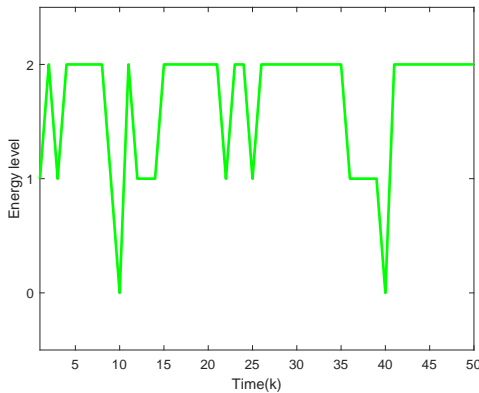


Fig. 6: Energy storage of the sensor at each time instant.

of open-loop system. Fig. 3 displays the state response of the underlying system. It is clear that the proposed RMPC strategy effectively stabilizes the open-loop unstable system. Although the state response of the underlying system is stable by using the method of [19], the state curve oscillates repeatedly and the convergence speed is slower than the proposed method. Fig. 4 shows the corresponding control inputs. The smooth evolution of the input signals indicates satisfactory control performance despite the presence of both EHS and RR protocol constraints.

V. CONCLUSION

This paper has investigated the RMPC problem for systems equipped with an EHS and operating under an RR communication protocol, subject to state and input constraints. The sensor has been assumed to harvest energy from the environment with a certain probability, and the RR protocol has been employed to schedule the transmission order from the controller to the actuator so as to avoid data collisions. To address the resulting control problem, an auxiliary optimization problem with guaranteed solvability has been formulated to compute the controller gains, and sufficient conditions have been established to ensure the mean-square stability of the closed-loop system. Finally, the effectiveness of the proposed control scheme has been demonstrated through one illustrative example. The comparative results have shown that the proposed method can not only stabilize the considered system effectively, but can also achieve improved transient performance over the compared main-stream technique, especially in terms of reduced oscillation and faster convergence. These advantages stem from the predictive optimization mechanism, the dynamic output-feedback structure, and the explicit incorporation of the EHS and RR effects into the controller design. Future research will be directed towards extending the proposed framework to larger-scale multisensor systems with energy-harvesting capabilities [33] and to validating its effectiveness in more realistic application scenarios, both of which are becoming increasingly important in modern networked control and IoT applications.

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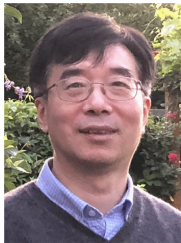
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