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Welfare-Improving Tax Disputes

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ABSTRACT

Tax law is often uncertain. Taxpayers and tax authorities can disagree over the tax position, eligibility for tax incentives, or sufficiency of evidence. Taxpayers can face uncertainty over sanctions when tax position is not sustained. In many jurisdictions uncertain tax position can be disputed in court. In this paper, I present an analysis of tax dispute over uncertain tax treatment of investment in the framework of rent-seeking contest with endogenous prize. I show that in the presence of tax uncertainty, a tax dispute can lead to a net welfare gain despite the litigation costs. These results provide economic efficiency rationale for tax disputes.

1 | Introduction

Tax law and tax liabilities are often uncertain for a variety of reasons. In many types of economic transactions, taxpayers and tax authorities can disagree over the tax position, eligibility for tax incentives, or sufficiency of evidence. Taxpayers can face uncertainty over sanctions when tax position is not sustained. In some jurisdictions disagreement over tax treatment can be resolved through a tax dispute, with the outcome being uncertain at the time the taxable economic activity takes place. If litigation is costly, the litigation expenditures of the parties can be seen as a form of rent-seeking, ultimately leading to welfare loss. In this paper I analyze the decision of an investor about the allocation of a productive resource between different sectors when tax position is uncertain and can be disputed in court. I show that a tax dispute can lead to efficiency gains by mitigating distortion in the investment activity, with gains outweighing the cost of litigation. When an investor anticipates tax litigation and can exert costly efforts to change the future outcome of litigation in his or her favor, the investment allocation decision can improve economic efficiency in equilibrium. I derive the conditions under which this efficiency gain exceeds the litigation costs. Thus, in the presence of tax uncertainty, tax disputes can be socially desirable, with the tax litigation system and courts serving the social purpose of chanelling resources to their most productive use.

An important source of tax uncertainty is the possibility that taxpayers and tax authorities can interpret the law differently, leading to the uncertainty in tax position. A notorious example is the distinction between cakes and biscuits (cookies) for the purpose of the VAT rate which became a part of VAT tribunal concerning Jaffa Cakes in the UK in 1991.[1] McVitie's, the manufacturer, argued that Jaffa Cakes are cakes, which in the UK are zero-rated for VAT. The HMRC (the UK tax authority), however, argued that Jaffa Cakes were biscuits covered in chocolate, which would make them subject to VAT at the standard rate. The tribunal examined various aspects, including ingredients and recipe (Jaffa Cakes contain a sponge base, which is typical of cakes), texture and freshness (Jaffa Cakes harden when stale, like cakes, whereas biscuits soften), size and packaging (Jaffa Cakes are more like biscuits in their size and are often packaged in a way similar to biscuits), presentation (Jaffa Cakes are presented as a snack and are usually eaten with the fingers, whereas a cake is usually eaten with a fork), perception and marketing (the product name suggests it is a cake, but in shops it is generally displayed near biscuits). Thus, Jaffa

[1] https://www.gov.uk/hmrc-internal-manuals/vat-food/vfood6260

Cakes have some characteristics of cakes and some characteristics of biscuits. Ultimately, the tribunal ruled in favor of McVitie's, deciding that Jaffa Cakes are, indeed, cakes and, therefore, should be zero-rated for VAT. Two similar cases, brought to the tribunal by DuelFuel Nutrition Ltd[2] and by WM Morrison Supermarkets Plc,[3] on the VAT rate applicable to certain varieties of flapjacks, were ruled in 2024 in favor of the HMRC.

Challenging and defending tax position is costly for tax authorities and taxpayers. While tax disputes are costly and divert resources from productive activities, as an institutional arrangement they exist in many jurisdictions. There is an argument that they may be beneficial from the social viewpoint, if the parties' litigation efforts provide the court with additional information helping to make the correct decision (Posner, 2014). Tax uncertainty framework can be used also to evaluate the welfare effect of tax disputes and to establish whether it is possible that the benefits outweigh the litigation costs. Intuitively, a change in tax uncertainty can increase revenue and welfare if it reduces distortionary effect of inefficient taxes, or expands the tax base, or both. A tax dispute is welfare-improving if the welfare gain from the change in tax uncertainty is greater than the litigation costs.

In this paper I investigate the economic implications of uncertainty in tax law in a model of investment allocation between two sectors, one with potentially preferential tax treatment and another one without. Whether or not profits earned in the first sector will qualify for tax incentives is uncertain at the time of the investment decision. The investor, or the taxpayer, and the tax authority can dispute the tax return in court. A tax differential between two sectors distorts the investment decision towards the sector with the lower expected tax rate. In general, the allocation is inefficient unless uncertainty exactly equalizes the expected tax rates between the two sectors. A tax dispute can reduce the distortion in the investment decision if it moves the allocation closer to the optimal level in equilibrium. I show that over a wide range of the model parameters the equilibrium welfare gain from the distortion reduction are higher than the total cost of litigation. Thus, I demonstrate an economic efficiency rationale for tax disputes. To my best knowledge, this paper is the first instance in the literature to provide the welfare analysis of tax disputes in the context of tax uncertainty.

In this paper, I take the statutory tax rates and the corresponding sectoral categories as given, reflecting an institutional arrangement where tax policy is set by the legislature, while its enforcement is delegated to tax authorities.[4] I do not investigate the political economy behind the specific tax differentials or the reasons why the law remains incompletely specified. Instead, I focus on the administrative and judicial reality: given these rates, the tax authority must decide which positions to challenge, and the taxpayer must decide how to respond. This perspective highlights how the tax dispute system can serve as a deliberate tool for administrative fine-tuning, where the litigation process effectively clarifies legislative ambiguity and has a potential to reduce allocative inefficiency.

In the next section I outline the research on tax uncertainty to which this paper is most closely related and the ways in which this paper contributes to the literature. The remaining sections present an analysis of a model of investment allocation under uncertainty over the effective profit tax rates. Section 3 presents a benchmark framework where a tax return is not disputed. Section 4 presents the framework where the taxpayer and the tax authority can dispute tax position in court assuming that each side pays its own legal costs regardless of the outcome. I analyze the equilibrium in two different scenarios: when the taxpayer's investment decision precedes the decision on the litigation expenditures, and when the two decisions are made simultaneously. Section 5 illustrates that costly litigation can lead to net welfare gains when each party pays only its own litigation cost irrespective of the court's decision. Section 6 verifies the result for a legal system where the losing party pays the legal costs of the winning party, in addition to its own costs. Section 7 outlines possible extensions and alternative assumptions. Full details of all mathematical derivations and the computing codes used to produce numerical illustrations are available upon request.

2 | Tax Uncertainty: Evidence and Related Literature

Uncertainty in tax law has become a major issue in recent years, with important implications for capital allocation decisions. As documented by Devereux (2016) in a survey of senior tax professionals in large businesses and tax advice firms, between 2010 and 2015 uncertainty in corporation tax law increased in 20 of 21 major countries analyzed in the study. In their answers, *uncertainty about the effective tax rate on profit* was the third most important determinant of investment and location decisions, more important than the *anticipated effective tax rate* itself. Moreover, '*unpredictable or inconsistent treatment by tax authority*' appeared as the single most important factor of uncertainty. Increasingly, business decisions affecting tax obligations bear uncertain implications for compliance with tax law.

Brok (2019) has constructed a measure of legal uncertainty in corporate income tax law, based on legal literature and the outcomes of court cases in ten countries over 7 years. Using this measure, he found evidence that legal uncertainty has significant effect on the financing and location decisions of companies. Jacob and Schütt (2020) demonstrated that tax uncertainty affects the firm valuation, while Jacob et al. (2022) found that tax uncertainty can significantly alter large capital investment decisions by firms.

Uncertainty can arise over eligibility for preferential tax treatment. For example, in many jurisdictions tax incentives are used to boost research and development (R&D). This policy creates tax uncertainty when it is not clear which expenses qualify for R&D tax incentives and whether evidence presented by the taxpayer is sufficient. Cowx (2021) showed that R&D tax incentive uncertainty has negative effect on R&D investment. Similarly, preferential tax treatment of economic activities with environmental

[2] https://financeandtax.decisions.tribunals.gov.uk/judgmentfiles/j12960/TC%2009055.pdf
[3] https://financeandtax.decisions.tribunals.gov.uk/judgmentfiles/j12994/TC%2009095.pdf
[4] I am grateful to the anonymous reviewer for pointing out the institutional interpretation of the nature of tax uncertainty.

goals creates tax uncertainty when there is a room for interpretation of eligibility criteria (Greene and Braathen 2014). More recently, development of digital technologies has raised concerns about the uncertainty in tax treatment of cryptoassets and transactions involving decentralized finance (OECD 2020; HMRC 2023). These findings suggest that tax law uncertainty can have wide economic implications, with questions then arising about welfare consequences.

Uncertainty in tax law is an example of a more general concept of legal uncertainty that is the subject of both academic and practical interest in law and economics (Raskolnikov 2017). D'Amato (1983) defined legal certainty as the predictability of the official reaction to an agent's action: for example, if the reaction is the court decision "win" or "lose," the law is the most uncertain when the probability of "winning" (or "losing") is 0.5, and legal certainty is higher, the closer is this probability to either zero or one.

One can formalize the description of uncertainty in law using D'Amato's interpretation, by considering the probability, denoted p, that a particular economic activity or a transaction is legal under the law. The law is certain if it unambiguously states whether or not the activity is legal, that is, if p is either zero or one. This can be interpreted as a discrete rule. Uncertainty in law corresponds to p between zero and one, and the closer is p to 0.5, the higher is the uncertainty about the legality of this activity. Tax law uncertainty does not necessarily refer to the dichotomy of legal (compliance) and illegal (noncompliance) actions of taxpayer, and it is often over the tax rates or tax base. For example, p can be the probability that a particular transaction qualifies for a preferential tax treatment.

Various sources of tax uncertainty have been analyzed in the empirical literature. Brühne and Schanz (2022) summarize the definitions of tax uncertainty and tax risk[5] and the perception of tax risk by practitioners (tax directors and tax risk specialists, as well as regulators and representatives of a tax authority) reported in a number of empirical studies published between 2000 and 2021. The list of sources of tax uncertainty features uncertain nature of deductions or credits, ambiguity in tax law about whether a tax position will be allowed, and a possibility that tax position will be litigated and will not be sustained if challenged by tax authority.

For a theoretical model, one can consider the situation where the uncertainty about the tax position can arise from several sources: the tax authority may or may not challenge the tax return; if it does, the challenge may or may not be successful; if it is successful, the extent of penalties imposed might also be uncertain. All of these can be formalized as the uncertainty over the the *effective* tax rate. Thus, the model developed in this paper is based on the assumption of uncertainty over the tax rates.

This paper is close in spirit to the theoretical analysis of economic effects of random tax policies in several alternative frameworks. Bizer and Judd (1989) constructed a dynamic model with two sources of uncertainty, random shocks to production technology and random tax policies. They showed that in this economy uncertainty in tax rates increases tax revenue at a relatively low efficiency cost, thus increasing social welfare. Intuitively, uncertainty in future tax rates encourages investment and reduces the distortion created by the capital income tax. A similar intuition is behind the result obtained by Weiss (1976) and, independently, Stiglitz (1982): random labor income taxation can increase ex-ante expected utility of risk-averse agents through its effect on labor supply. Niemann (2004, 2011) showed that random tax rates on cash flows has ambiguous effect on investment: depending on the structure of the cash flow and depreciation, tax rate uncertainty can accelerate or delay investment; the analysis covers both risk-neutral and risk-averse investors. In another setting, with risk-neutral firms, Hines and Keen (2021) demonstrated that a mean-preserving spread in input taxes can raise producers' profits and improve the efficiency of the tax system. In their model, the tax uncertainty, as opposed to a single, certain tax rate, effectively widens the range of policy instruments for the government and works as a screening device, so that higher (lower) tax rates apply to the activities less (respectively, more) responsive to the tax. This reduces the deadweight loss created by the distortionary taxation of inputs.

A similar mechanism generates improvement in allocative efficiency in my model: under an exogeneous distortionary tax uncertainty a tax dispute can reduce the loss of welfare created by distortionary tax differential by moving the uncertainty towards the efficient level. The rational response of an investor to uncertain tax rates determines simultaneously the tax base and the effective tax rate applied to the taxable investment activity. An anticipation of a tax dispute changes the investment decision. Using a modified version of the Tullock contest (Tullock 1975), I show that welfare gains resulting from improved efficiency of investment allocation can outweigh the total cost of the parties' litigation efforts. Thus, a legal system that allows costly tax disputes can be justified from the viewpoint of economic efficiency.

The system of tax disputes exists in many jurisdictions. While costly tax disputes divert resources from productive activities, they are an important part of maintaining an effective tax system. One compelling argument for tax disputes is that they can lead to court rulings or settlements that clarify the meaning of complex or ambiguous tax laws, making them easier to understand and follow in the future. The model presented in this paper offers an economic efficiency rationale for tax disputes. It shows that, in the presence of distortion of the tax base and uncertainty about the effective tax rates, a costly tax dispute can lead to net welfare gains by changing the degree of uncertainty in such a way that the distortion is reduced or eliminated.

My paper contributes to two strands of literature. First, it contributes to the literature on the economic effects of tax uncertainty by embedding an investment allocation decision in a setting with uncertain tax position and tax disputes. This framework reflects the contemporary environment where uncertainty about tax treatment of profits is one of the most important factors of investment decisions by large businesses

[5] Brühne and Schanz (2022) note that in the literature "tax uncertainty" and "tax risk" are used interchangeably.

(Devereux 2016; Brok 2019; Jacob et al. 2022). Second, it contributes to the literature on application of Tullock contest framework in the context of litigation by modeling an environment where the prize (the pre-tax profit), the sharing rule (the split between tax revenue and after-tax profit), and the productivity of the contestants' efforts (the effect of litigation costs on tax revenue and after-tax profit) are all endogenous. In the standard model of contest with endogenous prize and productive efforts the value of the prize is assumed to increases in the aggregate efforts of the parties, the efforts are always excessive in equilibrium, leading to the deadweight loss (Chung 1996; Baye and Hoppe 2003; Damianov et al. 2018; Hirai 2020). Under an alternative assumption, when the value of the prize is assumed to increase in the aggregate expense of the contestants, the equilibrium value can be above, or below, or equal to the efficient level.[6] In this paper, the parties' efforts enter the prize function separately, and the direction of their effect is endogenous. The privately optimal efforts (litigation expenses) can be either productive or destructive in the equilibrium, depending on the initial uncertainty and other model parameters,—that is, the prospect of a dispute can lead to a higher or a lower pre-tax profit, by affecting the investment allocation. In turn, productive equilibrium efforts can move the uncertainty closer towards the efficient (welfare-maximizing) level, so that the resulting increase in the pre-tax profit outweighs the aggregate cost of litigation.

3 | Tax Uncertainty Without Disputes

A taxpayer, an individual investor or a firm, decides how to allocate one unit of resources between two productive activities, or sectors, denoted 0 and 1. Investment in either sector returns a certain level of taxable profit. Profits generated by the investment in sector 0 are subject to tax at rate τ_0 with certainty. The investor's tax position in the sector 1 is uncertain. With probability p the effective tax rate that will apply to the profit earned in this sector is τ_L, and with probability $(1-p)$ the effective tax rate is $\tau_H > \tau_L$.[7] For example, τ_0 could be the 'standard' statutory tax rate, whereas the lower effective tax rate τ_L could reflect tax preferentials, such as exemptions, tax credits or tax reliefs (these could be tax incentives for R&D or for investment with environmental goals, and so on), and the higher effective tax rate τ_H could correspond to the full tax due when exemptions and reliefs are not applicable, including interest and any fines or penalties that might be imposed in addition to the standard rate, for example, fines for negligence. Thus, it is assumed that $\tau_H > \tau_0 > \tau_L$. I will refer to sector 0 as "standard," and to sector 1 as "tax-saving." There are no effects on third parties, and the purpose of taxation is purely to raise revenue. Following Hines and Keen (2021), I assume that the taxpayer is risk-neutral and there is no hard budget constraint or limited liability.[8]

As a benchmark, I consider first the situation in which the actions of the taxpayer and the tax authority have no effect on tax uncertainty. Let λ be the proportion of resources invested in the tax-saving sector, $0 \le \lambda \le 1$. The investor chooses λ to maximize the expected after-tax profit,

$$\Pi(\lambda) \equiv (1-\tau_0)\pi_0(1-\lambda) + [p(1-\tau_L) + (1-p)(1-\tau_H)]\pi_1(\lambda),$$

where π_0 is the profit function of the standard sector activity and π_1 is the profit function of the tax-saving sector activity.

Assumption 1. $\pi_i(0) = 0, \pi_i'(\lambda) > 0, \pi_i''(\lambda) < 0,$ for $\lambda \in [0, 1], i \in \{0, 1\}$.

Assumption 1 states that both profit functions are strictly increasing and strictly concave over the relevant domain; in general, the technology and the cost of production in these two activities can be different, so that $\pi_1(\lambda) \neq \pi_0(\lambda)$.

I am interested in the interior solution, denoted λ^*, such that $0 < \lambda^* < 1$. In an interior solution, the profit-maximizing allocation λ^* satisfies the optimality condition

$$\begin{aligned}[p(1-\tau_L) + (1-p)(1-\tau_H)]\pi_1'(\lambda^*) \\ = [1-\tau_0]\pi_0'(1-\lambda^*).\end{aligned} \tag{1}$$

This condition states that the after-tax profit is maximized when the expected marginal returns to investment in two sectors are equal.

The welfare is defined as the total surplus generated by the investment. In this framework it is the pre-tax profit,

$$W(\lambda) = \pi(\lambda) \equiv \pi_0(1-\lambda) + \pi_1(\lambda).$$

and it is divided between the investor, or the taxpayer, whose payoff is the after-tax profit, and the tax authority, whose payoff is the tax revenue.

The efficient (socially optimal) allocation of investment is the allocation which maximizes the welfare, or the total surplus. In an interior optimum, $0 < \lambda^E < 1$, it satisfies the optimality condition

$$\pi_1'(\lambda^E) = \pi_0'(1-\lambda^E). \tag{2}$$

This condition states that the welfare is maximized when the marginal pre-tax returns in investment in two sectors are equal.

Expected tax revenue,

$$T = \tau_0\pi_0(1-\lambda) + [p\tau_L + (1-p)\tau_H]\pi_1(\lambda)$$

is maximized at $\lambda = \lambda^T$ such that

$$[p\tau_L + (1-p)\tau_H]\pi_1'(\lambda^T) = \tau_0\pi_0'(1-\lambda^T). \tag{3}$$

[6] I am grateful to the anonymous reviewer for pointing this out. See further discussion in Section 5.
[7] Since the taxpayer and the tax authority know the probability distribution, this is not a Knightian uncertainty. Strictly speaking, the described situation is one of tax *risk*. Following Bizer and Judd (1989) and Hines and Keen (2021), I use *uncertainty* as it is commonly referred to in the legal literature and in the information theory.
[8] The potential effect of relaxing these assumptions is discussed in Section 7.

This condition states that the expected marginal revenues in two sectors are equalized.

Let β denote the value of p which induces the efficient allocation of investment in the private optimum, $\lambda^* = \lambda^E$. Combining conditions (1) and (2) gives

$$\beta = \frac{\tau_H - \tau_0}{\tau_H - \tau_L}. \tag{4}$$

It is straightforward to show that $p = \beta$ also maximizes the expected revenues for the tax authority. Intuitively, the efficient uncertainty eliminates investment distortion caused by the tax differential. It maximizes the expected tax base (the aggregate pre-tax profit), and, since the expected marginal tax rates are equalized between the two activities, it also maximizes the tax revenue. This leads to the first result.

Proposition 1. *In the absence of tax disputes, the uncertainty that equalises the expected tax rates between the tax-saving and standard activities,*

$$p\tau_L + (1 - p)\tau_H = \tau_0, \tag{5}$$

maximizes welfare and the tax revenue

The efficient (welfare-maximizing) uncertainty equalises the net marginal benefits of the two investment opportunities by equating the expected tax rate in the tax-saving sector 1 to the certain tax rate in sector 0. One can see that, if in sector 1 the tax rate is certain, either at τ_L ($p = 1$) or at τ_H ($p = 0$), the profit-maximizing allocation is different from the efficient allocation. This leads to the deadweight loss of welfare as long as the certain tax rates differ between the two sectors. Similarly, uncertain tax rates create distortion and deadweight loss unless they satisfy the efficiency condition (5). Since an arbitrary uncertainty over the tax rates is, in general, inefficient, it may be possible to reduce the distortion of investment allocation by changing the degree of uncertainty. In what follows, I investigate the possibility of reducing distortion when the tax uncertainty changes as a result of a tax dispute.

4 | Tax Disputes

In many jurisdictions, the tax authority can dispute the tax position claimed by the taxpayer and demand application of a higher effective tax rate, which could include interest and a fine or penalty for underpayment of tax. At the same time, a taxpayer has the right to challenge the decision of the tax authority. Either side can bring the case to the court, along with additional information, or evidence in support of their claims, possibly with the use of tax expert advice. The balance of evidence can change the likelihood of the taxpayer winning the case. In other words, the efforts of the two sides in a tax dispute can change the uncertainty over the tax position. This, in turn, will change the investment allocation in equilibrium and, thus, the equilibrium welfare. Suppose, the existing uncertainty is distortionary. The question of interest is under what conditions a tax dispute can reduce the distortion and whether the welfare will increase as a result. To analyze the tax uncertainty and welfare implications of tax disputes I embed the model presented in the previous section in a version of Tullock's framework of rent-seeking contest.

The tax authority is aware of the investment options available to the taxpayer and can challenge the tax position in court. Knowing about this possibility, the taxpayer can acquire additional information or hire a tax advisor. The tax authority can also acquire additional information or hire a tax expert, if it decides to take the case to the court. The efforts of the parties affect the probability of the court's decision in favor of the taxpayer. With probability $q \in [0, 1]$ the court will declare that the profit from investment in sector 1 is eligible for the effective tax rate of τ_L, and with probability $(1 - q)$ it will be declared ineligible, in which case the effective tax rate (inclusive of fines or penalties) of τ_H will apply. As in the previous section, $\tau_H > \tau_0 > \tau_L$, where τ_0 is the tax rate applied with certainty to the investment in sector 0. Depending on the resources spent by the parties on presenting their case, the ex-post (after tax dispute) probability q of the tax position being sustained, can be higher or lower than p, the ex-ante (before tax dispute) probability.

As in the previous section, the taxpayer invests fraction λ of her capital in the activity in sector 1 with potential tax savings and the rest in sector 0. The tax authority challenges the tax position at cost I_R of hiring a tax advisor or collecting evidence to be presented at the court. The taxpayer pays cost I_T of information and expert advice on the tax-saving opportunity. I will refer to I_R and I_T as the litigation costs or the tax dispute costs, that is any information obtained by the parties about the tax position contributes to the evidence they present to the judge. The taxpayer and the tax authority choose I_R and I_T, each to maximize their own objective. If the two sides take the case to the court, the judge reviews evidence from the taxpayer and the tax authority and makes a decision. Regardless of the outcome, each party pays only its own litigations costs; this is referred to as the American rule (under the English rule, analyzed in Section 6, the losing party pays all legal costs of both parties).

The probability q that the judge's decision will be in favor of the taxpayer depends on the ex-ante probability p that the activity is eligible for tax saving, and on the litigation efforts of the parties in the court, measured by the cost of evidence, I_T and I_R, acquired by the taxpayer and the tax authority.

To illustrate the properties of the equilibrium, I assume that the effect of the evidence presented in court on the probability of the taxpayer winning the case is described by the following:

Assumption 2.

$$q(p, I_T, I_R) = (1 - \theta)p + \theta f(I_T, I_R), \theta \in (0, 1); \tag{6}$$

$$f(I_T, I_R) = \frac{\delta I_T}{\delta I_T + I_R}, \delta > 0. \tag{7}$$

Assumption 2 states that the probability that after hearing the two parties the judge will decide in the favor of the taxpayer is the weighted average of the ex-ante probability and an adjustment function, $f(I_T, I_R)$. A higher ex-ante probability p makes it more likely that the decision will be in the taxpayer's favor. The adjustment in the probability of allowing tax saving is modeled as in Tullock (1975), and it is increasing in the taxpayer's effort and decreasing in the tax authority's effort. Thus, for a given p, the probability of winning the case for the taxpayer is higher, the higher are her own litigation efforts and the lower are the litigation efforts of the tax authority.

Parameter δ measures taxpayer's litigation ability relative to that of tax authority, or, alternatively, the weight of the taxpayer's evidence relative to the weight of the tax authority's evidence: $\delta > 1$ ($\delta < 1$) means that, for the same nominal cost of evidence and expert advice, the taxpayer's litigation efforts are more (less) effective in the courts than the tax authority's efforts. Parameter θ reflects the responsiveness of the judge to the evidence presented by the parties; higher (lower) θ means higher (lower) responsiveness. The two parameters can also be viewed as the characteristics or the design parameters of the tax dispute system. Thus, an increase (decrease) in δ can be interpreted as making it easier (harder) for the taxpayer to defend their tax position. Similarly, an increase (decrease) in θ could mean a higher (lower) degree of evidence-based discretion exercised by the judge in the tax dispute case.

I am interested in an interior solution, that is, the equilibrium in which the filed tax return is taken to court because the expected net gain for each party from litigation is nonnegative. I further assume that an interior equilibrium ($q \in (0, 1)$, $\lambda \in (0, 1)$, $I_T > 0$, $I_R > 0$) exists and focus on the analysis of its properties.

Given the probability q that the judge will decide in favor of the taxpayer, the payoff V_R of the tax authority is the expected tax revenue less the litigation cost,

$$V_R = -I_R + \tau_0\pi_0(1-\lambda) + [q\tau_L + (1-q)\tau_H]\pi_1(\lambda).$$

The payoff V_T of the taxpayer is the expected net of tax profit less the litigation cost,

$$\begin{aligned} V_T = &-I_T + (1-\tau_0)\pi_0(1-\lambda) \\ &+ [q(1-\tau_L) + (1-q)(1-\tau_H)]\pi_1(\lambda). \end{aligned}$$

Even though tax returns are typically audited some time after the investment activity has taken place (e.g., in the UK the typical lag is up to 18 months), it is plausible to assume that, when the case is taken to the court, the taxpayer and the tax authority do not observe each other's efforts in acquiring evidence to support their positions in the tax dispute. In the game-theoretic setup, it means that the decisions by the parties on their litigation expenditures are taken simultaneously. For the timing of the investment decision I analyze two possible scenarios: when the taxpayer makes it simultaneously with the litigation expenditure decision or prior to that. I refer to the former situation as the simultaneous decisions, or simultaneous choice, and to the latter as the sequential decisions, or sequential choice.

4.1 | Simultaneous Decisions

In this section I assume that all decisions are made at the same time.

The tax authority chooses I_R to maximize V_R, taking the taxpayer's litigation cost, I_T, and investment choice, λ, as given. Setting $\frac{dV_R}{dI_R} = 0$ gives the private optimality condition for an interior equilibrium ($I_R > 0$):

$$[\tau_H - \tau_L]\pi_1(\lambda)\frac{\partial q}{\partial I_R} = -1. \tag{8}$$

The taxpayer chooses the investment allocation, λ, and the information expenditure, I_T, simultaneously to maximize V_T, taking I_R as given. The private optimality conditions for an interior equilibrium ($I_T > 0, 0 < \lambda < 1$) are given by

$$(\tau_H - \tau_L)\pi_1(\lambda)\frac{\partial q}{\partial I_T} = 1, \tag{9}$$

$$\frac{\pi_1'(\lambda)}{\pi_0'(1-\lambda)} = \frac{1-\tau_0}{q(\tau_H - \tau_L) + 1 - \tau_H}. \tag{10}$$

Solving Equations (8)–(10) simultaneously, along with Equations (6) and (7), gives the equilibrium levels of I_T, I_R, and λ as functions of p and the tax rates.

The welfare equals the surplus from the productive activity net of litigation costs:

$$W = V_T + V_R = \pi_0(1-\lambda) + \pi_1(\lambda) - I_T - I_R. \tag{11}$$

In equilibrium,

$$I_T^* = I_R^* = (\tau_H - \tau_L)\pi_1(\lambda^*)\frac{\theta\delta}{[1+\delta]^2}, \tag{12}$$

$$q^* = (1-\theta)p + \frac{\theta\delta}{1+\delta}, \tag{13}$$

which, upon substitution in Equation (11), gives the equilibrium welfare,

$$W^* = \pi_0(1-\lambda^*) + \pi_1(\lambda^*)\left[1 - 2\frac{\theta\delta}{[1+\delta]^2}(\tau_H - \tau_L)\right]. \tag{14}$$

The net payoff to the tax authority in the equilibrium is given by

$$\begin{aligned} V_R^* = \tau_0\pi_0(1-\lambda^*) + \Bigg[&q^*\tau_L + (1-q^*)\tau_H \\ &- (\tau_H - \tau_L)\frac{\theta\delta}{[1+\delta]^2}\Bigg]\pi_1(\lambda^*). \end{aligned} \tag{15}$$

4.2 | Sequential Decisions

Alternatively, one can assume that the investment decision is made before the tax authority decides to challenge the tax position. In this case the parties move sequentially. In the first stage the taxpayer makes the investment decision, and in the second stage the taxpayer and the tax authority choose their litigation expenditures. Thus, the investment decision will anticipate the equilibrium probability of the tax position being sustained in court. The game is solved by backward induction.

The solution for the optimal choice of litigation expenditure in the second stage is the same as in the model with simultaneous decisions, given by Equations (12) and (13).

The net payoff of the taxpayer in the first stage is then given by

$$V_T = (1-\tau_0)\pi_0(1-\lambda) + \left[\left[(1-\theta)p + \frac{\theta\delta^2}{[1+\delta]^2}\right](\tau_H - \tau_L) + (1-\tau_H)\right]\pi_1(\lambda), \tag{16}$$

and the taxpayer's investment choice is described by

$$\frac{\pi_1'(\lambda)}{\pi_0'(1-\lambda)} = \frac{1-\tau_0}{\left[(1-\theta)p + \frac{\theta\delta^2}{[1+\delta]^2}\right](\tau_H - \tau_L) + 1 - \tau_H}. \tag{17}$$

Solving this equation gives λ^*, which can then be substituted into Equation (12) to obtain I_T^* and I_R^* and into Equations (14) and (15) to calculate W^* and V_R^*.

5 | Welfare-Improving Tax Disputes

There is no reason for the tax uncertainty to be optimal at the time when taxpayer makes the investment decision: indeed, the probability being exactly at the welfare-maximizing level is a knife-edge case. Given that, in general, it can be either above or below the optimal level, the allocation is suboptimal and there is a welfare loss. A tax dispute can change the uncertainty, but it will not necessarily improve the welfare. On the one hand, it can move the probability closer to the optimal level, which would reduce investment distortion and thus lead to a higher profit. On the other hand, it requires that the parties spend resources on litigation. The total effect on the welfare is ambiguous. In this section I investigate the possibility that a dispute leads to a net welfare gain.

The tax dispute framework analyzed in this paper is an extension of Tullock's (1975) model to a setting of contest with an endogenous shared prize. In the classical model of a winner-takes-all symmetric contest with endogenous prize (Chung, 1996) it is assumed that the rent-seeking efforts are productive, meaning that the size of the prize is increasing in the aggregate efforts. Thus, each party's effort exerts simultaneously a negative and a positive externality on the expected payoff of each other party. The negative externality of each contestant's effort comes from the effort diminishing the probability of other parties winning the prize. The positive externality comes from the effort increasing the size of the prize for everyone. One of the main results in Chung (1996) is that in equilibrium the aggregate efforts are always excessive. Thus, rent-seeking leads to welfare loss even when the efforts are productive. This result was extended to a more general asymmetric setting, with stronger and weaker players (Damianov et al. 2018) and when players have different valuation of the prize and different (strictly increasing and weakly concave in expenses) efforts when the elasticity of the prize with respect to the aggregate efforts is less than one (Hirai 2020).

In this model, the "prize“ is the pre-tax profit from investment, and it is shared between the taxpayer and the tax authority in the litigation contest, where the taxpayer is the "stronger" ("weaker") player when δ is greater (less) than one. Because the choice of litigation efforts determines the court's decision about the effective tax rates, it thus affects the equilibrium allocation of investment and the resulting profits. Hence, the pre-tax profit depends on the parties' anticipated litigation costs. In the contest framework, the size of the prize in this model is, therefore, endogenous. The sharing rule is also endogenous, since the split of the prize (the pre-tax profits) between the parties (tax bill and after-tax profits) depends on the probability of the taxpayer winning the dispute.

Furthermore, as shown below, in this framework the parties' efforts can be either productive or destructive in equilibrium. That is, the direction of the effect of the aggregate effort on the size of the prize is endogenous. This is different from the standard contest literature where the direction of the effect is exogenous.

Another feature of this model is that when the parties' privately optimal efforts are productive in equilibrium, they can be either higher or lower than the socially optimal level. Finally, when the efforts are productive, the equilibrium welfare can be higher than in the absence of rent-seeking. Thus, the legal system that allows tax disputes can contribute to economic efficiency.

To see what drives this result, note that the model in Hirai (2020) includes, in particular, the family of power functions, $f_i(x_i) = a_i x_i^{r_i}$, with $r_i > 0, a_i > 0$, where x_i is player i's expense in the contest, $f_i(\cdot)$ is i's effort, the probability of i's winning the prize is $\frac{f_i(x_i)}{\sum_{j=1}^n f_i(x_i)}$, when the size of the prize is a strictly concave function of the *aggregate effort*, $\sum_{j=1}^n f_i(x_i)$. However, one can show that when the prize, instead, is a concave function of the *aggregate expense*, $\sum_{j=1}^n x_i$, as, for example, $(\sum_{j=1}^n x_i)^\alpha$ for some $\alpha \in (0,1)$, in equilibrium the aggregate expense, and, hence, the size of the prize, depending on the values of the model parameters, can be excessive ($r_i = r > \alpha$), efficient ($r_i = r = \alpha$), or deficient ($r_i = r < \alpha$). In other words, the welfare properties of the equilibrium may depend on the concavity of the effort relative to the concavity of the prize function.[9] In this paper, the efforts (I_T, I_R) are linear in expense, while the prize $\pi(\lambda(q(I_T, I_R)))$, is non-linear and depends on the parties' expenses separately. This opens a possibility that, as the model parameters change, the efforts can become productive or destructive, and the size of the prize can get closer to the efficient level or move further away.

5.1 | Productive and Destructive Equilibrium Litigation Efforts

As in the previous section, let p denote an arbitrary ex ante probability and let q denote the ex post probability.

[9] I am grateful to the anonymous reviewer for pointing out this feature and providing this example.

With either simultaneous or sequential moves, in equilibrium

$$q^* = [1 - \theta]p + \frac{\theta\delta}{1 + \delta}. \tag{18}$$

Note that the no-dispute case corresponds to $\theta = 0$: if a dispute does not change the ex ante probability neither party will exert litigation efforts, $I_t^* = I_R^* = 0$ (see equations (10)–(14)).

Let $\epsilon_i(\lambda) \equiv \frac{\lambda \pi_i'(\lambda)}{\pi(\lambda)}$ be the elasticity of profit in sector $i \in 0, 1$, and let $\psi_i(\lambda) \equiv \frac{\lambda \pi_i''(\lambda)}{\pi_i'(\lambda)}$ be the elasticity of marginal profit in sector i. Then, by Assumption 1, $\epsilon_i(\lambda) > 0$ and $\psi_i(\lambda) < 0$ for $\forall\, \lambda \in (0, 1)$.

Further, observe that at $\theta = 0$

$$\frac{\partial \pi(\lambda^*)}{\partial \theta} = \pi_1(\lambda^*) \frac{\epsilon_1(\lambda^*)}{-\psi_1(\lambda^*) - \frac{\lambda^*}{1-\lambda^*}\psi_0(1 - \lambda^*)} \frac{(\beta - p)(\phi - p)}{\frac{1-\tau_0}{\tau_H - \tau_L}\left(p + \frac{1-\tau_H}{\tau_H - \tau_L}\right)}, \tag{19}$$

where $\pi(\lambda^*) \equiv \pi_0(1 - \lambda^*) + \pi_1(\lambda^*)$ is the pre-tax profit, or the size of the prize, in equilibrium, $\phi = \frac{\delta}{1+\delta}$ under simultaneous decisions and $\phi = \frac{\delta}{(1+\delta)^2}$ under sequential decisions. This equation is obtained by differentiating Equations (10) and (17) for the simultaneous and the sequential case, respectively, and setting $\theta = 0$.

A dispute is enabled when $\theta > 0$. Thus, if $\frac{\partial \pi(\lambda^*)}{\partial \theta} > 0$ at $\theta = 0$, by continuity, there exists a neighborhood to the right from $\theta = 0$ where $\pi(\lambda^* | \theta) > \pi(\lambda^* | 0)$. That is, in this neighborhood the litigation efforts are productive in equilibrium. One can see from Equation (19) that the necessary and sufficient condition is $(\beta - p)(\phi - p) > 0$. This leads to the following result:

Proposition 2. *Let Assumption* 1 *hold. Then there exists* $\xi > 0$ *such that for* $\forall\, \theta \in (0, \xi)$ *the litigation efforts are productive in equilibrium if, and only if the ex ante probability* p *satisfies either* (*i*) $p < \min\{\beta, \phi\}$ *or* (*ii*) $p > \max\{\beta, \phi\}$.

Intuitively, starting from arbitrary ex ante p, a dispute increases the pre-tax profit if it shifts the probability towards its profit-maximizing value, β. The feature of the model is that whether the efforts in the contest are productive or destructive is determined endogenously in equilibrium. This is different from the standard models in the literature on contests with endogenous prize, where efforts are productive (or destructive) by assumption.

5.2 | Welfare Gains

Productive litigation efforts increase the pre-tax profit but not necessarily the net welfare because of the litigation cost. An important question is whether a net welfare increase is possible.

Observe that at $\theta = 0$

$$\frac{\partial W(\lambda^*)}{\partial \theta} = \pi_1(\lambda^*)\left[\frac{\epsilon_1(\lambda^*)}{-\psi_1(\lambda^*) - \frac{\lambda^*}{1-\lambda^*}\psi_0(1 - \lambda^*)} \frac{(\beta - p)(\phi - p)}{(1 - \tau_0)\left(p + \frac{1-\tau_H}{\tau_H - \tau_L}\right)} - \frac{2\delta(\tau_H - \tau_L)}{(1 + \delta)^2}\right]. \tag{20}$$

(The notations are the same as in the previous section.) This equation is obtained by differentiating Equation (14) with respect to θ, taking into account Equations (10) and (17) for the simultaneous and the sequential case, respectively, and setting $\theta = 0$.

A dispute is enabled when $\theta > 0$. Thus, if $\frac{\partial W(\lambda^*)}{\partial \theta} > 0$ at $\theta = 0$, by continuity, there exists a neighborhood to the right from $\theta = 0$ such that $W(\lambda^* | \theta) > W(\lambda^* | 0)$. In other words, it is possible that allowing tax disputes can increase the net welfare. The necessary condition is $(\beta - p)(\phi - p) > 0$, i.e. the litigation efforts are productive. This leads to the following result:

Proposition 3. *Let Assumption* 1 *hold and let the ex-ante probability* p *satisfy either* (*i*) $p < \min\{\beta, \phi\}$ *or* (*ii*) $p > \max\{\beta, \phi\}$. *Then there exists* $\varphi > 0$ *such that for* $\forall\, \theta \in (0, \varphi)$ *tax dispute raises welfare in equilibrium if, and only if*

$$\frac{-\psi_1(\lambda^*) - \frac{\lambda^*}{1-\lambda^*}\psi_0(1 - \lambda^*)}{\epsilon_1(\lambda^*)} < \frac{(1 + \delta)^2}{2\delta} \frac{(\beta - p)(\phi - p)}{(1 - \tau_0)\left(p + \frac{1-\tau_H}{\tau_H - \tau_L}\right)}. \tag{21}$$

For an arbitrary $\theta \in (0, 1)$, a dispute raises the total welfare when an increase in the pre-tax profits is higher than the total litigation cost in equilibrium (see Equation (12)):

$$\pi_0(1 - \lambda(q^*)) + \pi_1(\lambda(q^*)) - [\pi_0(1 - \lambda(p)) + \pi_1(\lambda(p))] > 2(\tau_H - \tau_L) \frac{\theta\delta}{[1 + \delta]^2} \pi_1(\lambda(q^*)). \tag{22}$$

The welfare effect depends on the shape of the profit function. For the numerical example illustrated below I used $\pi_1(\lambda) = \pi_0(\lambda) = \lambda^\gamma$, $\gamma \in (0, 1)$. In this case condition (21) takes the form

$$1 + \left(\frac{p(\tau_H - \tau_L) + 1 - \tau_H}{1 - \tau_0}\right)^{1/(1-\gamma)} < \frac{\gamma}{1 - \gamma} \frac{(1 + \delta)^2}{2\delta} \frac{(\beta - p)(\phi - p)}{(1 - \tau_0)\left(p + \frac{1-\tau_H}{\tau_H - \tau_L}\right)}.$$

The simulations show that the endogenously productive efforts in tax dispute lead to the net welfare gains over a substantial range of the model parameters. In all figures chosen for illustration $\tau_0 = 0.2$, $\tau_H = 0.5$, $\tau_L = 0.1$, and $\gamma = 0.9$.

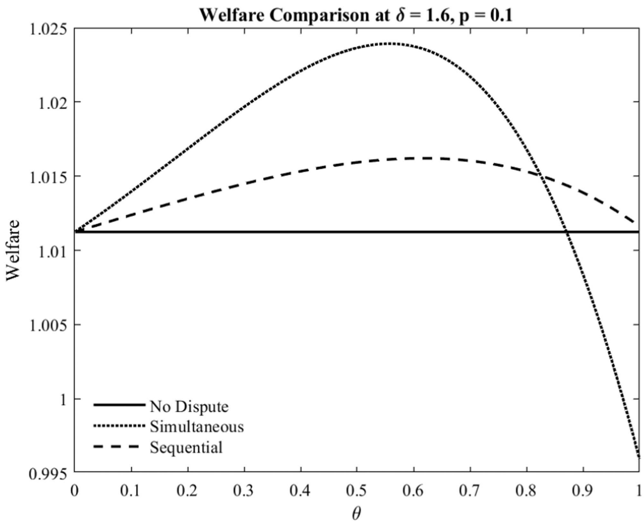

FIGURE 1 | Welfare without and with tax disputes under simultaneous and sequential choices, for $\pi(x) = \pi_0(x) = x^\gamma$, evaluated at $\gamma = 0.9, \tau_0 = 0.2, \tau_H = 0.5, \tau_L = 0.1$.

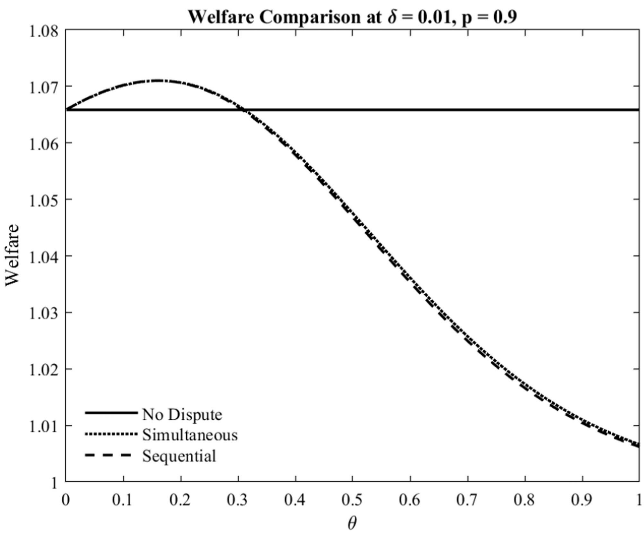

FIGURE 2 | Welfare without and with tax disputes under simultaneous and sequential choices, for $\pi(x) = \pi_0(x) = x^\gamma$, evaluated at $\gamma = 0.9, \tau_0 = 0.2, \tau_H = 0.5, \tau_L = 0.1$.

Figures 1 and 2 depict how the equilibrium welfare levels change with θ for two cases, $(\delta = 1.6, p = 0.1)$ and $(\delta = 0.01, p = 0.9)$, to illustrate the necessary condition for the tax disputes be welfare-improving. In both cases welfare is strictly increasing at $\theta = 0$. In the first case, the ex ante probability is farther away from the optimal value $\beta = 0.75$, and hence, a dispute delivers welfare gain over a larger range of θ, and this gain is higher, compared to the second case. Also, in the second case (small δ) the welfare levels under simultaneous and sequential choice are almost identical. This is because the investment allocations for small δ do not differ very much between the two scenarios (compare Equations (9) and (17) for δ close to zero). At the same time, in the first case (large δ) welfare gain is higher under simultaneous choice than under

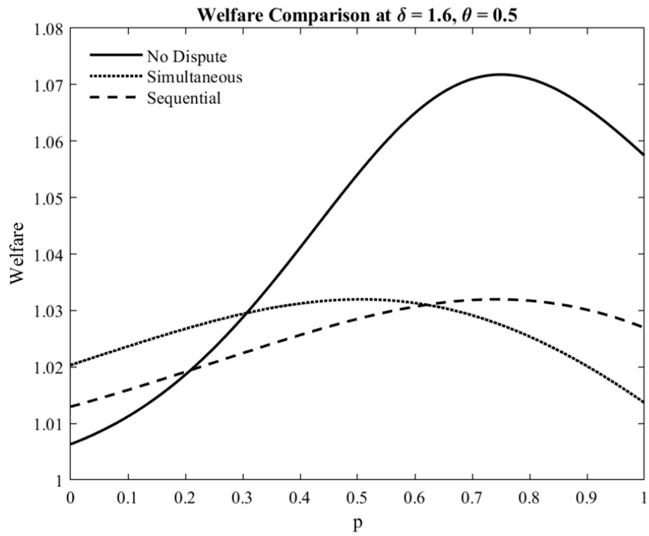

FIGURE 3 | Welfare without and with tax disputes under simultaneous and sequential choices, for $\pi(x) = \pi_0(x) = x^\gamma$, evaluated at $\gamma = 0.9, \tau_0 = 0.2, \tau_H = 0.5, \tau_L = 0.1$.

sequential choice for small and intermediate values of θ (ex ante probability has relatively high weight in the court's decision, whereas litigation efforts matter less) and turns into a loss for θ close to one (ex ante probability matter little).

Furthermore, for this configuration of parameters, a dispute delivers welfare gain over the full range of θ in the sequential case. One plausible explanation is that under sequential choice, the investment allocation is fixed when taxpayer chooses litigation effort evidence, and so the taxpayer does not internalize the full contemporaneous feedback from their own litigation choice to the movement in the success probability.[10] This, however, is reversed when the effect of the ex ante probability on the court's decision is small and the ex post probability is almost entirely determined by the parties' litigation efforts. Sequential choice enables the taxpayer to manipulate the tax authority's choice of the litigation effort to their own advantage by choosing investment in the first stage.

Figures 3 and 4 illustrate the possibility of net welfare gain over a range of values of the ex ante probability p for two values of θ: intermediate (0.5) and low (0.1). The first parametrisation, with relatively high δ (1.6), falls under condition (i) in Proposition 3; that is, it allows to obtain a range of cases in which the litigation efforts are productive in equilibrium when the ex ante probability is relatively low. In this case a tax dispute moves the probability up closer to the efficient level. The second parametrisation, with relatively low δ (0.01), falls under condition (ii) in Proposition 3: when the ex ante probability is relatively high, a dispute brings it down closer to the efficient level.

In the last two figures the curves trace the combinations of p and δ for which the welfare levels with and without disputes are equal, under simultaneous choice and under sequential choice

[10] I am grateful to the anonymous reviewer for this point.

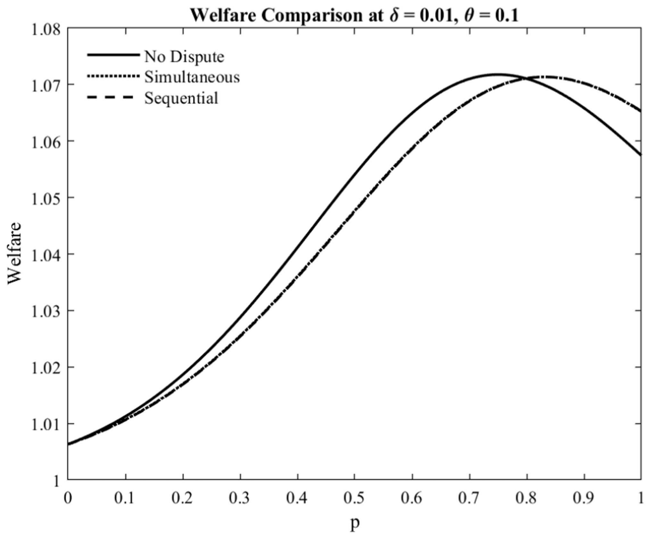

FIGURE 4 | Welfare without and with tax disputes under simultaneous and sequential choices, for $\pi(x) = \pi_0(x) = x^\gamma$, evaluated at $\gamma = 0.9$, $\tau_0 = 0.2$, $\tau_H = 0.5$, $\tau_L = 0.1$.

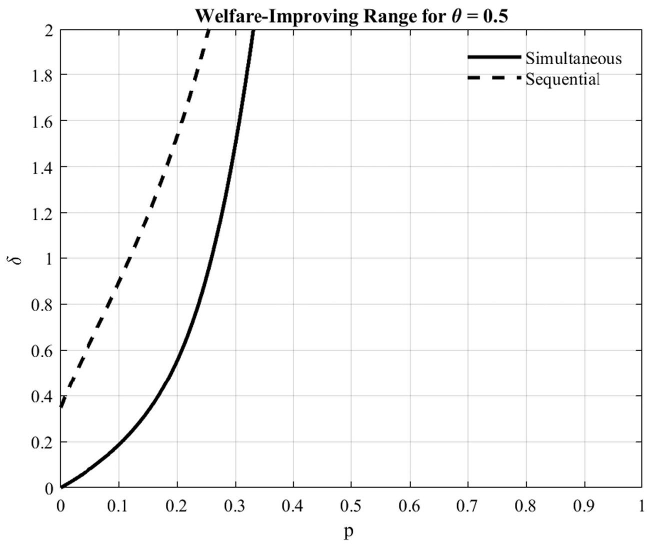

FIGURE 5 | Welfare effect of tax disputes under simultaneous and sequential choices: net gains (above the respective curve) and losses (below the respective curve) relative to no-dispute case, for $\pi(x) = \pi_0(x) = x^\gamma$, evaluated at $\gamma = 0.9$, $\tau_0 = 0.2$, $\tau_H = 0.5$, $\tau_L = 0.1$, and $\theta = 0.5$.

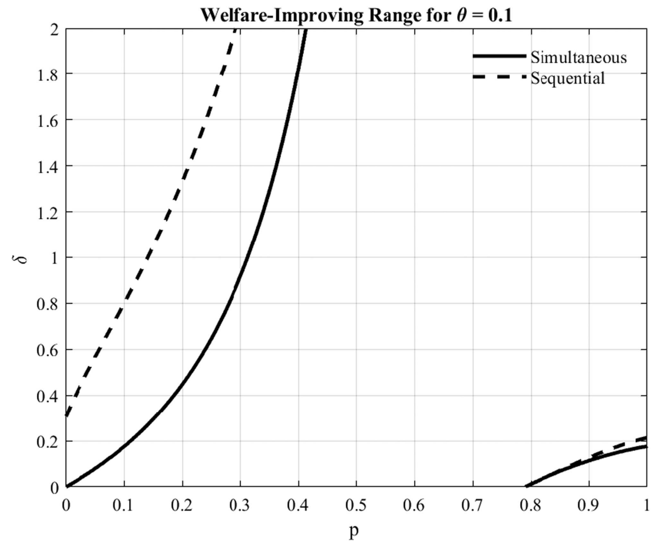

FIGURE 6 | Welfare effect of tax disputes under simultaneous and sequential choices: net gains (above the respective curve on the left and below on the right) and losses (between the respective curves) relative to no-dispute case, for $\pi(x) = \pi_0(x) = x^\gamma$, evaluated at $\gamma = 0.9$, $\tau_0 = 0.2$, $\tau_H = 0.5$, $\tau_L = 0.1$, and $\theta = 0.1$.

assumptions. In Figure 5, plotted for $\theta = 0.5$, tax disputes increase the total welfare in equilibrium over the range of p and δ above the respective curve and decrease it below the curve. Thus, for intermediate θ, if p is relatively low, a tax dispute is more likely to achieve welfare gain when δ is sufficiently high. In Figure 6, plotted for $\theta = 0.1$, in both scenarios there are two welfare-improving areas: above the respective curve on the left and below the curve on the right. That is, when θ is low, in addition to low-p-high-δ combinations tax disputes raise welfare for high-p-low-δ combinations, and the former region is also larger than for intermediate θ. Additional numerical investigation of the effect of θ appears to confirm the pattern that lower θ increases the range of $\{p, \delta\}$ where tax disputes achieve welfare gain, but this change is relatively small. One can also see that the welfare-increasing range of parameters is larger for the simultaneous choice than for the sequential choice for both values of θ in the low-p-high-δ, and the opposite is true in the high-p-low-δ area for low θ. The intuition here is similar to the one described above for the relative welfare gains.

Overall, the findings suggest that tax disputes are socially desirable in a variety of circumstances. When the judge puts relatively low weight on the parties' evidence in court, welfare improvement is likely in two scenarios. First, when the probability of the tax rate being low (in favor of the taxpayer) is small but it is relatively easy for the taxpayer to defend their position in court, strong litigation efforts push the probability sufficiently close to the efficient level, so that the gain outweighs the cost. The second scenario is the reverse of the first. When the probability of tax rate favoring the taxpayer is high but it is relatively difficult for them to defend their position in court, the latter discourages excessive litigation efforts and, hence, also makes it possible for the gain from a shift towards the efficient probability level to outweigh the litigation cost. The first scenario becomes less likely, while the second scenario disappears when the weight on the parties' evidence in court rises. Intuitively, higher weight on evidence encourages excessive litigation efforts, and, as a result, the opportunities for welfare improvement shrink.

6 | Robustness to Litigation System Design

The analysis presented so far assumes that each party pays only its own litigation expenses, regardless of the outcome, i.e. the so-called American rule. Under the so-called English rule, the losing party reimburses the winner for legal costs, in addition to paying its own costs (Baye et al. 2005). In this section I present the results obtained in this alternative setting under simultaneous choice and show that despite the change in the incentive structure, it is still possible for a tax dispute to achieve welfare improvement.[11]

The objective functions of the taxpayer and the tax authority under the English Rule are:

$$V_T = -(1-q)(I_T + I_R) + (1-\tau_0)\pi_0(1-\lambda) + [q(1-\tau_L) + (1-q)(1-\tau_H)]\pi_1(\lambda), \tag{23}$$

and

$$V_R = -q(I_T + I_R) + \tau_0\pi_0(1-\lambda) + [q\tau_L + (1-q)\tau_H]\pi_1(\lambda). \tag{24}$$

The welfare is the same as before, that is, pre-tax profit net of litigation costs.

When all decisions are made simultaneously, the first-order conditions for an interior equilibrium are given by the following equations:

$$\frac{\pi_0'(1-\lambda)}{\pi_1'(\lambda)} = \frac{q(\tau_H - \tau_L) + 1 - \tau_H}{1-\tau_0}, \tag{25}$$

$$\frac{1}{1-q}\frac{\partial q}{\partial I_T} = \frac{1}{(\tau_H - \tau_L)\pi_1(\lambda) + I_t + I_R}, \tag{26}$$

$$-\frac{1}{q}\frac{\partial q}{\partial I_R} = \frac{1}{(\tau_H - \tau_L)\pi_1(\lambda) + I_T + I_R}. \tag{27}$$

Here, again, the focus is on an interior solution exists, denoted λ^*, I_T^*, and I_R^*, assuming it exists. As in the previous section, to show that tax dispute can increase equilibrium welfare I calculate $\frac{\partial W}{\partial \theta}$ evaluated in equilibrium and set $\theta = 0$. The derivatives of λ^*, I_T^*, and I_R^* are obtained by differentiating the first-order conditions.

Straightforward rearrangement gives the following expression:

$$\left.\frac{\partial W^*}{\partial \theta}\right|_{\theta=0} = A(1-\delta)\pi_1(\lambda^*)\left[1 - \frac{-\psi_1(\lambda^*) - \frac{\lambda^*}{1-\lambda^*}\psi_0(1-\lambda^*)}{\epsilon_1(\lambda^*)} \frac{p}{p + \frac{1-\tau_H}{\tau_H - \tau_L}}\left(1 - (\beta - p)\frac{\tau_H - \tau_L}{1-\tau_0}\right)\right],$$

where $A \equiv \frac{(1-p)(1-2p(1-p))}{1-p+\delta p}(\tau_H - \tau_L) \geq 0 \ \forall\ p \in (0,1)$ and $A = 0$ in a knife-edge case $p = 1/2$. Therefore, in general, it is possible to have $\frac{\partial W^*}{\partial \theta}|_{\theta=0} > 0$ in two cases:

$$0 < \delta < 1; \quad \frac{-\psi_1(\lambda^*) - \frac{\lambda^*}{1-\lambda^*}\psi_0(1-\lambda^*)}{\epsilon_1(\lambda^*)} \frac{p}{p + \frac{1-\tau_H}{\tau_H - \tau_L}}\left(1 - (\beta - p)\frac{\tau_H - \tau_L}{1-\tau_0}\right) < 1$$

$$\delta > 1; \quad \frac{-\psi_1(\lambda^*) - \frac{\lambda^*}{1-\lambda^*}\psi_0(1-\lambda^*)}{\epsilon_1(\lambda^*)} \frac{p}{p + \frac{1-\tau_H}{\tau_H - \tau_L}}\left(1 - (\beta - p)\frac{\tau_H - \tau_L}{1-\tau_0}\right) > 1$$

Whether or not these inequalities can hold for some configuration of the model parameters depends on the shape of the profit functions. Moreover, one could argue that the first case is more likely for $p < \beta$, and the second case is more likely for $p > \beta$. For the power profit function, $\pi_1(\lambda) = \pi_0(\lambda) = \lambda^\gamma, \gamma \in (0,1)$ these conditions simplify to

$$0 < \delta < 1; \quad \frac{1-\gamma}{\gamma}\left[1 + \left(\frac{p(\tau_H - \tau_L) + 1 - \tau_H}{1-\tau_0}\right)^{1/(1-\gamma)}\right] \frac{p}{p + \frac{1-\tau_H}{\tau_H - \tau_L}}\left(1 - (\beta - p)\frac{\tau_H - \tau_L}{1-\tau_0}\right) < 1$$

$$\delta > 1; \quad \frac{1-\gamma}{\gamma}\left[1 + \left(\frac{p(\tau_H - \tau_L) + 1 - \tau_H}{1-\tau_0}\right)^{1/(1-\gamma)}\right] \frac{p}{p + \frac{1-\tau_H}{\tau_H - \tau_L}}\left(1 - (\beta - p)\frac{\tau_H - \tau_L}{1-\tau_0}\right) > 1$$

Since the expressions in the left-hand side of the second inequalities in each line are strictly positive, neither of these conditions can be ruled out in principle, and it is possible to find a configuration of parameters that satisfies these inequalities, thus leading to $\frac{\partial W^*}{\partial \theta}|_{\theta=0} > 0$. Therefore, by continuity, $\exists\ \varepsilon > 0$ such that $W(\lambda^*|\theta) > W(\lambda^*|0)\ \forall\ \theta \in (0, \varepsilon)$: a tax dispute can be welfare-improving.

7 | Concluding Remarks

I have shown in this paper that a dispute over an uncertain tax position can improve economic efficiency. In a model of

[11] A similar calculation and reasoning applies to the sequential choice case; the expressions are slightly lengthier without offering much of additional insight, and are therefore omitted.

investment allocation decision and uncertain tax rates, a costly tax dispute leads to net welfare gains, despite the litigation costs, by changing the degree of uncertainty in the direction that induces a more efficient allocation of the taxpayer's productive investment. I identify the conditions when litigation efforts increase the pre-tax profit by improving investment allocation sufficiently to outweigh the cost. The rise in welfare is achieved in the spirit of the second-best policy, by reducing the initial inefficiency in allocation, even though the instrument itself is unproductive. These results provide economic efficiency rationale for tax disputes and outline the circumstances where tax disputes are socially desirable. Costly litigation efforts destroy the rent from higher profit. The more weight in the court is given to the parties' evidence, the higher is the incentive for rent-dissipating activities. The net gain is more likely to be achieved either when the initial allocation is very far from being efficient or when the high cost discourages excessive litigation efforts.

The model can be further extended to explore the welfare properties of a continuum of fee-shifting rules (Baye et al. 2005), or to analyze distributional consequences of tax uncertainty in a setting with heterogeneous taxpayers facing different costs of evidence in tax disputes (Kaplow 1998). Another interesting extension of the model would be introducing the market for tax advice (Li et al. 2023) to compare the outcomes of different market structures on the welfare and revenue and to investigate the effect of market regulation when the price of tax advice is determined endogenously in the equilibrium.

In this paper, for analytical tractability I focus on a single risk-neutral taxpayer and a risk-neutral tax authority, and assume that litigation cost does not reduce the amount of resources available for productive investment. This is consistent with the interpretation of the taxpayer as a profit-maximizing firm without hard budget constraint. Thus, the uncertainty matters through distortion of the expected marginal incentives to invest. An interesting question is whether welfare improvement can be obtained in a setting with risk-averse agents, where uncertainty adds risk premium to the marginal distortion, and the taxpayer is facing hard budget constraint or limited liability.[12] On the one hand, the expected utility loss from uncertainty, or mean-preserving spread in the tax rates, for risk-averse agents can be greater than for risk-neutral agents. At the same time, with risk-averse agents the rent dissipation tends to fall as the degree of risk aversion increases (see, e.g., the results in competitive rent-seeking setting by Hillman and Katz 1984 and in the Tullock contest setting by Long and Vousden 1987, Skaperdas and Gan 1995, and Cornes and Hartley 2003, 2012). The analysis of how these two effects interact in a tax dispute setting where both the prize and the shares are determined endogenously, as in this paper, is left for future research.

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The author declares no conflicts of interest.

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Appendix A

Efficiency in the Long-Run

In this section, I outline how the framework can be extended to a repeated interaction between taxpayers and tax authorities. If the results of tax disputes become publicly available, taxpayers and tax authorities can use this information to update their beliefs about the probabilities of the effective tax rates. Let us assume that, once a sufficiently large number of tax dispute outcomes for a particular investment opportunity is known, the ex ante probability, p, is updated to the ex post value, q. The latter could be estimated, for example, by the proportion of disputes resolved in favor of the investor. If the institutional environment and the strategic behavior of the investors and the tax authorities remain the same, then, assuming $q_t = (1-\theta)p_t + \frac{\theta\delta}{1+\delta}$ and $p_{t+1} = q_t$, one can solve for $p_{\text{LR}} \equiv \lim_{t\to\infty} p_t$. It can be easily seen that in the long run the probability converges to a constant,

$$p \to p_{\text{LR}} \equiv \frac{\delta}{1+\delta}.$$

Parameters θ and δ are, effectively, two characteristics of tax disputes, which measure the weight of the evidence presented by the two parties in the court (θ) and the weight of taxpayer's evidence relative to the tax authority's evidence (δ), or the ability of the taxpayer in exerting litigation efforts. An interesting question is, then, about the set of (θ, δ) that maximize long-run welfare or tax revenue. To answer this question one needs to calculate welfare- and revenue-maximizing p. The resulting expressions are shown in Table A1.

In addition, Table A2 shows the relationship between θ and δ, as the main characteristics of tax dispute leading to maximal welfare and maximal revenue in the long run in a repeated setting. One can see that it is only possible ($\theta > 0$) when $\frac{\delta}{1+\delta} < \beta$, i.e. in case (*i*) of Proposition 3. Table A3 shows that the relationship between θ and δ in the welfare-maximizing tax dispute is negative (it is plausible to assume that $\tau_0 < 1/2$). The intuition here is the same as for the one-off interaction: the easier it is for the taxpayer to litigate (the higher the value of δ), the less weight the judge should assign to the parties' evidence in court (the lower the value of θ should be) to discourage excessive litigation expenses and thus to "keep the lid" on rent dissipation.

TABLE A1 | Ex ante interior welfare- and revenue-maximizing probabilities; $\beta = (\tau_H - \tau_0)/(\tau_H - \tau_L)$.

	Maximal welfare	Maximal revenue
No tax disputes	$p = \beta$ (both)	
Simultaneous choice	$p = \frac{\beta}{1-\theta} - \frac{\theta}{1-\theta}\frac{\delta}{1+\delta}\left[1 + \frac{2[1-\tau_0]}{1+\delta}\right]$	$p = \frac{\beta}{1-\theta} - \frac{\theta}{1-\theta}\left[\frac{\delta}{1+\delta}\right]^2\left[1 + \frac{2-\tau_0}{\delta}\right]$
Sequential choice	$p = \frac{\beta}{1-\theta} - \frac{\theta}{1-\theta}\left[\frac{\delta}{1+\delta}\right]^2\left[1 + \frac{2[1-\tau_0]}{\delta}\right]$ (both)	

TABLE A2 | Welfare- and revenue-maximizing tax dispute parameters; $\beta = (\tau_H - \tau_0)/(\tau_H - \tau_L)$.

	Maximal long-run welfare	Maximal long-run revenue
Simultaneous choice	$\theta = \frac{1+\delta}{2[1-\tau_0]}\left[\beta\frac{1+\delta}{\delta} - 1\right]$	$\theta = \frac{1+\delta}{1-\tau_0}\left[\beta\frac{1+\delta}{\delta} - 1\right]$
Sequential choice	$\theta = \frac{1+\delta}{1-2\tau_0}\left[\beta\frac{1+\delta}{\delta} - 1\right]$ (both)	

TABLE A3 | Relationship between θ and δ in the long run.

	Maximal long-run welfare	Maximal long-run revenue
Simultaneous choice:	$\frac{\partial\theta}{\partial\delta} = -\frac{\beta + [1-\beta]\delta^2}{2[1-\tau_0]}$	$\frac{\partial\theta}{\partial\delta} = -\frac{\beta + [1-\beta]\delta^2}{[1-\tau_0]\delta^2}$
Sequential choice:	$\frac{\partial\theta}{\partial\delta} = -\frac{\beta + [1-\beta]\delta^2}{[1-2\tau_0]\delta^2}$ (both)	