Reputation-Aware Distributed Filtering for Nonlinear Bias-Corrupted Systems Over Sensor Networks Under Event-Triggered Mechanism

Chaoqing Jia, Zidong Wang, Jun Hu, and Hongli Dong

***Abstract*—This paper is concerned with the reputation-aware Kalman-type distributed filtering (RAKTDF) problem for a class of nonlinear bias-corrupted systems (NBCSs) with a dynamical event-triggered mechanism (DETM). First, a representative unknown input, namely the dynamical bias, is introduced, which is evolved by a dynamical equation with Gaussian white noise. A DETM is employed to regulate the frequency of data transmission so that data conflict and network congestion are avoided. In order to identify and eliminate abnormal data from neighbors, a trust-based scoring strategy, referred to as the reputation-aware mechanism, is modeled and utilized to improve the accuracy of the filtering algorithm. The RAKTDF algorithm is recursively developed such that the covariance upper bound of the filtering error dynamic (CUBFED) is derived, after which the filter gain is determined by minimizing the trace of the CUBFED. Furthermore, a sufficient condition is established to guarantee the boundedness of the filtering error dynamic. Finally, an illustrative example is provided to verify the effectiveness of the proposed RAKTDF algorithm.**

***Index Terms*—Distributed filtering, wireless sensor networks, dynamical bias, reputation-aware scheme, event-triggered mechanism.**

I. Introduction

The past few decades have been marked by tremendous progress in wireless sensor networks (WSNs), which has primarily been attributed to their extensive applications in target surveillance, smart homes, hazardous/fault detection, weather forecasting, underwater vehicles, micro-environmental monitoring, and numerous other domains [1], [4], [16]. As is commonly acknowledged, WSNs are generally composed of many spatially distributed micro-sensors that are equipped with sensing, computing, and wireless communication capabilities, where each sensor is typically connected to other sensors under a predefined network topology. It has been particularly noted that the design of distributed filtering algorithms has been regarded as a fundamental and pivotal task in WSNs, and considerable attention has been devoted to this topic in recent years; see [22], [25], [26], [43], [51], [53] for further details.

It should be pointed out that no data processing center is included within the framework of distributed filtering algorithms. Instead, each sensor node is regarded as a fusion center, being responsible for measuring the system states, collecting local information, and facilitating communication with neighbors; see [14], [36], [37], [52], [54]. Recently, the commonly adopted distributed filtering approaches have encompassed, but are not restricted to, H_∞ distributed filtering [6], [24], [38], [45], set-membership distributed filtering [21], [28], [46], and Kalman-type distributed filtering [2], [18], [32], [41], [55]. Among these, Kalman-type distributed filtering has frequently been employed to address the filtering problem subject to Gaussian noises with known statistical characteristics. For instance, event-triggered distributed Kalman-type filtering problems have been investigated in [8] and [17], where boundedness analyses have specifically been provided.

The inherent distributed nature of WSNs has posed significant challenges in identifying and removing abnormal neighboring information caused by hardware malfunctions, sudden environmental changes, or malicious attacks. To ensure desirable filtering accuracy, extensive efforts have been devoted to the development of filtering algorithms under various malicious attacks [11], including denial-of-service attacks, deception attacks, replay attacks, and others. For instance, a consensus filtering algorithm has been proposed in [15] for a class of linear time-invariant systems subject to replay attacks, where a sufficient condition has been established to guarantee the detectability of such attacks. In addition, the trust-based scoring mechanism has been regarded as an effective strategy for identifying and mitigating untrustworthy data, in which a predefined scoring principle is provided to assign different reputation scores, reflecting both data credibility and historical performance [33], [34]. Despite the remarkable potential of the aforementioned trust-based scoring mechanism, only a limited number of results have been reported on reputation-aware distributed filtering problems. In this context, an initial attempt has been made in this work to address such research,

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thereby narrowing the noticeable gap.

In engineering practice, dynamical systems may be subjected to various disruptive inputs, including electromagnetic perturbations, cyber attacks, air resistance, component faults, and imperfect calibrations. Dynamical bias, a distinctive type of disturbance caused by unknown inputs, is characterized by a dynamical equation driven by Gaussian white noise. The presence of dynamical bias has considerably increased the difficulty and complexity of designing distributed filtering algorithms. As a result, extensive research related to the filtering problem with dynamical bias has been conducted and reported; see [12], [20], [29], [39] for more details and the references therein. At present, state-bias augmentation has been regarded as a typical technique for properly handling stochastic bias. For instance, the delay-compensation filtering problem for a class of coupled complex networks with dynamical bias has been addressed in [13], where an augmented model involving both system state and stochastic bias has been constructed. Moreover, the filter gain has been parameterized to ensure the optimality of the developed filtering algorithm.

For the purpose of improving transmission efficiency and avoiding resource occupation, the so-called event-triggered scheduler has frequently been utilized to govern data transmission through a pre-designed scheduling principle [3], [5], [9], [10], [27], [30], [40], [47], [48]. The prevalent event-triggered strategies have primarily been categorized into static and dynamical types, which provide a suitable compromise between shared resource utilization and filtering performance. To date, extensive studies have been conducted on the design of filtering algorithms subject to event-triggered mechanisms [31], [35], [44], [49]. For instance, the remote state estimation problem has been investigated in [7] for a class of time-varying systems with packet loss, where a static-type event-triggered strategy has been employed to reduce communication frequency and guarantee estimation accuracy. Furthermore, a stochastic event-triggered parameter has been constructed in [23], on the basis of which the event-triggered function has been designed and an attack-resistant remote filtering algorithm has been developed.

Motivated by the aforementioned discussions, a reputation-aware Kalman-type distributed filtering (RAKTDF) algorithm suitable for a kind of nonlinear bias-corrupted systems (NBCSs) with dynamical event-triggered mechanism (DETM) is recursively designed. The key difficulties and challenges encountered can be summarized as follows:

1) a reputation-aware-based evaluation mechanism is given to distinguish and eliminate untrusted data affected by sudden environmental changes or equipment malfunctions;
2) the covariance upper bound of the filtering error dynamic (CUBFED) is recursively obtained, whose trace is minimized by selecting the filter gain in the sense of minimum mean-square error;
3) a performance analysis is elaborated to keep the filtering error uniformly bounded.

In address of the difficulties and challenges outlined above, the main motivations and contributions of this paper are accordingly summarized as follows.

1) An intuitive reputation-aware model is constructed to suppress and get rid of the untrustworthy information that typically stems from hardware failures or environmental changes, where a mean-based credibility calculation principle is determined to assign the corresponding reputation scores to the neighbors.
2) By dealing with the reputation mechanism, dynamical bias, DETM, and noise interference, the recursion with respect to the CUBFED is obtained. In addition, the trace of CUBFED as an optimization index is minimized at each step via parameterizing the filter gain.
3) In terms of mild assumptions, a sufficient criterion is provided to ensure that the filtering error dynamic is uniformly bounded.

The organization of this paper is arranged as follows. Section II provides the problem formulation, concentrating on the RAKTDF problem for NBCSs subject to the reputation mechanism, dynamical bias, and DETM. The RAKTDF algorithm is presented in Section III, where the CUBFED is derived recursively and the filter gain is parameterized by optimizing the algorithm performance. Section IV contains the performance evaluation, in which the uniformly bounded property of the CUBFED is ensured. Section V presents a simulation example to verify the effectiveness of the proposed RAKTDF algorithm. Finally, the conclusion is given in Section VI.

Notations. The symbols in this paper are quite standard. $\mathbb{E}\{*\}$ denotes the mathematical expectation. The superscripts $(*)^{-1}$ and $(*)^T$ denote the inverse and transpose operations, respectively. The notation $\mathrm{tr}(*)$ represents the sum of the diagonal elements of a square matrix. 0 and I with appropriate dimensions denote the zero matrix and identity matrix, respectively.

II. Problem Formulation

In this paper, WSNs consisting of N sensor nodes are utilized to collect and process the target data, where the topology is denoted by a directed graph $\mathcal{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ with $\mathscr{V} = \{1, 2, \ldots, N\}$ being the set of nodes, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ representing the set of edges, and $\mathscr{A} = [a_{iz}]_{N\times N}$ $(a_{iz} \geq 0)$ denoting the adjacency matrix. The condition $a_{iz} > 0 \Leftrightarrow (i, z) \in \mathscr{E}$ indicates that the i-th node is able to receive data from its neighbor z. Furthermore, $\mathscr{N}_i = \{z \in \mathscr{V} | (i, z) \in \mathscr{E}\}$ denotes the neighbor set of the i-th node, and $|\mathscr{N}_i|$ represents the number of neighbors.

Consider the following NBCSs:

$$\vec{x}_{\ell+1} = \vec{A}_\ell \vec{x}_\ell + f(\vec{x}_\ell) + \vec{F}_\ell b_\ell + \vec{B}_\ell \vec{\omega}_\ell, \tag{1}$$

$$y_{i,\ell} = \vec{C}_{i,\ell} \vec{x}_\ell + \upsilon_{i,\ell}, \tag{2}$$

where $\vec{x}_\ell \in \mathbb{R}^{n_x}$ is the system state and $y_{i,\ell} \in \mathbb{R}^{n_y}$ denotes the measurement output. $\vec{A}_\ell$, $\vec{B}_\ell$. $\vec{F}_\ell$ and $\vec{C}_{i,\ell}$ are the system matrices with suitable dimensions. $\vec{\omega}_\ell \in \mathbb{R}^{n_\omega}$ is a sequence of zero-mean Gaussian white noise with covariance $\vec{Q}_\ell > 0$, and $\upsilon_{i,\ell}$ is a sequence of zero-mean Gaussian white noise with covariance $R_{i,\ell} > 0$.

For each $u_\ell \in \mathbb{R}^{n_x}$ and $d_\ell \in \mathbb{R}^{n_x}$, the nonlinearity $f(\vec{x}_\ell)$ satisfies $f(0) = 0$ and

$$\|f(u_\ell) - f(d_\ell) - H_\ell(u_\ell - d_\ell)\| \leq \alpha_\ell \|u_\ell - d_\ell\|, \tag{3}$$

where H_ℓ and α_ℓ are known parameters with proper dimensions. $b_\ell \in \mathbb{R}^{n_b}$ describes the dynamical bias satisfying

$$b_{\ell+1} = G_\ell b_\ell + \xi_\ell, \tag{4}$$

where G_ℓ is the known bias transition matrix and ξ_ℓ is a zero-mean noise sequence with covariance Θ_ℓ. In this paper, assume that $\vec{\omega}_\ell$, $\upsilon_{i,\ell}$ and ξ_ℓ are mutually independent.

The following notations are defined:

$$x_\ell = \begin{bmatrix} \vec{x}_\ell \\ b_\ell \end{bmatrix}, \ f(x_\ell) = \begin{bmatrix} f(\vec{x}_\ell) \\ 0 \end{bmatrix}, \ \omega_\ell = \begin{bmatrix} \vec{\omega}_\ell \\ \xi_\ell \end{bmatrix},$$
$$A_\ell = \begin{bmatrix} \vec{A}_\ell & \vec{F}_\ell \\ 0 & G_\ell \end{bmatrix}, \ C_\ell = \begin{bmatrix} \vec{C}_{i,\ell} & 0 \end{bmatrix},$$
$$B_\ell = \begin{bmatrix} \vec{B}_\ell & 0 \\ 0 & I \end{bmatrix}, \ Q_\ell = \begin{bmatrix} \vec{Q}_\ell & 0 \\ 0 & \Theta_\ell \end{bmatrix}.$$

By letting $x_\ell = [\vec{x}_\ell^T \quad b_\ell^T]^T$, the following system can be obtained:

$$x_{\ell+1} = A_\ell x_\ell + f(x_\ell) + B_\ell \omega_\ell, \tag{5}$$
$$y_{i,\ell} = C_{i,\ell} x_\ell + \upsilon_{i,\ell}. \tag{6}$$

To alleviate the communication burden, the DETM is adopted to adjust the communication frequency, where a time-varying triggering threshold is constructed. The triggering function is defined as

$$\hbar(\eta_{i,\ell}, \gamma_{i,\ell}, y_{i,\ell}) = \eta_{i,\ell}^T \eta_{i,\ell} - \gamma_{i,\ell} y_{i,\ell}^T \Lambda_{i,\ell} y_{i,\ell}, \tag{7}$$

where $\eta_{i,\ell} = y_{i,\theta_\ell} - y_{i,\ell}$ with y_{i,θ_ℓ} being the latest released measurement and θ_ℓ being the latest released instant. $\Lambda_{i,\ell} > 0$ is a known weighted matrix. $\gamma_{i,\ell}$ is the time-varying parameter designed as

$$\gamma_{i,\ell+1} = \bar{\gamma} + \frac{\eta_{i,\ell}^T \eta_{i,\ell}}{\psi + \eta_{i,\ell}^T \eta_{i,\ell}} (\gamma_{i,\ell} - \bar{\gamma}), \tag{8}$$

where $\bar{\gamma} \in [0, 1]$ with $0 \le \gamma_{i,\ell} \le \bar{\gamma}$ and $\psi > 0$ are known constants. The current measurement $y_{i,\ell}$ is released only when $\hbar(\eta_{i,\ell}, \gamma_{i,\ell}, y_{i,\ell}) > 0$, and the triggered instant is then updated as

$$\theta_{\ell+1} = \inf\{\ell \in \mathbb{N} | \ell > \theta_\ell, \hbar(\eta_{i,\ell}, \gamma_{i,\ell}, y_{i,\ell}) > 0\}.$$

Remark 1: On one hand, due to the characteristic drift of physical components or the uncertainty inputs (including electromagnetic perturbations, malicious attacks, air resistance, vibration response deviation and imperfect calibrations), the dynamical bias occurs frequently. If dynamical bias is not appropriately handled, it will seriously result in the filtering performance degeneration. In this paper, the dynamical bias is typically described in (4) by a dynamical equation driven by Gaussian white noise and is handled by joint state-bias augmentation technique in (5). On the other hand, a DETM is employed to govern data communication right for the purpose of reducing access frequency and relieving channel burden, where a time-varying triggered threshold is considered. Actually, the dynamical evolution of triggered threshold introduced in (8) is an extended version of the static situation. Obviously, the DETM will degenerate into a static-triggered mechanism when $\gamma_{i,0} = \bar{\gamma}$. Compared with the time-triggered situation, the DETM depends on the historical triggered information so as to avoid unnecessary occupation and waste of channel resources.

The following reputation-aware distributed filter is constructed:

$$\hat{x}_{i,\ell+1|\ell} = A_\ell \hat{x}_{i,\ell|\ell} + f(\hat{x}_{i,\ell|\ell}), \tag{9}$$
$$\hat{x}_{i,\ell+1|\ell+1} = \hat{x}_{i,\ell+1|\ell} + \Xi_{i,\ell+1}(y_{i,\theta_{\ell+1}} - C_{i,\ell+1}\hat{x}_{i,\ell+1|\ell}) + \sum_{z \in \mathscr{N}_i} a_{iz} \varphi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell}), \tag{10}$$

where $\hat{x}_{i,\ell|\ell}$ and $\hat{x}_{i,\ell+1|\ell}$ denote the estimation and prediction of x_ℓ at the i-th sensor, respectively. $\Xi_{i,\ell+1}$ is the filter gain to be determined, and $\varphi_{iz,\ell+1}$ represents the reputation parameter designed later.

A reputation scoring mechanism is proposed and modeled to enhance filtering accuracy by removing unreasonable data from neighbors. The scoring rule is defined as

$$\vec{\psi}_{iz,\ell+1} = 1 - \sum_{j \in \mathscr{N}_i \cup \{i\}} \frac{\|\hat{x}_{z,\ell+1|\ell} - \hat{x}_{j,\ell+1|\ell}\|}{|\mathscr{N}_i|} \tag{11}$$

and otherwise $\vec{\psi}_{iz,\ell+1} = 0$, with $\vec{\psi}_{iz,\ell+1}$ denoting the score assigned by node i to node z. $\hat{x}_{z,\ell+1|\ell}$ and $\hat{x}_{j,\ell+1|\ell}$ represent the local predictions at the z-th and j-th sensors, respectively. The value in (11) is subsequently normalized as

$$\psi_{iz,\ell+1} = \frac{\vec{\psi}_{iz,\ell+1} - \min_{j \in \mathscr{N}_i} \vec{\psi}_{ij,\ell+1}}{\zeta_{i,\ell+1}}, \tag{12}$$

where

$$\zeta_{i,\ell+1} = \max_{j \in \mathscr{N}_i} \vec{\psi}_{ij,\ell+1} - \min_{j \in \mathscr{N}_i} \vec{\psi}_{ij,\ell+1}.$$

Finally, the reputation value from node i to node z is determined as

$$\varphi_{iz,\ell+1} = \begin{cases} \psi_{iz,\ell+1}, & \text{if} \quad \zeta_{i,\ell+1} \ge \pi_{i,\ell+1} \\ \dfrac{1}{|\mathscr{N}_i|}, & \text{otherwise} \end{cases} \tag{13}$$

where $\pi_{i,\ell+1}$ is a positive scalar.

Remark 2: Within the framework of distributed filtering, the current filter is allowed to receive local information from its neighbors through a given communication topology. However, when hardware failures and cyber attacks are considered, the actual data received by the current filter are often untrustworthy, thereby leading to the deterioration of the filtering algorithm performance. Therefore, the real-time trust-based scoring strategy, namely the reputation-aware scheme, is typically employed in (11)–(13) to identify and discard the untrustworthy information through the shared and open wireless networks. Specifically, a mean-based reputation calculation principle is firstly presented in (11), where a node with a larger average deviation is regarded as an untrustworthy unit and is assigned a low reputation value. Then, a min-max normalization is adopted in (12) to map reputation score given in (11) to the range of 0 and 1. The parameter $\psi_{iz,\ell+1}$ gradually increases as the credibility increases, signifying that the current information is completely trustworthy when $\psi_{iz,\ell+1} = 1$. If $\psi_{iz,\ell+1} = 0$, the current information from neighbors is regarded as the unreliable data contaminated by equipment failure or malicious attacks and is discarded in the distributed filter. Finally, an adjustable parameter $\pi_{i,\ell+1}$ is introduced in (13) to characterize the uncertainty and robustness in many practical problems. In scenario $\zeta_{i,\ell+1} < \pi_{i,\ell+1}$, the real-time reputation score will degrade to a fixed weight in the traditional distributed filtering problems.

III. Main Results

This section is devoted to presenting the CUBFED and to designing the filter gain through the minimization of the trace of the CUBFED. To proceed, the following lemma is introduced.

Lemma 1: [36] There exist $h, g \in \mathbb{R}^n$ and real number $o > 0$ such that

$$hg^T + gh^T \leq ohh^T + o^{-1}gg^T. \tag{14}$$

First, the prediction and filtering errors are respectively expressed as

$$\begin{aligned} e_{i,\ell+1|\ell} =& x_{\ell+1} - \hat{x}_{i,\ell+1|\ell} \\ =& (A_\ell + H_\ell)e_{i,\ell|\ell} + \Upsilon_{i,\ell} + B_\ell \omega_\ell \end{aligned} \tag{15}$$

and

$$\begin{aligned} e_{i,\ell+1|\ell+1} =& x_{\ell+1} - \hat{x}_{i,\ell+1|\ell+1} \\ =& (I - \Xi_{i,\ell+1}C_{i,\ell+1})e_{i,\ell+1|\ell} - \Xi_{i,\ell+1}\eta_{i,\ell+1} \\ & - \Xi_{i,\ell+1}\upsilon_{i,\ell+1} - \sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,\ell+1}\psi_{iz,\ell+1} \\ & \times (\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell}) - \sum_{z\in\mathcal{N}_i} a_{iz} \\ & \times (1 - \hbar_{i,\ell+1})\frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell}), \end{aligned} \tag{16}$$

where $\Upsilon_{i,\ell} = f(x_\ell) - f(\hat{x}_{i,\ell|\ell}) - H_\ell e_{i,\ell|\ell}$ and $\hbar_{i,\ell+1}$ satisfies

$$\hbar_{i,\ell+1} = \begin{cases} 1, & \text{if } \zeta_{ij,\ell+1} \geq \pi_{ij,\ell+1} \\ 0. & \text{otherwise} \end{cases}$$

Next, the covariance matrices of the prediction and filtering errors are recursively derived as follows.

Lemma 2: The prediction error covariance and the filtering error covariance satisfy, respectively,

$$\begin{aligned} P_{i,\ell+1|\ell} =& (A_\ell + H_\ell)P_{i,\ell|\ell}(A_\ell + H_\ell)^T + B_\ell Q_\ell B_\ell^T \\ & + \mathbb{E}\big\{\Upsilon_{i,\ell}\Upsilon_{i,\ell}^T\big\} + H_{1,\ell} + H_{1,\ell}^T, \end{aligned} \tag{17}$$

and

$$\begin{aligned} P_{i,\ell+1|\ell+1} =& (I - \Xi_{i,\ell+1}C_{i,\ell+1})P_{i,\ell+1|\ell}(I - \Xi_{i,\ell+1}C_{i,\ell+1})^T \\ & + \Xi_{i,\ell+1}\mathbb{E}\{\eta_{i,\ell+1}\eta_{i,\ell+1}^T\}\Xi_{i,\ell+1}^T + \hbar_{i,\ell+1}^2 \\ & \times \mathbb{E}\Bigg\{\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg] \\ & \times \Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg]^T\Bigg\} \\ & + (1 - \hbar_{i,\ell+1})^2|\mathcal{N}_i|^{-2}\mathbb{E}\Bigg\{\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}(\hat{x}_{z,\ell+1|\ell} \\ & - \hat{x}_{i,\ell+1|\ell})\Bigg]\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg]^T\Bigg\} \\ & + \Xi_{i,\ell+1}R_{i,\ell+1}\Xi_{i,\ell+1}^T - J_{1,\ell+1} - J_{1,\ell+1}^T \\ & - J_{2,\ell+1} - J_{2,\ell+1}^T - J_{3,\ell+1} - J_{3,\ell+1}^T \\ & + J_{4,\ell+1} + J_{4,\ell+1}^T + J_{5,\ell+1} + J_{5,\ell+1}^T \\ & + J_{6,\ell+1} + J_{6,\ell+1}^T, \end{aligned} \tag{18}$$

where

$$\begin{aligned} H_{1,\ell} =& \mathbb{E}\big\{(A_\ell + H_\ell)e_{i,\ell|\ell}\Upsilon_{i,\ell}^T\big\}, \\ J_{1,\ell+1} =& \mathbb{E}\left\{(I - \Xi_{i,\ell+1}C_{i,\ell+1})e_{i,\ell+1|\ell}\eta_{i,\ell+1}^T\Xi_{i,\ell+1}^T\right\}, \\ J_{2,\ell+1} =& \mathbb{E}\Bigg\{(I - \Xi_{i,\ell+1}C_{i,\ell+1})e_{i,\ell+1|\ell}\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,\ell+1} \\ & \times \psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg]^T\Bigg\}, \\ J_{3,\ell+1} =& \mathbb{E}\Bigg\{(I - \Xi_{i,\ell+1}C_{i,\ell+1})e_{i,\ell+1|\ell}\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\frac{1}{|\mathcal{N}_i|} \\ & \times (1 - \hbar_{i,\ell+1})(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg]^T\Bigg\}, \\ J_{4,\ell+1} =& \mathbb{E}\Bigg\{\Xi_{i,\ell+1}\eta_{i,\ell+1}\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,\ell+1}\psi_{iz,\ell+1} \\ & \times (\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg]^T\Bigg\}, \\ J_{5,\ell+1} =& \mathbb{E}\Bigg\{\Xi_{i,\ell+1}\eta_{i,\ell+1}\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}(1 - \hbar_{i,\ell+1})\frac{1}{|\mathcal{N}_i|} \\ & \times (\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg]^T\Bigg\}, \\ J_{6,\ell+1} =& \mathbb{E}\left\{\Xi_{i,\ell+1}\eta_{i,\ell+1}\upsilon_{i,\ell+1}^T\Xi_{i,\ell+1}^T\right\}. \end{aligned}$$

Proof: According to the definitions of the covariance matrices, it follows that

$$\begin{aligned} P_{i,\ell+1|\ell} =& \mathbb{E}\{e_{i,\ell+1|\ell}e_{i,\ell+1|\ell}^T\} \\ =& (A_\ell + H_\ell)P_{i,\ell|\ell}(A_\ell + H_\ell)^T + B_\ell Q_\ell B_\ell^T \\ & + \mathbb{E}\big\{\Upsilon_{i,\ell}\Upsilon_{i,\ell}^T\big\} + H_{1,\ell} + H_{1,\ell}^T + H_{2,\ell} \\ & + H_{2,\ell}^T + H_{3,\ell} + H_{3,\ell}^T \end{aligned}$$

and

$$\begin{aligned} P_{i,\ell+1|\ell+1} =& \mathbb{E}\{e_{i,\ell+1|\ell+1}e_{i,\ell+1|\ell+1}^T\} \\ =& (I - \Xi_{i,\ell+1}C_{i,\ell+1})P_{i,\ell+1|\ell}(I - \Xi_{i,\ell+1}C_{i,\ell+1})^T \\ & + \Xi_{i,\ell+1}\mathbb{E}\{\eta_{i,\ell+1}\eta_{i,\ell+1}^T\}\Xi_{i,\ell+1}^T + \hbar_{i,\ell+1}^2 \\ & \times \mathbb{E}\Bigg\{\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg] \\ & \times \Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Bigg]^T\Bigg\} \\ & + (1 - \hbar_{i,\ell+1})^2|\mathcal{N}_i|^{-2}\mathbb{E}\Bigg\{\Bigg[\sum_{z\in\mathcal{N}_i} a_{iz}(\hat{x}_{z,\ell+1|\ell} \end{aligned}$$

$$
\begin{aligned}
&- \hat{x}_{i,\ell+1|\ell})\Big]\Big[\sum_{z\in\mathcal{N}_i} a_{iz}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]^T\Bigg\}\\
&+ \Xi_{i,\ell+1}R_{i,\ell+1}\Xi_{i,\ell+1}^T - J_{1,\ell+1} - J_{1,\ell+1}^T\\
&- J_{2,\ell+1} - J_{2,\ell+1}^T - J_{3,\ell+1} - J_{3,\ell+1}^T + J_{4,\ell+1}\\
&+ J_{4,\ell+1}^T + J_{5,\ell+1} + J_{5,\ell+1}^T + J_{6,\ell+1} + J_{6,\ell+1}^T\\
&+ J_{7,\ell+1} + J_{7,\ell+1}^T - J_{8,\ell+1} - J_{8,\ell+1}^T\\
&+ J_{9,\ell+1} + J_{9,\ell+1}^T + J_{10,\ell+1} + J_{10,\ell+1}^T,
\end{aligned}
$$

where

$$
\begin{aligned}
H_{2,\ell} =&\mathbb{E}\big\{(A_\ell + H_\ell)e_{i,\ell|\ell}\omega_\ell^T B_\ell^T\big\},\ H_{3,\ell} = \mathbb{E}\big\{\Upsilon_{i,\ell}\omega_\ell^T B_\ell^T\big\},\\
J_{7,\ell+1} =&\mathbb{E}\Bigg\{\sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,\ell+1}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\\
&\times\Big[\sum_{z\in\mathcal{N}_i} a_{iz}(1-\hbar_{i,\ell+1})\frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,\ell+1|\ell}\\
&- \hat{x}_{i,\ell+1|\ell})\Big]^T\Bigg\},\\
J_{8,\ell+1} =&\mathbb{E}\left\{\Xi_{i,\ell+1}\upsilon_{i,\ell+1}e_{i,\ell+1|\ell}^T(I - \Xi_{i,\ell+1}C_{i,\ell+1})^T\right\},\\
J_{9,\ell+1} =&\mathbb{E}\Bigg\{\Xi_{i,\ell+1}\upsilon_{i,\ell+1}\Big[\sum_{z\in\mathcal{N}_i} a_{iz}\hbar_{i,\ell+1}\psi_{iz,\ell+1}\\
&\times(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]^T\Bigg\},\\
J_{10,\ell+1} =&\mathbb{E}\Bigg\{\Xi_{i,\ell+1}\upsilon_{i,\ell+1}\Big[\sum_{z\in\mathcal{N}_i} a_{iz}\frac{1}{|\mathcal{N}_i|}\\
&\times(1-\hbar_{i,\ell+1})(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]^T\Bigg\}.
\end{aligned}
$$

It should be noted that $H_{2,\ell}$, $H_{3,\ell}$, $J_{7,\ell+1}$, $J_{8,\ell+1}$, $J_{9,\ell+1}$, and $J_{10,\ell+1}$ are all zero matrices. The proof is thus complete. ■

Remark 3: Note that the dynamical bias, DETM, and reputation-aware strategy are comprehensively considered and analyzed in the design of RAKTDF algorithm for NBCSs, which makes the covariance of filtering error dynamic inaccurate. Accordingly, a CUBFED will be derived by dealing with the cross-terms regarding DETM condition, reputation parameter and noise perturbation. Furthermore, the trace of the obtained CUBFED will be chosen as an optimization index to parameterize the filter gain in the sense of minimum mean-square error.

The following theorem gives the recursion of CUBFED and designs the filter gain.

Theorem 1: Consider the NBCSs in (5)–(6) and the constructed reputation-aware filter in (9)–(10). For scaling parameters $o_t > 0$ $(t = 1, 2, \ldots, 8)$ and $\alpha_\ell > 0$, assume that the following two matrix equations

$$
\begin{aligned}
\mathcal{P}_{i,\ell+1|\ell} =&(1+o_1)(A_\ell + H_\ell)\mathcal{P}_{i,\ell|\ell}(A_\ell + H_\ell)^T + (1+o_1^{-1})\\
&\times\alpha_\ell^2\mathrm{tr}(\mathcal{P}_{i,\ell|\ell})I + B_\ell Q_\ell B_\ell^T
\end{aligned}\tag{19}
$$

and

$$
\begin{aligned}
&\mathcal{P}_{i,\ell+1|\ell+1}\\
=&(1+o_2+o_3+o_4)(I - \Xi_{i,\ell+1}C_{i,\ell+1})\mathcal{P}_{i,\ell+1|\ell}\\
&\times(I - \Xi_{i,\ell+1}C_{i,\ell+1})^T + (1+o_2^{-1}+o_5+o_6+o_7)\\
&\times\Xi_{i,\ell+1}\vec{\Theta}_{i,\ell+1}\Xi_{i,\ell+1}^T + (1+o_7^{-1})\Xi_{i,\ell+1}R_{i,\ell+1}\\
&\times\Xi_{i,\ell+1}^T + (1+o_3^{-1}+o_5^{-1})\hbar_{i,\ell+1}\Big[\sum_{z\in\mathcal{N}_i} a_{iz}\\
&\times\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]\Big[\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,\ell+1}\\
&\times(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]^T + (1+o_4^{-1}+o_6^{-1})\\
&\times(1-\hbar_{i,\ell+1})|\mathcal{N}_i|^{-2}\Big[\sum_{z\in\mathcal{N}_i} a_{iz}(\hat{x}_{z,\ell+1|\ell}\\
&- \hat{x}_{i,\ell+1|\ell})\Big]\Big[\sum_{z\in\mathcal{N}_i} a_{iz}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]^T,
\end{aligned}\tag{20}
$$

admit solutions $\mathcal{P}_{i,\ell+1|\ell} > 0$ and $\mathcal{P}_{i,\ell+1|\ell+1} > 0$ under initial constraint $\mathcal{P}_{i,0|0} \geq P_{i,0|0} > 0$, where

$$
\begin{aligned}
\vec{\Theta}_{i,\ell+1} =&\gamma_{i,\ell+1}\mathrm{tr}\big((C_{i,\ell+1}\vec{\Phi}_{i,\ell+1|\ell+1}C_{i,\ell+1}^T\\
&+ R_{i,\ell+1})\Lambda_{i,\ell+1}\big)I,\\
\vec{\Phi}_{i,\ell+1|\ell} =&(1+o_8)\mathcal{P}_{i,\ell+1|\ell} + (1+o_8^{-1})\hat{x}_{i,\ell+1|\ell}\\
&\times\hat{x}_{i,\ell+1|\ell}^T.
\end{aligned}\tag{21}
$$

Then, $\mathcal{P}_{i,\ell+1|\ell+1}$ serves as an upper bound of $P_{i,\ell+1|\ell+1}$. Furthermore, if the selected filter gain satisfies

$$
\begin{aligned}
&\Xi_{i,\ell+1}\\
=&(1+o_2+o_3+o_4)\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T\Big[(1+o_2+o_3+o_4)\\
&\times C_{i,\ell+1}\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T + (1+o_2^{-1}+o_5+o_6+o_7)\\
&\times\vec{\Theta}_{i,\ell+1} + (1+o_7^{-1})R_{i,\ell+1}\Big]^{-1},
\end{aligned}\tag{22}
$$

then the minimization of the trace of (20) is guaranteed, yielding

$$
\begin{aligned}
&\mathcal{P}_{i,\ell+1|\ell+1}\\
=&(1+o_2+o_3+o_4)\mathcal{P}_{i,\ell+1|\ell} - (1+o_2+o_3+o_4)^2\\
&\times\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T\Big[(1+o_2+o_3+o_4)C_{i,\ell+1}\\
&\times\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T + (1+o_2^{-1}+o_5+o_6+o_7)\\
&\times\vec{\Theta}_{i,\ell+1} + (1+o_7^{-1})R_{i,\ell+1}\Big]^{-1}C_{i,\ell+1}\mathcal{P}_{i,\ell+1|\ell}\\
&+ (1+o_3^{-1}+o_5^{-1})\hbar_{i,\ell+1}^2\Big[\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,\ell+1}\\
&\times(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]\Big[\sum_{z\in\mathcal{N}_i} a_{iz}\psi_{iz,\ell+1}\\
&\times(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]^T + (1+o_4^{-1}+o_6^{-1})\\
&\times(1-\hbar_{i,\ell+1})^2|\mathcal{N}_i|^{-2}\Big[\sum_{z\in\mathcal{N}_i} a_{iz}(\hat{x}_{z,\ell+1|\ell}\\
&- \hat{x}_{i,\ell+1|\ell})\Big]\Big[\sum_{z\in\mathcal{N}_i} a_{iz}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell})\Big]^T.
\end{aligned}\tag{23}
$$

Proof: Taking (14) into account, it follows that

$$H_{1,\ell}+H_{1,\ell}^T \leq o_1(A_\ell+H_\ell)P_{i,\ell|\ell}(A_\ell+H_\ell)^T + o_1^{-1}\mathbb{E}\left\{\Upsilon_{i,\ell}\Upsilon_{i,\ell}^T\right\}, \tag{24}$$

where $o_1>0$ is a known scalar. By substituting (24) into (17), one obtains

$$P_{i,\ell+1|\ell} \leq (1+o_1)(A_\ell+H_\ell)P_{i,\ell|\ell}(A_\ell+H_\ell)^T+(1+o_1^{-1}) \times \mathbb{E}\left\{\Upsilon_{i,\ell}\Upsilon_{i,\ell}^T\right\}+B_\ell Q_\ell B_\ell^T. \tag{25}$$

It follows from (3) that

$$\mathbb{E}\{\Upsilon_{i,\ell}\Upsilon_{i,\ell}^T\} \leq \mathbb{E}\{\|\Upsilon_{i,\ell}\|^2\}I \leq \alpha_\ell^2 \mathrm{tr}(P_{i,\ell|\ell})I, \tag{26}$$

where $\alpha_\ell>0$ is a known scalar. Hence, we can derive

$$P_{i,\ell+1|\ell} \leq (1+o_1)(A_\ell+H_\ell)P_{i,\ell|\ell}(A_\ell+H_\ell)^T+(1+o_1^{-1}) \times \alpha_\ell^2\mathrm{tr}(P_{i,\ell|\ell})I+B_\ell Q_\ell B_\ell^T. \tag{27}$$

On the other hand, recalling the Lemma 1 results in

$$\begin{aligned}
&-J_{1,\ell+1}-J_{1,\ell+1}^T\\
\leq& o_2(I-\Xi_{i,\ell+1}C_{i,\ell+1})P_{i,\ell+1|\ell}(I-\Xi_{i,\ell+1}C_{i,\ell+1})^T\\
&+o_2^{-1}\mathbb{E}\{\Xi_{i,\ell+1}\eta_{i,\ell+1}\eta_{i,\ell+1}^T\Xi_{i,\ell+1}^T\},
\end{aligned}\tag{28}$$

$$\begin{aligned}
&-J_{2,\ell+1}-J_{2,\ell+1}^T\\
\leq& o_3(I-\Xi_{i,\ell+1}C_{i,\ell+1})P_{i,\ell+1|\ell}(I-\Xi_{i,\ell+1}C_{i,\ell+1})^T\\
&+o_3^{-1}\sum_{z\in\mathcal{N}_i}a_{iz}\hbar_{i,\ell+1}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\\
&\times\left[\sum_{z\in\mathcal{N}_i}a_{iz}\hbar_{i,\ell+1}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]^T,
\end{aligned}\tag{29}$$

$$\begin{aligned}
&-J_{3,\ell+1}-J_{3,\ell+1}^T\\
\leq& o_4(I-\Xi_{i,\ell+1}C_{i,\ell+1})P_{i,\ell+1|\ell}(I-\Xi_{i,\ell+1}C_{i,\ell+1})^T\\
&+o_4^{-1}\sum_{z\in\mathcal{N}_i}a_{iz}\frac{1}{|\mathcal{N}_i|^2}(1-\hbar_{i,\ell+1})(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\\
&\times\left[\sum_{z\in\mathcal{N}_i}a_{iz}(1-\hbar_{i,\ell+1})(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]^T,
\end{aligned}\tag{30}$$

$$\begin{aligned}
&J_{4,\ell+1}+J_{4,\ell+1}^T\\
\leq& o_5\mathbb{E}\{\Xi_{i,\ell+1}\eta_{i,\ell+1}\eta_{i,\ell+1}^T\Xi_{i,\ell+1}^T\}+o_5^{-1}\sum_{z\in\mathcal{N}_i}a_{iz}\hbar_{i,\ell+1}\\
&\times\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\left[\sum_{z\in\mathcal{N}_i}a_{iz}\hbar_{i,\ell+1}\right.\\
&\left.\times\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]^T,
\end{aligned}\tag{31}$$

$$\begin{aligned}
&J_{5,\ell+1}+J_{5,\ell+1}^T\\
\leq& o_6\mathbb{E}\{\Xi_{i,\ell+1}\eta_{i,\ell+1}\eta_{i,\ell+1}^T\Xi_{i,\ell+1}^T\}+o_6^{-1}\sum_{z\in\mathcal{N}_i}a_{iz}\\
&\times(1-\hbar_{i,\ell+1})\frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\left[\sum_{z\in\mathcal{N}_i}a_{iz}\right.\\
&\left.\left.\times(1-\hbar_{i,\ell+1})\frac{1}{|\mathcal{N}_i|}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]^T\right\},
\end{aligned}\tag{32}$$

$$\begin{aligned}
&J_{6,\ell+1}+J_{6,\ell+1}^T\\
\leq& o_7\mathbb{E}\left\{\Xi_{i,\ell+1}\eta_{i,\ell+1}\eta_{i,\ell+1}^T\Xi_{i,\ell+1}^T\right\}+o_7^{-1}\Xi_{i,\ell+1}\\
&\times R_{i,\ell+1}\Xi_{i,\ell+1}^T,
\end{aligned}\tag{33}$$

where $o_\mu>0$ $(\mu=1,2,\ldots,7)$ are known scalars. Substituting (28)–(33) into (18) leads to

$$\begin{aligned}
&P_{i,\ell+1|\ell+1}\\
\leq&(1+o_2+o_3+o_4)(I-\Xi_{i,\ell+1}C_{i,\ell+1})P_{i,\ell+1|\ell}\\
&\times(I-\Xi_{i,\ell+1}C_{i,\ell+1})^T+(1+o_2^{-1}+o_5+o_6+o_7)\\
&\times\Xi_{i,\ell+1}\mathbb{E}\{\eta_{i,\ell+1}\eta_{i,\ell+1}^T\}\Xi_{i,\ell+1}^T+(1+o_7^{-1})\\
&\times\Xi_{i,\ell+1}R_{i,\ell+1}\Xi_{i,\ell+1}^T+(1+o_3^{-1}+o_5^{-1})\\
&\times\hbar_{i,\ell+1}^2\left[\sum_{z\in\mathcal{N}_i}a_{iz}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]\\
&\times\left[\sum_{z\in\mathcal{N}_i}a_{iz}\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]^T\\
&+(1+o_4^{-1}+o_6^{-1})(1-\hbar_{i,\ell+1})^2|\mathcal{N}_i|^{-2}\left[\sum_{z\in\mathcal{N}_i}a_{iz}\right.\\
&\left.\times(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]\left[\sum_{z\in\mathcal{N}_i}a_{iz}(\hat{x}_{z,\ell+1|\ell}\right.\\
&\left.-\hat{x}_{i,\ell+1|\ell})\right]^T.
\end{aligned}\tag{34}$$

For a known scaling scalar $o_8>0$, it holds that

$$\begin{aligned}
\mathbb{E}\{x_{i,\ell+1}x_{i,\ell+1}^T\}=&(1+o_8)P_{i,\ell+1|\ell+1}+(1+o_8^{-1})\\
&\times\hat{x}_{i,\ell+1|\ell+1}\hat{x}_{i,\ell+1|\ell+1}^T\\
\triangleq&\Phi_{i,\ell+1|\ell+1}.
\end{aligned}\tag{35}$$

From the triggering condition, we have

$$\begin{aligned}
&\mathbb{E}\left\{\eta_{i,\ell+1}\eta_{i,\ell+1}^T\right\}\\
\leq&\mathbb{E}\left\{\mathrm{tr}\left(\eta_{i,\ell+1}^T\eta_{i,\ell+1}\right)I\right\}\\
\leq&\mathbb{E}\left\{\mathrm{tr}\left(\gamma_{i,\ell+1}y_{i,\ell+1}^T\Lambda_{i,\ell+1}y_{i,\ell+1}\right)I\right\}\\
\leq&\gamma_{i,\ell+1}\mathrm{tr}\big((C_{i,\ell+1}\Phi_{i,\ell+1|\ell+1}C_{i,\ell+1}^T+R_{i,\ell+1})\\
&\times\Lambda_{i,\ell+1}\big)I\\
\triangleq&\Theta_{i,\ell+1}.
\end{aligned}\tag{36}$$

Hence, we obtain

$$\begin{aligned}
&P_{i,\ell+1|\ell+1}\\
\leq&(1+o_2+o_3+o_4)(I-\Xi_{i,\ell+1}C_{i,\ell+1})P_{i,\ell+1|\ell}\\
&\times(I-\Xi_{i,\ell+1}C_{i,\ell+1})^T+(1+o_2^{-1}+o_5+o_6+o_7)\\
&\times\Xi_{i,\ell+1}\Theta_{i,\ell+1}\Xi_{i,\ell+1}^T+(1+o_7^{-1})\Xi_{i,\ell+1}R_{i,\ell+1}\\
&\times\Xi_{i,\ell+1}^T+(1+o_3^{-1}+o_5^{-1})\hbar_{i,\ell+1}^2\left[\sum_{z\in\mathcal{N}_i}a_{iz}\right.\\
&\left.\times\psi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]\left[\sum_{z\in\mathcal{N}_i}a_{iz}\psi_{iz,\ell+1}\right.\\
&\left.\times(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]^T+(1+o_4^{-1}+o_6^{-1})\\
&\times(1-\hbar_{i,\ell+1})^2|\mathcal{N}_i|^{-2}\left[\sum_{z\in\mathcal{N}_i}a_{iz}(\hat{x}_{z,\ell+1|\ell}\right.\\
&\left.-\hat{x}_{i,\ell+1|\ell})\right]\left[\sum_{z\in\mathcal{N}_i}a_{iz}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\right]^T.
\end{aligned}\tag{37}$$

By combining (19), (20), (27), and (37), together with mathematical induction, it is concluded that (20) is a CUBFED, i.e., $\mathcal{P}_{i,\ell+1|\ell+1} \geq P_{i,\ell+1|\ell+1}$. Furthermore, the filter gain is designed by calculating the following partial derivative:

$$\begin{aligned}&\frac{\partial \mathrm{tr}(\mathcal{P}_{i,\ell+1|\ell+1})}{\partial \Xi_{i,\ell+1}}\\ =&\frac{\partial}{\partial \Xi_{i,\ell+1}}\mathrm{tr}\Big\{(1+o_2+o_3+o_4)(I-\Xi_{i,\ell+1}C_{i,\ell+1})\\ &\times \mathcal{P}_{i,\ell+1|\ell}(I-\Xi_{i,\ell+1}C_{i,\ell+1})^T+(1+o_2^{-1}+o_5\\ &+o_6+o_7)\Xi_{i,\ell+1}\vec{\Theta}_{i,\ell+1}\Xi_{i,\ell+1}^T+(1+o_7^{-1})\\ &\times \Xi_{i,\ell+1}R_{i,\ell+1}\Xi_{i,\ell+1}^T\Big\}. \end{aligned} \tag{38}$$

By setting $\frac{\partial \mathrm{tr}(\mathcal{P}_{i,\ell+1|\ell+1})}{\partial \Xi_{i,\ell+1}}=0$, it is obtained that

$$\begin{aligned}&\Xi_{i,\ell+1}\\ =&(1+o_2+o_3+o_4)\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T\Big[(1+o_2+o_3+o_4)\\ &\times C_{i,\ell+1}\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T+(1+o_2^{-1}+o_5+o_6+o_7)\\ &\times \vec{\Theta}_{i,\ell+1}+(1+o_7^{-1})R_{i,\ell+1}\Big]^{-1}. \end{aligned} \tag{39}$$

Substituting (39) into (20) gives

$$\begin{aligned}&\mathcal{P}_{i,\ell+1|\ell+1}\\ =&(1+o_2+o_3+o_4)\mathcal{P}_{i,\ell+1|\ell}-(1+o_2+o_3+o_4)^2\\ &\times \mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T\Big[(1+o_2+o_3+o_4)C_{i,\ell+1}\\ &\times \mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T+(1+o_2^{-1}+o_5+o_6+o_7)\\ &\times \vec{\Theta}_{i,\ell+1}+(1+o_7^{-1})R_{i,\ell+1}\Big]^{-1}C_{i,\ell+1}\mathcal{P}_{i,\ell+1|\ell}\\ &+(1+o_3^{-1}+o_5^{-1})\hbar_{i,\ell+1}^2\Big[\sum_{z\in\mathscr{N}_i}a_{iz}\psi_{iz,\ell+1}\\ &\times(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\Big]\Big[\sum_{z\in\mathscr{N}_i}a_{iz}\psi_{iz,\ell+1}\\ &\times(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\Big]^T+(1+o_4^{-1}+o_6^{-1})\\ &\times(1-\hbar_{i,\ell+1})^2|\mathscr{N}_i|^{-2}\Big[\sum_{z\in\mathscr{N}_i}a_{iz}(\hat{x}_{z,\ell+1|\ell}\\ &-\hat{x}_{i,\ell+1|\ell})\Big]\Big[\sum_{z\in\mathscr{N}_i}a_{iz}(\hat{x}_{z,\ell+1|\ell}-\hat{x}_{i,\ell+1|\ell})\Big]^T. \end{aligned} \tag{40}$$

Thus, the proof is complete. ■

Remark 4: So far, a novel RAKTDF algorithm has been proposed for NBCSs subject to DETM, in which a mathematical model with respect to anti-interference reputation has been established to eliminate the deviated data caused by sensor failures or malicious attacks. A CUBFED involving the reputation parameter, the event-triggered threshold, and the noise information has been recursively derived by solving matrix equations. Moreover, the filter gain has been determined through minimization of the trace of the CUBFED, thereby ensuring the optimality of the developed RAKTDF algorithm. The execution steps of the RAKTDF algorithm are shown as follows.

TABLE I

THE RAKTDF ALGORITHM FOR NBCSS.

Algorithm : RAKTDF algorithm for NBCSs.
Step 1 : Let $\ell=0$ and initialize parameters.
Step 2 : Calculate $\hat{x}_{i,\ell+1|\ell}$ based on (9).
Step 3 : Obtain $\mathcal{P}_{i,\ell+1|\ell}$ in terms of (19).
Step 4 : Parameterize filter gain via (22).
Step 5 : Derive reputation values by (11)–(13) and compute $\hat{x}_{i,\ell+1|\ell+1}$ by means of (10).
Step 6 : Determine $\mathcal{P}_{i,\ell+1|\ell+1}$ by (20).
Step 7 : Let $\ell=\ell+1$ and return to *Step* 2.

IV. BOUNDEDNESS ANALYSIS

In this section, a sufficient criterion is established to guarantee that the filtering error remains uniformly bounded.

Assumption 1: The matrix $\vec{C}_{i,\ell+1}$ is of full row rank.

Assumption 2: For each i and ℓ, there exist positive constants $\bar{a}$, $\underline{b}$, $\bar{b}$, $\bar{h}$, $\underline{q}$, $\bar{q}$, $\underline{\gamma}$, $\underline{\lambda}$, $\underline{c}$, $\bar{c}$, $\underline{r}$ and $\bar{r}$ such that

$$\begin{aligned}&A_\ell A_\ell^T\le \bar{a}I,\ \ \underline{b}I\le B_\ell B_\ell^T\le \bar{b}I,\ \ H_\ell H_\ell^T\le \bar{h}I,\\ &\underline{q}I\le Q_\ell\le \bar{q}I,\ \ \gamma_{i,\ell}\ge\underline{\gamma},\ \ \Lambda_{i,\ell}\ge\underline{\lambda}I,\\ &\underline{c}I\le C_{i,\ell}C_{i,\ell}^T\le \bar{c}I,\ \ \underline{r}I\le R_{i,\ell}\le \bar{r}I.\end{aligned}$$

Theorem 2: For the augmented NBCSs (5)–(6) together with the designed reputation-aware distributed filter (9)–(10), if there exists a scalar $\bar{p}>0$ such that

$$\overline{\sigma}_1+2(1+o_2+o_3+o_4)\overline{\sigma}_2+\overline{\sigma}_3\le\bar{p}, \tag{41}$$

then $\mathcal{P}_{i,\ell+1|\ell+1}$ is guaranteed to remain uniformly bounded under the initial condition $\mathcal{P}_{i,0|0}\le\bar{p}I$, where

$$\begin{aligned}\overline{\sigma}_1=&2(1+o_2+o_3+o_4)\bar{p}_1+4(1+o_3^{-1}+o_5^{-1})|\mathscr{N}_i|^2\\ &\times\bar{p}_1+4(1+o_4^{-1}+o_6^{-1})\bar{p}_1,\\ \overline{\sigma}_2=&(1+o_2+o_3+o_4)^2\bar{c}\bar{p}_1^2\underline{c}^{-1}\underline{p}_1^{-1},\ \ \underline{p}_1=\underline{q}\underline{b},\\ \bar{p}_1=&2(1+o_1)\bar{p}(\bar{a}+\bar{h})+(1+o_1^{-1})\alpha_\ell^2(n_x+n_b)\bar{p}+\bar{b}\bar{q},\\ \overline{\sigma}_3=&\frac{(1+o_2+o_3+o_4)^2\bar{c}\bar{p}_1^2}{(1+o_2^{-1}+o_5+o_6+o_7)[(1+o_8)\underline{c}\underline{p}_1+\underline{r}]\underline{\gamma}\underline{\lambda}}.\end{aligned}$$

Proof: By means of mathematical induction, starting from $\mathcal{P}_{i,0|0}\le\bar{p}I$, it is assumed that $\mathcal{P}_{i,\ell|\ell}\le\bar{p}I$ holds. It remains to be shown that $\mathcal{P}_{i,\ell+1|\ell+1}\le\bar{p}I$. To begin with, by Assumption 2, it follows that

$$\begin{aligned}\mathcal{P}_{i,\ell+1|\ell}\le&2(1+o_1)\bar{p}(\bar{a}+\bar{h})I+(1+o_1^{-1})\alpha_\ell^2(n_x+n_b)\\ &\times\bar{p}I+\bar{b}\bar{q}I\\ \triangleq&\bar{p}_1I\end{aligned} \tag{42}$$

and

$$\mathcal{P}_{i,\ell+1|\ell}\ge B_\ell Q_\ell B_\ell^T\ge\underline{q}\underline{b}I\triangleq\underline{p}_1I. \tag{43}$$

The following notations are introduced:

$$\begin{aligned}\Omega_{i1,\ell+1}=&C_{i,\ell+1}\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T,\\ \Omega_{i2,\ell+1}=&(1+o_2^{-1}+o_5+o_6+o_7)\vec{\Theta}_{i,\ell+1}\\ &+(1+o_7^{-1})R_{i,\ell+1},\\ \Omega_{i,\ell+1}=&(1+o_2+o_3+o_4)\Omega_{i1,\ell+1}+\Omega_{i2,\ell+1}.\end{aligned}$$

From Lemma 1, it can be obtained that

$$(I-\Xi_{i,\ell+1}C_{i,\ell+1})\mathcal{P}_{i,\ell+1|\ell}(I-\Xi_{i,\ell+1}C_{i,\ell+1})^T$$

$$\leq 2\mathcal{P}_{i,\ell+1|\ell} + 2\Xi_{i,\ell+1}C_{i,\ell+1}\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T\Xi_{i,\ell+1}^T. \tag{44}$$

Substituting (44) into (20) gives

$$\begin{aligned}
&\mathcal{P}_{i,\ell+1|\ell+1} \\
\leq &\big[2(1+o_2+o_3+o_4)\bar{p}_1 + 4(1+o_3^{-1}+o_5^{-1})\hbar_{i,\ell+1}^2 \\
&\times |\mathscr{N}_i|^2\bar{p}_1 + 4(1+o_4^{-1}+o_6^{-1})(1-\hbar_{i,\ell+1})^2\bar{p}_1\big]I \\
&+ 2(1+o_2+o_3+o_4)\Xi_{i,\ell+1}\Omega_{i1,\ell+1}\Xi_{i,\ell+1}^T \\
&+ \Xi_{i,\ell+1}\Omega_{i2,\ell+1}\Xi_{i,\ell+1}^T.
\end{aligned} \tag{45}$$

Notice that

$$\begin{aligned}
&\Xi_{i,\ell+1}\Omega_{i1,\ell+1}\Xi_{i,\ell+1}^T \\
\leq &(1+o_2+o_3+o_4)^2\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T\Omega_{i1,\ell+1}^{-1} \\
&\times C_{i,\ell+1}\mathcal{P}_{i,\ell+1|\ell} \\
\leq &\frac{(1+o_2+o_3+o_4)^2\bar{c}\bar{p}_1^2}{\underline{c}\underline{p}_1}I \\
\triangleq &\bar{\sigma}_2 I
\end{aligned} \tag{46}$$

and

$$\begin{aligned}
&\Xi_{i,\ell+1}\Omega_{i2,\ell+1}\Xi_{i,\ell+1}^T \\
\leq &(1+o_2+o_3+o_4)^2\mathcal{P}_{i,\ell+1|\ell}C_{i,\ell+1}^T\Omega_{i2,\ell+1}^{-1} \\
&\times C_{i,\ell+1}\mathcal{P}_{i,\ell+1|\ell} \\
\leq &\frac{(1+o_2+o_3+o_4)^2\bar{c}\bar{p}_1^2}{(1+o_2^{-1}+o_5+o_6+o_7)\underline{\gamma}\underline{\lambda}[(1+o_8)\underline{c}\underline{p}_1+\underline{r}]}I \\
\triangleq &\bar{\sigma}_3 I.
\end{aligned} \tag{47}$$

From (46), (47), and Assumption 2, it follows that

$$\begin{aligned}
\mathcal{P}_{i,\ell+1|\ell+1} \leq &\big[2(1+o_2+o_3+o_4)\bar{p}_1 + 4(1+o_3^{-1} \\
&+ o_5^{-1})|\mathscr{N}_i|^2\bar{p}_1 + 4(1+o_4^{-1}+o_6^{-1})\bar{p}_1 \\
&+ 2(1+o_2+o_3+o_4)\bar{\sigma}_2 + \bar{\sigma}_3\big]I.
\end{aligned} \tag{48}$$

According to (41), it is obtained that

$$\mathcal{P}_{i,\ell+1|\ell+1} \leq \bar{p}I. \tag{49}$$

Thus, the proof of this theorem is complete. ■

Remark 5: In the framework of distributed filtering over WSNs, the unit-to-unit communication under given topologies is one of the typical features. In other words, the current filter can receive the data from neighbouring units by means of shared wireless channel, as mentioned in [15], [18], [32], [42]. Note that most results focus on the distributed filtering problems with certain connection coefficients, i.e., each data packet from neighbors is assigned with same weight. Distinctly, this paper is dedicated to developing the reputation evaluation strategy to identify and eliminate the untrustworthy data potentially caused by hardware abnormality or security vulnerability. Comparing with the existing literature, this paper has presented several distinctive novelties and advantages, extending the current research frontier.

1) A novel real-time reputation-aware scheme has been established to counter uncertainty and unreliability, where the individual reputation scores are dynamically allocated to the neighbors. It should be highlighted that the reputation scoring principle is capable of discriminating and discarding untrustworthy data corrupted by external disturbance or component malfunctions, thereby improving the precision and the robustness of filtering algorithm.
2) In the simultaneous presence of reputation evaluation mechanism, DETM, dynamical bias, and noise interference, the recursion of CUBFED has been determined by solving matrix equations.
3) The filter gain has been parameterized at each sampling instant by minimizing the filtering performance index, ensuring the stability and suboptimality of the proposed RAKTDF algorithm.
4) A sufficient criterion has been elaborated rigorously to guarantee that the filtering error dynamic is uniformly bounded, demonstrating the validity and adaptability of the introduced RAKTDF algorithm in the event of uncertainty and unreliability.

Remark 6: The complexity analysis of distributed filtering algorithms is of essential theoretical and practical significance, which directly reflects the operational efficiency in resource-constrained networked systems. In this paper, for the augmented system (5)–(6) and the designed reputation-aware distributed filter (9)–(10), let $n = n_x + n_b$, $m = n_y$, $\omega = n_w + n_b$, and $d = |\mathcal{N}_i|$. On the basis of analyzing the matrix operations implemented in the RAKTDF algorithm, such as matrix addition, multiplication, inversion, and transposition, the computational complexity incurred by each node during each iteration can be decomposed into the following primary components: $\mathcal{O}_1 = \mathcal{O}\big(f(\hat{x}_{i,\ell|\ell})\big)$, $\mathcal{O}_2 = \mathcal{O}\big(2n^3+n^2\omega+n\omega^2\big)$, $\mathcal{O}_3 = \mathcal{O}\big(m^3+m^2n+n^2m\big)$, $\mathcal{O}_4 = \mathcal{O}\big(n^2m+m^2n+m^3\big)$, $\mathcal{O}_5 = \mathcal{O}\big(n^2m+m^2n+dn\big)$, and $\mathcal{O}_6 = \mathcal{O}\big(d^2n\big)$. Therefore, the total computational complexity is $\mathcal{O} = \max_{\kappa=1,2,\ldots,6}\{\mathcal{O}_\kappa\}$. It should be noted that the computational complexity is closely related to the system dimension and increases monotonically with the increase of system dimensionality. Fortunately, the computational complexity of matrices remains an active research topic with ongoing progress and improvements.

V. Simulation Results

In this section, a simulation example is given to verify the validity and superiority of the developed RAKTDF algorithm over heterogeneous WSNs. The connection topologies involving 8 sensors are described by

$$\begin{aligned}
\mathscr{V} =&\{1,2,3,4,5,6,7,8\}, \\
\mathscr{E} =&\{(1,2),(1,3),(1,4),(2,1),(2,4),(3,1),(3,4), \\
&(4,2),(5,1),(5,6),(6,1),(6,2),(7,3),(7,4), \\
&(7,5),(8,6),(8,7)\}.
\end{aligned}$$

Among them, sensors 1 to 6 possess measurement and sensing capabilities, and the corresponding filter is constructed in (9)–(10). Conversely, sensors 7 and 8 are unable to perform measurement and sensing, and a dedicated filter is designed by

$$\begin{aligned}
\hat{x}_{i,\ell+1|\ell} =&A_\ell\hat{x}_{i,\ell|\ell} + f(\hat{x}_{i,\ell|\ell}), \\
\hat{x}_{i,\ell+1|\ell+1} =&\hat{x}_{i,\ell+1|\ell} + \sum_{z\in\mathscr{N}_i} a_{iz}\varphi_{iz,\ell+1}(\hat{x}_{z,\ell+1|\ell} - \hat{x}_{i,\ell+1|\ell}).
\end{aligned}$$

The system parameters are given by

$$\begin{aligned}
&\vec{A}_\ell = \begin{bmatrix} -0.4+0.5\sin(\ell) & 0.2 \\ 0.7-0.1\sin(\ell)\cos(\ell) & 0.2(1+\cos(\ell)) \end{bmatrix}, \\
&\vec{B}_\ell = \begin{bmatrix} -0.1 \\ 0.4-0.1\sin(\ell) \end{bmatrix}, \ \vec{F}_\ell = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}, \\
&\vec{C}_{1,\ell} = \begin{bmatrix} 1.6 & 1.6 \end{bmatrix}, \ \vec{C}_{2,\ell} = \begin{bmatrix} 1.8 & 1.8 \end{bmatrix}, \\
&\vec{C}_{3,\ell} = \begin{bmatrix} 2 & 2 \end{bmatrix}, \ \vec{C}_{4,\ell} = \begin{bmatrix} 1.7 & 1.7 \end{bmatrix}, \\
&\vec{C}_{5,\ell} = \begin{bmatrix} 1.3 & 1.3 \end{bmatrix}, \ \vec{C}_{6,\ell} = \begin{bmatrix} 1.5 & 1.5 \end{bmatrix}.
\end{aligned}$$

The nonlinearity is characterized by

$$f(\vec{x}_\ell) = [0.5\sin(\vec{x}_{1,\ell}) \ \ 0.5\cos(\vec{x}_{2,\ell})]^T,$$

where $\vec{x}_{1,\ell}$ and $\vec{x}_{2,\ell}$ are the 1-th and 2-th elements of $\vec{x}_\ell$, respectively.

The developed RAKTDF algorithm is initialized by $\hat{x}_{i,0|0} = \mathbb{E}\{x_0\} = \mathbb{E}\{[x_{1,0} \ \ x_{2,0} \ \ b_0]^T\} = [0.4 \ \ 0.4 \ \ 0.4]^T$ $(i = 1,2,\ldots,8)$, $\mathcal{P}_{1,0|0} = 2I$, $\mathcal{P}_{2,0|0} = \mathcal{P}_{8,0|0} = 3I$, $\mathcal{P}_{3,0|0} = \mathcal{P}_{5,0|0} = 5I$, $\mathcal{P}_{4,0|0} = \mathcal{P}_{6,0|0} = 6I$, $\mathcal{P}_{7,0|0} = 4I$, $\gamma_{1,0} = \gamma_{5,0} = 0.2$, $\gamma_{2,0} = \gamma_{6,0} = 0.3$, $\gamma_{3,0} = 0.1$ and $\gamma_{4,0} = 0.4$. The statistical characteristics of noise are described as $\Theta_\ell = 1$, $\vec{Q}_\ell = 0.3$ and $R_{i,\ell} = 0.4$. The remaining parameters are selected as $G_\ell = 0.2$, $o_1 = o_4 = o_5 = 0.2$, $o_2 = 0.6$, $o_3 = 0.7$, $o_6 = 0.3$, $o_7 = 0.5$, $o_8 = 0.4$, $\alpha_\ell = 0.8$, $H_\ell = I$, $\bar{\gamma} = 0.1$, $\psi = 0.2$, $\Lambda_{1,\ell} = 0.1$, $\Lambda_{2,\ell} = 0.2$, $\Lambda_{3,\ell} = 0.5$, $\Lambda_{4,\ell} = \Lambda_{5,\ell} = 0.4$, $\Lambda_{6,\ell} = 0.6$, $\pi_{1,\ell} = \pi_{7,\ell} = 0.01$, $\pi_{2,\ell} = \pi_{8,\ell} = 0.02$, $\pi_{3,\ell} = 0.04$, $\pi_{4,\ell} = 0.05$, $\pi_{5,\ell} = 0.03$ and $\pi_{6,\ell} = 0.06$.

The simulation results are displayed in Figs. 1–10. The system state and estimation trajectories over heterogeneous WSNs are presented in Figs. 1–4. The relationship between mean-square errors (MSEi) and the corresponding upper bounds are illustrated in Figs. 5–6, which indicates that each MSEi always stays below the respective upper bound.

Subsequently, a thorough comparative experiment is presented to illustrate the superiority of the proposed reputation-aware evaluation scheme. Specifically, it is assumed that the 4-th sensor is subjected to zero-mean malicious attacks with covariance $\gimel$. Based on the communication topologies of the considered heterogeneous WSNs, filters 1, 2, 3 and 7 will inevitably receive the contaminated data from neighbor 4. To validate the resilience of the proposed reputation-aware mechanism against malicious attacks, a comparative simulation is presented with the conventional distributed filtering method as mentioned in [19], [50]. In terms of $\gimel = 300$, the mean-square errors of the conventional distributed filtering method ($\varphi_{iz,\ell} = 0.8$) and the RAKTDF method are respectively plotted in Figs. 7–10. In addition, Table II is presented to further demonstrate the advantages of the proposed RAKTDF method, where the log(MSEp) $(p = 1,2,3,7)$ are respectively represented by M_p. The first and second elements of each vector in the table characterize the MSE of the reputation-based filtering algorithm and the traditional one, respectively. It is evident that the proposed filtering algorithm with a reputation score mechanism achieves higher estimation accuracy than the conventional filtering algorithm.

TABLE II
LOG(MSEp) $(p = 1,2,3,7)$ UNDER DIFFERENT CASES.

ℓ	11	12	13	14	15
M_1	$\begin{bmatrix}-1.7\\6.4\end{bmatrix}$	$\begin{bmatrix}-2.6\\7.6\end{bmatrix}$	$\begin{bmatrix}-2.3\\7.4\end{bmatrix}$	$\begin{bmatrix}-1.5\\6.3\end{bmatrix}$	$\begin{bmatrix}-0.2\\7.6\end{bmatrix}$
M_2	$\begin{bmatrix}-1.5\\6.5\end{bmatrix}$	$\begin{bmatrix}-2.7\\7.9\end{bmatrix}$	$\begin{bmatrix}-2\\6.6\end{bmatrix}$	$\begin{bmatrix}-1.6\\6.2\end{bmatrix}$	$\begin{bmatrix}0\\7.4\end{bmatrix}$
M_3	$\begin{bmatrix}-1.7\\6.6\end{bmatrix}$	$\begin{bmatrix}-2.7\\7.9\end{bmatrix}$	$\begin{bmatrix}-2\\6.5\end{bmatrix}$	$\begin{bmatrix}-2\\6.4\end{bmatrix}$	$\begin{bmatrix}0.1\\7.4\end{bmatrix}$
M_7	$\begin{bmatrix}-1.5\\6.9\end{bmatrix}$	$\begin{bmatrix}-2.9\\8.3\end{bmatrix}$	$\begin{bmatrix}-1.7\\7.9\end{bmatrix}$	$\begin{bmatrix}-3.6\\6.9\end{bmatrix}$	$\begin{bmatrix}0.5\\7.1\end{bmatrix}$
ℓ	16	17	18	19	20
M_1	$\begin{bmatrix}0.5\\8.1\end{bmatrix}$	$\begin{bmatrix}1.1\\5.4\end{bmatrix}$	$\begin{bmatrix}-0.2\\7.2\end{bmatrix}$	$\begin{bmatrix}-1.5\\7.7\end{bmatrix}$	$\begin{bmatrix}-1.5\\8\end{bmatrix}$
M_2	$\begin{bmatrix}0.6\\7.8\end{bmatrix}$	$\begin{bmatrix}1.2\\5.3\end{bmatrix}$	$\begin{bmatrix}-0.2\\4.3\end{bmatrix}$	$\begin{bmatrix}-1.6\\7.1\end{bmatrix}$	$\begin{bmatrix}-1.4\\7\end{bmatrix}$
M_3	$\begin{bmatrix}0.6\\7.8\end{bmatrix}$	$\begin{bmatrix}1\\5.2\end{bmatrix}$	$\begin{bmatrix}-0.2\\4.2\end{bmatrix}$	$\begin{bmatrix}-1.8\\7.1\end{bmatrix}$	$\begin{bmatrix}-1.3\\7\end{bmatrix}$
M_7	$\begin{bmatrix}-0.5\\7.8\end{bmatrix}$	$\begin{bmatrix}1.1\\5.5\end{bmatrix}$	$\begin{bmatrix}-0.2\\7.2\end{bmatrix}$	$\begin{bmatrix}-1\\7.7\end{bmatrix}$	$\begin{bmatrix}1.5\\7.7\end{bmatrix}$

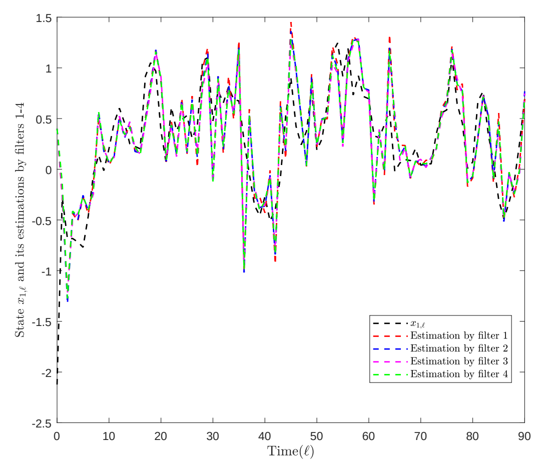

Fig. 1. State $x_{1,\ell}$ and its estimations by filters 1–4.

It should be noted that the attack intensity is a critical factor that significantly affects the estimation accuracy of distributed filtering algorithms. The stronger the attack intensity, the more significant the role played by the proposed reputation-aware mechanism. Specifically, as parameter $\gimel$ increases, the ability of the reputation-aware mechanism to accurately discriminate and eliminate attack signals is enhanced. Consequently, the probability of successfully identifying and mitigating attack signals also rises, which is quantitatively illustrated in Table III. Here, the components of the vectors in the table indicate the counts or probabilities that $\varphi_{14,\ell}$, $\varphi_{24,\ell}$, $\varphi_{34,\ell}$ and $\varphi_{74,\ell}$ are zero, respectively.

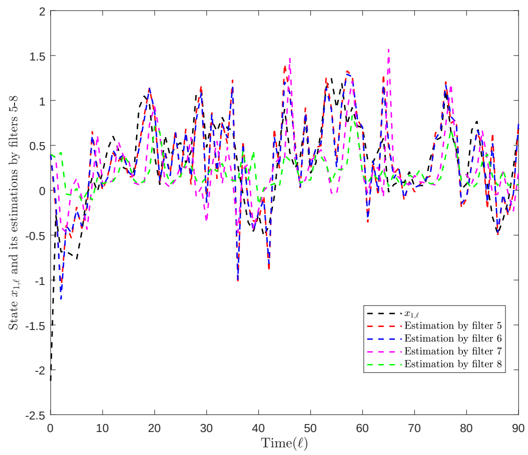

Fig. 2. State $x_{1,\ell}$ and its estimations by filters 5–8.

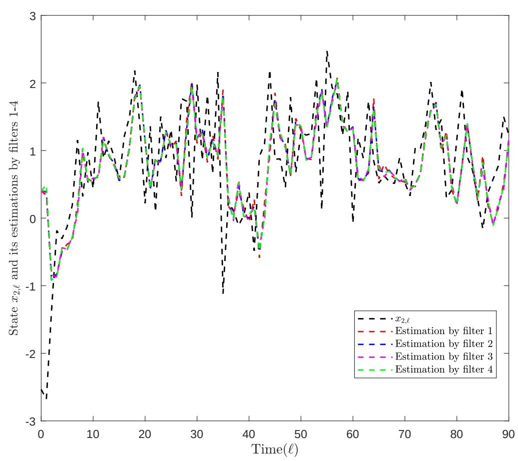

Fig. 3. State $x_{2,\ell}$ and its estimations by filters 1–4.

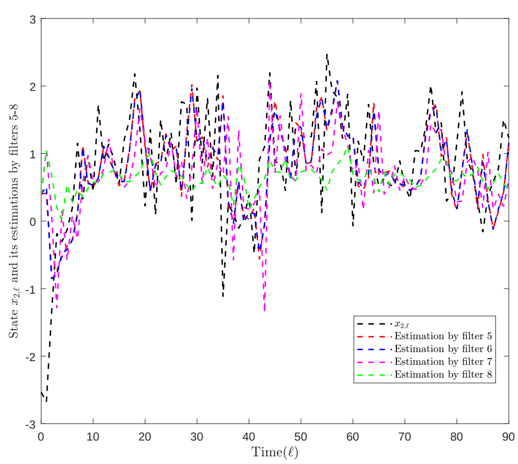

Fig. 4. State $x_{2,\ell}$ and its estimations by filters 5–8.

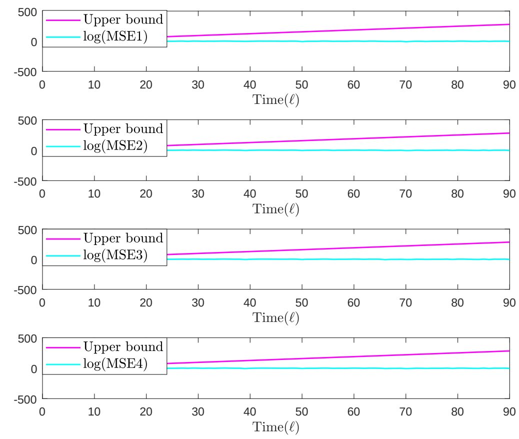

Fig. 5. log(MSEϕ_1) ($\phi_1 = 1, 2, 3, 4$) and their upper bounds.

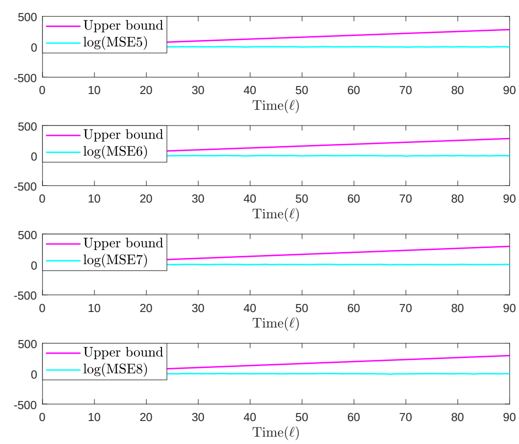

Fig. 6. log(MSEϕ_2) ($\phi_2 = 5, 6, 7, 8$) and their upper bounds.

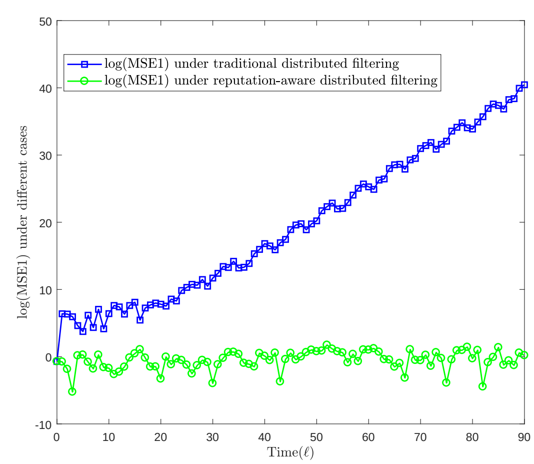

Fig. 7. log(MSE1) under different cases.

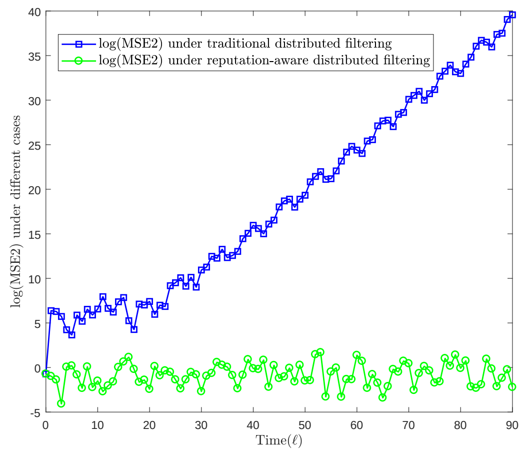

Fig. 8. log(MSE2) under different cases.

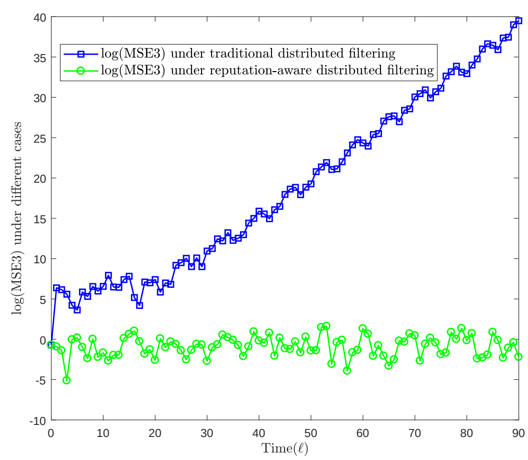

Fig. 9. log(MSE3) under different cases.

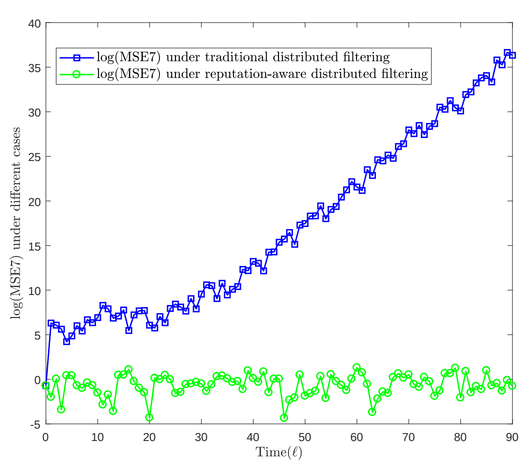

Fig. 10. log(MSE7) under different cases.

TABLE III

THE SUCCESSFUL IDENTIFICATION CASES UNDER DIFFERENT $\gimel$.

$\gimel$	The number of attack removed	Probability
0.01	[70 56 40 65]	[0.78 0.62 0.44 0.72]
0.05	[79 65 57 79]	[0.88 0.72 0.63 0.88]
0.1	[81 74 66 83]	[0.90 0.82 0.73 0.92]
0.4	[85 77 76 84]	[0.94 0.86 0.84 0.93]
0.9	[86 79 79 84]	[0.96 0.88 0.88 0.93]

VI. CONCLUSION

This paper has investigated the RAKTDF problem for a class of NBCSs subject to dynamical bias and DETM. A DETM with a time-varying threshold has been employed to reduce communication frequency and prevent data collisions. A reputation mechanism has been incorporated to eliminate abnormal data, thereby ensuring desirable filtering performance and improving algorithm robustness. The RAKTDF algorithm has been developed, where the CUBFED has been derived through two recursive matrix equations and the filter gain has been parameterized in the sense of minimum mean-square error. Moreover, a sufficient criterion for boundedness analysis has been established under moderate assumptions. Finally, a numerical simulation has been carried out to demonstrate the effectiveness of the proposed RAKTDF algorithm.

Future research may focus on extending the proposed framework to large-scale NBCSs with multi-hop communication structures, incorporating correlated or stealthy attack models, and reducing the computational complexity of the filtering scheme to facilitate real-time implementation. Another promising direction lies in integrating learning-based methods to adaptively adjust the reputation mechanism and event-triggering thresholds in highly dynamic environments.

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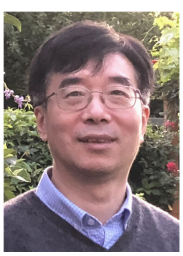

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